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Washington Project

Division 5, National Defense Research Committee
of the
Office of Scientific Research and Development

REPRESENTATION OF LONGITUDINAL STABILITY AND CONTROL OF HOMING GLIDE BOMBS BY AN ELECTRO-MECHANICAL MODEL

By
Harold K. Skramstad
National Bureau of Standards

A Report to Division 5
from the
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REPRESENTATION OF LONGITUDINAL STABILITY AND CONTROL OF HOMING GLIDE BOMBS BY AN ELECTRO-MECHANICAL MODEL

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A report from the National Bureau of Standards
to
Division 5, National Defense Research Committee
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Office of Scientific Research and Development

Approved for National Bureau of Standards by
Lyman J. Briggs, Director.

C O N F I D E N T I A L

Preface

The work described in this report is pertinent to the projects designated by the War Department Liaison officer as AC-1, AC-36, and AC-42 and to the projects designated by the Navy Department Liaison officer as NO-115, NO-174, and NO-235. This work was carried out and reported by National Bureau of Standards under a transfer of funds from OSRD with the co-operation of the Washington Radar Group of the Massachusetts Institute of Technology and Section Re4g of the Bureau of Ordnance, Navy Department.

Table of Contents

REPRESENTATION OF LONGITUDINAL STABILITY AND CONTROL OF HOMING GLIDE BOMBS BY AN ELECTRO-MECHANICAL MODEL

1. Derivation of Pitching Motion Equation

The general longitudinal stability characteristics of gliders are described in another paper entitled "Analysis of the Longitudinal Stability of Homing Glide-Bombs with Application to Navy SWOD Mark 7 and Mark 9". (Reference 1). The equations governing the motion are obtained by considering the effects of small displacements from an equilibrium condition and describe motions of the glider resulting from small displacements from this equilibrium condition. These equations are:

$$\left.\begin{aligned} L\Delta\gamma + 2Dv + D_\alpha\Delta\alpha + D_\delta\delta + V\frac{dv}{dt} &= 0 \\ -D\Delta\gamma + 2Lv + L_\alpha\Delta\alpha + L_\delta\delta - V\frac{d\gamma}{dt} &= 0 \\ M_q\dot{\theta} + M_\alpha\Delta\alpha + M_\delta\delta + M_{\dot{\alpha}}\dot{\alpha} + M_{\dot{\delta}}\dot{\delta} - \ddot{\theta} &= 0 \end{aligned}\right\} \qquad (1)$$

where

- θ = angle between longitudinal axis of glider and the horizontal,
- α = angle of attack of glider,
- γ = angle between flight path and horizontal,
- W = weight of glider,
- V = velocity along flight path,

ρ = air density,

S = wing area,

C_L = lift coefficient,

C_D = drag coefficient,

C_m = pitching moment coefficient,

m = mass,

B = moment of inertia about lateral axis through the center of gravity,

δ = angular displacement of pitch control surface,

q = $\frac{d\theta}{dt}$ = rate of pitch,

c = mean aerodynamic chord,

Δ is a prefix denoting a small displacement of the following quantity from an equilibrium condition

The following quantities are defined:

$$\left.\begin{array}{lll} L = \frac{\frac{1}{2}\rho S V^2 C_L}{m} & D = \frac{\frac{1}{2}\rho S V^2 C_D}{m} & M = \frac{\frac{1}{2}\rho S V^2 c\, C_m}{B} \\ L_\alpha = \frac{\partial L}{\partial \alpha} & D_\delta = \frac{\partial D}{\partial \delta} & M_\delta = \frac{\partial M}{\partial \delta} \\ L_\delta = \frac{\partial L}{\partial \delta} & M_\alpha = \frac{\partial M}{\partial \alpha} & M_{\dot\alpha} = \frac{\partial M}{\partial \dot\alpha} \\ D_\alpha = \frac{\partial D}{\partial \alpha} & M_q = \frac{\partial M}{\partial q} & M_{\dot\delta} = \frac{\partial M}{\partial \dot\delta} \\ & & v = \Delta V/V \end{array}\right\} \quad (2)$$

It was seen as a result of the discussion in Reference 1 that V varied only very slowly under all conditions of flight, and thus, in any study of the behavior of a glider over intervals of less than 20 or 30 seconds, it is permissible to neglect variations in V and consider only variations in γ , α , and δ . In any case, long-period variations in V can be treated independently.

If we neglect variations in V , the first of equations (1) disappears, and equations (1) become:

$$\left.\begin{array}{l} -D\Delta\gamma + L_\alpha\Delta\alpha + L_\delta\delta - V\dfrac{d\gamma}{dt} = 0 \\ M_q\dot{\theta} + M_\alpha\Delta\alpha + M_\delta\delta + M_{\dot{\alpha}}\dot{\alpha} + M_{\dot{\delta}}\dot{\delta} - \ddot{\theta} = 0 \end{array}\right\} \quad (3)$$

It should be noted that equilibrium between the forces along the flight path is no longer necessary and is actually seldom attained during a flight, owing to the small damping of disturbances in V (Ref. 1).

For simplicity, let us write:

$$a = \frac{L_\alpha}{V}, \quad b = \frac{L_\delta}{V}, \quad f = \frac{D}{V}. \quad (4)$$

The first equation of (3) above becomes

$$\frac{d\gamma}{dt} = a\Delta\alpha + b\delta - f\Delta\gamma. \quad (5)$$

Since $\theta = \gamma + \alpha$, we may write

$$\dot{\theta} - \dot{\alpha} = a\Delta\alpha + b\delta + f\Delta\alpha - f\Delta\theta$$

Note that $\dot{\alpha} = \frac{d\alpha}{dt}$ and also $\frac{d(\Delta\alpha)}{dt}$, and $\dot{\theta} = \frac{d(\Delta\theta)}{dt}$ as well as $\frac{d\theta}{dt}$.
The variables in the following equations are $\Delta\alpha$ and $\Delta\theta$.
Solving for $\dot{\alpha}$:

$$\dot{\alpha} + (a+f)\Delta\alpha = \dot{\theta} + f\Delta\theta - b\delta \quad (6)$$

Integrating:

$$\Delta\alpha = e^{-(a+f)t}\int\dot{\theta}e^{(a+f)t}dt + fe^{-(a+f)t}\int\Delta\theta e^{(a+f)t}dt - be^{-(a+f)t}\int\delta e^{(a+f)t}dt. \quad (7)$$

Substituting these values of $\Delta\alpha$ and α in the second of equations (3), we obtain:

$$\left.\begin{aligned}\ddot{\theta} &= (M_q + M_{\dot\alpha})\dot\theta + (M_\delta - b M_{\dot\alpha})\delta + f M_\alpha \beta\theta + M_{\dot\delta}\dot\delta \\ &+ \left\{M_\alpha - (a+f) M_{\dot\alpha}\right\}\left\{e^{-(a+f)t}\int \dot\theta e^{(a+f)t} dt + f e^{-(a+f)t}\int \beta\theta e^{(a+f)t} dt \right. \\ &\left. - b e^{-(a+f)t}\int \delta e^{(a+f)t} dt\right\}\end{aligned}\right\} \quad (8)$$

In general f is small compared to a or b, and may be neglected in the equations. Making this approximation, we obtain

$$\begin{aligned}&\ddot\theta + (-M_q - M_{\dot\alpha})\dot\theta + (-M_\alpha + a M_{\dot\alpha}) e^{-at}\int \dot\theta e^{at} dt = \\ &(M_\delta - b M_{\dot\alpha})\delta + M_{\dot\delta}\dot\delta + b(-M_\alpha + a M_{\dot\alpha}) e^{-at}\int \delta e^{at} dt .\end{aligned} \quad (9)$$

This is the equation that describes the angular pitching motion of a glider produced by any arbitrary motion of the elevons.

2. Description of Electro-mechanical Model

In order to represent the pitching motion of a glider by a model, it is necessary that the same equation describe both the motion of the model and the motion of the glider. A model constructed at the National Bureau of Standards consists of a circular table free to rotate about a vertical axis. (See Fig. 1). This table may be used to carry the gyros or position or motion-sensitive elements of the control system to be tested. On this same axis is a cylinder which rotates inside a concentric cylinder with a small separation. The space between the cylinders is filled with oil to produce viscous damping of the motion of the table. Since the spacing of the cylinders is small and the oil is sufficiently viscous, the damping force is quite accurately proportional to the first power of the angular velocity of the table.

Geared to the table are two synchros. One of these, called a "table synchro", is used to produce a voltage depending upon the position of the table, and the other is called a "torque synchro" and is used to apply a torque to the table. A third synchro (shown in Figure 2) designated as a "servo synchro" is used to produce a voltage depending upon the deflection of the pitch control surface of the glider being simulated by the model.

Let I = moment of inertia of table and associated system,

μ = damping coefficient of table,

L = torque applied to table by torque synchro,

and ϕ = angular displacement of table.

The equation of motion of the table is thus given by:

$$I \frac{d^2\phi}{dt^2} + \mu \frac{d\phi}{dt} = L(t). \qquad (10)$$

Let us now examine the wiring diagram shown in Figure 3. The first synchro, or table synchro, has its rotor coil connected to 115 volts A.C. The armature is positioned so that at a certain position of the table, called the zero position, the amplitudes of A.C. appearing between S1 and S2 and that between S2 and S3 are equal. These two A.C. voltages are rectified by the twin triode T2 connected as diodes.

In the "zero position", we thus have equal D.C. voltages built up across the condensers C3 and C4, and thus the D.C. voltage across the two condensers C3 and C4 in series is zero. However, when the table is displaced from its zero position, the A.C. voltages developed between S1 and S2 and between S2 and S3 are no longer equal, and after rectification, the D.C. potentials across C3 and C4 are no longer equal. The D.C. potentials thus developed across the two condensers in series are found to be sensibly proportional to the angular displacement of the table, up to a displacement of the order of 20°, and with a polarity depending upon the sense of the displacement.

