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METHOD FOR CLASSIFYING A RANDOM PROCESS  
FOR DATA SETS IN ARBITRARY DIMENSIONS

TO WHOM IT MAY CONCERN:

BE IT KNOWN THAT FRANCIS J. O'BRIEN, JR. and CHUNG T. NGUYEN, employees of the United States Government, citizens of the United States of America, residents respectively of Newport, County of Newport, State of Rhode Island, and Austin, County of Travis, State of Texas, have invented certain new and useful improvements entitled as set forth above of which the following is a specification:

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1 Attorney Docket No. 78586

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METHOD FOR CLASSIFYING A RANDOM PROCESS

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FOR DATA SETS IN ARBITRARY DIMENSIONS

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STATEMENT OF GOVERNMENT INTEREST

7

The invention described herein may be manufactured and used  
8 by or for the Government of the United States of America for  
9 Governmental purposes without the payment of any royalties  
10 thereon or therefore.

11

12

CROSS REFERENCE TO RELATED PATENT APPLICATIONS

13

Related applications include the following copending

14

applications: application of F. J. O'Brien, Jr. entitled

15

"Detection of Randomness in Sparse Data Set of Three Dimensional

16

Time Series Distributions," serial number 10/679,866, filed 6

17

October 2003 (Attorney Docket No. 83996); application of F. J.

18

O'Brien, Jr. entitled "Enhanced System for Detection of

19

Randomness in Sparse Time Series Distributions," filed 3 March

20

2004 (Attorney Docket No. 83995); application of F. J. O'Brien,

21

Jr. entitled "Method for Detecting a Spatial Random Process Using

22

Planar Convex Polygon Envelope," filed on even date with the

23

present application (Attorney Docket No. 83047); application of

24

F. J. O'Brien, Jr. entitled "Multi-Stage Planar Stochastic

25

Mensuration," filed on even date with the present invention

1 (Attorney Docket No. 83992); and application of F. J. O'Brien,  
2 Jr. entitled "Method for Sparse Data Two-Stage Stochastic  
3 Mensuration," filed on even date with the present application  
4 (Attorney Docket No. 84264).

5  
6 BACKGROUND OF THE INVENTION

7 (1) Field of the Invention

8 The present invention relates generally to the field of  
9 sonar signal processing and, more particularly, to determining  
10 whether d-dimensional data sets are random or non-random in  
11 nature.

12  
13 (2) Description of the Prior Art

14 Naval sonar systems require that signals be classified  
15 according to structure; i.e., periodic, transient, random or  
16 chaotic. For instance, in many cases it may be highly desirable  
17 and/or critical to know whether data received by a sonar system  
18 is simply random noise, which may be a false alarm, or is more  
19 likely due to detection of sound energy emitted from a submarine  
20 or other vessel of interest. In the study of nonlinear dynamics  
21 analysis, scientists, in a search for "chaos" in signals or other  
22 physical measurements, often resort to "embedding dimensions  
23 analysis," or "phase-space portrait analysis." One method of  
24 finding chaos is by selecting the appropriate time-delay close to  
25 the first "zero-crossing" of the autocorrelation function, and

1 then performing delay plot analyses. Other methods for detection  
2 of spatial randomness are based on an approach sometimes known as  
3 "box counting" and/or "box counting enumerative" models. Other  
4 methods such as power spectral density (PSD) techniques may be  
5 employed in naval sonar systems. Methods such as these may be  
6 discussed in the subsequently listed patents and/or the above-  
7 cited related patent applications which are hereby incorporated  
8 by reference and may also be discussed in patents and/or  
9 applications by the inventors of the above-cited related patent  
10 applications and/or subsequently listed patents.

11 It is also noted that recent research has revealed a  
12 critical need for highly sparse data set time distribution  
13 analysis methods and apparatus separate and apart from those  
14 adapted for treating large sample distributions. It is well  
15 known that large sample methods often fail when applied to small  
16 sample distributions, but that the same is not necessarily true  
17 for small sample methods applied to large data sets. Very small  
18 data set distributions may be defined as those with less than  
19 about ten (10) to thirty (30) measurement (data) points.

