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NATIONAL DEFENSE RESEARCH COMMITTEE

OF THE

OFFICE OF SCIENTIFIC RESEARCH AND DEVELOPMENT

Progress Report on "Theory of Detonation Waves" to April 1, 1942

by

John von Neuman
Institute for Advanced Study
Princeton, New Jersey

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"Theory of Detonation Waves"

April 1, 1942
by
John von Neuman
Institute for Advanced Study
Princeton, New Jersey

Endorsement from E. B. Wilson, Jr., Chairman Subsection B-1-B, to G. B. Kistiakowsky, Chairman, Section B-1. Forwarding report and noting:

> "This is an important contribution to the fundamental theory of the process of detonation and for the first time throws considerable light on the question of the validity of the Chapman-Jouguet condition. It is shown that it is possible that cases may arise where this condition is not valid. This is important in the problem of calculating theoretically the detonation velocity, pressure, temperature, etc. of an explosive."

Table of Contents

SUMMARY

The mechanism by which a stationary detonation wave maintains itself and progresses through the explosive is investigated.

Reasons are found which support the following hypothesis: The detonation wave initiates the detonation in the neighboring layer of the intact explosive by the discontinuity of material velocity which it produces. This acts like a very vehement mechanical blow ($\sim$ 1,500 m sec^{-1}), and is probably more effective than high temperature.

The velocity of the detonation wave is determined by investigating all phases of the reaction, and not only (as usually done heretofore) the completed reaction. The result shows when the so-called Chapman-Jouguet hypothesis is true, and what formulae are to be used when it is not true.

Detailed computations will follow.

Objectives, Methods and General Principles of this Report

1. The purpose of this report is to give a consequent theory of the mechanism by which a stationary detonation wave maintains itself and progresses through an explosive. Such a theory must explain how the head of the detonation wave initiates the reaction (and the detonation) in the intact explosive, and how a well-determined constant velocity of this wave arises. Both aims can be achieved only after overcoming certain characteristic difficulties.

2. Regarding the first objective, this ought to be said: We do not undertake to give a theory of the initiation of a detonation in general. The viewpoints which we bring up may throw some light on that question too, but our primary aim is to understand the mechanism of the existing stationary detonation wave. Here the detonation of each layer of the intact explosive is initiated by the stationary detonation wave which has already engulfed its neighboring layer.

In gaseous explosions this may be explained by the high temperature of the extremely compressed gas in the detonation zone. In solid explosives this explanation is hardly available: The detonation wave is very narrow in space and moving with very high velocity, so the chemical reaction (which supports the detonation) cannot be treated as instantaneous. Hence the head of the detonation wave corresponds to the just incipient reaction, and contains accordingly only an infinitesimal fraction of gas. The not-yet-reacted solid explosive in it is presumably cold. At any rate the high gas temperatures in the wave head no longer dominate the picture, and therefore cannot detonate the intact explosive.

We shall see that there is more likelihood in this: There must be a discontinuous change of velocity of the bulk of the substance which crosses the wave head. This acts as a violent blow, delivered at velocities of ~1,500 m $\sec^{-1}$ under typical conditions--which is probably at least as effective as temperatures of ~ 2,500°-3,000° centigrade.[1]

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[1] We are thus disregarding the possibility that the detonation is propagated by special kinds of particles (ions, etc.), moving ahead of the wave head. Indeed, if the views which we propose are found to be correct, no such special particles will be needed to explain the detonation wave.

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3. As to the second objective, the present literature is dominated by the so-called Chapman-Jouguet hypothesis. This hypothesis provides a definite value for the velocity of the detonation wave, but its theoretical foundations are not satisfactory. The experimental evidence is altogether reasonably favorable to this hypothesis, but it is not easy to appraise because of the slight information we possess concerning the physical properties of the substances involved under the extreme conditions in a detonation.

All theories on this subject are based on the Rankine-Hugoniot equations of a shock wave, which are actually applications of the conservation theorems of mass, momentum, and energy. The Chapman-Jouguet hypothesis is based on a consideration of these principles in the completed reaction only

We shall apply them to all intermediate phases of the reaction. This necessitates the investigation of the entire family of all so-called Rankine-Hugoniot curves, corresponding to all phases of the reaction. By doing this we shall succeed in determining the velocity of the detonation wave. We find that the Chapman-Jouguet hypothesis is true for some forms of the above-mentioned family of curves, and not for others. We obtain precise criteria, which determine when it is true, and also a general method to compute the velocity of the detonation wave which is always valid.

4. All these discussions are made in a general, as far as feasible qualitative, way, avoiding detailed computations. Specific computations which determine the detonation velocity for definite types of explosives will be made subsequently. In this connection the question is of importance whether the configuration of the family of curves mentioned above, for which the Chapman-Jouguet hypothesis fails to be true, ever occurs for real explosives. This question will be considered together with the first-mentioned computations. In all these discussions the compressibilities and specific heats of solids and gases under detonation conditions are the decisive factors.

The properties of a detonation which has not yet reached its stationary wave stage will also be investigated later. It is to be hoped that this will connect the present theory with the difficult questions of initiating a detonation: of "primers" and "boosters."

The present work is restricted to plane waves in absolutely confined explosives. The effects of spacial expansion on spherical waves and in the case of partial confinement will be considered later.

The author finally wishes to express his thanks to E. Bright Wilson, and to R. H. Kent, for whose valuable suggestions and discussions he is greatly indebted.

Analytic Table of Contents

1-3 Formulation of the Problem

1. In studying the mechanism of a detonation the following approximate picture suggests itself.

Imagine that the detonation wave moves across the explosive in parallel planes, i.e., everywhere in the same direction. Choose this direction as the negative x-axis. Then conditions are constant in all planes parallel to the y, z-plane. The explosive must be confined by some cylindrical boundary, i.e., by the same boundary curve in each one of the above parallel planes. Assume this confinement to be absolutely unyielding. Assume that all motions connected with the detonation take place in the direction of the x-axis, i.e., that there are no transversal components parallel to the y, z-plane.

The limitations of this picture are obvious, but it is useful for a first orientation. It ought to be essentially correct for a heavily confined stick of explosive, detonated at one end surface.[1]

Under these assumptions the process of detonation can be treated one-dimensionally; i.e., we may disregard the coordinates y, z and treat everything in terms of the coordinate x and of the time t.

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[1] It may even be approximately applicable to an unconfined stick of explosive, since the great velocity of the detonation wave--i.e., the brevity of the available time interval--makes the inertia of a solid explosive itself act as a confinement.

In a higher approximation, however, lack of confinement causes corrections which must be determined. This problem was considered by G. I. Taylor and H. Jones in the British reports RC 193 and RC 247 (1941). The authors, however, used a picture of the mechanism of detonation that differs from the one we shall evolve. It is proposed to take up this subject from our point of view in a subsequent report.

