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FIBER OFTIC MEASUREMENT OF TOWED ARRAY POSITION

TO ALL WHOM IT MAY CONCERN:

BE IT KNOWN THAT ANTHONY A. RUFFA, employee of the United States Government, citizen of the United States of America, and resident of Hope Valley, County of Washington, State of Rhode Island has invented certain new and useful improvements entitled as set forth above of which the following is a specification:

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1	Attorney Docket No. 79864
2	
3	FIBER OPTIC MEASUREMENT OF TOWED ARRAY POSITION
4	
5	STATEMENT OF GOVERNMENT INTEREST
6	The invention described herein may be manufactured and used
7	by or for the Government of the United States of America for
8	governmental purposes without the payment of any royalties
9	thereon or therefor.
10	
11	BACKGROUND OF THE INVENTION
12	(1) Field of the Invention
13	The present invention relates to arrays towed through water
14	by vessels and more particularly to an improved tow cable in
15	which the cable has integrated sensors allowing the curvature of
16	the tow cable to be measured.
17	(2) Description of the Prior Art
18	In naval operations, an array is towed behind a vessel for
19	gathering information, such as the location of enemy vessels or
20	the depth of the ocean. A typical array comprises an exterior
21	hose wall fabricated from rugged, insulated material, and a
22	plurality of information gathering wires communicating with
23	acoustical sensors disposed within the protective hose wall.
24	The wires or fibers of the towed array transmit information via
	1

the conducting wires or optical fibers of the tow cable to a
 microprocessor within the vessel for a readout of gathered data.
 The array also includes sensors that measure the depth, heading
 and roll of the array.

During operation of the towed array, the tow cable attached 5 to the array is extended from the hull of the submarine or 6 surface vessel. Since no segment of the tow cable or only a 7 minimal portion of the tow cable can be viewed on surface. 8 vessels, an accurate position of the array is difficult to 9 discern after the tow cable is deployed or "let out" from the 10 winch of the array handling system. Also, as the tow cable is 11 deployed in the ocean, the cable will yaw and pitch at varying 12 Because the tow cables can be several kilometers in 13 angles. length, errors in measurement caused by these angles can lead to 14 large errors in estimated position of the array. 15

In Chen et al. (U.S. Patent No. 6,256,090), a shape 16 measurement system of flexible bodies, such as a towed array, is 17 achieved by utilizing Bragg grating sensors along an optical 18 fiber path for time, spatial and wave division multiplexing and 19 strain-to-shape structural analysis measurements. By utilizing 20 these measurements, the shape of flexible bodies such as towed 21 arrays can be ascertained with the result of accurate 22. mathematical processing of data transmitted from the information 23 gathering wires of the towed array. Accurate mathematical 24

processing by the microprocessor manipulates the gathered
 information to an understandable and readable format.

While Bragg grating sensors can be applied to determine the 3 shape measurement of towed arrays, this application is not 4 easily transferable for using Bragg grating sensors on the tow 5 cable to determine the location of the towed array. In a first 6 example, towed arrays are a minute fraction of the length of the 7 cable that tows the array. Using the same amount of Bragg 8 9 grating sensors spaced along the tow cable would not provide an 10 accurate measurement of the tow cable position and increasing the amount of Bragg grating sensors on the tow cable to the 11 spacing of the array is impractical for a tow cable that can be 12 as long as 8000 feet. In a second example, the technology of 13 the cited reference is not easily transferable to the tow cable 14 for the array because of the strain encountered by the tow cable 15 16 due to its curvature at the array connection and at the winch of the towed array handling system. Bragg grating sensors bonded 17 to a polyurethane array body, as disclosed by the cited 18 reference, cannot compensate fully to these strains with the 19 result of inaccurate information gathering when the cable 20 strains are measured. 21

As such, there is presently a need for a tow cable using Bragg grating sensor technology to accurately determine the position of the towed array by accounting for the changes in the

strain of the tow cable while limiting the number of Bragg
grating sensors to an amount for adequate information gathering.
The Bragg grating sensor technology should be able to measure
the curvature of the cable at the winch of the handling system
and should be able to measure the curvature of the cable at the
connection to the array.

7 8

SUMMARY OF THE INVENTION

9 Accordingly, it is a general purpose and primary object of 10 the present invention to provide an improved tow cable in which 11 the position of the tow cable and the position of the array 12 towed by the cable is attainable by measured values.

