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CHARACTERISTICS OF CAVITY RESONATORS MADE OF FUSED QUARTZ

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CHARACTERISTICS OF CAVITY RESONATORS MADE OF FUSED QUARTZ

by

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August 1947

Problem No. 39R09-14

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CONTENTS

ABSTRACT

Previous investigations have shown that the frequency dispersions of cavity resonators are caused by the expansitivity anomalies common to all metals. In an investigation of non-metallic materials it was found that fused quartz also has an anomaly but that the magnitude of this anomaly is several orders smaller than the anomalies of metals. On this basis it is predicted that resonators made of fused quartz will be improved approximately 280 times over the equivalent metal cavity resonators.

It is shown that resonant cavities can be designed employing fused quartz as dielectric or as mechanical boundary. The fused quartz dielectric cavity resonators are approximately half the size of the equivalent air dielectric cavity resonators and are easier to fabricate but have a lower Q.

PROBLEM STATUS

Status of problem at present is in experimental stage.

AUTHORIZATION

The studies herein reported were requested by BuShips Ltr. (918Bb) R-918, 330000-12984, dated 3 Oct. 1946; Improvement of the Stability of Reference Cavities.

CHARACTERISTICS OF CAVITY RESONATORS MADE OF FUSED QUARTZ

INTRODUCTION

The causes for the frequency dispersions exhibited by invar cavity resonators have been investigated and summarized in three previously published NRL reports*. It has been found that the "stable invar" from which the resonators were fabricated has an expansitivity anomaly of 8.7×10^{-8} in/in/°C of temperature cycle. This anomaly causes a frequency dispersion of approximately 2.5 parts in 10^5 for a temperature range of 60 degrees Centigrade. The investigations also revealed that all metals have expansitivity anomalies which make them unsatisfactory for the construction of highly accurate cavity resonators.

In the investigation of other materials, suitable for the construction of cavity resonators of high accuracy, fused quartz was found to have some very desirable characteristics. The advantages of fused quartz for cavity resonators are discussed in this report.

RESULTS OF THE INVESTIGATION

Physical and Electrical Characteristics of Fused Quartz. - Fused quartz has a thermal expansion coefficient of 0.54×10^{-6} in/in/°C over the temperature range from -50°C to 200°C and increases only slightly outside this range. However, its primary advantage as applied to cavity resonators is its apparently low expansitivity anomaly. Although some experimenters have obtained values as great as $15 \times 10^{-9}t$ (t = range of the temperature cycle in degrees centigrade) for the anomaly, most experimenters agree that a value of $1.6 \times 10^{-9}t$ in the range from -50°C to 450°C is probably the true value.†

The frequency deviation for an uncompensated resonator constructed from fused quartz having a coefficient of thermal expansion of 0.54×10^{-6} in/in/°C and op-

* *Wuerffel, H. L. and Schlesinger, L. - NRL Report R-2640, Effect of Temperature on the Resonant Frequency of Modified Model 1541 (TFX-10GA) Cavities, 21 Sept. 1945; NRL Report R-2834, Causes for Frequency Instability in the Type 1605 Resonant Cavities, 3 June 1946; and NRL Report R-3056, Frequency Instability in Cavity Resonators made of Metal, 27 Jan. 1947.*

† *Sosman, Robert B. - The Properties of Silica, New York Chemical Catalog Company, 1927.*

erating at a frequency of 9310 Mc is 0.30 Mc for the range from 0°C to 60°C, or 0.005 Mc/°C. This resonator would probably have a frequency dispersion of 14.9 cycles/°C of temperature cycle (1.6 parts in 10^9), or 0.89 kc for a temperature cycle of 60°C (1 part in 10^7). This is based upon the value of $1.6 \times 10^{-9}t$ in/in anomaly agreed upon by most experimenters. Since the average frequency dispersion of the invar resonators for a 60°C temperature cycle was 0.25 Mc, the fused quartz resonator should exhibit an improvement by a factor of 280. A frequency dispersion as great as 8.38 kc for a temperature cycle of 60°C is highly improbable but cannot be entirely ignored since a few experimenters reported anomalies of $15 \times 10^{-9}t$ in/in.

Fused quartz is more elastic and has a higher tensile and compressive strength than glass. It has a melting point of approximately 1700°C and can be drawn, etched, and plated by the same methods that are used for glass. Fused quartz at temperatures slightly higher than the melting temperature resembles a mass of wriggling threads arranged in a random manner. Slow cooling causes these threads to become aligned and transforms them into crystals. Conversly, rapid cooling fixes them in a random arrangement. If the fused quartz is maintained at a temperature between 1000°C and the melting temperature, crystalization will gradually take place.

