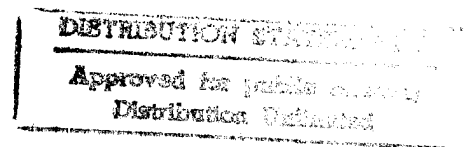


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Inventor Francis J. O'Brien, Jr.

NOTICE

The above identified patent application is available for licensing. Requests for information should be addressed to:

OFFICE OF NAVAL RESEARCH
DEPARTMENT OF THE NAVY
CODE OCCC3
ARLINGTON VA 22217-5660



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DTIC QUALITY INSPECTED 1

1 Navy Case No. 77580

2
3 SITE AND WORKSPACES LAYOUT

4 PROCESS EMPLOYING MDS AND A PDI FORMULA IN WHICH
5 DENSITY IS BASED ON AREA OF CIRCUMSCRIBING-CONVEX-HULLS
6

7 STATEMENT OF GOVERNMENT INTEREST

8 The invention described herein may be manufactured and used
9 by or for the Government of the United States of America for
10 governmental purposes without the payment of any royalties
11 thereon or therefor.
12

13 CROSS-REFERENCE TO RELATED APPLICATIONS

14 The instant application is related to two-pending U.S.
15 Patent Applications entitled SITE AND WORKSPACES LAYOUT PROCESS
16 EMPLOYING MDS AND A PDI FORMULA IN WHICH DENSITY IS CALCULATED
17 USING MEASURED SPANS OF CIRCUMSCRIBING-CONVEX-HULLS (Navy Case
18 No. 77581); and SITE AND WORKSPACES LAYOUT PROCESS EMPLOYING MDS
19 AND A PDI FORMULA IN WHICH DENSITY IS CALCULATED USING A UNIT
20 LATTICE SUPERPOSED OVER CIRCUMSCRIBING-CONVEX-HULLS (Navy Case
21 No. 77585) having same filing date.
22

23 BACKGROUND OF THE INVENTION

24 (1) Field of the Invention

25 The present invention relates to improvements in the art of
26 producing optimized layouts of objects in functional

1 organizations, including the location of building units of
2 multiple building functional facilities and location of personnel
3 work stations and equipments (collectively workplace elements")
4 in functional workspaces. In one of its aspect, the invention
5 relates to a non-metric multidimensional scaling (MDS) matrices
6 process which optimizes these layouts according to patterns of
7 inter-object operational criteria. In another of its aspects it
8 relates to the measurement of crowdedness of the objects
9 ("population density index"). In still another of its aspect it
10 relates to an application of principle of the computational
11 geometry branch of topology to the measurement of crowdedness.

12 (2) Description of the Prior Art

13 A paper, T. Tullis, B.B. Sperling and A.L. Steinberg (1986),
14 "The Use of Multidimensional Scaling for Facilities Layout: An
15 Application to the Design of the Space Station", outlines a
16 process employing the multidimensional scaling (MDS) methodology
17 to modify an experimental layout of a workspace of a naval vessel
18 from at an analytical viewpoint of optimizing performance of a
19 set of operational criteria associated with the function of the
20 workspace on the naval vessel. However, the disclosed process
21 make no provision for any consideration of crowdedness
22 (population density) in connection with the modification.

23 An abandoned U.S. Patent Application Serial No. 07/754,779
24 filed 30 August 1991 (which is accessible to the public by virtue
25 of it being referred to in. inter alia, a U.S. Patent 5,235,506
26 to F.J. O'Brien, Jr.) discloses a process for calculating a form

1 of a population density index (PDI). This abandoned application
2 goes on to make the observation that use of PDI data in MDS would
3 provide additional data for facilities layout. However, there is
4 no disclosure or teaching of how to employ PDI data with MDS.
5 Also, the form of density calculation of employed by the PDI
6 equation disclosed therein is based upon the area of the full
7 bounds of the quadrilateral the workspace in which a layout
8 configuration is located. As will become apparent, of in
9 accordance with the present invention a different form density
10 calculation is employed in the PDI equation.

11 Other related references included U.S. Patent 5,402,335 to
12 F.J. O'Brien, Jr. which discloses a process for producing
13 optimized layouts including of calculation of a non-metric PDI
14 MDS matrix (column 24, lines 29-32, therein) which is then
15 representing an MDS matrix of a normative ("best") a non-metric
16 MDS matrix of other inter-object matrices (column 24, lines 35-
17 38) are combined. As will become apparent of in accordance with
18 the present invention, one never generates a non-metric PDI
19 matrix and a totally different form interaction between PDI and
20 MDS is involved. Also, the earlier mentioned U.S. Patent
21 5,235,506, and an abandoned U.S. Patent Application 07/756,264,
22 file August 30, 1991 (but publicly available by virtue of a
23 reference thereto, inter alia, in U.S. Patent 5,235,506), each
24 include further observations that used of PDI data in MDS would
25 provide additional data for facilities layout. However, these
26 observations were also made without description of a process of

1 using PDI with MDS. As a further distinction, the PDI formula
2 disclosed in U.S. Patent 5,235,506, and that disclosed in
3 abandoned application 07/756,264 have respective limitations of
4 applicability to (I) a restricted number of objects and (ii)
5 situations where an approximate density index is acceptable.
6 Further each of the U.S. Patent 5,402,335, U.S. Patent 5,235,506
7 and abandoned application 07/756,264 disclose only forms of
8 density calculations in their PDI formula which are based upon
9 area of the full bounds of a quadrilateral workspace form, i.e.,
10 different from density calculation in PDI equation of the present
11 invention.

12

13 SUMMARY OF THE INVENTION

14 The invention provides a process for producing layouts of
15 building units, and layouts of personnel and equipment stations
16 (collectively "workplace elements") within the building units.
17 The building-unit configurations are to be located at a
18 quadrilaterally shaped facility-wide tier ("global area") and the
19 work element configurations are to be located at a
20 quadrilaterally shaped workspace tier ("subarea"). A layout
21 analyst becomes knowledgeable in or acts in concert with some
22 knowledgeable in the function of the facility and workplace
23 (single or in such concert called "expert"). The expert prepares
24 best intuitive experimental configurations of the building units
25 in the global area and workplace elements in their subarea, which
26 are termed "experts normative configuration". The analyst also

1 collects data regarding inter-building-unit and inter-workplace-
2 element operational criteria, such as for example inter-building-
3 unit-transition-frequency and shared usages of building units.
4 Using the well known non-metric MDS methodology, and separately
5 processing the global area and subarea tiers, the expert's
6 normative configuration is combined with individual matrices sets
7 of the different operational criteria producing a set of
8 normative configuration and operational criterion MDS matrices.
9 Also using well known MDS methodology and again with global area
10 and subarea tier separately processed, all the sets of normative
11 configuration and operational criterion matrices for building
12 units and workplace elements are combined, with uniform weighting
13 assigned to items of the configuration, and during the combining
14 step are interactively non-linearly stressed to produce "least
15 bad fit" configuration for performance of the operating criteria.
16 "Convex hulls" (in the mathematical sense of the term) are
17 circumscribed about the "least bad fit" and experts normative
18 configurations, respectively, and the polygon areas of the
19 interior of the hull are calculated using equation 8a, set forth
20 later in this specification. Population density indices (PDIs)
21 based upon a novel PDI formula (equation 9, later in the
22 specification) are calculated. The equation for the novel PDI
23 calculates a density related term using the above calculated
24 polygon area (rather than area bounded by the quadrilaterally
25 shaped global area and subarea, as was used in the prior art).
26 The PDI for the "least bad fit" configuration is compared with

1 the PDI for the "expert's normative" configuration to determine
2 if the "least bad fit" configuration which represents a
3 configuration presenting least stress in meeting the operational
4 criteria is acceptable in light of impact on PDI vis-a-vis the
5 PDI of the expert's experimental configuration.

6 Accordingly, the principal objects of the present invention
7 are:

- 8 (1) To provide a novel process which aids in laying out the
9 location of objects in a functional facility of objects
10 (e.g., multibuilding facility, workspace) which
11 involves both MDS methodology optimization based upon
12 takes into consideration both inter-object operational
13 criteria and consideration crowdedness (PDI).
- 14 (2) To provide a novel process as aforesaid wherein the
15 measure of PDI takes into consideration an observation
16 that human activity tends to clusters away from
17 boundaries of facility sites or workspace areas.
- 18 (3) To provide a novel process in accordance with the first
19 above said object for laying out two tiers of a
20 functional facility, namely a process tier of locating
21 building units in a quadrilateral facility site and a
22 another process tier of locating personnel workstations
23 and equipments (collectively "workplace elements") in
24 quadrilateral workspaces within a building unit.

