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**DTIC QUALITY INSPECTED 2**

2  
3 GEODETTIC POSITION ESTIMATION FOR UNDERWATER ACOUSTIC SENSORS

4  
5 STATEMENT OF GOVERNMENT INTEREST

6 The invention described herein may be manufactured and used  
7 by or for the Government of the United States of America for  
8 governmental purposes without the payment of any royalties  
9 thereon or therefore.

10  
11 BACKGROUND OF THE INVENTION

12 (1) Field of the Invention.

13 The present invention relates generally to a system and  
14 method for estimating the geodetic position of acoustic sensors,  
15 and more particularly to a system and method for estimating  
16 position which considers timing and sound velocity biases as  
17 parameters to be estimated, thus precluding biasing errors from  
18 propagating to the sensor coordinates.

19 (2) Description of the Prior Art

20 Underwater acoustic tracking ranges, as typically operated,  
21 utilize the well known principle of hyperbolic multilateration  
22 similar to radio frequency (RF) navigation systems. The accuracy  
23 and limitations of these systems has been well documented. The  
24 underwater acoustic environment, however, creates some unique

1 differences. For example, both the RF and acoustic systems make  
2 timing measurements and convert them to range measurements. The  
3 nature of underwater acoustic wave propagation results in non-  
4 dispersive wave propagation, significantly smaller propagation  
5 velocities, shorter propagation distances, longer transit times  
6 and severe refraction through the stratified ocean. In addition,  
7 both types of tracking systems require precise knowledge of the  
8 relative and geodetic location of the reference sensors  
9 (transmitters and receivers). For a land based RF system, this  
10 is accomplished using well known, conventional terrestrial survey  
11 techniques. For a satellite based RF system, it requires  
12 techniques adapted for that environment. Historically, two  
13 methods have been employed for surveying underwater acoustic  
14 sensors, commonly referred to as the Vanderkulk and Spherical  
15 Least Squares methods, the main difference between the two  
16 methods being the use of geodetic position information for the  
17 acoustic sources. However, both techniques are extensions of the  
18 conventional, well known trilateration survey technique. A  
19 trilateration survey consists of making range measurements from  
20 reference points to points to be surveyed. The ranging  
21 measurements are obtained by making timing measurements and  
22 converting them to range measurements based upon a presumed  
23 knowledge of the propagation velocity of the transmitted acoustic  
24 signal. The major source of error for the traditional survey

1 methods is systematic error or bias. This type of error can  
2 manifest itself both in the timing measurements and in the  
3 assumptions of the sound velocity. Timing biases are normally  
4 removed by making system timing measurements and removing the  
5 systematic component before processing, the systematic component  
6 normally being a timing delay. Systematic errors in the acoustic  
7 propagation velocity, however, are more difficult to deal with  
8 because they cannot be measured and are generally functions of  
9 both space and time. For underwater acoustic multilateration  
10 tracking systems, the nature of the acoustic propagation is such  
11 that ray theory is considered valid. If one further restricts  
12 the propagation to direct monotonic acoustic ray paths, the  
13 propagation velocity is characterized by an Effective Sound  
14 Velocity (ESV). The ESV is that velocity which when multiplied  
15 by the transit time between two underwater points, yields the  
16 geometric or slant range between them. To avoid systematic (non-  
17 random) errors in the ESV calculation, an unbiased measurement  
18 for the Sound Speed Profile (SSP) is prerequisite. However, a  
19 typical measured SSP may have a 2 meter/second bias error.  
20 Consequently, the traditional methods may suffer significant  
21 errors on estimating the sensor coordinates.



1 measurements, requiring calculation of the Effective Sound  
2 Velocity. Bias errors are precluded from propagating to the  
3 sensor coordinates by considering timing and sound velocity  
4 biases, in addition to the sensor coordinates, as parameters to  
5 be estimated.