The elasticity and transparency of fused quartz is determined by the elasticity and the degree of interlacing of the threads. In clear fused quartz the threads are of a finer texture and are more closely interwoven than they are in translucent fused quartz, thus making the clear material more elastic. When the temperature of the material is increased to a temperature approaching the melting point, and a rod or tube is then formed by the extrusion method, the threads tend to become oriented axially. This causes the material to have a transverse coefficient of expansion slightly different from the longitudinal coefficient. This condition is undesirable. Fused quartz which has been rapidly cooled to a temperature lower than the melting point and then extruded should be selected.

Unlike invar, the electrical and physical properties of fused quartz from different "melts" seem to be identical. Clear fused quartz and translucent fused quartz apparently have the same electrical and thermal properties.

Design Considerations for Fused Quartz Resonators. - Cavity resonators made of fused quartz can be of two types, one type using the material as the shell and the other type using the material as dielectric. One of each type is now being constructed for experimental purposes.

The cavity resonator illustrated by Figure 1 consists of a fused quartz cylinder ground precisely to a precalculated diameter and length and then silver-plated on all surfaces except two coupling ports of precalculated diameter placed diametrically oposite each other a precalculated distance from the ends of the cylinder. This resonator is designed to operate on the TE_{011} mode of oscillation.

The dielectric constant of fused quartz has been measured to be 3.75 with a precision of 0.3 percent in the temperature range from 25°C to 85°C.† Since, for a

† *Ibid.*

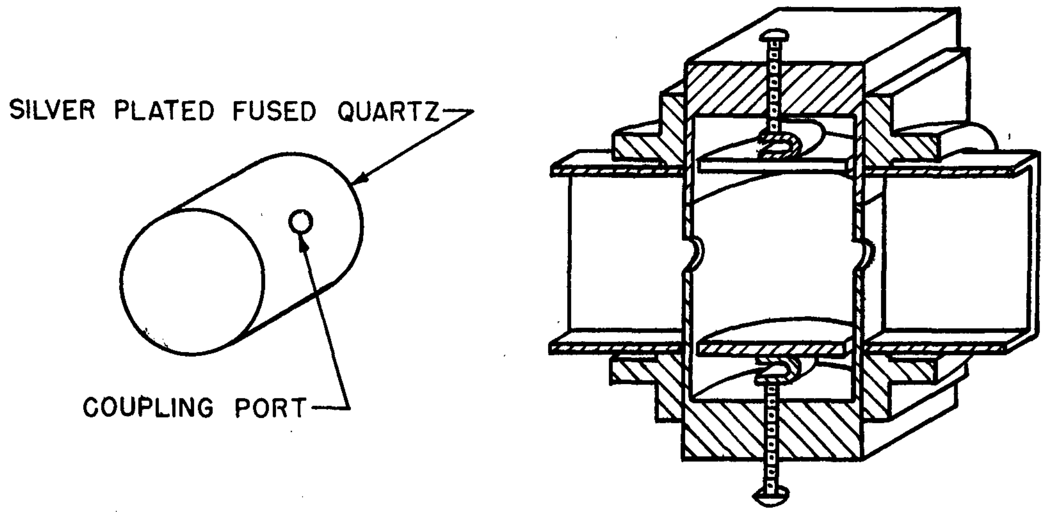

Figure 1. The TE_{011} Mode, Fused Quartz Dielectric, Cavity and Holder

given frequency, the radius and length of a fused quartz cavity resonator are proportional to the inverse square root of the dielectric constant, the resonator will be 0.515 times the size of an equivalent air dielectric resonator. The frequency variations in the resonator caused by variations in dielectric constant with changes in temperature cannot be predicted with an acceptable degree of accuracy, since the present day methods of measuring the dielectric constant are accurate to only 3 parts in 10^3. For an acceptable degree of accuracy the dielectric constant must not vary more than 3 parts in 10^5 throughout the temperature range from 0°C to 60°C. Conversely, by comparing an air dielectric fused quartz resonator with a solid dielectric resonator the dielectric constant of the material may be determined with a precision of 3 parts in 10^5. This appears to offer a new and improved method of measuring dielectric constant.

