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PRELIMINARY INVESTIGATION OF SUPERSONIC DIFFUSERS

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ADVANCE CONFIDENTIAL REPORT

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SUMMARY

The deceleration of air from supersonic velocities in channels has been studied. It has become apparent that a normal shock in the diverging part of the diffuser is probably necessary for stable flow, and ways of minimizing the intensity of this shock have been developed. The effect of various geometrical parameters, especially contraction ratio in the entrance region, on the performance of supersonic diffusers has been investigated.

By the use of these results, diffusers were designed which, starting without initial boundary layer, recovered 90 percent of the kinetic energy in supersonic air streams up to a Mach number of 1.85.

INTRODUCTION

The deceleration of air from supersonic to subsonic velocities is a problem that is encountered in the design of high-speed rotary compressors and supersonic air intakes. The efficiency of the supersonic diffusers used to accomplish this deceleration has an important effect on the performance of these machines. The present study is intended to provide information upon which to design efficient supersonic diffusers for use in cases in which the flow starts without initial boundary layer.

The available data on supersonic diffusers are very meager and are reviewed by Crocco in reference 1. This review indicates that, in the deceleration of air from supersonic velocities, the total-head losses are so large as to impair seriously the efficiency of machines employing this process. The experiments reported in reference 1 were primarily designed to serve the needs of supersonic wind tunnels, and therefore only diffusers starting with initial boundary layer were considered.

FLOW IN A SUPERSONIC DIFFUSER

Stability.- In a Laval nozzle the gases start at a low velocity, are accelerated to the velocity of sound in the converging part of the nozzle, and are accelerated to supersonic velocities in the diverging part of the nozzle. The supersonic velocities reached can be calculated approximately from the isentropic-mass-flow curve of figure 1 and the geometry of the nozzle. It is well known that, for shock-free flow, experiment is in good agreement with this one-dimensional isentropic theory although, since the boundary layer thickens in the diverging part of the nozzle, the Mach numbers reached may be a little lower than the values calculated. Two-dimensional nozzles can be designed by the Prandtl-Busemann method (reference 2) to give essentially shock-free expansions, which can be obtained experimentally provided no moisture-condensation effects are present.

It might be supposed that the flow in a nozzle designed by the Prandtl-Busemann method could be reversed and, if proper allowance were made for boundary-layer displacement thickness, a smooth deceleration through the speed of sound obtained. A flow of this type is, however, unstable in the sense that it is unattainable in practice. Consider that a flow of this type has been established. (See fig. 2(a).) In this flow pattern the mass flow per unit area through the throat is the maximum possible for the given state and velocity of the gas entering the diffuser. As long as the flow entering the diffuser is supersonic, the entering mass flow would be unaffected by events downstream. A transient disturbance propagated upstream from the subsonic region would, however, reduce the mass flow at least temporarily in the velocity-of-sound region. Thus, a disturbance would result in an accumulation of air ahead of the throat. The perturbation of the original isentropic flow produced by this accumulation of air would prevent the mass flow from returning to its initial maximum value; thus, air would continue to accumulate ahead of the throat until the mass flow entering the diffuser was reduced. In the case of a supersonic diffuser immersed in a supersonic stream as in the experimental arrangement described later, this would necessitate the formation of a normal shock ahead of the diffuser and, in other arrangements, would likewise necessitate drastic changes in the flow pattern. From the discussion of the starting of supersonic flows in diffusers given later, it will be seen that these changes are irreversible (certainly in the experimental arrangement described later and probably in most other arrangements). It therefore appears that isentropic deceleration through the speed of sound in channels is unstable and unattainable in practice.

In a series of preliminary attempts to produce an approximation to isentropic deceleration through the speed of sound, it was found that supersonic flow could not be started into diffusers designed to produce this flow. In diffusers with a larger throat area, the normal shock jumped from a position ahead of the diffuser to a position in the diverging part of the diffuser. Flows of the type shown in figure 2(b), which involve a normal shock in the diverging part of the diffuser, were found to be stable.

Contraction ratio and losses.- An important part of the losses in a supersonic diffuser are associated with the dissipation accompanying the normal shock in the diverging part of the diffuser. It is therefore important to consider the factors that determine its intensity. As in a Laval nozzle, the position of the shock wave is controlled by the back pressure on the diffuser and moves upstream as the back pressure is increased. When the back pressure forces the shock to a point close to the minimum area of the diffuser, the shock Mach number approaches its lowest value and the associated losses are minimized. The magnitude of these minimum losses depends upon how much the air entering the diffuser is slowed up by the time it reaches the minimum cross section. The more the entrance area of the diffuser can be contracted, the lower the Mach number of the normal shock and the greater the efficiency of the diffuser. It is therefore valuable to consider what determines the maximum contraction ratio that can be used. (Contraction ratio is defined as the ratio of the area at the entrance of a diffuser to the area at its minimum section. See fig. 2(b).)

In most applications, the establishment of supersonic flow is preceded by a normal shock traveling downstream. If this normal shock is to move into the diffuser at a given entrance Mach number and thus establish supersonic flow, the throat of the diffuser must be large enough to permit the passage of the mass flow in a stream tube having an area that corresponds to the entrance area of the diffuser and a total head that corresponds to the value behind a normal shock at the entrance Mach number. Thus, if the throat area has a minimum value for a given entrance Mach number, the Mach number at the throat will be close to 1 when there is a normal shock ahead of the diffuser. An approximation to the contraction ratio that produces this condition can be found from conventional one-dimensional-flow theory. The conditions after the normal shock are known from the usual normal-shock equations and it is necessary merely to find the stream tube contraction, which increases the Mach number at the throat to 1. Since the mass flow per unit area at the Mach number of 1 for a given stagnation temperature is proportional to the total head, the maximum permissible contraction ratio is equal to the contraction ratio that would be required for an isentropic compression to the Mach number of 1 (from the initial supersonic conditions) multiplied by the total-head ratio across the normal shock. The maximum theoretical contraction ratio that permits starting of supersonic flow is computed in this way in appendix A and is shown in figure 3. If the throat area were reduced after supersonic flow had been established or if the flow through the diffuser were started by temporarily increasing the entrance Mach number to a value greater than the design value, a less intense shock and lower losses could probably be obtained. In these cases, the lowest limit of the shock intensity would be provided by stability considerations.

For diffusers in which the geometry (particularly the throat area) cannot be varied and in which the supersonic flow cannot be started by temporarily increasing the entrance Mach number, the minimum-loss diffusion occurs with the shock just downstream from the minimum section. The Mach number preceding such a shock (with isentropic flow assumed) can be found from the computed contraction ratio (fig. 3) and equation (2) of appendix A. The total-head loss across a normal shock at this Mach number (equation (4), appendix A) is then an approximation to the minimum losses (with boundary-layer losses neglected) in a supersonic diffuser subject to the foregoing starting restrictions. The performances of diffusers obtained in this way are given in figure 4.

