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POINT STRESS LAMINATE ANALYSIS

Dr. D. L. Reed

Advanced Composites Division
Air Force Materials Laboratory
Wright-Patterson Air Force Base, Ohio

GENERAL DYNAMICS
Fort Worth Division

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Prepared by

Dr. D. L. Reed

Prepared for

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ABSTRACT

This report presents a point stress analysis of a laminate under inplane loads, moments, and temperature effects. The formulation presents the usual lamination theory whereby the laminate constitutive relation is derived from the constitutive relation for each layer in the laminate. Once the laminate relation has been formulated, it is used to determine midplane strains and curvatures which arise due to inplane stress and moment resultants. The midplane strains and curvatures are then used to determine the strains and thus the stresses in each layer of the laminate. The thermal analysis assumes a constant temperature through the thickness. Inplane stress and moment resultants caused by the temperature are calculated and added to the other known loads.

A simplified transverse shear analysis is presented. This analysis will predict the shear stress distribution across the laminate thickness from known values of the shear resultants Q_x and Q_y.

The background necessary to compute a laminate interaction diagram is presented. A laminate interaction diagram depicts allowable average stresses ($\bar{\sigma}_x$, $\bar{\sigma}_y$, and $\bar{\tau}_{xy}$) for a particular laminate based upon the maximum strain theory of failure.

The analyses which are presented have been programmed in Fortran IV as procedure SQ5. This procedure is described in the

Appendix and a sample problem is presented.] Some results obtained from using the procedure are also presented. An original laminate analysis program, U65, was revised and modified in writing procedure SQ5.

(pp) (fig.) (tbls..) (ref.)

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LIST OF ILLUSTRATIONS

N O M E N C L A T U R E

$[A]$	inplane stiffness coefficients
$[B]$	coupling coefficients between inplane and bending resultants
$[D]$	bending stiffness coefficients
$[A']$, $[B']$, $[C']$, $[D']$	submatrices of the inverted laminate constitutive relation
E_{11}	modulus of elasticity in lamina fiber direction
E_{22}	modulus of elasticity normal to lamina fiber direction
G_{12}	shear modulus of elasticity
h_k	coordinate from midplane to kth layer
$[N]$	inplane stress resultants
$[M]$	moment resultants
$[N^T]$	thermally induced inplane stress resultants
$[M^T]$	thermally induced moment resultants
Q_x, Q_y	plate transverse shear resultants
Q_{ij}	elements of stiffness matrix of layer in natural axis system
$\bar{Q}_{ij}$	elements of stiffness matrix of layer in x-y axis system
T	temperature
u, v, w	x, y, z displacements
u_o, v_o	x, y midplane displacements
$[\alpha]$	thermal expansion coefficients

NOMENCLATURE (Continued)

$\left\|\epsilon_{1-2}\right\|$	strains in natural axis system of a particular layer
$\left\|\epsilon_{x-y}\right\|$	strains in laminate axis system
$\left\|\epsilon^{o}_{x-y}\right\|$	midplane strains in laminate axis system
$\left\|k\right\|$	plate curvature
ν_{12}, ν_{21}	Poisson's ratios
$\left\|\sigma_{1-2}\right\|$	stresses in natural axis system of a particular layer
$\left\|\sigma_{x-y}\right\|$	stresses in laminate axis system
$\left\|\bar{\sigma}_{x-y}\right\|$	average stresses in laminate axis system
τ_{xz}, τ_{yz}	transverse shear stresses

SECTION I

INTRODUCTION

Until recently the point stress analysis of a laminate has been limited to inplane analyses and inplane applications. Recent composite laminate applications have required a combined inplane and bending point stress analysis. Initial laminated composite applications were, for example, sandwich plate skins which can be assumed to remain flat and thus eliminate curvature terms. With the expanding use and applications of composite elements came a need for a coupled inplane and bending point stress analysis. The present analysis presents the usual lamination theory which allows the derivation of the complete laminate constitutive relation from basic lamina properties. Lamination theory and the current notation in the field may be found in several references, for example: Primer on Composite Materials: Analysis, by Ashton, Halpin and Petit(1)*.

Allowable stress curves or interaction diagrams are important in the design of laminated structures. An interaction diagram for average inplane stresses is three-dimensional and is thus depicted in two-dimensions with the third variable $\bar{\tau}_{xy}$ appearing as cutoff lines. This type of curve or curves for combined

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*The numbers in parenthesis refer to the reference list at the end of the report.

inplane and bending stresses would become either too specialized or too difficult to present for normal design purposes.

Two other features which form a part of a laminate point stress analysis are thermally induced stresses and transverse shear stresses. The thermal stress formulation follows the work of Tsai(2) by calculating the thermally induced inplane stress and moment resultants. The transverse shear analysis is formulated by making some simplifying assumptions with respect to the classical theory of laminated plates.

The analyses described above should bring together the basic analytical background necessary to perform a complete linear point stress analysis of a laminated composite. The analysis presented in Sections II through VII has been programmed and is described in detail in the appendix. Section VIII describes the type of output which may be obtained with the computer program.

SECTION II

FORMULATION OF LAMINATE CONSTITUTIVE EQUATIONS

2.1 LAMINA CONSTITUTIVE EQUATION

The constitutive relation for an orthotropic layer in a state of plane stress may be written as follows:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} \tag{1}$$

where,

$$\begin{aligned} Q_{11} &= E_{11}/(1 - \nu_{12}\ \nu_{21}) \\ Q_{22} &= E_{22}/(1 - \nu_{12}\ \nu_{21}) \\ Q_{12} &= \nu_{21}\ E_{11}/(1 - \nu_{12}\ \nu_{21}) = \nu_{12}\ E_{22}/(1 - \nu_{12}\ \nu_{21}) \\ Q_{66} &= G_{12} \\ Q_{16} &= Q_{26} = 0. \end{aligned} \tag{2}$$

E_{11}, E_{22}, ν_{12}, and G_{12} are the four independent elastic constants in the 1-2 axis system of the layer. Thus the stresses σ_1, σ_2, τ_{12} and the strains ϵ_1, ϵ_2, γ_{12} are also in the layer axis system (see Figure 1). Transforming Equation 1 into the laminate x-y axis system results in

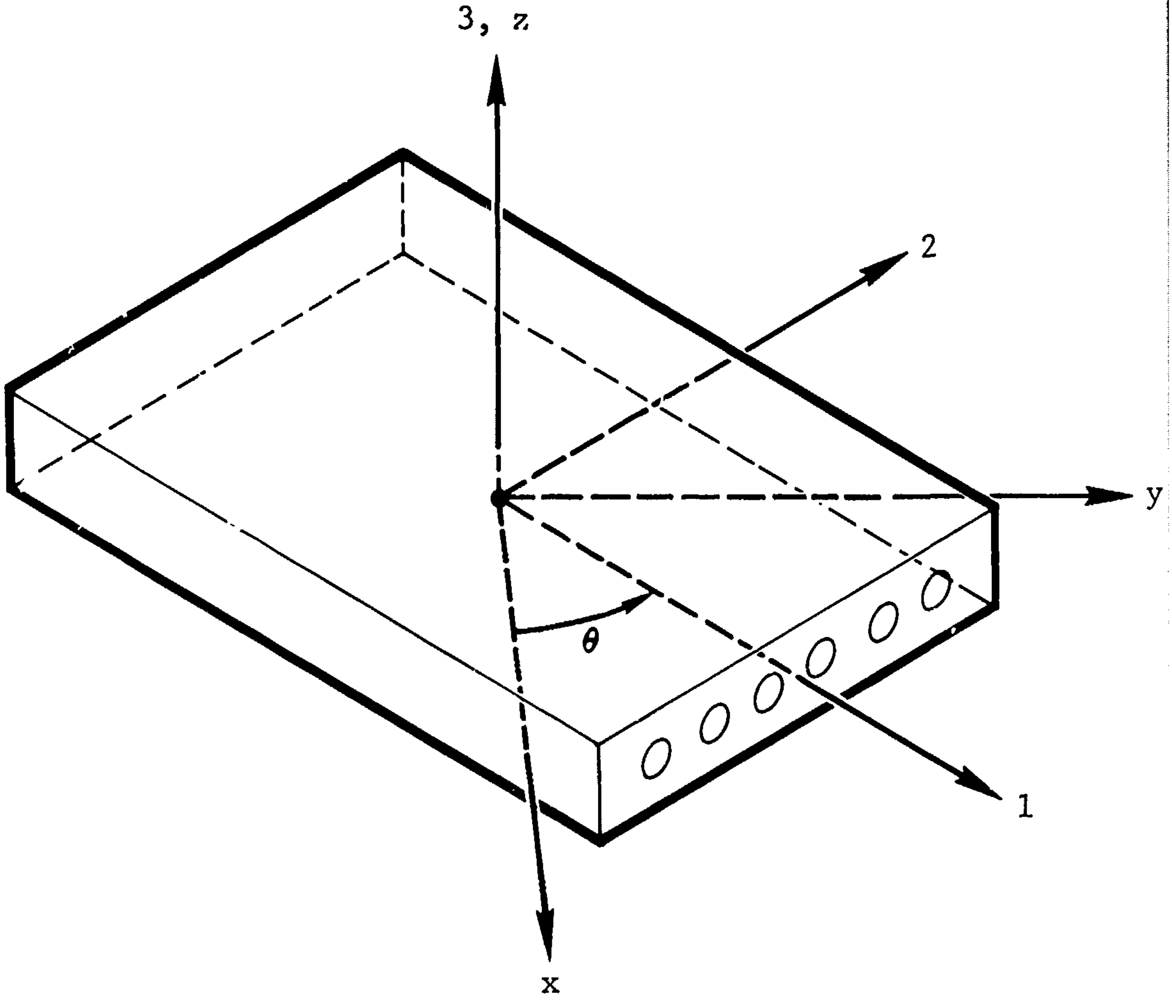

Figure 1 Lamina (1-2) and Laminate (x-y) Axis System

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix}_k \qquad (3)$$

where the $\bar{Q}_{ij}$ are the transformed stiffnesses and k presents the kth layer of the laminate. This transformation represents a rotation of 1-2 system into the x-y system through the angle θ. Equation 3 may also be written as,

$$\left[\sigma\right]_k = \left[\bar{Q}\right]_k \left[\epsilon\right]_k . \qquad (4)$$

2.2 STRAIN-DISPLACEMENT EQUATIONS

The displacements at any point of a cross-section may be written

$$\begin{aligned} u &= u_o - z\frac{\partial w}{\partial x} \\ v &= v_o - z\frac{\partial w}{\partial y} \\ w &= w_o \end{aligned} \qquad (5)$$

where u, v, and w represent the displacements in the x, y and z directions respectively. The midplane displacements are given by u_o, v_o, and w_o. The strain-displacement relations are given as

$$\begin{aligned} \epsilon_x &= \frac{\partial u}{\partial x} \\ \epsilon_y &= \frac{\partial v}{\partial y} \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} . \end{aligned} \qquad (6)$$

Now substituting Equations 5 into Equations 6:

$$\begin{aligned} \epsilon_x &= \frac{\partial u_o}{\partial x} - z\frac{\partial^2 w}{\partial x^2} \\ \epsilon_y &= \frac{\partial v_o}{\partial y} - z\frac{\partial^2 w}{\partial y^2} \\ \gamma_{xy} &= \frac{\partial u_o}{\partial y} + \frac{\partial v_o}{\partial x} - 2z\frac{\partial^2 w}{\partial x \partial y} \end{aligned} \tag{7}$$

or

$$\begin{aligned} \epsilon_x &= \epsilon_y^o + z\, k_x \\ \epsilon_y &= \epsilon_y^o + z\, k_y \\ \gamma_{xy} &= \gamma_{xy}^o + z\, k_{xy} \end{aligned} \tag{8}$$

where, ϵ_x^o, ϵ_y^o, γ_{xy}^o represent midplane strains and k_x, k_y, k_{xy} represent plate curvatures. These equations may be written as,

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \epsilon_x^o \\ \epsilon_y^o \\ \gamma_{xy}^o \end{bmatrix} + z \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix}$$

or

$$\left| \epsilon \right| = \left| \epsilon^o \right| + z \left| k \right| . \tag{9}$$

Now, substituting Equation 9 into Equation 4 results in,

$$\left| \sigma \right|_k = \left| \bar{Q} \right|_k \left| \epsilon^o \right| + z \left| \bar{Q} \right|_k \left| k \right| . \tag{10}$$

Equation 10 may be used to calculate the stresses at any point z and thus in any layer of the laminate if the midplane strains $|\epsilon^{o}|$ and curvatures $|k|$ are known.

2.3 LAMINATE CONSTITUTIVE EQUATIONS

With the exception of defining the stress (N_x, N_y, N_{xy}) and moment (M_x, M_y, M_{xy}) resultants, the background material for the formulation of the laminate constitutive equations has been presented. The stress and moment resultants represent a system which is statically equivalent to the stress system that is acting on the laminate. These stress and moment resultants are shown in Figure 2. They are defined as follows:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} dz \tag{11}$$

and,

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} zdz \quad . \tag{12}$$

By substituting Equation 10 into Equations 11 and 12 and separating the continuous integral into a sum of discrete

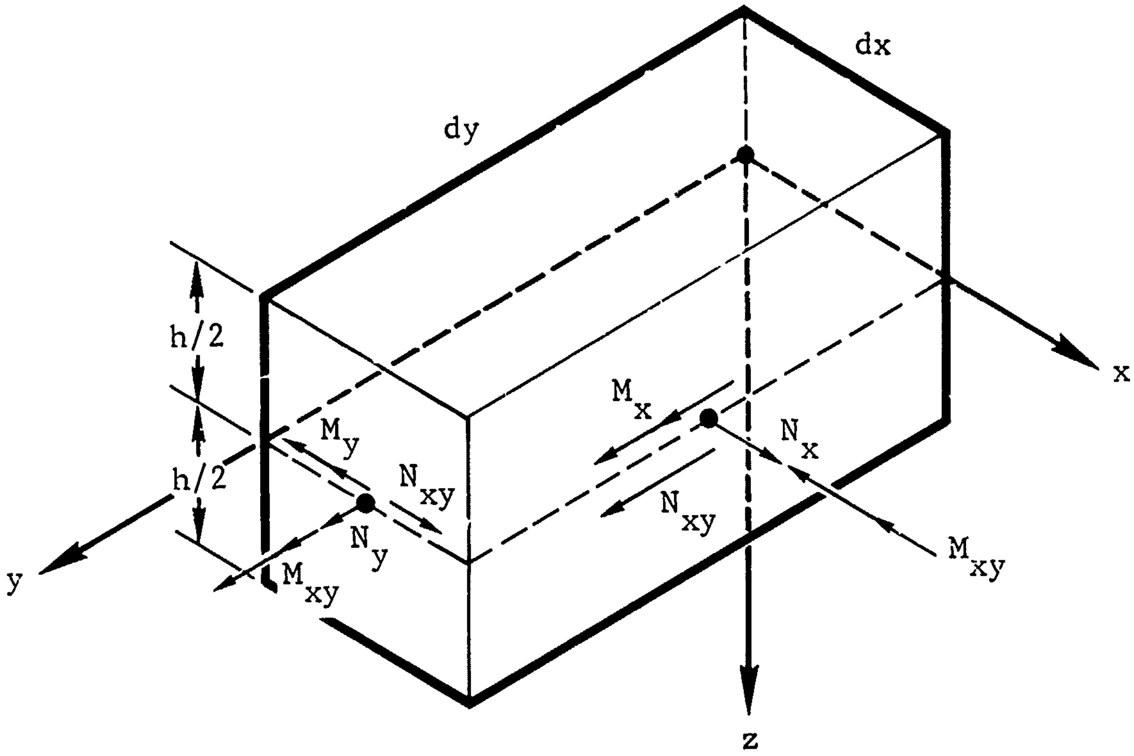

Figure 2 Stress and Moment Resultants

integrals across each layer of an n layered laminate results in:

$$\left[N \right] = \sum_{k=1}^{n} \left\{ \int_{h_{k-1}}^{h_k} \left[\bar{Q} \right]_k \left[\epsilon^o \right] dz + \int_{h_{k-1}}^{h_k} \left[\bar{Q} \right]_k \left[k \right] z \, dz \right\} \qquad (13)$$

and

$$\left[M \right] = \sum_{k=1}^{n} \left\{ \int_{h_{k-1}}^{h_k} \left[\bar{Q} \right]_k \left[\epsilon^o \right] zdz + \int_{h_{k-1}}^{h_k} \left[\bar{Q} \right]_k \left[k \right] z^2 \, dz \right\}. \qquad (14)$$

The notation for a particular lamina within a laminate is shown in Figure 3. Since $\left[\epsilon^o \right]$ and $\left[k \right]$ are constant across the laminate and $\left[\bar{Q} \right]_k$ is constant within any layer, the integrals in Equations 13 and 14 may be evaluated. Equations 13 and 14 thus may be reduced to the following,

$$\left[N \right] = \left[A \right] \left[\epsilon^o \right] + \left[B \right] \left[k \right] \qquad (15)$$

and,

$$\left[M \right] = \left[B \right] \left[\epsilon^o \right] + \left[D \right] \left[k \right] \qquad (16)$$

where

$$A_{ij} = \sum_{k=1}^{n} (Q_{ij})_k \, (h_k - h_{k-1}) \qquad (17)$$

$$B_{ij} = 1/2 \sum_{k=1}^{n} (Q_{ij})_k \, (h_k^2 - h_{k-1}^2) \qquad (18)$$

$$D_{ij} = 1/3 \sum_{k=1}^{n} (Q_{ij})_k \, (h_k^3 - n_{k-1}^3). \qquad (19)$$

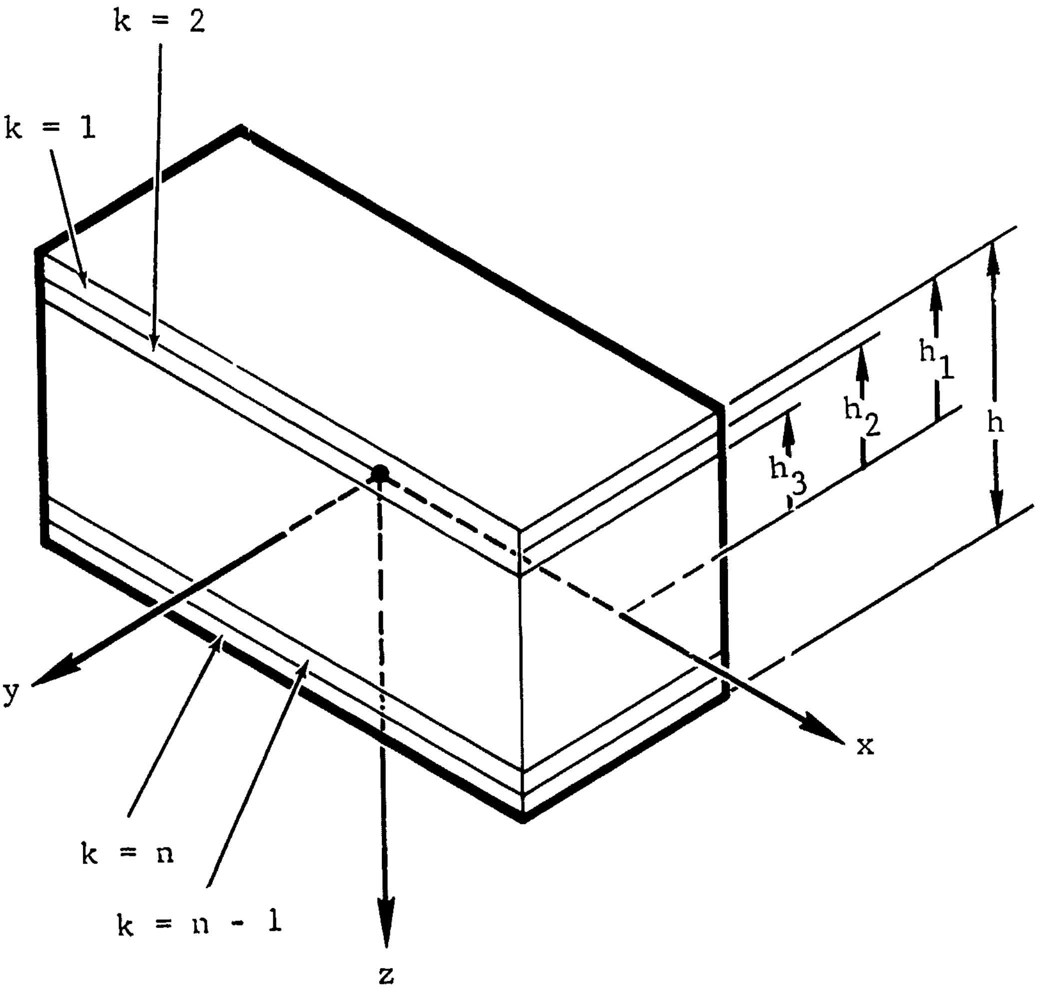

Figure 3 Lamina Notation

Combining Equations 15 and 16 results in:

$$\begin{bmatrix} N \\ \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ \\ B & D \end{bmatrix} \begin{bmatrix} \epsilon^{o} \\ \\ k \end{bmatrix} . \qquad (20)$$

Equation 20 is the total constitutive relation for a laminated plate. The coupling of inplane and bending is apparent in Equation 20 by the presence of the B submatrix. For a mid-plane symmetric laminate the B matrix is zero and thus the actions of bending and stretching uncouple.