The construction of an air-dielectric fused quartz cavity resonator is considerably more complex and expensive than the construction of a fused-quartz-dielectric resonator. However, the air-dielectric cavity has the advantage of being tuneable. The inner surfaces of the resonator as well as the outer surfaces must be plated in order to avoid auxiliary modes of oscillation in the walls of the resonator. The inner and outer silvered surfaces must be connected together electrically. The silver plating must be sufficiently thick to permit the end pieces to be bonded to the cylindrical tube and thin enough to prevent separation of the silver plating from the inner surface of the tube. This type of resonator is shown in Figure 2.

The Effect of Fused Quartz on Q. - The Q of a cavity resonator is equal to 2π times the ratio of the maximum energy stored within the cavity resonator to the power lost in the dielectric, in the walls, and through the coupling holes of the resonator, during one electric cycle. For a fused quartz dielectric TE_{011} mode resonator, the maximum energy stored is‡

‡ *Sarbacher, R. I. and W. S. Edson - Hyper and Ultrahigh Frequency Engineering, New York, John Wiley and Sons, Inc., 1943.*

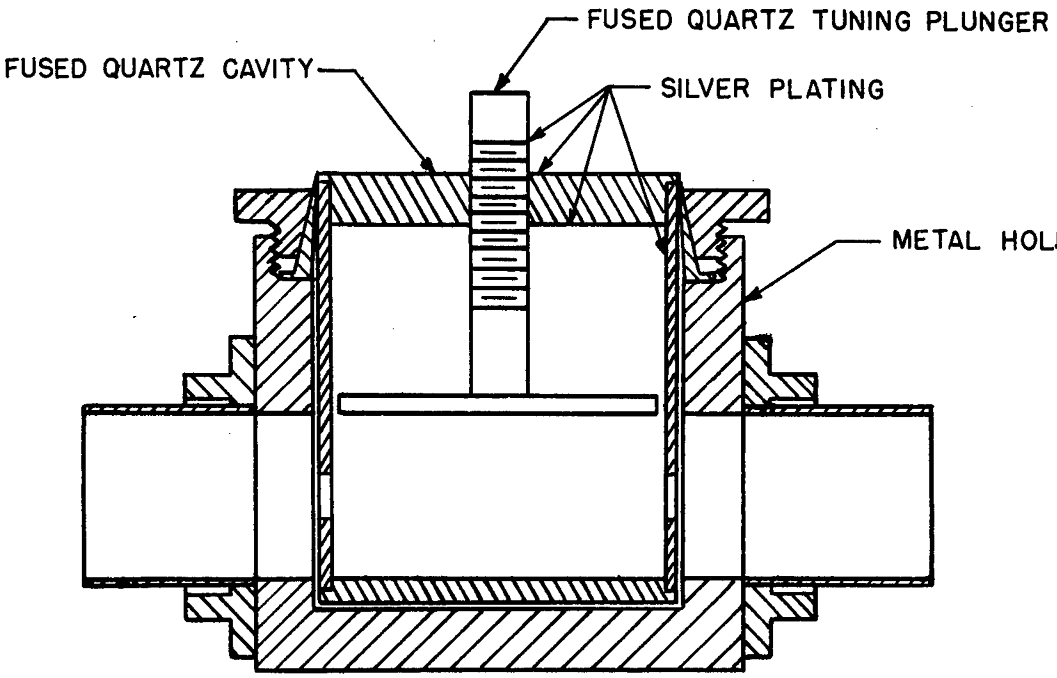

Figure 2. The TE_{011} Mode, Air Dielectric, Cavity and Holder

$$W_e = \frac{k_e e_0}{2} \int_{\phi=0}^{2\pi} \int_{r=0}^{a} \int_{x=0}^{x_0} \left| E'\phi r \right|^2 r\,d\phi\,dr\,dx$$

$$= \left(\frac{a}{r'_{01}}\right)^2 VA'^2 \quad \mu_0 k_e \left(\frac{\omega}{c}\right)^2 \quad J_0^2\left(r'_{01}\right)$$

where

k_e = dielectric constant of medium within the resonator

e_0 = electric inductive capacity of free space

μ_0 = magnetic inductive capacity of free space

ω = $2\pi f$

c = velocity of light

J_0 = Bessel function of zero order of first kind

r'_{01} = root of the first rank of the derivative of J_0

V = volume of the resonator

A' = constant resulting from the general solution of the field equations for the propagation of waves in a cylindrically bounded medium

r = variable distance from the origin to the point of reference in the radial direction

a = radius of the resonator

x = variable distance from the origin to the point of reference in the longitudional direction

x_0 = length of the resonator

ϕ = variable angular distance from a ray chosen arbitrarily for the development to the point of reference