20 Examples of exemplary patents related to the general field  
21 of the endeavor of analysis of sonar signals include:

22 United States Patent No. 5,675,553, issued October 7, 1997,  
23 to O'Brien, Jr. et al., discloses a method for filling in missing  
24 data intelligence in a quantified time-dependent data signal that  
25 is generated by, e.g., an underwater acoustic sensing device. In

1 accordance with one embodiment of the invention, this quantified  
2 time-dependent data signal is analyzed to determine the number  
3 and location of any intervals of missing data, i.e., gaps in the  
4 time series data signal caused by noise in the sensing equipment  
5 or the local environment. The quantified time-dependent data  
6 signal is also modified by a low pass filter to remove any  
7 undesirable high frequency noise components within the signal. A  
8 plurality of mathematical models are then individually tested to  
9 derive an optimum regression curve for that model, relative to a  
10 selected portion of the signal data immediately preceding each  
11 previously identified data gap. The aforesaid selected portion  
12 is empirically determined on the basis of a data base of signal  
13 values compiled from actual undersea propagated signals received  
14 in cases of known target motion scenarios. An optimum regression  
15 curve is that regression curve, linear or nonlinear, for which a  
16 mathematical convergence of the model is achieved. Convergence  
17 of the model is determined by application of a smallest root-  
18 mean-square analysis to each of the plurality of models tested.  
19 Once a model possessing the smallest root-mean-square value is  
20 derived from among the plurality of models tested, that optimum  
21 model is then selected, recorded, and stored for use in filling  
22 the data gap. This process is then repeated for each subsequent  
23 data gap until all of the identified data gaps are filled.

24 United States Patent No. 5,703,906, issued December 30,  
25 1997, to O'Brien, Jr. et al., discloses a signal processing

1 system which processes a digital signal, generally in response to  
2 an analog signal which includes a noise component and possibly  
3 also an information component representing three mutually  
4 orthogonal items of measurement information represented as a  
5 sample point in a symbolic Cartesian three-dimensional spatial  
6 reference system. A noise likelihood determination sub-system  
7 receives the digital signal and generates a random noise  
8 assessment of whether or not the digital signal comprises solely  
9 random noise, and if not, generates an assessment of degree-of-  
10 randomness. The noise likelihood determination system controls  
11 the operation of an information processing sub-system for  
12 extracting the information component in response to the random  
13 noise assessment or a combination of the random noise assessment  
14 and the degree-of-randomness assessment. The information  
15 processing system is illustrated as combat control equipment for  
16 submarine warfare, which utilizes a sonar signal produced by a  
17 towed linear transducer array, and whose mode operation employs  
18 three orthogonally related dimensions of data, namely: (i) clock  
19 time associated with the interval of time over which the sample  
20 point measurements are taken, (ii) conical angle representing  
21 bearing of a passive sonar contact derived from the signal  
22 produced by the towed array, and (iii) a frequency characteristic  
23 of the sonar signal.

24 United States Patent No. 5,966,414, issued October 12, 1999,  
25 to Francis J. O'Brien, Jr., discloses a signal processing system

1 which processes a digital signal generated in response to an  
2 analog signal which includes a noise component and possibly also  
3 an information component. An information processing sub-system  
4 receives said digital signal and processes it to extract the  
5 information component. A noise likelihood determination sub-  
6 system receives the digital signal and generates a random noise  
7 assessment that the digital signal comprises solely random noise,  
8 and controls the operation of the information processing sub-  
9 system in response to the random noise assessment.

10 United States Patent No. 5,781,460, issued July 14, 1998, to  
11 Nguyen et al., discloses a chaotic signal processing system which  
12 receives an input signal from a sensor in a chaotic environment  
13 and performs a processing operation in connection therewith to  
14 provide an output useful in identifying one of a plurality of  
15 chaotic processes in the chaotic environment. The chaotic signal  
16 processing system comprises an input section, a processing  
17 section and a control section. The input section is responsive  
18 to input data selection information for providing a digital data  
19 stream selectively representative of the input signal provided by  
20 the sensor or a synthetic input representative of a selected  
21 chaotic process. The processing section includes a plurality of  
22 processing modules each for receiving the digital data stream  
23 from the input means and for generating therefrom an output  
24 useful in identifying one of a plurality of chaotic processes.  
25 The processing section is responsive to processing selection



1 information to select one of the plurality of processing modules  
2 to provide the output. The control module generates the input  
3 data selection information and the processing selection  
4 information in response to inputs provided by an operator.

5 United States Patent No. 5,963,591, issued October 5, 1999,  
6 to O'Brien, Jr. et al., discloses a signal processing system  
7 which processes a digital signal generally in response to an  
8 analog signal which includes a noise component and possibly also  
9 an information component representing four mutually orthogonal  
10 items of measurement information representable as a sample point  
11 in a symbolic four-dimensional hyperspatial reference system. An  
12 information processing and decision sub-system receives said  
13 digital signal and processes it to extract the information  
14 component. A noise likelihood determination sub-system receives  
15 the digital signal and generates a random noise assessment of  
16 whether or not the digital signal comprises solely random noise,  
17 and if not, generates an assessment of degree-of-randomness. The  
18 noise likelihood determination system controls whether or not the  
19 information processing and decision sub-system is used, in  
20 response to one or both of these generated outputs. One  
21 prospective practical application of the invention is the  
22 performance of a triage function upon signals from sonar  
23 receivers aboard naval submarines, to determine suitability of  
24 the signal for feeding to a subsequent contact localization and  
25 motion analysis (CLMA) stage.

