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A COMPUTER MODEL PREDICTING THE THERMAL RESPONSE TO MICROWAVE RADIATION

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Final Report for Period January 1980 - November 1980

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USAF SCHOOL OF AEROSPACE MEDICINE
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NOTICES

This final report was submitted by personnel of the Biomathematics Modeling Branch, Data Sciences Division, USAF School of Aerospace Medicine, Aerospace Medical Division, AFSC, Brooks Air Force Base, Texas, under job order 2312-V7-02.

This report has been reviewed and is approved for publication.

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Partial differential equations	Electromagnetic radiation
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Newton cooling law boundary condition	

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

We compare the theoretical predictions of the temperature excursions that would be included in a simulated, spherically symmetric autothermally regulated bio-structure by a source of microwave radiation with experimental measurements. The predictions were made by using the divergence of the Poynting vector as a source term for the heat equation, and the measurements were made for a single-layer structure filled with simulated muscle material with a Vitek probe; these measurements were made by John Burr and were described in a

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20. ABSTRACT (Continued)

publication which appeared in Vol. BME-27, Nov. 8, of the IEEE Transactions on Biomedical Engineering. The results of these measurements are discussed in this report.

We describe a shooting method for solving the eigenvalue and eigenfunction determination problem for a multilayered, penetrable, spherically symmetric, autothermally regulated, simulated biostructure when there is heat removal by blood flow in some but possibly not all of the layers. This requires study of a new type of special function.

While originally our computer program experienced difficulty when the frequency of the incoming radiation was as high as 10 GHz or when the radius of the sphere bounding the ball of biotissue was as large as 48 cm, we have overcome this problem with a hybrid scheme for computing spherical Bessel functions.

Our computer program also permits the computation of temperature excursions that would be experienced by the simulated biostructure when the source of radiation is pulsed in a complex way. We develop exact formulas which enable us to express the expansion coefficients of the temperature in terms of integrals with respect to the spatial coordinates only. To save computing time, the points that will be used in the Gaussian quadrature are determined in advance and care is taken to make certain that no calculation is needlessly repeated.

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COMPUTER MODEL PREDICTING THE THERMAL RESPONSE TO MICROWAVE RADIATION OF A SIMULATED BIOLOGICAL STRUCTURE

1. INTRODUCTION

This paper describes a method of computing the thermal response of an autothermally regulated body such as a biological body to a source of microwave radiation. The description of this method is divided into five parts. It includes (1) a discussion of the symbols (and their units) used in developing the solution, (2) the induced electromagnetic field distribution and the power density distribution that represents the source term for the heat transfer problem, (3) the modified heat operator eigenvalues and eigenfunctions associated with a Newton cooling law boundary condition, (4) the computation of temperature excursions induced by microwave radiation including complex pulse heating schemes, and (5) a discussion of the spreading of temperature distributions with time in three types of simulated biological structures. These are discussed in Sections 3.1-3.5 respectively.

In [2] we developed a computer model to determine the temperature distribution in a penetrable, homogeneous, and spherically symmetric body that has been irradiated by microwave radiation. Heat removal by blood flow could be considered, provided that only one layer was used in the model. In the present paper a shooting method for solving the eigenvalue and eigenfunction determination problem for a multilayered, penetrable, but spherically symmetric scatterer is solved when the heat equation describing the microwave heating includes the possibility of blood-flow-heat-removal terms in some, but not necessarily all layers. This innovation is described in Section 3.

Also, originally our program in [2] experienced some difficulty in computing expansion coefficients used in determining the induced electric field when the frequency of the incoming radiation was high (>10 GHz) or when the radius of the outer sphere was as large as 48 cm; the procedure by which we overcame this difficulty is described in Section 2. Some experimental microwave bioenvironmentalists look for a nonthermal microwave effect and consequently attempt to control temperature in their microwave exposure systems by using a complex microwave pulse heating scheme with a low duty factor. Section 4 therefore contains a description of a formula which permits one to express the expansion coefficients associated with a complex temporal heating pattern in terms of integrals with respect to only the spatial variables. Finally we note that several people--including MacLatchy and Clements [9] and Washisu and Fukai [11] have proposed microwave-induced temperature excursions as a nonperturbing method of measuring or estimating field strengths. Consequently, in Section 3.5 of this paper we have included a discussion of the potential and limitations of this method of field measurement. Also, because microwave heating may be used to treat tumors in humans (c.f. Zimmer et al. [12]), we give in Section 3.5 some new computer calculations showing possible thermal effects on simulated biological structures.

2. RESULTS AND DISCUSSION

In this paper the authors extend the computer model which generated the results of [2]. We consider as before that a plane wave irradiates a spherically symmetric structure in the manner indicated in Figure 2.1. We allow, however, time profiles similar to that of the PAVE PAWS radar so that heating from any radar emission can be estimated directly. We note that with this capability and the possibility of estimating temperature derivatives that one can, by solving the equations of thermoelasticity, describe radar acoustic effects in a quantitative way. This is important in view of large efforts by other branches of the armed services to study this effect. We can also, by going to more general geometries and using an integral equation method, describe quantitatively the effect of microwave radiation on biochemical processes and fetal development which are strictly thermal in nature.

Figures 2.2-2.10 compare computer model predictions with measurements made by John G. Burr at Brooks AFB and give a comparison of our ability to predict spatial variations in temperature for two radiofrequencies (RF), 1.2 GHz and 2.5 GHz, and for a short 30-s and for a longer 3-min exposure. The capability of predicting the thermal response to pulsed radiation is demonstrated in Figure 2.11.

We now discuss the effect of blood flow in removing heat from a living system subjected to microwave-radiation-induced thermal excursions. John Burr and Jerome Krupp of the Radiation Sciences Division of the USAF School of Aerospace Medicine reported in [3] the results of Figure 2.12 showing temperature measurements in the head of a living and dead Macaca mulatta. In Figure 2.13 we use blood flow rates supplied in [9] to estimate the effect of the blood flow term in giving a lower predicted value of a radiation-induced temperature increase.

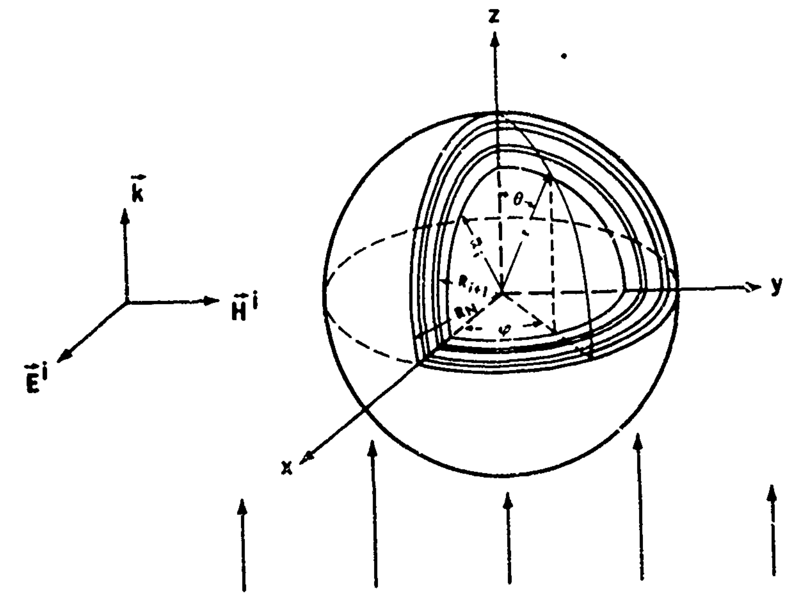

Figure 2.1. Electromagnetic plane wave impinging on a cranial model composed of an inner core sphere and N concentric spherical shells.

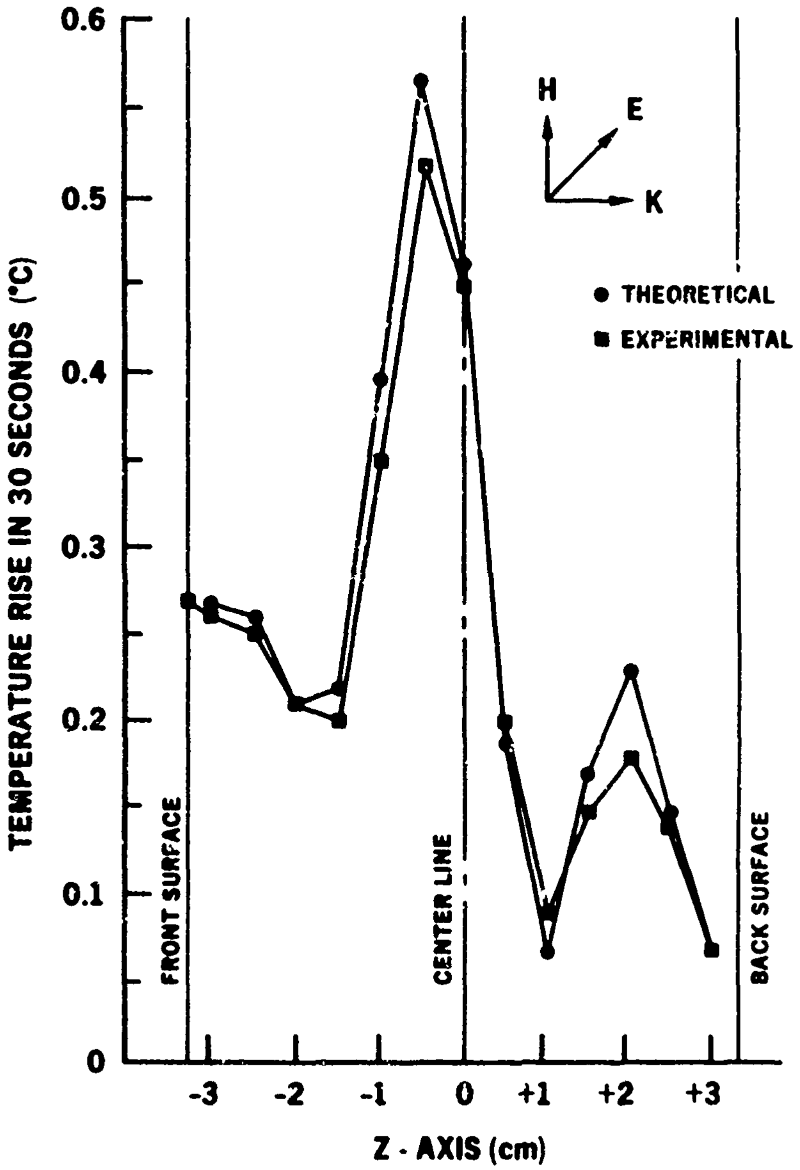

Figure 2.2. Temperature rise along the z-axis of a 3.3-cm radius homogeneous muscle-equivalent sphere exposed to 1.2 GHz, continuous wave (CW), 70 mW/cm^2, RF in the far field for 30 s.

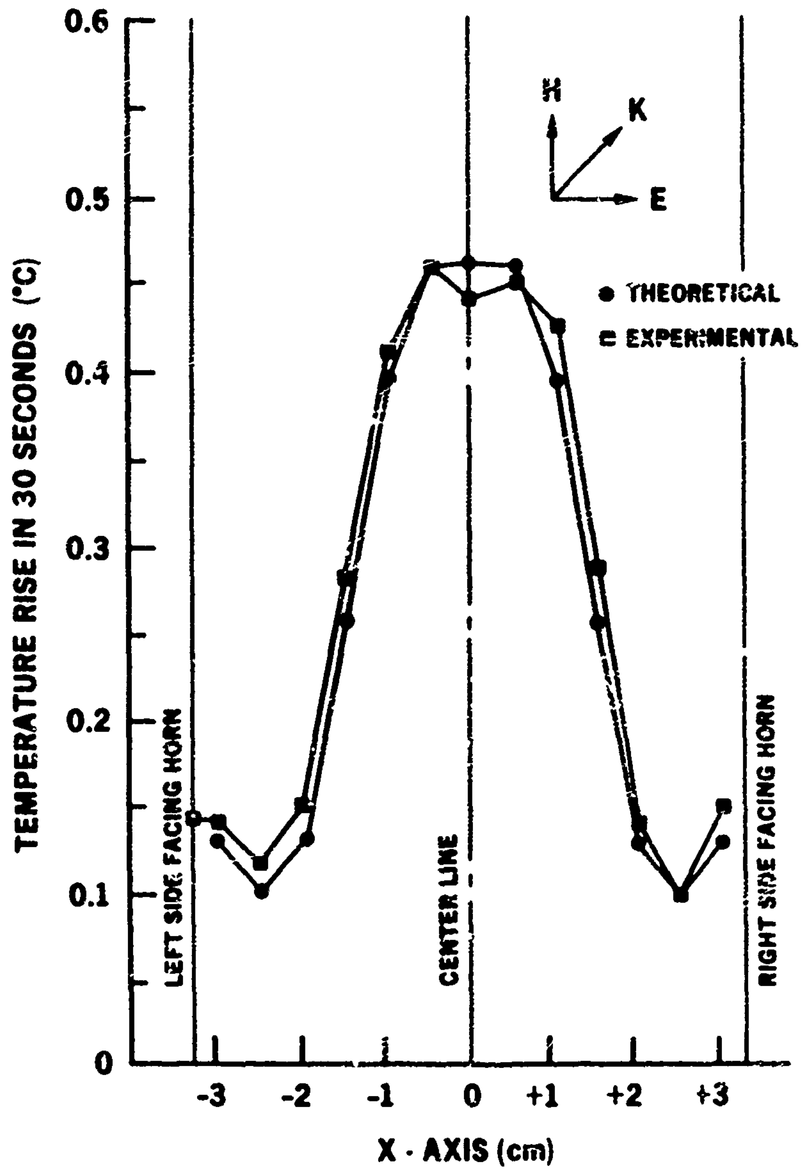

Figure 2.3. Temperature rise along the x-axis of a 3.3-cm radius homogeneous muscle-equivalent sphere exposed to 1.2 GHz, CW, 70 mW/cm^2, RF in the far field for 30 s.

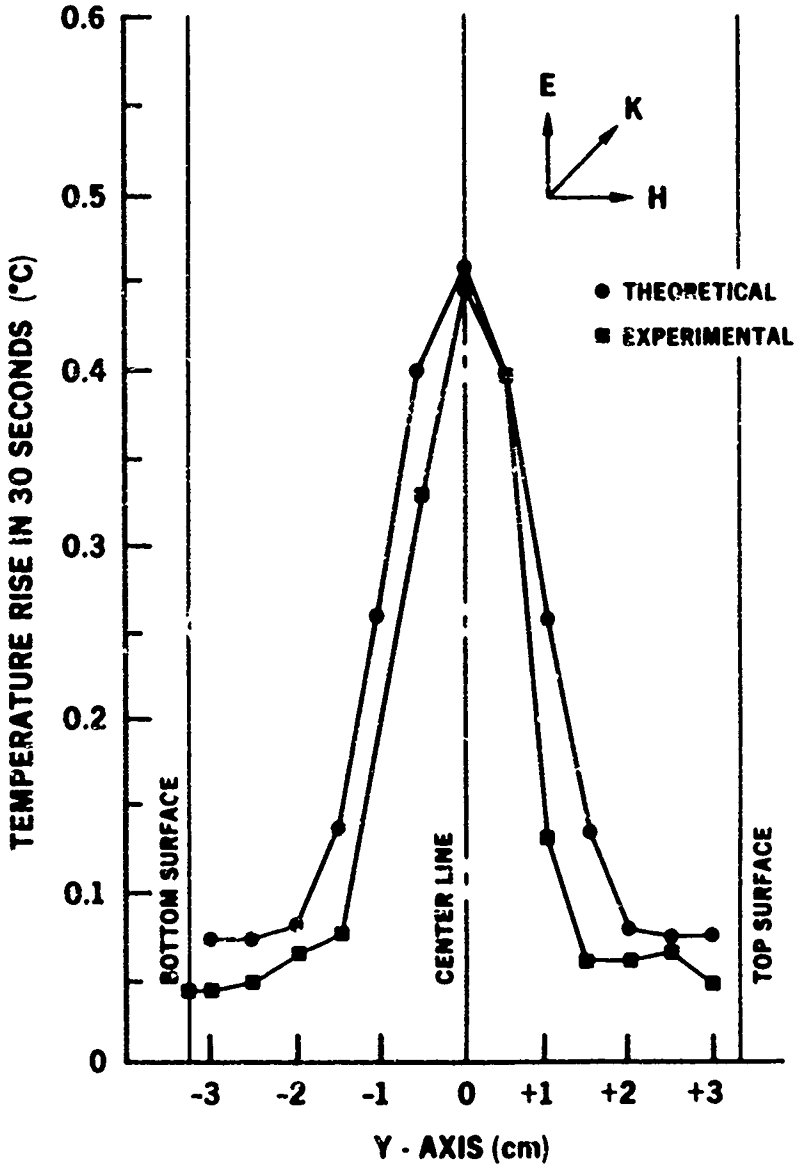

Figure 2.4. Temperature rise along the y-axis of a 3.3-cm radius homogeneous muscle-equivalent sphere exposed to 1.2 GHz, CW, 70 mW/cm^2, RF in the far field for 30 s.

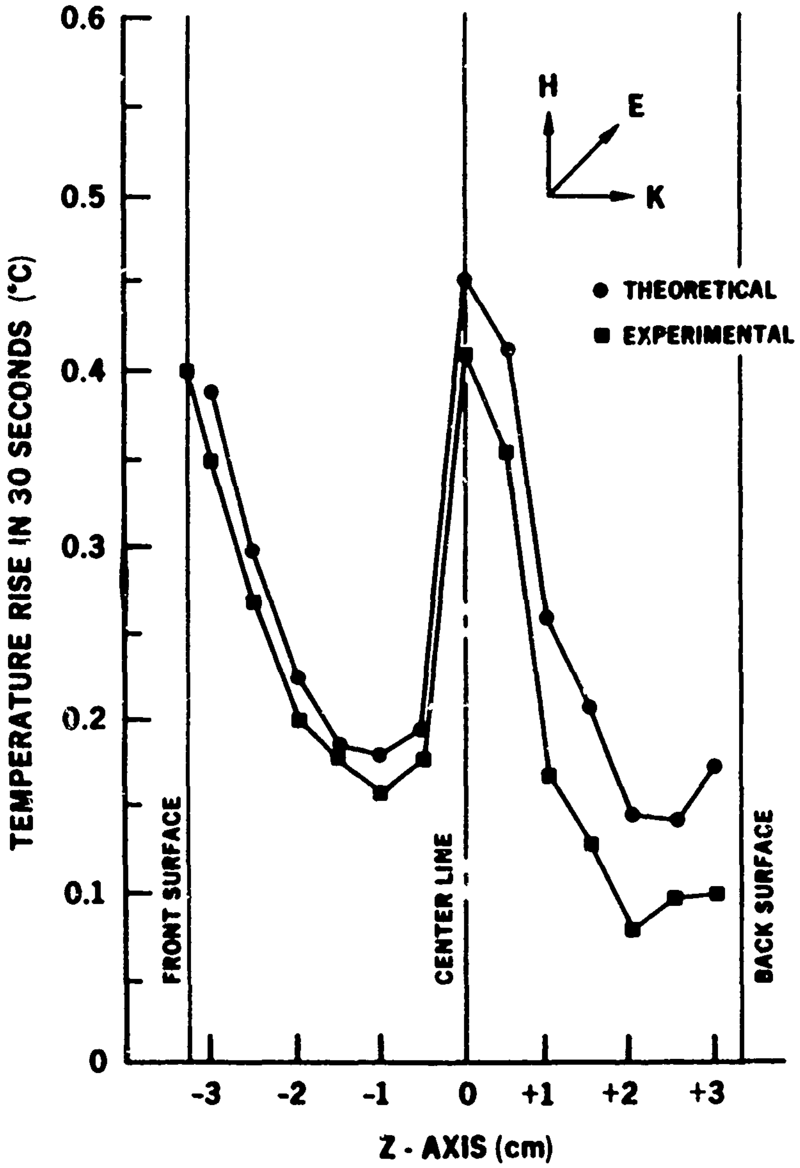

Figure 2.5. Temperature rise along the z-axis of a 3.3-cm radius homogeneous muscle-equivalent sphere exposed to 2.5 GHz, CW, 100 mW/cm^2, RF in the far field for 30 s.

