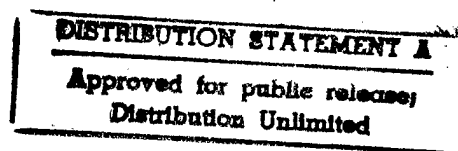Se rial Number   986,979

Filing Date   8 December 1997

Inventor   Francis J. O'Brien, Jr.

NOTICE

The above identified patent application is available for licensing. Requests for information should be addressed to:

OFFICE OF NAVAL RESEARCH
DEPARTMENT OF THE NAVY
CODE OOCC
ARLINGTON   VA   22217-5660

19980428 101

DTIC QUALITY INSPECTED 3

1   Navy Case No. 78677

2

3   METHOD FOR CHARACTERIZING

4   CONGESTION OF OBJECTS IN THREE DIMENSIONS

5

6   STATEMENT OF GOVERNMENT INTEREST

7   The invention described herein may be manufactured and used

8   by or for the Government of the United States of America for

9   governmental purposes without the payment of any royalties

10   thereon or therefore.

11

12   BACKGROUND OF THE INVENTION

13   (1) Field of the Invention

14   This invention relates to a method for providing layouts of

15   a three dimensional workplace and more particularly to a method

16   for determining the physical density or crowding of a three

17   dimensional workplace so as to optimize the layout of the

18   workspace.

19   (2) Description of the Prior Art

20   In the inventor's previous patent entitled "Process Which

21   Aids to the Laying Out of Locations of a Limited Number of 100,

22   Personnel and Equipments in Functional Organization", U.S. Patent

23   No. 5,235,506, which is incorporated into this disclosure in its

24   entirety by reference, a process is described whereby the

1   relationship among objects in a particular two dimensional work

2   area can be accurately determined to minimize crowding. A

3   crowding index, or Population Density Index (PDI), for the work

4   area, termed PDI_{act}, is calculated and compared to theoretical

5   minimum (PDI_{min}) and maximum (PDI_{max}) values, such that $PDI_{min} <$

6     $PDI_{act} < PDI_{max}$. The formula for calculating PDI_{act} is as follows:

$$PDI_{act} = \frac{1}{\overline{d_{act}}}\sqrt{\frac{n}{A}} \qquad\qquad 1$$

7   where

8   n = number of objects within the work area;

9   A = the geometric area of the work area; and

10        $\overline{d}_{act}$ = average Euclidean distance among all possible pairs

11   of n objects within the work area.

12   The values of PDI_{min} and PDI_{max} are given as follows:

$$PDI_{min} = \frac{1}{\overline{\Delta}}\frac{n}{A} \qquad\qquad 2$$

13   and

$$PDI_{max} = \frac{1}{c\overline{\Delta}}\sqrt{\frac{n}{A}} \qquad\qquad 3$$

15   where

16        $\overline{\Delta}$ = the average Euclidean distance of all possible pairs of

17   points for a unit lattice, i.e., a lattice of n points

18   uniformly distributed in area A; and

19   c = an arbitrary constant which corresponds to the minimum

20   possible spacing between the objects, e.g., personnel

1   standing shoulder to shoulder within a work area would

2   be spaced approximately one foot from head to head, so

3   c would be equal to one foot.

4 It can be seen that PDI_{min} corresponds to a uniform distribution

5 of n points within the work area, while PDI_{max} corresponds to a

6 uniform distribution of n points within the work area with a

7 minimum distance c between each horizontal and vertical point.

8 With these values determined, PDI_{act} can be optimized to provide

9 the optimum layout for the work area. Two dimensional layouts

10 suffice for a large number of work areas, such as office spaces

11 and typical manufacturing areas. However, for applications where

12 optimum layouts of three dimensional spaces are required, the

13 previous methods are inadequate. Such applications may include

14 space station layouts, robotic manufacturing facilities where

15 robots can operate in any orientation and nano technology

16 structures such as molecular machines. The conventional physical

17 science density measure of the ratio of mass to volume may be

18 used to provide an index of crowding within a three dimensional

19 space. However, this density measure fails to capture the

20 spatial relationship of objects in the space, i.e., moving

21 objects around within the space would not vary the density

22 measure.