2. We restrict ourselves further by assuming also that the detonation wave has reached its stationary stage; that is, that it moves along without any change of its structural details.

Then, among other things, the wave velocity must be constant in time. We now modify the frame of reference by making it--i.e., the origin of the coordinate--move along with the wave. To be specific, let the origin of the x-coordinate be at every moment at the head of the wave. The intact explosive is to the left of this. In other words:

(2-A) The intact explosive occupies the space $x < 0$

(2-B) The detonation--i.e., the chemical reaction underlying it--sets in at $x = 0$. This is the <u>head</u> of the detonation wave.

(2-C) The entire process of detonation--i.e., the chemical reaction mentioned above--occurs in successive stages in the space $x > 0$. The part of this space which it occupies is the <u>reaction</u> zone. The remoter parts are occupied by the completely reacted (burnt) products of the detonation,[1] <u>the burnt gases.</u> This is the region <u>behind</u> the detonation wave.

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[1] This may be true exactly, or only asymptotically.

---

3. The detonation wave has a constant velocity with respect to (and directed toward) the intact explosive, say D. Since the detonation wave is at rest in our frame of reference, this means that the intact explosive has the velocity D with respect to (and directed toward) the fixed head of the wave. That is, at every point $x < 0$ (in (2-A)) the velocity of matter is D. ($D > 0$, in the direction of the positive x-axis!)

At every point $x > 0$ (in (2-C)) we have a velocity of matter $u = u(x)$ and a fraction $n = n(x) > 0, \leq 1$ expressing to what extent the chemical reaction has been completed there. (I.e., at $x > 0$ a unit mass contains n parts of burnt gas and $1 - n$ parts of intact explosive.) During the time dt matter in this region moves by $dx = u\,dt$. So if the reaction velocity (under the physical conditions prevailing at x) is $\alpha = \alpha(x)$ then we have

(3-1) $$\frac{dn}{dx} = \frac{(1 - n)\alpha}{u}$$ [1]

separately stated: $\frac{dn}{dt} = (1-n)\alpha \qquad \frac{dx}{dt} = u$

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[1] In assuming the existence of a well-defined and stationary reaction velocity (i.e., of one which is a numerical constant in time), a definite physical hypothesis is being made: That of (at least) kinetic quasi equilibrium at every point of the reaction. This hypothesis is, however, a natural one to make, unless there is definite evidence to the contrary. And there does not seem to be any so far.

As x increases from 0 toward $+\infty$, n increases from 0 toward 1. According to the details of the situation, n may or may not reach the value (completely burnt gas) for a finite x (cf. footnote 1 on p. 7). We assume for the sake of simplicity that the former is the case, i.e., that the reaction is exactly completed for a finite x.

To conclude the description of condition in our phenomenon: At every point $x < 0$ (in (2-A)) we have the same physical characteristics, say pressure P_o and specific volume V_o. These describe the intact explosive. At every point $x > 0$ (in (2-C)) we have a pressure $P = P(x)$ and a specific volume $V = V(x)$.

The nature of the chemical reaction is expressed by a functional relationship

(3-2) $$\alpha = A\,(n, P, V),$$

where we assume A (n, P, V) to be a known function. The stability of the intact explosive requires

(3-3) $$A\ (0,\ P_o,\ V_o) = 0.$$

The nature of the explosive and its mixtures with the burnt gas is expressed--as far as it interests us--by its caloric equation for each $n \geqq 0,\ \leqq 1$, i.e., by the functional relationship determining its inner energy per unit mass

(3-4) $$e = E\ (n,\ P,\ V)$$

where we assume E (n, P, V) to be a known function.

4-7 Discussion of the Equations. The Chapman-Jouguet Hypothesis

4. The substance which passes from the intact explosive state--the space x [illegible] 0 in (2-A)--through the various phases of the chemical reaction underlying the detonation--the space $x > 0$ in (2-C)--must fulfill the conditions of conservation of mass, momentum and energy. Indeed these are the mechanical conditions for the stationarity of the detonation wave.[1)]

Denote the mass flow--the amount of matter crossing the wave head per second,[2)] i.e., the amount of matter detonating per second,[2)] by μ. Then the conditions of conservation become:

(4-1) [Mass] $D = \mu V_0, \; u = \mu V,$

(4-2) [Momentum] $\mu (D - u) = P - P_0,$

(4-3) [Energy] $\mu \left(\tfrac{1}{2} D^2 + E(0, P_0, V_0) - \tfrac{1}{2} u^2 - E(n, P, V) \right) = uP - D P_0.$

(4-1), (4-2), (4-3) together with (3-1), (3-2), determine everything.

But (3-1), (3-2) merely establish the scale of conversion of the two variables x and n into each other. If we are satisfied to use n--instead of x--as the independent variable, then we can rely upon (4-1), (4-2), (4-3) alone. If we obtain from them $P = P(n)$, $V = V(n)$ and $u = u(n)$, then (3-1), (3-2) may be stated as

$$x = \int_0^n \frac{u \, dn}{(1 - n) \, A(n, P, V)}, \tag{4-4}$$

i.e., as a conversion formula in the above sense.

[1)] This is the classical procedure of Rankine and Hugoniot. The application to all intermediate phases of the reaction for the purpose of a structural analysis was suggested by G. I. Taylor and H. Jones, loc. cit. on p. 6.

[2)] And per unit surface of the y, z-plane.

This is the procedure which we shall use.

5. The algorithm of solving (4-1), (4-2), (4-3) is well known.

We consider P_0, V_0 as given, D, μ as unknown parameters of the problem,[1] and P, V, u as unknown functions of n[2] (4-1) expresses D, u in terms of μ, V (V_0 is given!), so that equations (4-2), (4-3) remain. These are easily transformed into

$$(5\text{-}1) \qquad \frac{P - P_0}{V_0 - V} = \mu^2 ,$$

$$(5\text{-}2) \qquad \tfrac{1}{2}(P + P_0)(V_0 - V) = -E(0, P_0, V_0) + E(n, P, V).$$

Thus μ determines--for every $n > 0, \leqq 1$ - P, V by the implicit equations (5-1), (5-2).

The strange thing in all this is that one unknown parameter--say -- remains undetermined. But the stationary state of the detonation wave of a definite chemical reaction ought to possess--if it exists at all--unambiguously determined characteristics. Hence it should be possible to formulate some further condition which completes the determination of μ.

Before we consider this question, however, let us return to (5-1), (5-2).

Their solution can be illustrated by a familiar graphical method (Figure 1).