BRIEF DESCRIPTION OF THE DRAWINGS

A more complete understanding of the invention and many of the attendant advantages thereto will be readily appreciated as the same becomes better understood by reference to the following detailed description when considered in conjunction with the accompanying drawings wherein:

FIGS. 1A-1F are flow charts of an embodiment of process for laying out multibuilding facilities and workspaces subareas within the building;

FIGS. 2, 3, 4, 5A and 5B are diagrammatics useful with respect to a hereinafter presented discussion of mathematical and logical theory underlying the measure of crowdedness in employee in the process of FIGS. 1A-1F;

FIGS. 6A and 6B are diagrammatics useful in understanding process steps 26 and 26' in FIGS. 1C and 1E respectively; and

FIGS. 7, 8, and 9A-9D are diagrammatics useful with respect to a hereinafter presented discussion of mathematical and logical theory related to one of the critical parameters PDI_{NORM} employed in calculations performed in the process depicted in FIG. 1.

DESCRIPTION OF THE PREFERRED EMBODIMENT

Reference is now made to FIGS. 1A-1F. In accordance with the present invention there is provided a process 10, FIG. 1A, comprising a first tier of the process consisting of groups of steps 12a and 12b, FIGS. 1B and 1C, and a second tier of the process consisting of groups of steps 14a and 14b, FIGS. 1D and

1 1E. Groups of steps 12a and 12b relate to producing layouts of,
2 building units (including temporary structures such as tents of a
3 facility to be located in a designated quadrilateral site
4 ("global area")). The aim of process 10 is to produce layouts
5 which are nearly optimal with respect to the functions being
6 performed in the facility (first tier) and in a workspace (second
7 tier). In both the cases of the facility and workspace there has
8 been collected data regarding operational criteria as among the
9 building units and workplace elements. The above-identified
10 paper by Tullis et al. discusses and lists operational criteria
11 useful for the first tier (process step groups 12a, 12b). They
12 include: inter-building transition frequency, sequential activity
13 dependencies, inter-building travel distance, shared support
14 functions, privacy requirements of work/living spaces and volume
15 flow among buildings. The Tullis et al paper also discusses and
16 lists individual workspace operational criteria useful for the
17 second tier (process step groups 14a, 14b). These include:
18 human transition frequency, sequential activity dependencies,
19 shared support equipment, visual/auditory interference potential
20 and privacy requirements. The total number of criteria which
21 process 10 can accommodate is unlimited. Among the types of
22 functional facilities for which appropriate facility level
23 operational criteria exist are collectible are: (1) mobile
24 military field hospitals; (2) fixed hospital compounds; (3)
25 aircraft rework depots; (4) industrial facilities for carrying on
26 complex production modes; (5) facilities involved with production

1 of nuclear energy or production of hazardous nuclear materials;
2 (6) college campuses; and (7) prison compounds. The
3 corresponding functional workspace within these functional
4 facilities for which for which appropriate workspace level
5 operational criteria exist are: (1) and (2) in hospitals-triage
6 centers, surgery centers, x-ray centers, etc.; (3) in aircraft
7 rework depots-workspaces for aircraft breakdown, component
8 overhaul, parts and tool storage, reassembly; (6) in college
9 campuses-classrooms, food service areas, labs, libraries, etc.;
10 and (7) in prison compound-cell blocks, food service areas,
11 laundry areas, etc.

12 Final actual construction of a facility, and any appropriate
13 construction and fitting out of the workspace takes place,
14 constitutes a realization tier 16, FIG. 1F of process 10.

15 Reference is now made to FIGS. 2, 3, 4, 5A and 5B. The
16 present and succeeding eleven (11) paragraphs constitute
17 discussion of the mathematical and logical theory aspects of
18 measuring or modeling (mathematical sense) two-dimensional space.
19 A computational example is also provided.

20 The conventional formula to measure or model two-dimensional
21 discrete spatial density, i.e., population density or physical
22 crowding is defined as the average number of objects (n) per unit
23 area of space (A):

24
$$D = \frac{n}{A} \quad (1)$$

1 This definition has severe shortcomings since actual spatial
2 orientation within a specified area is disregarded. As an
3 example of this shortcoming, refer to FIG. 1 which displays three
4 different configurations of objects or density points. In each
5 case, the "perceived density" of the four points is obviously
6 different. Since the number of points and area are identical in
7 each depiction, there is a constant value of .25 for population
8 density. FIG. 2 depicts geometrically the population
9 demographer's model of population density shown for the
10 distributions in FIG. 1. FIG. 2 shows that each point occupies
11 four space units (such as feet); hence, population density or
12 physical crowdedness ($D=n/A$) equals one object per four square
13 feet. FIG. 2 represents the model for each depiction of FIG. 1.
14 However, large differences in perceived physical crowding clearly
15 exist among the three configurations shown in FIG. 1.

16 See the paper, F. J. O'Brien, "A Crowding Index for Finite
17 Populations", Perceptual and Motor Skills, February 1990, 70, pp.
18 3-11, by this reference hereby incorporated herein in its
19 entirety discloses a formula to exploit the difference shown in
20 FIG. 2 for more accurately representing crowding. This is
21 accomplished by taking the actual spatial orientation of objects
22 into account. Also see US Patents 5,235,506 and 5,402,335, each
23 incorporated herein in its entirety by reference.

24 This formula, referred to as the Population Density Index
25 (PDI) (and synonymous with "Crowding Index), is as follows:

$$PDI = \frac{1}{d} \sqrt{\frac{n}{A}} \quad (2)$$

where

n = number of objects

A = the geometric area, and

d = average Euclidean distance among all possible pairs of n objects.

Basically, the above proposed formula is a generalization of the bivariate Euclidean distance formula. The derivation of the proposed density formula is patterned on the well known square-root law used in the physical sciences. It may be noted in passing that PDI has the conceptual meaning:

$$\frac{\text{Average distance of one pair of points}}{\text{Average distance of all possible pairs of points}} \quad (3)$$

Assume two objects are plotted on an X, Y Cartesian coordinate system with a fixed origin O. The mathematical distance between the two objects is measurable by simple analytic geometry using the Pythagorean distance formula:

$$d_{12} = [(X_1 - X_2)^2 + (Y_1 - Y_2)^2]^{1/2} \quad (4)$$

where (X_1, Y_1) ; (X_2, Y_2) represent each object's coordinates.

If, now, we conceive of n objects, each given coordinates within the same geometric plane such as a room, it is possible to generalize the above formula to obtain an average Euclidean distance among the n objects. The average Euclidean distance of n points, considered pairwise, is given by:

$$\bar{d} = \frac{2 \sum_{i < j} d_{ij}}{n(n-1)} \quad (5)$$

where d_{ij} is the Euclidean distance between any two objects.
Note that for $n=2$ objects, $d; d_{12}$ are equivalent.

The last step in deriving a density index is to scale \bar{d} to adjust for a given number of objects residing within a specific area. A proposed general formula based on the square-root inverse law for distances incorporating size of area and the number of objects is:

$$\Delta = \bar{d} \sqrt{\frac{A}{n}} \quad (6)$$

where

A = the geometric area in which objects reside, and
 n = the number of objects within one area.

Dimensional analysis, as well as empirical Monte Carlo simulation investigations, of Δ shows that the units are:

$$\frac{A}{\sqrt{n}}; \frac{ft^2}{\sqrt{n}} \quad (7)$$

Essentially, Δ is the average pairwise Euclidean distance among n objects scaled for a given unit area. As will become evident in the following numerical example, Δ is inversely related to the average geometric distances among n points. Calculating the reciprocal of Δ , $1/\Delta$ will make the relationship monotonically increasing, that is, the more densely packed the objects, the

1 higher the value of the index. This reciprocal of Δ , or $1/\Delta$,
2 is arbitrarily referred to as the population density index, or
3 PDI. The units for PDI are $\frac{\sqrt{n}}{A}$.

4 A computational example is provided with the aid of FIG. 3.
5 For four points, there are $4 \times 3/2 = 6$ pairwise distances to
6 calculate. The coordinate points for the 4 units are (1,1),
7 (1,3), (2,4) and (3,2). The area shown is 16 units. Applying
8 Δ , Calculating the reciprocal of Δ and multiplying by 10 to give
9 integer results, $PDI=2.3$.

10
$$\Delta = \frac{2(d_{12}+d_{13}+d_{14}+d_{23}+d_{24}+d_{34})}{4(3)} \sqrt{\frac{A}{n}} = \sqrt{\frac{16}{4}}(2.22) = 4.4 \quad (8)$$

11 The Δ index appears to be valid even when areas differ by a
12 large amount. To demonstrate this, consider FIGS. 4A and 4B.
13 The average Euclidean distances are identical (1.6) in each
14 situation depicted. The smaller value of Δ in FIG. 4A (3.7) is
15 in accord with the basic interpretation of Δ , that is, the
16 smaller the value of Δ , the more densely packed are the points
17 relative to the allowed area. The results also correspond to the
18 intuitive notion of density.

19 The proposed crowding index, Δ or PDI, should be
20 interpreted as a relative measure much like a standard deviation
21 in statistics. The theoretical mathematical minimum value of Δ

1 or PDI is always 0, a condition realizable with dimensionless
2 points but not realizable with solid objects such as people.