It should be pointed out that these theoretical considerations are derived with the tacit assumption that conditions in a plane perpendicular to the axis of the channel are constant; that is, one-dimensional flow is assumed. For example, the occurrence of oblique shocks at the entrance of a diffuser would slightly alter these conditions; in particular, the normal shock in the diverging part of the diffuser would have a somewhat reduced intensity and the theoretical efficiency would be somewhat higher. It is considered, however, that the general features would not be much altered by the departures from one-dimensional flow that would occur in diffusers such as those discussed in the experimental part of this report.

EXPERIMENTAL TECHNIQUE

In order to investigate experimentally the properties of constant-geometry supersonic diffusers, the apparatus shown schematically in figure 5 was designed and constructed. The settling chamber was connected to a supply of dry compressed air controlled by a valve in such a way that the chamber pressure could be held constant at any desired value. The air left the chamber through interchangeable two-dimensional nozzles that were designed to give parallel flow at various desired Mach numbers. The feather-edge tip of the diffuser (fig. 6) was held in the center of the supersonic jet at the exit of the nozzle. The experimental arrangement was designed to study the operation of supersonic diffusers that started without initial boundary layer. This condition was studied for two reasons: (1) It is the simplest defined boundary-layer condition to obtain experimentally, and (2) it is considered to approximate more closely than any other the boundary-layer conditions that occur at the entrance to supersonic diffusers used in compressors. A long subsonic diffuser cone behind the supersonic diffuser tip was provided to complete the diffusion process. The valve behind the cone was used to control the back pressure in the subsonic portion of the diffuser and an orifice was used to measure the mass flow through the diffuser. The surface in the supersonic diffuser tips was machined steel, whereas the cone in the subsonic portion was rolled and finished heavy sheet steel.

In order to compare the efficiencies of the various diffuser combinations tested, two quantities were required: (1) the percentage of the total head that the diffuser recovered and (2) the entrance Mach number at which the diffuser attained this recovery.

Because the losses in well-designed supersonic nozzles are small, the absolute pressure in the settling chamber was assumed to be the total head before diffusion.

This pressure was measured with a large mercury manometer. The total head after diffusion can be assumed equal to the static pressure at the end of the subsonic diffuser cone without appreciable error, inasmuch as the kinetic energy at the end of the cones was of the order of 0.16 percent of the entering kinetic energy. A mercury manometer was used to measure the difference between the total heads before and after diffusion. These two measurements were sufficient to determine the percentage of total head recovered.

The mass flow per unit area and the stagnation conditions are sufficient to determine the Mach number at any point. (See equation (2), appendix A.) The Mach number at which a diffuser was operating was determined by measuring the mass flow through the diffuser, which had a known entrance area, and by measuring the settling chamber pressure and temperature that correspond to stagnation conditions.

Two other observations were made. The pressure just inside the supersonic tip of the diffuser was measured to make sure that the shock had passed down the diffuser and that supersonic flow existed in the contracting portion. The flow in the nozzle and into the diffuser was observed with a schlieren system to check visually whether the shock had entered the diffuser.

In order to make a test, the nozzle was brought up to design speed by increasing the pressure in the settling chamber p_o to some value that was held constant throughout the test. The throttling valve behind the diffuser cone was open and the shock passed down the diffuser, if the contraction ratio permitted, and stopped at some place in the diffuser cone. The throttling valve was then slowly closed, thus increasing the pressure at the end of the cone p_f and pushing the shock upstream to lower and lower Mach numbers. When the shock had been moved upstream as far as possible, that is, just downstream from the minimum section of the diffuser, p_f reached its maximum value. Although p_f was increased during this process, the mass flow through the diffuser was not affected because the flow was supersonic into the diffuser tip. When the valve was closed farther, the shock wave passed the minimum section and suddenly moved out in front of the diffuser.

The mass flow immediately dropped (and continued to drop as the valve was closed farther) and the pressure inside the diffuser tip immediately jumped to a subsonic value.

The results of a typical test are presented in figure 7. The breaks in the mass-flow and tip-pressure curves give an excellent indication of when the diffuser was operating at maximum efficiency and when it failed to act as a supersonic diffuser. The slight change in mass flow while the diffuser was operating was due to the fact that the pressure in the settling chamber varied slightly from the beginning to the end of the test run. The curves indicate that a given diffuser may have any value of total-head recovery, up to a certain maximum, depending upon the position of the shock. Therefore, the obvious method of comparing the performance of a number of diffusers is to compare their maximum recoveries.

RESULTS AND DISCUSSION

The primary design parameter of a supersonic diffuser is its contraction ratio, which determines the minimum Mach number at which the supersonic diffuser operates and the amount of compression that the entering air undergoes before it must negotiate the normal shock. If the contraction ratio of a diffuser is increased, the minimum Mach number at which it operates theoretically increases as shown in figure 3. The minimum Mach numbers at which a number of diffusers were observed to operate and the Mach numbers at which they first failed to operate are shown in figure 3. The points so plotted give excellent agreement with the theoretical contraction-ratio curve.

As was pointed out previously, the effect of contraction ratio upon the performance of a supersonic diffuser should be approximately as shown in figure 4. The observed performances of three diffusers with different contraction ratios are plotted in figure 8. The effect of contraction ratio is very similar to the approximate theoretical results shown in figure 4. The indicated discrepancy between experimental and theoretical results is probably chiefly due to losses in the subsonic portion of the diffuser.

After the contraction ratio of a supersonic diffuser has been fixed according to the minimum Mach number at which it must operate, two other parameters - the entrance-cone angle and the exit-cone angle - may be considered.

Owing to the difficulty of measuring the exact entrance angles on the small diffusers tested, the data evaluating the effect of the entrance-cone angle are not considered quantitative and are not presented herein. The trend observed, however, was that the larger the entrance-cone angle, the better the performance of the diffuser. Further experiment is needed to determine the optimum entrance-cone angles although, for the three diffusers of figure 8, the entrance-cone angles are probably so close to the optimum that no large gain in recovery could be expected from a change in this parameter. In the diffusers tested, the internal shape was faired in a smooth curve between the entrance cone and the exit cone. The curve was close to a circular arc and started very near the leading edge of the entrance cone.

Two diffusers of equal contraction ratio and entrance-cone angle but different exit-cone angle were tested. The performances of the two diffusers with exit-cone angles of 5° and 3° are plotted in figure 9. The diffuser with an exit-cone angle of 3° was found to give consistently higher recoveries. As is pointed out in reference 3, the boundary layer is thick after a normal shock and therefore the pressure recovery in the subsonic cone must be slow to prevent separation. The slightly different shape of the performance curve of these diffusers when compared with the other diffusers reported (fig. 8) may be due to the fact that, although the two diffusers correspond closely to each other except for exit-cone angles, they do not correspond to the other three diffusers.