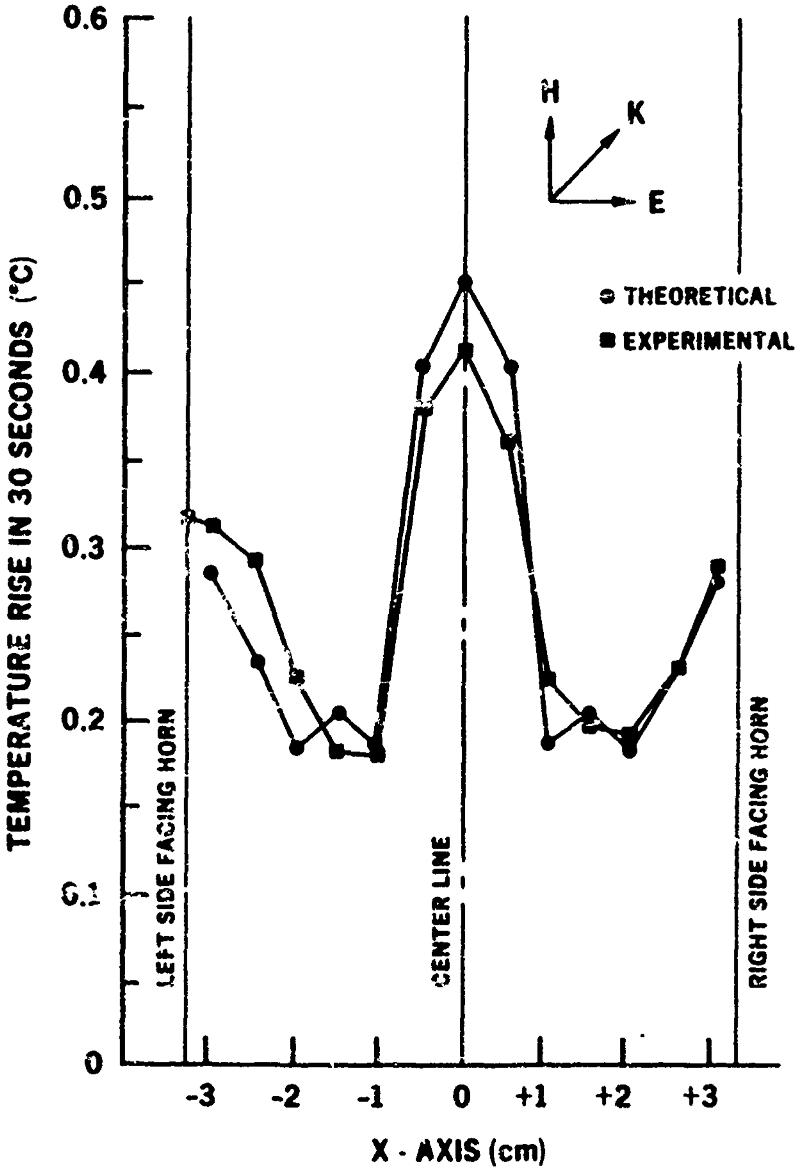

Figure 2.6. Temperature rise along the x-axis of a 3.3-cm radius homogeneous muscle-equivalent sphere exposed to 2.5 GHz, CW, 100 mW/cm^2, RF in the far field for 30 s.

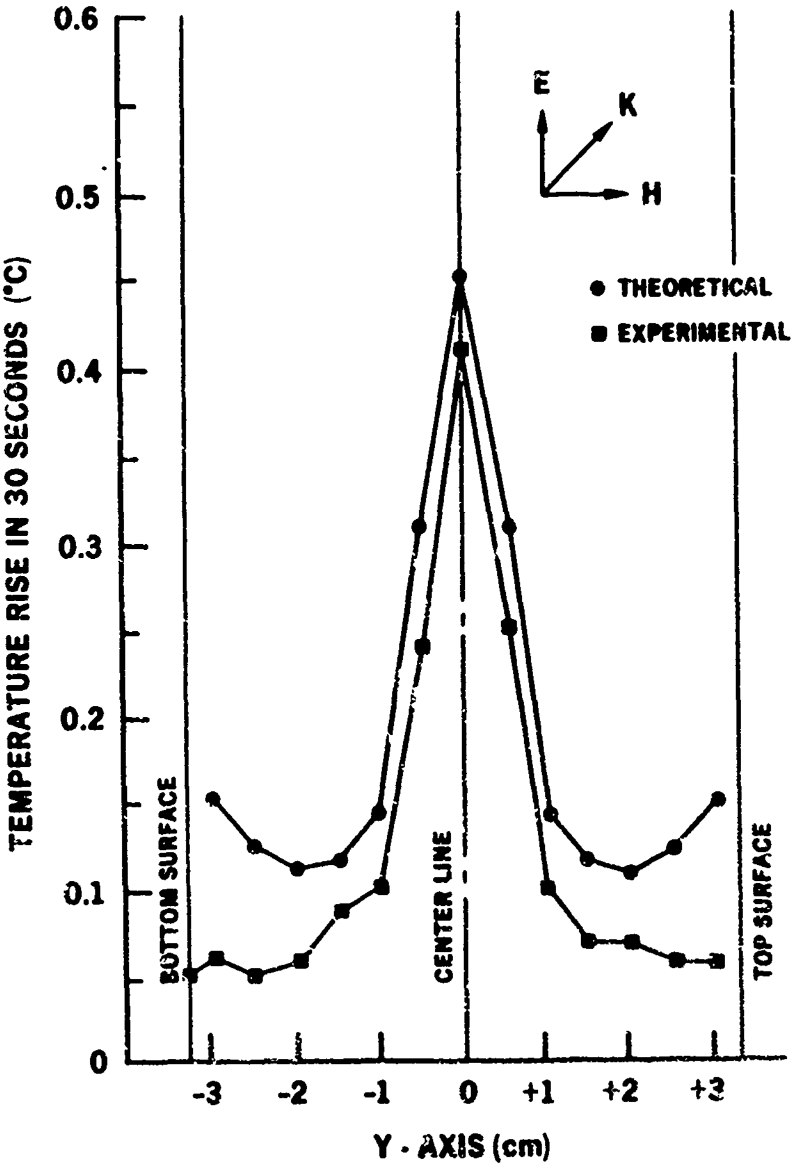

Figure 2.7. Temperature rise along the y-axis of a 3.3-cm radius homogeneous muscle-equivalent sphere exposed to 2.5 GHz, CW, 100 mW/cm^2, RF in the far field for 30 s.

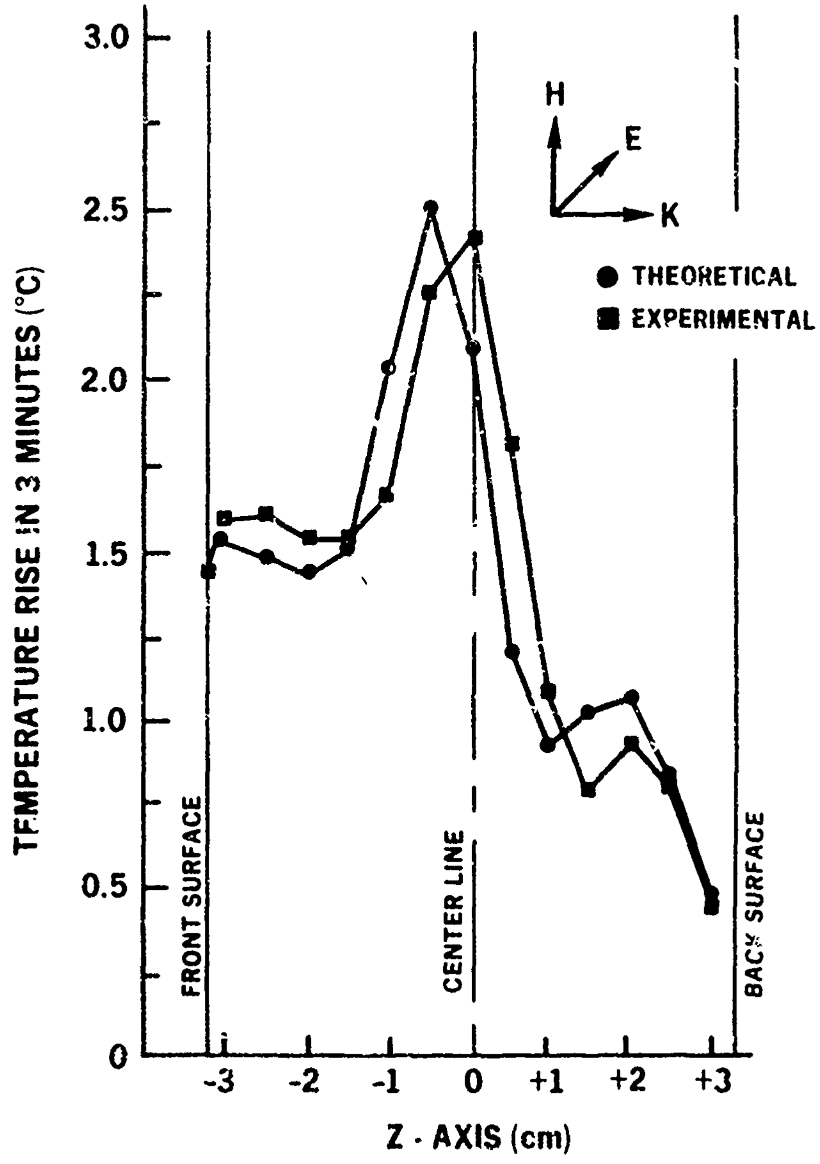

Figure 2.8. Temperature rise along the z-axis of a 3.3-cm radius homogeneous muscle-equivalent sphere exposed to 1.2 GHz, CW, 70 mW/cm^2, RF in the far field for 3 min.

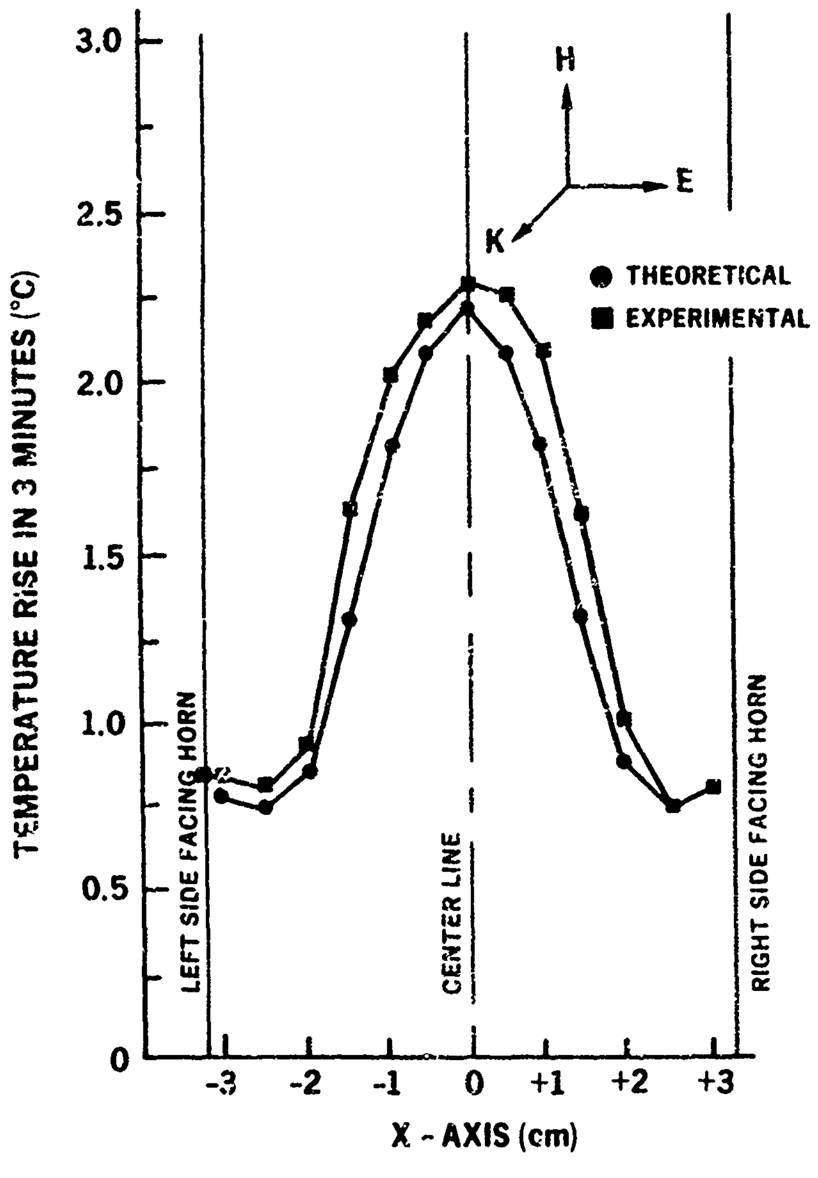

Figure 2.9. Temperature rise along the x-axis of a 3.3-cm radius homogeneous muscle-equivalent sphere exposed to 1.2 GHz, CW, 70 mW/cm^2, RF in the far field for 3 min.

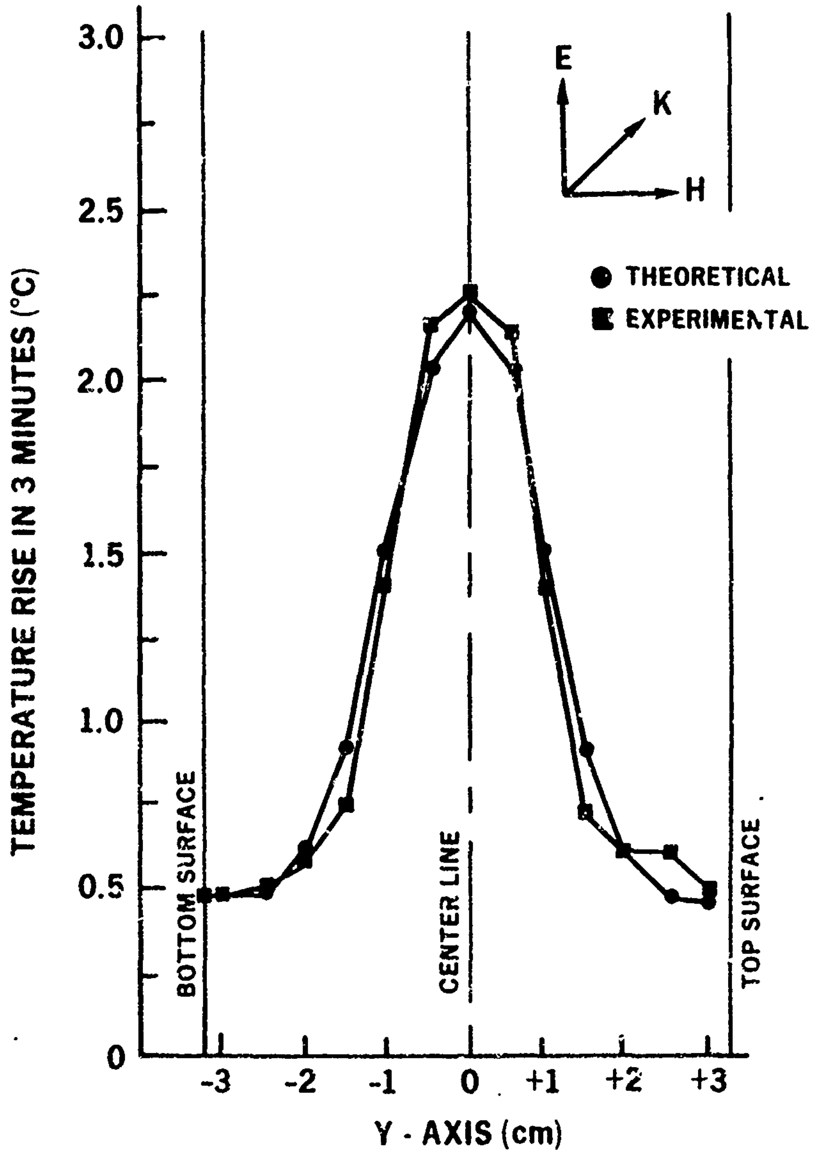

Figure 2.10. Temperature rise along the y-axis of a 3.3-cm radius homogeneous muscle-equivalent sphere exposed to 1.2 GHz, CW, 70 mW/cm^2, RF in the far field for 3 min.

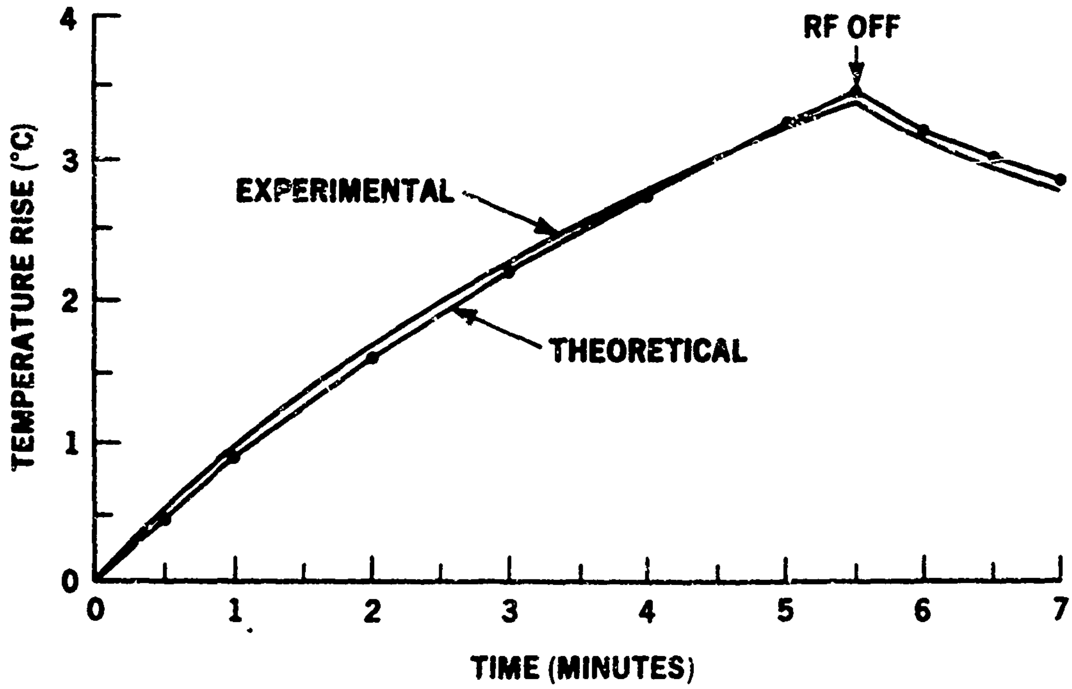

Figure 2.11. The predicted and measured temperature excursion versus time at the center of a 3.3-cm radius homogeneous muscle-equivalent sphere at 1.2 GHz, CW, 70 mW/cm^2.

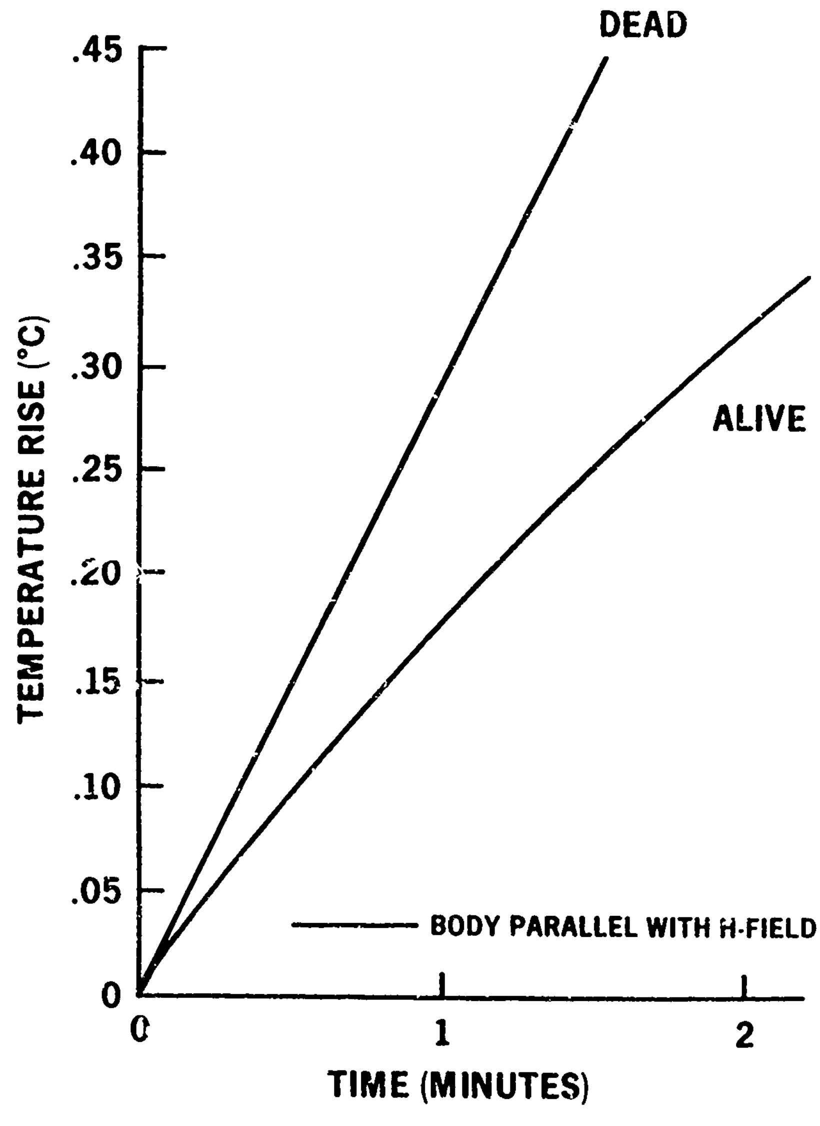

Figure 2.12. Temperature excursion in the midbrain of a living (blood flow case) and dead (no blood flow case) Macaca mulatta (rhesus monkey) head exposed to 70 mW/cm^2, CW, RFR in the far field, 1.2 GHz [3].