Let us denote the voltage between the top end of R7 and the arm of R7 by E_1. E_1 will be proportional to the voltage across C3 and C4 in series, and thus proportional to the table displacement ϕ. Thus we may write:

$$E_1 = -c_1 \phi. \qquad (11)$$

The constant c_1 depends on the setting of R7.

The servo synchro (Fig. 2) is connected electrically the same as the table synchro, and produces a D.C. potential across C1 and C2 in series sensibly proportional to the displacement of the control surface and a polarity depending upon the sense of the displacement.

Let us denote the voltage between the common terminal of R5 and R6 and the movable arm of R5 by E_2. This voltage will be proportional to the voltage across C1 and C2 in series, which is in turn proportional to the control surface displacement. Thus we may write

$$E_2 = a_2 \delta. \qquad (12)$$

Assume switch S_4 thrown to the left or "normal" position. Let us denote the voltage across C6 by E_2', the current in R8 by I_2, the resistance of R8 by R_8, and the capacitance of C6 by C_6.

We have the following relations:

$$E_2' = E_2 - I_2 R_8 = \frac{\int I_2 dt}{C_6},$$

$$\text{and } \frac{dE_2'}{dt} = \frac{I_2}{C_6} = \frac{E_2 - E_2'}{R_8 C_6}.$$

Solving for E_2':

$$E_2' = e^{-\frac{t}{R_8 C_6}} \int \frac{E_2}{R_8 C_6} e^{\frac{t}{R_8 C_6}} dt. \qquad (13)$$

In a similar manner, let us denote the voltage across R9 by E_1', the current in R9 by I_1, the resistance of R9 by

R_9, and the capacitance of C7 by C_7. The following relations are obtained:

$$E_1' = I_1 R_9 ,$$

$$E_1 - E_1' = \frac{\int I_1 dt}{C_7} ,$$

$$\text{and } \frac{dE_1}{dt} - \frac{dE_1'}{dt} = \frac{I_1}{C_7} = \frac{E_1'}{R_9 C_7}$$

Solving for E_1' :

$$E_1' = e^{-\frac{t}{R_9 C_7}} \int \frac{dE_1}{dt} e^{\frac{t}{R_9 C_7}} dt. \qquad (14)$$

Denote the voltage from the common terminal of R5 and R6 and the movable arm of R6 by E_3. This voltage will be proportional to the voltage across C1 and C2 in series, and thus proportional to the control-surface displacement. Thus we may write

$$E_3 = c_3 \delta. \qquad (15)$$

A generator is arranged so that its armature (Fig. 2) moves with the control surface and thus produces a voltage proportional to the rate of change of control-surface position. If we denote this voltage by E_4, we have

$$E_4 = c_4 \dot{\delta}. \qquad (16)$$

From Figure 3, it is seen that, with switch S4 to the left, the voltage applied between the two grids of T3, denoted by E_g, is given by

$$E_g = E_1' + E_2' + E_3 + E_4 . \qquad (17)$$

T3 is an amplifier stage that drives tubes T6, T7, T8, and T9 in push-pull parallel. It can be seen from an examination of the circuit that the current in the stator winding of the torque synchro will be proportional to the voltage applied between the two grids of T3.

Since the sum of the cathode currents of tubes T6, T7, T8, and T9 is constant, the current through the rotor coil of the torque synchro remains constant. The torque produced by the torque synchro will thus be proportional to the current through the stator winding, and thus proportional to the potential between the two grids of T3.

We may write then for the torque applied to the table:

$$\begin{aligned} L &= K E_g \\ &= K[E_1' + E_2' + E_3 + E_4] \\ &= K\left[e^{-\frac{t}{R_9 C_7}} \int \frac{dE_1}{dt} e^{\frac{t}{R_9 C_7}} dt + e^{-\frac{t}{R_8 C_6}} \int \frac{E_2}{R_8 C_6} e^{\frac{t}{R_8 C_6}} dt + E_3 + E_4\right] \qquad (18) \\ &= K\left[-e^{-\frac{t}{R_9 C_7}} c_1 \int \frac{d\phi}{dt} e^{\frac{t}{R_9 C_7}} dt + e^{-\frac{t}{R_8 C_6}} \frac{c_2}{R_8 C_6} \int \delta e^{\frac{t}{R_8 C_6}} dt + c_3 \delta + c_4 \dot{\delta}\right]. \end{aligned}$$

The equation of motion of the table becomes:

$$\begin{aligned} \frac{d^2\phi}{dt^2} + \frac{\mu}{I}\frac{d\phi}{dt} + \frac{K c_1}{I} e^{-\frac{t}{R_9 C_7}} \int \frac{d\phi}{dt} e^{\frac{t}{R_9 C_7}} dt = \\ \frac{K c_2}{I R_8 C_6} e^{-\frac{t}{R_8 C_6}} \int \delta e^{\frac{t}{R_8 C_6}} dt + \frac{c_3 \delta}{I} + \frac{c_4 \dot{\delta}}{I}. \end{aligned} \qquad (19)$$

Let us compare this equation with equation (9), which describes the pitching motion of the glider:

$$\frac{d^2\theta}{dt^2} + (-M_q - M_{\dot{\alpha}})\frac{d\theta}{dt} + (-M_\alpha + aM_{\dot{\alpha}})e^{-at}\int \frac{d\theta}{dt} e^{at}\,dt = (M_\delta - bM_{\dot{\alpha}})\delta + M_{\dot{\delta}}\dot{\delta} + b(-M_\alpha + aM_{\dot{\alpha}})e^{-at}\int \delta e^{at}\,dt. \tag{20}$$

The motion of the table and the pitching motion of a glider will be governed by the same equation if the following conditions are satisfied:

$$\left.\begin{array}{ll} \text{I.} & \dfrac{\mu}{I} = -M_q - M_{\dot{\alpha}} \\ \text{II.} & \dfrac{1}{R_7 C_7} = \dfrac{1}{R_8 C_8} = a \\ \text{III.} & \dfrac{Kc_1}{I} = -M_\alpha + aM_{\dot{\alpha}} \\ \text{IV.} & \dfrac{Kc_3}{IR_8 C_8} = b(-M_\alpha + aM_{\dot{\alpha}}) \\ \text{V.} & \dfrac{c_2}{I} = M_\delta - bM_{\dot{\alpha}} \\ \text{VI.} & \dfrac{c_4}{I} = M_{\dot{\delta}} \end{array}\right\} \tag{21}$$

The six conditions of Equations (21) are easily satisfied, the procedure for making the adjustments being described in

Part 3 of this Report. Once the model is so adjusted, its angular motion for any arbitrary motion of the simulated control surface will correspond in detail to the actual pitching motion of a glider whose pitch-control surface is operated in the same manner. Thus the model may be used to study the performance of a glider equipped with any given stabilization system by mounting the gyros or motion-sensitive elements of the system on the model table, and connecting the output so that the servo synchro moves in accord with the resulting motion of the control surface on the glider.

A much larger model could be used to study the performance of the control system of a homing glider by mounting the gyros and intelligence device on the model table, and placing a simulated target at a distance from the model, so that error signals as a function of angle of rotation of the model may be obtained. The information usually obtained from intelligence devices is a signal (e.g. electrical potential) proportional to the angular displacement of the glider from a zero (on course) position. This may be obtained directly in the small model of Figure 1 from the table synchro without the necessity of actually mounting and operating the intelligence device. Referring to Figure 3, it is seen that a voltage proportional to the displacement of the table from the zero position is obtained across R23. The manner in which the intelligence information is coupled to the control system determines how this voltage is used. Its application to SWOD Mark 7 and Mark 9 is shown in Figure 3, and will be discussed in Part 5.

3. Method of Adjusting Constants of Model

The following section describes the method used to adjust the constants of the table to satisfy the six conditions given by Equation (21).

Condition I may be satisfied by proper adjustment of the level or viscosity of the oil between the concentric cylinders. The procedure used is as follows: A thread is wound around a pulley of small diameter on the shaft of the table. The thread then passes over a pulley, and a scale pan and weights are tied to the end. With the torque synchro turned off, the equation of motion of the table will be given by

$$I \frac{d^2\phi}{dt^2} + \mu \frac{d\phi}{dt} = wr,$$

where w is the weight of the scale pan and weights and r is the radius of the pulley. If at time $t = 0$, the table is released from rest, we obtain

$$\frac{d\phi}{dt} = \frac{wr}{\mu}\left[1 - e^{-\frac{\mu t}{I}}\right]$$

After a few seconds the second term in brackets is negligible, and a constant angular velocity is obtained, given by

$$\frac{d\phi}{dt} = \left(\frac{wr}{I}\right)\left(\frac{I}{\mu}\right).$$

The desired value of μ/I, and the known values of w, r, and I are substituted in the right member of the above equation, and the viscosity of the oil or its level then adjusted until this computed value of $d\phi/dt$ is obtained.

Condition II is satisfied by choosing R8, R9, C6, and C8 of proper values to satisfy this condition. The values given in Figure 3 ($R_8C_6 = R_9C_7 = 1.0$ sec) correspond closely to the values for SWOD Mark 7 and 9 for a typical flight condition. When the equipment is used to study motions of gliders with different values of a, the values of R8, R9, C6, and C8 must be changed accordingly.