The total-head recoveries measured in the experiments were transformed into energy efficiencies. The energy efficiency η is defined as the percentage of available kinetic energy recovered in the diffusion process or the kinetic energy of an expansion from the pressure at rest after diffusion p_f to the pressure at the entrance of the diffuser p_e divided by the kinetic energy of an expansion from the initial chamber pressure p_o to p_e. Because no external work is done, the whole process of expansion and diffusion is a throttling process and the stagnation temperature T_o is the same after diffusion as in the settling chamber. The equation for the energy efficiency may be written

$$\eta = \frac{2c_p\left[T_o - T_o\left(\frac{p_e}{p_f}\right)^{R/c_p}\right]}{2c_p\left[T_o - T_o\left(\frac{p_e}{p_o}\right)^{R/c_p}\right]}$$

The symbols are defined in appendix B. When $\frac{c_p}{R} = 3.5$,

$$\eta = 1 - \frac{5}{M^2}\left[\left(\frac{p_o}{p_f}\right)^{1/3.5} - 1\right] \qquad (1)$$

where M is the Mach number of the flow entering the diffuser.

The efficiencies obtained by equation (1) are compared in figure 10 with the typical efficiencies (converted to efficiency as defined in equation (1))of the work previously done with supersonic diffusers presented by Crocco in reference 1, the efficiency of a normal shock (combined with compression to rest without further loss), and the approximate maximum theoretical efficiency for constant-geometry diffusers previously derived. Figure 10 shows that the normal-shock efficiency may be exceeded and that energy recoveries of over 90 percent can be obtained up to a Mach number of 1.85; thus, the results presented for supersonic diffusers in reference 1 are far too conservative for diffusers that have no initial boundary layer.

CONCLUDING REMARKS

An investigation of the deceleration of air in channels from supersonic to subsonic velocities was conducted. A channel flow involving the shock-free deceleration of a gas stream through the local speed of sound was found to be unstable. A stable flow probably involves a normal shock in the diverging part of the diffuser. The losses involved in this normal shock can be minimized by making the throat area as small as possible for a given entrance Mach number. The maximum contraction ratio that permits starting of supersonic flow at a given entrance Mach number has been calculated and checked very closely by experiment.

With the use of these results, diffusers were designed which, starting without initial boundary layer, recovered over 90 percent of the kinetic energy in supersonic air streams up to a Mach number of 1.85.

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APPENDIX A

CALCULATION OF MAXIMUM PERMISSIBLE CONTRACTION RATIO

It can be shown that the mass flow per unit area at Mach number M is

$$\frac{\rho V}{\rho_o a_c} = M\left(1 + \frac{\gamma-1}{2}M^2\right)^{-\frac{1}{2}\left(\frac{\gamma+1}{\gamma-1}\right)} \tag{2}$$

where the symbols are defined in appendix B.

The isentropic area-contraction ratio from a Mach number M to the local velocity of sound is then

$$\frac{(\rho V)_{M=1}}{(\rho V)_M} \tag{3}$$

where ρV is computed from equation (2).

When air crosses a shock wave, its stagnation temperature is unchanged; hence, the reduction in possible mass flow per unit area, from equation (2) and the perfect gas law, is proportional to the total-head loss across the shock. The total-head ratio p_3/p_o across a normal shock wave can be shown to be

$$\frac{p_3}{p_o} = \frac{\left(\frac{\gamma + 1}{\gamma - 1}\right)^{\frac{\gamma+1}{\gamma-1}} M^{\frac{2\gamma}{\gamma-1}}}{\left(\frac{2}{\gamma - 1} + M^2\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{2\gamma}{\gamma - 1}M^2 - 1\right)^{\frac{1}{\gamma-1}}} \tag{4}$$

Multiplying equation (4) by expression (3) gives the maximum contraction ratio that permits supersonic flow to start in a diffuser. This quantity is plotted in figure 2.

APPENDIX B

SYMBOLS

γ	ratio of specific heat at constant pressure to specific heat at constant volume
ρ	density
a	velocity of sound
V	velocity
M	Mach number
c_p	specific heat at constant pressure
R	gas constant
η	efficiency
p_e	pressure at entrance of diffuser
p_f	pressure at rest after diffusion
p_o	initial chamber pressure
p_3	total head after normal shock wave
p_d	pressure at internal leading edge of supersonic diffuser (see fig. 7)
M_d	design Mach number of supersonic diffuser; that is, minimum starting Mach number of diffuser with given contraction ratio
τ	entrance angle of diffuser (see fig. 6)
θ	exit angle of diffuser
b, c	dimensions used in fig. 2
C_R	contraction ratio (see fig. 2(b))

S passage area

T temperature

The subscript o refers to initial stagnation conditions.