1 United States Patent No. 6,397,234, issued May 28, 2002, to  
2 O'Brien, Jr. et al., discloses a method and apparatus are  
3 provided for automatically characterizing the spatial arrangement  
4 among the data points of a time series distribution in a data  
5 processing system wherein the classification of said time series  
6 distribution is required. The method and apparatus utilize a  
7 grid in Cartesian coordinates to determine (1) the number of  
8 cells in the grid containing at least-one input data point of the  
9 time series distribution; (2) the expected number of cells which  
10 would contain at least one data point in a random distribution in  
11 said grid; and (3) an upper and lower probability of false alarm  
12 above and below said expected value utilizing a discrete binomial  
13 probability relationship in order to analyze the randomness  
14 characteristic of the input time series distribution. A labeling  
15 device also is provided to label the time series distribution as  
16 either random or nonrandom.

17 United States Patent No.5,144,595, issued September 1, 1992,  
18 to Graham et al., discloses an adaptive statistical filter  
19 providing improved performance target motion analysis noise  
20 discrimination includes a bank of parallel Kalman filters. Each  
21 filter estimates a statistic vector of specific order, which in  
22 the exemplary third order bank of filters of the preferred  
23 embodiment, respectively constitute coefficients of a constant,  
24 linear and quadratic fit. In addition, each filter provides a  
25 sum-of-squares residuals performance index. A sequential

1 comparator is disclosed that performs a likelihood ratio test  
2 performed pairwise for a given model order and the next lowest,  
3 which indicates whether the tested model orders provide  
4 significant information above the next model order. The optimum  
5 model order is selected based on testing the highest model  
6 orders. A robust, unbiased estimate of minimal rank for  
7 information retention providing computational efficiency and  
8 improved performance noise discrimination is therewith  
9 accomplished.

10 United States Patent No. 5,757,675, issued May 26, 1998, to  
11 O'Brien, Jr., discloses an improved method for laying out a  
12 workspace using the prior art crowding index, PDI, where the  
13 average interpoint distance between the personnel and/or  
14 equipment to be laid out can be determined. The improvement lies  
15 in using the convex hull area of the distribution of points being  
16 laid out within the workplace space to calculate the actual  
17 crowding index for the workspace. The convex hull area is that  
18 area having a boundary line connecting pairs of points being laid  
19 out such that no line connecting any pair of points crosses the  
20 boundary line. The calculation of the convex hull area is  
21 illustrated using Pick's theorem with additional methods using  
22 the Surveyor's Area formula and Hero's formula.

23 United States Patent No. 6,466,516, issued October 5, 1999,  
24 to O'Brien, Jr. et al., discloses a method and apparatus for  
25 automatically characterizing the spatial arrangement among the

1 data points of a three-dimensional time series distribution in a  
2 data processing system wherein the classification of the time  
3 series distribution is required. The method and apparatus  
4 utilize grids in Cartesian coordinates to determine (1) the  
5 number of cubes in the grids containing at least one input data  
6 point of the time series distribution; (2) the expected number of  
7 cubes which would contain at least one data point in a random  
8 distribution in said grids; and (3) an upper and lower  
9 probability of false alarm above and below said expected value  
10 utilizing a discrete binomial probability relationship in order  
11 to analyze the randomness characteristic of the input time series  
12 distribution. A labeling device also is provided to label the  
13 time series distribution as either random or nonrandom, and/or  
14 random or nonrandom within what probability, prior to its output  
15 from the invention to the remainder of the data processing system  
16 for further analysis.

17 The above cited art, while extremely useful under certain  
18 circumstances, does not provide sufficient flexibility in  
19 processing different dimensionalities of data sets of sonar data.  
20 Consequently, those of skill in the art will appreciate the  
21 present invention which addresses these and other problems.

22

#### 23 SUMMARY OF THE INVENTION

24 Accordingly, it is an object of the invention to provide a  
25 method for classifying data sets in arbitrary dimensions.

1           It is another object of the present invention to provide  
2 automated measurement of the d-dimensional spatial arrangement  
3 among either a large sample or a very small number of points,  
4 objects, measurements or the like whereby an ascertainment of the  
5 noise degree (i.e., randomness) of the time series distribution  
6 may be made.