SECTION III

CALCULATION OF LAMINA STRESSES AND STRAINS FOR AVERAGE INPLANE STRESSES

In order to evaluate the stresses and strains in the lamina of a laminate when average inplane stresses are known, the constitutive equation is assumed to be uncoupled. Thus, Equation 20 results in:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} \epsilon^o{}_x \\ \epsilon^o{}_y \\ \gamma^o{}_{xy} \end{bmatrix} . \tag{21}$$

This equation is converted to an average stress analysis equation by dividing by the laminate thickness t, thus:

$$\begin{bmatrix} \bar{\sigma}_x \\ \bar{\sigma}_y \\ \bar{\tau}_{xy} \end{bmatrix} = \begin{bmatrix} A/t \end{bmatrix} \begin{bmatrix} \epsilon^o{}_x \\ \epsilon^o{}_y \\ \gamma^o{}_{xy} \end{bmatrix} . \tag{22}$$

The input average stresses may be input at some angle to the laminate axis. These stresses are first rotated into the laminate axis system to obtain the stresses in Equation 22. Therefore for a given set of average laminate stresses, Equation 22 may be used to calculate the laminate and thus the lamina strains in the laminate axis system. These strains are next rotated into the particular lamina natural axis system. The lamina constitutive equation (Equation 1) may then be used to convert the lamina strains into stresses.

SECTION IV

INTERACTION DIAGRAMS

A laminate interaction diagram is shown in Figure 4. This diagram is based on the maximum strain theory of failure for each lamina in the laminate and depicts allowable average stresses for a particular laminate. This diagram is in reality three dimensional in $\bar{\sigma}_x$, $\bar{\sigma}_y$, and $\bar{\tau}_{xy}$, where the bar indicates average stresses. The laminate interaction diagram thus represents a way of checking stress levels from a conventional stress analysis. If the stress state falls inside the envelope no lamina in the laminate will fail in any mode of the maximum strain theory of failure. These diagrams may be developed for many different laminates and used by a designer in setting thicknesses and orientations.

In order to determine these diagrams, all combinations of unit average stresses are applied to a specified laminate. Next, the strains ϵ_1, ϵ_2, and γ_{12} are determined for each lamina in the laminate for all combinations of the unit stresses. These strains are in the natural axis system of the particular lamina. These strains are calculated as described in Section III. Now, since these lamina strains were produced by unit average laminate stresses, the stresses can be ratioed up to some allowable stress if allowable lamina strains are known. Thus, for a particular shear stress, an allowable set of $\bar{\sigma}_x$ and $\bar{\sigma}_y$ is obtained for each type of failure in each lamina of the laminate.

By plotting these $\bar{\sigma}_x$ and $\bar{\sigma}_y$ intercepts for the various failure modes for all layers in the laminate, an interaction diagram is obtained. Figure 4 shows the various failure mode cutoffs for a particular laminate. This diagram is the minimum envelope of all the failure mode lines. This procedure is repeated for shear increments of ± 10,000 psi from zero to a maximum allowable. The maximum allowable shear stress is obtained from the procedure of applying unit stresses.

In the past, the computer had been used to compute the $\bar{\sigma}_x$ and $\bar{\sigma}_y$ intercepts for the various failure modes. These modes were then hand plotted to produce the desired interaction diagram. A search routine to compute the final coordinates of the interaction diagram for a laminate has been written, and is part of the program described in Appendix I.

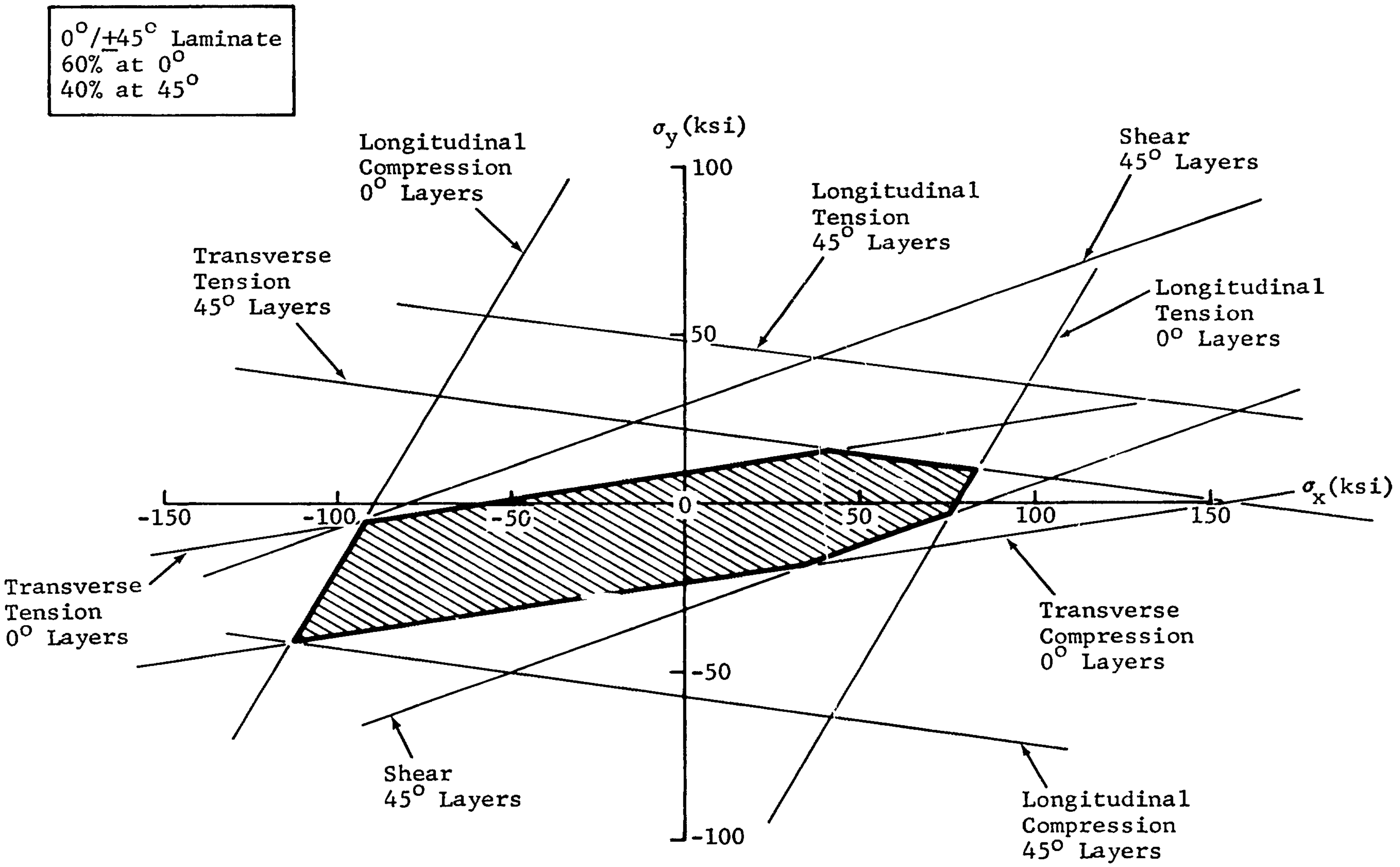

Figure 4 Laminate Interaction Diagram

SECTION V

COMPLETE POINT STRESS ANALYSIS

A complete point stress analysis of a laminate under an arbitrary set of loads includes both inplane and bending loads. The inverted form of Equation 20 is used for this analysis:

$$\begin{bmatrix} \epsilon^o \\ \\ k \end{bmatrix} = \begin{bmatrix} A' & B' \\ \\ C' & D' \end{bmatrix} \begin{bmatrix} N \\ \\ M \end{bmatrix}. \qquad (23)$$

If the laminate is midplane symmetric, the submatrix B in Equation 20 is zero. With this matrix zero, the B' and C' matrices in Equation 23 are zero, and thus the inplane and bending effects uncouple. With known inplane stress resultants (N_x, N_y and N_{xy}) and moments (M_x, M_y and M_{xy}), Equation 23 may be used to calculate the midplane strains ($\epsilon^o{}_x$, $\epsilon^o{}_y$ and $\gamma^o{}_{xy}$) and curvatures (k_x, k_y and k_{xy}). The state of strain at any point across the thickness of the laminate may now be determined by using Equation 9. Since the $|\epsilon|_k$ vector is still in the x-y coordinate system of the laminate, it must be transformed into the natural axis system for the particular lamina in question. The particular lamina constitutive relation, Equation 1, may now be used to compute lamina stresses. These lamina stresses and or strains may then be used to calculate margins of safety from a failure criteria. This completes the point stress analysis in that the complete state of stress and strain has been determined in every layer of the laminate.

SECTION VI

THERMALLY INDUCED STRESSES

The thermal expansion problem can be approached by calculating the thermally induced inplane stress $\left|N^T\right|$ and moment $\left|M^T\right|$ resultants using

$$\left|N^T\right| = (-T)\int_{-h/2}^{h/2} \left|Q\right| \left|\alpha\right| dz, \tag{24}$$

and

$$\left|M^T\right| = (-T)\int_{-h/2}^{h/2} \left|Q\right| \left|\alpha\right| z\ dz, \tag{25}$$

as presented in Reference (2). The $\left|Q\right|$ and $\left|\alpha\right|$ matrices in the above equations are the lamina stiffness matrix and the vector of thermal expansion coefficients respectively in the lamina natural axis system. The product of $\left|Q\right|$ and $\left|\alpha\right|$ must be rotated into the laminate x-y coordinate system before the integration is carried out. With the lamination temperature assumed as the zero stress state, -T, is the change from this lamination temperature. Note that (-T) is outside the integral, thus assuming a constant temperature across the thickness of the laminate. After the thermally induced stress $\left|N^T\right|$ and moment $\left|M^T\right|$ resultants have been found by using Equations 24 and 25, the point stress analysis proceeds as described in Section V. Thus with this type of formulation, the thermally induced inplane and moment resultants may be considered separately or added to corresponding resultants produced from other types of loadings.

SECTION VII

INTERLAMINAR SHEAR STRESSES

The interlaminar shear calculations for τ_{xz} and τ_{yz} were approached by making some simplifying assumptions. These assumptions will be pointed out in the following discussion. The shear resultants Q_x and Q_y were obtained from Reference (3) as,

$$
\begin{aligned}
Q_x = {} & B_{11}\, u^o{}_{,xx} + 2B_{16}\, u^o{}_{,xy} + B_{66}\, u^o{}_{,yy} + B_{16}\, v^o{}_{,xx} \\
& + (B_{12} + B_{66})\, v^o{}_{,xy} + B_{26}\, v^o{}_{,yy} - D_{11}\, w_{,xxx} \\
& - 3D_{16}\, w_{,xxy} - (D_{12} + 2D_{66})\, w_{,xyy} - D_{26}\, w_{,yyy}
\end{aligned}
\tag{26}
$$

and

$$
\begin{aligned}
Q_y = {} & B_{16}\, u^o{}_{,xx} + (B_{12} + B_{66})\, u^o{}_{,xy} + B_{26}\, u^o{}_{,yy} + B_{66}\, v^o{}_{,xx} \\
& + 2B_{26}\, v^o{}_{,xy} + B_{22}\, v^o{}_{,yy} - D_{16}\, w_{,xxx} \\
& - (D_{12} + 2D_{66})\, w_{,xxy} - 2D_{26}\, w_{,xyy} - D_{22}\, w_{,yyy}
\end{aligned}
\tag{27}
$$

where B_{ij} and D_{ij} are the same terms as in Equations (18) and (19) and u^o, v^o and w are the midplane deflections. Equations 26 and 27 reduce to the following for midplane symmetric laminates:

$$
Q_x = -D_{11}\, w_{,xxx} - 3D_{16}\, w_{,xxy} - (D_{12} + D_{66})\, w_{,xyy} - D_{26}\, w_{,yyy}, \tag{28}
$$

and

$$Q_y = -D_{16}\, w_{,xxx} - (D_{12} + 2D_{66})\, w_{,xxy} - 2D_{26}\, w_{,xyy} - D_{22}\, w_{,yyy}. \quad (29)$$

Next, the cross-derivative terms are neglected resulting in

$$Q_x = -D_{11}\, w_{,xxx} - D_{26}\, w_{,yyy}, \quad (30)$$

and

$$Q_y = -D_{16}\, w_{,xxx} - D_{22}\, w_{,yyy}. \quad (31)$$

Now by using Q_x and Q_y as known or input data, Equations 30 and 31 may be solved to obtain expressions for $w_{,xxx}$ and $w_{,yyy}$:

$$w_{,xxx} = \frac{1}{D}\left[-D_{22}\, Q_x + D_{26}\, Q_y\right] \quad (32)$$

and

$$w_{,yyy} = \frac{1}{D}\left[D_{16}\, Q_x - D_{11}\, Q_y\right] \quad (33)$$

where

$$D = D_{11}\, D_{22} - D_{16}\, D_{26}. \quad (34)$$

The interlaminar shear stresses are given as

$$\tau_{xz}^{(k)} = \frac{z^2}{2}\left[\bar{Q}_{11}^{(k)} w_{,xxx} + \bar{Q}_{26}^{(k)} w_{,yyy}\right] + f^{(k)}(x,y), \quad (35)$$

and

$$\tau_{yz}^{(k)} = \frac{z^2}{2}\left[\bar{Q}_{16}^{(k)} w_{,xxx} + \bar{Q}_{22}^{(k)} w_{,yyy}\right] + g^{(k)}(x,y) \quad (36)$$

after cross-derivative terms and inplane deformation terms are neglected (Reference 3). The $\bar{Q}_{ij}$ terms are the lamina stiffness terms rotated into the x-y coordinate system. The functions $f^{(k)}(x,y)$ and $g^{(k)}(x,y)$ are determined by using the boundary conditions that τ_{xz} and τ_{yz} are zero at the surface of the plate. The final form of $\tau_{xz}^{(k)}$ and $\tau_{yz}^{(k)}$ is

$$\tau_{xz}^{(k)} = \left(\frac{1}{8}\right) (4Z^2-h^2) \left[\bar{Q}_{11}^{(k)} \; w,_{xxx} + \bar{Q}_{26}^{(k)} \; w,_{yyy}\right], \tag{37}$$

and

$$\tau_{yz}^{(k)} = \left(\frac{1}{8}\right) (4Z^2-h^2) \left[\bar{Q}_{16}^{(k)} w,_{xxx} + \bar{Q}_{22}^{(k)} \; w,_{yyy}\right] \tag{38}$$

where h is the total laminate thickness. Thus by solving Equations (32) and (33) for $w,_{xxx}$ and $w,_{yyy}$, Equations (37) and (38) result in values of τ_{xz} and τ_{yz} at any point of the cross-section. The shear stresses resulting from the use of Equations (37) and (38) are based on two assumptions: (1) midplane symmetric laminates, and (2) neglect of the effects of the cross-derivative terms which appear in the Q_x and Q_y equations. The effect of the midplane symmetric assumption is clearly not significant in many cases since most laminates used in actual design are midplane symmetric. The effect of neglecting the cross-derivative term is the same as assuming the plate acts like an uncoupled beam in both directions. It is felt that this is not a serious assumption for the first pass effort at obtaining interlaminar shear stresses.

SECTION VIII

ANALYTICAL RESULTS

The analysis described in the preceeding sections has been programmed for an IBM 360-65 digital computer as program SQ5. The original program U65 was written by M. E. Waddoups. The following is a brief paragraph describing the results obtained for each of the major contributions of the program.

8.1 INTERACTION DIAGRAM

Figure 3 shows an interaction diagram obtained from the procedure SQ5. As stated earlier, the program prints out the $\bar{\sigma}_x$ and $\bar{\sigma}_y$ coordinates of the corners of the interaction diagram. The user then plots these points and connects them with straight lines to obtain the interaction diagram for a particular $\bar{\tau}_{xy}$ value. The $\bar{\tau}_{xy}$ value is printed out along with the $\bar{\sigma}_x$ and $\bar{\sigma}_y$ coordinates.

Lamina strain allowables must be input along with the usual lamina properties such as thickness and orientation in order to compute the interaction diagram coordinates.

8.2 BENDING ANALYSIS

In order to check the bending analysis subroutine in SQ5, data from a standard 0° flexure test was used. SQ5 predicted the expected Mc/EI strain to be 7026 μin/in., while experimentally a value of 7100 μin/in. was obtained with the use of strain gages. A test program which will include cross-ply beams will be initiated at a later date.

8.3 INTERLAMINAR SHEAR

The interlaminar shear stress distribution calculations in SQ5 have been checked for a midplane symmetric laminated beam. The distribution checked very close to the distribution obtained from a photoelastic coating on an experimental beam. A midplane symmetric laminate and beam action were the two basic assumptions in the interlaminar shear stress derivation, thus very good results were expected and obtained for this situation.

8.4 THERMAL EXPANSION ANALYSIS

The thermal analysis section of SQ5 has been checked by comparison with a $\pm 15^\circ$ glass laminate by Tsai (Reference 2). The coefficients of thermal loads (N^T and M^T) obtained from SQ5 check the results of Tsai.

This analysis also produces the laminate coefficients of thermal expansion. As an example of the accuracy obtained, SQ5 predicted an α_1 of 3×10^{-6} for a $0^\circ/\pm 60^\circ$ boron laminate while a value of 3.25×10^{-6} has been obtained experimentally.

SECTION IX

SUMMARY

An existing computer program, U65, has been updated and expanded in several respects. The major changes are as follows: (1) a point stress bending analysis using the full laminate constitutive equation has been included, (2) thermally induced moments and inplane stress resultants may be included in a point stress analysis, and (3) a simplified interlaminar shear stress analysis based on beam action and midplane symmetry has been added. The overall program was also modified to make it more efficient from the users point of view as well as machine efficiency.

Several basic checks were performed and the program should now become the laminate analysis program for use in linear analyses.

APPENDIX I

DESCRIPTION OF COMPUTER PROGRAM SQ5

The analysis presented in Sections II through VII has been programmed as computer program SQ5. The forerunner of the present program was U65. The program SQ5 consists of a main program and seven subroutines, four of which were added in producing SQ5. In summary, U65 was modified as follows in producing the computer program SQ5:

1. The input was completely revised.
2. The input data was written out as the first item of output
3. The input and output were updated to the current notation of Reference 1
4. A point stress bending analysis was added
5. A laminate thermal stress analysis was added
6. A search routine for the interaction diagram coordinates was added (written by R. W. McMickle)
7. A simplified interlaminar shear stress analysis routine was added.
8. Multiple option capability was added whereby many parts of the program can be used with a single problem input

The function of each subroutine is described below. A description of each card entry will be given in Appendix II.