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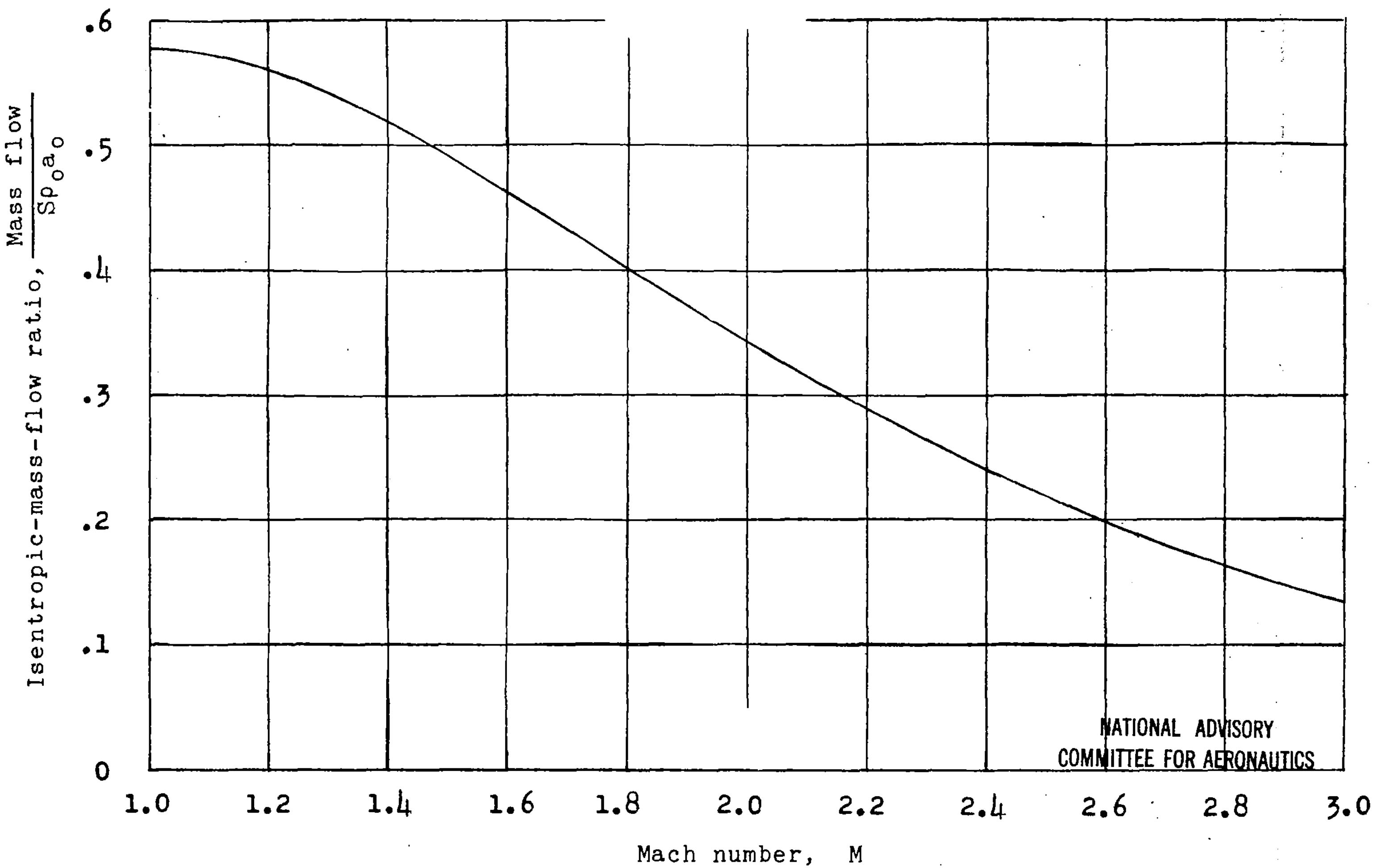
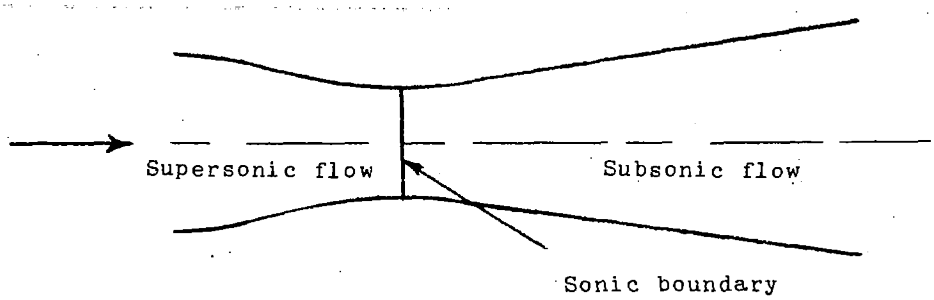
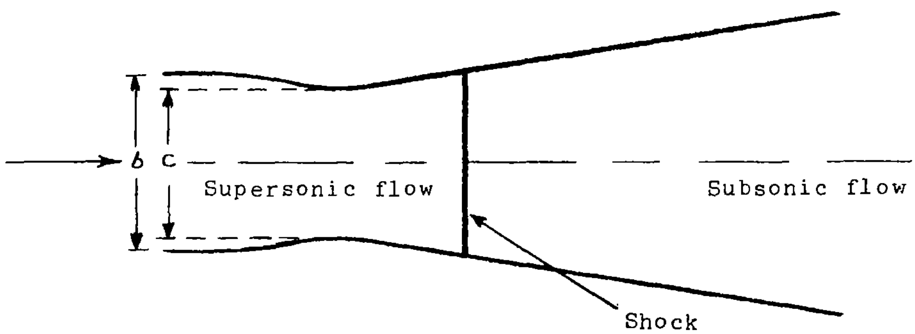


Figure 1.- Isentropic-mass-flow ratio as a function of Mach number. Mass flow measured through area S at Mach number M; ρ_o, density; a_o, speed of sound at initial stagnation conditions.



(a) Reversed Laval nozzle with isentropic flow (unstable).



(b) Stable supersonic diffuser flow. (For circular diffuser $\frac{b^2}{c^2} = C_R$, where C_R is contraction ratio.)

Figure 2.- Flow in a converging-diverging diffuser.