3 The maximum value depends on the number of objects and the
4 geometric area. Beyond three or four objects, it becomes
5 difficult and perhaps meaningless to attempt calculating a
6 precise maximum value of Δ or PDI. For these reasons,
7 hypothetical minimum and maximum bounds of the PDI formula are
8 derived below and presented as an integral component of the
9 disclosure of the last named application in the section "cross-
10 reference" above. Three additional properties derived from the
11 square-root law for average distances of Δ or PDI appear to be
12 critical to the usefulness and interpretability of the index: 1)
13 for constant area, PDI varies directly with the number of
14 objects; 2) for a constant number of objects, PDI varies
15 indirectly with area; and 3) for a constant number of objects and
16 constant area, PDI varies indirectly with distance. Small sample
17 Monte Carlo simulations performed by the inventor have supported
18 these square-root properties for the PDI formula. The values of
19 PDI computed from randomly selected uniform distributions
20 correlated .96 with the conventional formula for population
21 density (n/A). In addition, the PDI formula can be evaluated on
22 three key scientific criteria. First, the model is very simple.
23 It connects population density to three key variables -
24 distance, number of points and area - through an equation that
25 can be readily calculated. Second, the formula is justified by
26 mathematical analysis. The inverse square-root properties of the

1 index stated as conjectures are very reasonable and provide a
2 context for prediction and explanation of observed results.
3 Monte Carlo simulations support each conjecture, thereby
4 providing preliminary justification until large scale simulations
5 can be conducted. Third, the formula has been tested and
6 verified by empirical research. The use of the formula in
7 hypothetical military settings has produced results that were
8 readily interpretable and which correlated with qualitative
9 estimates of crowding made by independent expert observers.

10 The population density formula of US Patent No. 5,402,335
11 attempts to express differences such as those shown in FIG. 1
12 more accurately than the conventional population density formula.
13 Since the index can vary widely, as indicated in FIG. 1, it was
14 necessary to develop a new model to predict minimum and maximum
15 bounds of the population density index values. The model was
16 then expanded to accommodate any number of density points. Such
17 models are discussed in US Patent No. 5,402,335.

18 Referring now to FIGS. 6A and 6B, in accordance with the
19 present invention a novel form of population density index (PDI)
20 is employed in the group of facility layout process steps 12b,
21 FIG. 1C, and in the group of subarea process steps 14b, FIG. 1E.
22 This novel form of PDI is calculated using the polygon area of a
23 convex hull (the word "hull" being use in its mathematical sense
24 as a geometric structure formed by linear segments connecting the
25 perimetrical, or outer points of a configuration of points). In
26 the present discussion, wherein it is treated on a generalized

1 basis it is represented by the symbol PDI_{poly} . Hereinbelow where
2 it is discussed relative to specific forms of its in groups of
3 process steps 12b and 14b it is variously represented by the
4 symbols $PDI_{CPOSMDs}$, PDI_{NORM} , $PDI'_{CPOSMDs}$ and PDI'_{NORM} .

5 Reference is now made to FIG. 6A which represents a
6 quadrilateral area containing a configuration of five density
7 point P_1 through P_5 , representing the centeroid of building units
8 or workplace elements in the context of process 10. Referring
9 now to FIG. 6B, a convex hull having four sides, S , may be
10 visualized as an elastic band wrapped around the outer or
11 perimetrical pins of a configuration of pins occupying the
12 positions of set of point P_1 through P_5 . The Polygon area, A_{poly} ,
13 of the convex hull is calculated by the so-called Surveyor's
14 formula (which is based on known x-y coordinates of objects in a
15 two-dimensional Cartesian space) as follows.

$$16 \quad A_{poly} = 1/2[(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + \dots + (x_ny_1 - x_1y_n)] \quad (8a)$$

17 A more generalized description of the Surveyor's formula is
18 provided in V.M. Anderson and E.M. Mishall "Introduction to
19 Surveying", Article 8.26.

20 PDI_{poly} is calculated as follows:

$$21 \quad PDI_{poly} = \frac{1}{\bar{d}} \sqrt{\frac{n}{A_{poly}}} \quad (8b)$$

22 where n = the number of objects in the configuration, and
23 \bar{d} = average Euclidean distance among all possible pairs of
24 points

1 Insofar as the invention is presently understood, the
2 calculation of density aspect of the PDI formula on the basis of
3 the polygon area, A_{poly} , bounded by the convex hull takes into
4 account the actual spatial distribution of the objects and
5 thereby provides a more accurate calculation of population
6 density than simply the area of the quadrilateral space which is
7 the basis of the calculation in above identified abandoned, but
8 available to the public, U.S. Patent Application 07/754,779 filed
9 30 August 1991, and other references list in the hereinabove
10 "Description of the Prior Art" section. This conclusion tends to
11 be corroborated by a newer trend of thought in the "human
12 factors" disciplines of engineering, namely that self adjustment
13 responses to crowdedness or congestion by humans at their work
14 stations is more related to the relative position of other
15 workplace elements (personnel/equipment) than to the bounds of
16 the workplace. Stated another way, it is thought that there is
17 an intrinsic human behavior characteristic for individuals to
18 cluster together away from the boundaries of their facility or
19 workspace area, as they adapt to required operational criteria.

20 The primary density metric employed in conjunction with
21 process 10 is a normative population density index, PDI_{norm} or
22 PDI'_{NORM} depending upon whether used in group of process steps
23 12b or 14b, respectively. It is calculated by the equation:

$$24 \quad \text{PDI}_{\text{NORM}}; \text{PDI}'_{\text{norm}} = \frac{1}{d} \sqrt{\frac{n}{A_{\text{poly}}}}. \quad (9)$$

1 quadrilateral area (global area/subarea) by the layout analysts,
2 based a human intuitive response to knowledge of the function and
3 human activity occurring in the quadrilateral area. More
4 specifically the planner forms a convex hull out of the
5 perimetrical or outer points (centeroid of objects), the average
6 inter-object distance, \bar{d} , and polygon area, A_{poly} , are computed,
7 and the value produced by equation (9) is deemed the normative
8 population density index, PDI_{NORM} . Preferably, PDI_{NORM} is derived
9 from an experimental configuration which is the best available
10 human judgment, or combinations of human judgments. Typically,
11 it is selected by the concerted effort of a skilled layout
12 planner and client, or client's representative, who through
13 experience developed knowledge and/or experience involving of the
14 facility/workspace function, and has a keen awareness of inter-
15 object requirements. The person, or combination of persons,
16 providing this best human judgment is hereinafter, and in the
17 appended claims sometimes referred to as the "expert".

18 References is now made to FIGS. 7, 8, 9A and 9B. The
19 present and succeeding nineteen (19) paragraphs constitute a
20 discussion of the mathematical and logical theory relating to
21 bounds and other properties of PDI_{NORM} , including computational
22 examples.

23 As will become apparent in connection with the description
24 of the details of group of product steps 12b and 14b, PDI_{norm} is
25 a critical parameter in producing layouts of building units and
26 workplace-elements in accordance with the present invention.

1 Reference is now made to the above-identified abandoned, but
2 publicly accessible U.S. Patent Application Serial No.
3 07/756,264, filed 30 August 1991, which is hereby incorporated by
4 reference. With respect to the method of that invention,
5 firstly, it is to be understood that it is postulated with a
6 sample size of density points and geometric area of interest, the
7 points are plotted in a uniform lattice. That is, a two
8 dimensional grid, distribution, in a checkerboard arrangement
9 with every consecutive horizontal and vertical point being
10 equidistant. Secondly, using this plotted distribution, two
11 theoretical indices are calculated - a lower bound density index
12 and an upper bound density index. Thirdly, the data regarding
13 personnel and/or equipments which are the subjects of the layout
14 are collected and the population density index values are
15 calculated. Parenthetically, the latter abandoned, but
16 accessible to the public, patent application presents a
17 mathematical proof that a density metric actual PDI
18 (PDI_{act}) therein (which correspond to the "norm" PDI (PDI_{norm})
19 herein) is bounded by minimum and maximum PDIs. Fourthly, the
20 "effective inter-point distance" index is calculated based on the
21 actual population density index values, and the research findings
22 are compared to the model indices.

23 The process disclosed in that abandoned application include
24 at least two process steps involving different "models" of
25 experimental configurations. This feature of a two-step model in
26 that process allows an evaluation of the different layout

1 solutions explored as part of a given layout task, and enables
2 comparisons with solutions provided in connection with other
3 layout tasks. In the second step, the lattice or uniform
4 distribution is an effective visual aid for demonstrating how
5 population density changes with dynamic human or equipment
6 positioning being relocatable at will.

7 In the second step, the lower bound of the population
8 density index can be calculated for any uniform arrangement of
9 points. The lower bound is based on a lattice of the integers
10 called a "unit lattice". In general, a unit lattice means a
11 uniform distribution of n points in area A such that $n = A$. This
12 implies that the inter-point distance of consecutive horizontal
13 and vertical points is always equal to 1. A non-unit lattice
14 will mean that n and A are not equal. In the special cases of n
15 = 2 and $n = 3$ objects, "unit lattice" means either a unit line
16 segment ($n = 2$) or, for $n = 3$, a Euclidean equilateral triangle
17 (perimeter = 3 units), each constructed in the interior of A . A
18 linear dimension is herein designated in feet.