MAIN Program

The MAIN program is used to read and write out the input data. The input data is written out with identifying information in order to facilitate a check of the problem data. Current notation is used for all the output data. Next, the main program computes the laminate constitutive relation (Equation 20).

The remainder of the main program decides which of the subroutines will be called according to a list of option keys which have been input.

Subroutine STEC

This subroutine computes laminate strains for all combinations of unit average stresses. These laminate strains are needed for interaction diagram calculations. If a point stress analysis using input average stresses is to be performed, this subroutine rotates the input stresses (they may be input at some angle to the laminate axis) into the laminate x-y axis and computes the corresponding laminate strains.

Subroutine SSRC

This subroutine rotates the laminate strains found in STEC into the natural axis system of each layer in the laminate. Using the lamina constitutive relation, these strains are used to calculate lamina stresses. Margins of safety are also calculated from the lamina strains. If an interaction diagram was called for, allowable lamina stresses are calculated as described in Section IV.

Subroutine SURFS

Subroutine SURFS calculates the $\bar{\sigma}_x$ and $\bar{\sigma}_y$ cutoff allowable stresses which are used in plotting an interaction diagram. First, the laminate strains found in subroutine STEC are rotated into the natural axis system of each lamina in the laminate. Now, since these strains were produced by unit stresses, allowable stresses can be calculated by ratioing with an allowable strain. This procedure is repeated for all combinations of unit stresses and for increments of $\bar{\tau}_{xy}$. $\bar{\tau}_{xy}$ is initially set equal to zero and then increased in increments of ±10,000 psi, until the maximum value is reached. The negative increments of $\bar{\tau}_{xy}$ are necessary only for non-rotationally symmetric laminates. The final coordinates of the interaction diagram reflect the minimum envelope for both + and - $\bar{\tau}_{xy}$ increments. The maximum value of $\bar{\tau}_{xy}$ was calculated in subroutine STEC.

Subroutine SURFS next calls subroutine ISECT which will be described in the following paragraph.

Subroutine ISECT

This subroutine was written by R. W. McMickle and is a highly specialized search routine for the final coordinates of the interaction diagram. ISECT is called one time for each increment of $\bar{\tau}_{xy}$, thus all the interaction diagram coordinates are printed for each $\bar{\tau}_{xy}$ interval. The subroutine uses the $\bar{\sigma}_x$ and $\bar{\sigma}_y$ intercepts for the various failure modes which were calculated in subroutine SURFS. The $\bar{\sigma}_x$ and $\bar{\sigma}_y$ intercepts are also printed and may be used to obtain the desired interaction diagram if the user wishes to see which of the modes control the various failure lines.

Subroutine BEND

Subroutine BEND first computes the inverse of the laminate constitutive equation (Equation 23). Next, the subroutine prints Equation 23 and uses it to calculate the laminate midplane strains and curvatures (see Section V). These quantities are then used to calculate the state of stress and strain in each lamina of the laminate.

Subroutine TEMP

Subroutine TEMP uses Equations 24 and 25 to calculate the thermally induced inplane stress and moment resultants. The laminate coefficients of thermal expansion are also calculated in this subroutine.

Subroutine SHEAR

Subroutine SHEAR first calculates the third derivatives of w with respect to x and y using Equations 32 and 33. Next, Equations 37 and 38 are used to calculate $\bar{\tau}_{xz}$ and $\bar{\tau}_{yz}$ at each lamina interface across the thickness of the laminate. This distribution is printed along with the corresponding z position within the laminate.

APPENDIX II

INPUT DATA DESCRIPTION

The input consists of problem card deck(s). Data contained in the problem deck(s) will consist of integers and real numbers. All integers must be right adjusted in the proper card field. Real numbers must contain a decimal point in the proper position. The general content of each card in a problem deck is as follows:

Columns	
1-66	Input data
67-72	Six digit job number obtained from the Computing Laboratory
73	The alphabetic letter "P"
74-75	Number each problem within a problem deck sequentially from 01
76-79	Number each card within a problem sequentially from 0001

Card Descriptions

Card 1:

Column	Contents
1	Blank
2-66	Problem title or identifying information which will be printed at the top of the first page of the problem output. Any alphabetic or numeric symbol may be used.

Card 2: (8I5)

Column	Contents
1-5	KEY 1 = 1-Program terminates after computing and writing out the elements of the constitutive matrices (See Equation 20) = 0-Program operation continues after computation of laminate data.
6-10	KEY 2 = 1-A point stress analysis will be made on input sets of $|N|$ and $|M|$. One card per load case must be added to the problem deck. This key must also be set equal to one if a thermal analysis is to be performed. = 0-No point stress or thermal analysis will be done.
11-15	KEY 3 = 1-A point stress analysis will be made of average stresses σ_α, σ_β, $\tau_{\alpha\beta}$, and θ. θ is the angle at which the stresses are applied. This analysis is for inplane loads only. = 2-An interaction diagram will be computed for the input laminate.
16-20	KEY 4 = 1-Thermally induced inplane $|N^T|$ and moment $|M^T|$ resultants will be computed for an input temperature. If KEY 4 = 1, KEY 2 must be set equal to 1.

Column	Content
	= 0-No thermal analysis will be made.
21-25	KEY 5 = 1-An interlaminar shear stress analysis will be made for input values of Q_x and Q_y. = 0-No interlaminar shear stress analysis will be made.
26-30	MA = Number of layers in the laminate (max. no. = 400).
31-35	NOMAT = Number of material types (max. no. = 400).
36-40	NCL = Number of loading cases. This applies to sets of $\|N\|$ and $\|M\|$, temperatures, and Q_x, Q_y. (max. no. = 10).

Third Group of Cards: (7F9.0)

Column	Contents
1-9	E1(1) - Modulus of elasticity along the first or 1 lamina axis.
10-18	E2(1) - Modulus of elasticity along the second or 2 lamina axis which is orthogonal to the 1 lamina axis.
19-27	U1(1) - First poisson's ratio
28-36	G(1) - Shear modulus of elasticity
37-45	ALPHA1(1) - Coefficient of thermal expansion in the 1 lamina direction.

Column	Contents
46-54	ALPHA2(1) - Coefficient of thermal expansion in the 2 lamina direction.
55-63	ALPHA6(1) - Shearing coefficient of thermal expansion.

Additional cards of this type are added for each type of material in the laminate up to NOMAT as input previously. A maximum 400 such cards may be used. Thus, a different material type may be assigned for each layer in the laminate up to the maximum number of layers which is allowed.

Fourth Group of Cards: (2I5, 2F10.0)

Column	Contents
1 - 5	LAY - Layer number
6-10	MATYPE(1) - Material type number
11-20	TH(1) - Counterclockwise angle from the laminate reference axes (x,y) to the lamina natural axes (1,2). The angle is input in degrees.
21-30	AT(1) - Lamina thickness.

Additional cards of this type are added for each lamina in the laminate up to MA as input previously. A maximum of 400 layers may be input as described.

Fifth Group of Cards: (6F10.0)

Column	Contents
1-10	CALE1(1) - Compression limit strain allowable in the 1 lamina direction.
11-20	CALE2(1) - Compression limit strain allowable in the 2 lamina direction.
21-30	CALE3(1) - Negative limit shear strain allowable.
31-40	TALE1(1) - Tension limit strain allowable in the 1 lamina direction.
41-50	TALE2(1) - Tension limit strain allowable in the 2 lamina direction.
51-60	TALE3(1) - Positive limit shear strain allowable.

Additional cards of this type are added for each type of material in the laminate up to NOMAT as input previously.

Sixth Group of Cards: (7F9.0) (Optional)

Column	Contents
1-9	N(1,1) - Inplane force resultant in the X-direction for load case 1 (lbs/in.).
10-18	N(1,2) - Inplane force resultant in the Y-direction for load case 1 (lbs/in.).
19-27	N(1,3) - Inplane shear force resultant for load case 1 (lbs/in.).
28-36	M(1,1) - M_x moment resultant for load case 1 (in.lbs./in.).

Column	Contents
37-45	M(1,2) - My moment resultant for load case 1 (in.lbs./in.).
46-54	M(1,3) - M_{xy} moment resultant for load case 1 (in.lbs./in.).
55-63	T(1) - Change in temperature for load case 1 (+ or - with respect to the lamination temperature).

Additional cards of this type are added for each load case up to NLC as input on Card 2. A maximum of 10 load cases may be input. This group of cards is optional in that it would be omitted if (1) laminate properties only were desired, (2) an interaction diagram only were desired, and (3) only interlaminar shear stresses were desired.

Seventh Card: (6F10.0) (Optional)

Column	Contents
1-10	SIG1 - Average laminate stress ${}^{\sigma}\alpha$ acting in α direction of an (α , β) system at some angle PH1 from the laminate (x,y) axis system.
11-20	SIG2 - Average laminate stress
21-30	SIG3 - Average laminate shearing stress
31-40	PH1 - Angle in degrees from the (α , β) system to the (x,y) axis system.

This card is input only if KEY3 = 1 and KEY1 = KEY2 = KEY4 = 0.

Eighth Group of Cards: (6F10.0) (Optional)

Column	Contents
1-10	QX(1) - X shear force resultant for load case 1 (lbs./in.).
11-20	QY(1) - Y shear force resultant for load case 1 (lbs./in.).
21-30	QX(2) - X shear force resultant for load case 2 (lbs./in.).
31-40	QY(2) - Y shear force resultant for load case 2 (lbs./in.).
41-50	QX(3) - X shear force resultant for load case 3 (lbs./in.).
51-60	QY(3) - Y shear force resultant for load case 3 (lbs./in.).

Additional cards are added as needed until the number of load cases (NLC) has been fulfilled. These cards are input only if KEY5 = 1.

Output Data

All the input data are printed with identifying information. This listing may be used as a check for input errors. The output data are printed with appropriate headings and information and is thus self-explanatory. Appendix I also contains information on the output of the various subroutines.

Restrictions

The number of layers (MA) and the number of material types (NOMAT) may range from 1 to a maximum of 400. The maximum number of loading conditions is set at 10. The other restrictions on the program are in the use of the KEY options. These options were discussed earlier, and Figure 5 contains a flow chart showing which combinations of output may be obtained with one problem input.

Estimate of Running Time

The run time may be estimated using:

$$T \text{ (minutes)} = 0.3 + (0.1) \cdot N$$

where,

N = number of problems.