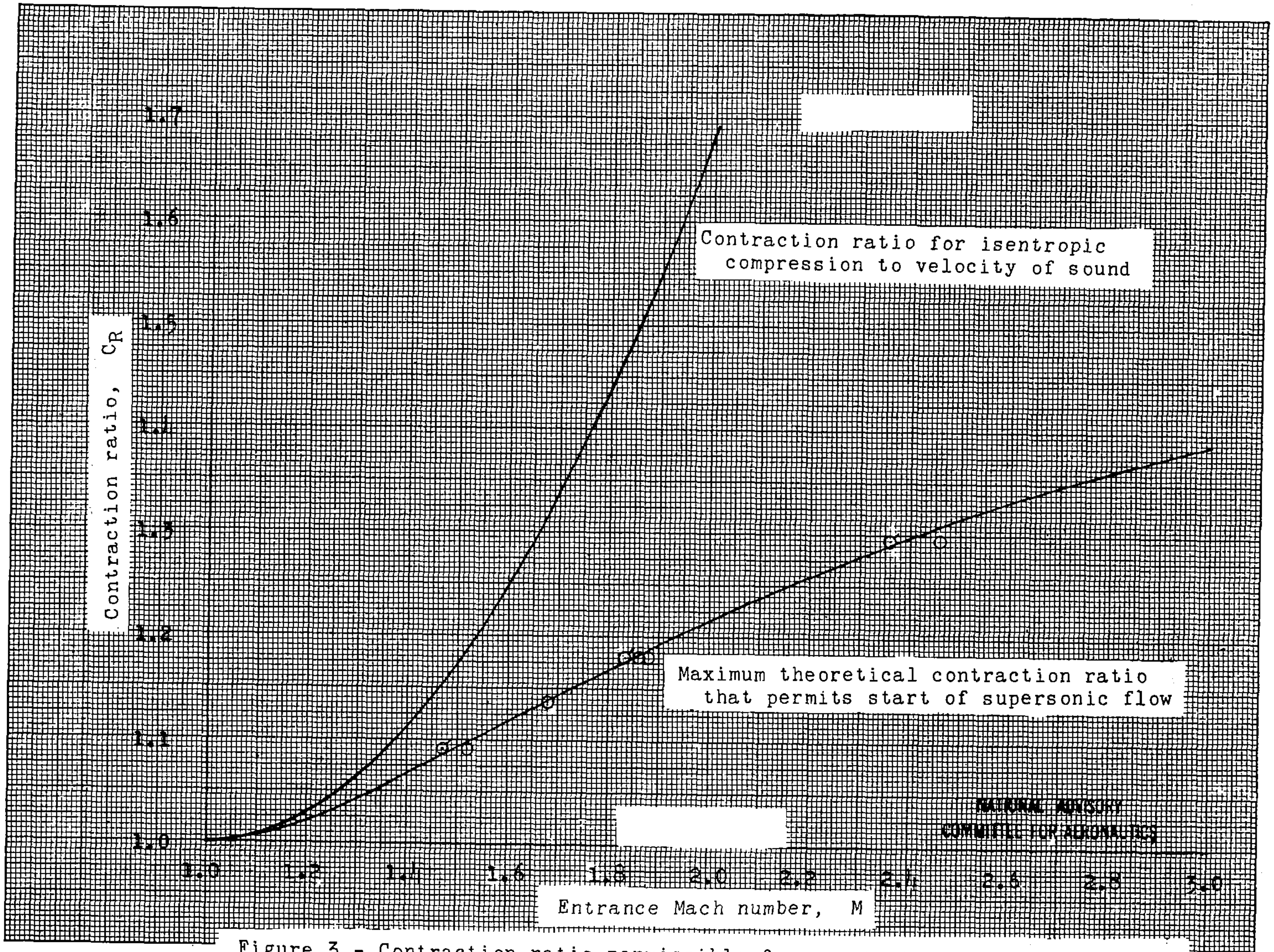


Figure 3.- Contraction ratio permissible for constant-geometry supersonic diffusers. Tailed symbols represent Mach numbers at which the diffusers failed to operate; plain symbols represent minimum Mach numbers at which diffusers operated.

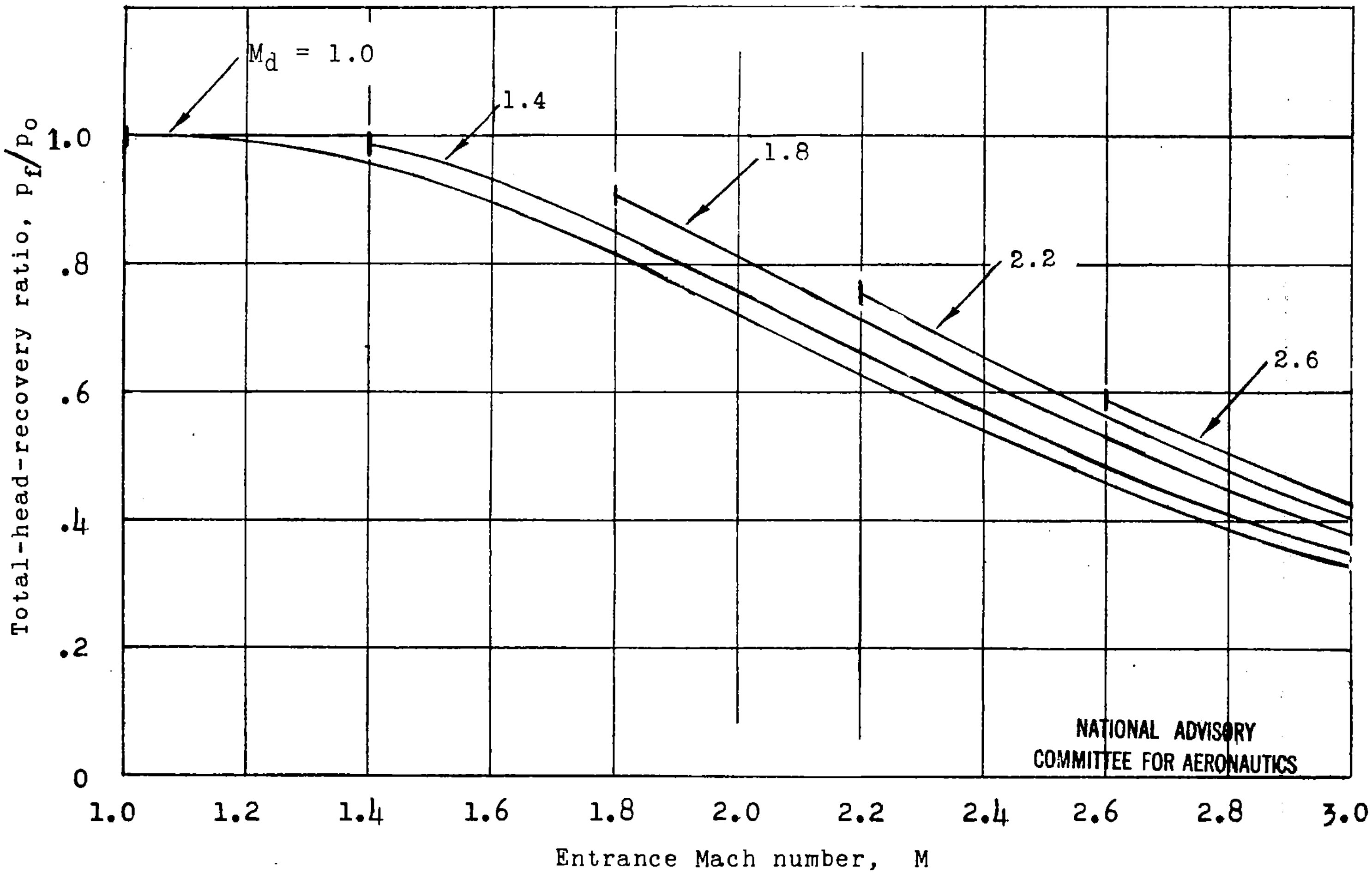


Figure 4.- Theoretical performance by approximate method of five supersonic diffusers with maximum theoretical contraction ratios corresponding to minimum starting or design Mach numbers of 1.0, 1.4, 1.8, 2.2, and 2.6. Vertical lines indicate design Mach numbers.