19 FIG. 7 depicts of a 2 X 2 unit lattice. Note that the area
20 is 4 ft^2 and that the number of points is 4, or $n = A$. The
21 horizontal and vertical distance between each of the consecutive
22 points is equal to 1. This is derived from a simple relation
23 that provides inter-point distances of lattices. Namely, if δ
24 denotes the inter-point distance, then

25
$$\delta = \sqrt{4 \frac{\text{ft}^2}{4}} = 1 \quad (10)$$

1 where δ is in feet.

2 The above example illustrates the approach that is used for
3 approximating inter-point distances for any lattice of n points
4 uniformly distributed in area A . The general formula to do this
5 is given by:

$$6 \quad \delta = \sqrt{\frac{A}{n}} \quad (11)$$

7 The next step, calculating the average Euclidean distance of
8 all possible pairs of points in FIG. 5, is given by:

$$9 \quad \bar{\Delta} = \frac{1+1+1+1+\sqrt{2}+\sqrt{2}}{6} = 1.14 \quad (12)$$

10 The average Euclidean distance for a unit lattice is called
11 $\bar{\Delta}$ to distinguish it from the general Euclidean distance given by
12 \bar{d} in the general population density index formula:

$$13 \quad PDI = \frac{1}{\bar{d}} \sqrt{\frac{n}{A}} \quad (13)$$

14 The reference parameter "lower bound estimate of the index
15 for a non-unit lattice" can now be derived. (Note that this
16 reference parameter is a hypothetical concept). It will be
17 appreciated that this is prior to conducting the density
18 analysis. At that time, an analyst involved with density of a
19 layout will have collected data providing a knowledge of the
20 number of points in the area, but the expected lower bound
21 Euclidean distance will not be known. The known average inter-
22 point distance relationship of equation:

$$\delta = \sqrt{\frac{A}{n}} \quad (14)$$

is used to calculate the average Euclidean distance for any non-unit lattice with the same number of points as the unit lattice distribution. The formula to calculate the average Euclidean distance associated with the minimum density (\bar{d}_{\min}) is given in the following equation:

$$\bar{d}_{\min} = \bar{\Delta} \sqrt{\frac{A}{n}} \quad (15)$$

That is, each coordinate point in the unit lattice is scaled by a constant, equal to δ in equation:

$$\delta = \sqrt{\frac{A}{n}} \quad (16)$$

to calculate the lower bound of the average Euclidean distance.

Thus, the lower-bound model of the population density index is obtained by replacing d in the general population density index formula:

$$PDI = \frac{I}{d} \sqrt{\frac{n}{A}} \quad (17)$$

by \bar{d}_{\min} in equation:

$$\bar{d}_{\min} = \bar{\Delta} \sqrt{\frac{A}{n}} \quad (18)$$

and simplifying. The result is the lower bound (PDI_{\min}):

$$PDI_{\min} = \frac{I}{\bar{\Delta}} \frac{n}{A} \quad (19)$$

1 To provide an example of calculating the population density
2 index lower-bound (PDI_{\min}), assume a study is being conducted on
3 12 people. Also assume the area involved in the study is a
4 square with area of 25 ft^2 . There are two logical choices in the
5 selection of a unit lattice of 12 points: 4X3 and 6X2 or
6 equivalently 3X4 and 6X2. A unit lattice of 4X3 or 3X4 will
7 provide an excellent estimate of the lower-bound value of the
8 index for this square. The only other logical choice for a
9 uniform distribution of 12 points is the unit lattice 6X2 or 2X6;
10 however, with this choice, the 12 points cannot be accommodated
11 into a 5-ft.X5-ft. area with inter-point distance $\delta=1.4$. FIG. 8
12 shows the 4X3 unit lattice. $\bar{\Delta}$ can be calculated using equation
13 46, below. Thus, the lower bound population density index based
14 on equation:

$$15 \qquad PDI_{\min} = \frac{1}{\bar{\Delta}} \frac{n}{A} \qquad (20)$$

16 is $(1/1.90) (12/25)$, which is approximately equal to 0.25. The
17 population density index can be no smaller than 0.25 units in a
18 lattice distribution consisting of 12(4X3 unit lattice) points
19 and area 25 ft^2 with inter-point distance of about 1.4.

20 The calculation of the upper bound of the index is based on
21 a further assumption. It is assumed that there is a minimum
22 (non-zero) inter-point distance between any two neighboring
23 points in a uniform distribution corresponding to some practical
24 lower limit of elbow room allowable between persons. The
25 distance value selected will correspond to a lattice distribution

1 that produces the maximum population density index value for the
2 given number of objects and area in a layout task. Selecting the
3 minimum inter-point distance is empirical. To exemplify the
4 derivation, assume that 1 ft is the minimum value. As is widely
5 accepted by persons skilled in the art to which the present
6 invention pertains, this distance might correspond to the nose-
7 to-nose distance of two persons positioned shoulder-to-shoulder.
8 Other values of minimum inter-point distance, appropriate to the
9 circumstances of the functional organization involved in a given
10 task at hand of laying out objects, are to be selected by the
11 layout analyst employing this invention. Then the task worker
12 computes it as a reasonable value to choose based on the known
13 opinion of experts. Other values could be chosen by this analyst
14 who could derive personally upper bounds for the population
15 density index using the derivation that follows.

16 The assumption that 1 ft is the practical minimum inter-
17 point distance translates into δ of $\delta = (A/n)^{1/2}$ being set to 1.

18 From this, it follows from the generalized average Euclidean
19 distance equation:

20
$$\bar{d}_{\min} = \bar{\Delta} \sqrt{\frac{A}{n}} \quad (21)$$

21 that the upper bound of the average Euclidean distance (\bar{d}_{\max}) is:

22
$$\bar{d}_{\max} = \bar{\Delta} \sqrt{\frac{A}{n}} = \bar{\Delta} \quad (22)$$

23 Substituting equation:

$$\bar{d}_{\max} = \bar{\Delta} \sqrt{\frac{A}{n}} = \bar{\Delta} \quad (23)$$

into the general population density index formula:

$$PDI = \frac{1}{\bar{d}} \sqrt{\frac{n}{A}} \quad (24)$$

gives the upper bound:

$$PDI_{\max} = \frac{1}{\bar{\Delta}} \sqrt{\frac{n}{A}} \quad (25)$$

Equation

$$PDI_{\max} = \frac{1}{\bar{\Delta}} \sqrt{\frac{n}{A}} \quad (26)$$

gives the expected reference parameter "upper bound of the population density index in a lattice" distribution (which is a hypothetical concept) assuming a 1-ft distance as the practical minimum value of inter-point elbow room. From the earlier example where $n = 12$ objects and area = 25 ft², PDI_{\max} is equal to:

$$PDI_{\max} = (1/1.90)(12/25)^{1/2} = 0.36. \quad (27)$$

That is, density can be no larger than .36 units when the 12 density points are distributed uniformly with a one foot distance between each horizontal or vertical point.

In general, if the selected inter-point distance is some arbitrary constant c , then PDI_{\max} is:

$$PDI_{\max} = \frac{1}{c\bar{\Delta}} \sqrt{\frac{n}{A}} \quad (28)$$

1 For example, if 1 in. is the selected value for c, then the
2 maximum population density index value is

3
$$PDI_{\max} = 12(1/1.90)(12/25)^{1/2} = 4.38. \quad (29)$$

4 The final index provided by the model is called the actual
5 effective inter-point distance or δ_{eff} . The effective inter-
6 point distance index translates the clustering of n points
7 observed in an actual study into a lattice distribution for which
8 an hypothetical inter-point distance, or effective inter-point
9 distance can be determined and compared with the theoretical
10 maximum inter-point distance of uniform dispersions in non-unit
11 lattices provided by equation:

12
$$\delta = \sqrt{\frac{A}{n}}. \quad (30)$$

13 The utility of this comparison resides in the fact that δ_{eff}
14 varies in accord with the relation $1 \leq \delta_{\text{eff}} \leq \delta$.