7           Yet another object of the present invention is directed to  
8 methods by which sonar signals may be classified heuristically as  
9 deterministic, chaotic or random in nature.

10          Yet another object of the present invention is to provide a  
11 useful method for classifying data produced by naval sonar,  
12 radar, and/or lidar in aircraft and missile tracking systems as  
13 indications of how and from which direction the data was  
14 originally generated.

15          These and other objects, features, and advantages of the  
16 present invention will become apparent from the drawings, the  
17 descriptions given herein, and the appended claims. However, it  
18 will be understood that above listed objects and advantages of  
19 the invention are intended only as an aid in understanding  
20 certain aspects of the invention, are not intended to limit the  
21 invention in any way, and do not form a comprehensive or  
22 exclusive list of objects, features, and advantages.

23          Accordingly, the present invention provides a method for  
24 characterizing a plurality of data sets in a d-dimensional  
25 Euclidean space. The data sets are based on a plurality of

1 measurements of physical phenomena such as sonar or radar data  
2 but may also comprise synthetic data generated by a random number  
3 generator for testing that the method is operating as expected.  
4 The method may comprise one or more steps such as, for example,  
5 reading in data points from a first data set in the d-dimensional  
6 Euclidean space to be characterized, creating a first virtual d-  
7 dimensional volume containing the data points of the first data  
8 set, and partitioning the first virtual d-dimensional volume into  
9 a plurality  $k$  of partitions. Other steps may comprise  
10 determining an expected number  $E(M)$  of the plurality  $k$  of  
11 partitions which contain at least one of the data points if the  
12 first data set is randomly dispersed, determining a number  $M$  of  
13 the plurality  $k$  of partitions which actually contain at least one  
14 of the data points, and statistically determining a range of  
15 values such that if the number  $M$  is within the range of values,  
16 then the first data set is automatically characterized as random  
17 in structure, and if the number is outside of the range of  
18 values, then the first data set is automatically characterized as  
19 non-random.

20 In one preferred embodiment, the plurality  $k$  of partitions  
21 may comprise a plurality  $k$  hypercuboidal subspaces. The d-  
22 dimensional Euclidean space may comprise any number  $d$  of  
23 dimensions and in a preferred embodiment may comprise three or  
24 four or more dimensions. The method may further comprise  
25 determining the sample size  $N$  of the data points and, if the

1 sample size N is less than approximately ten to thirty, then  
2 utilizing a discrete binomial distribution for determining the  
3 range of values. If the sample size N is greater than  
4 approximately ten to thirty, then utilizing a Poisson probability  
5 distribution for determining the range of values. For data  
6 within sample sizes of N from 10 to thirty it may be desirable to  
7 utilize two different types of statistical techniques for  
8 comparison purposes. In a preferred embodiment, the step of  
9 reading data points may further comprise reading in  $X_1, X_2, \dots, X_d$   
10 for d-dimensional vector data in the form of coordinate  
11 measurements to describe the data points. In a preferred  
12 embodiment, the method may further comprise constructing a  
13 closest fitting parallelepiped around the first data set. Other  
14 steps may comprise storing the characterization of the first data  
15 set, and then reading in data points from a second data set to be  
16 characterized. In one preferred embodiment, the method may  
17 further comprise utilizing one or more sonar arrays to produce  
18 the plurality of data sets.

19 The above and other novel features and advantages of the  
20 invention, including various novel details of construction and  
21 combination of parts will now be more particularly described with  
22 reference to the accompanying drawings and pointed out by the  
23 claims. It will be understood that the particular device and  
24 method embodying the invention is shown and described herein by  
25 way of illustration only, and not as limitations on the

1 invention. The principles and features of the invention may be  
2 employed in numerous embodiments without departing from the scope  
3 of the invention in its broadest aspects.

#### 4 5 BRIEF DESCRIPTION OF THE DRAWINGS

6 Reference is made to the accompanying drawings in which is  
7 shown an illustrative embodiment of the apparatus and method of  
8 the invention, from which its novel features and advantages will  
9 be apparent to those skilled in the art, and wherein:

10 FIG. 1 is a block diagram flow chart which provides a  
11 general overview of a presently preferred embodiment of a method  
12 in accord with the present invention; and

13 FIG. 2 is a block diagram flow chart which provides  
14 additional details of a presently preferred embodiment in accord  
15 with the present invention.