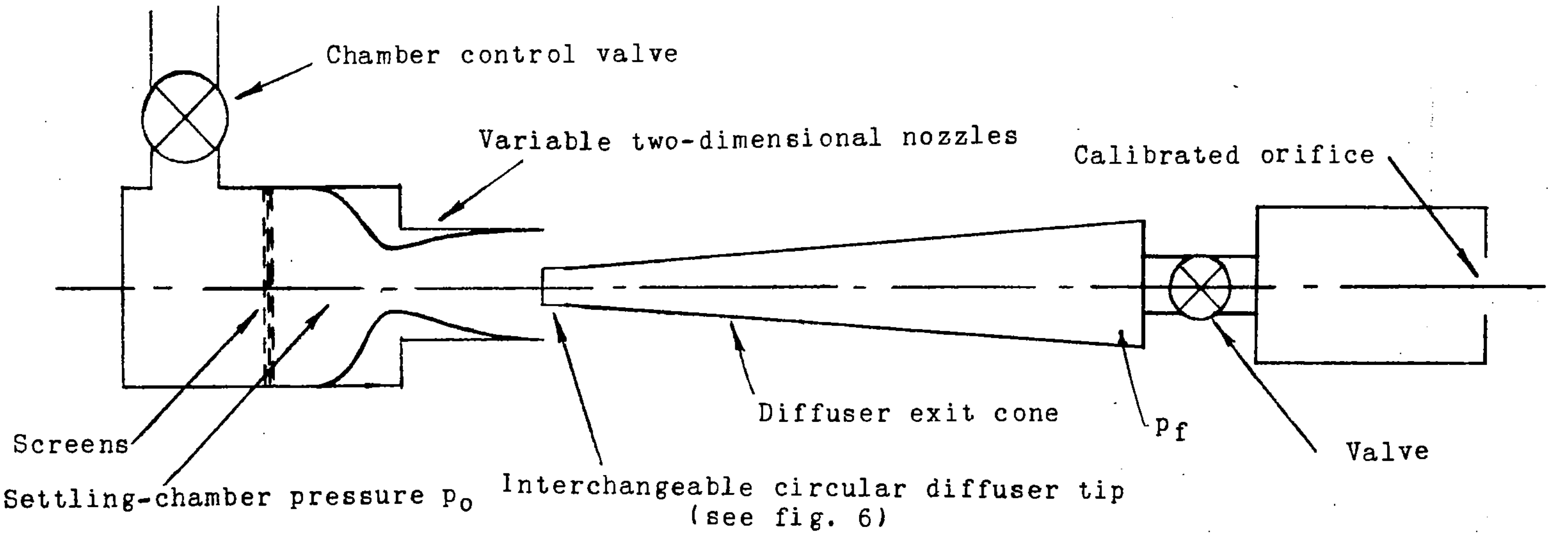


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Figure 5.- Schematic diagram of apparatus used to test supersonic diffusers.

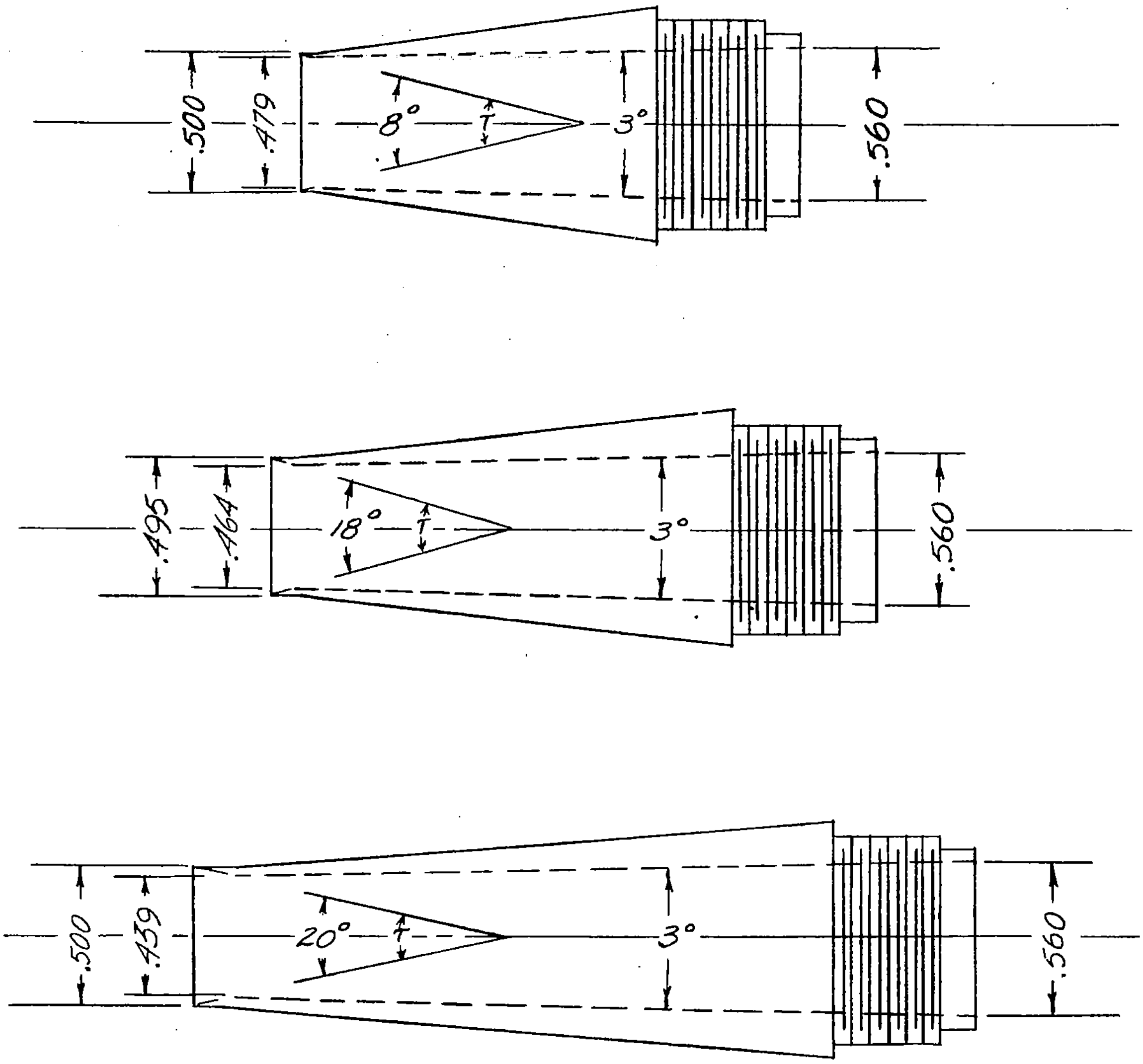

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Figure 6.- Interchangeable circular diffuser tips for which performances are shown in figures 8 and 10. These different tips were screwed into a permanent cone having an exit angle of 3°. τ, entrance-cone angle.