15 The effective uniform - distance index can be derived as
16 follows. A generalized version of equation (30) provides the
17 average Euclidean distance for any lattice. For \bar{d}_{\min} equation (22)
18 is calculated directly. For \bar{d}_{\max} , $(A/n)^{1/2}$ is set equal to some
19 hypothetical constant (such as 1). Now, a reference parameter
20 "inter-point distance" such as $\delta = (A/n)^{1/2}$ can be conceived
21 theoretically as a variable number for any population density index
22 value in a study with n persons and area A. An inter-point
23 distance such as δ is strictly determined by the average Euclidean
24 distance and vice versa. Hence, the following equation expresses

1 the hypothetical relationship between the actual Euclidean distance
2 d_{act} of an observed population density index value and another
3 reference parameter δ_{eff} .

$$4 \quad \bar{d}_{act} = \bar{\Delta} \delta_{eff} \quad (31)$$

5 solving for δ gives:

$$6 \quad \delta_{eff} = \frac{\bar{d}_{act}}{\bar{\Delta}} \quad (32)$$

7 From the earlier discussion, it is obvious that δ_{eff} expresses the
8 ratio of the actual PDI relation to the maximum PDI and expresses
9 it in a average inter-point distances. Equation(32) can be stated
10 explicitly in terms of the norm population density index (PDI_{norm})
11 of the general population density index formula (equation (33)
12 hereinabove):

$$13 \quad \delta_{eff} = \frac{1}{PDI_{norm} \bar{\Delta}} \sqrt{\frac{n}{A}} \quad (33)$$

14 Either equation (32) or equation (33) provides the effective inter-
15 point distance in a population density study. To illustrate the
16 concept of effective inter-point distance, assume that in the
17 example described earlier with 12 persons in an area of 25 ft²
18 (refer to FIG. 9C), that the calculated norm PDI value (PDI_{norm}) is
19 0.30. If a uniform distribution of the 12 points is constructed,
20 the inter-point hypothetical uniform distance that preserves the
21 actual average Euclidean distance δ_{eff} based on equation (33) is
22 equal to $(1/0.30)(1/1.90)(12/25)^{1/2} = 1.2\text{ft.}$

1 That is, a population density index value of 0.30 means that
2 the 12 persons can be arranged theoretically in a uniform
3 distribution such that the hypothetical inter-point uniform
4 distance is about 1.2 feet. This value can be compared with

$$5 \qquad \delta_{\text{eff}} = \sqrt{\frac{25}{12}} \approx 1.4 \qquad (34)$$

6 which corresponds to the hypothetical inter point maximum uniform
7 dispersion of 12 persons in a 25 ft² area. When this is done, the
8 percent increase in density associated with a population density
9 index of 0.30 is approximately 20 percent (1.2 vs 1.4ft). In
10 summary, δ_{eff} gives a useful (visual) mathematical summarization of
11 discrete spatial density, translated into terms of uniform inter-
12 point distance language. FIGS. 9A-9D are diagrammatic depictions
13 representing the employment of actual effective inter-point
14 distance δ_{eff} as a model to visualize the translation of
15 mathematical summarizations of discrete density into terms of
16 inter-point distances. To collect all of the information contained
17 in the model, these four figures are presented as a summary for the
18 example employing 12 persons within an area of 25ft². The
19 assumptions and findings for this example were as follows. First,
20 n was selected as 12 and the area was 25 ft². Secondly, a unit
21 lattice of 4X3 was determined to be appropriate for the calculation
22 of the unit lattice Euclidean distance $\bar{\Delta}$ (1.90, see FIG. 8). In
23 the third step of the model, lower and upper bounds of the
24 population density index were calculated to be 0.25 and 0.36 units,
25 respectively. The lower and upper bounds of the index were shown

1 to be describable in terms of uniform inter-point distances. In
2 the examples, those values are 1.4 (FIG. 9A) and 1.0 (FIG. 9B) feet
3 for the lower and upper bounds, respectively. It was then
4 demonstrated how to translate the PDI_{norm} value into a uniform
5 inter-point distance using equation (11). The data points with a
6 PDI_{norm} value of 0.30 (FIG. 9C) were then translated into a uniform
7 distribution of points ("effective distance" which as calculated to
8 be 1.2 feet (FIG. 9D).

9 The present and succeeding paragraphs describe the functions
10 that a computer program using non-metric multidimensional scaling
11 (MDS) algorithms perform in the practice of process 10. The inputs
12 to an MDS algorithm program are preferably a single matrix
13 representing an initial experimental, two dimensional, Cartesian
14 plot of a configuration of building units located in the
15 quadrilateral global area selected to receive the building units of
16 a facility, and a set matrices representing degrees of association
17 among all possible pairs of these building units for various inter-
18 building operational criteria associated with the function the
19 facility performs. (For examples of such operational criteria were
20 refer to the earlier overview description of process 10 which was
21 provided in connection with FIGS. 1A-1F). Briefly, the MDS
22 algorithm computer program performs a transformation of the initial
23 experimental configuration of the building units into an output
24 Cartesian plot of a configuration of the building units which is
25 nearly optimized in its composite performance of the various
26 inputted operational criteria. A figurative analogy (simplified

1 only one set of operational criteria data) would be the MDS
2 algorithm receiving as it inputs someone's best guess of a two-
3 dimensional Cartesian plot of places in city which are frequent
4 pairs of termini of taxi trips, and a table of amounts of fare paid
5 for such trips (operational criteria data). Using the experimental
6 plot and the data, the MDS algorithms would convert even a grossly
7 distorted initial inaccurate plot into an output plot with accurate
8 relative geographic locations of the places.

9 As earlier mentioned it is preferred that the initial
10 experimental configuration of building units in the quadrilateral
11 designated facility site be produced based on an intuitive
12 "expert's" (as defined above) response to knowledge of the function
13 served by the facility. It is to be appreciated that optionally
14 the MDS algorithm could function with plural different experiment
15 plots being processed and/or with different set of operational
16 criterion data that are poor or even arbitrary. The data of the
17 experimental configuration and the sets operation criteria data
18 must be conventionally translated into compatible matrix sets.
19 Appropriate inter-building units operational criteria data is
20 identified from an analysis of the facility's function, and then
21 collected by such techniques as extraction from records of facility
22 operation, desk audits, time and motion type studies and time-lapse
23 photography. Each set of data is translated into matrix
24 expressions of degrees of association of the individual operational
25 criteria between all possible pairs of building units of the
26 configuration. The set of building units of configuration are

1 preferably uniformly weighted, and represented in the plot as
2 symbolic points (zero dimensional abstract points).

3 The MDS software algorithms individually combine the
4 experimental configuration matrix of data with each respective
5 matrix representing a set of inter-building unit degrees of
6 association of a operational criteria, producing a set of non-
7 metric, building-unit and operational criterion MDS matrices. The
8 MDS algorithm program then combines all the matrices of this set
9 into a composite-facility-operational-criteria-Cartesian-
10 configuration-of-building units; preferably using a non-linear,
11 least-square stressing function. The algorithm in effect follows
12 a progression of matrices operations starting with an operation
13 upon adjacent matrices, an then operation on compatibility matrices
14 and finally an operation upon similarity or "distance" matrices.
15 What is happening is a measurement of a "badness of fit" interacts
16 with a measure of composite operational criteria, in a way that
17 causes a decrease of the stress function. "Distances" (as
18 mentioned earlier in an abstract senses) are taken to be the
19 measures or metrics of the operational criteria, in such a way that
20 the highest matrix score for any pair of objects represents a
21 strong requirement that they be placed close together, and the
22 lowest score indicates least importance in proximate locations.
23 The MDS algorithm software is applied first for building units in
24 the global area for grouped process steps 12a and 12b, and then for
25 workplace elements in one or more subareas within the building
26 units, groups of process steps, 14a and 14b.

1 Typically, after between 20 and 50 iterations, a suitable
2 stopping criteria related to gradient of stress decrease and/or
3 magnitude of the gradient is reached. At that time the then
4 coordinates of the objects represent a near optimum spatial layout
5 from the viewpoint of operational criteria, and a printed plot of
6 the global area or subarea showing the machine recommended location
7 of each object is presented for consideration. The final solution
8 is one that has a minimum "stress value".

9 Any of various well known of-the-shelf computer programs for
10 implementing MDS algorithms may be employed. The Matrix Laboratory
11 (MATLAB) software product can be conventionally adapted for MDS
12 algorithms. It is produced by Matworks, of Cambridge Massachusetts
13 is one suitably set of programs. A well known existing package
14 program is "KYST" which is described and discussed in the below
15 identified papers of Kruskal.

16 Further details regarding the methodology of employing MDS
17 algorithm computer program see A.T. Siegel, J.J. Wolf and J.
18 Piliotis (1982), "A New Method for the Scientific Layout of
19 Workspace". Applied Ergonomics, 18(2) 87-90; Kruskal, Non-Metric
20 "Multidimensional Scaling; A Numerical Method", Psychometrika, Vol.
21 29, No 2, June 1964; and Kruskal "Multidimensional Scaling by
22 Optimizing Goodness of Fit to a Non-metric Hypothesis:",
23 Psychometrika, Vol 29, No. 1 (1964), all of which are by these
24 present references incorporated herein in their entirety.

25 This plot serves as preliminary solution layout so that the
26 layout planner may then, at the planner's discretion, make a series

1 of minor manual adjustments to the computer-generated solution.
2 Such adjustments may be desirable to compensate for special needs
3 such a repair access or for physical constraining features of the
4 area such as the location of posts, stairways, and doorways, in the
5 existing area. Accordingly, the multistage method of the step two
6 in the exemplary description provides for the exercise of
7 workspace-user judgments as well as consideration of work imposed
8 conditions. Accordingly, this regard process 10 is computer aided,
9 rather than computer generated. The computer offers an initial
10 solution followed by a manual adjustment.