#### 6 7 BRIEF DESCRIPTION OF THE DRAWINGS

8 A more complete understanding of the invention and many of  
9 the attendant advantages thereto will be readily appreciated as  
10 the same becomes better understood by reference to the following  
11 detailed description when considered in conjunction with the  
12 accompanying drawings wherein corresponding reference characters  
13 indicate corresponding parts throughout the several views of the  
14 drawings and wherein:

15 FIG. 1 is a flow chart of the method of the present  
16 invention;

17 FIG. 2 is a schematic representation of a hydrophone  
18 location to be estimated;

19 FIG. 3 is a Sound Velocity Profile corresponding to FIG. 2;  
20 and

21 FIG. 4 is a Slant Range Residual Plot for data from FIGS. 2  
22 and 3.



1 without propagating to the position estimate of the underwater  
2 acoustic sensor. The hyperbolic model is thus:

$$3 \quad R_i = c_i(t_i^a - t_i^e + b_i) \quad ; \quad i = 1 \dots N. \quad (3)$$

4 This positioning model has not normally been employed for  
5 acoustic sensor position estimation because one has the ability  
6 to synchronize the transmitter and receiver and perform timing  
7 measurements so that this term is known a priori. However, the  
8 present invention extends the concept further. A fifth parameter  
9 representing an unknown effective sound velocity bias  $b_c$  is  
10 added. Consequently, a new positioning model with effective  
11 sound velocity bias estimation is thus:

$$12 \quad R_i = (c_i + b_c)(t_i^a - t_i^e + b_i) \quad ; \quad i = 1 \dots N. \quad (4)$$

13 It is noted that higher order terms could be added to the model  
14 if the mathematical nature of the ESV error dictates.

15 The models shown in Eq. (1), (3) and (4) are nonlinear  
16 systems of equations. All of the models shown above are nonlinear  
17 systems of equations. Eq. (1) contains three parameters to be  
18 estimated, namely  $(x, y, z)$ . Eq. (3) and (4) contain four  
19  $(x, y, z, b_t)$  and five  $(x, y, z, b_c, b_t)$  parameters, respectively. A  
20 common approach to solve this kind of nonlinear problem is based  
21 on a well known iterated Newton-Raphson method, suitable for  
22 implementation within a computer, as will be explained further  
23 hereinafter.

24 Defining a new variable  $t^t$  as the transit time where

1  $t^t = t^a - t^e$  and denoting a reference point  $(x_0, y_0, z_0, b_c^0, b_t^0)$  for  
 2  $(x, y, z, b_c, b_t)$ , a direct use of the Taylor series expansion for  
 3  $t_i^t$ , as a function of  $(x, y, z, b_c, b_t)$ , yields:

$$4 \quad t_i^t = t_i^0 + \frac{\partial_i^t}{\partial x} \Big|_0 (x - x_0) + \frac{\partial_i^t}{\partial y} \Big|_0 (y - y_0) + \frac{\partial_i^t}{\partial z} \Big|_0 (z - z_0) + \frac{\partial_i^t}{\partial b_c} \Big|_0 (b_c - b_c^0) + \frac{\partial_i^t}{\partial b_t} \Big|_0 (b_t - b_t^0), \quad (5)$$

5 where a super- or sub-script "0" indicates that the value is  
 6 calculated at the reference point  $(x_0, y_0, z_0, b_c^0, b_t^0)$ . For  
 7 instance,

$$8 \quad t_i^0 = \frac{R_i^0}{c_i^0 + b_c^0} - b_t^0, \quad (6)$$

9 in which:

$$10 \quad R_i^0 = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2} \quad (7)$$

11 and  $c_i^0$  is the ESV between  $(x_i, y_i, z_i)$  and  $(x_0, y_0, z_0)$ .

12 From EQ. (5), one can easily show

$$13 \quad \frac{\partial_i^t}{\partial b_c} \Big|_0 = - \frac{R_i^0}{(c_i^0 + b_c^0)^2} \quad (8)$$

14 and

$$15 \quad \frac{\partial_i^t}{\partial b_t} \Big|_0 = -1. \quad (9)$$

16 Due to the fact that both  $R_i$  and  $c_i$  are dependent on  $x$ , one  
 17 writes:

$$18 \quad \frac{\partial_i^t}{\partial x} \Big|_0 = \frac{1}{c_i^0 + b_c^0} \left( \frac{\partial R_i}{\partial x} \Big|_0 - \frac{R_i^0}{c_i^0 + b_c^0} \frac{\partial c_i}{\partial x} \Big|_0 \right). \quad (10)$$