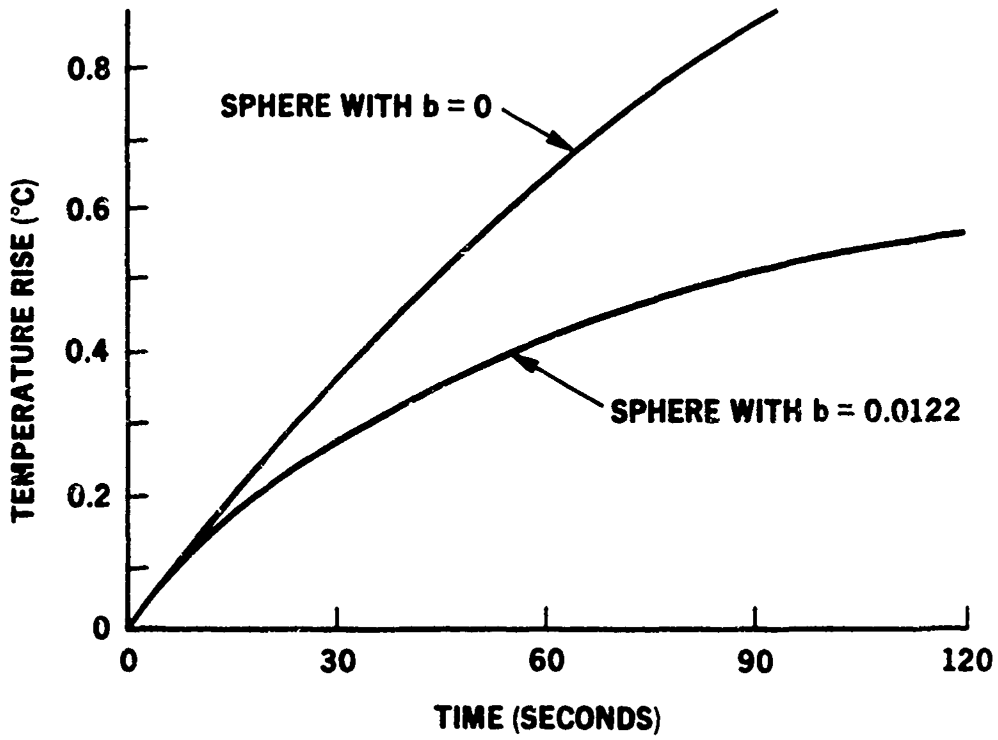

Figure 2.13. Effect of the blood flow term (b) on the temperature excursion in the center of a 4.5-cm radius homogeneous muscle-equivalent sphere.

We see that there are qualitative differences in the curvature of the temperature versus time curves in Figures 2.12 and 2.13 that are at this point unexplained although the measured and predicted values seem to be reasonably close.

Finally we give a comparison between our computer model and the simpler model developed by Kritikos and Schwan [5]. This comparison is given in Figures 2.14 and 2.15.

We note that consideration of the thought experiment depicted in Figure 2.16 makes it obvious that there is such a thing as a nonthermal effect. We consider a structure that will not drift in a microwave field but which will necessarily respond differently to a microwave field and to an equivalent amount of thermal energy. We consider a simple molecule with three charges in a row connected by two chemical bonds that we approximate by two linear springs with identical spring constants. The outside two moieties have charge q and the middle moiety has charge $-2q$.

Since all three masses are the same, the thermal energy of the solvent will act in the same way on all three moieties, but the electric field will exert twice as much force on the inner moiety.

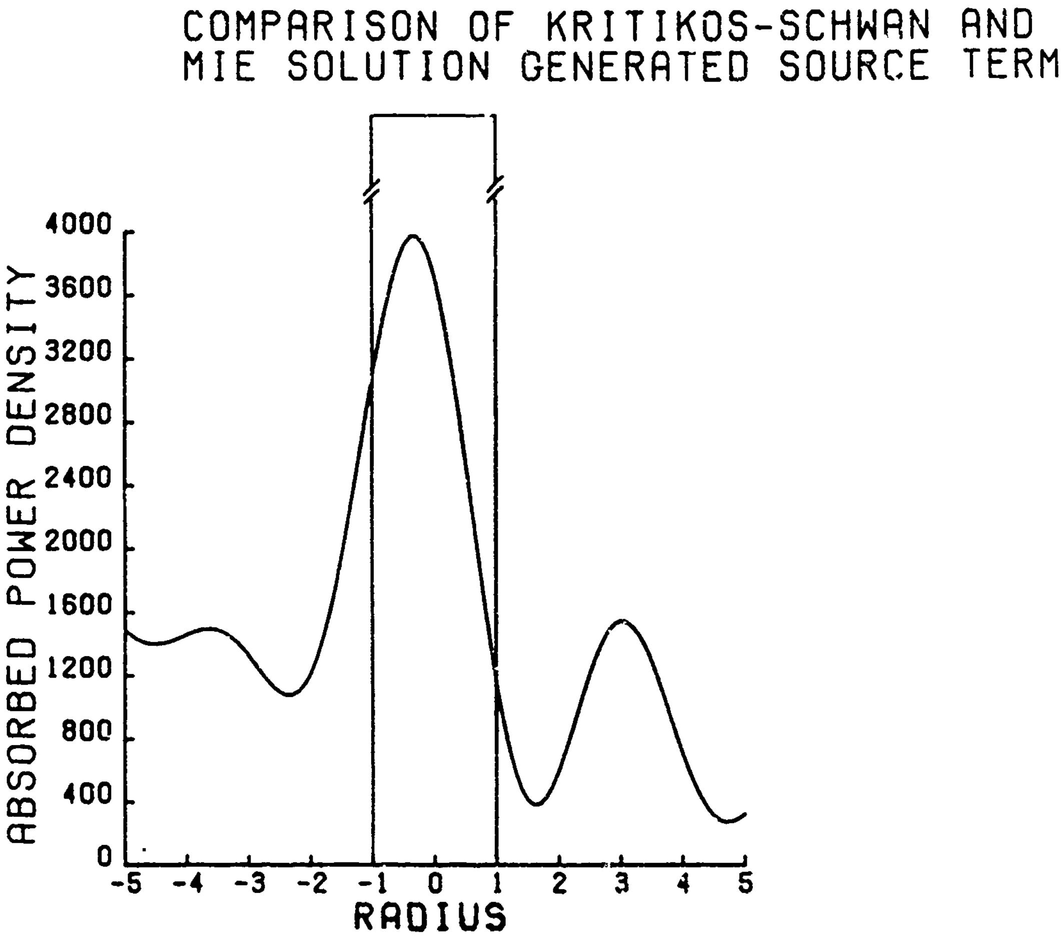

Figure 2.14. Comparison of the Kritikos and Schwan source term used in [5] and the Mie solution generated source term used in this paper. The magnitude of the Kritikos and Schwan source term is 10,000 W/m^3. The Mie solution assumes that the incident power is 10 mW/cm^2 (field strength = 194.09 V/m), that the frequency is 1000 MHz, that the real part of the relative permittivity is 34.4, that the ionic plus polarization current conductivity = $\sigma' + \omega\varepsilon_0\varepsilon''$ = 0.8 mhos/m, and that the outer boundary of this scattering body is a sphere whose radius is 5 cm.

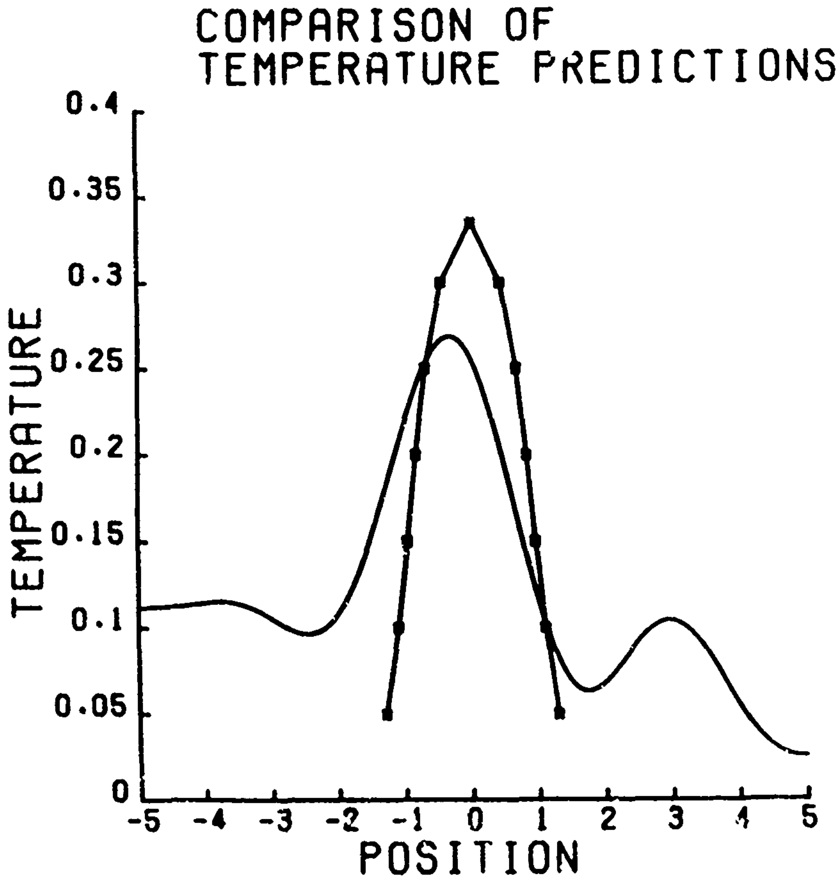

Figure 2.15. Comparison of the Kritikos-Schwan predictions in [5] (marked with an *) and our solution (smooth curve). We assumed, following Kritikos and Schwan, that the blood flow was normal ($b = 0.00186$ cal/cm^3/s) and that the exposure time was 200 s; we used the parameters $K = 0.001$ cal/cm/°C, $\rho = 1.0$ g/cm^3, and $c = 1.0$ cal/g · °C that were used in [5].

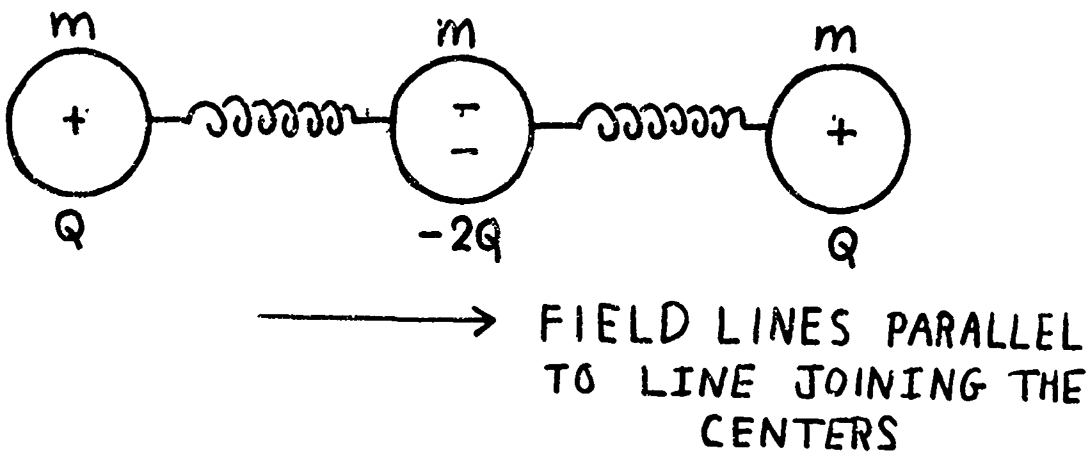

Figure 2.16. Electromagnetic field interaction model for which there would be a nonthermal effect.

3. MATHEMATICAL PRELIMINARIES

3.1. Notation

The variables used in this paper are

ENGLISH

A_i = dimensionless, coefficient of the radial eigenfunction that is regular at the origin,

$a_{(\ell,p)}$ = expansion coefficient for the odd regular vector wave functions,

$a_k^{(m,n)}(t)$ = the temperature decay factor of the solution u associated with the radial eigenvalue $\lambda_{(n,k)}$ and the Legendre transform $\mathcal{L}_n^m$,

$b(r)$ = the product of the number of grams of blood per gram of tissue per second, the tissue density in grams of tissue per cubic centimeter of tissue, and the specific heat of the blood (typically b = .0122),

B_i = dimensionless coefficient of the radial eigenfunction that is singular at the origin,

$b_{(\ell,p)}$ = expansion coefficient for the even regular vector wave functions,

$b_k^{(m,n)}(t)$ = the temperature decay factor of the source term S (associated with the radial eigenvalue $\lambda_{(n,k)}$ and the Legendre transform $\mathcal{L}_n^m$,

$c(r)$ = tissue specific heat in calories per gram degree centigrade (typically $c = .84$),

$\vec{E}$ = the electric field intensity in volts per meter (10 milliwatts per square centimeter corresponds to 194.087 volts per meter),

f = frequency in Hertz,

H = Newton cooling constant in calories per square centimeter per degree per second (typically $H = .0000572$),

$\vec{H}$ = magnetic field intensity in Henrys per meter (10 milliwatts per square centimeter corresponds to .5151 Henrys per meter)

h_n = spherical Hankel function = $j_n - iy_n$ that is used in expanding the electromagnetic fields in Tesseral harmonics,

$\underline{J}(\underline{S}(\lambda,r),r,n)$ = the radial eigenfunction, used in expanding the temperature, that is nonsingular at the origin,

j_n = the spherical Bessel function of order n,

K = thermal conductivity in calories per centimeter per degree centigrade per second (typically $K = .0012$),

m = the index of the finite cosine transform ($m = 0$ or 1 in our application),

n = the index of the Legendre polynomial used in expanding the field,

$P_n^m(\cos(\theta))$ = the associated Legendre polynomial,

r = the distance from the center of the scatterer in centimeters,

R_i = the radius of the ith bounding sphere in centimeters,

S = the source term for the heat equation in calories per cubic centimeter per second,

$\underline{S}(\lambda,r)$ = $(\lambda\rho(r)c(r) - b(r))/K(r)$,

$\underline{S}_i(\lambda)$ = a constant value of $\underline{S}(\lambda,r)$ occurring when $R_{i-1} < r < R_i$,

t = time in seconds,

u = temperature excursion above the ambient temperature,

y_n = spherical Bessel function of the second kind,

$Y_{n+1/2}$ = half order Bessel function of the second kind (Weber function of order n+1/2),

$\underline{Y}(\underline{S}(\lambda,r),r,n)$ = the radial eigenfunction, used in expanding the temperature, that is singular at the origin,

$Z_{(n,k)}(r)$ = the radial eigenfunction associated with the eigenvalue $\lambda_{(n,k)}$,

GREEK

$\alpha_{(\ell,p)}$ = expansion coefficient for the odd singular vector wave functions,

$\alpha_i(\lambda,R,n) = \underline{J}(\underline{S}_i(\lambda),r,n),$

$\tilde{\alpha}_i(\lambda,R,n) = K_i[(\partial/\partial r)\underline{J}(\underline{S}_i(\lambda),r,n)]$ evaluated at $r = R$,

$\beta_{(\ell,p)}$ = expansion coefficient for the even singular vector wave functions,

$\beta_i(\lambda,R,n) = \underline{Y}(\underline{S}_i(\lambda),R,n),$

$\tilde{\beta}_i(\lambda,R,n) = K_{i+1}[(\partial/\partial r)\underline{Y}(\underline{S}_i(\lambda),r,n)]$ evaluated at $r = R$,

$\Delta_i(\lambda,R_i,n) = \alpha_{i+1}(\lambda,R_i,n)\tilde{\beta}_{i+1}(\lambda,R_i,n) - \tilde{\alpha}_{i+1}(\lambda,R_i,n)\beta_{i+1}(\lambda,R_i,n),$

ε = permittivity in farads per meter,

θ = spherical coordinate--angle of ray to a point with the positive z-axis,

λ = eigenvalue associated with radial harmonics,

ρ = density in grams per cubic centimeter,

σ = conductivity in ohms per meter,

τ = dummy variable of integration used in expressing temperature decay factors as a convolution integral,

ϕ = spherical coordinate of the x-y plane,

ω = frequency in radians per second,

and

$\chi(N_p, T, T_{\underline{p}}, T_p)$ = a cutoff function for the temporal envelope of the pulse heating scheme.

MISCELLANEOUS

∂ = partial derivative symbol

ENGLISH SCRIPT

$\mathcal{B}(T_d, T_p, N_p, T_{\underline{p}}, T_R, t)$ = the pulse heating scheme temporal envelope function,

$\mathcal{C}_m$ = the finite cosine transform,

$\mathcal{L}_n^m$ = the Legendre transform,

$\mathcal{S}(T_p, T_{\underline{p}}, N_p, T_d, T)$ = part of a temperature decay factor associated with a heating pattern defined by equation (3.4.14),

$\mathcal{T}(T_p, T_{\underline{p}}, N_p, T_d, T)$ = part of a temperature decay factor associated with a complex pulse heating scheme defined by equation (3.4.15),

and

$\mathcal{T}(n,k)$ = the radial transform used in getting expansion coefficients to express a function of r in terms of radial eigenfunctions that satisfy the Newton cooling law boundary condition.

Other notation that is introduced in the text of the paper is defined and used locally.

3.2. Induced Electromagnetic Field Distribution

A new practical method of developing Tesseral harmonic expansion coefficients for the electromagnetic field induced in a penetrable scatterer with spherical symmetry is described here. Our numerical technique will permit us to use the Mie-solution method to determine the response of a body with a large size to a higher frequency radiation than we could with the standard methods described in the references of [1].