11 In summary, the MDS methodology reveals the underlying
12 structures in data sets and then presents them in a graphic format,
13 i.e., a geometric configuration/mapping, suitable for visual
14 analysis and interpretation.

15 With this background, the sequence of steps in the practice of
16 process 10 will be described in connection with flow charts of
17 process steps in FIGS. 1A-1F. In steps 18, 20 and 22, FIG 1B,
18 selection of the quadrilateral global area, set of building units
19 generation of an experts normative configuration of the building
20 units in the global area, and the collection of operational
21 criteria data are performed, or obtained, by the layout analyst.
22 In step 24, a set of non-metric, multidimensional scaling (MDS),
23 building units and operational criterion matrices are generated
24 following the MDS algorithm methodology (as hereinbefore described
25 in detail), and in step 24 this set of matrices is combined into 4
26 composite-facility-operational-criterion-Cartesian-configuration-

1 of-building units (again following MDS algorithm methodology
2 hereinbefore described in detail). In step 26, convex hulls are
3 circumscribed about both (i) the expert's normative experimental
4 configuration and (ii) the facility-operational-criteria-
5 configuration-of-building units and each respective polygon areas,
6 A_{poly} , of the polygon within the respective convex hulls is
7 calculated using Equation 8a. In step 28 calculation of composite-
8 facility-operational-criteria-MDS-matrix-population-density-of-
9 building units, $PDI_{CPOSMDS}$, is performed using equation 9. This
10 index constitutes a measure of the crowdedness of the resultant
11 configuration of building units after MDS methodology repositioned
12 the locations of the building units to provide least stress in
13 enabling the facility to compositely performance the operational
14 criteria. Also as part of step 28, a corresponding normative-
15 population-density-index PDI_{NORM} of the initial experts
16 configuration of the building units is calculated using equation
17 8b. In step 30 $PDI_{CPOSMDS}$ is compared with PDI_{NORM} , providing the
18 layout analyst with a qualitative indication of the impact upon
19 crowdedness of the readjustment of the experts initial
20 configuration by MDS methodology. The higher the value of PDI
21 index the more crowdedness exist in the configuration. The analyst
22 can thereby quantitatively consider whether the facility-
23 operational-criteria-building units-configuration is acceptable
24 relative to the expert's normative experimental configuration by
25 observing the ratio of the values. For example, if $PDI_{CPOSMDS}$ is
26 one-half of the PDI_{NORM} , the resulting ratio 2.0 would indicate the

1 MDS methodology not only reduced stress in the performance of the
2 operational criteria, but also significantly reduced crowdedness.
3 With these circumstances it is likely that MDS methodology induced
4 configuration would be acceptable to the layout analyst's client.
5 If desired, an arbitrary low threshold value of the ratio may be
6 used and the step automated. Facility operational-criteria-
7 building units-configurations having unsatisfactory $PDI_{CPOSMDs}$
8 indices result in discontinuing the MDS layout process, step 32.
9 Where configuration is acceptable optional manual adjustments are
10 made, as for example, to overcome physical constraints, step 34.

11 Passing to the tier of process 10 for the layout of workplace
12 elements (personnel workstations and equipments) in one or more
13 quadrilateral subareas of a building units or units, process steps
14 18', 20', 22', 24', 26', 28', 30', 32' and 34', FIGS. 1D and 1E are
15 performed utilizing workspace operational criteria data instead of
16 facility operation criteria.

17 In the event the MDS methodology induced facility layout and
18 one or more workplace layout are confirmed by PDI_{poly} comparison,
19 the project effort may proceed to ultimate construction of the
20 building units in locations within the global area and construction
21 and fitting out of workplace elements with one or more subareas,
22 step 36, FIG 1F, in accordance with facility and workspace
23 operational-criteria-building units-and-workplace-elements
24 configuration solutions provided by process 10.

25 It will be appreciated by those skilled in the art that
26 various process step or combinations of process step in addition

1 to those identified with the MDS algorithm methodology can be
2 implemented by using a suitably programmed general purpose
3 computer.

4 Many modifications of the presently disclosed invention will
5 become apparent to those of skill in the art without departing
6 from the inventive concepts.

1 Navy Case No. 77580

2
3 SITE AND WORKSPACES LAYOUT

4 PROCESS EMPLOYING MDS AND A PDI FORMULA IN WHICH
5 DENSITY IS BASED ON AREA OF CIRCUMSCRIBING-CONVEX-HULLS
6

7 ABSTRACT OF THE DISCLOSURE

8 A process is provided for producing layouts of building
9 units on a quadrilateral facility site, and layouts of personnel
10 workstations and items of equipment (collectively "workplace
11 elements") in quadrilateral subarea in the building units. There
12 are inter-building-unit, and inter-workplace-element, operational
13 criteria associated with the activity being performed in the
14 facility. The well known multi-dimensional scaling (MDS)
15 methodology is employed in optimizing building unit and workplace
16 element configurations to suit the operational criteria.
17 Measurement of population density index (PDI) is employed to
18 judge whether candidate configurations of building units and
19 workplace elements result in adverse crowdedness conditions. The
20 PDI employed for this purpose is novel. The novelty of the PDI
21 is its use of a "convex hull" (in the mathematical sense of the
22 term) circumscribed about the perimetrical objects in a
23 candidate configuration. The equation for the PDI then employ
24 the polygon interior of the convex hull in calculation of a
25 density related term of the equation.