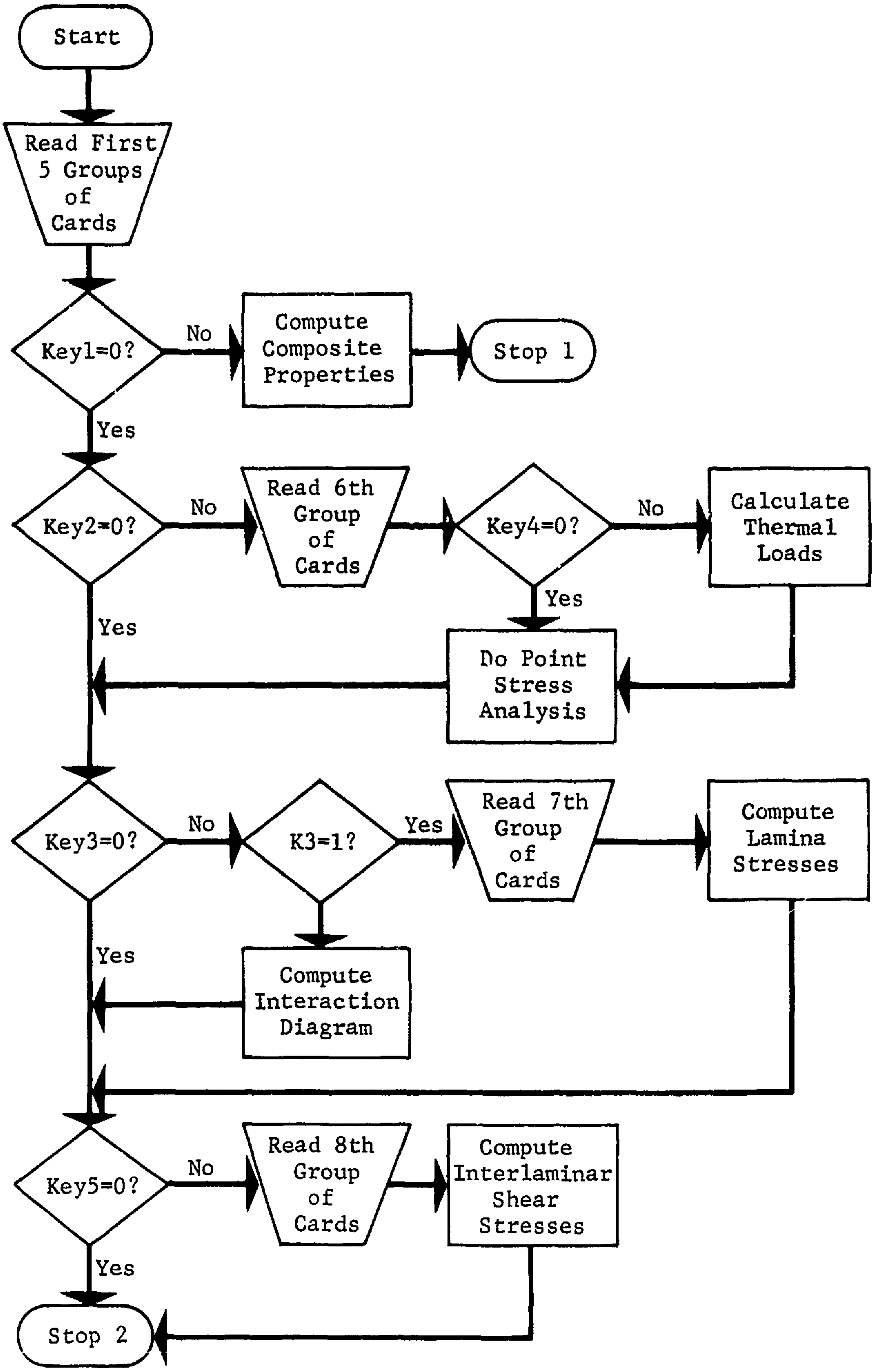

Figure 5 SQ5 Flow Diagram

APPENDIX III

PROGRAM LISTING

```
      COMMON AL(3,3), CALE1(400), CALE2(400), CALE3(400), TALE1(400),
     1 TALE2(400), TALE3(400), ADT(3,3), TH(400), Q11(400), Q12(400),
     2 Q22(400), Q66(400), BLE(18), A(3,3), B(3,3), D(3,3), AH(401),
     3 AT(400), E1(400), E2(400), U1(400), U2(400), G(400), SB1(18),
     4 QBAR(400,3,3), GAM(3,400,3), S1A(400), S2A(400), S3A(400),
     5 SJ(1200), S(50), X(50), Y(50), XN(50), YN(50), FX(3), FY(3),
     6 SIGX(1200), SIGY(1200), MATYPE(400)
      COMMON BSTAR(3,3), CSTAR(3,3), DSTAR(3,3), DSTARI(3,3), BDC(3,3),
     1 APRIME(3,3), BPRIME(3,3), CPRIME(3,3), DPRIME(3,3), ASTAR(3,3),
     2 BAB(3,3), Z(401), AI(3,3), E0(10,3), E(10,401,3), K(10,3),
     3 N(10,3), M(10,3), NT(10,3), MT(10,3), QQ11(400), QQ22(400),
     4 QQ12(400), QQ66(400), ALPHAC(400), TAL(3,400), TQA(3,400),
     5 ALPHA1(400), ALPHA2(400), ALPHA6(400), T(10), QX(10), QY(10)
      COMMON CO, CO2, SI, SI2, KEY1, KEY2, KEY3, KEY4, KEY5, SIG0, SIG1,
     1 SIG2, SIG3, PHI, CON, I, J, I2, I4, MA, NN, DAF, II, LDR, KK,
     2 I6, NLC, DAF3, DAF6, ATT, L, ML1, MB, DEL
      REAL K, N, M, NT, MT
      CALL GSTART (3HSQ5,LDR)
   10 CALL PROB
C
C *** READ IN PROBLEM TITLE                                             ***
C
      READ(5,1000)
      WRITE(6,1000)
C
C *** READ IN PROBLEM DATA                                              ***
C
      READ (5,1010) KEY1, KEY2, KEY3, KEY4, KEY5, MA, NOMAT, NLC
      WRITE(6,5000) KEY1, KEY2, KEY3, KEY4, KEY5, MA, NOMAT, NLC
   20 DO 30 I = 1,NOMAT
      READ (5,1025) E1(I), E2(I), U1(I), G(I), ALPHA1(I), ALPHA2(I),
     1 ALPHA6(I)
   30 CONTINUE
      WRITE (6,5090)
      WRITE (6,5020)
      WRITE(6,5030) (I,E1(I),E2(I),U1(I),G(I),ALPHA1(I),ALPHA2(I),
     1 ALPHA6(I), I = 1,NOMAT )
      WRITE (6,5090)
      WRITE (6,5040)
      DO 40 I = 1,MA
      READ (5,1030) LAY, MATYPE(I), TH(I), AT(I)
      WRITE(6,5050) LAY, MATYPE(I), TH(I), AT(I)
   40 CONTINUE
      READ (5,1020) (CALE1(I), CALE2(I), CALE3(I), TALE1(I), TALE2(I),
     1 TALE3(I),  I = 1, NOMAT )
      WRITE (6,5090)
      WRITE (6,1050)
      WRITE (6,1060) (I, CALE1(I), CALE2(I), CALE3(I), TALE1(I),
     1 TALE2(I), TALE3(I), I = 1, NOMAT )
C
C     LOCATE THE MIDDLE SURFACE
C
      MB = MA + 1
      DO 50 I2 = 1,MB
      AH(I2) = 0.0
   50 CONTINUE
```

```
      DO 60 I3 = 2,MB
      AH(I3) = AT(I3-1) + AH(I3-1)
   60 CONTINUE
      AHK = AH(MB)/2.0
      DO 70 I4 = 1,MB
      AH(I4) = AH(I4) - AHK
      ATT = 2.0*AHK
   70 CONTINUE
C
C     COMPUTE THE MODULI OF EACH LAYER
C
      DO 80 I5 = 1,MA
      I6 = MATYPE(I5)
      U2(I6) = E2(I6) / E1(I6)*U1(I6)
      DEL = 1.0 - U1(I6)*U2(I6)
      Q11(I5) = E1(I6) / DEL
      Q22(I5) = E2(I6) / DEL
      Q12(I5) = Q11(I5)*U2(I6)
      Q66(I5) = G(I6)
      CON = TH(I5)*0.0174533
      CO = COS(CON)
      CO2 = CO**2
      CO3 = CO**3
      CO4 = CO2 ** 2
      SI = SIN(CON)
      SI2 = SI**2
      SI3 = SI**3
      SI4 = SI2** 2
      SICO = SI2 * CO2
      QBAR(I5,1,1) = Q11(I5)*CO4 + 2.0*(Q12(I5) + 2.0*Q66(I5))*SICO +
     1               Q22(I5)*SI4
      QBAR(I5,1,2) = (Q11(I5) + Q22(I5) - 4.0*Q66(I5))*SICO +
     1               Q12(I5)*(SI4 + CO4)
      QBAR(I5,1,3) = (Q11(I5) - Q12(I5) - 2.0*Q66(I5))*CO3*SI +
     1               (Q12(I5) - Q22(I5) + 2.0*Q66(I5))*SI3*CO
      QBAR(I5,2,1) = QBAR(I5,1,2)
      QBAR(I5,2,2) = Q11(I5)*SI4 + 2.0*(Q12(I5) + 2.0*Q66(I5))*SICO +
     1               Q22(I5)*CO4
      QBAR(I5,2,3) = (Q11(I5) - Q12(I5) - 2.0*Q66(I5))*SI3*CO +
     1               (Q12(I5) - Q22(I5) + 2.0*Q66(I5))*CO3*SI
      QBAR(I5,3,1) = QBAR(I5,1,3)
      QBAR(I5,3,2) = QBAR(I5,2,3)
      QBAR(I5,3,3) = (Q11(I5) + Q22(I5) - 2.0*Q12(I5) - 2.0*Q66(I5))*
     1               SICO + Q66(I5)*(SI4 + CO4)
   80 CONTINUE
C
C     COMBINE THE LAMINA
C
      DO 100 I6 = 1,3
      DO  90 J6 = 1,3
      A(I6,J6) = 0.0
      B(I6,J6) = 0.0
      D(I6,J6) = 0.0
      ADT(I6,J6) = 0.0
   90 CONTINUE
  100 CONTINUE
```

```
      DO 130 I6 = 1,3
      DO 120 J6 = 1,3
      DO 110 NN = 1,MA
      A(I6,J6) = A(I6,J6) + QBAR(NN,I6,J6)*(AH(NN+1)-AH(NN))
      B(I6,J6) = B(I6,J6) + QBAR(NN,I6,J6)*(AH(NN+1)**2 - AH(NN)**2)
      D(I6,J6) = D(I6,J6) + QBAR(NN,I6,J6)*(AH(NN+1)**3 - AH(NN)**3)
  110 CONTINUE
  120 CONTINUE
  130 CONTINUE
      DO 150 I8 = 1,3
      DO 140 J8 = 1,3
      B(I8,J8) = B(I8,J8) / 2.0
      D(I8,J8) = D(I8,J8) / 3.0
      AOT(I8,J8) = A(I8,J8) / ATT
  140 CONTINUE
  150 CONTINUE
C
C     COMPUTE THE AL MATRIX
C
      DET = (AOT(1,1)*AOT(2,2)*AOT(3,3)) + (AOT(1,2)*AOT(2,3)*AOT(3,1))
     1 +   (AOT(1,3)*AOT(2,1)*AOT(3,2)) - (AOT(1,3)*AOT(2,2)*AOT(3,1))
     2 -   (AOT(1,1)*AOT(2,3)*AOT(3,2)) - (AOT(1,2)*AOT(2,1)*AOT(3,3))
      AL(1,1) = (AOT(2,2)*AOT(3,3) - AOT(2,3)*AOT(3,2)) / DET
      AL(1,2) = (AOT(2,3)*AOT(3,1) - AOT(2,1)*AOT(3,3)) / DET
      AL(1,3) = (AOT(2,1)*AOT(3,2) - AOT(2,2)*AOT(3,1)) / DET
      AL(2,2) = (AOT(1,1)*AOT(3,3) - AOT(1,3)*AOT(3,1)) / DET
      AL(2,3) = (AOT(1,2)*AOT(3,1) - AOT(1,1)*AOT(3,2)) / DET
      AL(3,3) = (AOT(1,1)*AOT(2,2) - AOT(1,2)*AOT(2,1)) / DET
      AL(2,1) = AL(1,2)
      AL(3,1) = AL(1,3)
      AL(3,2) = AL(2,3)
      DO 155 I = 1,3
      DO 155 J = 1,3
      AT(I,J) = AL(I,J)/ATT
  155 CONTINUE
      FE1 = 1./AL(1,1)
      FU1 =-FE1*AL(1,2)
      FE2 = 1./AL(2,2)
      FG  = 1./AL(3,3)
      FA1 = 0.
      FA2 = 0.
      IF(AL(1,3).NE.0.) FA1=1./AL(1,3)
      IF(AL(2,3).NE.0.) FA2=1./AL(2,3)
      WRITE(6,5060)
      WRITE(6,5070)
      WRITE (6,5080)(A(I,1),A(I,2),A(I,3),B(I,1),B(I,2),B(I,3),D(I,1),
     1 D(I,2),D(I,3), I = 1,3)
      WRITE(6,5090)
      WRITE(6,5100)
      WRITE (6,5110)(AOT(J,1), AOT(J,2), AOT(J,3), AL(J,1), AL(J,2),
     1 AL(J,3), J = 1,3 )
      WRITE (6,5120)
      WRITE (6,5130) FE1, FE2, FU1, FG
      IF (KEY1.GT.0) CALL BEND
      IF (KEY1.GT.0) GO TO 10
      IF (KEY2.EQ.0) GO TO 160
```

```
      DO 156 L = 1,NLC                                                  SQ50169
      READ (5,1025) N(L,1), N(L,2), N(L,3), M(L,1), M(L,2), M(L,3), T(L)SQ50170
  156 CONTINUE                                                          SQ50171
      IF (KEY4.EQ.0) GO TO 158                                          SQ50172
      CALL TEMP                                                         SQ50173
  158 CONTINUE                                                          SQ50174
      WRITE(6,1070)                                                     SQ50175
      DO 157  L = 1,NLC                                                 SQ50176
      WRITE(6,1080) L                                                   SQ50177
      WRITE(6,1090) N(L,1), M(L,1), N(L,2), M(L,2), N(L,3), M(L,3), T(L)SQ50178
  157 CONTINUE                                                          SQ50179
      CALL BEND                                                         SQ50180
  160 IF (KEY3.EQ.0) GO TO 180                                          SQ50181
      IF (KEY3.EQ.2) GO TO 170                                          SQ50182
      READ (5,1020) SIG1, SIG2, SIG3, PHI                               SQ50183
      WRITE (6,1040) SIG1, SIG2, SIG3, PHI                              SQ50184
  170 CALL STEC                                                         SQ50185
      CALL SSRC                                                         SQ50186
      IF (KEY3.EQ.1) GO TO 180                                          SQ50187
      CALL SURFS                                                        SQ50188
  180 CONTINUE                                                          SQ50189
      IF (KEY5.EQ.0) GO TO 10                                           SQ50190
      READ (5,1020) ( QX(I), QY(I), I = 1,NLC )                         SQ50191
      WRITE(6,5140)                                                     SQ50192
      WRITE(6,5150) ( I, QX(I), QY(I), I = 1,NLC)                       SQ50193
      CALL SHEAR                                                        SQ50194
      GO TO 10                                                          SQ50195
C                                                                       SQ50196
 1000 FORMAT (56H                                                       SQ50197
     1                         )                                        SQ50198
 1010 FORMAT (8I5)                                                      SQ50199
 1020 FORMAT (6F10.0)                                                   SQ50200
 1025 FORMAT (7F9.0)                                                    SQ50201
 1030 FORMAT (2I5,2F10.0)                                               SQ50202
 1040 FORMAT (1H1,5X,'*** INPUT AVERAGE STRESSES ***'//5X,'SIGMA-1 = ', SQ50203
     1 F10.2,5X,'SIGMA-2 = ',F10.2,5X,'TAUXY = ',F10.2,5X,'ANGLE TO STRESQ50204
     2SS STATE = ',F10.5 / )                                            SQ50205
 1050 FORMAT(///,' *** ALLOWABLE STRAIN DATA ***'//1X,'MATYPE',3X,      SQ50206
     1 'LIMIT STRAIN',7X,                                               SQ50207
     2'LIMIT STRAIN',7X,'LIMIT STRAIN',7X,'LIMIT STRAIN',7X,'LIMIT STRAISQ50208
     3N',7X,'LIMIT STRAIN'/10X,'1 - DIRECTION',6X,'2 - DIRECTION',9X,   SQ50209
     4'SHEAR',11X,'1 - DIRECTION',6X,'2 - DIRECTION',9X,'SHEAR'/11X,    SQ50210
     5'COMPRESSION',8X,'COMPRESSION',8X,'NEGATIVE',12X,'POSITIVE',11X,  SQ50211
     6'POSITIVE',11X,'POSITIVE'// )                                     SQ50212
 1060 FORMAT (1X,I3,8X,F7.4,12X,F7.4,11X,F7.4,13X,F7.4,12X,F7.4,12X,    SQ50213
     1 F7.4 / )                                                         SQ50214
 1070 FORMAT(1H1,10X,'*** INPUT DATA FOR COMBINED N - M ANALYSIS ***'//)SQ50215
 1080 FORMAT(5X,'LOAD CASE NUMBER ',I2 / )                              SQ50216
 1090 FORMAT( 5X,'NX  = ',F10.0,10X,'MX  = ',F10.0 //                   SQ50217
     1          5X,'NY  = ',F10.0,10X,'MY  = ',F10.0 //                 SQ50218
     2          5X,'NXY = ',F10.0,10X,'MXY = ',F10.0 ////               SQ50219
     3          5X,'TEMPERATURE = ',F10.0 //////// )                    SQ50220
 5000 FORMAT (////15X,' *** INPUT DATA ***' ///                         SQ50221
     1 5X,'KEY1 = ',I5 // 5X,'KEY2 = ',I5 // 5X,'KEY3 = ',I5 //         SQ50222
     2 5X,'KEY4 = ',I5 // 5X,'KEY5 = ',I5 //                            SQ50223
     3 5X,'THE NUMBER OF LAYERS IN THE LAMINATE IS  ',I2 //             SQ50224
```

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     4 5X,'THE NUMBER OF MATERIAL TYPES IS  ',I2 //                    SQ50225
     5 5X,'THE NUMBER OF LOADING CONDITIONS IS  ',I2 // )              SQ50226
 5020 FORMAT (1H0,'*** MATERIAL DATA ***' // )                         SQ50227
 5030 FORMAT ( 5X,'MATYPE', 5X,'E1',14X,'E2',14X,'U1',15X,'G',15X,'ALPHASQ50228
     11',10X,'ALPHA2',10X,'ALPHA6' // ( 6X,I3, 1X,E15.7, 1X,E15.7, 1X,  SQ50229
     2 E15.7, 1X,E15.7, 1X,E15.7, 1X,E15.7, 1X,E15.7 )  )              SQ50230
 5040 FORMAT(1H1,'*** LAYER DATA ***'//10X,'LAYER NO.   MATYPE',7X,'ORIESQ50231
     1NTATION',11X,'THICKNESS'/ )                                      SQ50232
 5050 FORMAT (5X,2I10,2F20.5)                                          SQ50233
 5060 FORMAT (1H1,///15X,'*** OUTPUT DATA ***'////10X,'COMPOSITE PROPERTSQ50234
     1IES'/// )                                                        SQ50235
 5070 FORMAT (1H ,15X,'A MATRIX',35X,'B MATRIX',35X,'D MATRIX'// )     SQ50236
 5080 FORMAT (1H ,E12.5,2X,E12.5,2X,E12.5,5X,E12.5,2X,E12.5,2X,E12.5,5X,SQ50237
     1E12.5,2X,E12.5,2X,E12.5/ )                                       SQ50238
 5090 FORMAT (///)                                                     SQ50239
 5100 FORMAT (1H ,15X,'(A/T) MATRIX',25X,'(A/T) INVERSE MATRIX'///)    SQ50240
 5110 FORMAT (1H ,E12.5,2X,E12.5,2X,E12.5,5X,E12.5,2X,E12.5,2X,E12.5 /) SQ50241
 5120 FORMAT (1H ,///5X,'AVERAGE LAMINATE ELASTIC CONSTANTS'// )       SQ50242
 5130 FORMAT(1H ,'EX =',E12.5,2X,'EY =',E12.5,2X,'UX =',E12.5,2X,'GXY ='SQ50243
     1,E12.5 //// )                                                    SQ50244
 5140 FORMAT (1H1,10X,'*** SHEAR FORCES ***' /// 5X,'LOAD CASE', 6X,   SQ50245
     1 'QX', 8X, 'QY' //  )                                            SQ50246
 5150 FORMAT ( 8X,I2,4X,F10.0,F10.0 )                                  SQ50247
C                                                                      SQ50248
      END                                                              SQ50249
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      SUBROUTINE STEC                                                   SQ50250
      COMMON AL(3,3), CALE1(400), CALE2(400), CALE3(400), TALE1(400),   SQ50251
     1 TALE2(400), TALE3(400), ACT(3,3), TH(400), Q11(400), Q12(400),   SQ50252
     2 Q22(400), Q66(400), BLF(18), A(3,3), B(3,3), D(3,3), AH(401),    SQ50253
     3 AT(400), E1(400), E2(400), U1(400), U2(400), G(400), SB1(18),    SQ50254
     4 QBAR(400,3,3), GAM(3,400,3), S1A(400), S2A(400), S3A(400),       SQ50255
     5 SJ(1200), S(50), X(50), Y(50), XN(50), YN(50), FX(3), FY(3),     SQ50256
     6 SIGX(1200), SIGY(1200), MATYPE(400)                              SQ50257
      COMMON BSTAR(3,3), CSTAR(3,3), DSTAR(3,3), DSTARI(3,3), BDC(3,3), SQ50258
     1 APRIME(3,3), BPRIME(3,3), CPRIME(3,3), DPRIME(3,3), ASTAR(3,3),  SQ50259
     2 BAB(3,3), Z(401), AI(3,3), E0(10,3), E(10,401,3), K(10,3),       SQ50260
     3 N(10,3), M(10,3), NT(10,3), MT(10,3), QQ11(400), QQ22(400),      SQ50261
     4 QQ12(400), QQ66(400), ALPHAC(400), TAL(3,400), TQA(3,400),       SQ50262
     5 ALPHA1(400), ALPHA2(400), ALPHA6(400), T(10), QX(10), QY(10)     SQ50263
      COMMON CO, CO2, SI, SI2, KEY1, KEY2, KEY3, KEY4, KEY5, SICO, SIG1,SQ50264
     1 SIG2, SIG3, PHI, CON, I, J, I2, I4, MA, NN, DAF, II, LDR, KK,    SQ50265
     2 I6, NLC, DAF3, DAF6, ATT, L, ML1, MB, DFL                        SQ50266
      REAL K, N, M, NT, MT                                              SQ50267
C                                                                       SQ50268
C     ROTATE THE AVERAGE STRESSES TO THE REFERENCE AXIS                 SQ50269
C                                                                       SQ50270
      IF(KEY3.NE.1) GO TO 10                                            SQ50271
      CON = PHI*0.0174533                                               SQ50272
      CO = COS(CON)                                                     SQ50273
      CO2 = CO**2                                                       SQ50274
      SI = SIN(CON)                                                     SQ50275
      SI2 = SI**2                                                       SQ50276
      SICO = SI*CO                                                      SQ50277
      CIG1 = SIG1*CO2 + SIG2*SI2 - 2.*SIG3*SICO                         SQ50278
      CIG2 = SIG1*SI2 + SIG2*CO2 + 2.*SIG3*SICO                         SQ50279
      CIG3 = SIG1*SICO - SIG2*SICO + SIG3*(CO2-SI2)                     SQ50280
C                                                                       SQ50281
C     COMPUTE THE LAMINATE STRAINS                                      SQ50282
C                                                                       SQ50283
   10 MX = 1                                                            SQ50284
      IF(KEY3.EQ.2)MX=6                                                 SQ50285
      DO 20 I=1,MX                                                      SQ50286
      NA = 3*I - 2                                                      SQ50287
      IF(KEY3.EQ.1) GO TO 30                                            SQ50288
      MZ = I                                                            SQ50289
      IF(I.GE.4) MZ = I-3                                               SQ50290
      CIG1 = 0.                                                         SQ50291
      CIG2 = 0.                                                         SQ50292
      CIG3 = 0.                                                         SQ50293
      IF(I.GE.4) GO TO 40                                               SQ50294
      GO TO (12,14,16), MZ                                              SQ50295
   12 CIG1 = 1.0                                                        SQ50296
      GO TO 30                                                          SQ50297
   14 CIG2 = 1.0                                                        SQ50298
      GO TO 30                                                          SQ50299
   16 CIG3 = 1.0                                                        SQ50300
      GO TO 30                                                          SQ50301
   40 GO TO (42,44,46), MZ                                              SQ50302
   42 CIG1 = -1.0                                                       SQ50303
      GO TO 30                                                          SQ50304
   44 CIG2 = -1.0                                                       SQ50305
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      GO TO 30                                                          SQ50306
   46 CIG3 = -1.0                                                       SQ50307
   30 BLF(NA)   = AL(1,1)*CIG1+AL(1,2)*CIG2+AL(1,3)*CIG3                SQ50308
      BLF(NA+1) = AL(2,1)*CIG1+AL(2,2)*CIG2+AL(2,3)*CIG3                SQ50309
      BLF(NA+2) = AL(3,1)*CIG1+AL(3,2)*CIG2+AL(3,3)*CIG3                SQ50310
   20 CONTINUE                                                          SQ50311
      RETURN                                                            SQ50312
      END                                                               SQ50313
```