1 Since

$$\frac{\partial R_i}{\partial x} \Big|_0 = \frac{x_0 - x_i}{R_i^0}, \quad (11)$$

3 one can rewrite EQ. (10) to be

$$\frac{\partial \alpha_i}{\partial x} \Big|_0 = \frac{1}{c_i^0 + b_c^0} \left( \frac{x_0 - x_i}{R_i^0} - \frac{R_i^0}{c_i^0 + b_c^0} \frac{\partial \alpha_i}{\partial x} \Big|_0 \right). \quad (12)$$

5 Because there is no explicit expression available for  $c_i$  as a

6 function of  $x$ , one cannot analytically evaluate  $\frac{\partial \alpha_i}{\partial x} \Big|_0$ .

7 Therefore, one has to rely on a numerical method to obtain it.

8 One possible way to obtain an approximation for  $\frac{\partial \alpha_i}{\partial x} \Big|_0$  is stated

9 below. First, one estimates the ESVs corresponding to the target

10 position at  $(x_0 + \varepsilon, y_0, z_0)$  and at  $(x_0 - \varepsilon, y_0, z_0)$ , respectively, denoted

11 as  $c_i^0(x_0^+)$  and  $c_i^0(x_0^-)$ , where  $\varepsilon$  is a small length quantity (say 1

12 meter). Then, one calculates  $\frac{\partial \alpha_i}{\partial x} \Big|_0$  as:

$$\frac{\partial \alpha_i}{\partial x} \Big|_0 = \frac{c_i^0(x_0^+) - c_i^0(x_0^-)}{2\varepsilon} \quad (13)$$

14 It is well known in solving Eq. (10) to assume that  $\frac{\partial \alpha_i}{\partial x} \Big|_0$

1 is negligibly small. This assumption is particularly true where  
 2 the elevation angle associated with  $(x_i, y_i, z_i)$  and  $(x_0, y_0, z_0)$  is  
 3 large. As a result, one can simplify Eq. (12) to be:

$$4 \quad \left. \frac{\partial \alpha_i'}{\partial x} \right|_0 = \frac{1}{c_i^0 + b_c^0} \frac{x_0 - x_i}{R_i^0}. \quad (14)$$

5 Exercising the same mathematics used in deriving Eq. (12), one  
 6 obtains:

$$7 \quad \left. \frac{\partial \alpha_i'}{\partial y} \right|_0 = \frac{1}{c_i^0 + b_c^0} \left( \frac{y_0 - y_i}{R_i^0} - \frac{R_i^0}{c_i^0 + b_c^0} \left. \frac{\partial \alpha_i'}{\partial y} \right|_0 \right) \quad (15)$$

8 and

$$9 \quad \left. \frac{\partial \alpha_i'}{\partial z} \right|_0 = \frac{1}{c_i^0 + b_c^0} \left( \frac{z_0 - z_i}{R_i^0} - \frac{R_i^0}{c_i^0 + b_c^0} \left. \frac{\partial \alpha_i'}{\partial z} \right|_0 \right), \quad (16)$$

10 and following the same numerical procedure as in Eq. (13):

$$11 \quad \left. \frac{\partial \alpha_i'}{\partial y} \right|_0 = \frac{c_i^0(y_0^+) - c_i^0(y_0^-)}{2\varepsilon} \quad (17)$$

12 and

$$13 \quad \left. \frac{\partial \alpha_i'}{\partial z} \right|_0 = \frac{c_i^0(z_0^+) - c_i^0(z_0^-)}{2\varepsilon}, \quad (18)$$

14 where, following the same definitions as given for  $c_i^0(x_0^+)$  and  
 15  $c_i^0(x_0^-)$ ,  $c_i^0(y_0^+)$ ,  $c_i^0(y_0^-)$ ,  $c_i^0(z_0^+)$  and  $c_i^0(z_0^-)$  are ESVs  
 16 corresponding to the target positions at  $(x_0, y_0 + \varepsilon, z_0)$ ,  
 17  $(x_0, y_0 - \varepsilon, z_0)$ ,  $(x_0, y_0, z_0 + \varepsilon)$  and  $(x_0, y_0, z_0 - \varepsilon)$ , respectively.