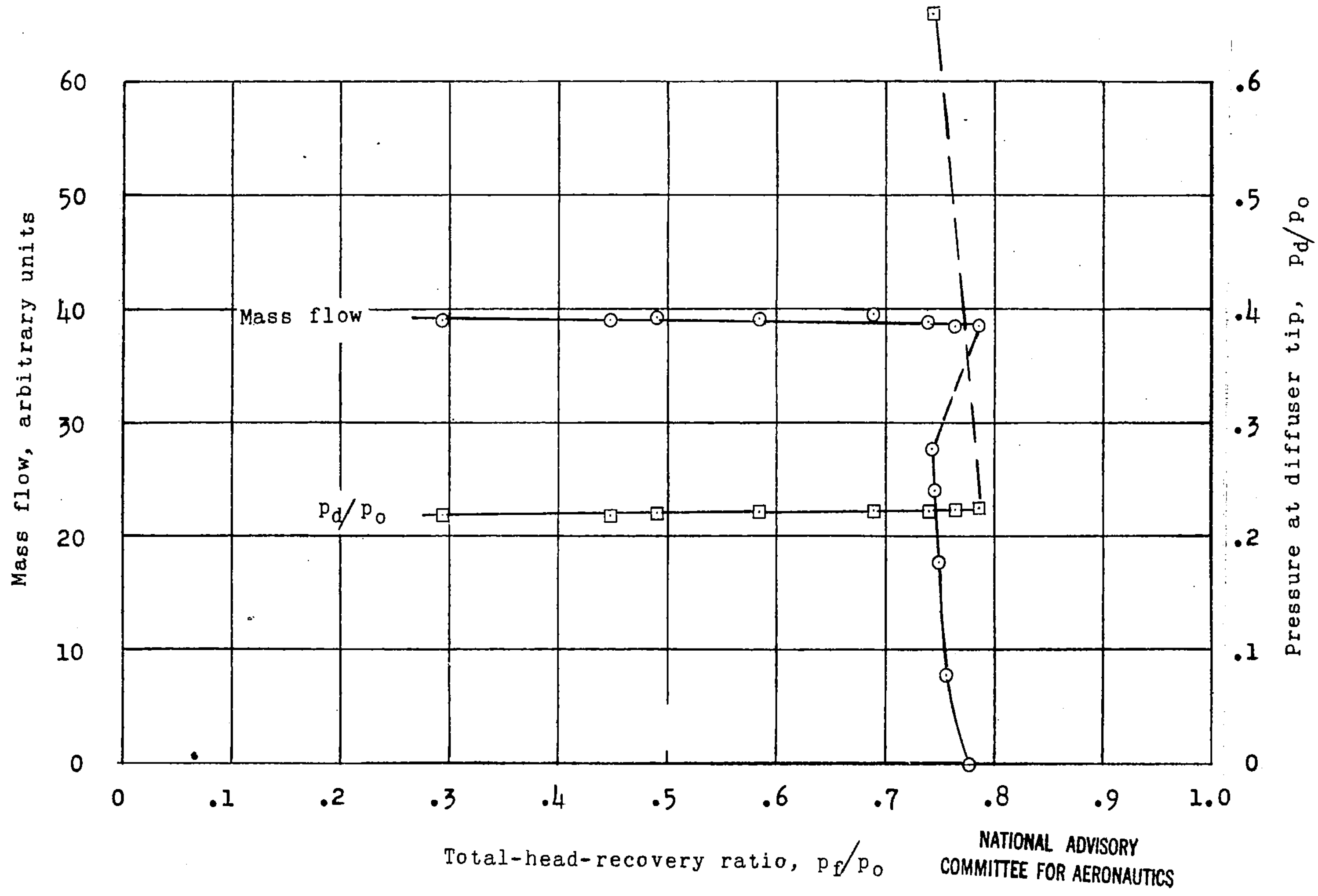

Figure 7.- Performance of a supersonic diffuser during typical test.

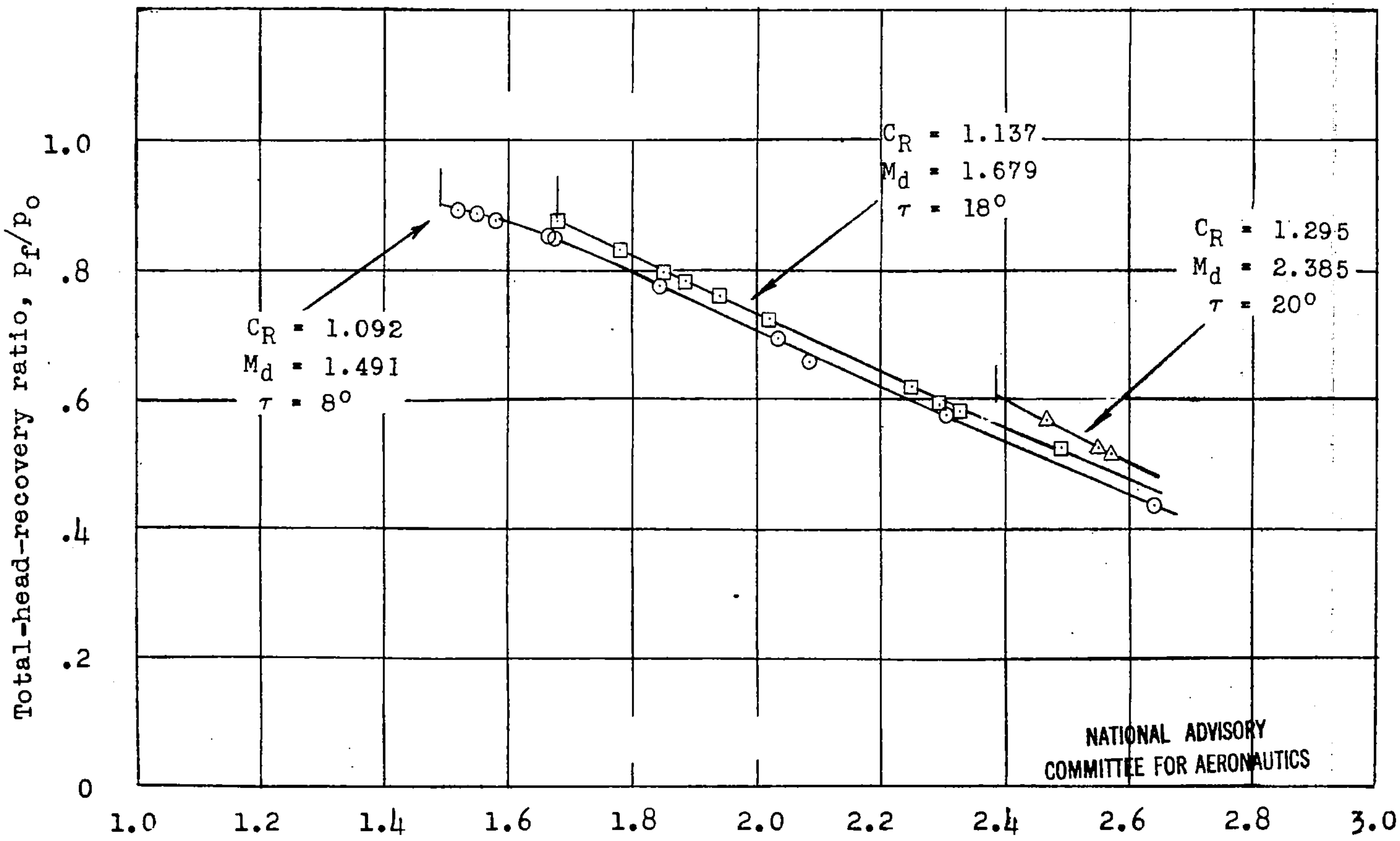

Figure 8.- Performance of the three diffusers shown in figure 6 with exit-cone angles of 3°. τ, entrance-cone angle.