```
      SUBROUTINE SSRC
      COMMON AL(3,3), CALE1(400), CALE2(400), CALE3(400), TALE1(400),
     1 TALE2(400), TALE3(400), AOT(3,3), TH(400), Q11(400), Q12(400),
     2 Q22(400), Q66(400), BLF(18), A(3,3), B(3,3), D(3,3), AH(401),
     3 AT(400), E1(400), E2(400), U1(400), U2(400), G(400), SB1(18),
     4 QBAR(400,3,3), GAM(3,400,3), S1A(400), S2A(400), S3A(400),
     5 SJ(1200), S(50), X(50), Y(50), XN(50), YN(50), FX(3), FY(3),
     6 SIGX(1200), SIGY(1200), MATYPE(400)
      COMMON BSTAR(3,3), CSTAR(3,3), DSTAR(3,3), DSTARI(3,3), BDC(3,3),
     1 APRIME(3,3), BPRIME(3,3), CPRIME(3,3), DPRIME(3,3), ASTAR(3,3),
     2 BAB(3,3), Z(401), AI(3,3), EO(10,3), E(10,401,3), K(10,3),
     3 N(10,3), M(10,3), NT(10,3), MT(10,3), QQ11(400), QQ22(400),
     4 QQ12(400), QQ66(400), ALPHAC(400), TAL(3,400), TQA(3,400),
     5 ALPHA1(400), ALPHA2(400), ALPHA6(400), T(10), QX(10), QY(10)
      COMMON CO, CO2, SI, SI2, KEY1, KEY2, KEY3, KEY4, KEY5, SICO, SIG1,
     1 SIG2, SIG3, PHI, CON, I, J, I2, I4, MA, NN, DAF, II, LDR, KK,
     2 I6, NLC, DAF3, DAF6, ATT, L, ML1, MB, DEL
      REAL K, N, M, NT, MT
C
C     SET INDEX
C
      N1 = 1
      IF(KEY3.EQ.2)N1=6
      WRITE (6,6000)
      DO 80 I1=1,N1
      N2 = 3*I1 - 2
C
C     COMPUTE THE INPUT STRESS LEVEL
C
      SB1(N2)   = BLF(N2)*AOT(1,1)+BLF(N2+1)*AOT(1,2)+BLF(N2+2)*AOT(1,3)
      SB1(N2+1) = BLF(N2)*AOT(2,1)+BLF(N2+1)*AOT(2,2)+BLF(N2+2)*AOT(2,3)
      SB1(N2+2) = BLF(N2)*AOT(3,1)+BLF(N2+1)*AOT(3,2)+BLF(N2+2)*AOT(3,3)
      WRITE(6,50)
      WRITE(6,60) SB1(N2), SB1(N2+1), SB1(N2+2)
C
C     COMPUTE THE STRESSES AND STRAINS IN EACH LAYER
C
      WRITE(6,10)
      DO 20 I2=1,MA
      I6 = MATYPE(I2)
      CON = TH(I2)*0.0174533
      CO = COS(CON)
      SI = SIN(CON)
      CO2 = CO**2
      SI2 = SI**2
      SICO = SI*CO
      EE1 = BLF(N2)*CO2+BLF(N2+1)*SI2+BLF(N2+2)*SICO
      EE2 = BLF(N2)*SI2+BLF(N2+1)*CO2-BLF(N2+2)*SICO
      EE3 = -2.*BLF(N2)*SICO+2.*BLF(N2+1)*SICO+BLF(N2+2)*(CO2-SI2)
      SS1 = Q11(I2) * EE1  +  Q12(I2)  * EE2
      SS2 = Q12(I2) * EE1  +  Q22(I2)  * EE2
      SS3 = Q66(I2) * EE3
      EU1 = TALE1(I6)
      IF(EE1.LE.0.) EU1 = CALE1(I6)
      EU2 = TALE2(I6)
      IF(EE2.LE.0.) EU2 = CALE2(I6)
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```
      EU3 = TALE3(I6)                                                   SQ50370
      IF(EE3.LE.0.) EU3 = CALE3(I6)                                     SQ50371
      IF(KEY3-1) 30,30,40                                               SQ50372
   30 AMAR1 = 100.                                                      SQ50373
      IF(EE1.NE.0.) AMAR1 = EU1/EE1 - 1.0                               SQ50374
      AMAR2 = 100.0                                                     SQ50375
      IF(EE2.NE.0.) AMAR2 = EU2/EE2 - 1.0                               SQ50376
      AMAR3 = 100.0                                                     SQ50377
      IF(EE3.NE.0.) AMAR3 = EU3/EE3 - 1.0                               SQ50378
      WRITE(6,70)I2,SS1,SS2,SS3,EE1,EE2,EE3,AMAR1,AMAR2,AMAR3           SQ50379
      GO TO 20                                                          SQ50380
   40 IF(EE1.EQ.0.) GO TO 41                                            SQ50381
      S1A(I2) = EU1/EE1                                                 SQ50382
      GO TO 42                                                          SQ50383
   41 S1A(I2)= 1000000.0                                                SQ50384
   42 IF(EE2.EQ.0.) GO TO 43                                            SQ50385
      S2A(I2)= EU2/EE2                                                  SQ50386
      GO TO 44                                                          SQ50387
   43 S2A(I2)= 1000000.0                                                SQ50388
   44 IF(EE3.EQ.0.) GO TO 45                                            SQ50389
      S3A(I2) = EU3 / EE3                                               SQ50390
      GO TO 46                                                          SQ50391
   45 S3A(I2) = 1000000.0                                               SQ50392
   46 SD = 1.                                                           SQ50393
      IF(I1.GE.4)SD=-1.                                                 SQ50394
      SD1=S1A(I2)*SD                                                    SQ50395
      SD2=S2A(I2)*SD                                                    SQ50396
      SD3=S3A(I2)*SD                                                    SQ50397
      WRITE(6,70)I2,SS1,SS2,SS3,EE1,EE2,EE3,SD1,SD2,SD3                 SQ50398
   20 CONTINUE                                                          SQ50399
      IF(KEY3.NE.2) GO TO 80                                            SQ50400
      DA = S1A(1)                                                       SQ50401
      DB = S2A(1)                                                       SQ50402
      DC = S3A(1)                                                       SQ50403
      IF (MA .EQ. 1) GO TO 95                                           SQ50404
      DO 90 I4=2,MA                                                     SQ50405
      IF(S1A(I4).LE.DA) DA = S1A(I4)                                    SQ50406
      IF(S2A(I4).LE.DB) DB = S2A(I4)                                    SQ50407
      IF(S3A(I4).LE.DC) DC = S3A(I4)                                    SQ50408
   90 CONTINUE                                                          SQ50409
   95 CONTINUE                                                          SQ50410
      DAF = DA                                                          SQ50411
      IF(DB.LE.DAF) DAF =DB                                             SQ50412
      IF(DC.LE.DAF) DAF =DC                                             SQ50413
      WRITE(6,100) DAF                                                  SQ50414
      IF (I1 .EQ. 3) DAF3 = DAF                                         SQ50415
      IF (I1 .EQ. 6) DAF6 = DAF                                         SQ50416
   80 CONTINUE                                                          SQ50417
      RETURN                                                            SQ50418
C                                                                       SQ50419
   10 FORMAT (2X,'LAYER',5X,'SIG-1',3X,'SIG-2',7X,'TAU-12',8X,'STRAIN-1'SQ50420
     1 ,5X,'STRAIN-2',5X,'GAMMA-12',3X,'ALLO - MAR-1',3X,'ALLO - MAR-2',SQ50421
     2 3X,'ALLO - MAR-12' // )                                          SQ50422
   50 FORMAT(////)                                                      SQ50423
   60 FORMAT(1H ,'   COMPOSITE AVERAGE STRESSES AT THE REFERENCE AXES', SQ50424
     13X,'SIGX = ',E12.5,5X,'SIGY = ',E12.5,5X,'SIGXY = ',E12.5 // )    SQ50425
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   70 FORMAT ( 3X,I2,4X,E11.4,2X,E11.4,2X,E11.4,3X,E11.4,2X,E11.4,2X,      SQ50426
     1 E11.4,2X,E11.4,4X,E11.4,4X,E11.4  / )                                SQ50427
  100 FORMAT (1H0,'ABSOLUTE VALUE OF THE MAXIMUM STRESS = ',E12.4 )        SQ50428
 6000 FORMAT (1H1)                                                          SQ50429
C                                                                           SQ50430
      END                                                                   SQ50431
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```
      SUBROUTINE SURFS
      COMMON AL(3,3), CALE1(400), CALE2(400), CALE3(400), TALE1(400),
     1 TALE2(400), TALE3(400), AOT(3,3), TH(400), Q11(400), Q12(400),
     2 Q22(400), Q66(400), BLF(18), A(3,3), B(3,3), D(3,3), AH(401),
     3 AT(400), E1(400), E2(400), U1(400), U2(400), G(400), SB1(18),
     4 QBAR(400,3,3), GAM(3,400,3), S1A(400), S2A(400), S3A(400),
     5 SJ(1200), S(50), X(50), Y(50), XN(50), YN(50), FX(3), FY(3),
     6 SIGX(1200), SIGY(1200), MATYPE(400)
      COMMON BSTAR(3,3), CSTAR(3,3), DSTAR(3,3), DSTARI(3,3), BDC(3,3),
     1 APRIME(3,3), BPRIME(3,3), CPRIME(3,3), DPRIME(3,3), ASTAR(3,3),
     2 BAR(3,3), Z(401), AI(3,3), E0(10,3), E(10,401,3), K(10,3),
     3 N(10,3), M(10,3), NT(10,3), MT(10,3), QQ11(400), QQ22(400),
     4 QQ12(400), QQ66(400), ALPHAC(400), TAU(3,400), TOA(3,400),
     5 ALPHA1(400), ALPHA2(400), ALPHA6(400), T(10), QX(10), QY(10)
      COMMON CO, CO2, SI, SI2, KEY1, KEY2, KEY3, KEY4, KEY5, SICO, SIG1,
     1 SIG2, SIG3, PHI, CON, I, J, I2, I4, MA, NN, DAF, II, LDR, KK,
     2 I6, NLC, DAF3, DAF6, ATT, L, ML1, MB, DEL
      REAL K, N, M, NT, MT
      WRITE(6,10)
      DO 200 J=1,MA
      BETA = TH(J) * 0.0174533
      CO = COS(BETA)
      SI = SIN(BETA)
      CO2 = CO ** 2
      SI2 = SI ** 2
      SICO = SI * CO
      DO 100 I=1,3
      N22 = 3 * I - 2
      GAM(1,J,I) = BLF(N22)*CO2 + BLF(N22+1)*SI2 + BLF(N22+2)*SICO
      GAM(2,J,I) = BLF(N22)*SI2 + BLF(N22+1)*CO2 - BLF(N22+2)*SICO
      GAM(3,J,I) =-2.*BLF(N22)*SICO + 2.*BLF(N22+1)*SICO + BLF(N22+2) *
     1             (CO2 - SI2)
  100 CONTINUE
  200 CONTINUE
      DO 400 ITAU = 1,2
      IF (ITAU .EQ. 1) DAF = DAF3
      IF (ITAU .EQ. 2) DAF = DAF6
      KAB = DAF * 0.00010 + 2.0
      DO 340 KAA = 1,KAB
      II = 0
      WRITE(6,325)
      WRITE(6,30)
      AAK = KAA - 1
      TAUXY = AAK * 10000.0
      IF(TAUXY.GE.DAF) TAUXY = DAF*0.99
      IF (ITAU.EQ.2) TAUXY = -TAUXY
      DO 330 J=1,MA
      I6 = MATYPE(J)
      FX(1) = TALE1(I6)
      FX(2) = TALE2(I6)
      FX(3) = TALE3(I6)
      FY(1) = CALE1(I6)
      FY(2) = CALE2(I6)
      FY(3) = CALE3(I6)
      DO 320 I=1,3
      DD = TAUXY * GAM(I,J,3)
```

```
      Q1 = FX(I) - Q0                                                   S050488
      Q2 = FY(I) - Q0                                                   S050489
      XIP = 0.1E15                                                      S050490
      XIN = 0.1E15                                                      S050491
      IF(GAM(I,J,1).EQ.0.) GO TO 210                                    S050492
      XIP = Q1 / GAM(I,J,1)                                             S050493
      XIN = Q2 / GAM(I,J,1)                                             S050494
  210 YIP = 0.1E15                                                      S050495
      YIN = 0.1E15                                                      S050496
      IF(GAM(I,J,2).EQ.0.) GO TO 220                                    S050497
      YIP = Q1 / GAM(I,J,2)                                             S050498
      YIN = Q2 / GAM(I,J,2)                                             S050499
  220 IBALL = - I                                                       S050500
      WRITE(6,230) J, XIP, YIP, TAUXY, I                                S050501
      WRITE(6,230) J, XIN, YIN, TAUXY, IBALL                            S050502
      II = II + 2                                                       S050503
      SIGX(II-1) = XIP                                                  S050504
      SIGY(II-1) = YIP                                                  S050505
      SIGX(II) = XIN                                                    S050506
      SIGY(II) = YIN                                                    S050507
  320 CONTINUE                                                          S050508
      WRITE(6,325)                                                      S050509
  330 CONTINUE                                                          S050510
      CALL ISECT                                                        S050511
      WRITE(6,1000) TAUXY, (I, X(I), Y(I), I=1,KK)                      S050512
  340 CONTINUE                                                          S050513
  400 CONTINUE                                                          S050514
      RETURN                                                            S050515
C                                                                       S050516
   10 FORMAT(////4X,'YIELD SURFACE COORDINATES'//)                      S050517
   30 FORMAT(3X,'PLY NO.   SIGX INTERCEPT  SIGY INTERCEPT       TAUXY   S050518
     1  MODE'//)                                                        S050519
  230 FORMAT(1H ,3X,I3,6X,E12.5,4X,E12.5,4X,E12.5,4X,I2)                S050520
  325 FORMAT(//)                                                        S050521
 1000 FORMAT(1H0,'THE INTERACTION YIELD COORDINATES'/' FOR TAUXY = ',   S050522
     1E12.5,' ARE' //'   I        X(I)            Y(I)'//(I4,2E15.5/))  S050523
C                                                                       S050524
      END                                                               S050525
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```
      SUBROUTINE ISECT
      COMMON AL(3,3), CALE1(400), CALE2(400), CALE3(400), TALE1(400),
     1 TALE2(400), TALE3(400), ADT(3,3), TH(400), Q11(400), Q12(400),
     2 Q22(400), Q66(400), BLF(18), A(3,3), B(3,3), D(3,3), AH(401),
     3 AT(400), E1(400), E2(400), U1(400), U2(400), G(400), SB1(18),
     4 QBAR(400,3,3), GAM(3,400,3), S1A(400), S2A(400), S3A(400),
     5 SJ(1200), S(50), X(50), Y(50), XN(50), YN(50), FX(3), FY(3),
     6 SIGX(1200), SIGY(1200), MATYPE(400)
      COMMON BSTAR(3,3), CSTAR(3,3), DSTAR(3,3), DSTARI(3,3), BDC(3,3),
     1 APRIME(3,3), BPRIME(3,3), CPRIME(3,3), DPRIME(3,3), ASTAR(3,3),
     2 BAB(3,3), Z(401), AI(3,3), EO(10,3), E(10,401,3), K(10,3),
     3 N(10,3), M(10,3), NT(10,3), MT(10,3), QQ11(400), QQ22(400),
     4 QQ12(400), QQ66(400), ALPHAC(400), TAL(3,400), TQA(3,400),
     5 ALPHA1(400), ALPHA2(400), ALPHA6(400), T(10), QX(10), QY(10)
      COMMON CO, CO2, SI, SI2, KEY1, KEY2, KEY3, KEY4, KEY5, SIGC, SIG1,
     1 SIG2, SIG3, PHI, CON, I, J, I2, I4, MA, NN, DAF, II, LDR, KK,
     2 I6, NLC, DAF3, DAF6, ATT, L, ML1, MB, DEL
      REAL K, N, M, NT, MT
      KK= 4
      X(1) = 2000000.0
      X(2) = 0.0
      X(3) =-2000000.0
      X(4) = 0.0
      Y(1) = 0.0
      Y(2) = 2000000.0
      Y(3) = 0.0
      Y(4) =-2000000.0
      X(5) = 2000000.0
      Y(5) = 0.0
      S(1) = -1.0
      S(2) =  1.0
      S(3) = -1.0
      S(4) =  1.0
      DO 1000 J=1,II
      IF(ABS(SIGX(J)).GT.0.000100)GO TO 15
      WRITE(6,2100)
      WRITE(6,3000)
      GO TO 600
   15 SJ(J) = - SIGY(J)/SIGX(J)
      ICOUNT = 0
      KCOUNT = 0
      NCOUNT = 0
      DO 40 I=1,KK
      IB = 0
      IP1 = I + 1
      ZZ = SJ(J) - S(I)
      Z1 = ABS(ZZ / S(I))
      IF(Z1.LT.0.000100)GO TO 40
      D1 = SJ(J)*(Y(I) - S(I) * X(I)) - SIGY(J)*S(I)
      D2 = Y(I) - S(I)*X(I) - SIGY(J)
      YY = D1 / ZZ
      XX = D2 / ZZ
      X1 = AMAX1(X(I),X(IP1))
      X2 = AMIN1(X(I),X(IP1))
      Y1 = AMAX1(Y(I),Y(IP1))
      Y2 = AMIN1(Y(I),Y(IP1))
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      IF(ABS(XX-X(I)).GT.10.0. OR.ABS(YY-Y(I)).GT.10.0) GO TO 18         SQ50582
      IF(ICOUNT.EQ.0) NCOUNT = 1                                         SQ50583
      IF(ICOUNT.EQ.1) KCOUNT = 1                                         SQ50584
      GO TO 25                                                           SQ50585
   18 IF(ABS(XX-X(IP1)).GT.10.0. OR.ABS(YY-Y(IP1)).GT.10.0)GO TO 20      SQ50586
      IB = 1                                                             SQ50587
      IF(ICOUNT.EQ.0) NCOUNT = 1                                         SQ50588
      IF(ICOUNT.EQ.1) KCOUNT = 1                                         SQ50589
      GO TO 25                                                           SQ50590
   20 IF(XX.LT.X1.AND.XX.GT.X2) GO TO 25                                 SQ50591
      GO TO 40                                                           SQ50592
   25 IF(ICOUNT.EQ.1) GO TO 30                                           SQ50593
      IF(IB.EQ.0) GO TO 27                                               SQ50594
      IBAR1 = I+1                                                        SQ50595
      GO TO 29                                                           SQ50596
   27 IBAR1 = I                                                          SQ50597
   29 XX1 = XX                                                           SQ50598
      YY1 = YY                                                           SQ50599
      ICOUNT = 1                                                         SQ50600
      GO TO 40                                                           SQ50601
   30 XAL = ABS(XX1-XX)                                                  SQ50602
      YAL = ABS(YY1-YY)                                                  SQ50603
      ALTH2 = XAL**2 + YAL**2                                            SQ50604
      IF(ALTH2.LT.625.0) KCOUNT = 0                                      SQ50605
      IF(ALTH2.LT.625.0) GO TO 40                                        SQ50606
      IF(IB.EQ.0) GO TO 35                                               SQ50607
      IBAR2 = I+1                                                        SQ50608
      GO TO 36                                                           SQ50609
   35 IBAR2 = 1                                                          SQ50610
   36 XX2 = XX                                                           SQ50611
      YY2 = YY                                                           SQ50612
      ICOUNT = 2                                                         SQ50613
      GO TO 50                                                           SQ50614
   40 CONTINUE                                                           SQ50615
      IF(ICOUNT.LT.2) GO TO 1000                                         SQ50616
   50 JCOUNT = 1                                                         SQ50617
      IF(SIGX(J)) 100,120,120                                            SQ50618
  100 IF(SIGY(J)) 105,110,110                                            SQ50619
  105 NQUAD = 3                                                          SQ50620
      GO TO 150                                                          SQ50621
  110 NQUAD = 2                                                          SQ50622
      GO TO 150                                                          SQ50623
  120 IF(SIGY(J)) 125,130,130                                            SQ50624
  125 NQUAD = 4                                                          SQ50625
      GO TO 150                                                          SQ50626
  130 NQUAD = 1                                                          SQ50627
  150 MCOUNT = 0                                                         SQ50628
      KKK = KK + 1                                                       SQ50629
      DO 300 I = 1,KKK                                                   SQ50630
      GO TO (200,280), JCOUNT                                            SQ50631
  200 IF(I.LT.IBAR1) GO TO 300                                           SQ50632
      GO TO (210,220,230,240), NQUAD                                     SQ50633
  210 IF(XX1.LT.XX2.OR.YY1.GT.YY2) GO TO 260                             SQ50634
      GO TO 250                                                          SQ50635
  220 IF(XX1.LT.XX2.OR.YY1.LT.YY2) GO TO 260                             SQ50636
      GO TO 250                                                          SQ50637
```

```
230 IF(XX1.GT.XX2.OR.YY1.LT.YY2) GO TO 260
    GO TO 250
240 IF(XX1.GT.XX2.OR.YY1.GT.YY2) GO TO 260
250 LCOUNT = 1
    GO TO 270
260 LCOUNT = 2
270 JCOUNT = 2
    GO TO 300
280 IF(I.GT.IBAR2) GO TO 300
    MCOUNT = MCOUNT + 1
300 CONTINUE
    IF(LCOUNT.EQ.1) MCOUNT = MCOUNT + NCOUNT
    IF(LCOUNT.EQ.1) NODES = MCOUNT
    IF(LCOUNT.EQ.2) MCOUNT = MCOUNT - KCOUNT
    IF(LCOUNT.EQ.2) NODES = KK- MCOUNT
    KNEW = KK+ 2 - NODES
    XN(1) = XX1
    YN(1) = YY1
    IF(LCOUNT.EQ.1) GO TO 320
    DO 310 I=1,MCOUNT
    XN(I+1) = X(IBAR1 + I)
    YN(I+1) = Y(IBAR1 + I)
310 CONTINUE
    XN(KNEW) = XX2
    YN(KNEW) = YY2
    GO TO 400
320 XN(2) = XX2
    YN(2) = YY2
    IX = KK - IBAR2
    IF(IBAR2.EQ.KK)GO TO 340
    DO 330 I=1,IX
    N1 = I + 2
    M1 = IBAR2 + I
    XN(N1) = X(M1)
    YN(N1) = Y(M1)
330 CONTINUE
340 NN= IX + 2
    DO 350 I=1,IBAR1
    MM = NN + I
    XN(MM) = X(I)
    YN(MM) = Y(I)
350 CONTINUE
400 KK= KNEW
    YN(KK+1)= YN(1)
    XN(KK+1)= XN(1)
    X(KK+1) = XN(1)
    Y(KK+1) = YN(1)
    DO 410 I=1,KK
    X(I) = XN(I)
    Y(I) = YN(I)
    DX = XN(I+1) - XN(I)
    IF(ABS(DX).GT.0.000001)GO TO 450
    WRITE(6,2020)
    WRITE(6,3000)
    GO TO 600
450 DY = YN(I+1) - YN(I)
```