1 Simplifying Eq. (15) and Eq. (16) when the associated  
2 elevation angle is large yields:

$$3 \quad \left. \frac{\partial a'_i}{\partial y} \right|_0 = \frac{1}{c_i^0 + b_c^0} \frac{y_0 - y_i}{R_i^0} \quad (19)$$

4 and

$$5 \quad \left. \frac{\partial a'_i}{\partial z} \right|_0 = \frac{1}{c_i^0 + b_c^0} \frac{z_0 - z_i}{R_i^0} \quad (20)$$

6 The well known iterated Newton-Raphson method can now be  
7 applied. Referring now to FIG. 1, there is shown a flow chart of  
8 the method of the present invention. Writing the linearized N  
9 equations (Eq. 5) in five unknowns in matrix vector notation,  
10 step 100 yields:

$$11 \quad J\Delta x = J\Delta R, \quad (21)$$

12 where  $J$  is the well known Jacobian matrix, formed using Equations  
13 14, 19, 20, 8 and 9 for  $t$  with respect to  $x$ ,  $y$ ,  $z$ ,  $b_c$  and  $b_t$ ,  
14 respectively, and using data points 1 ...  $N$ ;  $\Delta x$  is the state error  
15 vector; and  $\Delta R$  is the difference between the measurement and the  
16 reference state vector. A first linearization point,  $x_0, y_0, z_0, b_c^0$   
17  $, b_t^0$ , is chosen to obtain the least squares estimate, given by:

$$18 \quad \Delta \bar{x} = (J^T J)^{-1} J \Delta R, \quad (22)$$

19 as provided for in the Newton-Raphson method at step 200. The  
20 least squares estimate is added to the linearization point to  
21 yield an improved estimate at step 300. If the error vector is  
22 greater than a predetermined tolerance value, step 400 returns to

1 step 100, using the updated parameter values. If the error  
2 vector is less than the predetermined tolerance value, step 400  
3 exits with the improved estimate. As noted previously, computer  
4 algorithms for implementing the Newton-Raphson method for solving  
5 linear equations in multiple unknowns are well known in the art.

6 As also noted earlier, the term  $\left. \frac{\partial c_i}{\partial x} \right|_0$  was taken to be

7 negligibly small, especially for large elevation angles. In  
8 practice, the evaluation of the spatial dependence of  $c_i$  is  
9 incorporated into the iteration of the linearized state variable  
10 and is performed automatically regardless of the elevation angle.

11 The following numerical simulation verifies the solution  
12 algorithm and clearly demonstrates the capability of this new  
13 method without the introduction of noise and uncertainty of field  
14 data. Referring to FIG. 2, consider hydrophone 10 cabled to  
15 shore (not shown) which is placed at a fixed location "L" on the  
16 seafloor such that its geodetic coordinates are 24.50° N  
17 latitude, 77.50° W longitude and an ellipsoid height of -1628.94  
18 meters. The geoid height is known to be -28.94 meters, thus the  
19 hydrophone 10 depth (orthometric height) is -1600.00 meters.  
20 Assume that a surface vessel, or ship 12, transited on a ship  
21 path relative to the hydrophone as shown by arrows 14 and 16,  
22 where (without loss of generality) the coordinate system has been  
23 transformed to an East, North, Up local tangent plane coordinate

1 system. The ship 12 is transmitting acoustic signals which  
 2 propagate to hydrophone 10 as indicates by lines 18 in FIG. 2 and  
 3 are detected by electronics (not shown) on shore. The signals  
 4 correspond to data points 1 ... N for use in forming the Jacobian  
 5 matrix. FIG. 3 shows a SVP plot for the area in question. Using  
 6 this SVP as a true SVP, the true transit times from the ship 12  
 7 to hydrophone 10 at each position can be calculated. If a  
 8 systematic timing bias error of 2 ms and an ESV bias error of  
 9 0.63 m/s is incorporated, a new set of (erroneous) acoustic  
 10 transit times can be calculated. Starting from a reference  
 11 location  $(x_0, y_0, z_0)$  several hundred meters from the true location,  
 12 the position of hydrophone 10 is estimated using both data sets  
 13 and all three positioning models (spherical least squares, SLS;  
 14 hyperbolic least squares, HLS; and the method of the present  
 15 invention referred to as hyperbolic least squares with bias  
 16 estimation, HLSBE) as summarized in Table 1 as follows:

17 **TABLE 1: Numerical Simulation Results**

	Model	x(m)	y(m)	z(m)	$b_t$ (ms)	$b_c$ (m/s)
1	SLS	0.00	0.00	-1600.00	-	-
2	HLS	0.00	0.00	-1600.00	0.0	-
3	HLSBE	0.00	0.00	-1600.00	0.0	0.0
4	SLS	0.00	0.00	-1606.78	-	-
5	HLS	0.00	0.00	-1597.09	4.1	-
6	HLSBE	0.00	0.00	-1600.00	2.0	0.63

1 Estimates numbered 1 through 3 correspond with the three  
2 different positioning models for acoustic data containing no bias  
3 error. As shown in Table 1, if no biases are present, each  
4 positioning model and its associated solution algorithm is  
5 sufficient for estimating the sensor location. Estimates 4  
6 through 6 utilize data having the timing delay and ESV bias. In  
7 this situation, however, only the method of the present  
8 invention, HLSBE, correctly estimates the sensor location in the  
9 presence of both a system timing bias and ESV bias.  
10 Traditionally, any systematic bias was estimated and eliminated  
11 through the analysis of slant range residuals. The residuals for  
12 estimates 4 through 6 associated with each model are shown in  
13 FIG. 4. Use of a residual plot shows that a bias is present, but  
14 gives no immediate insight as to the source or relative magnitude  
15 of the bias errors. Historically, errors were removed by  
16 undertaking a laborious and time consuming trial and error  
17 approach. The situation is further complicated for field data as  
18 the residual plot will have noise superimposed on any systematic  
19 trend event in the residuals. It is noted that the HLSBE model  
20 of the present invention yields not only a correct estimate for  
21 the sensor location, but also correct estimates for the timing  
22 and ESV biases.

23 The invention thus described provides an improved survey  
24 method for estimating the geodetic position of sensors placed at

1 fixed but unknown locations on the sea floor developed. This  
2 method eliminates the largest sources of error, timing and sound  
3 velocity bias error, by considering them as additional unknown  
4 parameters to be estimated. The method is shown to be very  
5 robust and insensitive to the uncertainties of SVP and timing  
6 bias error when estimating acoustic sensor location from  
7 experimental field data. In addition to providing accurate  
8 estimates of geodetic position, the method also provides correct  
9 estimates for the timing and ESV biases." For this new method, as  
10 well as all previous methods, care must be taken to  
11 simultaneously ensure that the measurements, mathematical model,  
12 and solution algorithm are compatible. Specifically, the new  
13 method will usually require a wider spatial coverage while  
14 collecting time of arrival measurements, because the successful  
15 use of this new model needs diverse effective sound velocities  
16 associated with those measurements.

17 Although the present invention has been described relative  
18 to a specific embodiment thereof, it is not so limited. For  
19 example, as noted previously, higher order terms could be added  
20 to the model if the mathematical nature of the ESV error so  
21 dictated.

22 Thus, it will be understood that many additional changes in  
23 the details and steps which have been herein described and  
24 illustrated in order to explain the nature of the invention, may

1 be made by those skilled in the art within the principle and  
2 scope of the invention.

1 Attorney Docket No. 78865

2

3 GEODETIC POSITION ESTIMATION FOR UNDERWATER ACOUSTIC SENSORS

4

5 ABSTRACT OF THE DISCLOSURE

6 A system and survey method for estimating the  
7 geodetic position of acoustic sensors placed at fixed but unknown  
8 locations on the seafloor is disclosed. Bottom mounted sensors  
9 are surveyed using an extension of the well known trilateration  
10 survey technique, i.e., making ranging measurements from  
11 reference points to the point to be surveyed. For acoustic  
12 sensors, these ranging measurements are obtained by transmitting  
13 an acoustic signal from a near surface projector and making  
14 corresponding timing and/or position measurements, requiring  
15 calculation of the Effective Sound Velocity. Bias errors are  
16 precluded from propagating to the sensor coordinates by  
17 considering timing and sound velocity biases, in addition to the  
18 sensor coordinates, as parameters to be estimated.