Condition III is satisfied by the following method. Switch S3 is closed, shorting out condenser C7, and E_3 is made zero by turning R6 to zero (the arm of R6 to the end common to R5). For this condition

$$E_1' = E_1 .$$

If we set δ fixed at 0, the equation of motion of the table now becomes

$$\frac{d^2\phi}{dt^2} + \frac{\mu}{I}\frac{d\phi}{dt} + \frac{Kc_1}{I}\phi = 0.$$

which represents a damped oscillation of period T, given by

$$T = \frac{2\pi}{\sqrt{\frac{Kc_1}{I} - \frac{\mu^2}{4I^2}}}$$

If the third condition is to be satisfied, we must have for the period of the oscillation

$$T = \frac{2\pi}{\sqrt{(+M_{\alpha} + a M_{\dot{\alpha}}) - \frac{(-M_{q} - M_{\dot{\alpha}})^2}{4}}} .$$

The table is now disturbed, and the setting of potentiometer R7 is varied until the period of oscillation is equal to the value computed from the above equation.

Condition IV is satisfied as follows: Since conditions II and III have been already satisfied, we combine conditions II, III and IV and obtain:

$$\frac{aKe_2}{I} = \frac{bKe_1}{I} .$$

The table, with S3 still closed and R6 turned to zero, is displaced a known small angle. The servo synchro is now turned through an angle equal to a/b times the angle through which the table is displaced, and potentiometer R5 is adjusted until meter M3 reads zero. This adjustment must be made slowly, allowing time for the potential on the condenser C5 to attain its final value. Since with M3 reading zero the table has no torque applied, equilibrium is established with

$$\delta = \frac{a}{b}\phi ,$$

and the condition is satisfied.

Condition V is satisfied as follows: Set ϕ equal to zero and R6 to zero (arm to end common to R5), adjust R5 to proper value under Condition IV, and determine the reading of M3 caused by a known deflection of the servo δ_1. Now set R5 to zero, and deflect the servo an amount δ_2 given by

$$\delta_2 = \delta_1 \frac{b(-M_\alpha + aM_{\dot{\alpha}})}{a(M_\delta + bM_{\dot{\alpha}})}$$

Now vary R6 until the same reading of M3 is obtained as previously. R5 is now returned to its original setting. Thus we have

$$\frac{Kc_2}{I}\delta_1 = \frac{c_3}{I}\delta_2 ,$$

and condition V is satisfied.

Condition VI is satisfied by setting R5 and R6 to zero position, setting ϕ to zero, operating the control surface at a known rate $\dot{\delta}$, and determining the deflection of M3. Then R5 is set to its proper value to satisfy condition IV, and the setting δ' of the control surface that produces the same deflection of M3 is determined. The generator gain control (not shown) is adjusted to make the following relation hold:

$$\frac{\dot{\delta}}{\delta'} = \frac{b(-M_\alpha + aM_{\dot{\alpha}})}{aM_{\dot{\delta}}}$$

The effect of the term in $\dot{\delta}$ is usually very small, and in most cases is negligible. For most tests, the term was neglected, and the generator omitted from the circuit.

4. The Pitch Control System of SWOD Mark 7 and Mark 9

Unlike the conventional type of airplane in which control in pitch is obtained by changing the angle of attack of the vehicle by means of elevators on the horizontal tail surface, control is obtained on SWOD Mark 7 and Mark 9 by means of elevons or flaps along the trailing edge of the main wing. These effectively change the lift of the wing without changing the angle of attack of the glider. This is accomplished by proper placement of the tail in the downwash of the main wing and proper location of the center of gravity so that practically no overall pitching moment results from movement of the elevons.

The elevons are moved in pitch by a servo-control unit (described in Reference 2) which moves the elevons at a constant rate and in a direction determined by contacts in the pitch gyro. The pitch gyro (described in Reference 3) is in effect a rate gyro oriented in the glider so as to be sensitive to rate of pitch. Electromagnets are connected to apply torques to the gimbal frame which are proportional to the error angle in pitch as obtained from the homing device. Electrical contacts are arranged on opposite sides of the gimbal frame so that one contact or the other is closed, depending upon the sign of the sum of the torque applied by the electromagnets and the torque due to precession of the gyro wheel.

Let $\Delta\theta$ represent the angular error in pitch (obtained from the homing device), $d\theta/dt$ the rate of pitch, and c, the rate of pitch in degrees per second that produces the same torque on the gimbal frame as an angular error of one degree. The operation of the gyro may be expressed mathematically as follows:

$$\frac{d\theta}{dt} + c\Delta\theta = S. \qquad (22)$$

When S is positive, that contact on the gyro will be closed that causes the servo-control unit to move the elevons upward; and when S is negative, the other contact will be closed, causing the elevons to move downward. The movement of the elevons is always in such a direction as to reduce the value of S to zero. Thus a hunting motion is set up, with the gyro contacts alternately closed, and the elevons moving alternately up and down.

If we neglect this hunting motion, and assume that S is, on the average, zero, we have

$$\frac{d\theta}{dt} + c\Delta\theta = 0. \qquad (23)$$

Integrating, and letting $\Delta\theta = \Delta\theta_0$ at $t = 0$, we have

$$\Delta\theta = \Delta\theta_0 e^{-ct}. \qquad (24)$$

This shows that any error in pitch should approach exponentially to zero at a rate depending upon c. This is found to be the case in flights of SWOD Mark 7 and Mark 9 under conditions where the period of the hunting oscillations is of the order of one second or less.

A detailed mathematical analysis of the hunting motion is very lengthy and complex, owing to the "off-on" character of the link between the gyro and servo-control unit. Multiplying equation (20) by e^{at} and differentiating, we have

$$\frac{d^3\theta}{dt^3} + (-M_q - M_{\dot\alpha} + a)\frac{d^2\theta}{dt^2} + (-M_\alpha - aM_q)\frac{d\theta}{dt} = (aM_\delta - bM_\alpha)\delta + (M_\delta - bM_{\dot\alpha} + aM_{\dot\delta})\dot\delta + M_{\dot\delta}\ddot\delta \qquad (25)$$

Integrating:

$$\frac{d^2\theta}{dt^2} + (-M_q - M_{\dot\alpha} + a)\frac{d\theta}{dt} + (-M_\alpha - aM_q)(\Delta\theta - \Delta\theta_0) = (aM_\delta - bM_\alpha)\int\delta\,dt + (M_\delta - bM_{\dot\alpha} + aM_{\dot\delta})\delta + M_{\dot\delta}\dot\delta. \qquad (26)$$

Assume that at $t = 0$, $\Delta\theta = \Delta\theta_0$, $\frac{d\theta}{dt} = \theta_0'$, and $\delta = \delta_0$. Let the elevons move from the initial position at $t = 0$ at a rate of K radians per second, the sign of K depending upon the direction of movement. Thus

$$\delta = \delta_0 \pm Kt, \qquad (27)$$

and equation (26) becomes

$$\frac{d^2\theta}{dt^2}+(-M_q-M_{\dot{\alpha}}+a)\frac{d\theta}{dt}+(-M_\alpha-aM_q)(\Delta\theta-\Delta\theta_0)=$$
$$(aM_\delta-bM_\alpha)\left(\delta_0 t\pm\frac{Kt^2}{2}\right)+(M_\delta-bM_{\dot{\alpha}}+aM_{\dot{\delta}})(\delta_0\pm Kt)\pm M_{\dot{\delta}}K. \quad (28)$$

This may be written in the form

$$\frac{d^2\theta}{dt^2}+2A\frac{d\theta}{dt}+B\Delta\theta=C+Dt+Et^2, \quad (29)$$

where A , B , C , D , and E are constants involving θ_0 , δ_0 , K , and the various aerodynamic coefficients.

Equation (29) has the following solution:

$$\Delta\theta=\left(\frac{C-2AD-2E}{B}+\frac{8A^2E}{B^2}\right)+\left(\frac{D-4AE}{B}\right)t+\frac{E}{B}t^2$$
$$+Fe^{-At}\cos\sqrt{B-A^2}\,t+Ge^{-At}\sin\sqrt{B-A^2}\,t, \quad (30)$$

where F and G are arbitrary constants to be evaluated. Differentiating, we obtain

$$\frac{d\theta}{dt}=\left(\frac{D-4AE}{B}\right)+\frac{2E}{B}t+(G\sqrt{B-A^2}-AF)e^{-At}\cos\sqrt{B-A^2}\,t$$
$$+(-F\sqrt{B-A^2}-AG)e^{-At}\sin\sqrt{B-A^2}\,t \quad (31)$$

Let us now obtain a new equation by substituting Equations (30) and (31) in Equation (22). This new equation must be solved for the smallest value of t, say t_1, for which $S = 0$. A time lag in the servo system is assumed, denoted by Δt, and the value of δ from Equation (27), the value of $\Delta\theta$ from Equation (30), and the value of $d\theta/dt$ from Equation (31) found at time $t = t_1 + \Delta t$. Next these values of δ, $\Delta\theta$, and $d\theta/dt$ are used as new values for δ_o, $\Delta\theta_o$, and θ_o', the sign of K is changed, and the value of t found which again reduces S to zero. This process is repeated for each cycle.

This procedure is too unwieldy for practical use, and thus the model presents a much more practical method for studying these oscillations.

5. Application of Model to Study of Control System Of SWOD Mark 7 and Mark 9

This model has been used to study the pitch-control system used in SWOD Mark 7 and Mark 9. Comparison of the data obtained with the model with actual flight tests of SWOD gliders shows that the model gives a good representation of the actual conditions of flight.