#### 16 17 DESCRIPTION OF THE PREFERRED EMBODIMENT

18 Referring now to the drawings and, more specifically to FIG.  
19 1, there is shown an overview of the present invention wherein a  
20 presently preferred embodiment is described by flow chart or  
21 method 10. In one embodiment, the method of flow chart 10 may be  
22 derived from a probability model for the random distribution of  
23 particles in space and events in time as may be termed an  
24 elementary stochastic (Poisson) process. Accordingly, method 10  
25 provides a generalized solution to detecting randomness in an



1 arbitrary dimension. The method of interest may be based on an  
2 elementary stochastic (Poisson) process coupled with statistical  
3 hypothesis testing procedures. One of the many uses of the  
4 method 10 is in the field of nonlinear dynamics when sample sizes  
5 are small. However, method 10 can be utilized for large samples  
6 as well. The present invention provides an analysis which is not  
7 limited to the everyday dimensions of 3-dimensional space.

8 Method 10 permits a determination of whether such d-  
9 dimensional distributions are merely instances of "pure  
10 stochastic randomness" or "pure deterministic randomness"  
11 (chaos). Thus, pure randomness, pragmatically speaking, is  
12 herein considered to be a time series distribution for which no  
13 function, mapping or relation can be constituted that provides  
14 meaningful insight into the underlying structure of the  
15 distribution, but which at the same time is not chaos.  
16 Randomness may also be defined in terms of a "random process" as  
17 measured by the probability distribution model used, such as a  
18 nearest-neighbor stochastic (Poisson) process. Method 10 of the  
19 present invention provides a novel means to determine whether the  
20 signal structure is random in nature in arbitrary dimensions.

21 The present invention as shown in method 10 is a logical  
22 alternative to other "distance models" and, under certain  
23 circumstances, the present method offers superior performance.  
24 The present invention incorporates herein by reference the above-  
25 cited related applications. Method 10 of the present invention

1 may, for instance, provide the naval sonar signal processing  
2 operator with greater flexibility for processing different  
3 dimensionalities of data sets.

4 In the novel spatial Poisson point-process method as shown  
5 in FIG. 1, one begins at 12 to analyze a sample distribution of  
6 data whereby it is assumed that particles (data points) exist  
7 that may be read in, as indicated at 14, in d-dimensional  
8 Euclidean space. Reference space 16 may be selected and/or  
9 determined, e.g., a quadrilateral or other multilateral area,  
10 hypercube, or space. The location of the particles (Cartesian  
11 coordinate measurements) as well as the total number of particles  
12 are considered random variables. The data type as indicated at  
13 18 may be synthetic data 20, such as statistically anticipated  
14 data, or may real world data as determined from measurements  
15 which may be input as indicated at 22. Synthetic data 20 may be  
16 utilized to verify operation of the method in properly  
17 classifying data sets as random.

18 In method 10, an analysis is made of the d-dimensional  
19 distribution of particles contained in a finite number of random  
20 subsets (small hypercubes covering the entire space). Within  
21 each hypercuboidal subspace (in d-dimensional space) one counts  
22 the numbers of particles contained therein. An R statistic 24 is  
23 determined by comparing the actual number of points to the  
24 expected number, as discussed hereinafter. A Poisson probability  
25 distribution governs the distribution of particles in each random

1 subset, as indicated at 26, as may be used in box counting  
2 techniques described in the related applications discussed  
3 hereinbefore. An equality is established between the elementary  
4 events of distance and the particle count. From this starting  
5 point, a single continuous distribution function is shown to  
6 equate a gamma distribution and the complement of a finite  
7 Poisson series, from which one obtains the probability  
8 distribution. Knowing the parametric values of the distribution  
9 (mean, variance) allows the researcher to appeal to the central  
10 limit theorem to test the randomness hypothesis to provide a  
11 solution for classification of the data and to store the result  
12 as indicated at 28.

13 For finite samples, the normal approximation formula is  
14 employed to test the hypothesis that the average sample subspace  
15 count, denoted  $M$  matches the theoretical mean of a random  
16 distribution, denoted  $E(M)$  for use in R Statistic 24. An  
17 exhaustive search in each level of dimensionally is then made to  
18 record and measure  $M$ . When the sample size  $N$  is very small ( $N <$   
19  $25$  to  $30$ ), then the exact discrete binomial probability  
20 distribution may be used at 26 instead of the normal  
21 approximation formula (derived from the central limit theorem).

22 In more detail, and with reference now to FIG. 2, a  
23 preferred embodiment 10 of the method steps may comprise  
24 beginning operations with a new distribution or sampling of data  
25 points, as indicated at 30.

1 Step 32 may comprise reading in  $X_1, X_2, \dots, X_d$  (d-dimensional  
2 vectors) data in the form of coordinate measurements. In step  
3 34, the number of measurements from step 32 is counted to give  
4 the sample size  $N$ .