The electric field induced in the pth interior region by a wave of the form

$$\vec{E}^i = E_0 \exp(-i\omega_0(t - \frac{x}{c})) \tag{3.2.1}$$

is given in the pth region by

$$\vec{E}_p = E_0 \sum_{\ell=1}^{\infty} i^{\ell} \frac{2\ell+1}{\ell(\ell+1)} \Big[a_{(\ell,p)} \vec{M}^{(o,1)}_{(1,\ell)} - ib_{(\ell,p)} \vec{N}^{(e,1)}_{(1,\ell)} + \alpha_{(\ell,p)} \vec{M}^{(o,3)}_{(1,\ell)}$$

$$- i\beta_{(\ell,p)} \vec{N}^{(e,3)}_{(1,\ell)} \Big] \tag{3.2.2}$$

where

$$\vec{M}^{(e,j)}_{(1,n)} = - \frac{1}{\sin(\theta)} Z^j_n(k_p r) P^1_n(\cos(\theta)) \sin(\phi) \vec{e}_\theta$$

$$- Z^j_n(k_p r)(d/d\theta)(P^1_n(\cos(\theta)))\cos(\phi)\vec{e}_\phi, \tag{3.2.3}$$

$$\vec{M}^{(o,j)}_{(1,n)} = \frac{1}{\sin(\theta)} z^{j}_{n}(k_p r)P^{1}_{n}(\cos(\theta))\cos(\phi)\vec{e}_\theta$$

$$- z^{j}_{n}(k_p r)(d/d\theta)P^{1}_{n}(\cos(\theta))\sin(\phi)\vec{e}_\phi, \qquad (3.2.4)$$

$$\vec{N}^{(e,j)}_{(1,n)} = \frac{n(n+1)}{k_p r} z^{j}_{n}(k_p r)P^{1}_{n}(\cos(\theta))\cos(\phi)\vec{e}_r$$

$$+ (\frac{1}{k_p r})(\partial/\partial r)(rz^{2}_{n}(k_p r))(d/d\theta)(P^{1}_{n}(\cos(\theta)))\cos(\phi)\vec{e}_\theta$$

$$- \frac{1}{((k_p r)\sin(\theta))}(\partial/\partial r)(rz^{j}_{n}(k_p r))P^{1}_{n}(\cos(\theta))\sin(\phi)\vec{e}_\phi \qquad (3.2.5)$$

and

$$\vec{N}^{(o,j)}_{(1,n)} = \frac{n(n+1)}{k_p r} z^{j}_{n}(k_p r)P^{1}_{n}(\cos(\theta))\sin(\phi)\vec{e}_r$$

$$+ \frac{1}{k_p r}(\partial/\partial r)(rz^{j}_{n}(k_p r))(d/d\theta)(P^{1}_{n}(\cos(\theta)))\sin(\phi)\vec{e}_\theta$$

$$+ \frac{1}{((k_p r)\sin(\theta))}(\partial/\partial r)(rz^{j}_{n}(k_p r))P^{1}_{n}(\cos(\theta))\cos(\phi)\vec{e}_\phi, \qquad (3.2.6)$$

where

$$k_p = \text{sign}\ (\omega)\sqrt{\frac{\mu\varepsilon\omega^2 + \sqrt{\mu^2\varepsilon^2\omega^4 + \mu^2\sigma^2\omega^2}}{2}} + i(\sqrt{\frac{-\mu\varepsilon\omega^2 + \sqrt{\mu^2\varepsilon^2\omega^4 + \mu^2\sigma^2\omega^2}}{2}}).$$

(3.2.7)

in (3.2.3) - (3.2.6) functions P_n^1 are the associated Legendre polynomials and

$$z_n^j(z) = \begin{cases} h_n^1(z) = (j_n+iy_n)(z) & \text{if} \quad j = 3 \\ j_n(z) & \text{if} \quad j = 0 \end{cases}, \tag{3.2.8}$$

where j_n and y_n are respectively the spherical Bessel functions of the first and second kind. Part of the difficulty is that we cannot use (3.2.7) to compute h_n^1 even if we know j_n and y_n exactly. For example, $z = u + iv$ implies that

$$\begin{aligned} h_0^1(z) = (1/z)[&\sin(u)(\cosh(v) - \sinh(v)) \\ &+ i\cos(u)(\sinh(v) - \cosh(v))] \end{aligned} \tag{3.2.9}$$

which is uncomputable on a digital computer if v is large enough so that $\cosh(v)$ and $\sinh(v)$ are indistinguishable.

A better way is the use of the formula

$$h_n^1(z) = i^{-n-1}z^{-1}\exp(iz)\sum_{k=0}^{n}(n+1/2,k)(-2iz)^k, \tag{3.2.10}$$

coupled with the Hankel symbol formula,

$$\begin{aligned} (n+1/2,k) &= \frac{(n+k)!}{k!(n-k)!} = \frac{(n+k)(n+k-1)\cdots(n+1)n(n-1)\cdots(n-(k-1))}{k!} \\ &= \prod_{i=1}^{k}\left(\frac{(n-(k-2i))(n-(k-(2i-1)))}{i}\right), \end{aligned} \tag{3.2.11}$$

when the complex number z is such that $(n+1/2,k)(-2iz)^k$ are of such a size that round-off error is not encountered in the computation of (3.2.10).

The reader can verify (3.2.10) by induction using the three-term recursion formula

$$h^1_{n+1} = \frac{(2n+1)}{z} h^1_n - h^1_{n-1} \tag{3.2.12}$$

is satisfied and by showing that (3.2.10) is true for $n = 0$ and $n = 1$ since from (3.2.8) we know that

$$h^1_0(z) = \frac{\sin(z)}{z} + i(-\frac{\cos(z)}{z}), \tag{3.2.13}$$

and

$$h^1_1(z) = (\frac{\sin(z)}{z^2} - \frac{\cos(z)}{z}) + i(-\frac{\cos(z)}{z^2} - \frac{\sin(z)}{z}). \tag{3.2.14}$$

For intermediate values of z, another method must be used to compute $h^1_n(z)$. We observe that equation (3.2.12) implies that

$$\frac{h^1_{n-2}(z)}{h^1_{n-1}(z)} = \frac{2n-1}{z} - \frac{h^1_n(z)}{h^1_{n-1}(z)} . \tag{3.2.15}$$

The basic idea is to write

$$a_{n+1/2} = \frac{h^1_{n-1}(z)}{h^1_n(z)} = \frac{H^1_{(n-1+1/2)}(z)}{H^1_{(n+1/2)}(z)} \tag{3.2.16}$$

and then observe that equations (3.2.15) and (3.2.16) imply that

$$a_{(n+1/2)} = \frac{2(n+1)-1}{z} - \frac{1}{a_{(n+1+1/2)}} \tag{3.2.17}$$

Hence, $\nu = n + 1/2$ and $n = \nu - 1/2$ and equation (3.2.17) imply that

$$a_\nu = \frac{2\nu}{z} - \frac{1}{a_{\nu+1}} \tag{3.2.18}$$

Thus, from (3.2.16) we get immediately a continued fraction expansion

$$a_\nu = \frac{2\nu}{z} - \cfrac{1}{\frac{2(\nu+1)}{z} + \frac{1}{a_{\nu+2}}} \tag{3.2.19}$$

et cetera, which by the following Lemma is always convergent.

Lemma (Wall [10], p. 50). We have uniform convergence of the continued fraction,

$$c = b_0 + \cfrac{a_1}{b_1 + \cfrac{a_2}{b_2 + \cfrac{a_3}{b_3 + \cfrac{a_4}{b_4 + \cdots}}}}\ , \tag{3.2.20}$$

if there exists constants $g_p \varepsilon (0,1)$ such that

$$\left| \frac{a_{p+1}}{b_p b_{p+1}} \right| \leq (1 - g_p) g_{p+1}. \tag{3.2.21}$$

In our situation

$$b_0 = \frac{2\nu}{z} \tag{3.2.22}$$

$$b_p = \frac{2(\nu+p)}{z} \quad p = 1,2,\ldots \tag{3.2.23}$$

and

$$a_p = 1 \qquad p = 1,2,\ldots \tag{3.2.24}$$

Thus, for every z there is an $N_z > 0$ such that if $p > N_z$ then

$$\left| \frac{a_{p+1}}{b_p b_{p+1}} \right| \leq \frac{|z|^2}{4(\nu+p)(\nu+p+1)} \leq (1-g_p)g_{p+1} \tag{3.2.25}$$

provided that N_z is such that $(N_z+\nu) \geq |z|$ and $g_p = 1/2$ for all p. Thus in view of the Lemma the continued fraction expansion theoretically converges for all $z \neq 0$.

The idea then is not to compute the spherical Bessel functions of the second kind y_n at all, but rather use a direct method for obtaining the h_n^1. Observe that

$$h_n^1 = \left(\frac{h_n^1}{h_{n-1}^1}\right)\left(\frac{h_{n-1}^1}{h_{n-2}^1}\right)\cdots\left(\frac{h_1^1}{h_0^1}\right)\frac{(-i)\exp(iz)}{z} \tag{3.2.26}$$

The functions $j_n(z)$ used in the expansion are successfully computed by the methods of Lentz [8].

3.3. Heat Operator Eigenvalues and Eigenfunctions for a Newton Cooling Law Boundary Condition

3.3.1. The Radiative Heat Transfer Problem. From the E field determination of the preceding section, we develop an expression for a source

$$S = \mathrm{div}(\vec{E} \times \vec{H})/(10^6 \times 4.184) \tag{3.3.1}$$

of internal energy generation which is used as a term in the heat equation,

$$\rho c \frac{\partial u}{\partial t} - \mathrm{div}(K\ \mathrm{grad}(u)) + bu = S, \tag{3.3.2}$$

where ρc is the product of density and specific heat, b is a blood-flow cooling term, and K is the thermal conductivity. Assume that the scattering body is a union of material regions bounded by spheres $r = R_i$ for i in $\{1,\ldots,N\}$ (with $N \leq 6$ in our computer program) and

$$0 = R_0 < R_1 < \ldots < R_N. \tag{3.3.3}$$

Assume that $\rho(r)$, $c(r)$, and $K(r)$ have the constant values ρ_i, c_i, K_i respectively for $R_{i-1} < r < R_i$. Then for $R_{i-1} < r < R_i$ equation (3.2) may be written

$$\rho_i c_i (\partial/\partial t)u = K_i \left[\left(\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial u}{\partial r}\right)\right) + \frac{1}{r^2 \sin(\theta)} \left\{ \left(\frac{\partial}{\partial \theta}\right)\left(\sin(\theta) \frac{\partial u}{\partial \theta}\right) + \frac{1}{\sin(\theta)} \frac{\partial^2 u}{\partial \phi^2} \right\} \right] - b_i u + S, \tag{3.3.4}$$

where the initial condition is that

$$u(r,\theta,\phi,0) = 0, \tag{3.3.5}$$

continuity of temperature and heat flux implies

$$\lim_{\varepsilon \to 0} u(R_i-\varepsilon,\theta,\phi,t) = \lim_{\varepsilon \to 0} u(R_i+\varepsilon,\theta,\phi,t), \tag{3.3.6}$$

and

$$\lim_{\varepsilon \to 0} K_i(\partial/\partial r)u(R_i-\varepsilon,\theta,\phi,t) = \lim_{\varepsilon \to 0} K_{i+1}(\partial/\partial r)u(R_i+\varepsilon,\theta,\phi,t), \tag{3.3.7}$$

and the Newton cooling law implies that

$$K_N(\partial/\partial r)u(R_N,\theta,\phi,t) + Hu(R_N,\theta,\phi,t) = 0. \tag{3.3.8}$$

We define the finite cosine transform of the temperature excursion $u(r,\theta,\phi,t)$ by the rule,

$$(\mathcal{C}_m u)(r,\theta,t) = (1/\pi)\int_{-\pi}^{\pi} u(r,\theta,\phi,t)\cos(m\phi)d\phi, \tag{3.3.9}$$

for positive integers m and

$$(\mathcal{C}_0 u)(r,\theta,t) = (1/2\pi)\int_{-\pi}^{\pi} u(r,\theta,\phi,t)d\phi. \tag{3.3.10}$$

We define the Legendre transform operator $\mathcal{L}_n^m$ on the temperature excursion $u(r,\theta,\phi,t)$ by the rules,

$$(\mathcal{L}_n^m u)(r,\phi,t) =$$

$$\left(\frac{2n+1}{1}\right)\left(\frac{(n-m)!}{(n+m)!}\right)\int_0^{\pi} u(r,\theta,\phi,t)P_n^m(\cos(\theta))\sin(\theta)d\theta \tag{3.3.11}$$

and

$$\mathcal{L}_n^0 u(r,\phi,t) = \left(\frac{2n+1}{1}\right)\int_0^{\pi} u(r,\theta,\phi,t)P_n(\cos(\theta))\sin(\theta)d\theta. \qquad (3.3.12)$$

Thus, if we combine (3.3.9), (3.3.10), (3.3.11), (3.3.12), and (3.3.4) we see that if ρ, c, and K are simply functions of r, then

$$(\partial/\partial t)\mathcal{L}_n^m\mathcal{C}_m u = \left(\frac{1}{\rho c r^2}\right)\{(\partial/\partial r)(r^2K(r))(\partial/\partial r)\mathcal{L}_n^m\mathcal{C}_m u$$
$$- K(r)n(n+1)\mathcal{L}_n^m\mathcal{C}_m u\} - (b/(\rho))\mathcal{L}_n^m\mathcal{C}_m u + \mathcal{L}_n^m\mathcal{C}_m(S/\rho c) \qquad (3.3.13)$$

Let us attempt to write

$$(\mathcal{L}_n^m\mathcal{C}_m u)(r,t) = \sum_{k=1}^{\infty} a_k^{(m,n)}(t)Z_{(n,k)}(r), \qquad (3.3.14)$$

and

$$\mathcal{L}_n^m\mathcal{C}_m(S/\rho c)(r,t) = \sum_{k=1}^{\infty} b_k^{(m,n)}(t)Z_{(n,k)}(r), \qquad (3.3.15)$$

where

$$(d/dr)(K(r)r^2(d/dr))Z_{(n,k)}(r) +$$
$$\{\lambda_{(n,k)}r^2\rho c - Kn(n+1) - br^2\}Z_{(n,k)}(r) = 0, \qquad (3.3.16)$$

$$\lim_{\varepsilon\to 0} Z_{(n,k)}(R_i+\varepsilon) = \lim_{\varepsilon\to 0} Z_{(n,k)}(R_i-\varepsilon) \qquad (3.3.17)$$

$$\lim_{\varepsilon\to 0} K(R_i+\varepsilon)(Z_{(n,k)})'(R_i+\varepsilon) = \lim_{\varepsilon\to 0} K(R_i-\varepsilon)(Z_{(n,k)})'(R_i-\varepsilon) \qquad (3.3.18)$$

for

$$i = 1,\ldots,N-1$$

where the $\lambda_{(n,k)}$ are positive numbers for which

$$(Z_{(n,k)})'(R_N) + (H/K_N)Z_{(n,k)}(R_N) = 0. \qquad (3.3.19)$$

Thus, we conclude that

$$a_k^{(m,n)'}(t) + \lambda_{(n,k)}a_k^{(m,n)}(t) = b_k^{(m,n)}(t) \qquad (3.3.20)$$

and consequently that

$$a_k^{(m,n)}(t) = \int_0^t \exp(-\lambda_{(n,k)}(t-\tau))b_k^{(m,n)}(\tau)d\tau. \qquad (3.3.21)$$

By defining for every function $g(r)$

$$\mathcal{T}_{(n,k)}(g) = \frac{\int_0^{R_N} g(r)Z_{(n,k)}(r)(\rho c)(r)r^2dr}{\int_0^{R_N} |Z_{(n,k)}(r)|^2(\rho c)(r)r^2dr} \qquad (3.3.22)$$

we see that

$$b_k^{(m,n)}(t) = \mathcal{T}_{(n,k)}\mathcal{L}_n^m\mathcal{C}_m(\frac{S}{\rho c}). \qquad (3.3.23)$$

3.3.2. Eigenvalue Determination. We wish to describe here a computer algorithm for computing the $\lambda_{(n,k)}$ and the $Z_{(n,k)}(r)$ when ρ, c, b, and K are nonnegative piecewise constant functions. While this is trivial when $b(r)$ is a constant function, the problem becomes interesting when $b(r)$ is positive in some intervals (R_{i-1},R_i) and is identically zero in others.

To begin with we define

$$\underline{S}(\lambda,r) = \frac{\lambda\rho(r)c(r) - b(r)}{K(r)} \tag{3.3.24}$$

and we obtain in each interval (R_{i-1},R_i) one of three different classes of solutions of the problem (3.3.16) - (3.3.19) depending on whether or not $\underline{S}(\lambda_{(n,k)},r)$ is uniformly positive, zero, or negative in this interval. We, therefore, write

$$Z(r,\lambda) = A_i\underline{J}(\underline{S}(\lambda,r),r,n) + B_i\underline{Y}(\underline{S}(\lambda,r),r,n) \tag{3.3.25}$$

and require that

$$\lim_{r\to 0} \frac{A_1\underline{J}(\underline{S}(\lambda,r),r,n)}{r^n} = 1 \tag{3.3.26}$$

and

$$B_1 = 0 \tag{3.3.27}$$

In general for $\underline{S}(\lambda,r) > 0$ we have setting

$$z = r\sqrt{\frac{\lambda\rho c-b}{K}} = r\sqrt{\underline{S}(\lambda,r)} \tag{3.3.28}$$

the fact that (3.3.16) is equivalent to

$$r^2(d/dr)^2Z_n + 2r(d/dr)Z_n + r^2\underline{S}(\lambda,r)Z_n - n(n+1)Z_n = 0 \tag{3.3.29}$$

and if we change variables by the rule

$$z = \sqrt{\underline{S}(\lambda,r)}r \tag{3.3.30}$$

and observe that

$$\frac{d}{dr} = \frac{dz}{dr}\frac{d}{dz} , \qquad (3.3.31)$$

we see that if

$$Z_{(n,k)}(r) = W(z), \qquad (3.3.32)$$

then

$$z^2W'' + 2zW' + (z^2-(n(n+1)W = 0, \qquad (3.3.33)$$

which implies that in (R_{i-1}, R_i) we have

$$W(z) = A_i j_n(z) + B_i y_n(z), \qquad (3.3.34)$$

where

$$j_n(z) = \left[\frac{z^n}{\prod_{k=1}^{n}(2k-1)}\right] \sum_{m=0}^{\infty} \frac{(-1)^m(z^2/2)m}{m!\prod_{k=0}^{m}(2k+1+2n)} . \qquad (3.3.35)$$

Thus, to make (3.3.34) consistent with (3.3.26) when $i = 1$, we must have

$$A_1 = 1/\{(\prod_{k=1}^{n+1}(2k-1))(\sqrt{\underline{S}(\lambda,r)})^n\}. \qquad (3.3.36)$$

If $\underline{S}(\lambda,r) = 0$ we have

$$\underline{J}(\underline{S}(\lambda,r),r,n) = r^n \qquad (3.3.37)$$

and

$$\underline{Y}(\underline{S}(\lambda,r),r,n) = r^{-n-1} \qquad (3.3.38)$$

If $\underline{S}(\lambda,r) = \underline{S}_i(\lambda) < 0$ in (R_{i-1},R_i) we have in this interval

$$\underline{J}(\underline{S}(\lambda,r),r,n) = \underline{J}(\underline{S}_i(\lambda),r,n)$$

$$= \sum_{m=0}^{\infty} \frac{(|\underline{S}_i(\lambda)|r^2/2)^m (\sqrt{|\underline{S}_i(\lambda)|}r)^n}{m!\left[\prod_{k=1}^{m}(2k+1)+2n)\right]\left[\prod_{k=0}^{n}(2k+1)\right]}, \qquad (3.3.39)$$

and

$$\underline{Y}(\underline{S}(\lambda,r),r,n) = \underline{Y}(\underline{S}_i(\lambda),r,n)$$

$$= \left(\sum_{m=0}^{\infty} \frac{(|\underline{S}_i(\lambda)|r^2/2)^m}{m!\prod_{k=1}^{m}(2k-1-2n)}\right)\left[\frac{\prod_{k=0}^{n}(2k-1)}{(\sqrt{|\underline{S}_i(\lambda)|}r)^{n+1}}\right]. \qquad (3.3.40)$$

The functions defined by (3.3.39) and (3.3.40) do not satisfy Bessel's differential equation, but they may be expressed in terms of Bessel functions of a purely imaginary argument. This is the way they are developed in our computer program.

We begin our search for eigenvalues by finding the unique solution $Z_n(r,\lambda)$ of equation (3.3.29) which satisfies the condition

$$\lim_{r\to 0} \frac{Z_n(r,\lambda)}{r^n} = 1 \qquad (3.3.41)$$

and the requirement that $Z_n(r,\lambda)$ and $K(r)(\partial/\partial r)Z_n(r,\lambda)$ be continuous on $[0, R_N)$. We then define a function of λ by the rule,

$$F_n(\rho,c,K,\lambda) = \lim_{r\to R_N}\{K_N(\partial/\partial r)Z_n(r,\lambda) + HZ_n(r,\lambda)\}. \qquad (3.3.42)$$

where R_N is the radius of the bounding sphere of the scattering body. We then use a root-finding routine to find for each n an ascending sequence,

$$k \to \lambda_{(n,k)} \qquad (3.3.43)$$

of positive real numbers such that $\lambda = \lambda_{(n,k)}$ implies that

$$F_n(\rho,c,K,\lambda) = 0. \qquad (3.3.44)$$

These numbers $\lambda_{(n,k)}$ are the eigenvalues associated with the heat transfer problem and have the units of reciprocal seconds. The numbers $t_{(n,k)} = 1/\lambda_{(n,k)}$ estimate the time needed for the (n,k)th mode of the temperature solution to decay to $(1/e)$ times its original value.

3.3.3. Eigenfunction Computation. We assume here that the eigenvalue $\lambda = \lambda_{(n,k)}$ that we are using to develop the radial eigenfunction is known and use the initial condition (3.3.26) and the regularity conditions (3.3.17) and (3.3.18) to uniquely determine the eigenfunction $Z_{(n,k)}$.