13 It is a further object of the present invention to provide 14 a tow cable having optical fibers with sensors in predetermined 15 groupings for measuring deflections in the cable proximal to the 16 towed array and for measuring deflections in the cable proximal 17 to a winch of the towed array handling system.

18 It is a still further object of the present invention to 19 provide a tow cable having optical fibers with sensors in 20 predetermined groupings which is simple in design and 21 inexpensive to manufacture.

To attain the objects described, there is provided a tow cable in which the curvature of the tow cable is measured from the use of the multiplexing capability intrinsic to the Bragg

grating sensors embedded in the fiber optics of the tow cable. 1 Specifically, the individual Bragg grating sensors measure the 2 strain on the tow cable at opposite sides of the cable such that 3 the shape of the cable can be determined by subtracting the 4 strain measurements, thereby measuring the curvature at the 5 location of the Bragg grating sensors. The measured curvature 6 in conjunction with strain-to-shape algorithms calculated by a 7 multiplexer or other data processors known to those skilled in 8 the art provide real-time position data of the tow cable and the 9 towed array. 10

In the manufacture of the cable, four or eight of the steel 11 wire strength member wires of the tow cable are substituted with 12 armored optical fibers containing multiple Bragg grating 13 The use of multiple Bragg grating sensors (each having sensors. 14 a different wavelength) produced on a single fiber allows the 15 use of only four or eight detecting fibers to measure the 16 curvature of the cable. The Bragg grating sensors are 17 positioned within the armored optical fiber at a spacing between 18 the sensors in which the spacing is small proximal to the 19 connection of the tow cable to the towed array for a first 20 grouping of the Bragg grating sensors. 21

The remaining groupings of Bragg grating sensors with close spacing between each of the Bragg grating sensors are positioned at predetermined lengths of the tow cable in which the tow cable

would be held at these lengths to a winch of the towed array 1 handling system. The winch area of the towed array handling 2 system is a critical area to determine the curvature since the 3 curvature at the winch is used to further determine the yaw and 4 pitch angles of the cable. Example predetermined lengths of 5 cable from the towed array to the winch area are approximately 6 The curvature of the 2500', 3000', 4000', 5000', etc. 7 remaining portions of the cable between the winch and the towed 8 array or between the predetermined lengths, when the cable is 9 let out, varies so slowly within scope that the sensor spacing 10 in the remaining portions can be much larger. The use of 11 armored fibers in the tow cable compensates for the strain 12 associated with the curvatures of the tow cable at the winch and 13 at the array connection and the grouping of Bragg grating 14 sensors reduces the amount of Bragg grating sensors to a 15 practical amount for measurement. 16

The angle of the tow cable to the winch can be determined accurately by measuring the shape of the cable wound on the winch and then determining the cable pitch and yaw relative to the shape of the cable on the winch. In conjunction with the depth from sensors in the towed array, these measurements can further be used to minimize the error in the estimated position of the towed array.

The above and other features of the invention, including 1 various and novel details of construction and combinations of 2 parts will now be more particularly described with reference to 3 the accompanying drawings and pointed out in the claims. It 4 will be understood that the particular devices embodying the 5 invention are shown by way of illustration only and not as the 6 limitations of the invention. The principles and features of 7 this invention may be employed in various and numerous 8 embodiments without departing from the scope of the invention. 9

- 10
- 11

BRIEF DESCRIPTION OF THE DRAWINGS

A more complete understanding of the invention and many of the attendant advantages thereto will be readily appreciated as the same becomes better understood by reference to the following detailed description when considered in conjunction with the accompanying drawings wherein:

FIG. 1 depicts an arrangement view of the tow cable of the present invention secured to a tow vessel and secured to a towed array;

FIG. 2 depicts the grouping of Bragg grating sensors on the tow cable according to the present invention where a first grouping is located proximal to the towed array of FIG. 1 and a second grouping is located at the 5000 foot mark of the tow cable with the view taken from reference line 2-2 of FIG. 1;