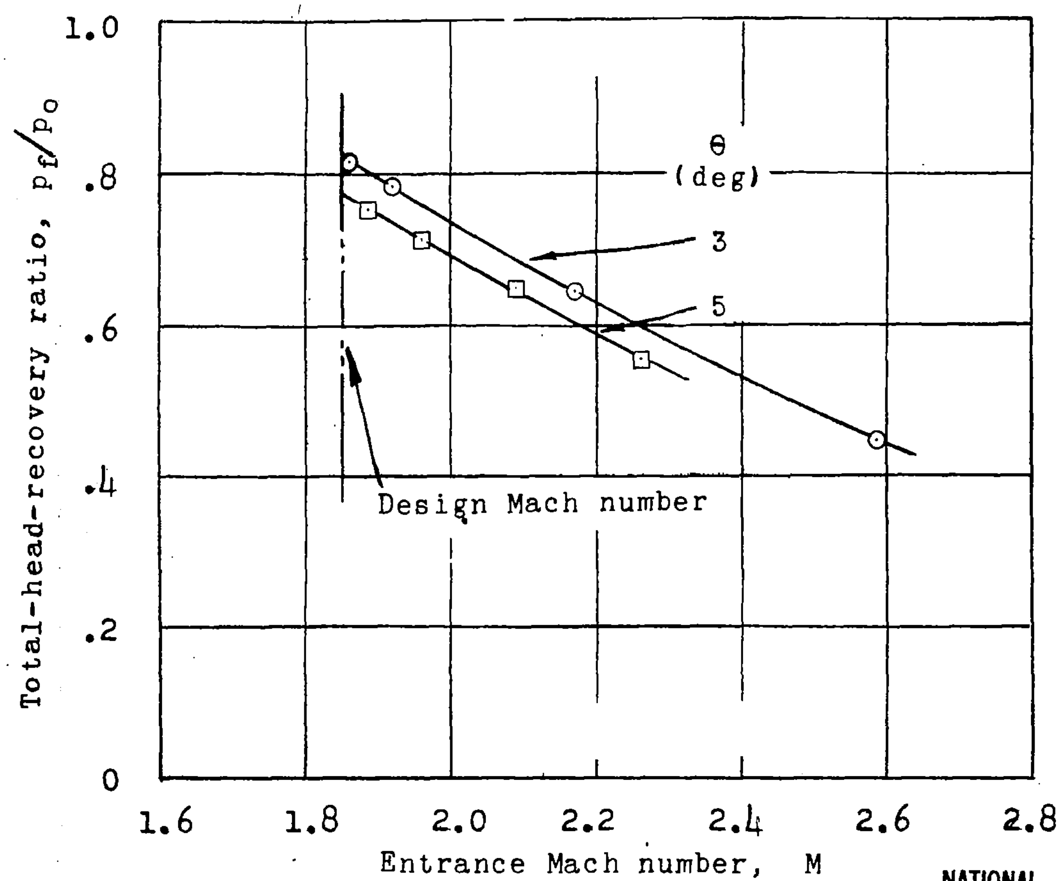

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Figure 9.- Effect of exit-cone angle on the performance of two supersonic diffusers with equal contraction ratios and entrance-cone angles. θ, exit-cone angle.

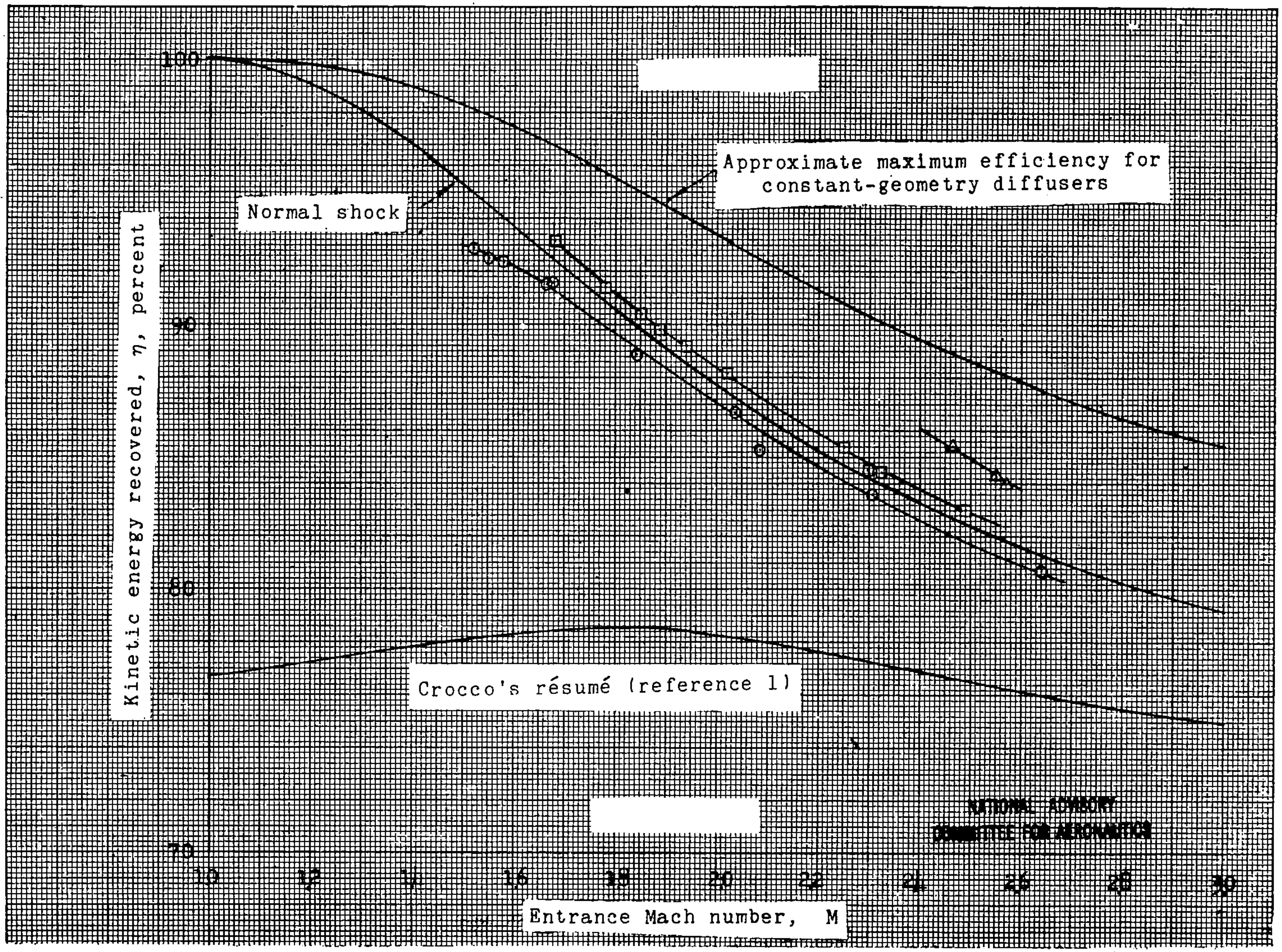

Figure 10.- Comparison on an energy basis of diffusers tested with theoretical results and with results of former experiments. Experimental points are from same tests as those shown in figure 8.

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ABSTRACT:

An investigation of the deceleration of air in channels from supersonic to subsonic velocities was conducted for use in high-speed rotary compressors and supersonic air intake design. A stable flow probably involves a normal shock in diverging part of diffuser. Losses caused by this shock are reduced by making throat area as small as possible for given entrance Macb number. Several geometrical features of diffusers were investigated. Using these results, very efficient diffusers were designed.

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