A first step in carrying this out is the determination of the eigenfunction coefficients A_i and B_i used in expressing the eigenfunction $Z(r,\lambda)$ by the relation (3.3.25). We observe that A_1 and B_1 are given by equations (3.3.26) and (3.3.27) and that if A_i and B_i are known, then

$$A_{i+1} = \frac{\det\begin{bmatrix} A_i\alpha_i(\lambda,R_i,n) + B_i\beta_i(\lambda,R_i,n) & \beta_{i+1}(\lambda,R_i,n) \\ A_i\tilde{\alpha}_i(\lambda,R_i,n) + B_i\tilde{\beta}_i(\lambda,R_i,n) & \tilde{\beta}_{i+1}(\lambda,R_i,n)\end{bmatrix}}{\Delta_i(\lambda,R_i,n)} \qquad (3.3.45)$$

$$B_{i+1} = \frac{\det\begin{bmatrix} \alpha_{i+1}(\lambda,R_i,n) & A_i\alpha_i(\lambda,R_i,n) + B_i\beta_i(\lambda,R_i,n) \\ \tilde{\alpha}_{i+1}(\lambda,R_i,n) & A_i\tilde{\alpha}_i(\lambda,R_i,n) + B_i\tilde{\beta}_i(\lambda,R_i,n)\end{bmatrix}}{\Delta_i(\lambda,R_i,n)} \qquad (3.3.46)$$

where

$$\alpha_i(\lambda,r,n) = \underline{J}(\underline{S}_i(\lambda),r,n), \qquad (3.3.47)$$

$$\tilde{\alpha}_i(\lambda,r,n) = K_i(\partial/\partial r)J(\underline{S}_i(\lambda),r,n), \qquad (3.3.48)$$

$$\beta_i(\lambda,r,n) = \underline{Y}(\underline{S}_i(\lambda),r,n), \qquad (3.3.49)$$

and

$$\tilde{\beta}_i(\lambda,r,n) = K_i(\partial/\partial r)\underline{Y}(\underline{S}_i(\lambda),r,n), \qquad (3.3.50)$$

defines the entries in the numerators of (3.3.45) and (3.3.46) and where

$$\Delta_i(\lambda,R_i,n) = \alpha_{i+1}(\lambda,R_i,n)\tilde{\beta}_{i+1}(\lambda,R_i,n) - \tilde{\alpha}_{i+1}(\lambda,R_i,n)\beta_{i+1}(\lambda,R_i,n) \qquad (3.3.51)$$

defines the determinant of the matrix multiplying the column vector whose entries are A_{i+1} and B_{i+1}. Thus, the relations (3.3.45) and (3.3.46) determine A_i and B_i for all $i \in \{1,\ldots,N\}$. Consequently, if $\lambda = \lambda_{(n,k)}$ the eigenfunction $Z_{(n,k)}(r)$ has for r in (R_{i-1},R_i) the explicit representation

$$Z_{(n,k)}(r) = A_i\underline{J}(\underline{S}_i(\lambda),r,n) + B_i\underline{Y}(\underline{S}_i(\lambda),r,n) \qquad (3.3.52)$$

where the form of the functions $\underline{J}$ and $\underline{Y}$ depend on whether or not

$$\underline{S}_i(\lambda) = \frac{\lambda\rho(r)c(r) - b(r)}{K(r)} \quad (R_{i-1} < r < R_i) \tag{3.3.53}$$

is positive, zero, or negative in the manner indicated in Section 3.2.

3.4. Details of the Temperature Computation Including Complex Pulse Heating Schemes

3.4.1. Series Expansion of the Temperature. In this section we describe the computational procedure for determining the solution u of equation (3.3.2) under the assumption that we know the eigenvalues $\lambda_{(n,k)}$ and eigenfunctions $Z_{(n,k)}$ described in Section (3.3.1). Now that this is done we express the solution $u(r,\theta,\phi,t)$ by the series

$$u(r,\theta,\phi,t) = \sum_{k=1}^{\infty}\sum_{n=0}^{\infty}\sum_{m=0}^{n} a_k^{(m,n)}(t)P_n^m(\cos(\theta))\cos(m\phi)Z_{(n,k)}(r) \tag{3.4.1}$$

where $a_k^{(m,n)}(t)$ is defined by equation (3.21) and

$$\begin{aligned} a_k^{(m,n)}(t) &= (\mathcal{T}_{(n,k)}\mathcal{L}_n^m\mathcal{C}_m u)(t) \\ &= \int_0^t b_k^{(m,n)}(\tau)\exp(-\lambda_{(n,k)}(t-\tau))d\tau \end{aligned} \tag{3.4.2}$$

with the operators $\mathcal{T}_{(n,k)}$, $\mathcal{L}_n^m$, and $\mathcal{C}_m$ being defined by equations (3.3.22), (3.3.11) - (3.3.12), and (3.3.9) - (3.3.10) respectively. Almost all of the computing time is taken up in the computation of the coefficients $b_k^{(m,n)}(t)$, defined by equation (3.3.23), that are used in expanding the source function $(S/\rho c)$. While each of these represents the result of a triple integration, the total running time is still only between 3 and 4 min on an IBM 360 for results which are good to within the capabilities of experimental measurement.

3.4.2. <u>Complex Pulse Heating Scheme</u>. We wish to consider a pulse heating scheme (e.g., Figure 3.4.1) in which a group of pulses with a duty time and period followed by a period of quiescence defines a function that is periodic with respect to the total duty time of the pulse group plus the length of time of the quiescent period.

More precisely the time profiles we consider include time harmonic radiation whose basic frequency is that of a radar transmitter multiplied by a function of time $(T_d, T_p, N_p, T_{\underline{p}}, T_R, t)$ defined for $0 < T_d < T_p \leq N_p T_p \leq T_{\underline{p}}$ and $T_R > 0$ by the initialization rule

$$B(T_d, T_p, N_p, T_{\underline{p}}, T_R, t) = \begin{cases} 0 & t > T_R, \\ 1 & 0 \leq t \leq T_d \text{ and } t \leq T_R, \\ 0 & T_d < t < T_p, \\ 0 & N_p T_p < t < T_{\underline{p}}, \end{cases} \tag{3.4.3}$$

and the periodicity rules,

$$B(T_d, T_p, N_p, T_{\underline{p}}, T_R, t+T_p) = B(T_d, T_p, N_p, T_{\underline{p}}, T_R, t) \tag{3.4.4}$$

if $t + T_p \leq T_R$ and $t + T_p \leq N_p T_p$ and

$$B(T_d, T_p, N_p, T_{\underline{p}}, T_R, t+T_{\underline{p}}) = B(T_d, T_p, N_p, T_{\underline{p}}, T_R, t) \tag{3.4.5}$$

if $t + T_{\underline{p}} \leq T_R$, where

T_d = the pulse duration,

T_p = the intrapulse group period,

N_p = the number of pulses per group,

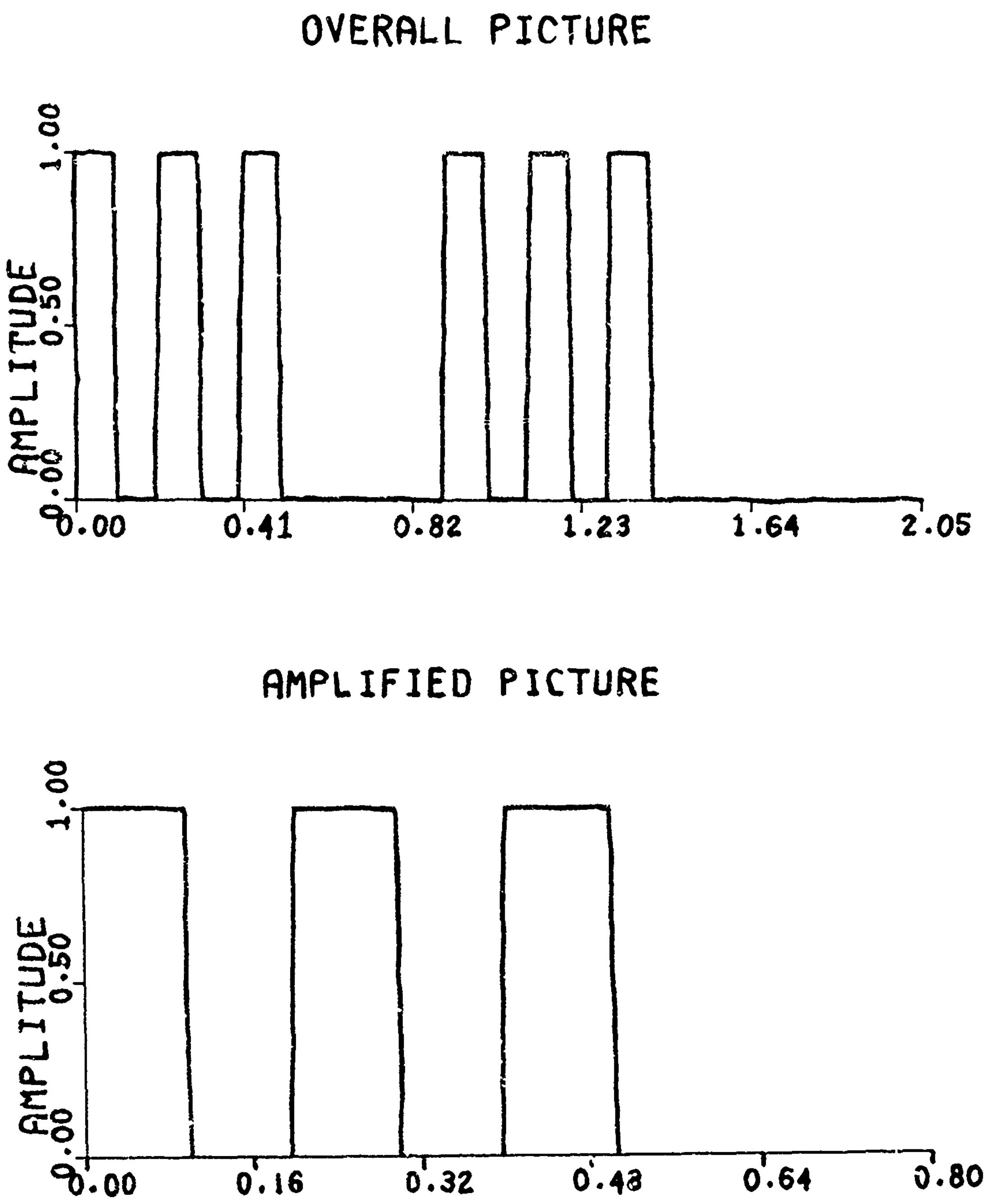

Figure 3.4.1. Complex pulse heating pattern typical of radar emissions with a burst of three pulses followed by a quiet period and with the pattern being repeated periodically. In the above figure we have N_p = 3 pulses per group, T_d = .1 millisecond (ms), T_p= .2 ms, T_p = .9 ms, T_R = 1.6 ms, and t = 2.05 ms.

$T_{\underline{p}}$ = the period,

T_R = the time that the source has been on,

and

t = the time of observation of the radiation effect.

The basic idea is to assume that T_d is large enough that the continuous-wave solution accurately predicts the electromagnetic field distribution and consequently the source term of the heat transfer equation. That is to say, if

$$b_k^{(m,n)}(t) = \underline{b}_k^{(m,n)} B(T_d, T_p, N_p, T_{\underline{p}}, T_R, t), \tag{3.4.6}$$

then $T = T_R$ implies that

$$\begin{aligned} a_k^{(m,n)}(t) = \underline{b}_k^{(m,n)} \Bigg(& \sum_{k=1}^{[T/T_{\underline{p}}]} \sum_{j=1}^{N_p} \int_{(j-1)T_p + (k-1)T_{\underline{p}}}^{(j-1)T_p + (k-1)T_{\underline{p}} + T_d} \exp(-\lambda(t-\tau))\,d\tau \\ & + \sum_{j=1}^{\min([(T - [T/T_{\underline{p}}]T_{\underline{p}})/T_p],N_p)} \left\{ \int_{[T/T_{\underline{p}}]T_{\underline{p}} + (j-1)T_p}^{[T/T_{\underline{p}}]T_{\underline{p}} + (j-1)T_p + T_d} \exp(-\lambda(t-\tau)\,d\tau \right\} \\ & + \int_{[T/T_{\underline{p}}]T_{\underline{p}} + (\min(N_p,[(T - [T/T_{\underline{p}}]T_{\underline{p}})/T_p]))T_p}^{\min(T,\{\{[T/T_{\underline{p}}]T_{\underline{p}} + (\min(N_p,[(T - [T/T_{\underline{p}}]T_{\underline{p}})/T_p]))T_p\} + T_d\})} \exp(-\lambda(t-\tau)\,d\tau \Bigg) \end{aligned} \tag{3.4.7}$$

where min is an abbreviation for the minimum function and [ ] denotes the greatest integer function([x] denotes the largest integer not exceeding x).

Some changes of variables and introduction of notation will make it easier to develop computer code to evaluate the preceding three integrals. We therefore define

$$r(k,j) = (j-1)T_p + (k-1)T_{\underline{p}}\ , \tag{3.4.8}$$

$$s(j) = [T/T_{\underline{p}}]T_{\underline{p}} + (j-1)T_p, \tag{3.4.9}$$

$$M(T,N_p) = \min(([T-[T/T_{\underline{p}}]T_{\underline{p}})/T_p],N_p), \tag{3.4.10}$$

$$\underline{t}(T,N_p,T_p,T_{\underline{p}}) = M(T,N_p)T_p + [T/T_{\underline{p}}]T_{\underline{p}}, \tag{3.4.11}$$

$$\chi = \chi(N_p,T,T_{\underline{p}},T_p) = \begin{cases} 0 & \text{if } [(T-[T/T_{\underline{p}}]T_{\underline{p}})/T_p] \geq N_p, \\ 1 & \text{otherwise} \end{cases} \tag{3.4.12}$$

$$\overline{t}(T,N_p,T_p,T_{\underline{p}}) = \min\ (T,\underline{t}(T,N_p,T_p,T_{\underline{p}})) + T_d\chi(N_p,T,T_{\underline{p}},T_p), \tag{3.4.13}$$

$$\mathcal{S}(T_p,T_{\underline{p}},N_p,T_d,T) = \sum_{k=1}^{[T/T_{\underline{p}}]} \sum_{j=1}^{N_p} \exp(\lambda r(k,j)) =$$

$$\left\{\frac{\exp(\lambda N_p T_p)-1}{\exp(\lambda T_p)-1}\right\}\left\{\frac{\exp(\lambda[T/T_{\underline{p}}]T_{\underline{p}})-1}{\exp(\lambda T_{\underline{p}})-1}\right\}, \tag{3.4.14}$$

and

$$\mathcal{T}(T_p,T_{\underline{p}},N_p,T_d,T) = \sum_{j=1}^{M(T,N_p)} \exp(\lambda s(j)) =$$

$$\exp(\lambda([T/T_{\underline{p}}]T_{\underline{p}}))\left(\frac{\exp(\lambda M(T,N_p)T_p)-1}{\exp(\lambda T_p)-1}\right). \tag{3.4.15}$$

Putting all this together we find that

$$a_k^{(m,n)}(t) = \underline{b}_k^{(m,n)}\{\exp(-\lambda t)\frac{\exp(\lambda T_d)-1}{\lambda}\{\ \mathcal{S}(T_p,T_{\underline{p}},N_p,T_d,T)$$

$$+\ \mathcal{T}(T_p,T_{\underline{p}},N_p,T_d,T)\} \qquad (3.4.16)$$

$$+\ \exp(-\lambda t)\frac{\exp(\lambda\overline{t}(T,N_p,T_p,T_{\underline{p}})) - \exp(\lambda\underline{t}(T,N_p,T_p,T_{\underline{p}}))}{\lambda}\}.$$

This completes the discussion of our temperature computation method.

3.5. Simulated Biostructures

3.5.1. Description of Structures to be Studied. In a previous paper [2], the authors made a study of the thermal response of a ball of muscle-equivalent material; this study is extended in this paper to multi-layer simulated biological structures. In [2] analytical results were compared with measurements made with Vitek-Model 101 Electrothermia monitor; in this paper we compare the shape of the thermal response curve with the electromagnetic power density curve which serves as a source term for the heat equation. We study a one-layer structure with blood flow at a higher frequency (4.5 GHz) than was considered in [2], three-layer simulated fetal structures with and without blood flow, and six-layer simulated cranial structure with blood flow.

For one-layer structures Figures 2.2.2-2.2.4 show the agreement between theory and experimental measurement; from the results described in Figure 3.5.1 we see that there are striking resonance effects in a simply one-layer structure exposed to 4.5-GHz radiation; we demonstrate by the results shown in Figures 3.5.2 and 3.5.3 that the temperature distribution curve has a shape very similar to that of a power density distribution--particularly when the exposure time is short.

Next we treat simulated fetal structures. M. J. Edwards [4] observation that microwave heating of rat embryos can cause teratogenic effects suggests that a quantitative analysis of a simulated fetal structure's response to microwave radiation may assist in the assessment of the potential hazard of a source of microwave radiation. We use a three-layer model whose layers consist of fetal tissue, amniotic fluid, and maternal tissue in simulating the response of the fetus to microwave radiation. Figure 3.5.4 shows the power density across the diameter of the three-layer structure; this diameter coincides with the z-axis of a coordinate system whose origin is the center of sphere; the wave is assumed to propagate in the direction of the positive z-axis. The temperature distributions across the parts of the x, y, and z-axes of the structure within its interior after a 1-hr exposure are given in Figures 3.5.5-3.5.7 where we include blood flow. Figures 3.5.8-3.5.15 show how this temperature distribution changes as exposure time increases when we don't assume removal of heat by blood flow.

Finally we consider a six-layer simulated cranial structure exposed to microwave radiation. Figure 3.5.16 shows the power density across a six-layer simulated structure exposed to 800-MHz radiation. Figures 3.5.17 and 3.5.18 show the thermal response of the structure to 800-MHz microwave radiation for 30-s and 1-min exposures.

3.5.2. Microwave Heating of a Muscle-Equivalent Sphere. In this section we study the manner in which a microwave-induced temperature profile is smoothed as exposure time increases. We conclude that short-time temperature measurements would serve as an adequate means of validating computer predictions of internal field distributions even when there are resonance effects which cause the power density profile (e.g., Figure 3.5.1) to have many relative maximums and minimums; this particular assertion is valid for continuous-wave exposure, but is not established for a general pulse exposure scenario. The thermal response for 5-s and 1-min exposures is shown in Figures 3.5.2 and 3.5.3. The electromagnetic field strength for the results portrayed in Figures 3.5.1-3.5.3 was 194.09 V/m or 10 mW/cm^2.

Table 3.5.1 defines the parameters used in making the computer runs.

TABLE 3.5.1. PARAMETERS FOR ONE-LAYER MUSCLE EQUIVALENT SPHERE EXPOSED TO 4500-MHZ RADIATION

ELECTRICAL PROPERTIES

Tissue type	Radius of bounding sphere (cm)	Relative permittivity	Conductivity (mhos/meter)
Muscle	3.3	48.25	2.75

THERMAL PROPERTIES
(centimeter-gram-second units)

Tissue type	Thermal conductivity	Density	Specific heat	Blood flow cooling
Muscle	.00126	1.050	.883	.00186

3.5.3. Microwave Heating of a Simulated Fetal Structure. To estimate the potential hazard of a source of microwave radiation, we have made a simulated fetal structure comprised of three tissue regions delimited by concentric spheres. The parameters used in the computer runs are given in Table 3.5.2. The field strength used in the runs was 194.09 v/m, which is equivalent to 10 mW/cm^2.

TABLE 3.5.2. PARAMETERS DEFINING A SIMULATED FETAL STRUCTURE EXPOSED TO 1000-MHz RADIATION

ELECTRICAL PROPERTIES

Tissue type	Radius of bounding sphere (cm)	Relative permittivity	Conductivity (mhos/meter)
Fetal	1.6	50.5	1.65
Amniotic fluid	2.8	72.0	2.00
Maternal	3.3	50.5	1.65

THERMAL PROPERTIES
(centimeter-gram-second units)

Tissue type	Thermal conductivity	Density	Specific heat	Blood flow cooling
Fetal	.00126	1.050	.883	.00186
Amniotic fluid	.00124	1.007	.998	.00000
Maternal	.00126	1.050	.883	.00186

We get some qualitative information (e.g., the shielding effect of the amniotic fluid) about the vulnerability of the fetus to microwave exposure from the computer results for the simple model given in this paper. Since Edwards [4] suggests that thermal pulses can affect cell cycles and that there are teratogenic effects associated with elevated fetal temperatures, it would probably be valuable to carry out this analysis for a whole-body model and use (1) the smallest fetal temperature known to cause abnormal fetal development and (2) the computer model for predicting fetal temperature excursions as a definitive way of stating that a particular source of microwave radiation is a potential health hazard.

Figure 3.5.4 shows the power density across a simulated fetal structure when the exposure was carried out in the manner described in Figure 2.1. Figures 3.5.5-3.5.7 show the thermal response of the simulated fetal structure after a 1-hr exposure, and Figures 3.5.8-3.5.15 show how the temperature distribution across the structure changes with time when there is no removal of heat by an autothermal regulatory process. While our autothermal regulatory process model is based on actual physiological parameters relating to blood flow, it can at best be considered phenomenological since we have not modeled the details of the flow of blood through vessels in the tissue and have in essence only added a dissipative term to the heat equation. However, Figures 3.1.5-3.5.7 suggest that there is in the blood flow case a net heating of the amniotic fluid due to the absence of autothermal regulation.