	1	FIG. 3 depicts a cross-sectional view of the location of
• •	2	the optical fibers with Bragg grating sensors on the tow cable
	3 .	of the present invention with a number of strength wires of the
	4	tow cable removed for purposes of clarification and with the
	5	view taken from reference line 3-3 of FIG. 1;
	6	FIG. 4 depicts a perspective view of the optical fibers
	7_	wound at a helical angle on the tow cable of the present
	8	invention;
	9	FIG. 5 depicts the basic operating principle of Bragg
	10	grating sensors;
	11	FIG. 6 depicts the power as a function of wavelength from a
	12	broadband source;
	13	FIG. 7 graphically depicts the spectrum reflected through
	14	Bragg grating sensors;
	15	FIG. 8 depicts measurement indicia on the tow cable for the
	16	purpose of determining the curvature measurement; and
	17	FIG. 9 depicts "N", the normal vector in the plane of
	18	curvature of the tow cable of the present invention and "B", the
	19	bi-normal vector perpendicular to the plane as set against a
	20	local coordinate system.
	21	DESCRIPTION OF THE PREFERRED EMBODIMENT
	22	Referring now to the drawings wherein like numerals refer
	23	to like elements throughout the several views, one sees that
		8

FIG. 1 depicts an arrangement view including the tow cable 10 of the present invention let out from a winch 12 of a vessel 14 in which the tow cable 10 tows an acoustic sensing array 16 through the ocean 18.

As shown in FIG. 2, the tow cable 10 is joined to the towed 5 array 16 and a roll sensor 19. The extending tow cable 10 6 includes a first grouping 20 of spatially separated Bragg 7 grating sensors 22 with each of the spatially separated Bragg 8 grating sensors 22 having a different grating wavelength, Λ_{i} , i =9 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10. The Bragg grating sensors 22 10 are positioned along the tow cable 10 with a spacing between the 11 Bragg grating sensors based on an equally divisible angle ϕ 12 between the X-axis and the vector tangent to the tow cable 10 in 13 the area of the first grouping 20. The curvature K is defined 14 by $\kappa = \left| \frac{d\phi}{ds} \right|$ where S is the cable scope. Since the tow cable 10 15 twists when it is let out, the spacing between the Bragg grating 16 sensors 22 is further defined as the area between a tangent 17 vector of the tow cable at it operational curvature and a 18 horizontal axis of the tow cable. 19

Because the upper limit to the number of Bragg grating sensors 22 in the tow cable 10 is based on the signal and/or data-processing capabilities of the multiplexing processor 23 (shown in FIG.1) and the expected strain range of the tow cable

1 10, the number of Bragg grating sensors can be decreased or 2 increased from the ten gratings indicated. However, by grouping 3 the Bragg grating sensors 22 at the connection of the towed 4 array 16, a more accurate determination can be made of the 5 curvature of the tow cable 10 at the connection.

As further shown in FIG. 2, a second grouping 24 of Bragg 6 grating sensors 22, each with different grating wavelengths $\Lambda_{i,\ i}$ 7 = 11, 12, 13, 14, 15, 16, 17, 18, 19, is placed along the tow 8 cable 10 at a position spaced significantly apart from the first 9 grouping 20. For example in FIG.1, the second grouping 24 is 10 positioned on the tow cable 10 at approximately 5000' from the 11 winch 12, a third grouping 26 with different grating pitches is 12 positioned at 4000' from the winch 12, a fourth grouping 28 with 13 different grating pitches is positioned at 3000' from the winch 14 12, a fifth grouping 30 with different grating pitches is 15 positioned at 2500', etc. from the winch 12. The positioning of 16 the groupings is based upon the tow cable 10 being stopped after 17 being let out at 2500', 3000', 4000', 5000', etc (within a range 18 five percent); however, the tow cable 10 can be let out at 19 different positions based upon the use of the array 16. The 20 winch 12 of the towed array handling system on the vessel 14 is 21 a critical area to have an accurate curvature measurement 22 determined by the Bragg grating sensors 22 since the curvature 23 at the winch 12 is further used to gather measurements in 24

determining the position of the array 16. Also, the tow cable
 10 can be deployed automatically at the positioned groupings by
 monitoring the wavelengths of the Bragg grating sensors 22.

Similar to the first grouping 20, the spacing between the 4 Bragg grating sensors 22 of the additional grouping of sensors 5 is based on an equally divisible angle ϕ between the X-axis and 6 the vector tangent to the tow cable 10 in the area of the 7 The curvature K is defined by $\kappa = \left| \frac{d\phi}{ds} \right|$ where S is the grouping. 8 cable scope and K is an operational curvature of the tow cable 9 10 in the area of the winch 12. The spacing between the sensors 10 11 is further defined as the area between a tangent vector of the tow cable 10 at it operational curvature and a horizontal axis 12 of the tow cable. 13