```
      IF(ABS(DY).GT.0.000001)GO TO 500
      WRITE(6,2110)
      WRITE(6,3000)
      GO TO 600
  500 S(I) = DY/DX
  410 CONTINUE
 1000 CONTINUE
  600 RETURN
C
 2100 FORMAT(1H0,'COMPUTATIONS ARE STOPPED.  A ZERO IS DETECTED FOR THE
     *VALUE OF SIGX')
 2020 FORMAT(1H0,'A LINE WITH A VERTICAL SLOPE IN THE INTERACTION PLOT W
     1AS DETECTED. FURTHER COMPUTATIONS FOR THIS INTERACTION PLOT WERE S
     2TOPPED'///)
 2110 FORMAT(1H0,'COMPUTATIONS ARE STOPPED.  A SLOPE OF ZERO WAS DETECTE
     *D IN THE INTERACTION CURVE')
 3000 FORMAT(1H0,'THE FOLLOWING INTERACTION YIELD COORDINATES SHOW INTER
     *MEDIATE VALUES DETERMINED',/1X,'BEFORE DETECTING A ZERO VALUE.  TH
     *ESE VALUES ARE TO BE USED FOR AN ERROR'/1X,'ANALYSIS ONLY'/)
C
      END
```

```
      SUBROUTINE BEND                                                   SQ50715
      COMMON AL(3,3), CALE1(400), CALE2(400), CALE3(400), TALE1(400),   SQ50716
     1 TALE2(400), TALE3(400), AOT(3,3), TH(400), Q11(400), Q12(400),   SQ50717
     2 Q22(400), Q66(400), BLF(18), A(3,3), B(3,3), D(3,3), AH(401),    SQ50718
     3 AT(400), E1(400), E2(400), U1(400), U2(400), G(400), SB1(18),    SQ50719
     4 QBAR(400,3,3), GAM(3,400,3), S1A(400), S2A(400), S3A(400),       SQ50720
     5 SJ(1200), S(50), X(50), Y(50), XN(50), YN(50), FX(3), FY(3),     SQ50721
     6 SIGX(1200), SIGY(1200), MATYPE(400)                              SQ50722
      COMMON BSTAR(3,3), CSTAR(3,3), DSTAR(3,3), DSTARI(3,3), BDC(3,3), SQ50723
     1 APRIME(3,3), BPRIME(3,3), CPRIME(3,3), DPRIME(3,3), ASTAR(3,3),  SQ50724
     2 BAB(3,3), Z(401), AI(3,3), E0(10,3), E(10,401,3), K(10,3),       SQ50725
     3 N(10,3), M(10,3), NT(10,3), MT(10,3), QQ11(400), QQ22(400),      SQ50726
     4 QQ12(400), QQ66(400), ALPHAC(400), TAL(3,400), TQA(3,400),       SQ50727
     5 ALPHA1(400), ALPHA2(400), ALPHA6(400), T(10), QX(10), QY(10)     SQ50728
      COMMON CO, CO2, SI, SI2, KEY1, KEY2, KEY3, KEY4, KEY5, SICO, SIG1,SQ50729
     1 SIG2, SIG3, PHI, CON, I, J, I2, I4, MA, NN, DAF, II, LDR, KK,    SQ50730
     2 I6, NLC, DAF3, DAF6, ATT, L, ML1, MB, DEL                        SQ50731
      REAL K, N, M, NT, MT                                              SQ50732
      DO 10 I = 1,3                                                     SQ50733
      DO 10 J = 1,3                                                     SQ50734
      BSTAR(I,J) = 0.0                                                  SQ50735
      CSTAR(I,J) = 0.0                                                  SQ50736
      DSTAR(I,J) = 0.0                                                  SQ50737
      DSTARI(I,J)= 0.0                                                  SQ50738
      BDC(I,J)   = 0.0                                                  SQ50739
      BAB(I,J) = 0.0                                                    SQ50740
      APRIME(I,J)= 0.0                                                  SQ50741
      BPRIME(I,J)= 0.0                                                  SQ50742
      CPRIME(I,J)= 0.0                                                  SQ50743
      DPRIME(I,J)= 0.0                                                  SQ50744
      ASTAR(I,J) = 0.0                                                  SQ50745
   10 CONTINUE                                                          SQ50746
      DO 30 I = 1,3                                                     SQ50747
      DO 30 J = 1,3                                                     SQ50748
      ASTAR(I,J) = AI(I,J)                                              SQ50749
      DO 20 L = 1,3                                                     SQ50750
      BSTAR(I,J) = BSTAR(I,J) + AI(I,L)*B(L,J)                          SQ50751
      CSTAR(I,J) = CSTAR(I,J) + B(I,L)*AI(L,J)                          SQ50752
   20 CONTINUE                                                          SQ50753
   30 CONTINUE                                                          SQ50754
      DO 50 I = 1,3                                                     SQ50755
      DO 50 J = 1,3                                                     SQ50756
      DO 40 L = 1,3                                                     SQ50757
      BAB(I,J) = BAB(I,J) + B(I,L)*BSTAR(L,J)                           SQ50758
   40 CONTINUE                                                          SQ50759
   50 CONTINUE                                                          SQ50760
      DO 60 I = 1,3                                                     SQ50761
      DO 60 J = 1,3                                                     SQ50762
      DSTAR(I,J) = D(I,J) - BAB(I,J)                                    SQ50763
      BSTAR(I,J) = -BSTAR(I,J)                                          SQ50764
   60 CONTINUE                                                          SQ50765
      DET = (DSTAR(1,1)*DSTAR(2,2)*DSTAR(3,3))                          SQ50766
     1     + (DSTAR(1,2)*DSTAR(2,3)*DSTAR(3,1))                         SQ50767
     2     + (DSTAR(1,3)*DSTAR(2,1)*DSTAR(3,2))                         SQ50768
     3     - (DSTAR(1,3)*DSTAR(2,2)*DSTAR(3,1))                         SQ50769
     4     - (DSTAR(1,1)*DSTAR(2,3)*DSTAR(3,2))                         SQ50770
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     5     - (DSTAR(1,2)*DSTAR(2,1)*DSTAR(3,3))                         SQ50771
      DSTARI(1,1) = (DSTAR(2,2)*DSTAR(3,3) - DSTAR(2,3)*DSTAR(3,2)) /DETSQ50772
      DSTARI(1,2) = (DSTAR(2,3)*DSTAR(3,1) - DSTAR(2,1)*DSTAR(3,3)) /DETSQ50773
      DSTARI(1,3) = (DSTAR(2,1)*DSTAR(3,2) - DSTAR(2,2)*DSTAR(3,1)) /DETSQ50774
      DSTARI(2,2) = (DSTAR(1,1)*DSTAR(3,3) - DSTAR(1,3)*DSTAR(3,1)) /DETSQ50775
      DSTARI(2,3) = (DSTAR(1,2)*DSTAR(3,1) - DSTAR(1,1)*DSTAR(3,2)) /DETSQ50776
      DSTARI(3,3) = (DSTAR(1,1)*DSTAR(2,2) - DSTAR(1,2)*DSTAR(2,1)) /DETSQ50777
      DSTARI(2,1) = DSTARI(1,2)                                         SQ50778
      DSTARI(3,1) = DSTARI(1,3)                                         SQ50779
      DSTARI(3,2) = DSTARI(2,3)                                         SQ50780
      DO 80 I = 1,3                                                     SQ50781
      DO 80 J = 1,3                                                     SQ50782
      DPRIME(I,J) = DSTARI(I,J)                                         SQ50783
      DO 70 L = 1,3                                                     SQ50784
      BPRIME(I,J) = BPRIME(I,J) + BSTAR(I,L)*DSTARI(L,J)                SQ50785
      CPRIME(I,J) = CPRIME(I,J) + DSTARI(I,L)*CSTAR(L,J)                SQ50786
   70 CONTINUE                                                          SQ50787
   80 CONTINUE                                                          SQ50788
      DO 100 I = 1,3                                                    SQ50789
      DO 100 J = 1,3                                                    SQ50790
      CPRIME(I,J) = -CPRIME(I,J)                                        SQ50791
      DO 90 L = 1,3                                                     SQ50792
      BDC(I,J) = BDC(I,J) + BPRIME(I,L)*CSTAR(L,J)                      SQ50793
   90 CONTINUE                                                          SQ50794
  100 CONTINUE                                                          SQ50795
      DO 110 I = 1,3                                                    SQ50796
      DO 110 J = 1,3                                                    SQ50797
      APRIME(I,J) = ASTAR(I,J) - BDC(I,J)                               SQ50798
  110 CONTINUE                                                          SQ50799
      WRITE (6,900)                                                     SQ50800
      WRITE (6,1000)                                                    SQ50801
      WRITE (6,1010) (APRIME(I,1), APRIME(I,2), APRIME(I,3), BPRIME(I,1)SQ50802
     1 , BPRIME(I,2), BPRIME(I,3) , I = 1,3)                            SQ50803
      WRITE (6,1030)                                                    SQ50804
      WRITE (6,1010) (CPRIME(I,1), CPRIME(I,2), CPRIME(I,3), DPRIME(I,1)SQ50805
     1 , DPRIME(I,2), DPRIME(I,3) , I = 1,3)                            SQ50806
      WRITE(6,1060)                                                     SQ50807
      WRITE (6,1020)                                                    SQ50808
      IF (KEY1.EQ.1) GO TO 200                                          SQ50809
      DO 135 L = 1,NLC                                                  SQ50810
      DO 130 I = 1,3                                                    SQ50811
      EO(L,I) = 0.0                                                     SQ50812
      K (L,I) = 0.0                                                     SQ50813
      DO 120 J = 1,3                                                    SQ50814
      EO(L,I) = EO(L,I) + APRIME(I,J)*N(L,J) + BPRIME(I,J)*M(L,J)       SQ50815
      K (L,I) = K (L,I) + CPRIME(I,J)*N(L,J) + DPRIME(I,J)*M(L,J)       SQ50816
  120 CONTINUE                                                          SQ50817
  130 CONTINUE                                                          SQ50818
  135 CONTINUE                                                          SQ50819
      WRITE (6,1080)                                                    SQ50820
      WRITE (6,1090)                                                    SQ50821
      DO 136  L = 1,NLC                                                 SQ50822
      WRITE (6,1100) L                                                  SQ50823
      WRITE (6,1110) EO(L,1), K(L,1), EO(L,2), K(L,2), EO(L,3), K(L,3)  SQ50824
  136 CONTINUE                                                          SQ50825
      DO 155 L = 1,NLC                                                  SQ50826
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      DO 150 I = 1,MA                                                   SQ50827
      Z(I) = 0.0                                                        SQ50828
      DO 140 J = 1,3                                                    SQ50829
      E(L,I,J) = 0.0                                                    SQ50830
  140 CONTINUE                                                          SQ50831
  150 CONTINUE                                                          SQ50832
  155 CONTINUE                                                          SQ50833
      MB = MA + 1                                                       SQ50834
      DO 160 I = 1,MB                                                   SQ50835
      Z(I) = AH(I)                                                      SQ50836
  160 CONTINUE                                                          SQ50837
      DO 185 L = 1,NLC                                                  SQ50838
      DO 180 I = 1,MB                                                   SQ50839
      DO 170 J = 1,3                                                    SQ50840
      E(L,I,J) = EO(L,J) + Z(I)*K(L,J)                                  SQ50841
  170 CONTINUE                                                          SQ50842
  180 CONTINUE                                                          SQ50843
  185 CONTINUE                                                          SQ50844
      DO 195 L = 1,NLC                                                  SQ50845
      WRITE (6,1050)                                                    SQ50846
      WRITE(6,1070) L                                                   SQ50847
      DO 190 I = 1,MB                                                   SQ50848
      ML1 = 0                                                           SQ50849
      J = I                                                             SQ50850
      IF ( Z(I) .GE. 0.0) J = I - 1                                     SQ50851
  186 IF ( ML1 .NE. 0 ) J = I                                           SQ50852
      I6 = MATYPE(J)                                                    SQ50853
      CON = TH(J)*0.0174533                                             SQ50854
      CO   = COS(CON)                                                   SQ50855
      SI   = SIN(CON)                                                   SQ50856
      CO2  = CO**2                                                      SQ50857
      SI2  = SI**2                                                      SQ50858
      SICO = SI*CO                                                      SQ50859
  187 CONTINUE                                                          SQ50860
      EE1  = E(L,I,1)*CO2 + E(L,I,2)*SI2 + E(L,I,3)*SICO                SQ50861
      EE2  = E(L,I,1)*SI2 + E(L,I,2)*CO2 - E(L,I,3)*SICO                SQ50862
      EE3  = -2.0*E(L,I,1)*SICO + 2.0*E(L,I,2)*SICO + E(L,I,3)*(CO2-SI2)SQ50863
      SS1=Q11(J)*(EE1-ALPHA1(I6)*T(L))+Q12(J)*(EE2-ALPHA2(I6)*T(L))     SQ50864
      SS2=Q12(J)*(EE1-ALPHA1(I6)*T(L))+Q22(J)*(EE2-ALPHA2(I6)*T(L))     SQ50865
      SS3=Q66(J)*(EE3-ALPHA6(I6)*T(L))                                  SQ50866
      EU1  = TALE1(I6)                                                  SQ50867
      IF (EE1.LE.0.0) EU1 = CALE1(I6)                                   SQ50868
      EU2  = TALE2(I6)                                                  SQ50869
      IF (EE2.LE.0.0) EU2 = CALE2(I6)                                   SQ50870
      EU3  = TALE3(I6)                                                  SQ50871
      IF (EE3.LE.0.0) EU3 = CALE3(I6)                                   SQ50872
      AMAR1 = 100.0                                                     SQ50873
      IF (EE1.NE.0.0) AMAR1 = EU1/EE1 - 1.0                             SQ50874
      AMAR2 = 100.0                                                     SQ50875
      IF (EE2.NE.0.0) AMAR2 = EU2/EE2 - 1.0                             SQ50876
      AMAR3 = 100.0                                                     SQ50877
      IF (EE3.NE.0.0) AMAR3 = EU3/EE3 - 1.0                             SQ50878
      WRITE (6,5000) Z(I), TH(J)                                        SQ50879
      WRITE (6,1040) I,SS1,SS2,SS3,EE1,EE2,EE3,AMAR1,AMAR2,AMAR3        SQ50880
      IF (Z(I) .LT. -0.0001 .OR. Z(I) .GT. 0.0001 ) GO TO 190           SQ50881
      IF ( ML1  .EQ.  1 ) GO TO 190                                     SQ50882
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      ML1 = 1                                                           SQ50883
      GO TO 186                                                         SQ50884
  190 CONTINUE                                                          SQ50885
  195 CONTINUE                                                          SQ50886
  200 CONTINUE                                                          SQ50887
      RETURN                                                            SQ50888
C                                                                       SQ50889
  900 FORMAT (1H1,10X,'*** BENDING OUTPUT DATA ***'//// )               SQ50890
 1000 FORMAT (27X,'A-PRIME MATRIX',40X,'B-PRIME MATRIX'// )             SQ50891
 1010 FORMAT (10X,E14.7,3X,E14.7,3X,E14.7,6X,E14.7,3X,E14.7,3X,E14.7 /) SQ50892
 1020 FORMAT (27X,'C-PRIME MATRIX',40X,'D-PRIME MATRIX' ////// )        SQ50893
 1030 FORMAT (//)                                                       SQ50894
 1040 FORMAT (3X,I2,4X,E11.4,2X,E11.4,2X,E11.4,3X,E11.4,2X,E11.4,2X,    SQ50895
     1 E11.4,2X,E11.4,4X,E11.4,4X,E11.4  / )                            SQ50896
 1050 FORMAT(1H1,' *** COMBINED BENDING AND MEMBRANE STRESSES, STRAINS, SQ50897
     1AND MARGINS OF SAFETY FOR EACH LAYER ***'///2X,'LAYER',5X,'SIG-1',SQ50898
     2 8X,'SIG-2',7X,'TAU-12',8X,'STRAIN-1',5X,'STRAIN-2',5X,'GAMMA-12',SQ50899
     3 6X,'MAR-1',10X,'MAR-2',10X,'MAR-12'  // )                        SQ50900
 1060 FORMAT ( / )                                                      SQ50901
 1070 FORMAT (10X,'LOAD CASE NUMBER ',I2 / )                            SQ50902
 1080 FORMAT ( //// )                                                   SQ50903
 1090 FORMAT (10X,'*** MID-PLANE STRAINS AND CURVATURES ***'/// )       SQ50904
 1100 FORMAT (5X,'LOAD CASE NUMBER = ', I2 // )                         SQ50905
 1110 FORMAT (5X,'E0 - X  = ',E15.7,10X,'K - X  = ',E15.7 //            SQ50906
     1        5X,'E0 - Y  = ',E15.7,10X,'K - Y  = ',E15.7 //            SQ50907
     2        5X,'E0 - XY = ',E15.7,10X,'K - XY = ',E15.7 /      )      SQ50908
 5000 FORMAT (10X,'Z = ',F10.6,5X,'THETA = ',F5.0 )                     SQ50909
C                                                                       SQ50910
      END                                                               SQ50911
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      SUBROUTINE TEMP
      COMMON AL(3,3), CALE1(400), CALE2(400), CALE3(400), TALE1(400),
     1 TALE2(400), TALE3(400), AOT(3,3), TH(400), Q11(400), Q12(400),
     2 Q22(400), Q66(400), BLE(18), A(3,3), B(3,3), D(3,3), AH(401),
     3 AT(400), E1(400), E2(400), U1(400), U2(400), G(400), SB1(18),
     4 QBAR(400,3,3), GAM(3,400,3), S1A(400), S2A(400), S3A(400),
     5 SJ(1200), S(50), X(50), Y(50), XN(50), YN(50), FX(3), FY(3),
     6 SIGX(1200), SIGY(1200), MATYPE(400)
      COMMON BSTAR(3,3), CSTAR(3,3), DSTAR(3,3), DSTARI(3,3), BDC(3,3),
     1 APRIME(3,3), BPRIME(3,3), CPRIME(3,3), DPRIME(3,3), ASTAR(3,3),
     2 BAB(3,3), Z(401), AI(3,3), FO(10,3), F(10,401,3), K(10,3),
     3 N(10,3), M(10,3), NT(10,3), MT(10,3), QQ11(400), QQ22(400),
     4 QQ12(400), QQ66(400), ALPHAC(400), TAL(3,400), TOA(3,400),
     5 ALPHA1(400), ALPHA2(400), ALPHA6(400), T(10), QX(10), QY(10)
      COMMON CO, CO2, SI, SI2, KEY1, KEY2, KEY3, KEY4, KEY5, SIGC, SIG1,
     1 SIG2, SIG3, PHI, CON, I, J, I2, I4, MA, NN, DAF, II, LDR, KK,
     2 I6, NLC, DAF3, DAF6, ATT, L, ML1, MB, DEL
      REAL K, N, M, NT, MT
C
C     COMPUTE THE TEMPERATURE INDUCED N AND M VECTORS
C
      DO    5 L = 1,NLC
      DO    4 I = 1,3
      NT(L,I) = 0.0
      MT(L,I) = 0.0
    4 CONTINUE
    5 CONTINUE
      DO 10 I = 1,MA
      QQ11(I) = 0.0
      QQ22(I) = 0.0
      QQ12(I) = 0.0
      QQ66(I) = 0.0
      ALPHAC(I) = 0.0
   10 CONTINUE
      DO 30 I = 1,3
      DO 20 J = 1,MA
      TOA(I,J) = 0.0
   20 CONTINUE
   30 CONTINUE
      DO 50 L = 1,NLC
      DO 40 I = 1,MA
      IM = MATYPE(I)
      U2(IM) = E2(IM) / E1(IM) * U1(IM)
      DEL = 1.0 - U1(IM)*U2(IM)
      QQ11(I) = E1(IM) / DEL
      QQ22(I) = E2(IM) / DEL
      QQ12(I) = QQ11(I)*U2(IM)
      QQ66(I) = G(IM)
C
C     COMPUTE QQ * ALPHA
C
      QALP11 = QQ11(I)*ALPHA1(IM) + QQ12(I)*ALPHA2(IM)
      QALP22 = QQ12(I)*ALPHA1(IM) + QQ22(I)*ALPHA2(IM)
      QALP66 = QQ66(I)*ALPHA6(IM)
      CON    = TH(I )*0.0174533
      CO     = COS(CON)
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      CO2   = CO**2
      SI    = SIN(CON)
      SI2   = SI**2
      SICO = SI * CO
C
C     TRANSFORM (QQ * ALPHA) INTO X - Y SYSTEM
C
      TQA(1,I) = QALP11 * CO2  + QALP22 * SI2  - 2.0 * QALP66 * SICO
      TQA(2,I) = QALP11 * SI2  + QALP22 * CO2  + 2.0 * QALP66 * SICO
      TQA(3,I) = QALP11 * SICO - QALP22 * SICO + QALP66 * (CO2 - SI2)
C
   40 CONTINUE
   50 CONTINUE
C
C     COMBINE THE LAMINA
C
      DO 80 L = 1,NLC
      DO 70 I = 1,3
      DO 60 J = 1,MA
      NT(L,I)  = NT(L,I) + TQA(I,J) * (AH(J+1) - AH(J))
      MT(L,I)  = MT(L,I) + TQA(I,J) * (AH(J+1)**2 - AH(J)**2)
   60 CONTINUE
   70 CONTINUE
   80 CONTINUE
      L = 1
      DO 86 I = 1,3
      DO 85 J = 1,3
      ALPHAC(I) = ALPHAC(I) + AI(I,J)*NT(L,J)
   85 CONTINUE
   86 CONTINUE
      DO 100 L = 1,NLC
      DO  90 I = 1,3
      MT(L,I) = 0.5*MT(L,I)
   90 CONTINUE
  100 CONTINUE
      WRITE (6,1000)
      WRITE (6,1010) (ALPHAC(I), I = 1,3)
      DO 105 L = 1,NLC
      WRITE (6,1020) NT(L,1), NT(L,2), NT(L,3), MT(L,1), MT(L,2),
     1 MT(L,3)
  105 CONTINUE
      DO 120 L = 1,NLC
      DO 110 I = 1,3
      N(L,I)   =  T(L) * NT(L,I) + N(L,I)
      M(L,I)   =  T(L) * MT(L,I) + M(L,I)
  110 CONTINUE
  120 CONTINUE
      RETURN
C
 1000 FORMAT (1H1,10X,'*** THERMAL EXPANSION DATA ***'//// )
 1010 FORMAT (5X,'THERMAL EXPANSION COEFFICIENT X  FOR COMPOSITE = ',
     1F15.7//5X, 'THERMAL EXPANSION COEFFICIENT Y  FOR COMPOSITE = ',
     2F15.7//5X, 'THERMAL EXPANSION COEFFICIENT XY FOR COMPOSITE = ',
     3F15.7///)
 1020 FORMAT ( 5X,'COEFFICIENT OF THERMAL FORCE  NX  = ',E15.7//
     1         5X,'COEFFICIENT OF THERMAL FORCE  NY  = ',E15.7//
```