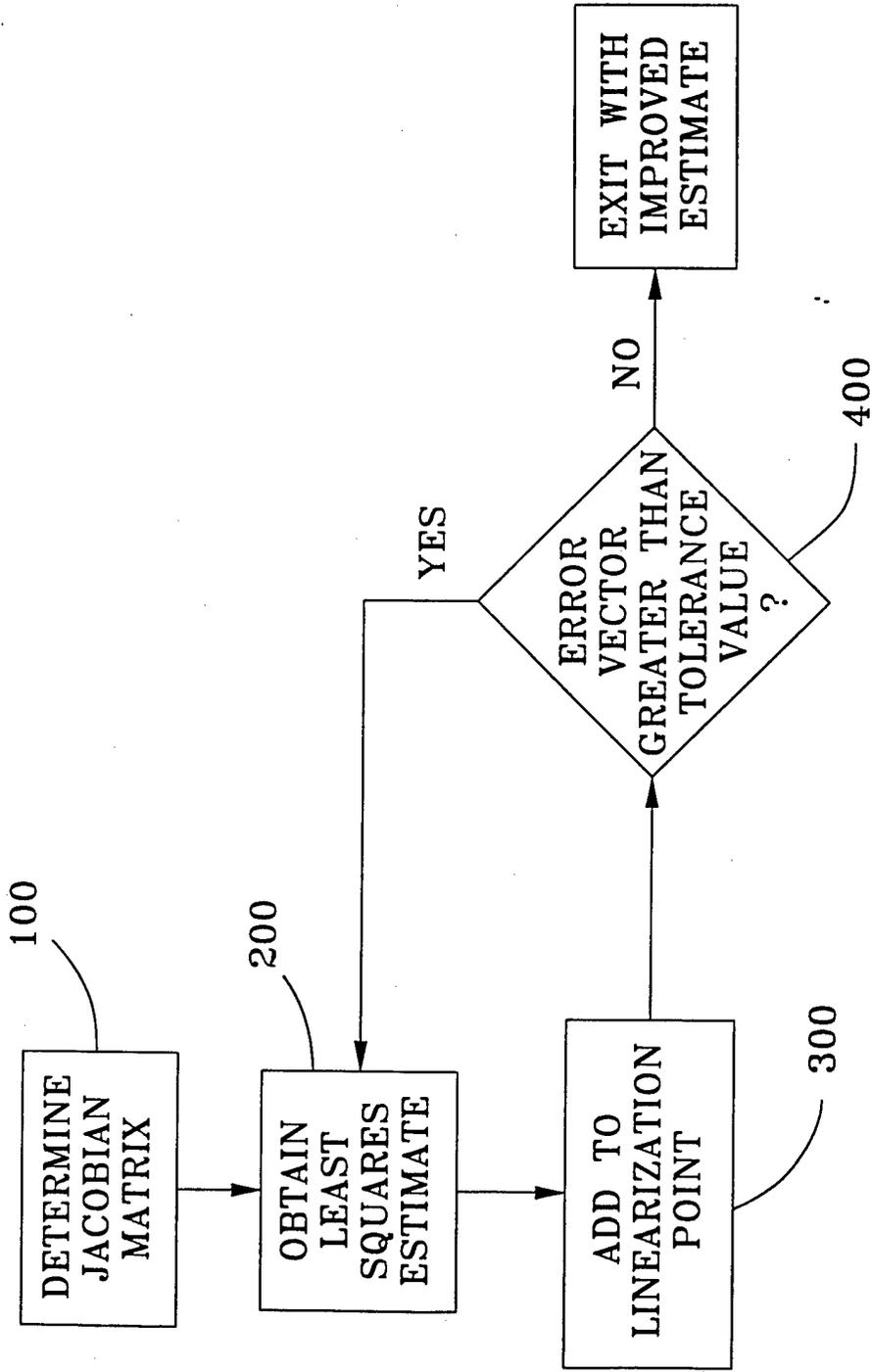


FIG. 1