The following table gives typical values of the aerodynamic constants of SWOD Mark 9 for a typical condition of flight. [See reference 1, equations (42) and (50)]

$$
\left.
\begin{array}{ll}
V = 415 \text{ ft sec}^{-1} & M_{\dot{\alpha}} = -0.7 \text{ sec}^{-1} \\
L_{\alpha} = 427 \text{ ft sec}^{-2} & M_{\dot{\delta}} = -0.14 \text{ sec}^{-1} \\
L_{\delta} = 85.3 \text{ ft sec}^{-2} & M_{\delta} = 0 \text{ sec}^{-2} \\
M_{\alpha} = -30.6 \text{ sec}^{-2} & a = 1.03 \text{ sec}^{-1} \\
M_{q} = -1.4 \text{ sec}^{-1} & b = 0.206 \text{ sec}^{-1}
\end{array}
\right\} \quad (32)
$$

Inserting these values in Equation (20), we have

$$
\frac{d^2\theta}{dt^2} + 2.1\frac{d\theta}{dt} + 29.9\, e^{-1.03t}\int \frac{d\theta}{dt} e^{1.03t} dt = + 0.14\delta - 0.14\dot{\delta} + 6.18\, e^{-1.03t}\int \delta e^{1.03t} dt. \quad (33)
$$

The six conditions of Equations (21) that must be satisfied to make the model represent this condition of flight are

$$
\begin{array}{llll}
\text{I.} & \dfrac{\mu}{I} = 2.1 & \text{IV.} & \dfrac{Kc_2}{IR_8C_6} = 6.18 \\
\text{II.} & \dfrac{1}{R_8C_6} = \dfrac{1}{R_9C_7} = 1.03 & \text{V.} & \dfrac{c_3}{I} = 0.14 \\
\text{III.} & \dfrac{Kc_1}{I} = 29.9 & \text{VI.} & \dfrac{c_4}{I} = -0.14
\end{array} \quad (34)
$$

The table is adjusted to satisfy these conditions by the method given in Part 3. The pitch gyro is mounted on the table so it is sensitive to rotation about the table axis (Figure 1). An overhead rubber tube is brought to the gyro to provide the necessary vacuum line for driving the gyro wheel, and overhead wires for necessary electrical connections. The contacts on the gyro are connected to the servo unit (Figure 2), which is mounted on a stand which provides a torsion load simulating the aerodynamic load on the elevons during flight. The servo synchro is connected to the servo unit by a crank arm so that its displacement will be equal to the elevon displacement.

The unit containing the electronic circuits associated with the equipment is shown in Figure 4, and the power supply for this unit in Figure 5.

Homing information is obtained from the table synchro. It can be seen by examination of the diagram in Figure 3 that a voltage proportional to the angular displacement of the table from its zero or on course position is obtained across R23. This voltage is amplified by a two-stage differential amplifier having the same constants as the differential amplifier used in the SWOD Mark 1 and Mark 2 equipment. The output is connected by overhead wires to the electromagnet coils of the gyro. The gain-control potentiometer R23 is adjusted so that the angular displacement of the table is the same as the angular error of the glider in flight that produces a given differential current in the gyro coils.

The experimental setup, showing all the units, is shown in Figure 6.

In order to obtain permanent records and facilitate the study, a two-channel Brush Oscillograph, Model OBC, was used. For one series of runs, the input of one channel was connected through an isolation transformer between S1 and S3 of the servo synchro, and the input to the other channel was connected between S1 and S3 of the table synchro. Records were thus obtained showing 60-cycle oscillations whose amplitudes, on one channel, were proportional to the displacement of the servo unit from the zero position, and on the other channel, were proportional to the displacement of the table from its zero position.

In another series of runs, one channel was connected to the servo synchro as previously, but the other channel was connected to a 60-cycle alternating voltage modulated so that its amplitude was proportional to the current through the torque synchro. For the flight condition of SWOD Mark 9 studied here, the pitching moments due to elevon displacement and rate of change of elevon displacement are small compared to the moments due to angle of attack changes, and the current through the torque synchro and thus the amplitude of the alternating voltage may be considered to be proportional to the deviation of the angle of attack from the trim position.

Records obtained using this oscillograph are shown in Figures 7 to 12 inclusive. The gain controls on the oscillograph are adjusted so that a definite value of the quantity being measured (shown in the left margin) is represented by a given number of vertical scale divisions. The distance traveled by the chart in one second is marked on each record.

The symbols ϕ , α , and δ on these figures denote quantities designated by ϕ , $\Delta\alpha$, and δ in this report. Φ represents the minimum error angle that produces saturation of the differential amplifier, G.R. (signifying gyro rate) is the rate of turn of the gyro which produces the same torque on its gimbal frame as saturation of the differential amplifier, RC is the value of R_8 C_6 and R_9 C_7 , δ_o the initial displacement of the control surface, and ϕ_o the initial displacement of the table. When values are not given for ϕ_o and δ_o , the record represents a steady state hunting condition.

The sense of the amplitude of the oscillations is not distinguished in this type of record, but by examination of the traces it is generally possible to determine whether or not a reversal of phase occurs at points where the amplitude is reduced to zero.

Figure 7 shows records obtained with the constants of the model adjusted to represent the conditions of Equation (34). The three records differ only in the value of Φ . The records with Φ = 6° and Φ = 4° show damped oscillations in ϕ with a period of about 7.5 seconds, the one with Φ = 6° being the more highly damped. The record with Φ = 2° shows a steady state undamped oscillation with a period of about 9.5 seconds. The effect of increasing Φ is to increase the damping of the oscillations. For actual flights, however, increasing Φ has the undesirable effect of reducing the value of c (Equation 22), i.e., the rate of correction of attitude ($d\theta/dt$) obtained for a given error angle $\Delta\theta$, so that errors due to winds, target motion, or gusts encountered near the target are increased. In SWOD Mark 7 and Mark 9 a compromise has been made, and Φ has been made equal to 4°.

Figure 8 shows records obtained under the same conditions as Figure 7, except that G.R., the gyro rate, has been changed to 2°/sec. The records obtained with Φ = 6° and Φ = 4° show practically a smooth return to on course position without overshoot. The case with Φ = 2° shows an oscillation with small damping. Reducing the gyro rate thus increases the damping of the oscillations, but also has the effect of reducing c as discussed above. G.R. has been made equal to 2°/sec. in the SWOD Mark 7 and Mark 9 gliders.

Figures 13, 14, and 15 show data obtained from analysis of camera records obtaining during actual flights. Figure 13 shows the flight of a Mark 13 glider with Mark 1 homing equipment (Navy SWOD Mark 8). For this flight Φ = 4° and G.R. = 2°/sec. The upper curve entitled "Elevon Position for Zero Differential" shows the average position of the elevons (δ) and the lower curve entitled "Apparent Angular Vertical Motion of Beacon" gives the angular deviation of the attitude of the glider from on course position ($\Delta\theta$) as functions of the time.

At the beginning of the flight a highly damped short period oscillation, sometimes called the "rapid incidence adjustment", is seen (Reference 1). This is superimposed on a rapid movement to on course position without overshoot. This figure may be compared with the second record in Figure 8, where the model is adjusted to approximately the same constants as used in this flight.

Figure 14 shows the flight of a Mark 12 glider with Mark 1 homing equipment (SWOD Mark 7). For this flight Φ = 6°, G.R. = 4°/sec. This flight shows an oscillation of about 10 seconds period, which however, damps out in a few cycles. This figure should be compared with the first record of Figure 7, in which the model is adjusted to approximately the same constants as used in this flight.

Figure 15 shows another flight of a Mark 12 glider with Mark 1 homing equipment. For this flight Φ = 2°, G.R. = 4°/sec. In addition to the types of curves given in Figures 13 and 14, a curve is shown labeled "Glide Meter", which gives the current in the gyro electromagnet coil which tends to close that contact which causes the elevons to move downward. This figure shows a long-period undamped oscillation and may be compared with the lower record of Figure 7 taken with the model adjusted to approximately the same constants.

Figures 9 and 10 show records obtained of δ and $\Delta\alpha$ as functions of the time. Records of long-period oscillations in δ as in the beginning of the first and in the third records of Figure 9 show oscillations in α of the same period as in δ, and also highly damped oscillations superimposed on them of shorter period corresponding to that of the "rapid incidence adjustment". These rapid oscillations are excited by the reversal of motion of the elevons, and are quite highly damped before being re-excited by the next reversal. When reversals of motion of the elevons take place with a period of the same order as that of the "rapid incidence adjustment" such as shown in the second records of Figures 9 and 10, these oscillations, although excited by each reversal in direction of motion of the elevons, are not separately distinguishable.

Although the details of the calculation will not be given here, it can be shown that $\Delta\alpha$ can be represented by an equation similar to that for $\Delta\theta$ in Equation (30), except that the right-hand member will not contain a term in t^2. The oscillations of the same period as δ are due to the reversals in sign of K, which are discussed following Equation (30). The highly damped oscillations of the period of the "rapid incidence adjustment" are given by terms in the expression for $\Delta\alpha$ which are equivalent to the last two terms in the right member of Equation (30).

The effect of changes in the value of RC used in the model is shown in Figures 11 and 12. Changing the value of RC is equivalent to changing L_α and L_δ for the glider whose flight is being represented by the model.

Records with RC = 1.5 seconds are shown in Figure 11. The oscillations are less damped and of longer period than for RC = 1 second. Records with RC = .83 seconds are shown in

Figure 13. Here the oscillations are more highly damped, and of shorter period. Thus it may be presumed that increasing L_α and L_δ will increase the damping and shorten the period of the hunting oscillations.

October 26, 1945

References

(1) Analysis of the Longitudinal Stability of Homing Glide-Bombs with Application to Navy SWOD Mark 7 and Mark 9, by Harold K. Skramstad, prepared for Division 5, National Defense Research Committee.

(2) The Development of Servo-Control Mechanisms for Homing Aero-Missiles, by Emmett C. Bailey and Wesley G. Spangenberg, prepared for Division 5, National Defense Research Committee.

(3) An Automatic Pilot for Homing Glide-Bombs, by John A. Hart, prepared for Division 5, National Defense Research Committee.