Plot the curve (5-2), the Rankine-Hugoniot curve, in the P, V-plane, together with the point P_0, V_0. Denote the angle between the direction of

---

[1] Not dependent upon x or n !

[2] Instead of x, cf. the end of 4.

the negative V-axis and the direction $P_0, V_0 \rightarrow P_1 V$ by φ. Then (5-2) states that P, V lies on the Rankine-Hugoniot curve, and (5-1) states that

$$(5\text{-}3) \qquad \mu = \sqrt{tg\,\varphi}\,, \qquad D = V_0 \sqrt{tg\,\varphi}$$

Thus the unknown parameter φ is substituted for the unknown parameter μ (or D).

This picture should be drawn for each $n > 0_1 \leqq 1$, with the same P_0, V_0 and φ but varying Rankine-Hugoniot curves, and so the corresponding P, V are obtained.[1]

Observe that all these considerations are also valid for n = 0, in which case they describe the conditions under which a discontinuity in our substance can exist--without making use of any chemical reaction at all. This phenomenon is particularly important in gases and in liquids. It is known as shock wave, and is actually an essential component of the theory of the detonation wave.

6. While φ is undetermined, it is not entirely arbitrary (Figure 2).

To begin with, (5-3) necessitates $tg\,\varphi > 0$, so φ must lie in the Quadrant I or III. We have accordingly:

Quadrant I: $P > P_0$, $V < V_0$ hence $\mu < D$.

Quadrant III: $P < P_0$, $V > V_0$ hence $\mu > D$.

In Quadrant I the burnt gases are carried along with the detonation wave (i.e., $D - \mu > 0$); their density and pressure are higher than those in the explosive. This is detonation proper.

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[1] And the $\mu = V \sqrt{tg\,\varphi}$

In Quadrant III the burnt gases are streaming out of the explosive (i.e., $D - u < 0$); their density and pressure are lower than those in the explosive. This is the process commonly known as burning.

We are interested in the process of detonation only, so we assume φ in Quadrant I.

Now our treatment of the chemical reaction (cf. in particular[1]) on page 8) compels us to postulate for each $n > 0, \leq 1$ the existence of a well-defined physical state, fulfilling the requirements of the theory of 4-5. Therefore φ must be at least the angle φ_n of the tangent from P_0, V_0 to the Rankine-Hugoniot curve. For $\varphi < \varphi_n$ the line on which P, V should lie does not intersect that curve at all. For $\varphi \geqq \varphi_n$ the situation is usually this: If $\varphi = \varphi_n$ then there exists exactly one value for P, V, the tangent point X; if $\varphi > \varphi_n$ then there exist exactly two values for P, V, the lower intersection point Y and the upper intersection point Z. We assume these qualitative conditions--as exhibited by Figure 2--to hold for the Rankine-Hugoniot curves of all n.

7. These considerations were originally applied to n = 1 only. (Cf. however footnote 1 on p. 10) The lower limit for φ is then $\varphi = \varphi_1$, the direction of the tangent to the Rankine-Hugoniot curve n = 1.

The classical theory of detonation is based on the assumption that the actual value of φ is this lower limit φ_1. This is the so-called hypothesis of Chapman and Jouguet.[1] Various theoretical motivations have

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[1] For this, and several references in what follows, cf. the report of G. B. Kistiakowsky and E. Bright Wilson, Jr., "The Hydrodynamic Theory of Detonation and Shock Waves," OSRD (1941). This report will be quoted as K. W. Concerning the Chapman-Jouguet hypothesis, cf. K. W., Sec. 5.

been proposed for this hypothesis, mostly based on considerations of stability.1) The experimental evidence is not easy to appraise, since the question of the validity of this hypothesis is intertwined with uncertainties concerning the caloric equations of the substances involved under the extreme conditions in a detonation.

We propose to carry out a theoretical analysis of the Chapman-Jouguet hypothesis by a study of the Rankine-Hugoniot curves for all $n \geqq 0, \leqq 1$, i.e., for all intermediate phases of the reaction. It will be seen that a proper understanding of the situation with the help of the curve $n = 1$ alone--as attempted always heretofore--is impossible. It is necessary to consider the family of all curves $n \geqq 0, \leqq 1$. By doing this we shall supply the missing condition mentioned in 5 (after (5-1), (5-2)), i.e., determine φ (that is μ, D).

For certain forms of the family of all curves $n \geqq 0, \leqq 1$ the Chapman-Jouguet hypothesis will turn out to be true; for other forms it is false and another (higher) value of φ will be found.

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1) For a summary cf. K. W., Sec. 5, pp. 8-11.

8-9 General Remarks

8. The discussion ar the end of 6 showed that we must have $\varphi \geqq \varphi_n$ for all $n > 0_1, \leqq 1$. Now the Chapman-Jouguet hypothesis is based on the considerations of the Rankine-Hugoniot curve $n = 1$ alone, and it consists of assuming $\varphi = \varphi_1$. Hence it is certainly unacceptable unless $\varphi_1 \geqq \varphi_n$ for all $n > 0_1, \leqq 1$. I.e.: *Unless φ_n assumes its maximum (in $0 \leqq n \leqq 1$) for $n = 1$*. Or in a more geometrical form: *Unless every line issuing from the point P_0, V_0 which intersects the curve $n = 1$ also intersects all other curves $n \geqq 0_1, \leqq 1$.*

We illustrate this condition by exhibiting two possible forms of the family of all curves $n \geqq 0_1, \leqq 1$ (Figures 3, 4):

The condition is fulfilled in Figure 3, but not in Figure 4.

From a purely geometrical point of view these two forms are not the only possible ones for the family of curves $n \geqq 0_1 \leqq 1$. We shall not attempt to give here a complete enumeration. The main question in this connection is, however, which forms of this family occur for real explosives. We shall take up this question in a subsequent investigation.

9. We shall obtain the missing condition for φ repeatedly mentioned before. This derivation will differ essentially from the existing analyses, since those make use of the curve $n = 1$ only. (For this, and for some of the remarks which follow, cf. footnote 1 on p. 14). There is nevertheless one element in those discussions which deserves our closer attention.

The discussions referred to treat the upper intersection points Z and the lower intersection points Y separately.[1] That is, in order to establish the Chapman-Jouguet hypothesis--i.e., the tangent point X--the Z and the Y are ruled out by two separate arguments. We saw in 8 that the Chapman-Jouguet hypothesis cannot be always true; hence the arguments in question cannot be conclusive.[1] Our considerations will prove that the exclusion of the Z on the curve n = 1 must be maintained, but not that of the Y.