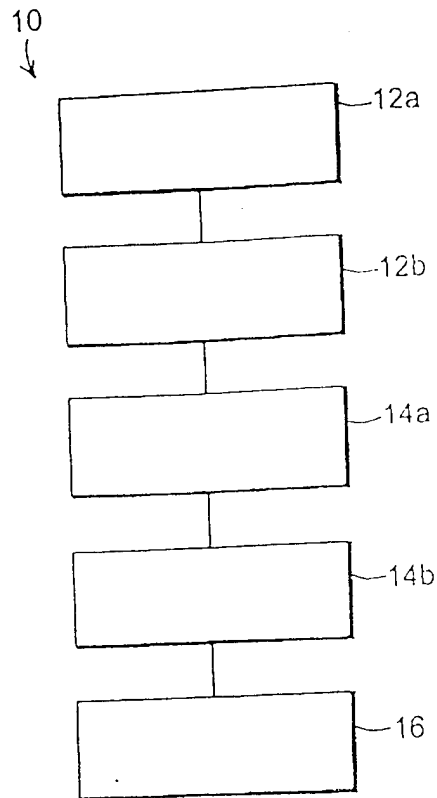


FIG. 1A

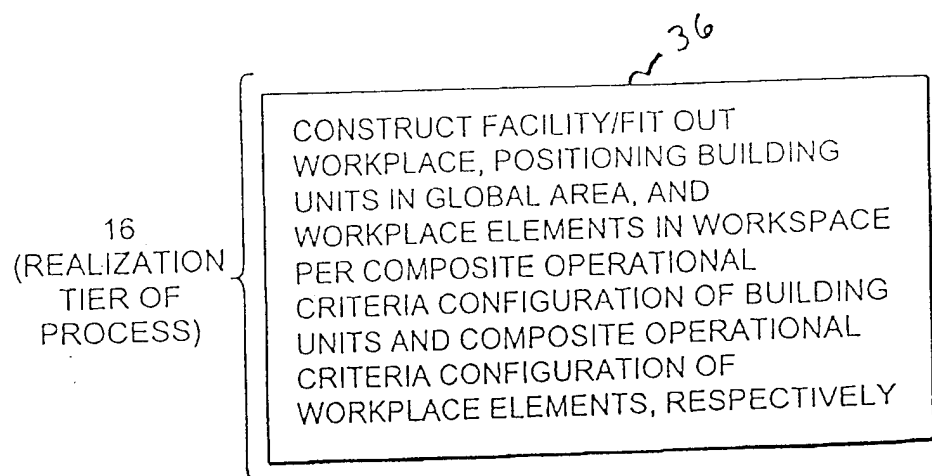


FIG. 1F

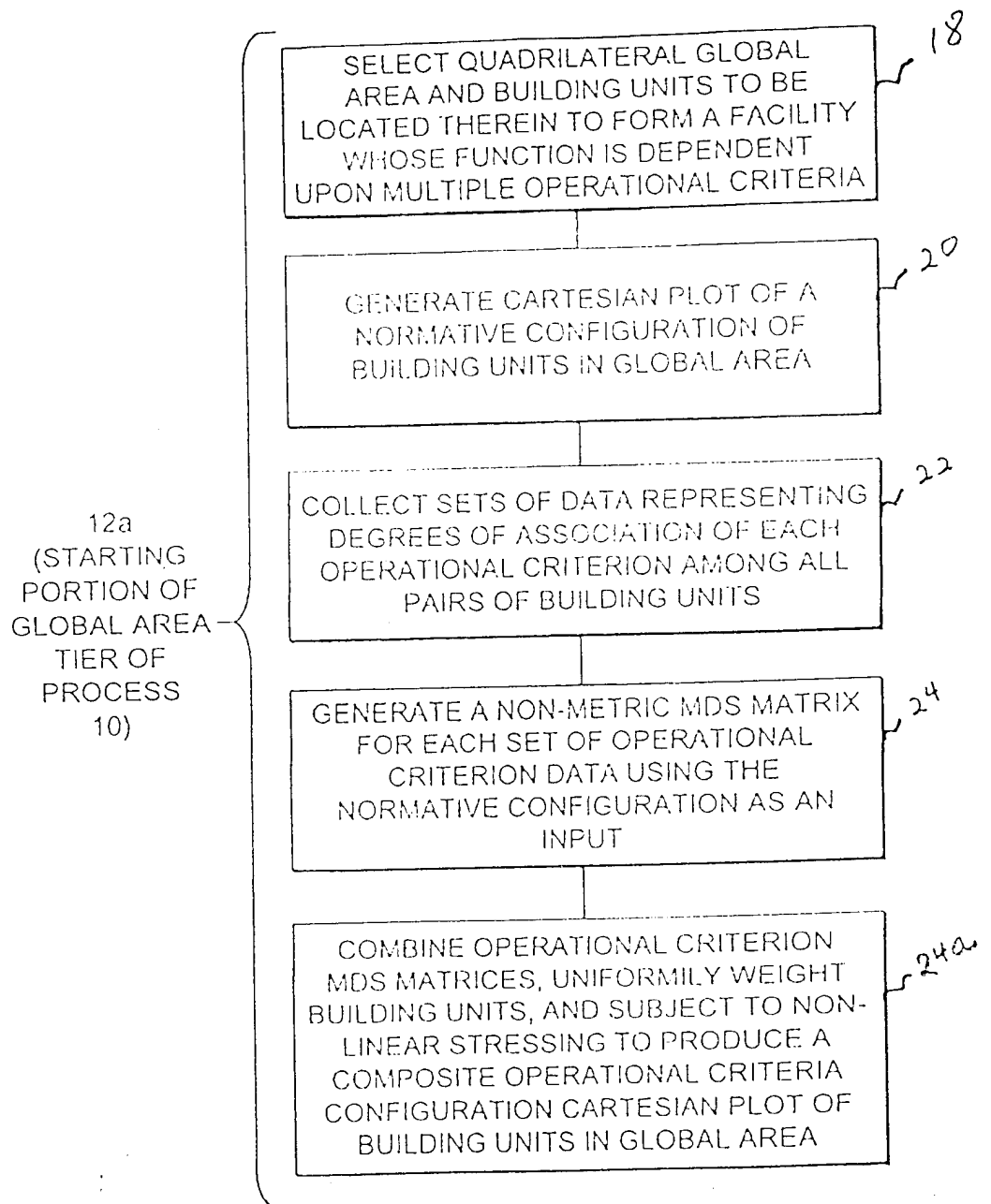


FIG. 1B

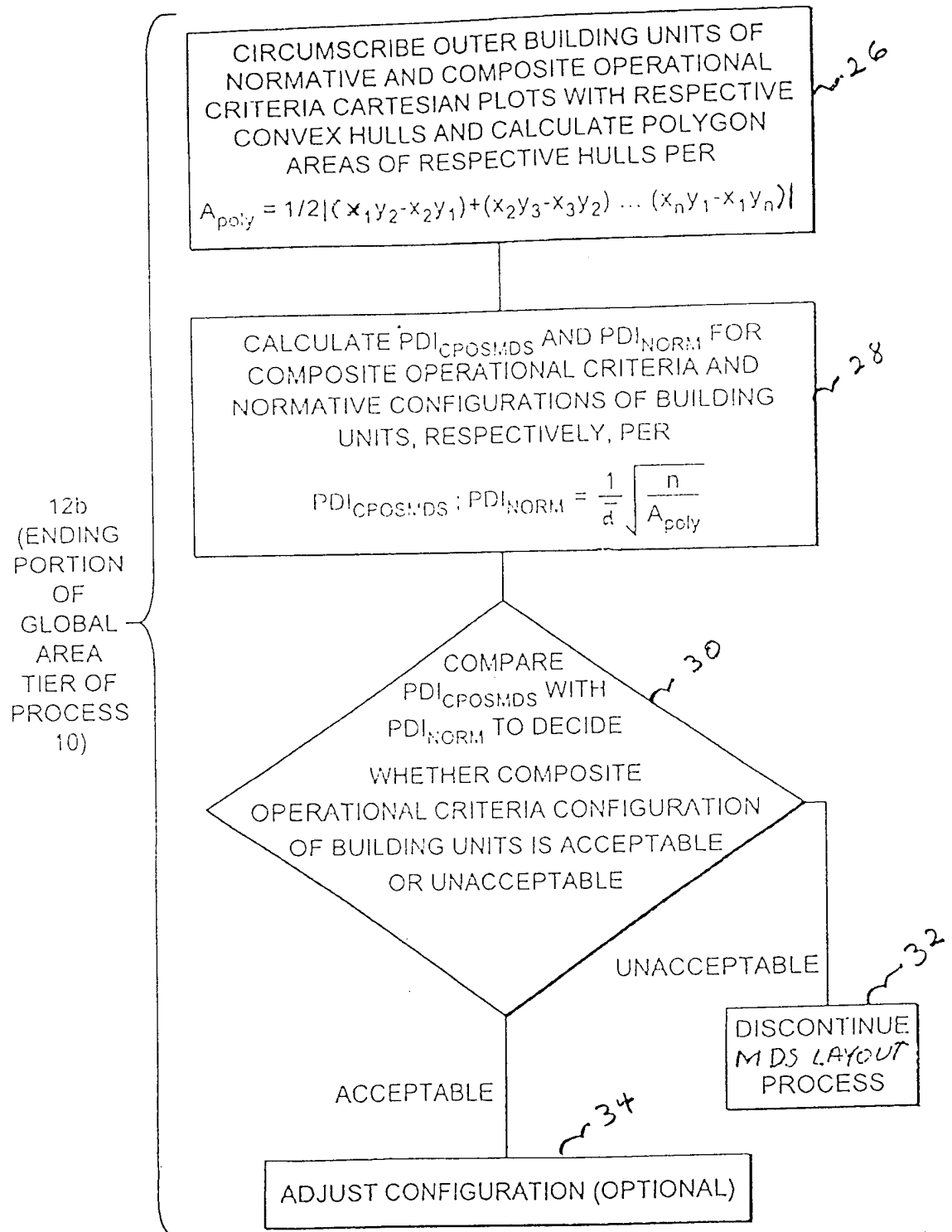


FIG. 1C

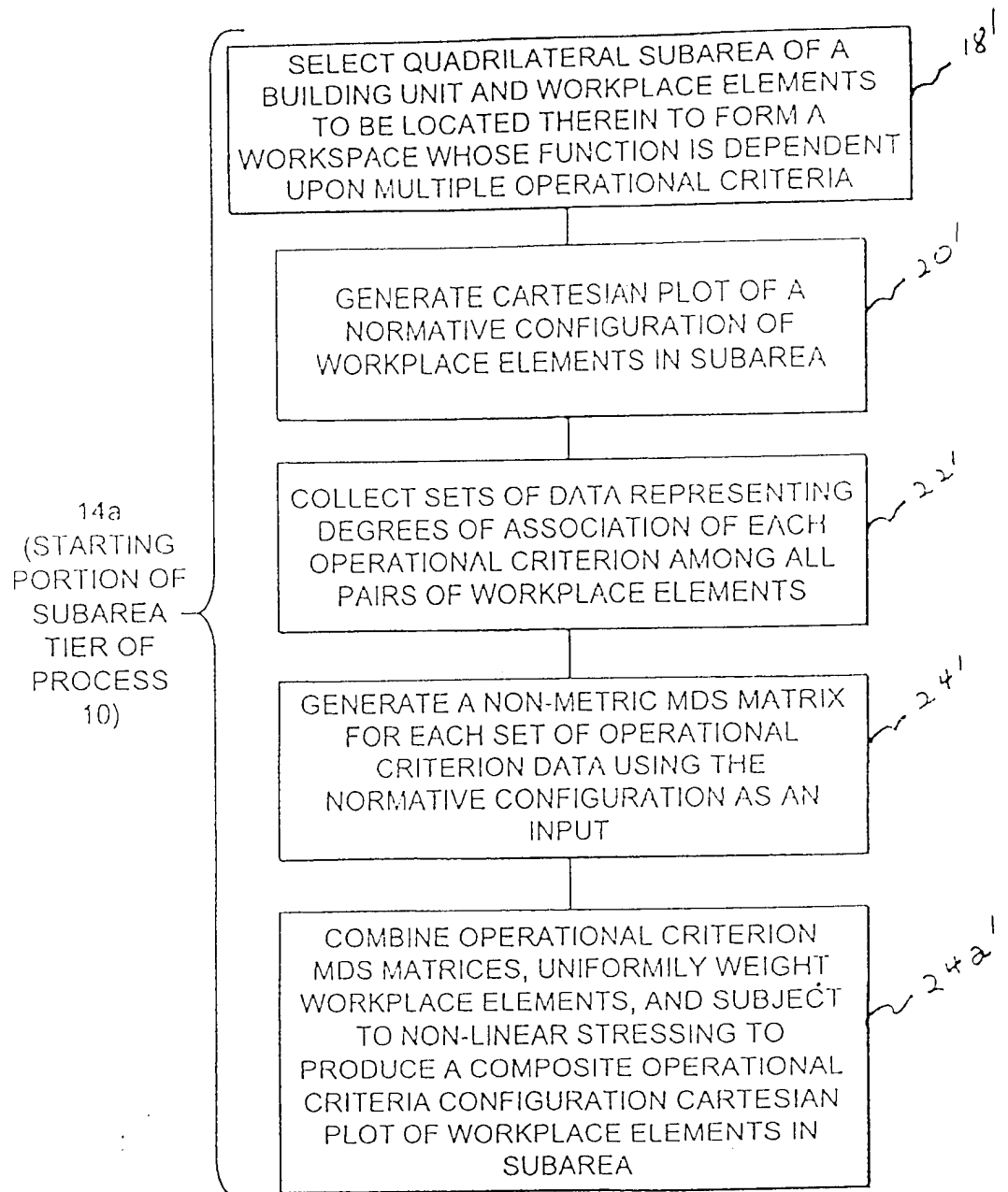


FIG. 1D

14b
(ENDING
PORTION
OF
SUBAREA
TIER OF
PROCESS
10)

CIRCUMSCRIBE OUTER WORKPLACE ELEMENTS
OF NORMATIVE AND COMPOSITE OPERATIONAL
CRITERIA CARTESIAN PLOTS WITH RESPECTIVE
CONVEX HULLS AND CALCULATE POLYGON
AREAS OF RESPECTIVE HULLS PER

$$A_{poly} = 1/2 \{ x_1 y_2 - x_2 y_1 \} + (x_2 y_3 - x_3 y_2) \dots (x_n y_1 - x_1 y_n) \}$$

CALCULATE $PDI_{CPOS:MDS}$ AND PDI_{NORM} FOR
COMPOSITE OPERATIONAL CRITERIA AND
NORMATIVE CONFIGURATIONS OF WORKPLACE
ELEMENTS, RESPECTIVELY, PER

$$PDI_{CPOS:MDS} : PDI_{NORM} = \frac{1}{2} \sqrt{\frac{n}{A_{poly}}}$$