The curvature of the remaining portion of the tow cable 10 14 between the winch 12 and the array 16 varies so slowly that the 15 spacing of the Bragg grating sensors 22 in the remaining portion 16 can be much larger, using the same guidelines, i.e., spacing the 17 Bragg gratings so that the change in ϕ (the angle between the 18 vector tangent to the cable and the X-axis is approximately 19 equal between sensors.) For example, a single Bragg grating 20 sensor 22 can be placed at 100 foot intervals between the second 21 grouping 24 at the 5000' length and the third grouping 26 at the 22 4000' length. 23

As shown in the cross-sectional view of FIG. 3, the doublearmored tow cable 10 includes eight armored optical fibers 31, 32, 33, 34, 35, 36, 37 and 38 replacing eight of the steel strength wires 39 of the cable. The number of steel strength wires 39 are shown in FIG. 3 for comparison and illustrative purposes wherein the actual number would be much larger and arrangement of steel strength wires would vary.

Each of the armored optical fibers 31, 32, 33, 34, 35, 36, 8 37 and 38 include the Bragg grating sensors 22 in the groupings 9 described for FIGS. 1 and 2. Furthermore, the armored optical 10 fibers 31, 32, 33, 34, 35, 36, 37 and 38 that replace the 11 strength wires 39 are wound in a similar manner as the strength 12 wires around a longitudinal axis 40 at a helical angle θ . See FIG 13 The helical angle θ is approximately 30 degrees; however, the 4. 14 helical angle is based upon manufacturer's specifications (i.e.: 15 the angle may decrease for larger tow cables). An optical fiber 16 placement at a helical angle θ to a longitudinal axis 40 reduces 17 the strain of the tow cable 10 at the large bending of the tow 18 cable 10 encountered at the winch 12. 19

The operation of the Bragg grating sensors 22 during towing of the array 16 is illustrated in FIGS. 5, 6 and 7. For an array position measurement, light 41 from a broadband source (shown in FIG.6) interacts with the Bragg grating sensors 22 (as

shown by the solid horizontal arrow in FIG. 5). Each index
periodicity 42 scatters a small amount of light such that only
narrowband energy centered at a single wavelength is reflected
(as shown by the dotted horizontal arrow).

5 The cumulative effect of optical scattering from the 6 periodicities 42 of the refractive index profile 43 is to 7 reflect narrowband energy centered at a single wavelength, 8 called the "Bragg wavelength" of intensity, P (shown in FIG. 6) 9 This Bragg wavelength is related to the grating pitch, Λ and the 10 mean refractive index n of the core 44 by the equation:

11

22

 $\lambda_B = 2\Lambda n$

The multiplexing of the Bragg grating sensors 22 includes the first grouping 20, the second grouping 24, etc... of FIGS. 1 and 2 with the spatially separated Bragg grating sensors 22 of each grouping, and with each grouping having different grating pitches ($\Lambda_I = 1, 2, 3...$) The output of the multiplexing Bragg grating sensors 22 is processed through the multiplexing signal processor 23.

When outputted, the reflected spectrum contains a series of peaks (as shown in FIG. 7), each associated with a different Bragg wavelength given by the equation:

$\lambda_{BI=} 2n\Lambda_I$

(2)

(1)

For example, the measurement field at Λ_2 is uniquely encoded as a perturbation Bragg wavelength, λ_{B2} with the multiplexing based on the optical wavelength of the Bragg grating sensors 22. Based upon the processing of the multiplexing operation using strain-based algorithms, the Bragg grating sensors 22 make shape measurements. Bragg grating technology is known in the prior art.

Since the tow cable 10 is deflected as it travels through 8 the water by the turning of the vessel 14 and by the vessel's 9 unsteady path due to wave encounters, the tow cable 10 becomes 10 strained and twisted due to these same forces being applied 11 thereon. As such, the Bragg grating sensors 22 can make shape 12 measurements of the strained and twisted towed cable 10 at the 13 winch 12, along the length of the tow cable and at the 14 attachment of the tow cable to the array 16. 15

In further explanation of strain measurements, a curvature determination is made with the strain measurements on opposite sides of the tow cable 10 as illustrated by FIG.8. The radius of curvature ρ of a section of the tow cable 10 initially having length L is related to the diameter D of the cable and the measured strain ε_1 and ε_2 on opposite sides of the diameter D and represented by the equations:

23

$$L(1 + \varepsilon_1) = \rho \theta; \qquad (3)$$

)

$$L(1 + \varepsilon_2) = (\rho - D) \theta.$$
(4)

2 Solving the equations for θ and ρ :

1

3

4

$$\rho = D \frac{1 + \varepsilon_1}{\varepsilon_1 - \varepsilon_2} \cong \frac{D}{\varepsilon_1 - \varepsilon_2} ; \qquad (5)$$

$$\theta = \frac{L}{D} \left(\varepsilon_1 - \varepsilon_2 \right) \tag{6}$$

These equations describe the curvature in a single plane. The three-dimensional tow cable 10 requires eight strain measurements to be determined at each section of the tow cable 10, described further below in use with Frenet formulas.