Thus the first part of our analysis will be restricted to the curve n = 1 and on it to the upper intersection points Z: proving that they cannot occur. In doing this we shall use a line of argument which is closely related to the traditional one referred to above, even to the detail of being based on a comparison of the detonation velocity to the sound velocity in the burnt gas behind it.[2] In spite of this similarity, however, the two argumentations are not the same: The traditional one is, as mentioned before, essentially one of stability,[2] while ours is purely cinematical. It is actually closer to a viewpoint which has been put forward recently by G. I. Taylor.[3]

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[1] Cf. our Figure 2, or K. W., Figure 5-1, p. 8

[1] Cf. also K. W., bottom of p. 11.

[2] Cf. K. W., pp. 8-9

[3] British report of G. I. Taylor, "Detonation Waves," RC 178 (1941). This report will be quoted as T.

In other parts of our analysis we shall have to consider all curves $n \gtreqless 0, \lesseqgtr 1$ and their positions relative to each other. This is essentially different from the existing analyses referred to above.

On one occasion we shall consider the possibility of a shock wave in the reaction zone and the change of entropy which it causes. This phenomenon has been investigated in connection with the Chapman-Jouguet hypothesis,[1] but in a different arrangement: In a stability consideration, in order to exclude (on the curve n = 1) the lower intersection points Y also, to prove the Chapman-Jouguet hypothesis. Our procedure again is not one based on stability, and besides we use it for a different purpose. Furthermore we shall find that under certain conditions the lower intersection points cannot be disregarded.

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[1] Cf. K. W., pp. 9, 11.

10-12 Conditions in the Completely Burnt Gas

10. Let us consider the state P, V at n = 1, which is a point on the Rankine-Hugoniot curve n = 1. This is the back end of the reaction zone, where the chemical reaction is just completed (cf. (2-C)). We use the frame of reference in which the detonation wave--and hence this particular point too--is at rest. Consider now the conditions behind this point P (Figure 5):

In order to complete the picture we must remember that the explosive was assumed to be completely confined (cf. 1); hence it is logical to assume a back wall behind it, say at Q.[1] This back wall is at rest in the original frame of reference in which the intact explosive is at rest; in our present frame of reference the intact explosive has the velocity D (cf. the beginning of 3), and therefore the same is true of the back wall Q.

So the burnt gases lie between the points P_1 Q, which move with the velocities 0, D, respectively. The gases stream across P with the velocity $\mu = V\sqrt{\mathrm{tg}\,\varphi}$ (cf. footnote 1 on p. 12), while Q is an enclosing wall.

The phenomenon is certainly not stationary in P_1 Q, since P and Q recede from each other; but we assumed it to be stationary in the reaction zone 0, P. In P_1 Q, on the other hand, no chemical reaction is going on: The gases there are completely burnt.

---

[1] If the explosive were unconfined at Q, the conclusions of these would probably be valid *a fortiori*, since they demonstrate the necessity of a rarefaction wave in the burnt gases under certain conditions (cf. 12). But we prefer to restrict ourselves to the case of absolute confinement.

The question is whether the necessity of fitting a gas-dynamically possible state of motion between P and Q imposes any restriction on the state at P, i.e., on the intersection point on the curve $n = 1$.

An answer can be obtained mathematically by integrating the differential equations of gas dynamics in P_1Q.1) But it can also be found by more qualitative considerations, which we shall now present.

11. Consider the stretch O_1Q in the original frame of reference in which the intact explosive and the back wall at Q are at rest. Throughout the entire period of time in which the detonation progressed from its start at Q to its present head at O the explosive to the left of O remained intact. Hence no substance was transferred from one side of O to the other,2) and so the total mass in O_1Q was not changed. Therefore the average density in O_1Q was not changed either--i.e., it is now, when the wave head is at O, the same as it was when the detonation started at Q--i.e., the same as in the intact explosive.

The specific volumes in O_1P are the V on the line of Figure 2, from P_o, V_o to X or Y or Z, as the case may be; hence all smaller than the specific volume V_o of the intact explosive. I.e., the density in O_1P is everywhere greater, as in the intact explosive.

Hence the average density in P_1Q is lower, as in the intact explosive, and <u>a fortiori</u> as at P. Consequently we can assert:

<u>The density at P is greater than in some places in P_1Q.</u>

---

1) This is actually contained in the computations of T., pp. 1-4.

2) O is now at rest with respect to the intact explosive!

Let P^1 be the place (in P_1Q) where the density begins to fall below its value at P.[1] The density, i.e., the specific volume V, is therefore constant in P_1P^1.

12. We now return to the frame of reference of 3-5 and 10, where the wave front 0—and so the geometrical locus of the reaction zone 0_1P—is at rest.

V is constant in P_1P^1; hence the same is true of u[2] and P,[3] and, along with P_1V, of the sound velocity c.[4]

By its definition, P^1 is the head of a rarefaction wave looked at from $0_1 P_1 P^1$. Hence moves with sound velocity towards $0_1 P_1 P^1$; i.e., the velocity of P^1 is $u-c$. We know that we may take this $u-c$ at P instead of P^1, and that $u-c$ taken at P is constant in time.[5] Hence $u-c < 0$ would imply that P^1 will reach and enter the zone 0_1P in a finite time, thus disturbing its stationarity. Hence $u-c \geq 0$, i.e., $c \leq u$ at P.

---

[1] P^1 is P, or to the right of P.

[2] Because of the conservation of mass, i.e., (4-1) or footnote 1 on p. 12.

[3] Because of the adiabatic law, which holds throughout P_1Q, since there are no chemical reactions or shock waves there.

[4] We measure c with respect to the gas, which itself moves with the velocity u.

[5] The zone 0_1P is stationary!

Let us now consider Figure 1, taking n = 1 for its curve. $u = V\sqrt{\operatorname{tg}\varphi}$ (cf. footnote 1 on p. 12), and c too can be expressed in terms of this figure. Denote the angle between the direction of the negative V-axis and the direction of the tangent of the curve at P_1V by χ. We claim that $c = V\sqrt{\operatorname{tg}\chi}$. This can be established by a direct thermodynamical computation,[1] and also by a more qualitative argument which we shall now give.

The line $P_0, V_0 \rightarrow P_1 V$ represents a detonation wave ending with the burnt gas. Therefore a line $P_1' V' \rightarrow P_1 V$ where both points $P_1' V'$ and $P_1 V$ lie on the curve n = 1, represents a discontinuity which begins and ends in the burnt gas, i.e., a shock wave in it.[2] If $P_1' V'$ moves very close to $P_1 V$, then this direction tends to the tangent at $P_1 V$. And the shock wave tends to a very small discontinuity, i.e., it becomes very weak. Now the velocity of a very weak disturbance is very nearly sound velocity. Hence c, χ are connected in the burnt gas in the same way as D, φ were in the intact explosive. So the $D = V_0\sqrt{\operatorname{tg}\varphi}$ of (4-1) becomes $c = V\sqrt{\operatorname{tg}\chi}$.