SUMMARY OF THE INVENTION

Accordingly, it is a general purpose and object of the present invention to provide a method for optimizing the layout of three dimensional workplaces.

It is a further object of the present invention that the present invention reflect the spatial relationship of objects within a three dimensional shape.

These objects are provided with the present invention by a method for measuring and recording the relative degree of physical density, congestion, or crowding of objects dispersed in a three-dimensional space. A Population Density Index (PDI_{act}) is obtained for the actual conditions of the objects within the space as determined from measurements taken of the objects. The PDI_{act} is compared with values of PDI_{min} and PDI_{max}, considered as minimum and maximum bounds, respectively, for PDI_{act} values. The objects within the space can then be repositioned to optimize PDI_{act} for the three dimensional space and the objects contained within the space, thus optimizing the layout of objects within the space.

BRIEF DESCRIPTION OF THE DRAWINGS

A more complete understanding of the invention and many of the attendant advantages thereto will be readily appreciated as the same becomes better understood by reference to the following

4

1   detailed description when considered in conjunction with the

2   accompanying drawings wherein corresponding reference characters

3   indicate corresponding parts throughout the several views of the

4   drawings and wherein:

5   FIG. 1A depicts a volume V of 10 x 10 x 10 units with n = 8

6   objects distributed therein;

7   FIG. 1B depicts the volume V with a hypothetical maximum

8   congestion of the objects;

9   FIG. 1C depicts the volume V with a hypothetical minimum

10   congestion of the objects;

11   FIG. 2 is a flow chart of the method of the present

12   invention; and

13   FIG. 3 is a detailed flow chart for selecting a conformal

14   lattice.

15

16   DESCRIPTION OF THE PREFERRED EMBODIMENT

17   Referring now to FIG. 1A, there are shown a number "n" of

18   objects "o" dispersed in a three dimensional space, or volume

19   "V". In the schematic representation of FIG. 1A, the objects are

20   represented as solid beads which are locatable within the space

21   by ordinary means of a Cartesian coordinate system with arbitrary

22   origin such that each object has a unique coordinate measurement

23   of location (x_i, y_i, z_i) in the space. It will be appreciated

24   that objects may be mobile within the space. In such instances,

1   the spatial location can be determined by using the desired

2   location of the objects or by locating the objects at a

3   particular point in time. Preferably, V is a parallelepiped,

4   i.e., a solid figure with six faces such that the planes of the

5   faces containing the two opposite sides are parallel. In

6   instances where the objects are not contained within a

7   parallelepiped, such as on open land, sea or air, then a

8   regularly shaped figure is constructed to circumscribe the

9   objects, e.g., the tightest fitting cube or parallelepiped. For

10   purposes of illustration, V is shown in FIG. 1A as a cube having

11   sides equal to 10 units each, i.e., X = 10, Y = 10 and Z = 10.

12   To characterize the congestion or crowding of the objects

13   within the space, the system and method of the present invention

14   utilizes the fundamental characterization measure of congestion

15   called the Population Density Index (PDI). Extending the two

16   dimensional PDI_{act} of equation 1 to three dimensions, we

17   substitute the volume of the space V for the area A of the two

18   dimensional space and use a cube root to obtain:

19
$$PDI_{act} = \frac{1}{d_{act}}\sqrt[3]{\frac{n}{V}}.$$
    4

20

21   To gauge the relative degree of congestion for a given

22   dispersion of objects provided by the formula PDI_{act}, or similar

23   dispersions of the same number of objects dispersed in the same

24   space, lower and upper limits of the bounds of possible

congestability, PDI_{min} and PDI_{max}, respectively, need to be determined. Referring now additionally to FIG. 1B, to model the upper limit of PDI_{act}, we inscribe a smaller volume, V_{min}, inside the original volume V. The smaller volume has the same relative dimensions and orientation as the original volume. V_{min} is obtained by assuming that the objects are uniformly pushed together so that each consecutive pair of objects (parallel to the axes of the parallelepiped circumscribing the objects) has the minimum distance between the objects. In the illustration of FIG. 1B, the distance is taken as 1 unit. The minimum distance, however, is a function of the physical size of the objects being arranged within the volume. For example, when arranging workers within a three dimensional work volume, the minimum distance could be taken as the center to center distance of workers standing shoulder to shoulder. The minimum distance illustrated in FIG. 1B corresponds to a minimum for the average distance between objects, i.e., to the minimum value for the $\overline{d}$ term of equation 4. Referring now additionally to FIG. 1C, the lower limit of congestability is obtained by uniformly distributing the objects within the volume, V, such that each consecutive pair of objects (again parallel to the axes of the parallelepiped circumscribing the objects) has the maximum distance between each object. It is to be noted that the arrangement of objects in FIG. 1C also maintains the relative dimensions and orientation of the original volume V. Essentially, FIG. 1C is an expanded version of FIG. 1B, i.e., the spatial arrangement of the objects is the same, but the distances between the objects have

1 increased. In the illustration of FIG. 1C, each object occupies

2 a cube 5 units per side, thus the maximum distance is seen to be

3 5 units. PDI_{max} and PDI_{min} are then obtained based on the

4 arrangements of objects in FIG. 1B and in FIG. 1C, respectively.

5 For the cubic volume and objects illustrated in FIGS. 1A

6 through 1C, the determination of V_{min} and the maximum distance

7 between objects is essentially intuitive. However, to

8 accommodate real world volumes and arrangements of objects the

9 determination of PDI_{min} and PDI_{max} must be generalized. Referring

10 now to FIG. 2, there is shown a flow diagram of the steps used to

11 obtain the congestability measures, PDI_{min}, PDI_{max} and PDI_{act}, for

12 an arrangement of objects in a volume. In step 50, one records

13 the activities of the objects and determines at which point in

14 time the congestability measures will be determined. For static

15 objects, the timing will not influence the results. However, in

16 a dynamic situation, the timing should be chosen to obtain the

17 most representative arrangements of objects. In step 100, the

18 parameters n and V are measured, where n is the number of

19 objects. In order to use the method of the present invention, n

20 must be a nonprime number. Where n is a prime number, such as

21 17, 19, etc., then n is taken as the next higher nonprime number;

22 e.g., $n = 18$ would be used in place of $n = 17$. V is the size of

23 the volume containing the n objects. Typically, the volume will

24 be a rectangularly shaped figure such as a room, cage, etc. with

25 measurable sides called X (horizontal length), Y (vertical width)

26 and Z (height), as illustrated by FIG. 1A. In applications of

27 the method in which the n points are not circumscribed by a

rectangularly shaped figure, then a rectangularly shaped figure which encloses the objects is constructed in any convenient manner and this construct is used in determining the congestability measures. The measurements can be taken in any convenient units, provided that consistent units are used throughout.

In step 150, the Cartesian coordinates of each of the objects is obtained. To obtain a unique set of coordinates for each object, the coordinates of a representative point for each object are used. For example, the coordinates of an object such as a worker may be taken at a point coincident with the middle of the worker's torso. Step 200 determines the arrangement of objects corresponding to the minimum and maximum spacing of the objects as illustrated in FIGS. 1B and 1C, respectively. The arrangement can be seen to be in the shape of a conformal lattice of R rows, C columns and H in height. The objective of step 200 is to determine a unique, finite, discrete, conformal RCH lattice of points for determining the hypothetical lower and upper bounds of the congestion metrics, PDI_{min} and PDI_{max}. Referring now to FIG. 3, there is shown a flow chart of the detailed steps required to determine a conformal RCH lattice.

The selection of an RCH lattice is based on number theory concepts. In particular, we are interested in sets and subsets of composite numbers that can be expressed as three-integer products. The value of n is first expressed, at step 202, in terms of the prime factors of the whole number:

$$n = \prod_{j=1}^{r} P_j^{\alpha_j} ,$$

5

where P_j represents the jth prime number and α_j is the number of occurrences of the jth prime number of n. For example, composite 30 can be decomposed into $P_1^{\alpha_1}P_2^{\alpha_2}P_3^{\alpha_3}$ = 2 x 3 x 5. Next, step 204 derives the total number of possible *RCH* product configurations of n ($n = R$ x C x H) in order to create the set of *RCH* configurations. This can be derived by letting $\tau(n)$ represent the total number of all possible three-way configurations of a composite integer n. Then it can be shown that the number of all possible three-way configurations of a composite integer n is obtained from equation 5 by the relation:

$$\tau(n) = \frac{1}{2^r} \prod_{j=1}^{r} (\alpha_j + 1)(\alpha_j + 2).$$

6

For example, n = 30 = 2 x 3 x 5 can be partitioned into $\tau(30)$ = $\frac{1}{2^3}(2)(3)(2)(3)(2)(3) = 27$ three-integer products, listed as follows:

1x 1x 30	2x 1x 15	3x 1x 10	5x 1x 6	6x 1x 5	10x 1x 3	15x 1x 2	30x 1x 1
1x 2x 15	2x 3x 5	3x 2x 5	5x 2x 3	6x 5x 1	10x 3x 1	15x 2x 1	
1x 3x 10	2x 5x 3	3x 5x 2	5x 3x 2				
1x 5x 6	2x 15x 1	3x 10x 1	5x 6x 1				
1x 6x 5							
1x 10x 3							
1x 15x 2							
1x 30x 1							

Next, at step 206, the set of the $\tau(n)$ configurations is examined
to determine if there are nontrivial configurations, i.e., those
configurations not having a factor of 1. It can be seen that
there are six nontrivial configurations for the above listing:
2 x 3 x 5; 3 x 2 x 5; 5 x 2 x 3; 2 x 5 x 3; 3 x 5 x 2; and
5 x 3 x 2. It will be appreciated that for some numbers (e.g.,
$n = 10$), no nontrivial configurations are possible. In such
cases, step 206 branches to step 208 to increment n to the next
higher nonprime number and return to step 202. Once n yields
nontrivial configurations, step 210 selects the *RCH* lattice
configuration with dimensions most commensurate with the exterior
dimensions of volume V, i.e., the relative order of the magnitude
of the *R*, *C* and *H* values for the lattice must match the relative
order of the magnitude of the *X*, *Y* and *Z* dimensions of volume V.
For example, if V = 8 x 6 x 10 $(Z \geq X \geq Y)$, then for a given
n, the selected *RCH* lattice is of order $H \geq R \geq C$.

In step 212, the initial uniform interpoint spacing
parameter δ, is determined. δ represents the relationship
between volume V and lattice *RCH*, such that:

$$\delta = \sqrt[3]{\frac{V}{n}} = \sqrt[3]{\frac{XYZ}{RCH}} . \hspace{3cm} 7$$

Next, at step 214, test for conformity of the dimensions of the
selected lattice to volume V dimensions. If any of the R, C, H
dimensions is nonconformal, i.e., if $(R - 1)\delta > X$, or $(C - 1)\delta > Y$,
or $(H - 1)\delta > Z$, then the lattice dimensions are compressed at

step 216 by adjusting δ using the relation:

$$\delta = min[X/(R - 1), \ Y/(C - 1), \ Z/(H - 1)] - \varepsilon, \qquad 8$$

where *min* indicates taking the minimum of the quantities within the brackets and ε is a small number, such that $\varepsilon^3 \cong 0$. Steps 214 and 216 are repeated until the lattice is conformal. When commensurability is achieved simultaneously by more than one lattice configuration, step 218 repeats steps 212 through 216 for each such configuration. Step 220 compares the value δ for each configuration and selects the *RCH* configuration with the maximum value of δ. Step 222 calculates the average distance of maximum congestion, $\bar{d}_{max}$ based on the relationship:

$$\bar{d}_{max} = \bar{\Delta}(RCH) = \frac{\sum_{k=1}^{7} T_k}{n(n-1)/2}, \qquad 9$$

where

$$T_1 = CH\sum_{i=1}^{R-1}(R-i)i \ ; \qquad 10a$$

$$T_2 = RH\sum_{j=1}^{C-1}(C-j)j \ ; \qquad b$$

$$T_3 = RC\sum_{k=1}^{H-1}(H-k)k \ ; \qquad c$$

$$T_4 = 2H\sum_{i=1}^{R-1}\sum_{j=1}^{C-1}(R-i)(C-j)\sqrt{i^2+j^2} \ ; \qquad d$$