9 Since Bragg grating sensors 22 have a low deviation for 10 error in strain measurements ($\cong 5\mu\epsilon$), the multiple Bragg grating 11 sensors 22 located in each of the groupings 20, 24, 26, 28 and 12 30 further reduce error in strain measurements along the tow 13 cable 10. In conjunction with the spatial averaging process 14 described above for each of the groupings, accurate curvature 15 measurements are possible.

The position of the array 16 can be found by solving the Frenet formulas (using the curvature data). Since the heading angle, pitch angle, roll angle and depth for the array 16 is often available as a result of sensors positioned on the array, an accurate position measurement for the array is achievable in conjunction with the curvature data accumulated for the tow cable 10.

As it is known in the art, the Frenet formulas uniquely 1 define the shape or position vector R of any three dimensional 2 curve along the arc length s, given the curvature κ ($\kappa = \frac{1}{\rho}$) and 3. the torsion τ (the rotation of the plane of curvature along the 4 length of the curve). The Frenet formulas are as follows:

5

$$\frac{d\mathbf{R}}{ds} = \mathbf{T}; \quad \frac{d\mathbf{T}}{ds} = \kappa \mathbf{N}; \quad \frac{d\mathbf{N}}{ds} = -\kappa \mathbf{T} + \tau \mathbf{B}; \quad \frac{d\mathbf{B}}{ds} = -\tau \mathbf{N}$$
(7)

The solution for R (the position vector) simultaneously 7 yields the solution for T (the tangent vector), N (the normal 8 vector in the plane of curvature), and \boldsymbol{B} (the binormal vector 9 perpendicular to the plane of curvature). These factors of the 10 Frenet formulas provide major measurements for the position of 11 the array 16. 12

The Frenet formulas can be solved for **R** if $\kappa(s)$ and $\tau(s)$ are 13 known for $0 \le s \le L$, and if the initial conditions are known for 14 **R**, **T**, **N** and **B**. Since $\kappa(s)$ can be measured indirectly by 15 measuring the strain with the Bragg grating sensors 22, it is 16 possible to determine the position vector \mathbf{R} if the twist angle 17 $\psi(s)$ is also known. 18

The heading/pitch measurements at both ends of typical 19 arrays including the array 16, provide a tangent vector ${m au}$ at 20 The roll measurements at both ends of the array 16 either end. 21 provide a rotational angle of one end of the array relative to 22

the other. During towing, the array 16 has roll(or twist) angles 1 that vary quadratically from one end of the array 16 to the 2 The tow cable 10, similarly encountering rolling forces, 3 other. also includes quadratically varying twist angles. 4

For the array 16, the roll/twist angle ψ varies with the 5 tension F according to the equation: 6

$$F = A \frac{d\psi}{ds} \tag{8}$$

(11)

Here A is determined by testing, i.e., by measuring the 8 amount of rotation as a section of the array 16 is put under 9 tension. 10

During a steady tow of the array 16, the tension F_0 will 11 depend on the accumulated tangential drag behind an aft roll 12 measurement of the array 16. With the distance s (re-designated 13 from the aft roll measurement) and D (re-designated as the speed 14 of the cable through the water) the relational tension F is 15 determined as follows: 16

$$F(s) = F_0 + \frac{1}{2} \rho C_T \pi D u^2 s$$
 (9)

18

17

7

The twist angle then varies as follows:

19
$$A \frac{d\psi}{ds} = F_0 + \frac{1}{2} \rho C_T \pi D u^2 s;$$
 (10)
20 $A d\psi = [F_0 + \frac{1}{2} \rho C_T \pi D u^2 s] ds;$ (11)