Thus our $c \lesseqqgtr u$ becomes $V\sqrt{\operatorname{tg}\chi} \lesseqqgtr V\sqrt{\operatorname{tg}\varphi}$, i.e., $\chi \lesseqqgtr \varphi$. In other words:

The direction $P_0, V_0 \rightarrow P_1 V$ cannot be less steep than the direction of the tangent to the Rankine-Hugoniot curve n = 1--i.e., the direction of the curve itself--at $P_1 V$.

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[1] Cf. K. W., pp. 9-11.

[2] Involving no chemical reaction!

One look at Figure 2 suffices to show that this means the exclusion of the upper intersection points Z on the curve $n = 1$.1) We restate this:

The state $P(n)$, $V(n)$ at $n = 1$ which lies on the curve $n = 1$ cannot be an upper intersection point.

---

1) As mentioned in footnote 3 on p. 16, the computations of T., pp. 1-4, prove the same thing. The state in P, Q (cf. Figure 5) for the tangent point X or a lower intersection point Y (cf. Figure 2) is exhibited by Figures 1-b and 1-a, respectively, in T.

13-15 Discontinuous and Continuous Changes in the Reaction Zone. Mechanism which Start the Reaction

13. We turn next to the consideration of the Rankine-Hugoniot curve for all $n \geqq 0, \leqq 1$.

The state at the point of the reaction zone with a given n is characterized by the data P(n), V(n) and $\mu(n)$ (cf. the end of 4). These points P(n), V(n) for $n \geqq 0, \leqq 1$ form a line Λ in the P, V -plane, the line representing the successive states across the reaction zone (cf. the end of 12).

This line Λ must intersect the curve of every $n \geqq 0, \leqq 1$. If this intersection occurs in a (unique) tangent point, then that is P(n), V(n); if it occurs in two intersection points--the upper and the lower--then one of those is P(n), V(n). Let us now follow the history of P(n), V(n) as n increases from 0 to 1.

To begin with, there is no absolute reason why P(n), V(n) should vary continuously with n. Consider now an n_0 where they are discontinuous. Follow the path of matter (in the detonation) which crosses that point of the reaction zone. It moves in the direction of the progressing chemical reaction, i.e., of an increasing n. The discontinuity of P(n), V(n) necessitates that these quantities have different limiting values, as n_0 is approached on the incoming side from below, and on the outgoing side from above. Both limiting positions of P(n), V(n) must be intersections of Λ with the curve n_0. So they are the upper and the lower intersection point;[1] the question is only: which is which?

The two intersection points of Δ on the curve n_0 represent the two sides of a shock wave;[2] indeed our above description of the situation at n_0 makes it clear that there is a shock wave at the point n_0 of the reaction zone.

Now it is well known--for thermodynamical reasons, but also intuitively plausible--that in a shock wave the high-pressure area always absorbs the substance of the low-pressure area, i.e., that matter passes from the low-pressure state into the high-pressure state.[3]

In the present case matter moves in the direction of increasing n (cf. above). Therefore the lower intersection point is the limiting position when n_0 is approached from below, and the upper intersection point is the limiting position when n_0 is approached from above.

Summing up:

The point P(n), V(n) varies continuously, except for possible jumps from the lower to the upper intersection point which occur--if at all--always in this direction with increasing n.

---

[1] And we do not have the tangent position!

[2] Cf. a similar discussion in 12.

[3] The thermodynamical reason is that the entropy is higher on the high-pressure side. Cf. K. W., p. 11. For a discussion of this property of shock waves cf. e.g. J. W. Rayleigh, "Aerial Plane Waves of finite Amplitude," Scientific Papers, Vol. 5, Cambridge (1912), particularly pp. 590-591.

14. Let us consider the conditions at $n = 0$, i.e., at the head of the wave. The state there adjoins the P_o, V_o of the intact explosive which lies on the curve $n = 0$.

Assume first that the variation of the $P(n)$, $V(n)$, at this point is continuous, i.e., that P_o, V_o is the limiting position when n approaches 0 from above.

In this case there is no agent to *start* the chemical reaction which supports the detonation. This reaction ought to set in, and quite vehemently at $n = 0$, i.e., for small values of $n > 0$. Or, if we use the independent variable x: at $x = 0$, i.e., for small values of $x > 0$. Now the assumed continuity means that the conditions in this critical zone--the beginning of the reaction zone--differ only insignificantly from those at P_o, V_o, i.e., throughout the intact explosive (in $x < 0$). Thus the reaction cannot start in this region. If it did--for any reason whatever--it should *a fortiori* have done so in the intact explosive (in $x < 0$). There was much more time available there, and yet, by assumption, no reaction!

This argument is qualitative, to be sure, but it is quite easy to amplify it mathematically.[1]

Thus we must have a discontinuity at $n = 0$. By the result of 13 this means:

---

[1] Consider the formula (4-4) which expresses x in terms of n. If we have continuity, i.e., if $n \to 0$ implies $P \to P_o$, $V \to V_o$, then (3-3) yields $A(n, P, V) \to 0$. So we must expect divergence of the integral (4-4) (near $n = 0$), and failure to obtain an acceptable x.

This is, of course, not essentially different from the text's verbal argument: We cannot make $x \to 0$--nor even $x \to$ finite for $n \to 0$--i.e., the reaction cannot be started on any finite interval.

P_o, V_o must be the lower intersection point, and as n increases (from 0 on), $P(n)$, $V(n)$ immediately jumps to the upper intersection point--say P^0, V^0--and goes on continuously from there.

Thus the reaction zone sets in with a shock wave, an abrupt increase of P, and with it an equally abrupt decrease of V (cf. (5-1)) and of u (cf. footnote 1 on p. 12).

These abrupt changes of P, V may start the reaction, particularly because they imply usually an increase of the temperature. But the change of u is even more remarkable. This decrease from D to u^0 means, of course, that the intact explosive (in $x<0$) receives a vehement blow, delivered by the wave head with the velocity.

$$w = D - u^0.$$

Using (5-1), (5-3) and footnote 1 on p. 12, we see that

$$w = D - u^0 = \left(1 - \frac{V^0}{V_o}\right)D = (V_o - V^0)\mu = \sqrt{(P^0 - P_o)(V_o - V^0)}. \tag{14-1}$$

This velocity is smaller than, but in the order of magnitude of, the detonation velocity D. It is comparable to the thermic agitation of a very high temperature; indeed it may be more effective, since it is delivered with one systematic velocity, and not in statistical disorder.[1] discontinuity of the velocity provides at any rate the mechanism needed to start the reaction.[2]

---

[1] For a typical high explosive, like TNT, D ~ 6,000 m $\sec^{-1}$, w ~ 1,500 m $\sec^{-1}$, so that the corresponding temperature would be ~ 2500° - 3000° centigrade.