3.5.4. Microwave Heating of a Simulated Cranial Structure. In this section we study the response of a simulated cranial structure to microwave radiation. The manner in which the structure is exposed is described in Figure 2.1. The power density in the simulated cranial structure is shown in Figure 3.5.16. The observed thermal response after 30-s and 1-min exposures to 800-MHz radiation is described in Figures 3.5.17 and 3.5.18.

The parameters used in carrying out these computations are described in Table 3.5.3.

TABLE 3.5.3. PARAMETERS DEFINING A SIX-LAYER SIMULATED CRANIAL STRUCTURE EXPOSED TO 800-MHz RADIATION

ELECTRICAL PROPERTIES

Tissue type	Radius of bounding sphere (cm)	Relative permittivity	Conductivity (mhos/meter)
Brain	2.68	33.76	0.960
CSF	2.88	79.47	1.740
Dura	2.93	45.64	1.230
Fat	3.13	5.61	0.096
Bone	3.20	5.61	0.096
Skin	3.30	45.64	1.230

THERMAL PROPERTIES
(centimeter-gram-second units)

Tissue type	Thermal conductivity	Density	Specific heat	Blood flow cooling
Skin	.0012300	1.0000	.900	.001002242
Fat	.0003822	.8500	.600	.000000000
Bone	.0027780	1.5000	.380	.000000000
Dura	.001230	1.0000	.900	.000000000
CSF	.001240	1.0069	.998	4.498×10^{-6}
Brain	.001260	1.0500	.883	.00743742

We see from the results described in Figures 3.5.17 and 3.5.18 that there is fairly rapid smoothing of the temperature distributions. Indeed, we have assumed mathematically that both the temperature excursion u and $K(x,y,z)\mathrm{grad}(u)$, where $K = K(x,y,z)$ is the thermal conductivity, are continuous. Thus, since in our calculation K is constant, we see that u and its derivatives are continuous. The electrical properties of the six tissue types are given in [7]. The model is capable of predicting the microwave-induced temperature excursion when there is blood flow in some of the layers, and typical values of these blood flow parameters can be obtained from [6].

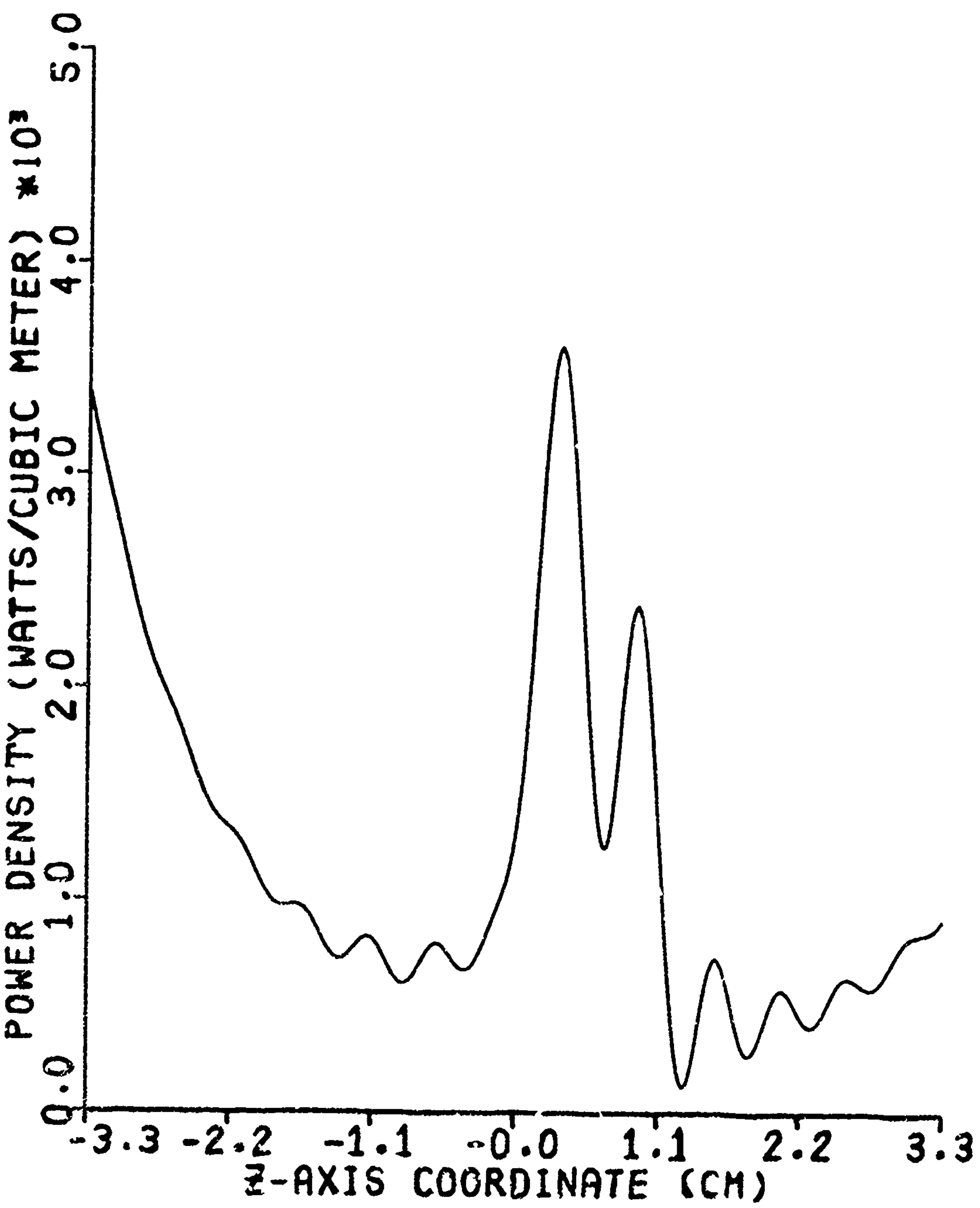

Figure 3.5.1. Power density induced in a muscle-equivalent sphere by 4.5-GHz continuous-wave radiation with a power of 10 mW/cm^2.

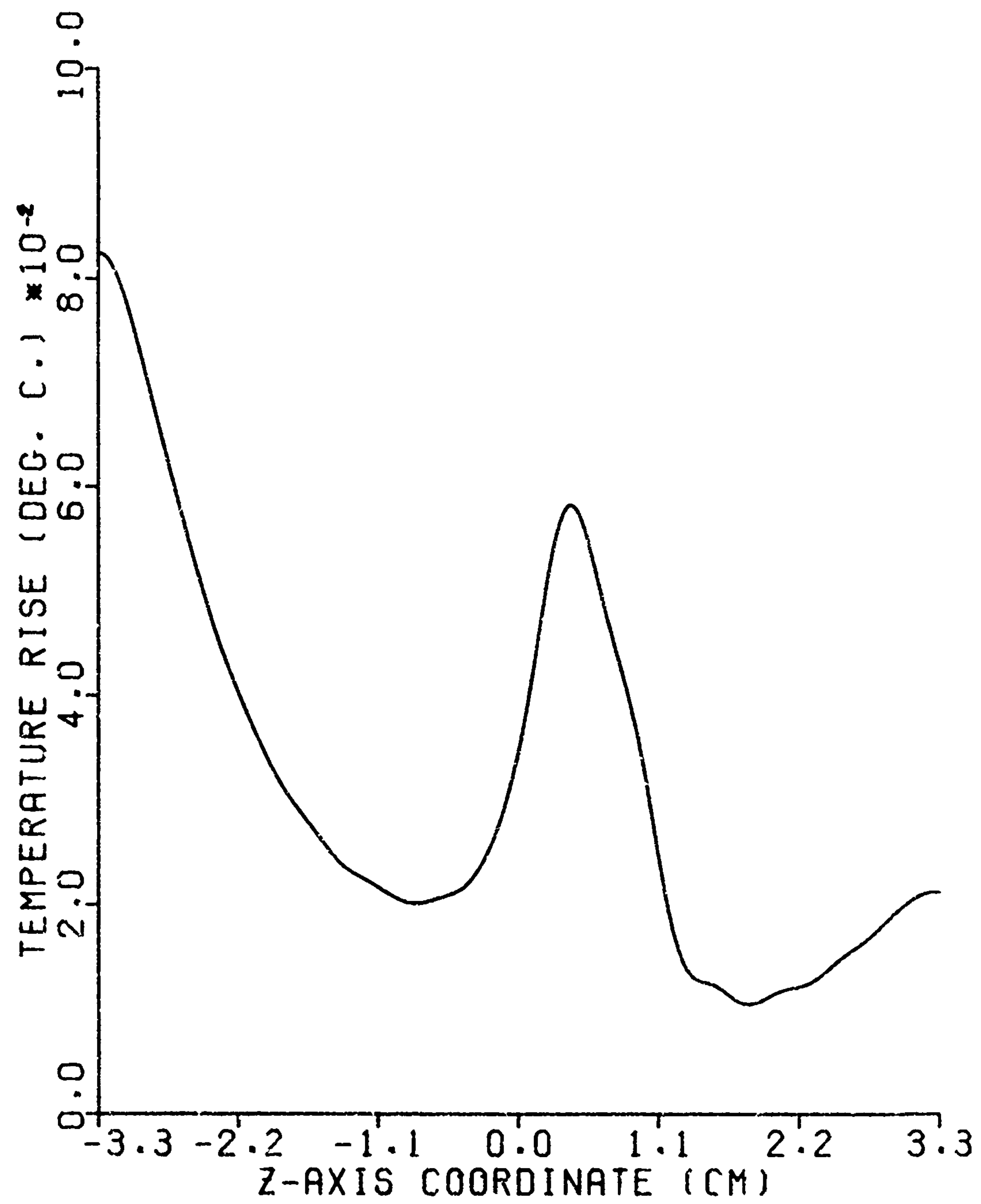

Figure 3.5.2. Thermal response of a muscle-equivalent sphere to a 1-min exposure to 4.5-GHz continuous-wave radiation with a power of 10 mW/cm^2. Parameters defining the problem are given in Table 3.5.1.

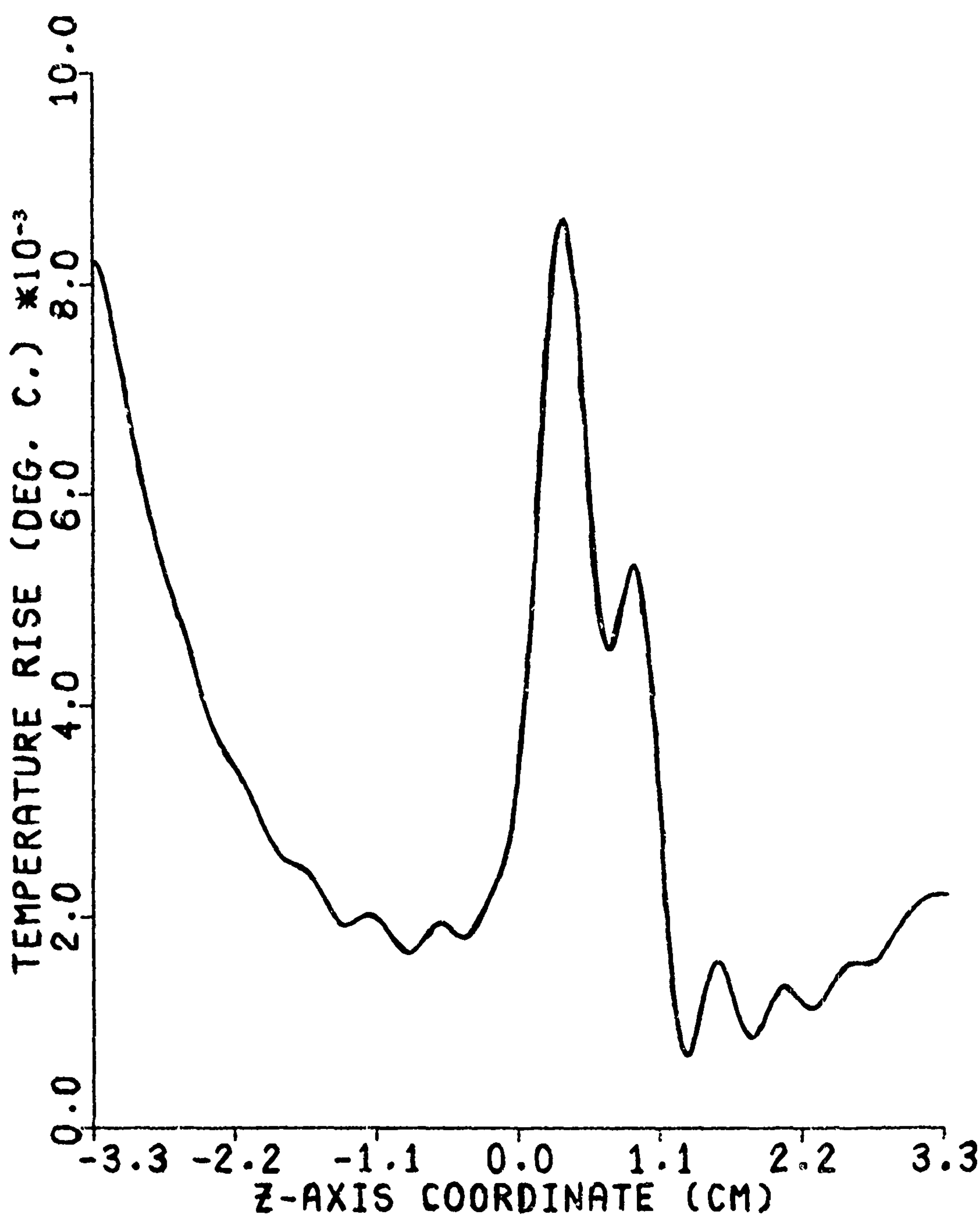

Figure 3.5.3. Thermal response of a muscle-equivalent sphere to a 5-s exposure of 4.5-GHz continuous-wave radiation with a power of 10 mW/cm^2. Parameters defining the problem are given in Table 3.5.1.

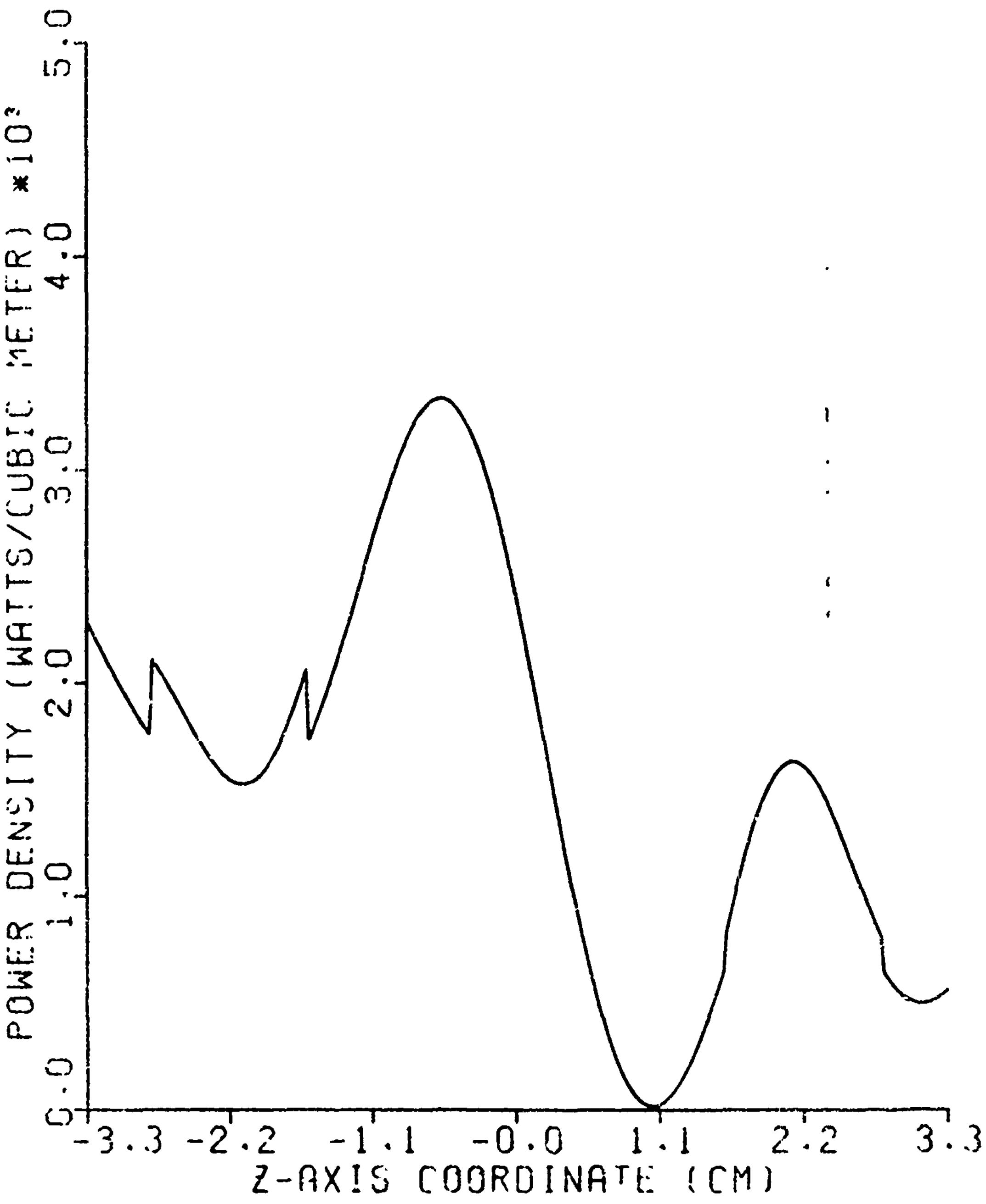

Figure 3.5.4. Power density across the z-axis of a simulated fetal structure exposed to 1-GHz continuous-wave microwave radiation with a power of 10 mW/cm^2. Parameters defining the problem are given in Table 3.5.2.

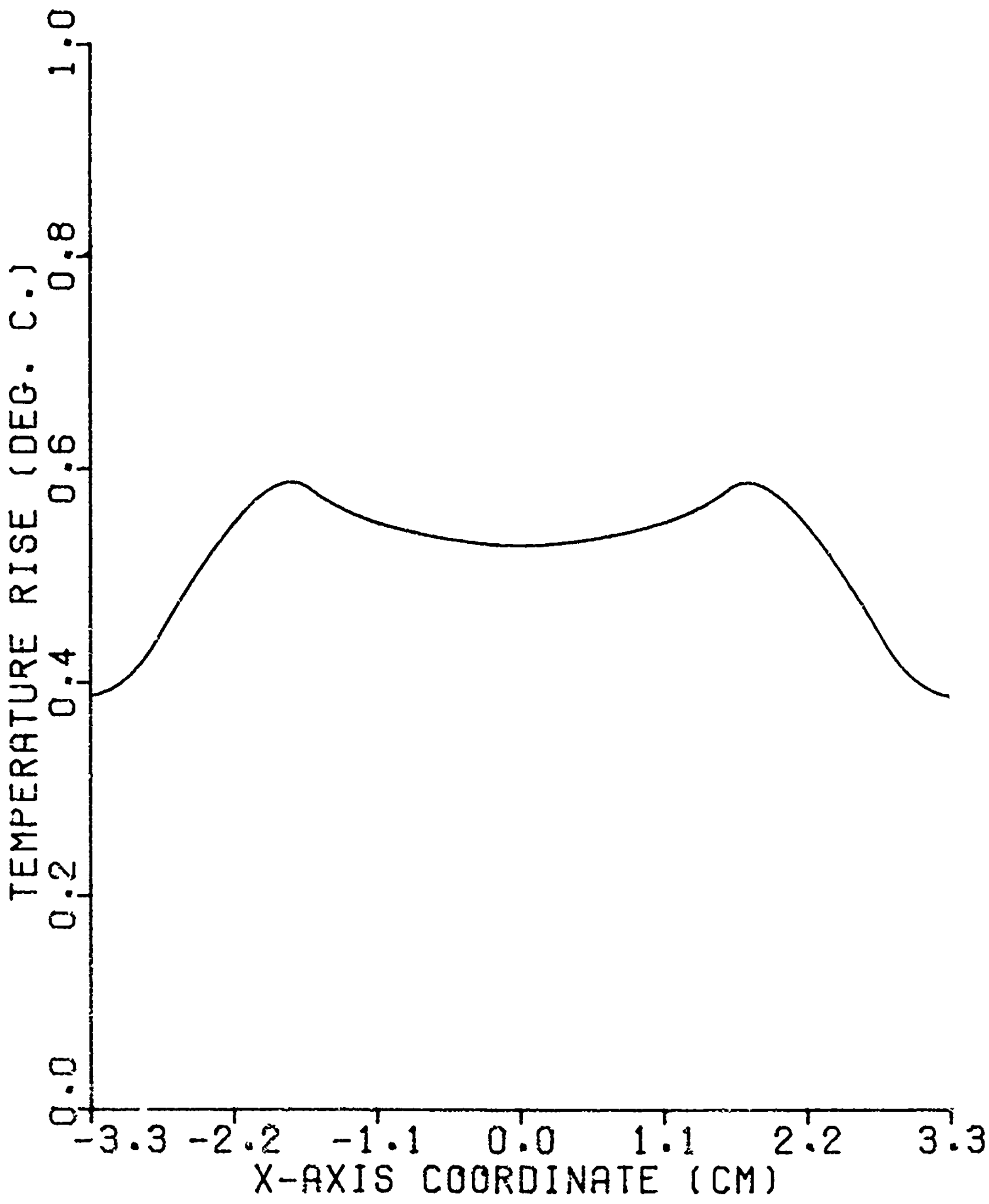

Figure 3.5.5. Thermal response of a simulated fetal structure to a 1-hr exposure to 1-GHz radiation with a power of 10 mW/cm^2. The temperature is computed across the x-axis. The orientation of the axes is given in Figure 2.1. The parameters defining the problem are given in Table 3.5.2.