$$T_5 = 2C\sum_{i=1}^{R-1}\sum_{k=1}^{H-1}(R-i)(H-k)\sqrt{i^2+k^2} \ ; \qquad e$$

$$T_6 = 2R\sum_{j=1}^{C-1}\sum_{k=1}^{H-1}(C-j)(H-k)\sqrt{j^2+k^2} \ ; \quad \text{and} \qquad f$$

$$T_7 = 4\sum_{i=1}^{R-1}\sum_{j=1}^{C-1}\sum_{k=1}^{H-1}(R-i)(C-j)(H-k)\sqrt{i^2+j^2+k^2} \ . \qquad g$$

12

1   The equations 10a-g are derived from the geometry of a

2   lattice of maximum congestion, i.e., a lattice with an interpoint

3   distance of one unit. Equations 10a-c provide the sum of

4   distances between objects for objects lying along a line parallel

5   to an axis. For example, equation 10a gives the sum of all the

6   distances between objects for objects lying along a line parallel

7   to the X axis with the sum multiplied by the total number of such

8   strings, *CH*. Similarly, each of the equations 10d-f provides the

9   sum of distances between objects within the same plane. Equation

10   10g provides the sum of distances between objects in separate

11   planes.

12   Step 224 checks if plural maxima δ occur, and returns to

13   step 222 to calculate $\overline{d}_{max}$ for each such δ, with step 226

14   selecting the *RCH* configuration associated with the smallest

15   value of $\overline{d}_{max}$.

16   Referring back to FIG. 2, step 250 computes the average

17   distance of minimum congestion based on the relationship:

18   $$\overline{d}_{min} = \delta \overline{\Delta}(RCH),$$ 11

19   i.e., the average distance of maximum congestion as determined at

20   step 222 is multiplied by the scaling parameter δ obtained from

21   step 212 or step 216. The PDI_{min} and PDI_{max} are then determined

22   from the relationships

23   $$PDI_{min} = \frac{1}{\overline{d}_{min}} \sqrt[3]{\frac{n}{V}}, \quad \text{and}$$ 12

24   $$PDI_{max} = \frac{1}{\overline{d}_{max}} \sqrt[3]{\frac{n}{V}}$$ 13

25   at steps 300 and 350, respectively.

13

1   At step 400, the actual Population Density Index, PDI_{act}, is
2   determined using equation 4, with $\bar{d}_{act}$ being the average
3   Euclidean distance between pairs of objects as previously
4   defined, but adapted to a three dimensional volume using the
5   relation:

$$\bar{d}_{act} = \frac{\sum_{i,j}^{n} \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}}{(n)(n-1)/2} .$$

6   14

7   One can then compare the PDI_{act} for a layout of objects
8   within a volume to theoretical maximum and minimum crowding
9   measures, PDI_{max} and PDI_{min} at step 450. A decision as to whether
10   to alter the parameters of the layout or volume is then made at
11   step 500. Depending on the crowding measures determined by the
12   method, the layout can be altered for the same number of objects
13   within the same volume to either increase or decrease crowding.
14   Alternatively, the number of objects, or the size of the volume,
15   or any combination of the layout, number of objects or size of
16   the volume can be altered and new crowding measures determined.
17   The steps are repeated until an optimum layout is determined,
18   i.e., the crowding measures fall within an arbitrary optimum
19   range of values.
20   What has thus been described is a method which can be used
21   to optimize the layout of objects in a three dimensional space.
22   The method measures the locations of the objects and the
23   dimensions of a parallelepiped volume containing the objects.
24   Based on the number of objects and the dimensions of the volume,
25   a theoretical minimum crowding measure, PDI_{min}, and a theoretical

14

maximum crowding measure, PDI_{max}, are determined. An actual

crowding measure, PDI_{act}, is determined using the average

Euclidean distance between pairs of objects in the space and is

compared to PDI_{min} and PDI_{max}. The layout, volume or number of

objects can then be varied and the method repeated until an

optimum layout is determined. The method provides an improved

density better measure over the standard ratio of mass to volume

by using the average Euclidean distance between objects in

determining PDIact, thus accounting for the spatial relationship

among the objects within the volume.