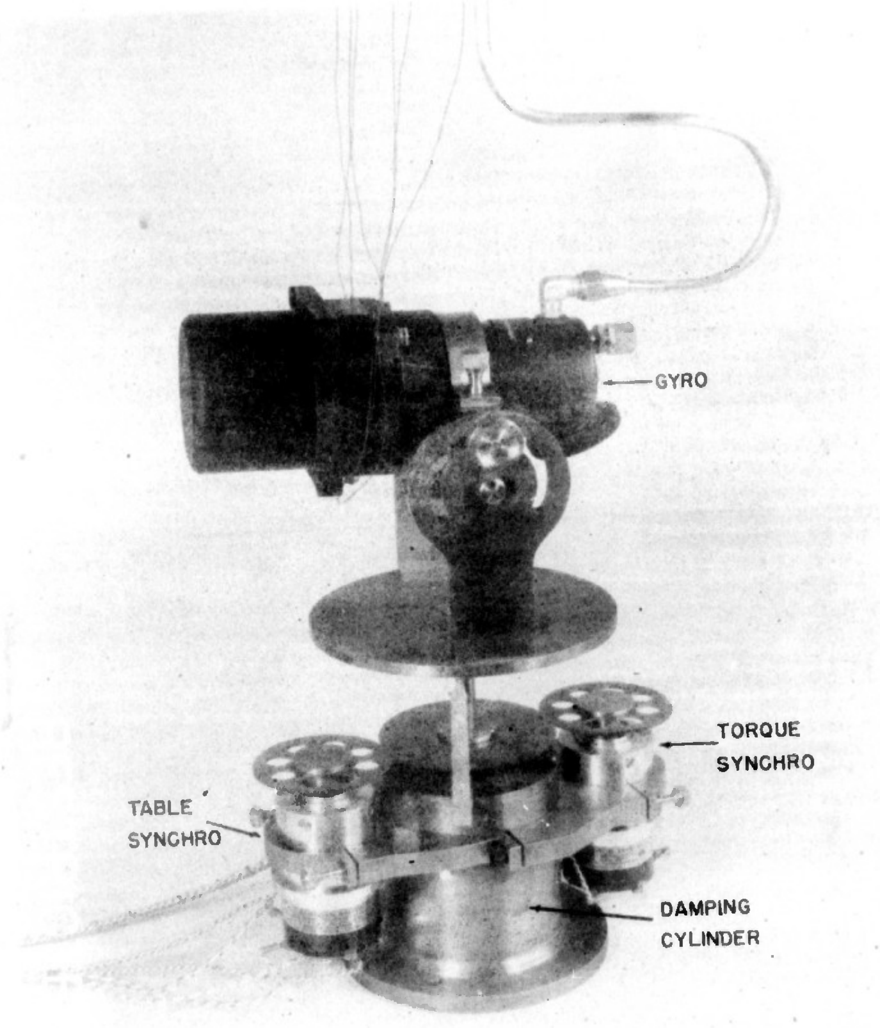


FIG. 1

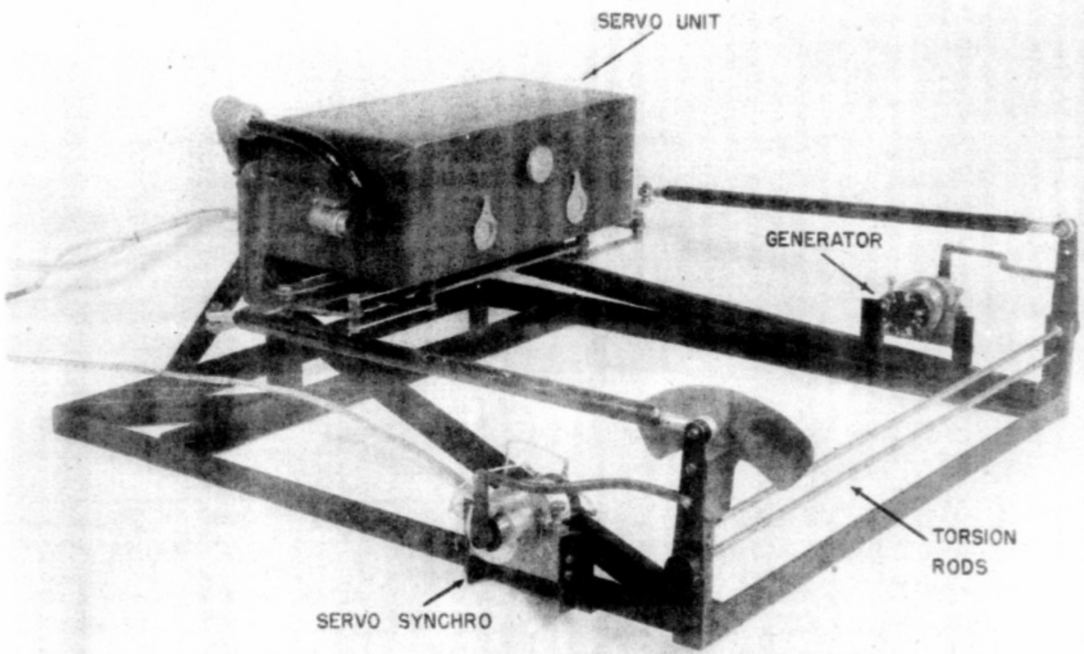
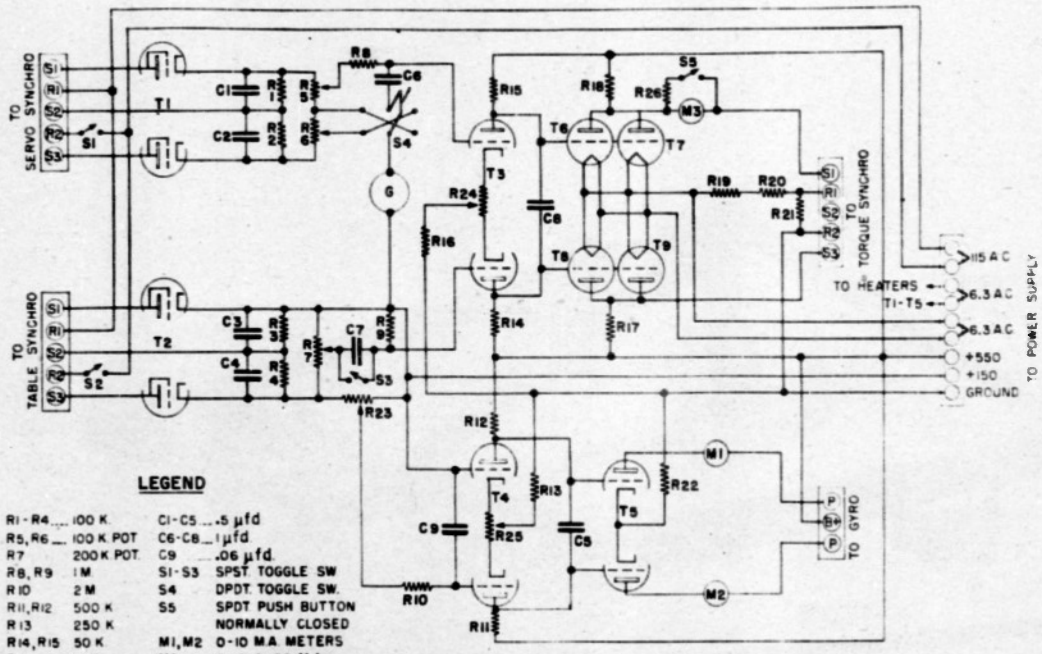


FIG. 2



LEGEND

R1-R4	100 K	C1-C5	.5 μfd
R5, R6	100 K POT	C6-C8	1 μfd
R7	200 K POT	C9	.06 μfd
R8, R9	1 M	S1-S3	SPST TOGGLE SW
R10	2 M	S4	DPDT TOGGLE SW.
R11, R12	500 K	S5	SPDT PUSH BUTTON
R13	250 K		NORMALLY CLOSED
R14, R15	50 K	M1, M2	0-10 M.A. METERS
R16	12 K	M3	0-15, 0-150 M.A.
R17, R18	500		ZERO CENTER METER
R19, R20	750	T1-T5	6SN7
R21	200	T6-T9	6A3
R22	50K		
R23	1M POT	G	GENERATOR
R24	500 POT		
R25	2 K		
R26	3.5		