[2] This view would necessitate a modification of (4-4), but in a favorable sense: towards even smaller changes of x in the neighborhood of n = 0.

15. We know from 13 that $P(n)$, $V(n)$ is immediately after a discontinuity at an upper intersection point. We also know from 12 that the latter cannot be the case at $n = 1$. We have:

$P(n)$, $V(n)$ is continuous at $n = 1$, in the position indicated in 12.

We are now informed about the behavior of $P(n)$, $V(n)$ at $n = 0$ and $n = 1$, and also at discontinuities for $n > 0, < 1$. There remains only the necessity of discussing its behavior when it varies continuously near an $n > 0, < 1$. We are particularly interested how--if at all--it can change under these conditions from an upper intersection point to a lower one, or *vice versa*.[1]

---

[1] The first change (for n increasing) actually excludes a discontinuity according to the result of 13.

Let us therefore consider such a change. If $P(n)$, $V(n)$ changes at $n = n_o$ $(n_o > 0, < 1)$ continuously from an upper intersection point to a lower one, or *vice versa*, then it is necessarily a tangent point at n_o.

At this juncture it becomes necessary to introduce the *envelope* of the family of all curves $n \geq 0, \leq 1$. This envelope may or may not exist (cf. Figures 4 and 3 for these two alternatives, respectively). At any rate, if the tangent point which we are now considering is not on the envelope,[2] then the position of the curves for the n near to n_o is this (Figure 6).

---

[2] This is meant to include the case where the envelope does not exist.

Thus the curves with $n > n_0$ are all on one side of the curve and the curves with $n < n_0$ are all on the other side. Consequently the line Λ does not intersect that one of these two curve systems which is on the concave side of the curve n_0. But Λ must intersect the curves of all n (cf. 6), So we have a contradiction.

Hence the envelope must exist, and our tangent point must lie on it. At this point Λ is tangent to the curve n_0. But the envelope has at each one of its points the same tangent as that curve n of the family which touches it there. Hence Λ is also a tangent for the envelope.

Summing up:

If $P(n)$, $V(n)$ changes at $n = n_0$ ($n_0 > 0, < 1$) continuously from an upper intersection point to a lower one (cf. footnote 1 on p. 27), then:

A) The envelope of the family of all curves n exists.

B) The point $P(n_0)$, $V(n_0)$ lies on the envelope.

C) The line Λ is at this point tangent to both the curve n_0 and to the envelope.

It is easy to visualize that if the point $P(n_0)$, $V(n_0)$ fulfills B), C), then the change from an upper to a lower intersection, or *vice versa*, can be effected continuously (Figure 7, Cases 1, 2).

But it is also possible to have B), C), and nevertheless no such change (Figure 7, Cases 3, 4).

16 Conclusions

16. The results of 12, 13, 14 and the two results of 15 permit us to form a complete picture of the variations of

By 14 $P(n)$, $V(n)$ begins, for small $n > 0$, as an upper intersection point. If it stays one for all $n > 0, < 1$, then 12 and the first result of 15 necessitate that it terminate at $n = 1$ as a tangent point. In this case $\mathcal{A}$ is a tangent to the curve $n = 1$.

If the above assumption is not true, then $P(n)$, $V(n)$ must change (as n increases) from an upper intersection point to a lower one, for some $n > 0, < 1$. By 13 this must occur continuously (cf. also footnote 1 on p. 27), and by the second result of 15 we have A)-C) there: The envelope exists, and $\mathcal{A}$ is tangent to it at the point in question. In this case $\mathcal{A}$ is tangent to the envelope.

So we see:

$\mathcal{A}$ <u>is tangent either to the curve $n = 1$ or to the (then necessarily existing) envelope</u>.

Since the line $\mathcal{A}$ comes from the given point P_o, V_o this condition leaves only a finite number of alternatives for it, i.e., for its angle φ with the direction of the negative V-axis. So we have obtained essentially the missing condition, referred to at the beginning of 9 and before.

Indeed, when the envelope does not exist (as in Figure 3), then we see that $\mathcal{A}$ must be tangent to the curve $n = 1$, proving the Chapman-Jouguet hypothesis. We restate this:

If the envelope does not exist--i.e., if the curves for all n do not intersect each other (cf. Figure 3)--then the Chapman-Jouguet hypothesis is true.

On the other hand we saw a case in 8 in which the Chapman-Jouguet hypothesis cannot be true. Then the envelope must exist, and 1 must be tangent to it.

We leave it to the reader to discuss the details of the solution in various typical cases, e.g. in Figure 3 (the Chapman-Jouguet hypothesis is true), and in Figure 4 (the Chapman-Jouguet hypothesis is not true). The results referred to at the beginning of 16 are all that is needed in all these cases.

Figure 1 (p. 11)

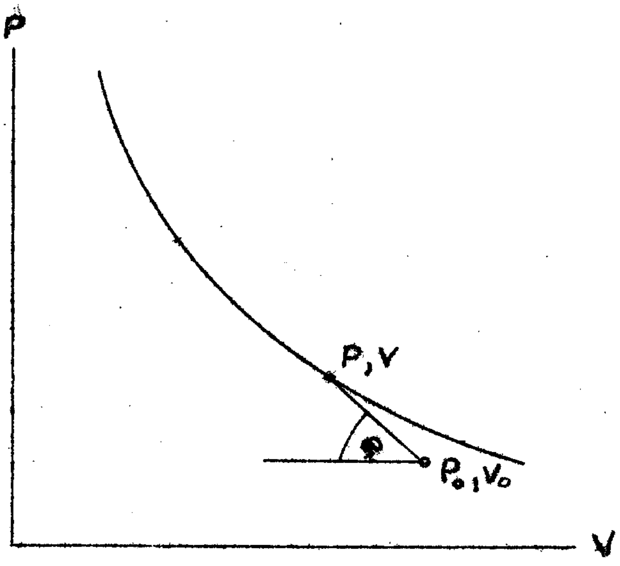


Figure 2 (p. 12)