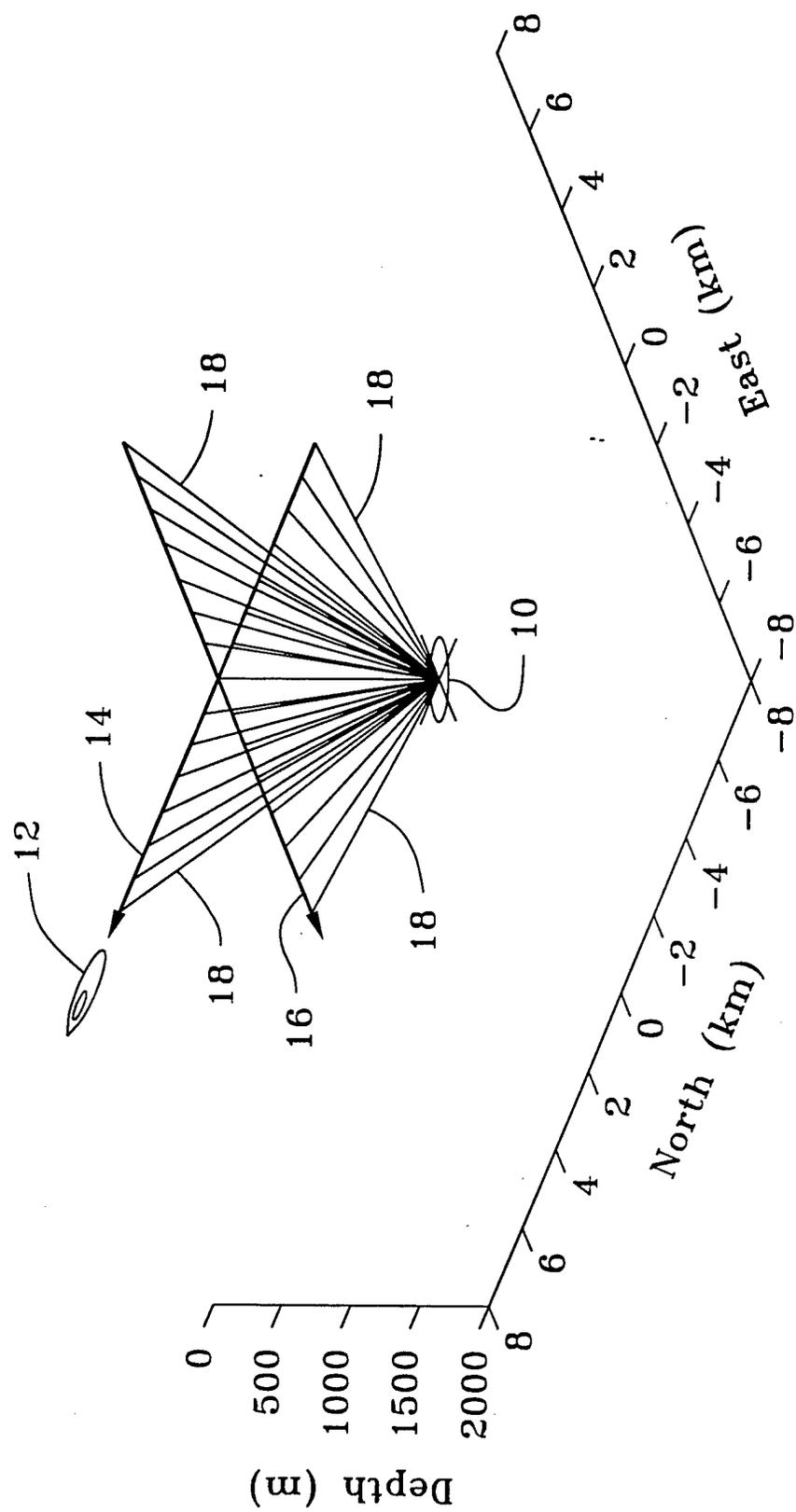


FIG. 2

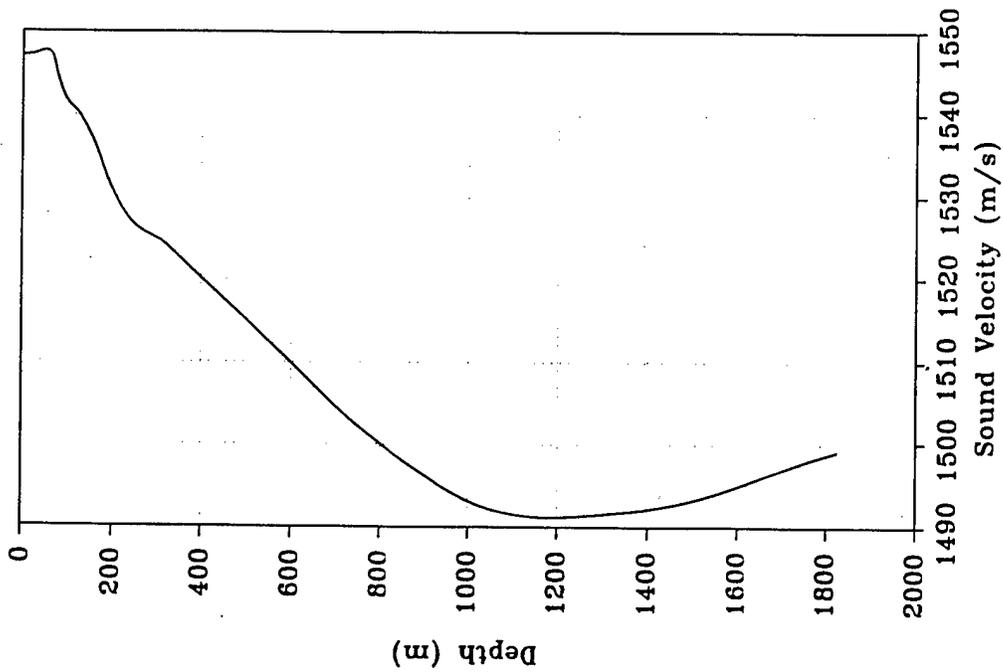