5 Step 36 involves building a d-dimensional window. This is  
6 accomplished by computing the following quantities from Step 32  
7 where (min is "minimum" and max is "maximum"):

$$8 \quad \min(X_1)\max(X_1), \min(X_2)\max(X_2), \dots, \min(X_d)\max(X_d).$$

9  
10 Then, the tightest fitting parallelepiped is determined or  
11 constructed, e.g., a prism or polyhedron whose bases are  
12 parallelograms, around the  $N$  data points. The volume of this  
13 tightest fitting window has a measure of volume,

14

$$15 \quad V = \prod_{i=1}^d (\max(X_i) - \min(X_i)) \quad (1)$$
$$= [(\max(X_1) - \min(X_1))(\max(X_2) - \min(X_2)) \cdots (\max(X_d) - \min(X_d))]$$

16

17 Step 38 involves partitioning the space or volume  $V$  into  $k$   
18 hypercuboidal or d-dimensional cuboids subspaces or partitions  
19 wherein each hypercuboidal subspace may be sized to have a  
20 selected expected number of data points, e.g., sized such that it  
21 is statistically expected to include one or at least one data  
22 point. Some examples of partitioning for other dimensional  
23 partitions and related methods are provided in the above-cited  
24 related applications listed hereinbefore.

1 As per step 40, compute the theoretical number of partitions  
2 expected to be non-empty if the d-dimensional point distribution  
3 were randomly dispersed:

$$4 \quad E(M) = k \left( 1 - e^{-\frac{N}{k}} \right) \quad (2)$$

5 where  $E(M)$  = "expected number" of the  $k$  subspace hypercubes to be  
6 non empty,  $M$  = actual number the  $k$  subspace hypercubes non empty,  
7  $e$  = the mathematical constant (2.71828 )

8 As per step 42, the standard error is given as:

$$9 \quad \sigma_m = \sqrt{k \left( e^{-\frac{N}{k}} \right) \left( 1 - e^{-\frac{N}{k}} \right)} \quad (3)$$

10 As per step 44, compute  $M$ , the actual number of non empty  
11 subspaces.

12 As per step 46, the R statistic from the foregoing quantities of  
13  $E(M)$  and  $M$  are now provided by the following equation:

$$14 \quad R = \frac{M}{E(M)} \quad (4)$$

15 In step 48, a Z-test is performed by computing the quantity:

$$16 \quad Z = \frac{M - E(M)}{\sigma_\mu} \quad (5)$$

17 As per step 50, the significance probability is then determined  
18 by evaluating the following definite integral by a Taylor series  
19 expansion:

$$20 \quad P(|Z| \leq z) = 1 - \int_{-|z|}^{|z|} (2\pi)^{-\frac{1}{2}} e^{-\frac{x^2}{2}} dx \quad (6)$$

1       The "probability of a false alarm" (pfa), as used in step  
2 52, may be set to a suitable constant, e.g., .05, or .01 or .001.  
3 The remaining steps occur depending upon the outcome of the  
4 decision loop of step 52.

5       If the probability  $P(|Z| \leq z)$  as per step 52 is less than or  
6 equal to the pfa (meaning that  $R \approx 1.0$ ), and the answer to step  
7 52 is YES, then the procedure may preferably store and record a  
8 solution, as indicated at 58, that the data is characterized as  
9 random as indicated at 60. The flow chart then goes to  
10 designated A step which, as can be seen in the flow chart, loops  
11 back or returns to begin step 30 for processing the next window  
12 of data.

13       However, if the probability  $P(|Z| \leq z)$  is not less than or  
14 equal to the pfa (meaning that  $R \neq 1.0$ ) as per step 52 whereby  
15 the answer is NO, then the procedure may preferably store and  
16 record a solution, as indicated at 54, with the data being  
17 characterized as non-random as indicated at 56. The flow chart  
18 then goes to A which as noted in the flow chart returns to begin  
19 30 for the next window of data.

20       As noted hereinbefore, the following ratio measure of sample  
21 to population means is as shown in step 46,

$$22 \qquad R = \frac{M}{E(M)} \qquad (7)$$

23       Under the null hypothesis, the sample M should be very close  
24 to  $E(M)$  in a large random distribution (i.e.,  $R = 1.0$ ). It can be

1 shown that the theoretical limits for  $R$  are  $0 \leq R \leq 2.0$ , where  $R$   
2  $< 1$  indicates the tendency of the points to cluster, and  $R > 1$   
3 indicates the tendency of the points to resemble a uniform  
4 distribution of hypercuboids.