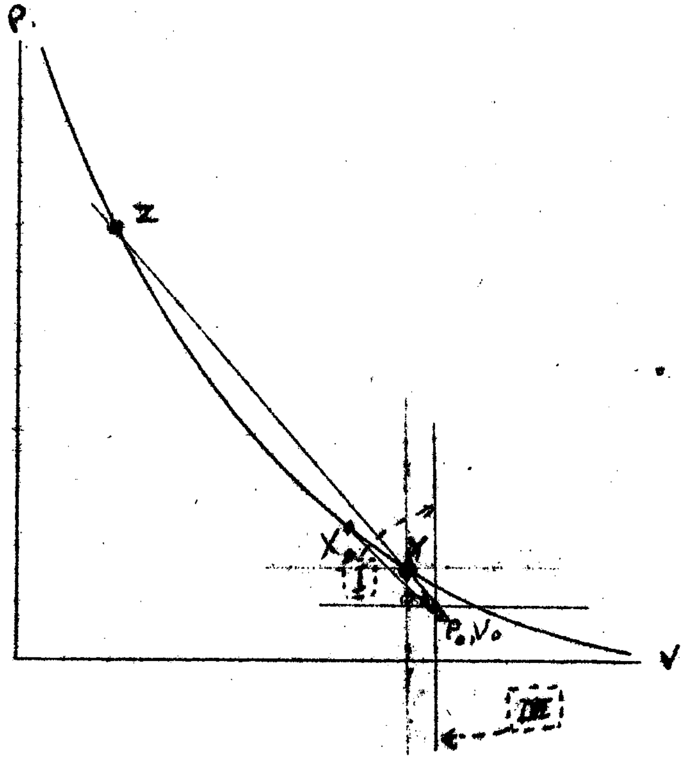


Figure 3 (p. 15)

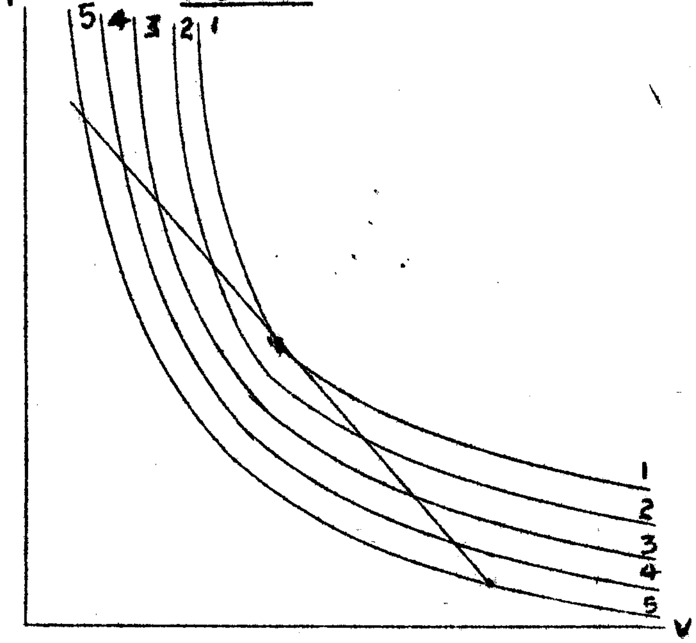
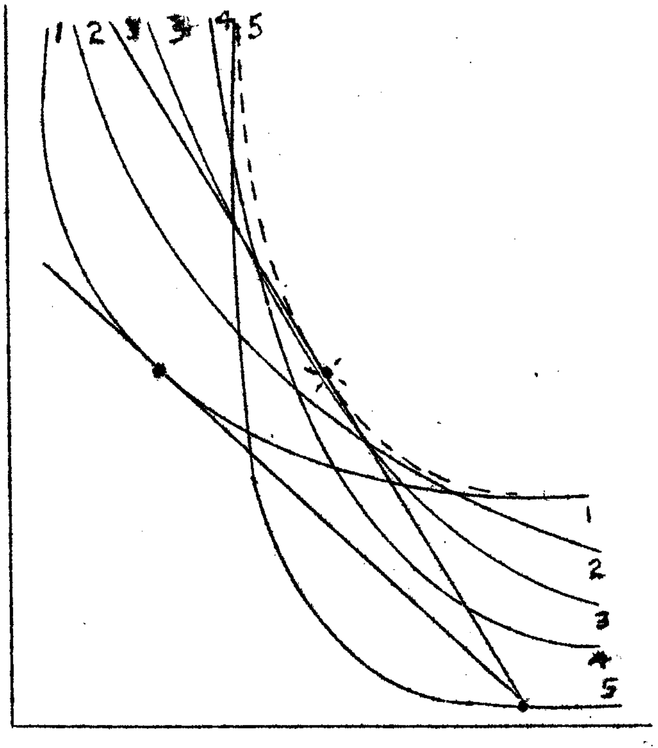


Figure 4 (p. 15)



Explanation of Figures 3, 4:

The curves 1-1, 2-2, 3-3, 4-4, 5-5 are Rankine-Hugoniot curves, corresponding to successive values of n decreasing from 1 to 0. (1-1 is n = 1, 5-5 is n = 0.)

The line  occupies the extreme (lowest) position which intersects the curve 1-1 (i.e., n = 1). This is the position determined by the Chapman-Jouget hypothesis.

In Figure 3 this line intersects all curves. In Figure 4 it does not; the extreme (lowest) line which intersects all curves is 

In Figure 4 the family of curves has an envelope, marked

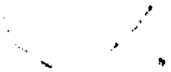


Figure 5 (pp. 18-20)

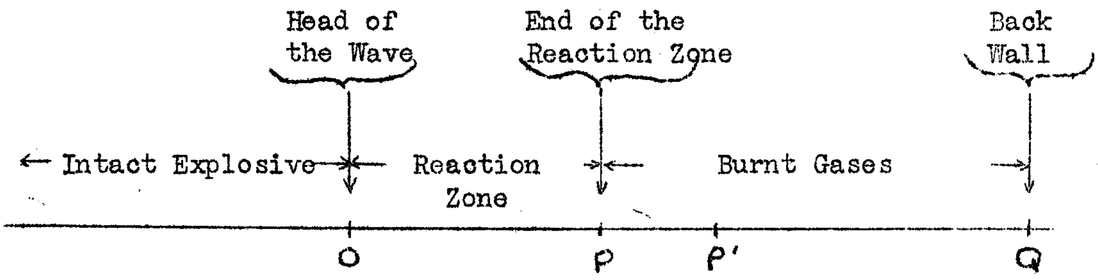


Figure 6 (p. 27)