PITCH CONTROL FLIGHT TABLE
AMPLIFIER UNIT

FIG. 3

12-1-44

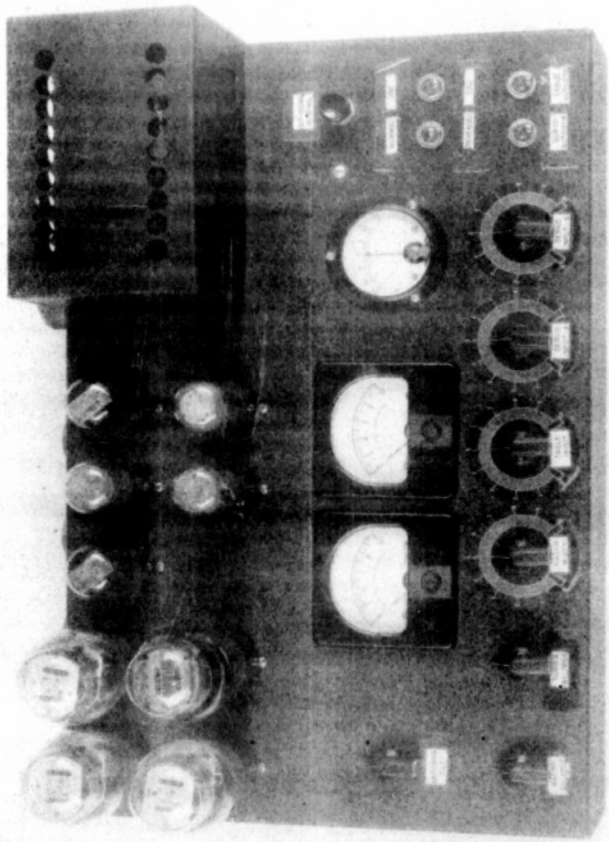


FIG 4

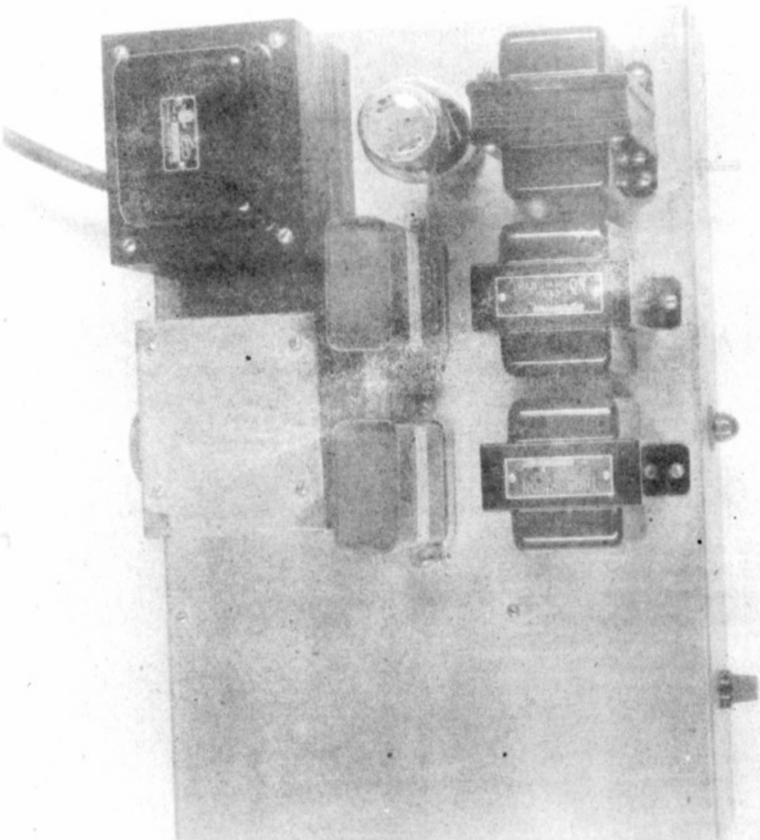
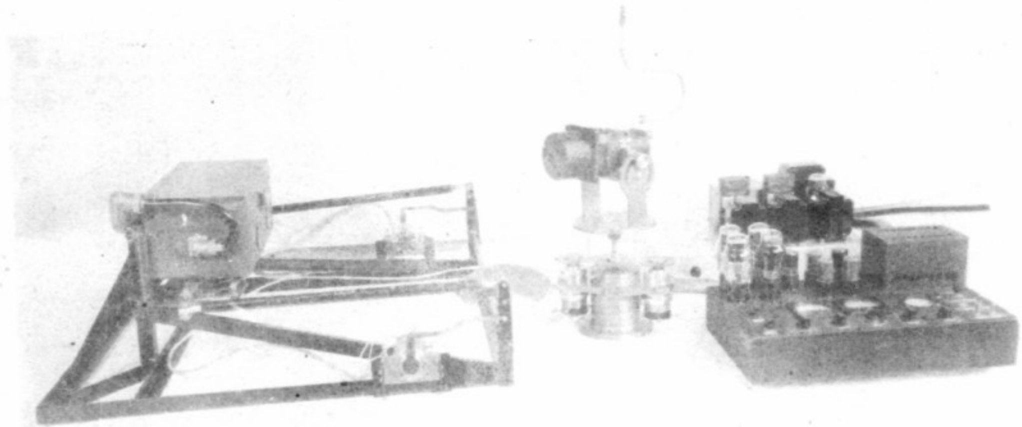


FIG.5



RESTRICTED

FIG. 6

Φ = 6°, GR = 4°/sec, RC = 1 sec, σ_0 = 0, Φ_0 = 5°
Φ = 4°, GR = 4°/sec, RC = 1 sec, σ_0 = 0, Φ_0 = 5°
Φ = 2°, GR = 4°/sec, RC = 1 sec, σ_0 = 0, Φ_0 = 5°
1 sec
5°
σ
Φ

FIG. 7

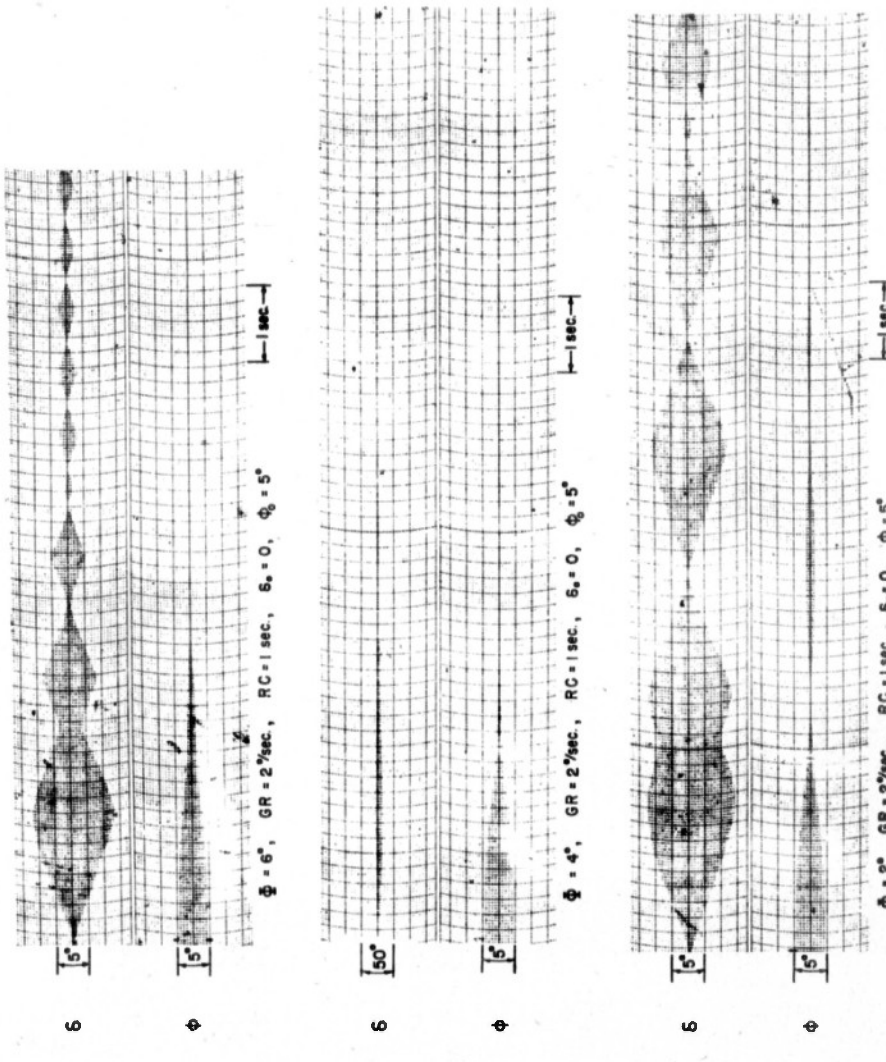
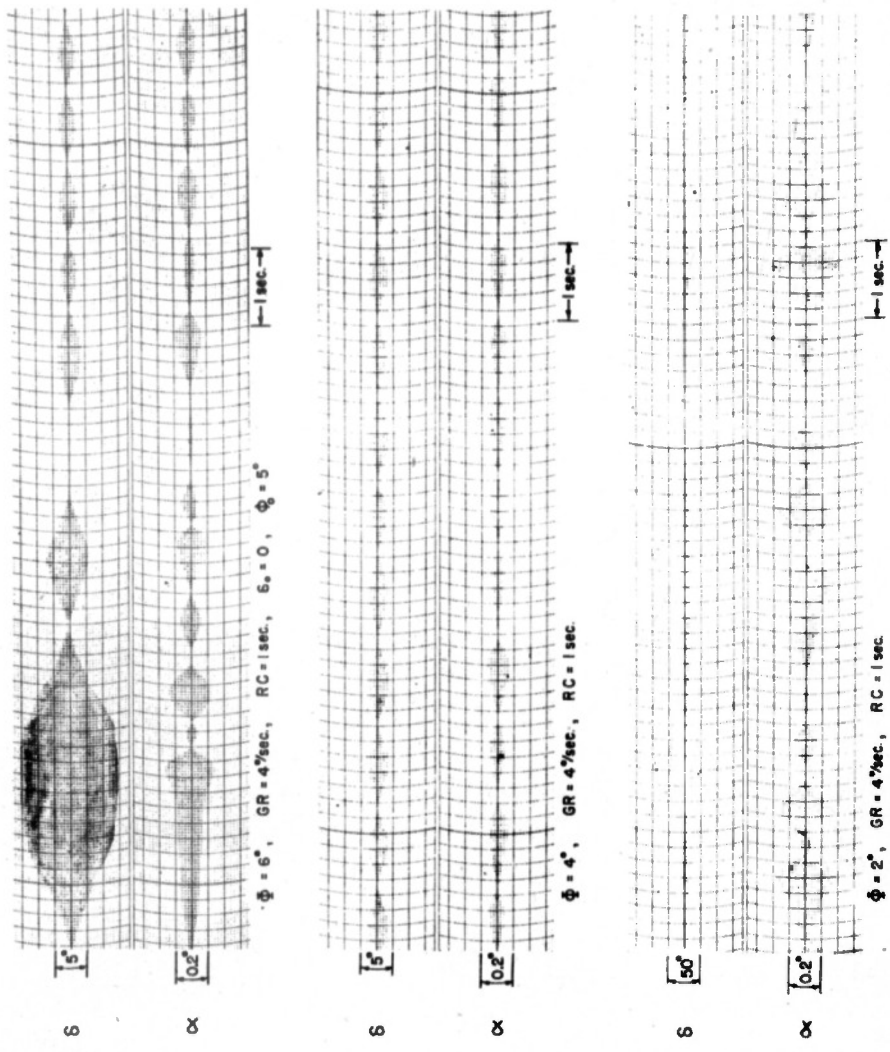

σ
Φ
5°
5°
Φ = 6°, GR = 2°/sec., RC = 1 sec., σ0 = 0, Φ0 = 5°
1 sec.
σ
Φ
50°
5°
Φ = 4°, GR = 2°/sec., RC = 1 sec., σ0 = 0, Φ0 = 5°
1 sec.
σ
Φ
5°
5°
Φ = 2°, GR = 2°/sec., RC = 1 sec., σ0 = 0, Φ0 = 5°
1 sec.