FIG. 3

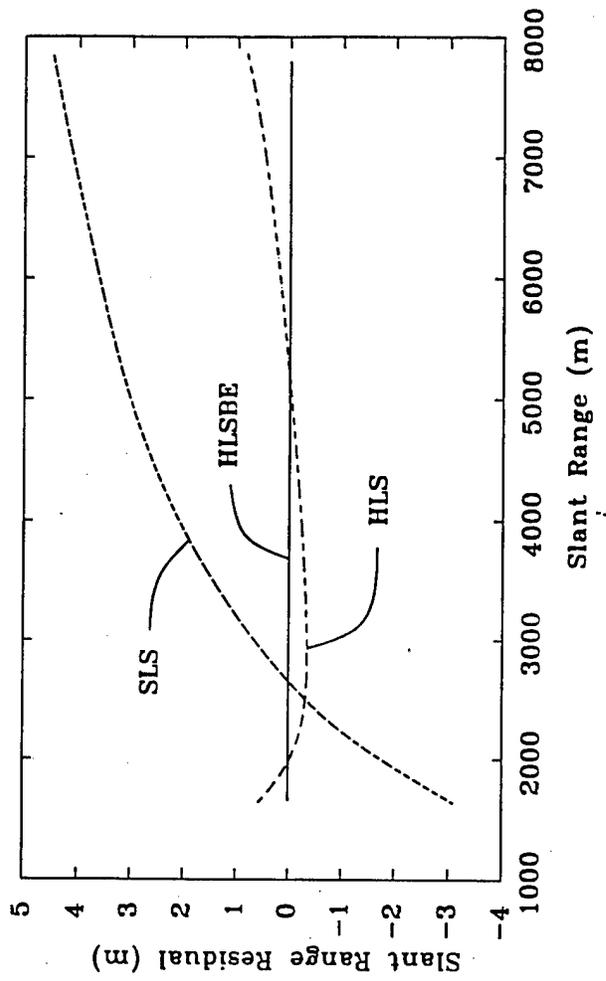


FIG. 4