COMPARE
 $PDI_{CPOS:MDS}$ WITH
 PDI_{NORM} TO DECIDE
WHETHER COMPOSITE
OPERATIONAL CRITERIA CONFIGURATION
OF WORKPLACE ELEMENTS IS
ACCEPTABLE OR
UNACCEPTABLE

UNACCEPTABLE

DISCONTINUE
MDS LAYOUT
PROCESS

ACCEPTABLE

ADJUST CONFIGURATION (OPTIONAL)

FIG. 1E

FIG. 2
(PRIOR ART)

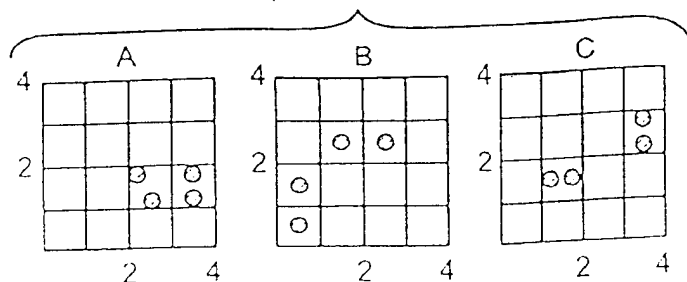


FIG. 3

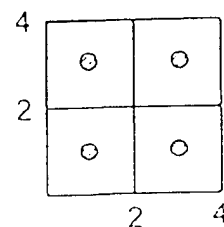


FIG. 4

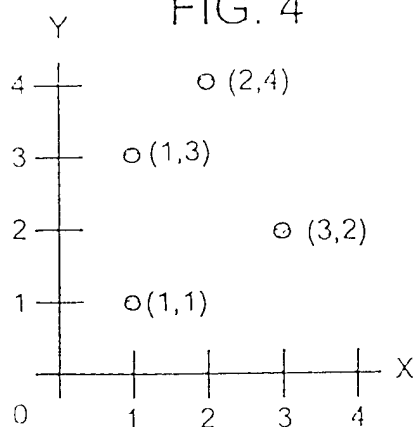


FIG. 5A.
(PRIOR ART)

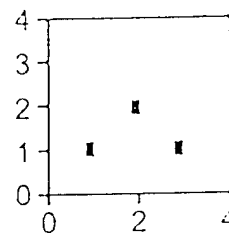


FIG. 5B.
(PRIOR ART)

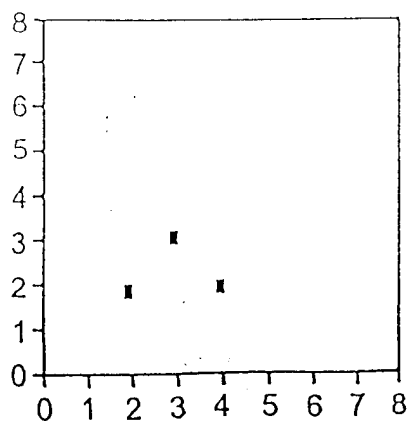
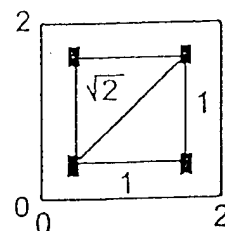


FIG. 7



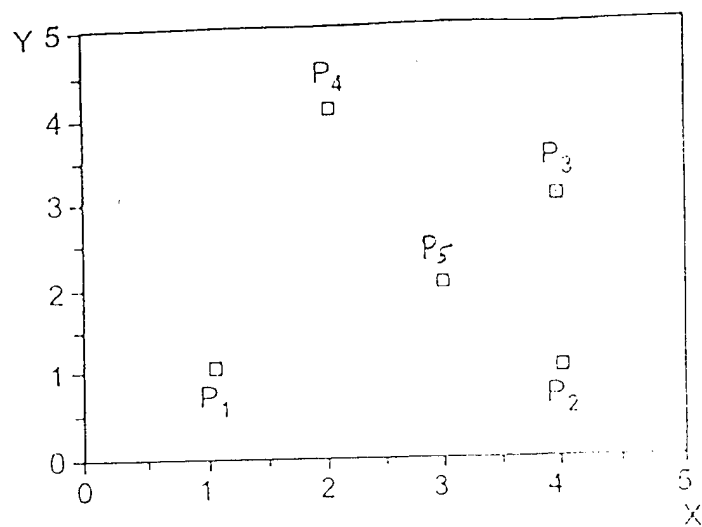


FIG. 6A

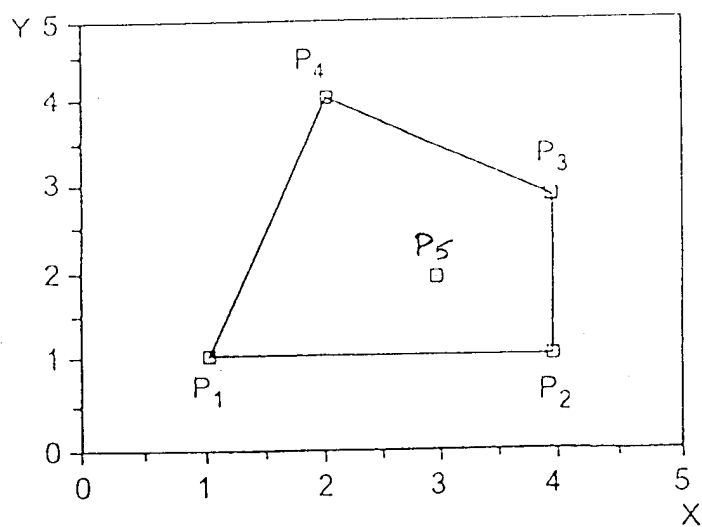


FIG. 6B

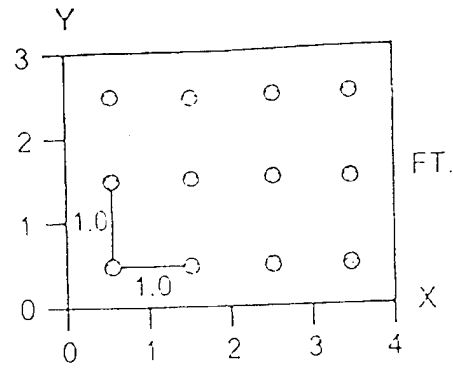


FIG. 8