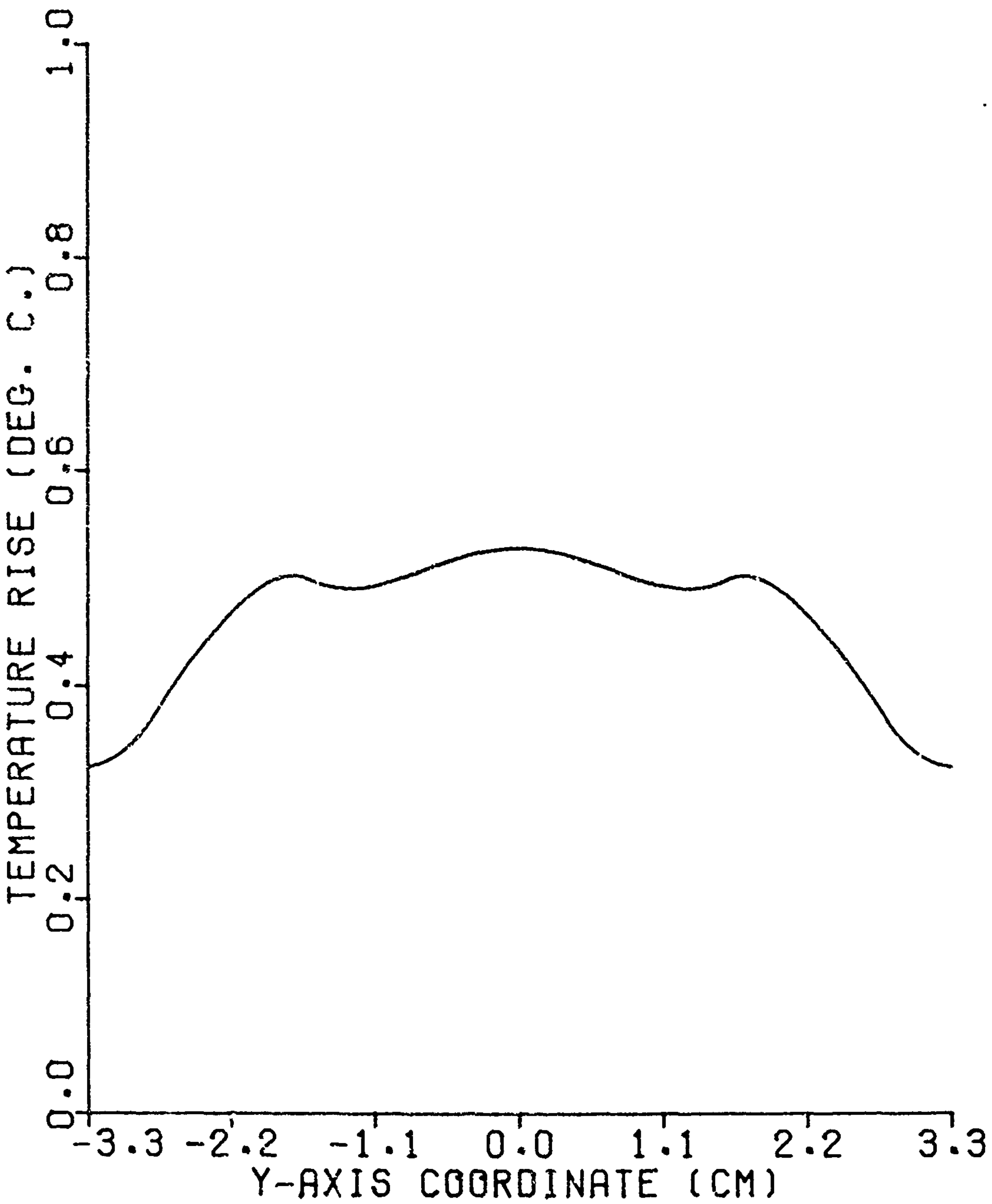

Figure 3.5.6. This is the same as Figure 3.5.5 except that the temperature is computed along the y-axis.

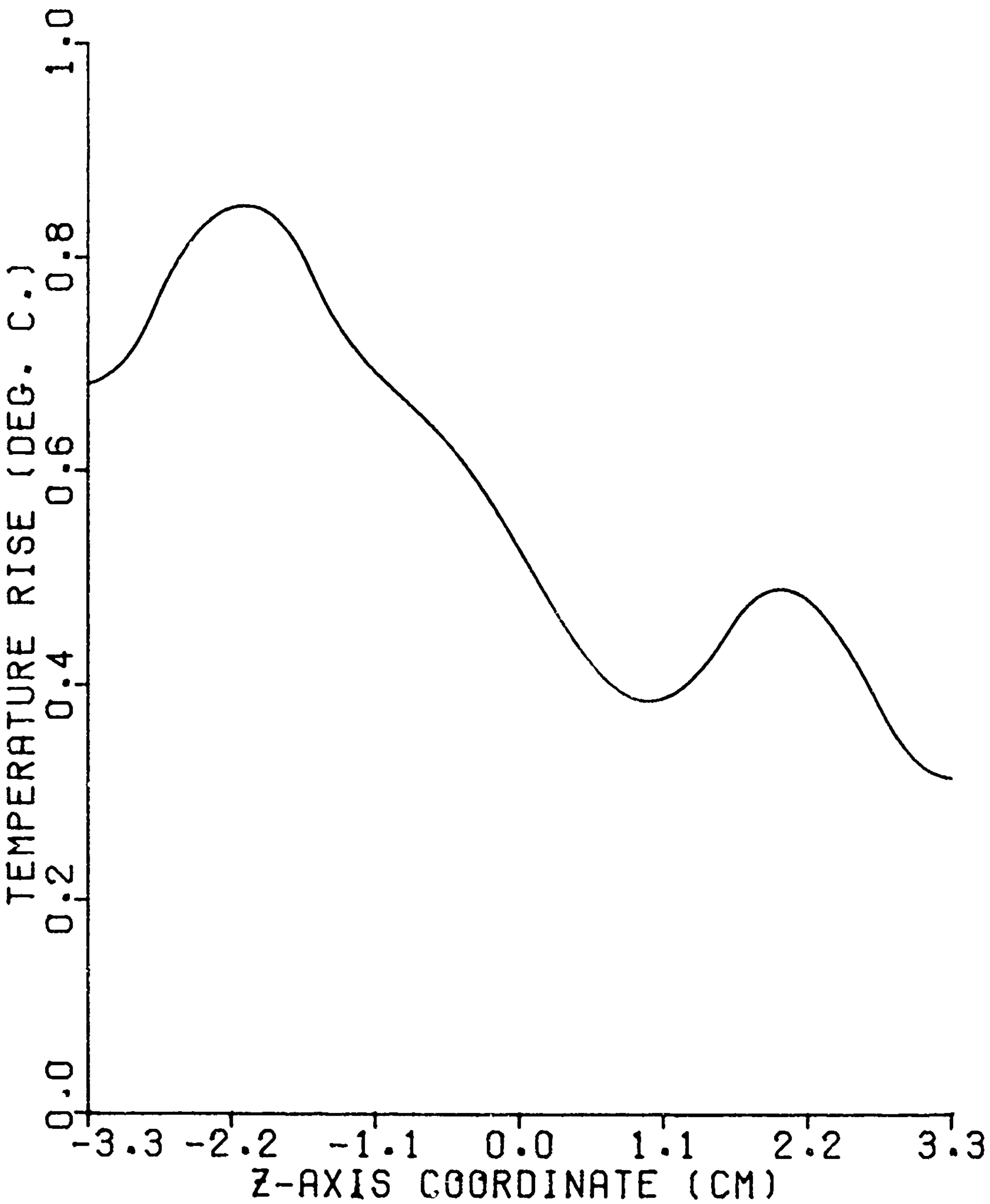

Figure 3.5.7. This is the same as Figure 3.5.5 except that the temperature is computed along the z-axis.

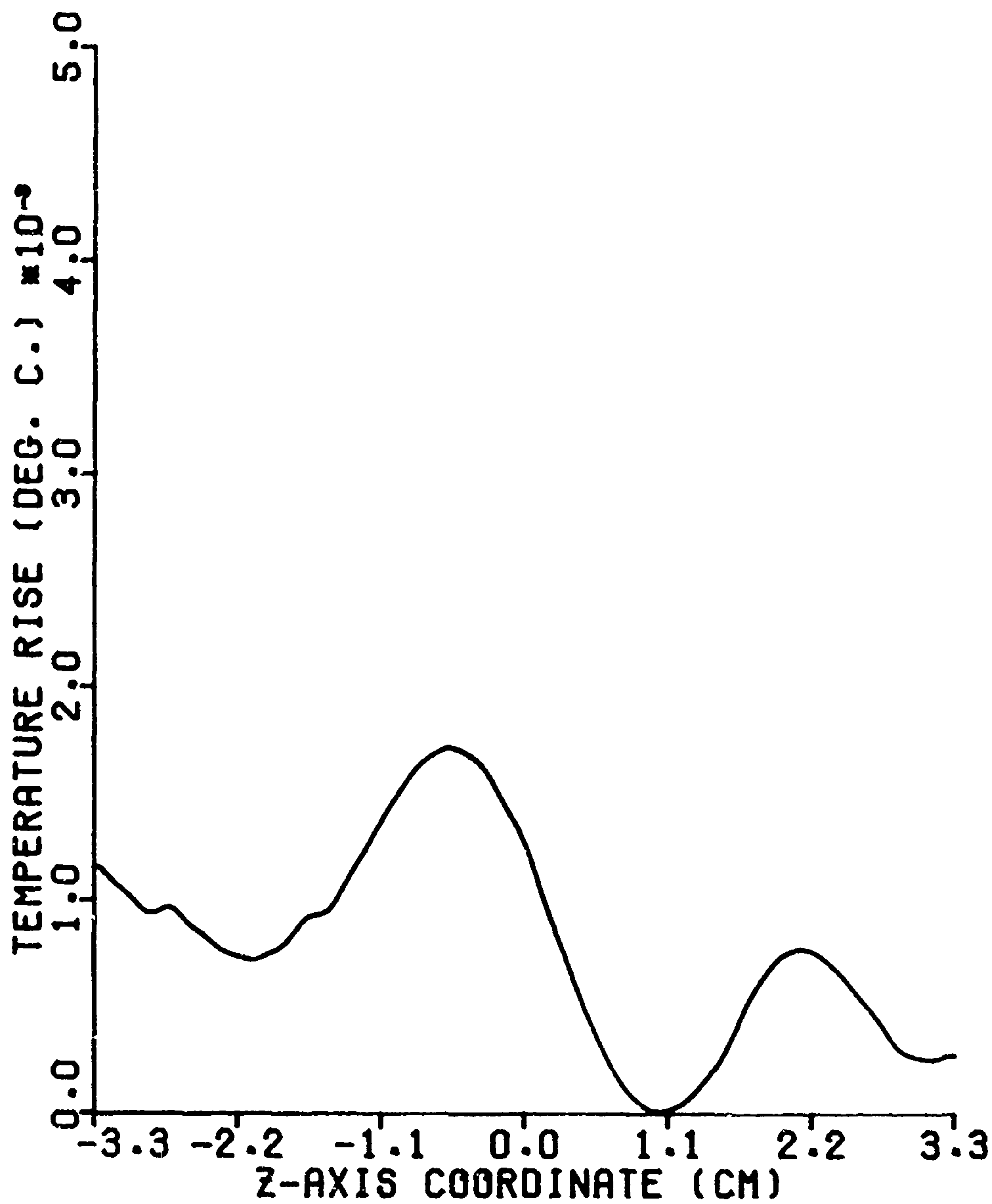

Figure 3.5.8. Temperature distribution along the z-axis for a simulated fetal structure exposed to 1-GHz (10 mW/cm^2) radiation for 1 s. The parameters defining the problem are given in Table 3.5.2, except that all blood flow terms are set to zero.

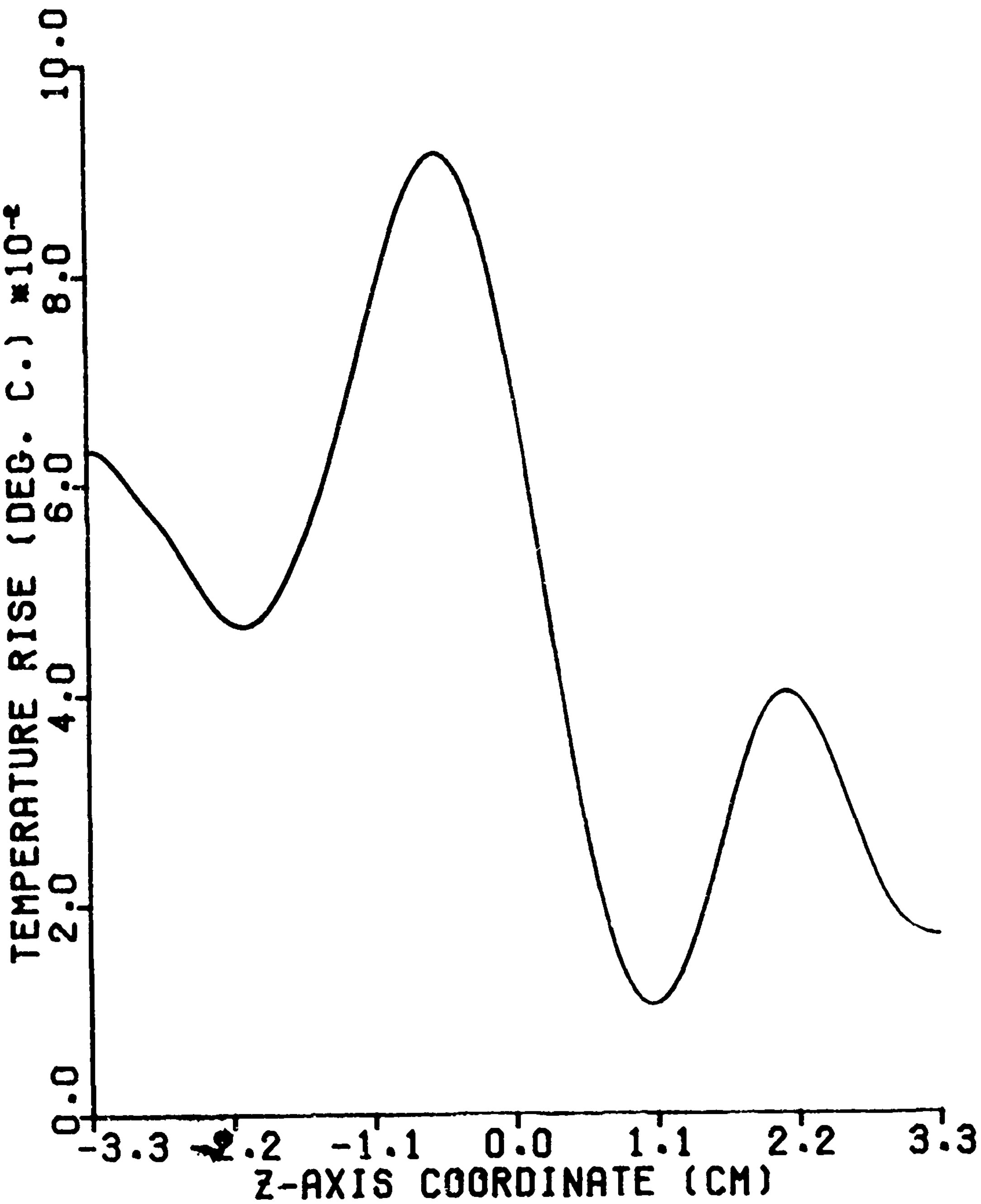

Figure 3.5.9. Temperature distribution along the z-axis of a simulated fetal structure exposed to 1-GHz (10 mW/cm^2) radiation for 1 min. The parameters defining the problem are given in Table 3.5.2, except that all blood flow terms are set to zero.

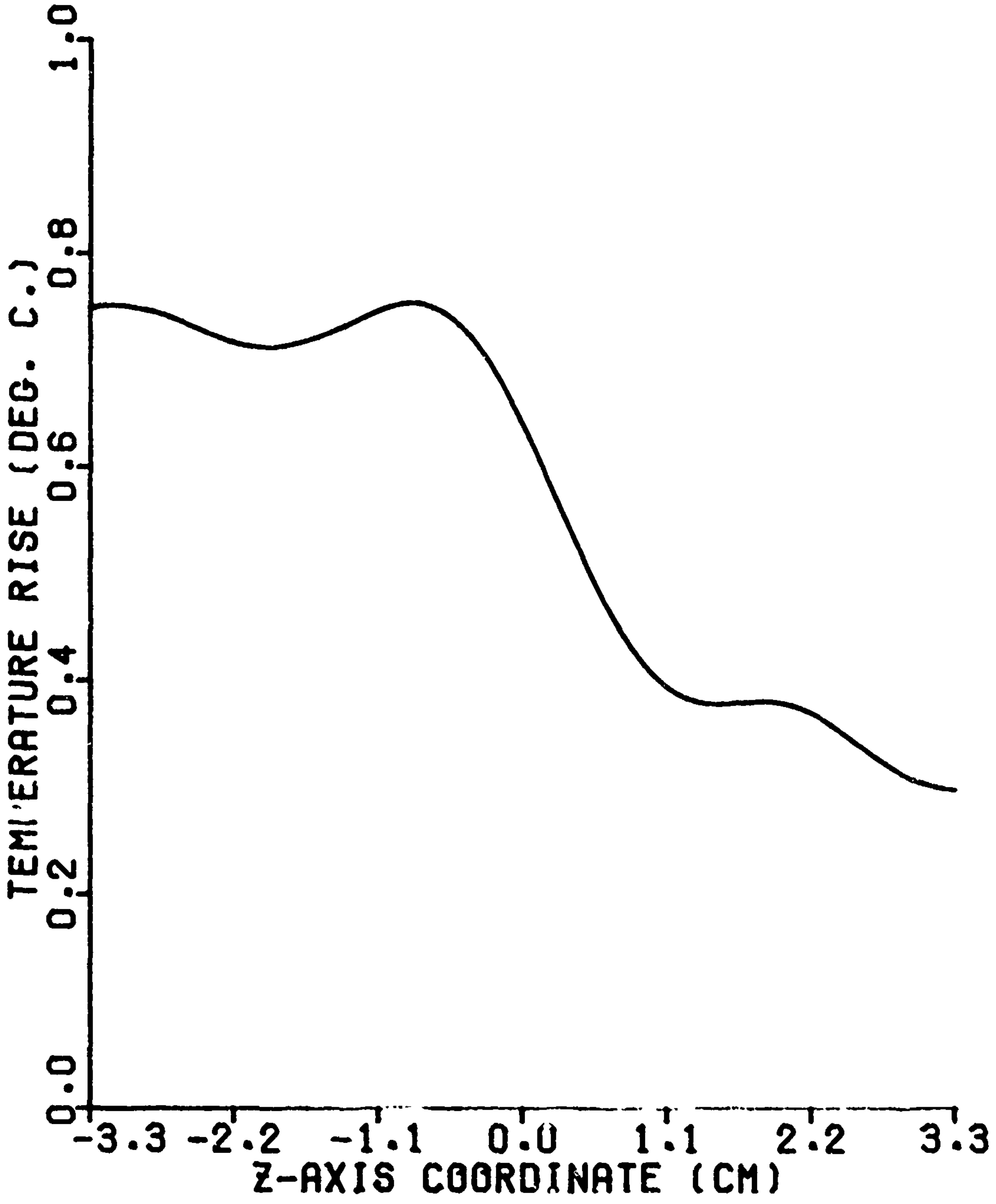

Figure 3.5.10. Temperature rise along the z-axis of a simulated fetal structure exposed to 1-GHz (10 mW/cm^2) radiation for 15 min. The parameters defining the problem are given in Table 3.5.2, except that all blood flow terms are set to zero.