```
     2          5X,'COEFFICIENT OF THERMAL FORCE  NXY = ',E15.7//        SQ51024
     3          5X,'COEFFICIENT OF THERMAL MOMENT MX  = ',E15.7//        SQ51025
     4          5X,'COEFFICIENT OF THERMAL MOMENT MY  = ',E15.7//        SQ51026
     5          5X,'COEFFICIENT OF THERMAL MOMENT MXY = ',E15.7///)      SQ51027
C                                                                        SQ51028
      END                                                                SQ51029
```

```
      SUBROUTINE SHEAR                                                  SQ51030
      COMMON AL(3,3), CALE1(400), CALE2(400), CALE3(400), TALE1(400),   SQ51031
     1 TALE2(400), TALE3(400), AOT(3,3), TH(400), Q11(400), Q12(400),   SQ51032
     2 Q22(400), Q66(400), BLF(18), A(3,3), B(3,3), D(3,3), AH(401),    SQ51033
     3 AT(400), E1(400), E2(400), U1(400), U2(400), G(400), SB1(18),    SQ51034
     4 QBAR(400,3,3), GAM(3,400,3), S1A(400), S2A(400), S3A(400),       SQ51035
     5 SJ(1200), S(50), X(50), Y(50), XN(50), YN(50), FX(3), FY(3),     SQ51036
     6 SIGX(1200), SIGY(1200), MATYPE(400)                              SQ51037
      COMMON BSTAR(3,3), CSTAR(3,3), DSTAR(3,3), DSTARI(3,3), BDC(3,3), SQ51038
     1 APRIME(3,3), BPRIME(3,3), CPRIME(3,3), DPRIME(3,3), ASTAR(3,3),  SQ51039
     2 BAB(3,3), Z(401), AI(3,3), E0(10,3), E(10,401,3), K(10,3),       SQ51040
     3 N(10,3), M(10,3), NT(10,3), MT(10,3), QQ11(400), QQ22(400),      SQ51041
     4 QQ12(400), QQ66(400), ALPHAC(400), TAL(3,400), TOA(3,400),       SQ51042
     5 ALPHA1(400), ALPHA2(400), ALPHA6(400), T(10), QX(10), QY(10)     SQ51043
      COMMON CO, CO2, SI, SI2, KEY1, KEY2, KEY3, KEY4, KEY5, SICO, SIG1,SQ51044
     1 SIG2, SIG3, PHI, CON, I, J, I2, I4, MA, NN, DAF, II, LDR, KK,    SQ51045
     2 I6, NLC, DAF3, DAF6, ATT, L, ML1, MB, DEL                        SQ51046
      REAL K, N, M, NT, MT                                              SQ51047
      MB = MA + 1                                                       SQ51048
      DETD = D(1,1) * D(2,2) - D(1,3) * D(2,3)                          SQ51049
      WRITE (6,5000)                                                    SQ51050
      WRITE (6,5010)                                                    SQ51051
      DO 70 L = 1,NLC                                                   SQ51052
C                                                                       SQ51053
C     COMPUTE THE THIRD DERIVATIVES OF W -- W.R.T. X AND Y              SQ51054
C                                                                       SQ51055
      D3WX = - (D(2,2) / DETD)*QX(L) + (D(2,3) / DETD)*QY(L)            SQ51056
      D3WY =   (D(1,3) / DETD)*QX(L) - (D(1,1) / DETD)*QY(L)            SQ51057
      ML1 = 0                                                           SQ51058
      ML2 = 0                                                           SQ51059
      DO 60 I = 1,MB                                                    SQ51060
      IF ( I .EQ. 1 ) GO TO 3                                           SQ51061
      IF ( I .EQ. MB) GO TO 3                                           SQ51062
      GO TO 5                                                           SQ51063
    3 ZS = AH(I)                                                        SQ51064
      J = I                                                             SQ51065
      SXZ = 0.0                                                         SQ51066
      SYZ = 0.0                                                         SQ51067
      GO TO 50                                                          SQ51068
    5 ZS = AH(I)                                                        SQ51069
      IF (ZS .LT. 0.0) GO TO 10                                         SQ51070
      IF (ZS .EQ. 0.0 .AND. ML1 .EQ. 0) GO TO 20                        SQ51071
      IF (ZS .GT. 0.0 .AND. ML1 .EQ. 0) GO TO 30                        SQ51072
      J = I                                                             SQ51073
      GO TO 40                                                          SQ51074
   10 J = I - 1                                                         SQ51075
      GO TO 40                                                          SQ51076
   20 J = I - 1                                                         SQ51077
      ML1 = 1                                                           SQ51078
      GO TO 40                                                          SQ51079
   30 ZS = 0.0                                                          SQ51080
      J = I - 1                                                         SQ51081
      ML1 = 1                                                           SQ51082
   40 CONTINUE                                                          SQ51083
      SXZ  =  ( QBAR(J,1,1)*D3WX + QBAR(J,2,3)*D3WY ) * (1.0 / 8.0) *   SQ51084
     1         ( 4.0*ZS**2 - ATT**2 )                                   SQ51085
```

```
      SYZ  =  ( QBAR(J,1,3)*D3WX + QBAR(J,2,2)*D3WY ) * (1.0 / 8.0) *    SQ51086
     1          ( 4.0*ZS**2 - ATT**2 )                                   SQ51087
   50 WRITE (6,5030)     ZS, SXZ, SYZ                                    SQ51088
      IF (ML2 .EQ. 1) GO TO 60                                           SQ51089
      IF (ZS .EQ. 0.0 .AND. ML1 .EQ.1 ) GO TO 55                         SQ51090
      GO TO 60                                                           SQ51091
   55 ML2 = 1                                                            SQ51092
      GO TO 5                                                            SQ51093
   60 CONTINUE                                                           SQ51094
   70 CONTINUE                                                           SQ51095
      RETURN                                                             SQ51096
C                                                                        SQ51097
 5000 FORMAT (////10X,'*** INTERLAMINAR SHEAR STRESSES ***' //// )       SQ51098
 5010 FORMAT (10X,'          Z            TAU-XZ          TAU-YZ' /// )  SQ51099
 5030 FORMAT (11X,2X,F11.5,6X,F7.0,8X,F7.0 // )                          SQ51100
C                                                                        SQ51101
      END                                                                SQ51102
```

APPENDIX IV

SAMPLE PROBLEM INPUT

```
  SAMPLE PROBLEM INTERACTION DIAGRAM -- 60/0 , 40/45 DEGREES       121530P010001
   0     1     2     0     1     4     1     1                       121530P010002
20000.000.2100000. 0.21      850000.  0.0       0.0       0.0      121530P010003
     1     1         0         0.30                                121530P010004
     2     1       +45         0.20                                121530P010005
     3     1       -45         0.20                                121530P010006
     4     1         0         0.30                                121530P010007
-0.006600 -0.006660 -0.010000 +0.005800 +0.002550 +0.010000        121530P010008
+100.0    0.0       0.0       0.0       0.0       0.0       0.0    121530P010009
+100.0     0.0                                                     121530P010010

                                                        CC = 0010
```

APPENDIX V

SAMPLE PROBLEM OUTPUT

```
GENERAL DYNAMICS                    360 PROCEDURE S05             PAGE 0001
FORT WORTH DIVISION                 PROBLEM 121530-01              01/12/70

 SAMPLE PROBLEM INTERACTION DIAGRAM -- 60/0 , 40/45 DEGREES


                *** INPUT DATA ***

    KEY1 =      0

    KEY2 =      1

    KEY3 =      2

    KEY4 =      0

    KEY5 =      1

    THE NUMBER OF LAYERS IN THE LAMINATE IS   4

    THE NUMBER OF MATERIAL TYPES IS   1

    THE NUMBER OF LOADING CONDITIONS IS   1



*** MATERIAL DATA ***

    MATYPE     E1              E2              U1              G               ALPHA1          ALPHA2          ALPHA6

      1    0.2000000E 08   0.2100000E 07   0.2100000E 00   0.8500000E 06   0.0             0.0             0.0
```

```
GENERAL DYNAMICS                     360 PROCEDURE SQ5               PAGE 0002
FORT WORTH DIVISION                  PROBLEM 121530-01                01/12/70

*** LAYER DATA ***

          LAYER NO.   MATYPE        ORIENTATION          THICKNESS

              1          1              0.0               0.30000
              2          1            45.00000            0.20000
              3          1           -45.00000            0.20000
              4          1              0.0               0.30000



*** ALLOWABLE STRAIN DATA ***

MATYPE   LIMIT STRAIN      LIMIT STRAIN      LIMIT STRAIN      LIMIT STRAIN      LIMIT STRAIN      LIMIT STRAIN
         1 - DIRECTION     2 - DIRECTION         SHEAR         1 - DIRECTION     2 - DIRECTION         SHEAR
          COMPRESSION       COMPRESSION        NEGATIVE           POSITIVE          POSITIVE          POSITIVE

  1        -0.0066           -0.0067           -0.0100            0.0058            0.0025            0.0100
```

```
GENERAL DYNAMICS                   360 PROCEDURE SQ5              PAGE 0003
FORT WORTH DIVISION                PROBLEM 121530-01               01/12/70


          *** OUTPUT DATA ***


     COMPOSITE PROPERTIES


          A MATRIX                                 B MATRIX                                   D MATRIX

0.14705E 08   0.22347E 07   0.0           0.50000E 00   0.0            -0.17983E 06   0.16026E 07   0.60810E 05   0.0
0.22347E 07   0.39147E 07   0.0           0.0           0.0            -0.17983E 06   0.60810E 05   0.19988E 06   0.0
0.0           0.0           0.26417E 07  -0.17983E 06  -0.17983E 06    0.0          0.0           0.0           0.94722E 05


          (A/T) MATRIX                        (A/T) INVERSE MATRIX

0.14705E 08   0.22347E 07   0.0           0.74466E-07  -0.42508E-07   0.0
0.22347E 07   0.39147E 07   0.0          -0.42508E-07   0.27971E-06   0.0
0.0           0.0           0.26417E 07   0.0           0.0           0.37855E-06


     AVERAGE LAMINATE ELASTIC CONSTANTS

FX = 0.13429E 08  EY = 0.35751E 07  UX = 0.57085E 00  GXY = 0.26417E 07
```

```
GENERAL DYNAMICS                           360 PROCEDURE SQ5                  PAGE 0004
FORT WORTH DIVISION                        PROBLEM 121530-01                   01/12/70

        *** INPUT DATA FOR COMBINED N - M ANALYSIS ***

    LOAD CASE NUMBER  1

    NX   =        100.        MX   =         0.

    NY   =          0.        MY   =         0.

    NXY  =          0.        MXY  =         0.


    TEMPERATURE =         0.
```

```
GENERAL DYNAMICS                      360 PROCEDURE SQ5                PAGE 0005
FORT WORTH DIVISION                   PROBLEM 121530-01                01/12/70

       *** BENDING OUTPUT DATA ***


                   A-PRIME MATRIX                                                      B-PRIME MATRIX

        0.7484948E-07   -0.3965840E-07   -0.1196803E-14     -0.2372022E-13    0.6139651E-14    0.6681114E-07
       -0.3965840E-07    0.3008633E-06    0.6341155E-15      0.1256796E-13   -0.3253044E-14    0.4959049E-06
       -0.1196803E-14    0.6341160E-15    0.4048606E-06      0.3197891E-07    0.3545241E-06   -0.1068273E-14


       -0.2372022E-13    0.1256796E-13    0.3197891E-07      0.6338108E-06   -0.1640532E-06   -0.2117280E-13
        0.6139651E-14   -0.3253044E-14    0.3545241E-06     -0.1640532E-06    0.5371868E-05    0.5480290E-14
        0.6681114E-07    0.4959049E-06   -0.1068273E-14     -0.2117280E-13    0.5480290E-14    0.1162553E-04


                   C-PRIME MATRIX                                                      D-PRIME MATRIX





       *** MID-PLANE STRAINS AND CURVATURES ***


    LOAD CASE NUMBER =   1

    EO - X  =   0.7484948E-05          K - X  =  -0.2372021E-11

    EO - Y  =  -0.3965839E-05          K - Y  =   0.6139650E-12

    EO - XY =  -0.1196803E-12          K - XY =   0.6681114E-05
```

```
GENERAL DYNAMICS                     360 PROCEDURE SQ5                 PAGE 0006
FORT WORTH DIVISION                  PROBLEM 121530-01                  01/12/70

 *** COMBINED BENDING AND MEMBRANE STRESSES, STRAINS, AND MARGINS OF SAFETY FOR EACH LAYER ***

 LAYER      SIG-1          SIG-2        TAU-12         STRAIN-1      STRAIN-2     GAMMA-12       MAR-1          MAR-2          MAR-12

         LOAD CASE NUMBER  1

         Z =  -0.500000       THETA =     0.
   1     0.1486E 03  -0.5C51E 01  -0.2839E 01    0.7485E-05  -0.3966E-05  -0.3341E-05   0.7739E 03     0.1678E 04     0.2993E 04

         Z =  -0.200000       THETA =    45.
   2     0.2301E 02   0.56C5E 01  -0.9733E 01    0.1091E-05   0.2428E-05  -0.1145E-04   0.5313E 04     0.1049E 04     0.8723E 03

         Z =    0.0           THETA =    45.
   3     0.3613E 02   0.4492E 01  -0.9733E 01    0.1760E-05   0.1760E-05  -0.1145E-04   0.3295E 04     0.1448E 04     0.8723E 03

         Z =    0.0           THETA =   -45.
   3     0.3613E 02   0.4492E 01   0.9733E 01    0.1760E-05   0.1760E-05   0.1145E-04   0.3295E 04     0.1448E 04     0.8723E 03

         Z =   0.200000       THETA =   -45.
   4     0.2301E 02   0.5605E 01   0.9733E 01    0.1091E-05   0.2428E-05   0.1145E-04   0.5313E 04     0.1049E 04     0.8723E 03

         Z =   0.500000       THETA =     0.
   5     0.1486E 03  -0.5C51E 01   0.2839E 01    0.7485E-05  -0.3966E-05   0.3341E-05   0.7739E 03     0.1678E 04     0.2993E 04
```

```
GENERAL DYNAMICS                      360 PROCEDURE SQ5                 PAGE 0007
FORT WORTH DIVISION                   PROBLEM 121530-0.                  01/12/70



  COMPOSITE AVERAGE STRESSES AT THE REFERENCE AXES   SIGX =  0.10000E 01    SIGY = -0.59605E-07    SIGXY =  0.0

LAYER     SIG-1        SIG-2        TAU-12        STRAIN-1     STRAIN-2     GAMMA-12     ALLO - MAR-1   ALLO - MAR-2   ALLO - MAR-12

  1     0.1477E 01  -0.5669E-01    0.0          0.7447E-07  -0.4251E-07    0.0          0.7789E 05     0.1567E 06     0.1000E 07
  2     0.3231E 00   0.4079E-01  -0.9943E-01    0.1598E-07   0.1598E-07  -0.1170E-06   0.3630E 06     0.1596E 06     0.8549E 05
  3     0.3231E 00   0.4079E-01   0.9943E-01    0.1598E-07   0.1598E-07   0.1170E-06   0.3630E 06     0.1596E 06     0.8549E 05
  4     0.1477E 01  -0.5669E-01    0.0          0.7447E-07  -0.4251E-07    0.0          0.7789E 05     0.1567E 06     0.1000E 07

ABSOLUTE VALUE OF THE MAXIMUM STRESS =    0.7789E 05



  COMPOSITE AVERAGE STRESSES AT THE REFERENCE AXES   SIGX = -0.29802E-06    SIGY =  0.10000E 01    SIGXY =  0.0

LAYER     SIG-1        SIG-2        TAU-12        STRAIN-1     STRAIN-2     GAMMA-12     ALLO - MAR-1   ALLO - MAR-2   ALLO - MAR-12

  1    -0.7302E 00   0.5713E 00    0.0         -0.4251E-07   0.2797E-06    0.0          0.1553E 06     0.9117E 04     0.1000E 07
  2     0.2436E 01   0.3028E 00   0.2739E 00    0.1186E-06   0.1186E-06   0.3222E-06   0.4890E 05     0.2150E 05     0.3103E 05
  3     0.2436E 01   0.3028E 00  -0.2739E 00    0.1186E-06   0.1186E-06  -0.3222E-06   0.4890E 05     0.2150E 05     0.3103E 05
  4    -0.7302E 00   0.5713E 00    0.0         -0.4251E-07   0.2797E-06    0.0          0.1553E 06     0.9117E 04     0.1000E 07