The method can also provide an alternative measure of

congestion when exact coordinate measures and thus average

distance between objects cannot be obtained or are too burdensome

to determine. Once a value for δ has been determined from either

step 212 or step 216, the volume is divided into smaller volumes

of size δ^3. This can be accomplished by stringing dividing lines

between opposite faces of the parallelepiped. The number m' of

smaller volumes containing one or more objects is then counted.

The alternative measure of congestion, termed V_o, is then taken

as the ratio of m' over the number of objects n:

$$V_o = m'/n. \hspace{3cm} 15$$

While this measure V_o omits interobject distance information, it

does provide meaningful information on object congestion in

three-dimensions by taking into account relative location,

whereas the conventional measure of density ignores such

information altogether.

1   Thus, it will be understood that additional changes in the

2 details and arrangement of steps, which have been herein

3 described and illustrated in order to explain the nature of the

4 invention, may be made by those skilled in the art within the

5 principle and scope of the invention.

6

1 Navy Case No. 78677

2

3   METHOD FOR CHARACTERIZING

4   CONGESTION OF OBJECTS IN THREE DIMENSIONS

5

6   ABSTRACT OF THE DISCLOSURE

7

8 A method is provided for measuring and recording the relative

9 degree of physical density, congestion, or crowding of objects

10 dispersed in a three-dimensional space. A Population Density

11 Index (PDI_{act}) is obtained for the actual conditions of the

12 objects within the space as determined from measurements taken of

13 the objects. The PDI_{act} is compared with values of a PDI_{min} and a

14 PDI_{max}, considered as minimum and maximum bounds, respectively,

15 for PDI_{act} values. The objects within the space are then

16 repositioned to optimize PDI_{act}, thus optimizing the layout of
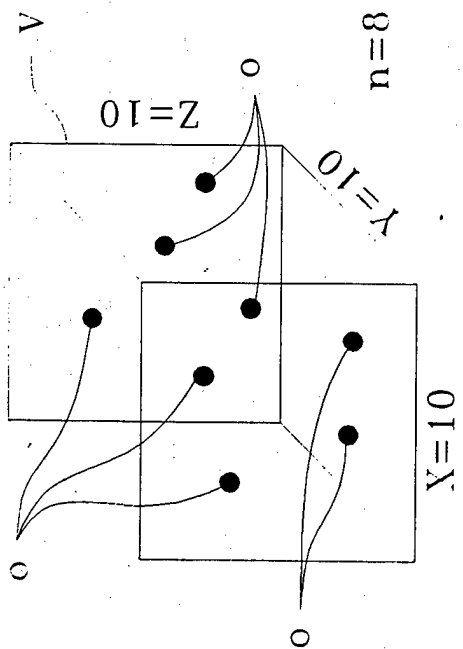
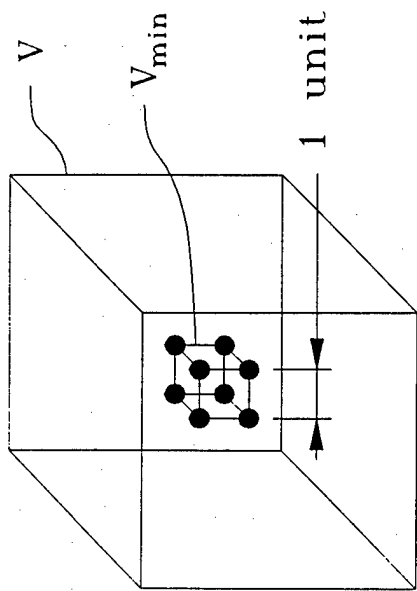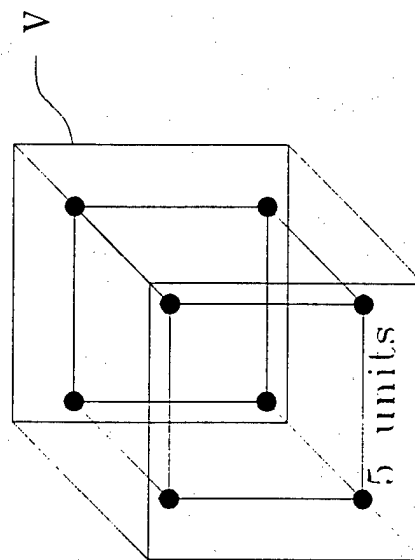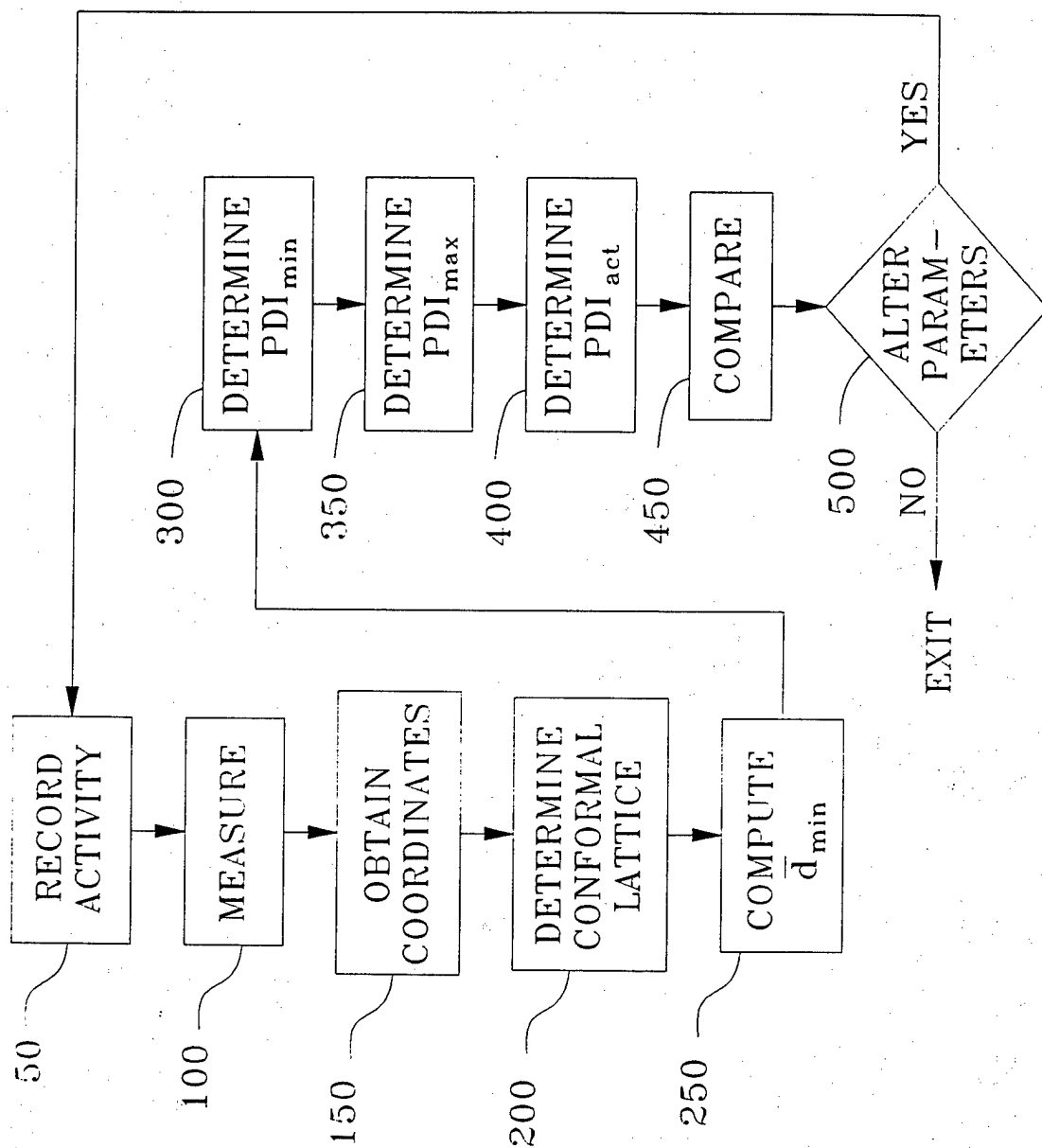17 objects within the space.

FIG. 1B

FIG. 1C

FIG. 1A

FIG. 2

FIG. 3