E' = amplitude of the electric intensity vector

Since for small power factors

$$\sigma_1 = p e_0 \mu_0 k_e$$

The power loss in the dielectric will be §

$$P_d = \frac{\sigma_1}{2}\int_{\phi=0}^{2\pi}\int_{r=0}^{a}\int_{x=0}^{x_0}\left|E_{\phi r}\right|^2 r\,d\phi\,dr\,dx$$

$$= p\mu_0^2\,k_e^2\left(\frac{a}{r'_{01}}\right)^2 VA'^2\left(\frac{\omega}{c}\right)^2 J_0^2\left(r'_{01}\right)$$

and the power loss in the resonator walls will be ‡

$$P_w = \eta\left[1/2\int_{x=0}^{x_0}\int_{\phi=0}^{2\pi}\left|H'_{tan}\right|^2 a\,d\phi\,dx + \int_{\phi=0}^{2\pi}\int_{r=0}^{a}\left|H'_{rr}\right|^2 r\,d\phi\,dr\right]$$

$$= \eta\left\{\left[A'^2 A_r J_0\left(r'_{01}\right)\right] + \left[4A'^2\,\beta_{01}^2\left(\frac{a}{r'_{01}}\right)^2 A_x\,J_0^2\left(r'_{01}\right)\right]\right\}$$

where

H' = magnetic intensity amplitude

η = $(\pi f \mu_2/\sigma_2)^{1/2}$

σ_1 = conductivity of the dielectic material

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§ *Stratton, J. A. - Electromagnetic Theory, New York, McGraw - Hill Book Co., 1941*

‡ *Ibid.*

σ_2 = conductivity of the metal walls = 6.1×10^7 mhos per meter for silver

p = power factor of the dielectric material = 8.93×10^{-5} for fused quartz at a frequency of 10^4 Mc.

μ_2 = magnetic inductive capicity of the metal walls

A_r = surface area of the metal cylinder

A_x = surface area of the circular end disks

$$\beta_{01} = \left(\frac{\omega}{c}\right)^2 - \left(\frac{r'_{01}}{a}\right)^2 = \left(\frac{\pi}{x_0}\right)^2$$

If it is assumed that $\mu_2 = \mu_0$ and that $x_0 = 2_a$, the Q of the cavity will be

$$Q = \frac{\omega W_E}{P_w + P_d} = \frac{\left(\frac{\mu_0}{e_0}\right)^{1/4}\left[\left(\frac{\pi}{2}\right)^2 + \left(r'_{01}\right)^2\right]^{5/4} a^{1/2}}{\left[\left(\frac{1}{2\sigma_2}\right)^{1/2}\left(r'^{\,2}_{01} + 2\pi^2\right)\right] + \left\{p k_e \mu_0^{7/4} e_0^{1/4}\left[\left(\frac{\pi}{2}\right)^2 + \left(r'_{01}\right)^2\right]^{3/4} a^{3/2}\right\}}$$

where the power loss through the coupling parts has been neglected.

This yields an unloaded Q of 21,800 for the TE_{011} mode fused-quartz-dielectric resonator as compared with a Q of 30,600 for the equivalent air-dielectric resonator. When the cavity is coupled to an external circuit the Q will be somewhat less than the value given above, depending upon the degree of coupling.

CONCLUSIONS

Because of the low expansitivity anomaly of fused quartz, it is predicted that the use of this material for the construction of cavity resonators will reduce the frequency dispersion to 1.6 parts in 10^9 per degree centigrade of temperature cycle or to 0.9 part in 10^7 for a temperature cycle of 60°C. It has been shown in previous reports 2, 3, and 4 that a frequency dispersion of 2.5 parts in 10^5 can be expected from invar cavity resonators for a temperature cycle of 60°C.

Cavity resonators constructed of fused quartz and not temperature compensated will have a frequency deviation of approximately 5 parts in 10^7 per °C.

Two types of cavity resonators are suggested, one type using fused quartz as the dielectric and the other using the same material for the shell. The solid dielectric resonators would be approximately half the size of the equivalent air dielectric resonators and should be much easier to fabricate but would have approximately 0.71 times the Q of the air dielectric resonators.

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