5 The primary utility of this method is in the field of signal  
6 processing and nonlinear dynamics in which it is of interest to  
7 know whether the measurement structure is random or chaotic. The  
8 present method may be used in the field of signal processing, and  
9 nonlinear dynamics analysis. The generalization of the entire  
10 method can be taken no higher, but its application for lower  
11 dimensions is an obvious component. When sample sizes are very  
12 small, the binomial probability model may be employed in place of  
13 the central limit theorem approximation formulas.

14 It will be understood that many additional changes in the  
15 details, steps, types of spaces, and size of samples, and  
16 arrangement of steps or types of test, which have been herein  
17 described and illustrated in order to explain the nature of the  
18 invention, may be made by those skilled in the art within the  
19 principles and scope of the invention as expressed in the  
20 appended claims.

2

3

METHOD FOR CLASSIFYING A RANDOM PROCESS

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FOR DATA SETS IN ARBITRARY DIMENSIONS

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ABSTRACT OF THE DISCLOSURE

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A method is provided for automatically characterizing data

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sets containing data points described by d-dimensional vectors

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obtained by measurements, such as with sonar arrays, as either

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random or non-random. The data points are located by the d-

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dimensional vectors in a d-dimensional Euclidean space which may

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comprise any number d of dimensions and may comprise more than

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three dimensions. Large or small sets of data may be analyzed.

14

A virtual volume is determined which contains data points from

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the maximum and minimums of the d-dimensional vectors. The

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virtual volume is then partitioned. The probability of each

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partition containing at least one data point for a random

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distribution is compared to a measurement of the number of

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partitions actually containing at least one data point whereby

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the data set is characterized as either random or non-random.