1
$$A(\psi - \psi_0) = F_0 s + \frac{1}{4} \rho C_7 \pi D u^2 s^2$$
 (12)
2 or, since both terms $u^2 s$, $u^2 s^2$ can be expressed as a constant
3 multiplied by u^2 ,
4 $\psi = \psi_0 + C_1 u^2 s + C_2 u^2 s^2$ (13)
5 Here the constant C_1 is $\frac{F_0}{A}$ and C_2 is $\frac{1}{4} \rho C_7 \pi D$.
6 The equation:
7 $\psi_L = \psi_0 + C_1 u^2 L + C_1 u^2 L^2$ (14)
8 with L based on the length of the array 16 can be used to
9 reduce the error of the effective speed. For example, if roll
10 measurements of the array 16 are independent and have a standard
11 deviation of σ_{ψ} , then the sum of the second and third terms above
12 has a standard deviation of $\sigma_{\psi}/\sqrt{2}$, and the error of $\psi(s)$ has a
13 constant standard deviation with respect to s .
14 Because the tow cable 10 of the present invention as well
15 as other tow cables known in the array hose at the end of the tow
17 cable, the twist angles are calculated from the strain
18 measurements at the groupings of the Bragg grating sensors 22
19 and from the roll measurement of the roll sensor 19 at the array
20 16.
21 For the measurement of the twist angles along the tow cable
22 10, the optical fibers 31, 32, 33, 34, 35, 36, 37 and 38 are

spiraled around the tow cable 10 with the helical angle θ . (See FIG. 4) Each strain measurement leads to an axial strain and a shear strain as follows:

4
$$\varepsilon_{ai} \cong \varepsilon_i \cos \theta$$
 (axial strain); (15)

5
$$\varepsilon_{si} \cong \varepsilon_i \sin \theta$$
 (shear strain); (16)

6

7

12

$$\kappa \cos \phi = \frac{\mathcal{E}_{a31} - \mathcal{E}_{a34}}{4r_1} + \frac{\mathcal{E}_{a32} - \mathcal{E}_{a33}}{4r_2};$$
 (17)

$$\kappa \sin \phi = \frac{\mathcal{E}_{a35} - \mathcal{E}_{a38}}{4r_1} + \frac{\mathcal{E}_{a36} - \mathcal{E}_{a37}}{4r_2}$$
 (18)

8 with twist angle measured from the eight strain 9 measurements with the equation (it is possible to infer the 10 twist with only four fibers such as fibers 31, 34, 35 and 38 11 however, the measurement will not be as accurate):

$$\frac{d\psi}{ds} = \frac{\mathcal{E}_{31} + \mathcal{E}_{34} + \mathcal{E}_{35} + \mathcal{E}_{38}}{8r_1} - \frac{\mathcal{E}_{32} + \mathcal{E}_{33} + \mathcal{E}_{36} + \mathcal{E}_{37}}{8r_2}$$
(19)

where the centers of the inner fibers 32, 33, 36, 37 are located at radius r_1 and the outer fibers 31, 34, 35 and 38 are located at radius r_2 . The wires at radius r_1 are helixed oppositely the wires at r_2 (so that the cable will be approximately torque balanced). Because of this, $\frac{d\phi}{ds}$ will be zero if a pure tension is applied to the cable. In relation to the coordinate systems described below, the

In relation to the coordinate systems described below, the twist angle rotates the local y-axis about the tangent vector T. Roll measurements have meaning only when the pitch angle α is

1	less than 90 degrees - a reasonable assumption for a towed arra	iy
2	that is nominally neutrally buoyant. The pitch angle α is	
3	defined as follows for $ \alpha < \pi/2$:	
4	$\sin \alpha = \mathbf{T} \cdot \hat{j} = \mathbf{T}_j \tag{2}$))
5	Here \hat{j} is the vertical unit vector in the global coordina	te
6	system. The unit vectors \hat{i} and \hat{k} are perpendicular to each	
7	other and to \hat{j} .	
8	When the roll is zero, the y-coordinate of the local	
9	coordinate system (aligned at the towed array 16) is	
10	perpendicular to the tangent vector \boldsymbol{T} , and in the plane define	d
11	by ${m T}$ and \hat{j} , allowing \hat{y} to be defined as follows:	
12	$\hat{y} = \frac{\left(\mathbf{T} + \hat{j}\right)\mathbf{x}\mathbf{T}}{\left \left(\mathbf{T} \times \hat{j}\right)\mathbf{x}\mathbf{T}\right },$ (2)	1)
13	where "x" denotes the vector cross product	
14	or	
15	$\hat{y} = \frac{-\mathbf{T}_{j}\mathbf{T}_{j}}{\sqrt{1 - \mathbf{T}_{j}^{2}j}} \hat{i} + \sqrt{1 - \mathbf{T}_{j}^{2}j} - \frac{\mathbf{T}_{j}\mathbf{T}_{k}}{\sqrt{1 - \mathbf{T}_{j}^{2}}} \hat{k} $ (2)	2)
16	Note that $\mathbf{T}_{j} \neq \pm 1$, again forbidding the magnitude of the pitch	
17	angle to be 90 degrees.	
18	The x-coordinate and the z-coordinate in the local	
19	reference frame are as follows:	
20	$\hat{x} = T$ (2)	:3)