FIG. 8



$\Phi = 6°$, GR = 4°/sec., RC = 1 sec., $\delta_o = 0$, $\dot{\Phi}_o = 5°$
$\Phi = 4°$, GR = 4°/sec., RC = 1 sec.
$\Phi = 2°$, GR = 4°/sec., RC = 1 sec.
1 sec.
5°
0.2°
50°
δ
α

FIG. 9

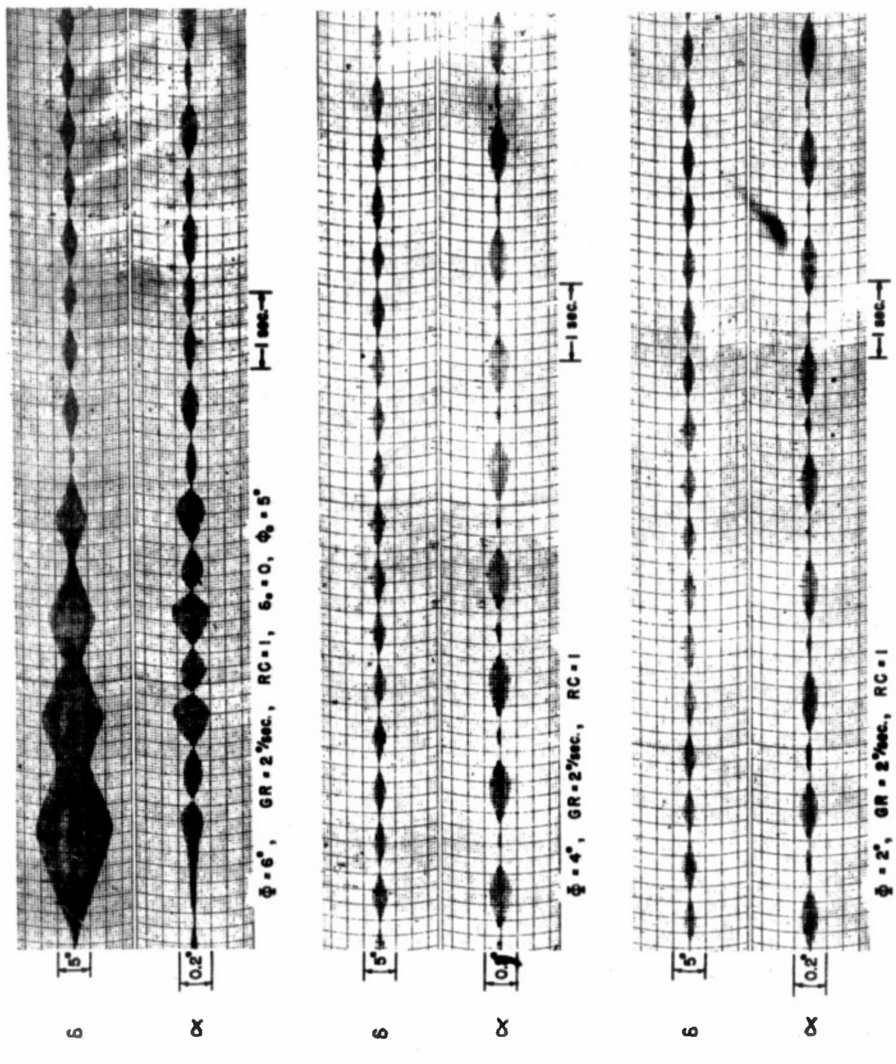
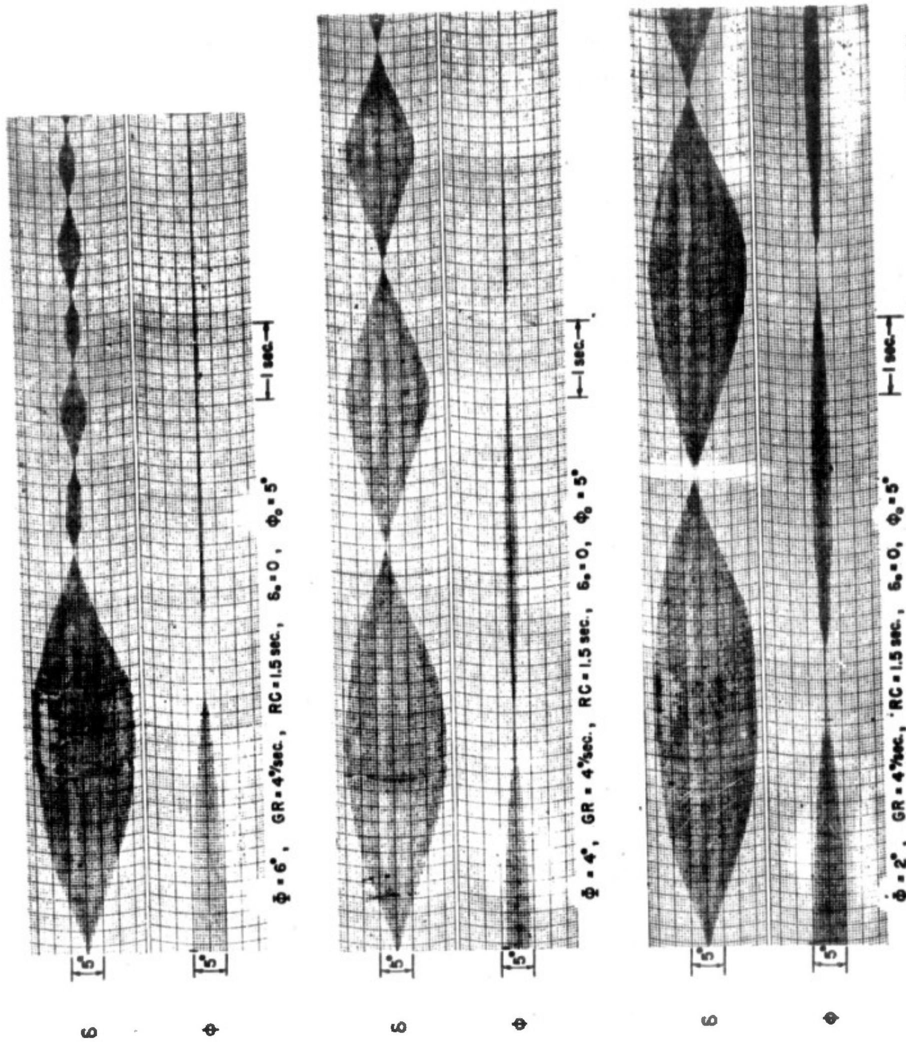


FIG. 10



$\Phi = 6°$, GR = 4°/sec., RC = 1.5 sec., $\delta_0 = 0$, $\Phi_0 = 5°$
$\Phi = 4°$, GR = 4°/sec., RC = 1.5 sec., $\delta_0 = 0$, $\Phi_0 = 5°$
$\Phi = 2°$, GR = 4°/sec., RC = 1.5 sec., $\delta_0 = 0$, $\Phi_0 = 5°$
1 sec.
5°
δ
Φ