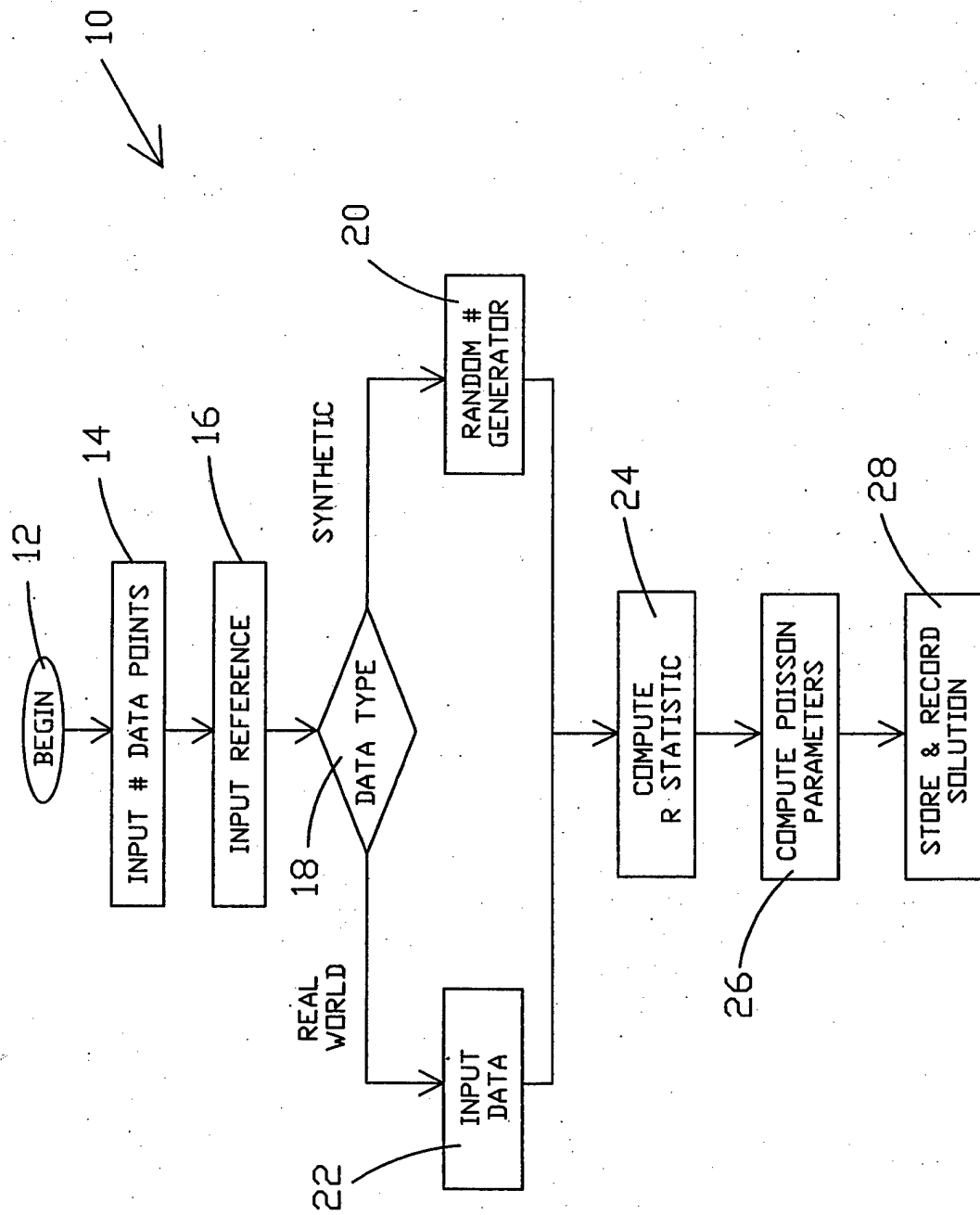


FIG. 1

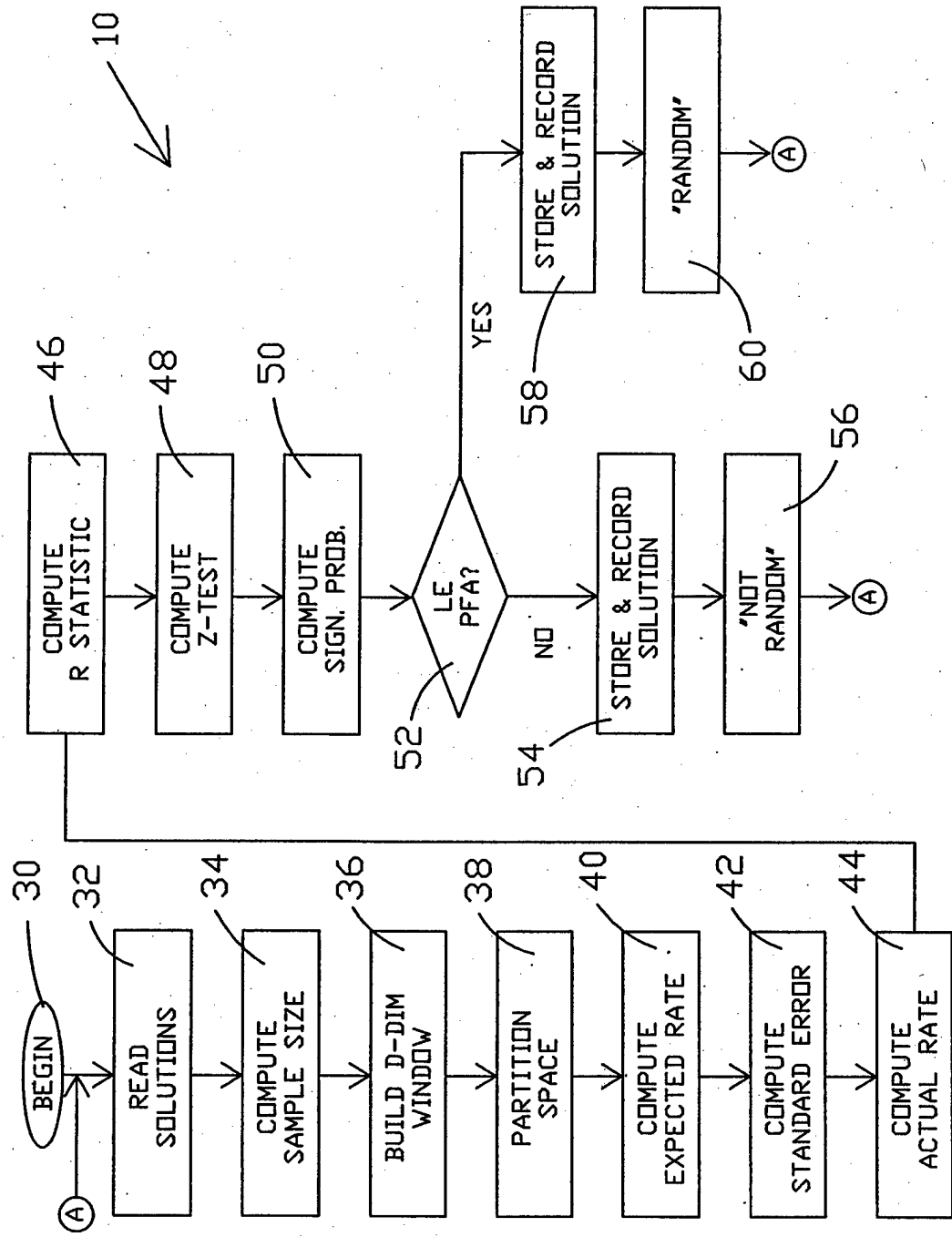


FIG. 2