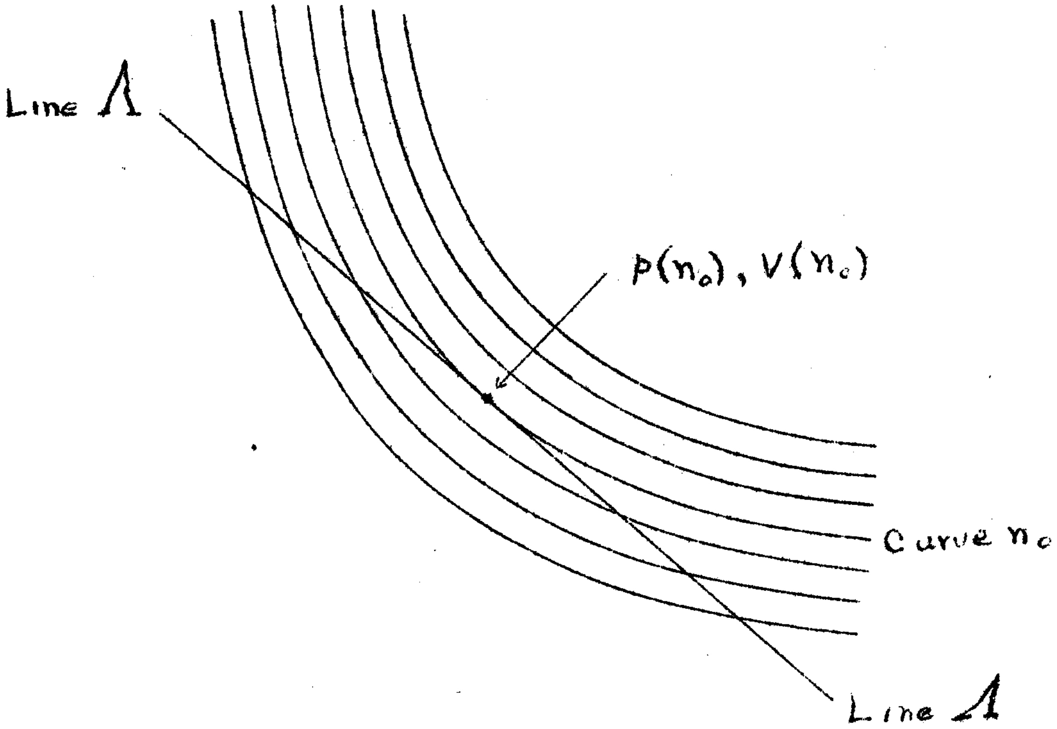
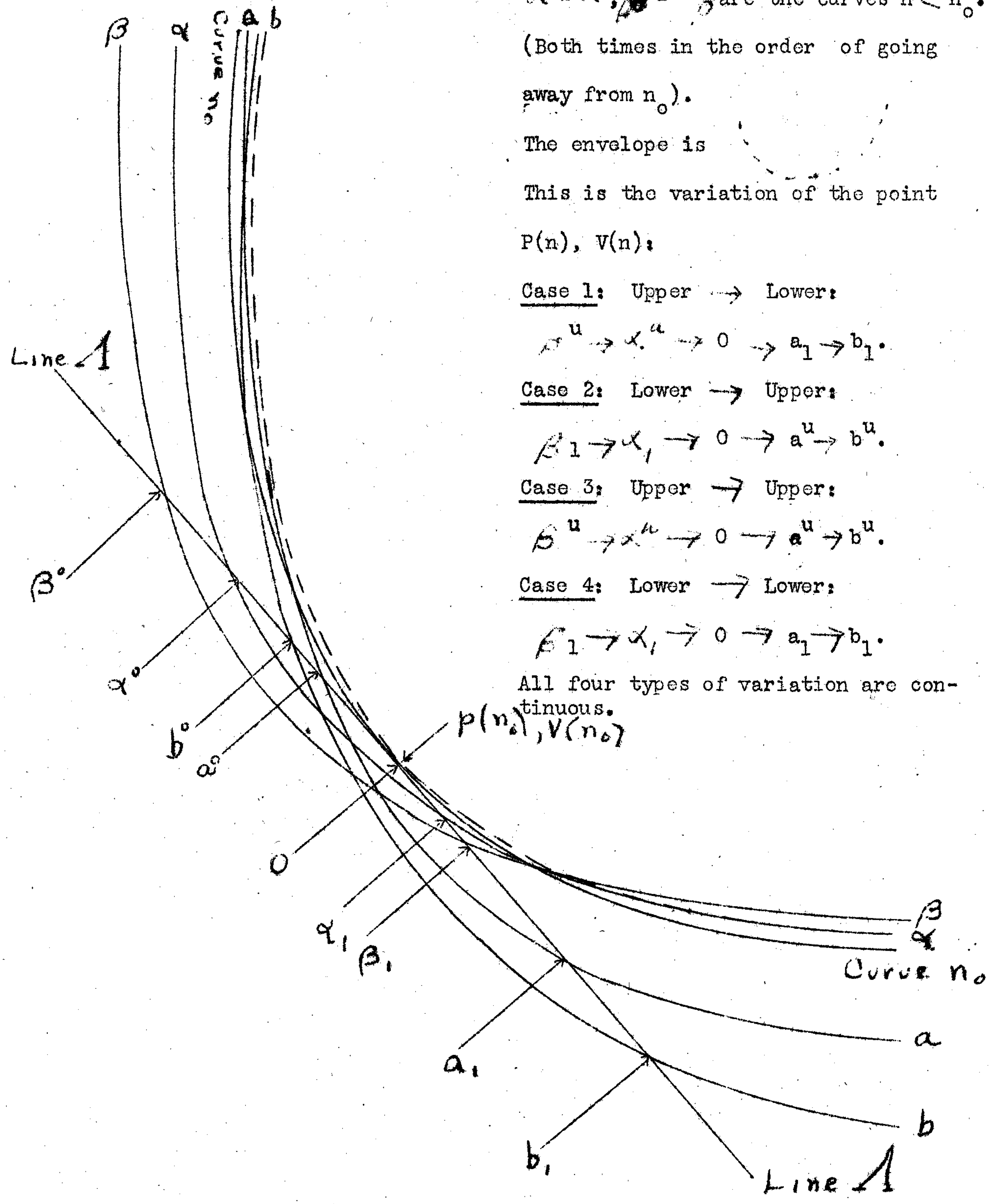


Figure 7 (p. 28)



Explanation of Figure 7:

a-a, b-b are the curves $n > n_0$.

$\alpha - \alpha$, $\beta - \beta$ are the curves $n < n_0$.

(Both times in the order of going away from n_0).

The envelope is

This is the variation of the point $P(n)$, $V(n)$:

Case 1: Upper → Lower:

$\beta^u \to \alpha^u \to 0 \to a_1 \to b_1$.

Case 2: Lower → Upper:

$\beta_1 \to \alpha_1 \to 0 \to a^u \to b^u$.

Case 3: Upper → Upper:

$\beta^u \to \alpha^u \to 0 \to a^u \to b^u$.

Case 4: Lower → Lower:

$\beta_1 \to \alpha_1 \to 0 \to a_1 \to b_1$.

All four types of variation are continuous.

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