FIG. 11

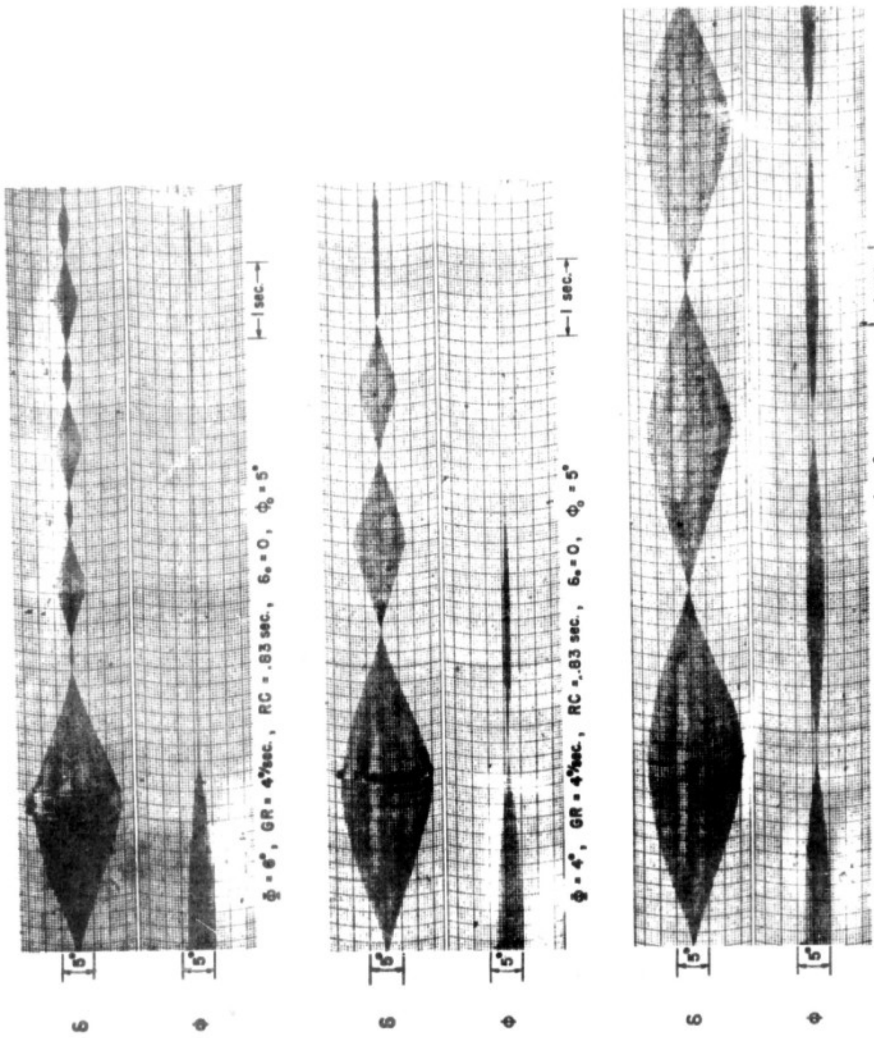

Φ = 6°, GR = 4°/sec., RC = .83 sec., θo = 0, Φo = 5°
Φ = 4°, GR = 4°/sec., RC = .83 sec., θo = 0, Φo = 5°
Φ = 2°, GR = 4°/sec., RC = .83 sec., θo = 0, Φo = 5°
1 sec.
5°
θ
Φ

FIG. 12

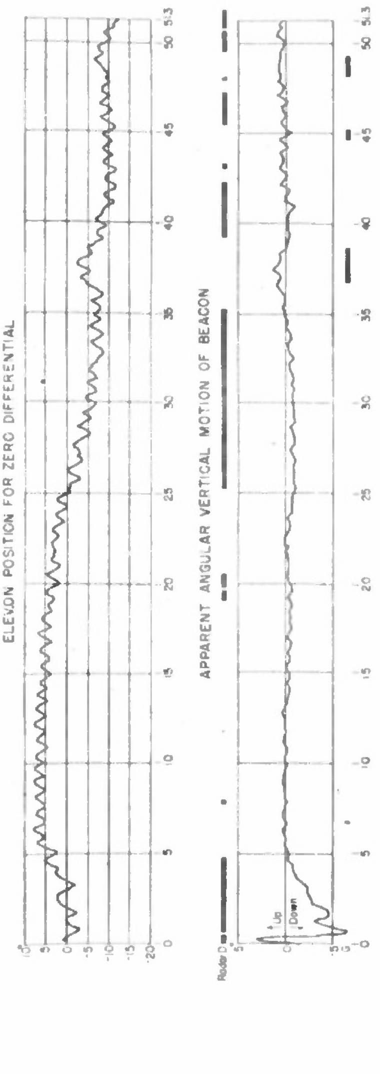

ELEVON POSITION FOR ZERO DIFFERENTIAL
APPARENT ANGULAR VERTICAL MOTION OF BEACON
Radar D
Up
Down

Fig.13

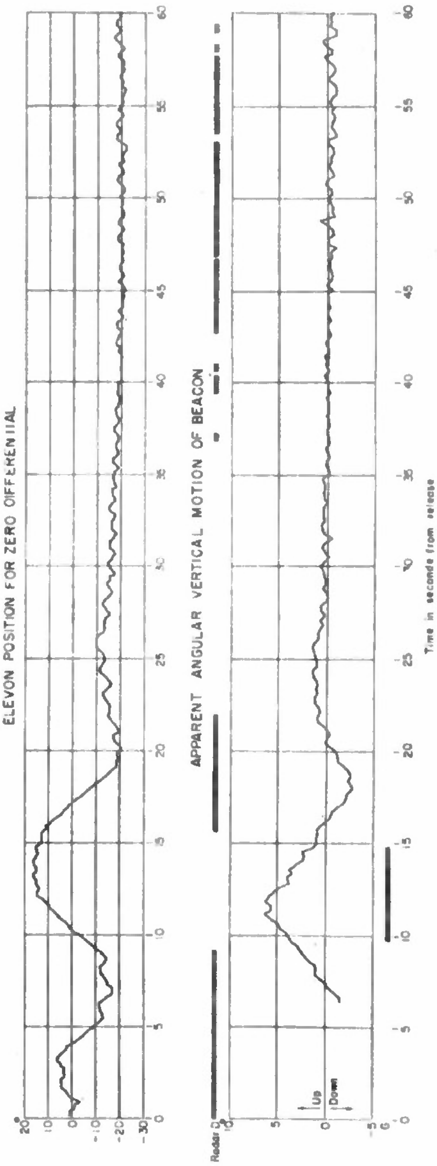

ELEVON POSITION FOR ZERO DIFFERENTIAL
APPARENT ANGULAR VERTICAL MOTION OF BEACON
Radar D
Up
Down
Time in seconds from release

Fig.14

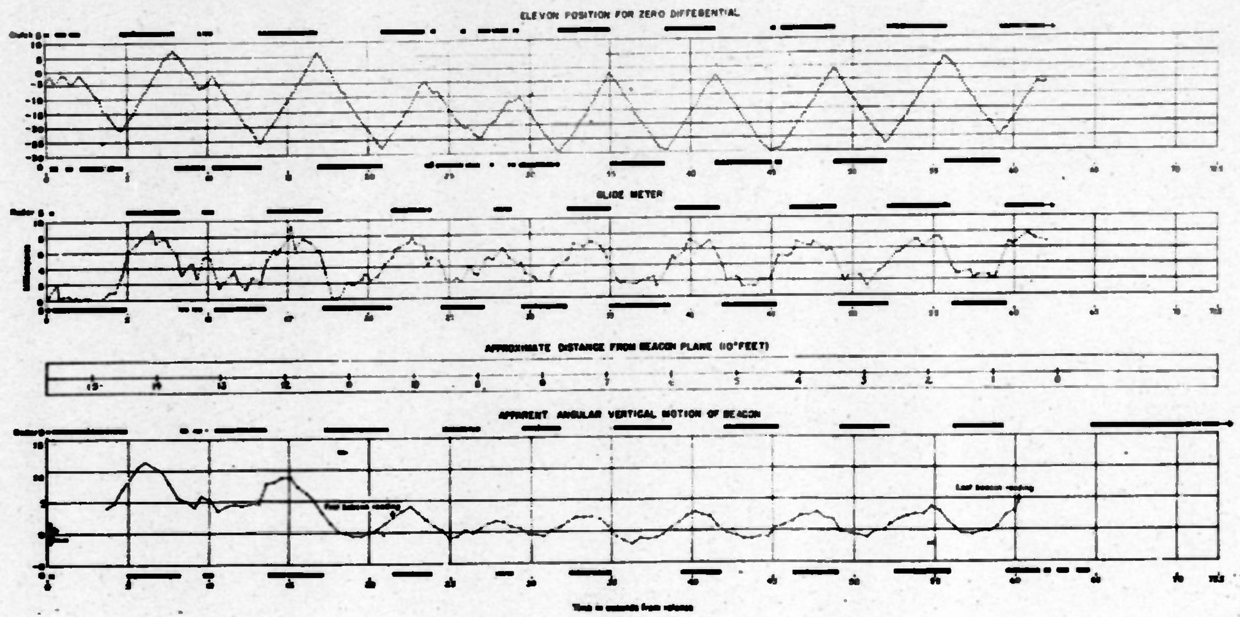

ELEVON POSITION FOR ZERO DIFFERENTIAL
GLIDE METER
APPROXIMATE DISTANCE FROM BEACON PLANE (10³FEET)
APPARENT ANGULAR VERTICAL MOTION OF BEACON
First beacon reading
Last beacon reading

FIG.15

REEL

C 62

FRAME

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ORIG. AGENCY NUMBER

REVISION

AUTHOR(S): Skramstad, H. K.

DIVISION: Guided Missiles (1)

SECTION: Performance (11)

CROSS REFERENCES: Bombs - Flight path (16700); Missiles, Guided - Stability (63650); SWOD Mark 7 (16700); SWOD Mark 9 (16700)

AMER. TITLE: Representation of longitudinal stability and control of homing glide bombs by an electro-mechanical model

FORG'N. TITLE:

ORIGINATING AGENCY: National Bureau of Standards, Washington, D. C.

TRANSLATION:

COUNTRY	LANGUAGE	FORG'N.CLASS	U. S.CLASS.	DATE	PAGES	ILLUS.	FEATURES
U.S.	Eng.		Confd'l		43	15	photos, graphs

ABSTRACT

The report shows the derivation of pitching motion equations for homing glide bombs, describes an electromechanical model, indicates a method of adjusting constants of model, and elaborates on the application of the results obtained to other types of bombs called SWOD Mark 7 and Mark 9 gliders. A servo-control unit and a pitch gyro are described, and it is shown how the error in pitch is considerably reduced by this arrangement.

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Robert Storer

Robert Storer
Chief, Records and Declassification Division

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CD

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28 JAN 2013

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Dear [redacted]:

We have reviewed the enclosed document in consultation with the Department of the Navy and have declassified it in full. If you have any questions, contact me by e-mail at Records.Declassification@whs.mil or by phone at 571-372-0483.

Sincerely,

Robert Storer

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Chief, Records and Declassification Division

Enclosures:
1. MDR request
2. Document 1