ABSOLUTE VALUE OF THE MAXIMUM STRESS =    0.9117E 04



  COMPOSITE AVERAGE STRESSES AT THE REFERENCE AXES   SIGX =  0.0            SIGY =  0.0            SIGXY =  0.10000E 01
```

```
GENERAL DYNAMICS                          360 PROCEDURE SQ5                PAGE 0008
FORT WORTH DIVISION                       PROBLEM 121530-01                 01/12/70

 LAYER      SIG-1          SIG-2         TAU-12        STRAIN-1       STRAIN-2      GAMMA-12     ALLO - MAR-1   ALLO - MAR-2   ALLO - MAR-12

   1     0.0           0.0           0.3218E 00    0.0            0.0           0.3785E-06    0.1000E 07     0.1000E 07     0.2642E 05
   2     0.3719E 01   -0.3155E 00   -0.1918E-06    0.1893E-06    -0.1893E-06   -0.2256E-12    0.3064E 05     0.3519E 05     0.4432E 11
   3    -0.3719E 01    0.3155E 00   -0.1918E-06   -0.1893E-06     0.1893E-06   -0.2256E-12    0.3487E 05     0.1347E 05     0.4432E 11
   4     0.0           0.0           0.3218E 00    0.0            0.0           0.3785E-06    0.1000E 07     0.1000E 07     0.2642E 05

ABSOLUTE VALUE OF THE MAXIMUM STRESS =    0.1347E 05


   COMPOSITE AVERAGE STRESSES AT THE REFERENCE AXES   SIGX = -0.10000E 01     SIGY =  0.59605E-07     SIGXY =  0.0

 LAYER      SIG-1          SIG-2         TAU-12        STRAIN-1       STRAIN-2      GAMMA-12     ALLO - MAR-1   ALLO - MAR-2   ALLO - MAR-12

   1    -0.1477E 01    0.5669E-01    0.0          -0.7447E-07     0.4251E-07    0.0          -0.8863E 05    -0.5999E 05    -0.1000E 07
   2    -0.3281E 00   -0.4079E-01    0.9943E-01   -0.1598E-07    -0.1598E-07    0.1170E-06   -0.4131E 06    -0.4168E 06    -0.8549E 05
   3    -0.3281E 00   -0.4079E-01   -0.9943E-01   -0.1598E-07    -0.1598E-07   -0.1170E-06   -0.4131E 06    -0.4168E 06    -0.8549E 05
   4    -0.1477E 01    0.5669E-01    0.0          -0.7447E-07     0.4251E-07    0.0          -0.8863E 05    -0.5999E 05    -0.1000E 07

ABSOLUTE VALUE OF THE MAXIMUM STRESS =    0.5999E 05


   COMPOSITE AVERAGE STRESSES AT THE REFERENCE AXES   SIGX =  0.29802E-06     SIGY = -0.10000E 01     SIGXY =  0.0

 LAYER      SIG-1          SIG-2         TAU-12        STRAIN-1       STRAIN-2      GAMMA-12     ALLO - MAR-1   ALLO - MAR-2   ALLO - MAR-12

   1     0.7302E 00   -0.5713E 00    0.0           0.4251E-07    -0.2797E-06    0.0          -0.1364E 06    -0.2381E 05    -0.1000E 07
   2    -0.2436E 01   -0.3028E 00   -0.2739E 00   -0.1186E-06    -0.1186E-06   -0.3222E-06   -0.5565E 05    -0.5615E 05    -0.3103E 05
   3    -0.2436E 01   -0.3028E 00    0.2739E 00   -0.1186E-06    -0.1186E-06    0.3222E-06   -0.5565E 05    -0.5615E 05    -0.3103E 05
```

```
GENERAL DYNAMICS                      360 PROCEDURE S95            PAGE 0009
FORT WORTH DIVISION                   PROBLEM 121530-01             01/12/70
   4     0.7302E 00  -0.5713E 00   0.0          0.4251E-07  -0.2797E-06   0.0         -0.1364E 06   -0.2381E 05   -0.1000E 07

ABSOLUTE VALUE OF THE MAXIMUM STRESS =    0.2381E 05


   COMPOSITE AVERAGE STRESSES AT THE REFERENCE AXES    SIGX =  0.0             SIGY =  0.0              SIGXY = -0.10000E 01

 LAYER      SIG-1          SIG-2          TAU-12          STRAIN-1        STRAIN-2        GAMMA-12    ALLO - MAR-1   ALLO - MAR-2   ALLO - MAR-12

   1     0.0            0.0           -0.3219E 00     0.0            0.0             -0.3785E-06  -0.1000E 07    -0.1000E 07    -0.2642E 05

   2    -0.3719E 01     0.3155E 00     0.1918E-06    -0.1893E-06     0.1893E-06      0.2256E-12  -0.3487E 05    -0.1347E 05    -0.4432E 11

   3     0.3719E 01    -0.3155E 00     0.1918E-06     0.1893E-06    -0.1893E-06      0.2256E-12  -0.3064E 05    -0.3519E 05    -0.4432E 11

   4     0.0            0.0           -0.3218E 00     0.0            0.0             -0.3785E-06  -0.1000E 07    -0.1000E 07    -0.2642E 05

ABSOLUTE VALUE OF THE MAXIMUM STRESS =    0.1347E 05


   YIELD SURFACE COORDINATES


  PLY NO.    SIGX INTERCEPT   SIGY INTERCEPT       TAUXY        MODE

     1         0.77888E 05    -0.13644E 06      0.0              1
     1        -0.88632E 05     0.15526E 06      0.0             -1
     1        -0.59988E 05     0.91166E 04      0.0              2
     1         0.15668E 06    -0.23810E 05      0.0             -2
     1         0.10000E 15     0.10000E 15      0.0              3
     1         0.10000E 15     0.10000E 15      0.0             -3


     2         0.36295E 06     0.48904E 05      0.0              1
     2        -0.41305E 06    -0.55649E 05      0.0             -1
     2         0.15959E 06     0.21501E 05      0.0              2
     2        -0.41631E 05    -0.55155E 05      0.0             -2
```

```
      2       -0.85489E 05      0.31035E 05      0.0            3
      2        0.85489E 05     -0.31035E 05      0.0           -3


      3        0.36299E 06      0.48904E 05      0.0            1
      3       -0.41305E 06     -0.55649E 05      0.0           -1
      3        0.15959E 06      0.21501E 05      0.0            2
      3       -0.41681E 06     -0.56155E 05      0.0           -2
      3        0.85489E 05     -0.31035E 05      0.0            3
      3       -0.85489E 05      0.31035E 05      0.0           -3


      4        0.77888E 05     -0.13644E 06      0.0            1
      4       -0.88632E 05      0.15526E 06      0.0           -1
      4       -0.59988E 05      0.91166E 04      0.0            2
      4        0.15668E 06     -0.23810E 05      0.0           -2
      4        0.10000E 15      0.10000E 15      0.0            3
      4        0.10000E 15      0.10000E 15      0.0           -3


THE INTERACTION YIELD COORDINATES
FOR TAUXY =  0.0          ARE

  I        X(I)             Y(I)

  1   -0.11105E 06    -0.40687E 05

  2    0.34230E 05    -0.18608E 05

  3    0.75901E 05    -0.34806E 04

  4    0.83723E 05     0.10221E 05

  5    0.43196E 05     0.15681E 05

  6   -0.91352E 05    -0.47665E 04

  7   -0.11180E 06    -0.40586E 05


  PLY NO.    SIGX INTERCEPT   SIGY INTERCEPT        TAUXY          MODE


      1        0.77888E 05     -0.13644E 06     0.10000E 05        1
      1       -0.88632E 05      0.15526E 06     0.10000E 05       -1
```

```
      1        -0.59988E 05      0.91166E 04      0.10000E 05       2
      1         0.15668E 06     -0.23810E 05      0.10000E 05      -2
      1         0.10000E 15      0.10000E 15      0.10000E 05       3
      1         0.10000E 15      0.10000E 15      0.10000E 05      -3


      2         0.24453E 06      0.32945E 05      0.10000E 05       1
      2        -0.53151E 06     -0.71608E 05      0.10000E 05      -1
      2         0.27804E 06      0.37460E 05      0.10000E 05       2
      2        -0.29835E 06     -0.40196E 05      0.10000E 05      -2
      2        -0.85489E 05      0.31035E 05      0.10000E 05       3
      2         0.85489E 05     -0.31035E 05      0.10000E 05      -3


      3         0.48144E 06      0.64862E 05      0.10000E 05       1
      3        -0.29460E 06     -0.39690E 05      0.10000E 05      -1
      3         0.41133E 05      0.55417E 04      0.10000E 05       2
      3        -0.53526E 06     -0.72114E 05      0.10000E 05      -2
      3         0.85489E 05     -0.31035E 05      0.10000E 05       3
      3        -0.85489E 05      0.31035E 05      0.10000E 05      -3


      4         0.77888E 05     -0.13644E 06      0.10000E 05       1
      4        -0.88632E 05      0.15526E 06      0.10000E 05      -1
      4        -0.59988E 05      0.91166E 04      0.10000E 05       2
      4         0.15668E 06     -0.23810E 05      0.10000E 05      -2
      4         0.10000E 15      0.10000E 15      0.10000E 05       3
      4         0.10000E 15      0.10000E 15      0.10000E 05      -3


THE INTERACTION YIELD COORDINATES
FOR TAUXY =  0.10000E 05 ARE

  I         X(I)              Y(I)

  1    -0.55387E 05    -0.32228E 05

  2     0.34230E 05    -0.18608E 05

  3     0.73483E 05    -0.43584E 04

  4    -0.12469E 05     0.72216E 04

  5    -0.91352E 05    -0.47665E 04

  6    -0.10334E 06    -0.25767E 05
```

```
PLY NO.    SIGX INTERCEPT  SIGY INTERCEPT       TAUXY       MODE

    1        0.77888E 05    -0.13644E 06      0.13338E 05      1
    1       -0.88632E 05     0.15526E 06      0.13338E 05     -1
    1       -0.59988E 05     0.91166E 04      0.13338E 05      2
    1        0.15668E 06    -0.23810E 05      0.13338E 05     -2
    1        0.10000E 15     0.10000E 15      0.13338E 05      3
    1        0.10000E 15     0.10000E 15      0.13338E 05     -3


    2        0.20499E 06     0.27618E 05      0.13338E 05      1
    2       -0.57105E 06    -0.76934E 05      0.13338E 05     -1
    2        0.31758E 06     0.42786E 05      0.13338E 05      2
    2       -0.25881E 06    -0.34869E 05      0.13338E 05     -2
    2       -0.85489E 05     0.31035E 05      0.13338E 05      3
    2        0.85489E 05    -0.31035E 05      0.13338E 05     -3


    3        0.52098E 06     0.70189E 05      0.13338E 05      1
    3       -0.25506E 06    -0.34363E 05      0.13338E 05     -1
    3        0.15959E 04     0.21501E 03      0.13338E 05      2
    3       -0.57480E 06    -0.77441E 05      0.13338E 05     -2
    3        0.85489E 05    -0.31035E 05      0.13338E 05      3
    3       -0.85489E 05     0.31035E 05      0.13338E 05     -3


    4        0.77888E 05    -0.13644E 06      0.13338E 05      1
    4       -0.88632E 05     0.15526E 06      0.13338E 05     -1
    4       -0.59988E 05     0.91166E 04      0.13338E 05      2
    4        0.15668E 06    -0.23810E 05      0.13338E 05     -2
    4        0.10000E 15     0.10000E 15      0.13338E 05      3
    4        0.10000E 15     0.10000E 15      0.13338E 05     -3


THE INTERACTION YIELD COORDINATES
FOR TAUXY =  0.13338E 05 ARE

  I        X(I)            Y(I)

  1   -0.36808E 05   -0.29404E 05
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  2     0.34230E 05    -0.18608E 05

  3     0.62782E 05    -0.82434E 04

  4    -0.31048E 05     0.43981E 04

  5    -0.91352E 05    -0.47565E 04

  6    -0.10052E 06    -0.20821E 05



  PLY NO.    SIGX INTERCEPT  SIGY INTERCEPT       TAUXY        MODE


      1          0.77888E 05    -0.13644E 06     -0.0               1
      1         -0.88632E 05     0.15526E 06     -0.0              -1
      1         -0.59988E 05     0.91166E 04     -0.0               2
      1          0.15668E 06    -0.23810E 05     -0.0              -2
      1          0.10000E 15     0.10000E 15     -0.0               3
      1          0.10000E 15     0.10000E 15     -0.0              -3


      2          0.36299E 06     0.48904E 05     -0.0               1
      2         -0.41305E 06    -0.55649E 05     -0.0              -1
      2          0.15959E 06     0.21501E 05     -0.0               2
      2         -0.41681E 06    -0.56155E 05     -0.0              -2
      2         -0.85489E 05     0.31035E 05     -0.0               3
      2          0.85489E 05    -0.31035E 05     -0.0              -3


      3          0.36299E 06     0.48904E 05     -0.0               1
      3         -0.41305E 06    -0.55649E 05     -0.0              -1
      3          0.15959E 06     0.21501E 05     -0.0               2
      3         -0.41681E 06    -0.56155E 05     -0.0              -2
      3          0.85489E 05    -0.31035E 05     -0.0               3
      3         -0.85489E 05     0.31035E 05     -0.0              -3


      4          0.77888E 05    -0.13644E 06     -0.0               1
      4         -0.88632E 05     0.15526E 06     -0.0              -1
      4         -0.59988E 05     0.91166E 04     -0.0               2
      4          0.15668E 06    -0.23810E 05     -0.0              -2
      4          0.10000E 15     0.10000E 15     -0.0               3
      4          0.10000E 15     0.10000E 15     -0.0              -3
```

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GENERAL DYNAMICS                                   360 PROCEDURE SQ5                    PAGE 0014
FORT WORTH DIVISION                                PROBLEM 121530-01                     01/12/70


THE INTERACTION YIELD COORDINATES
FOR TAUXY = -0.0           ARE

  I        X(I)              Y(I)

  1   -0.11105E 06    -0.40687E 05

  2     0.34230E 05    -0.18608E 05

  3     0.75901E 05    -0.34806E 04

  4     0.83723E 05     0.10221E 05

  5     0.43196E 05     0.15681E 05

  6   -0.91352E 05    -0.41665E 04

  7   -0.11180E 06    -0.40586E 05



 PLY NO.     SIGX INTERCEPT  SIGY INTERCEPT        TAUXY        MODE

    1          0.77888E 05    -0.13644E 06     -0.10000E 05      1
    1        -0.88632E 05      0.15526E 06     -0.10000E 05     -1
    1        -0.59988E 05      0.91166E 04     -0.10000E 05      2
    1          0.15668E 06    -0.23810E 05     -0.10000E 05     -2
    1          0.10000E 15     0.10000E 15     -0.10000E 05      3
    1          0.10000E 15     0.10000E 15     -0.10000E 05     -3


    2          0.48144E 06     0.64862E 05     -0.10000E 05      1
    2        -0.29460E 06    -0.39690E 05     -0.10000E 05     -1
    2          0.41133E 05     0.55417E 04     -0.10000E 05      2
    2        -0.53526E 06    -0.72114E 05     -0.10000E 05     -2
    2        -0.85489E 05      0.31035E 05     -0.10000E 05      3
    2          0.85489E 05    -0.31035E 05     -0.10000E 05     -3


    3          0.24453E 06     0.32945E 05     -0.10000E 05      1
    3        -0.53151E 06    -0.71608E 05     -0.10000E 05     -1
    3          0.27804E 06     0.37460E 05     -0.10000E 05      2
    3        -0.29835E 06    -0.40196E 05     -0.10000E 05     -2
    3          0.85489E 05    -0.31035E 05     -0.10000E 05      3
    3        -0.85489E 05      0.31035E 05     -0.10000E 05     -3
```

```
        4          0.77888E 05    -0.13644E 06     -0.10000E 05       1
        4         -0.88632E 05     0.15526E 06     -0.10000E 05      -1
        4         -0.59988E 05     0.91166E 04     -0.10000E 05       2
        4          0.15668E 06    -0.23810E 05     -0.10000E 05      -2
        4          0.10000E 15     0.10000E 15     -0.10000E 05       3
        4          0.10000E 15     0.10000E 15     -0.10000E 05      -3


THE INTERACTION YIELD COORDINATES
FOR TAUXY = -0.10000E 05 ARE

  I       X(I)           Y(I)

  1   -0.55387E 05   -0.32228E 05

  2    0.34230E 05   -0.18608E 05

  3    0.73483E 05   -0.43584E 04

  4   -0.12469E 05    0.72216E 04

  5   -0.91352E 05   -0.47665E 04

  6   -0.10334E 06   -0.25767E 05


  PLY NO.   SIGX INTERCEPT  SIGY INTERCEPT       TAUXY       MODE

        1          0.77888E 05    -0.13644E 06     -0.13338E 05       1
        1         -0.88632E 05     0.15526E 06     -0.13338E 05      -1
        1         -0.59988E 05     0.91166E 04     -0.13338E 05       2
        1          0.15668E 06    -0.23810E 05     -0.13338E 05      -2
        1          0.10000E 15     0.10000E 15     -0.13338E 05       3
        1          0.10000E 15     0.10000E 15     -0.13338E 05      -3


        2          0.52098E 06     0.70189E 05     -0.13338E 05       1
        2         -0.25506E 06    -0.34363E 05     -0.13338E 05      -1
        2          0.15959E 04     0.21501E 03     -0.13338E 05       2
        2         -0.57480E 06    -0.77441E 05     -0.13338E 05      -2
        2         -0.85489E 05     0.31035E 05     -0.13338E 05       3
        2          0.85489E 05    -0.31035E 05     -0.13338E 05      -3
```

```
     3      0.20499E 06    0.27618E 05    -0.13338E 05     1
     3     -0.57105E 06   -0.76934E 05    -0.13338E 05    -1
     3      0.31758E 06    0.42786E 05    -0.13338E 05     2
     3     -0.25881E 06   -0.34869E 05    -0.13338E 05    -2
     3      0.85489E 05   -0.31035E 05    -0.13338E 05     3
     3     -0.85489E 05    0.31035E 05    -0.13338E 05    -3


     4      0.77888E 05   -0.13644E 06    -0.13338E 05     1
     4     -0.88632E 05    0.15526E 06    -0.13338E 05    -1
     4     -0.59988E 05    0.91166E 04    -0.13338E 05     2
     4      0.15668E 06   -0.23810E 05    -0.13338E 05    -2
     4      0.10000E 15    0.10000E 15    -0.13338E 05     3
     4      0.10000E 15    0.10000E 15    -0.13338E 05    -3


THE INTERACTION YIELD COORDINATES
FOR TAUXY = -0.13338E 05 ARE

  I        X(I)           Y(I)

  1   -0.36808E 05   -0.29404E 05

  2    0.34230E 05   -0.18608E 05

  3    0.62782E 05   -0.82434E 04

  4   -0.31048E 05    0.43981E 04

  5   -0.91352E 05   -0.47665E 04

  6   -0.10052E 06   -0.20821E 05
```

```
      *** SHEAR FORCES ***

   LOAD CASE      QX         QY

       1          100.        0.


      *** INTERLAMINAR SHEAR STRESSES ***


                 Z           TAU-XZ          TAU-YZ


             -0.50000          0.              0.

             -0.20000        132.              0.

              0.0             52.             35.

              0.0             52.            -35.

              0.20000        132.              0.

              0.50000          0.              0.
```

REFERENCES

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3. J. E. Ashton and J. M. Whitney, *Theory of Laminated Plates*, Technomic Publishing Co., Inc., Stamford, Connecticut, 1970.

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