1
$$\hat{z} = \hat{x} \times \hat{y} = -\frac{\tau \kappa}{\sqrt{1-\tau_{j}^{2}}} \hat{x} + \frac{\tau_{i}}{\sqrt{1-\tau_{j}^{2}}} \kappa$$
 (24)
2 Now, N and B can be defined relative to the local coordinate
3 system (See FIG. 9)
4 In local coordinates, N can be written as follows:
5 $N = \frac{\delta_{bendr}}{\kappa \rho} \hat{y} + \frac{\delta_{bendr}}{\kappa \rho} \hat{z}$ (25)
6 $= \cos\phi \hat{y} + \sinh\phi \hat{z}$ (26)
7 where $\kappa = \frac{\sqrt{\epsilon_{bendr}^{2} - \epsilon_{bendr}^{2}}}{\rho}$ (27)
8 $\tan\phi = \frac{\delta_{bendr}}{\epsilon_{bendr}}$ (28)
9 Since ϵ_{bendy} and ϵ_{bendr} are measured, κ , and ϕ are known at
10 every point, substituting leads to a formula for N expressed in
11 the global coordinate system:
12 $N = -\frac{\tau_{\kappa} \sin\phi + \tau_{2}\tau_{j}\cos\phi}{\sqrt{1-\tau_{j}^{2}}} \hat{i} + \cos\phi\sqrt{1-\tau_{j}^{2}} \hat{j} + \frac{\tau_{i}\sin\phi - \tau_{j}\tau_{k}\cos\phi}{\sqrt{1-\tau_{j}^{2}}} \hat{\kappa}$ (29)
13 T is then solvable by the equation:
14 $\frac{d\tau}{ds} = \kappa N = -\frac{\tau_{k}\sin\phi + \tau_{k}\tau_{j}\cos\phi}{\sqrt{1-\tau_{j}^{2}}} \kappa \hat{i} + \kappa \cos\phi\sqrt{1-\tau_{j}^{2}} \hat{j} + \frac{\tau_{i}\sin\phi - \tau_{j}\tau_{k}\cos\phi}{\sqrt{1-\tau_{j}^{2}}} \kappa \hat{\kappa}$. (30)
15 Finally, the position vector R is determined by integrating

$$16 \quad \frac{d\mathbf{R}}{ds} = \mathbf{T}$$

When there is a roll of the array 16 and the tow cable 10, the nonzero roll angle is a rotation about the local x-axis or

21

(31)

T. As such, the roll of the tow cable 10 and the array 16
separately or together must be accounted for before N can be
determined. Since roll sensors are designed to tell where "up"
is and the roll sensor 19 indicates the roll at the connection
of the tow cable 10 to the array 16, the roll can relate N and B
to the global coordinate system as shown above.

During rolling of the tow cable 10 and the array 16, the local y and z coordinates are rotated thereby becoming new vectors y_r and z_r . The vectors are the basis of new local coordinate system (x_r, y_r, z_r) at the rolling point on the tow cable 10 where

$$x_r = T;$$
 (32)

(33)

13

 $\hat{y}_r \cdot \hat{y} = \hat{z}_r \cdot \hat{z} = \cos \psi$

14 where "." denotes the scalar dot product.

The new local coordinate system becomes identical to the first local coordinate system when the roll angle is zero. When the roll angle is nonzero, N and B are actually measured in the new coordinate system, which physically rotates with the Bragg grating sensors 22 on the tow cable 10 in relation to the roll sensor 19, i.e.,:

21 $N = \cos\phi \hat{y}_r + \sin\phi \hat{z}_r ;$ (34) 22 $B = -\sin\phi \hat{y}_r + \cos\phi \hat{z}_r$ (35)

so that they have to be rotated back into the zero-roll
 coordinate system. The nonzero roll coordinate system and the
 zero-roll system are related as follows:

$$4 \qquad \begin{pmatrix} x_r \\ y_r \\ z_r \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \psi & \sin \psi \\ 0 & -\sin \psi & \cos \psi \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$
(36)
5 Thus, \mathbf{N} becomes:
6 $\mathbf{N} = (\cos \phi \cos \psi - \sin \phi \sin \psi) \hat{y} + (\cos \phi \sin \psi + \sin \phi \cos \psi) \hat{z}$ (37)
7 $(\cos \phi \sin \psi + \sin \phi \cos \psi) \hat{z}$ (37)
8 or
8 or
9 (38)

9
$$\mathbf{N} = \cos(\phi + \psi) y + \sin(\phi + \psi) z, \qquad (30)$$

10

12
$$N = -\frac{\mathbf{T}_{i}\mathbf{T}_{j}\cos(\phi + \psi) + \mathbf{T}_{k}\sin(\phi + \psi)}{\sqrt{1 - \mathbf{T}_{j}^{2}}}\hat{i} + \cos(\phi + \psi)\sqrt{1 - \mathbf{T}_{j}^{2}}\hat{j}$$
13
$$+\frac{\mathbf{T}_{i}\sin(\phi + \psi) - \mathbf{T}_{j}\mathbf{T}_{k}\cos(\phi + \psi)}{\sqrt{1 - \mathbf{T}_{j}^{2}}}\hat{k}$$
(39)

14 Once
$$\kappa(s)$$
, $\psi(s)$ and $\phi(s)$ are measured, the following equations can
15 be solved for **T**:

$$16 \quad \frac{d\mathbf{T}}{ds} = \kappa \mathbf{N} = -\frac{\mathbf{T}_{i}\mathbf{T}_{j} \cos(\phi + \psi) + \mathbf{T}_{k}\sin(\phi + \psi)}{\sqrt{1 - \mathbf{T}_{j}^{2}}} \quad \kappa \mathbf{i} + \kappa \cos(\phi + \psi)\sqrt{1 - \mathbf{T}_{j}^{2}}\mathbf{j}$$

$$17 \quad + \frac{\mathbf{T}_{i}\sin(\phi + \psi) - \mathbf{T}_{j}\mathbf{T}_{k}\cos(\phi + \psi)}{\sqrt{1 - \mathbf{T}_{j}^{2}}} \quad \kappa \mathbf{k}.$$

23

(40)

1 The position vector \boldsymbol{R} is once again determined by integrating

(41)

 $2 \quad \frac{d\mathbf{R}}{ds} = \mathbf{T}$

To summarize, the twist angle $\psi(s)$ can be measured at the 3 roll sensor 19 or extrapolated from the roll sensor to the 4 groupings of Bragg grating sensors 22. Where there is an arc 5 length s, then $\kappa(s)$ is sufficient to determine R, the position 6 vector. By gathering measurements from the Bragg grating 7 sensors 22 and integrating the roll measurement at the roll 8 sensor 19 as an initial condition, the twist angles at the major 9 curvatures can be calculated with accuracy and as such the major 10 measurement factors of Frenet's formula are determinable leading 11 to the position of the array 16. Since the depth of the tow 12 cable 10 is known relative to the vessel 14, the difference in 13 the \hat{j} coordinate can be used to reduce error in a least squares 14 sense. 15

Thus by the present invention its objects and advantages are realized and although preferred embodiments have been disclosed and described in detail herein, its scope should be determined by that of the appended claims.

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FIBER OPTIC MEASUREMENT OF TOWED ARRAY POSITION

ABSTRACT OF THE DISCLOSURE

A tow cable connectable to an sensor array with Bragg 6 grating sensors embedded in helical-wound armored fibers of the 7 A first grouping of spaced apart sensors are positioned cable. 8 at the connection of the tow cable to the array. Additional 9 groupings of sensors are positioned at predetermined lengths of 10 the cable in which the cable would be held at these lengths to a 11 towed array handling winch. The sensor spacing in the remaining 12 portions of the cable is much larger than that in the sensor 13 The sensors measure the strain on the tow cable at groupings. 14 opposite points of the cable and further measure the shape of 15 the cable by strain-based algorithmic equations, thereby 16 measuring the position of the Bragg grating sensors coincident 17 with the position of a tow array. 18











FIG.5

 \sim

FIG.6













B

FIG.9