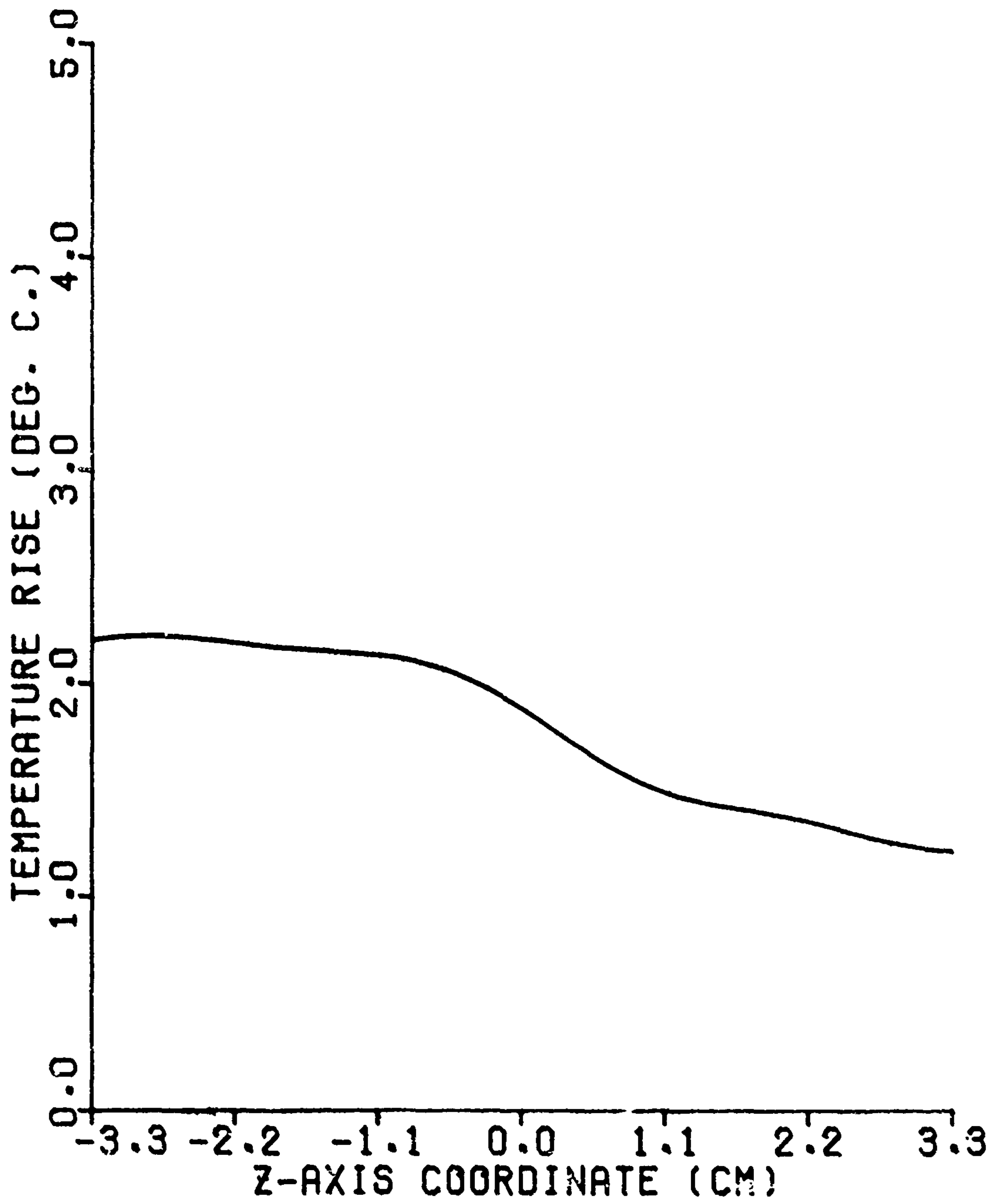

Figure 3.5.11. Temperature rise along the z-axis of a simulated fetal structure exposed to 1-GHz (10 mW/cm^2) radiation for 1 hr. The parameters defining this problem are given in Table 3.5.2, except that all blood flow terms are set to zero.

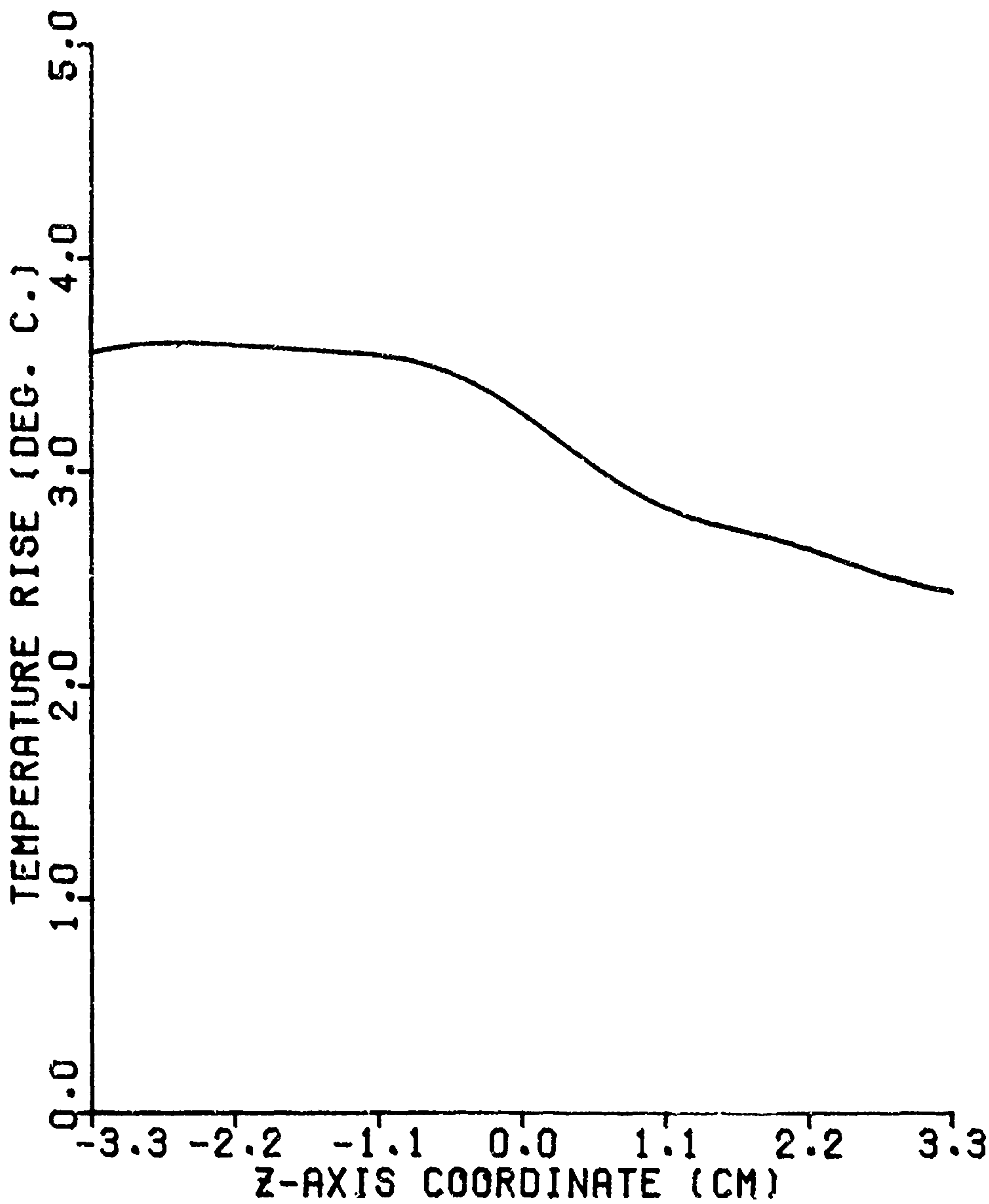

Figure 3.5.12. Temperature rise along the z-axis of a simulated fetal structure exposed to 1-GHz (10 mW/cm^2) radiation for 2 hr. The parameters defining this problem are given in Table 3.5.2, except that all blood flow terms are set to zero.

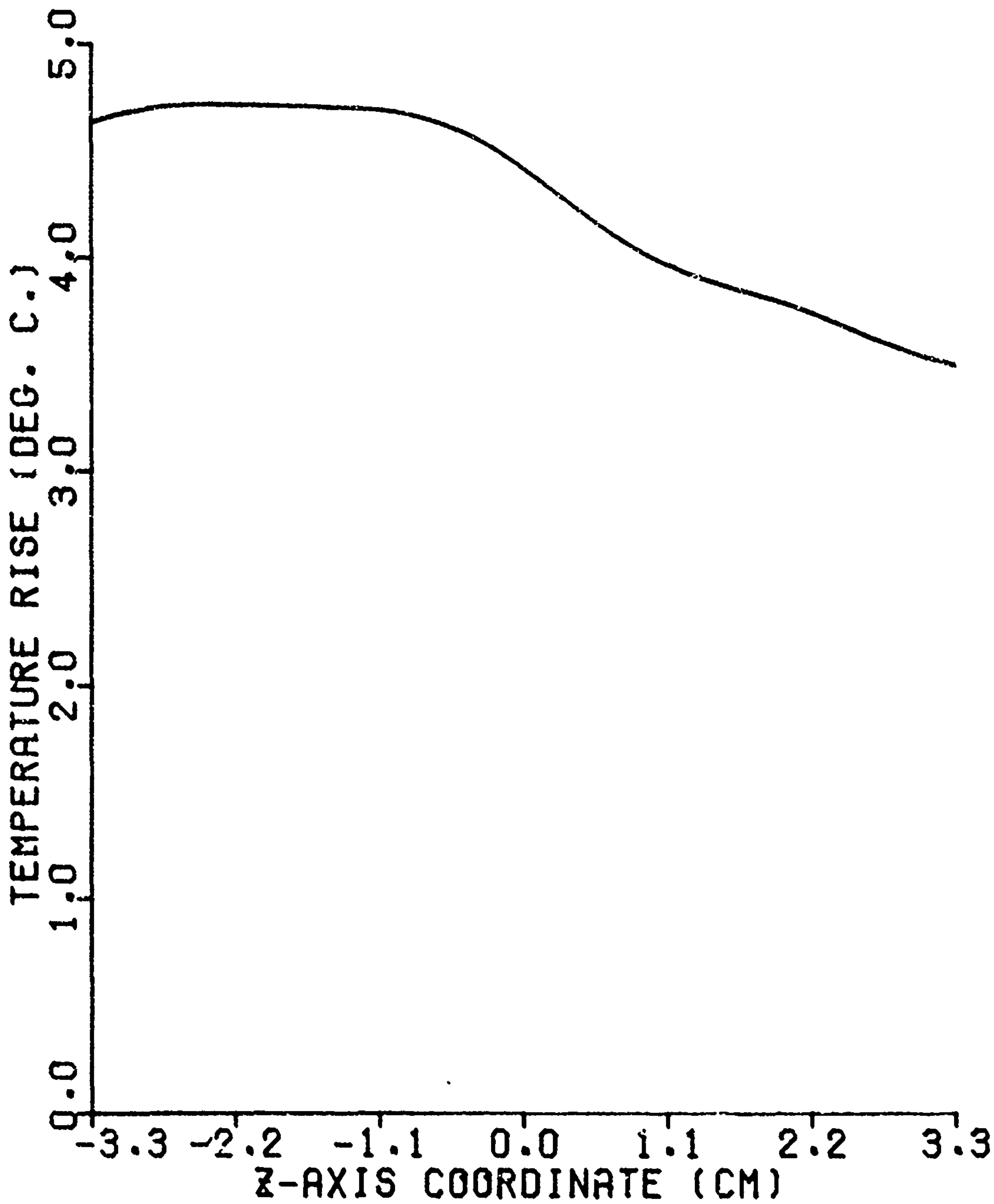

Figure 3.5.13. Temperature rise along the z-axis of a simulated fetal structure exposed to 1-GHz (10 mW/cm^2) radiation for 3 hr. The parameters defining this problem are given in Table 3.5.2, except that all blood flow terms are set to zero.

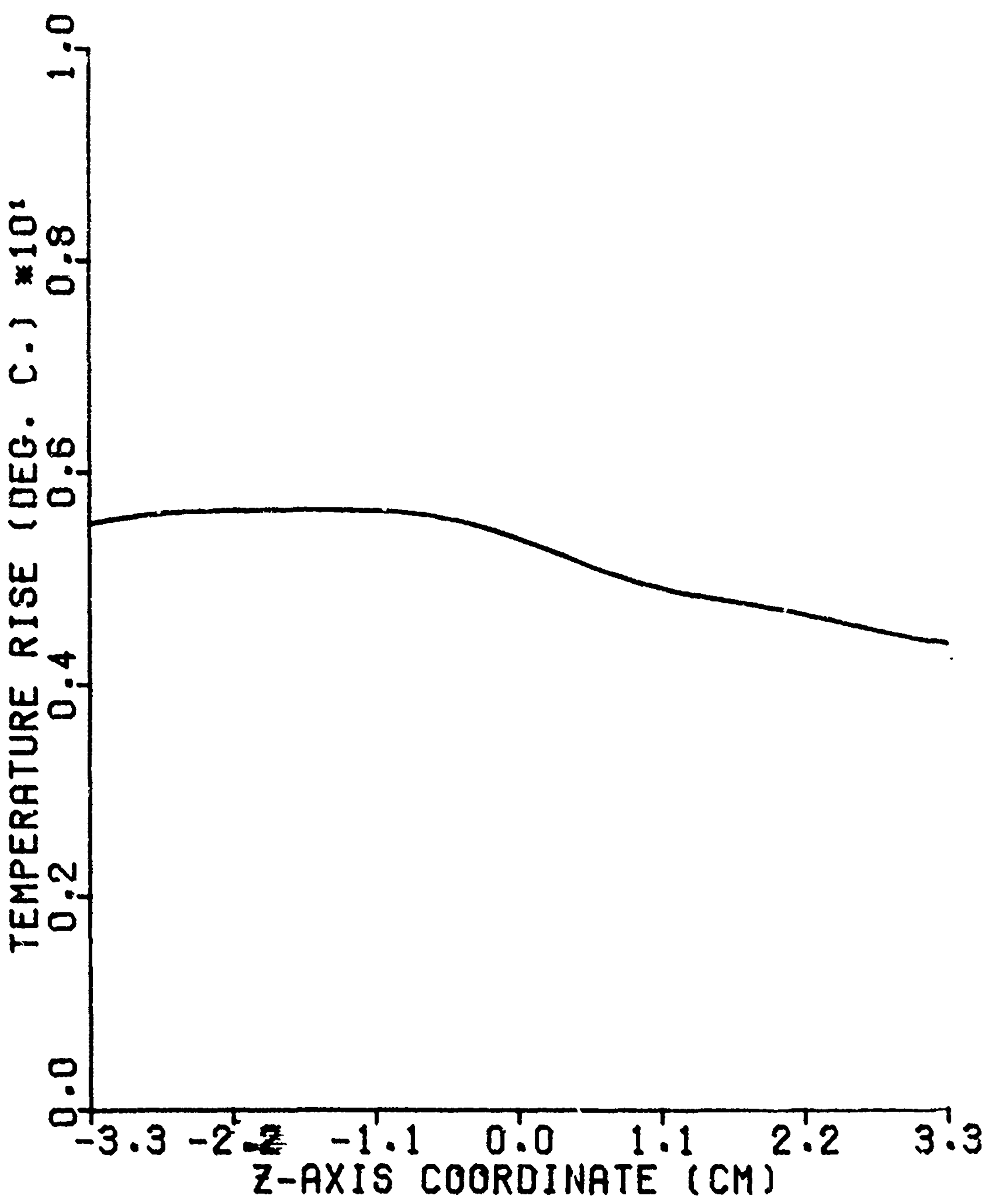

Figure 3.5.14. Temperature rise along the z-axis of a simulated fetal structure exposed to 1-GHz (10 mW/cm^2) radiation for 4 hr. The parameters defining this problem are given in Table 3.5.2, except that all blood flow terms are set to zero.

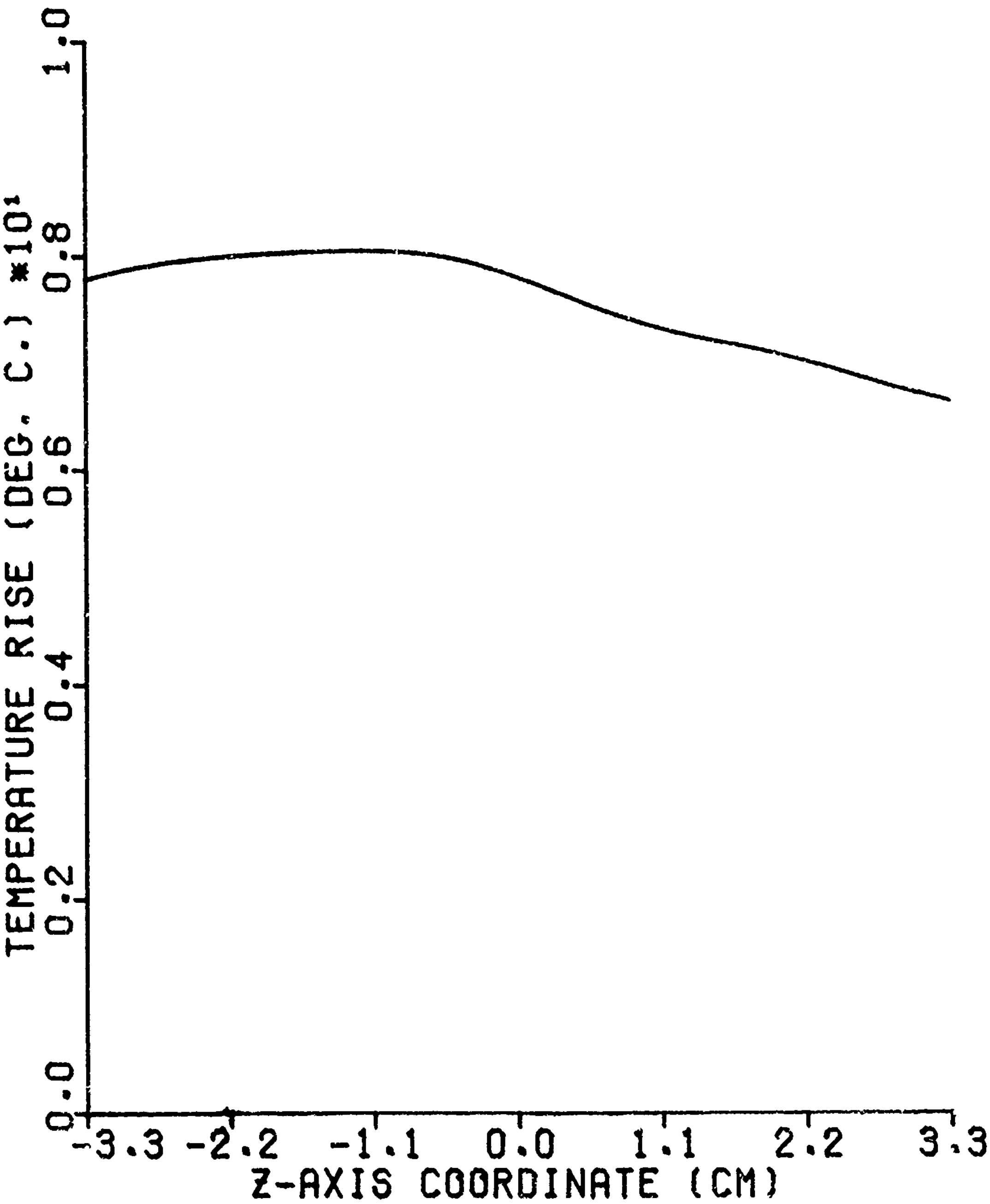

Figure 3.5.15. Temperature rise along the z-axis of a simulated fetal structure exposed to 1-GHz (10 mW/cm^2) radiation for 8 hr. The parameters defining this problem are given in Table 3.5.2, except that all blood flow terms are set to zero.

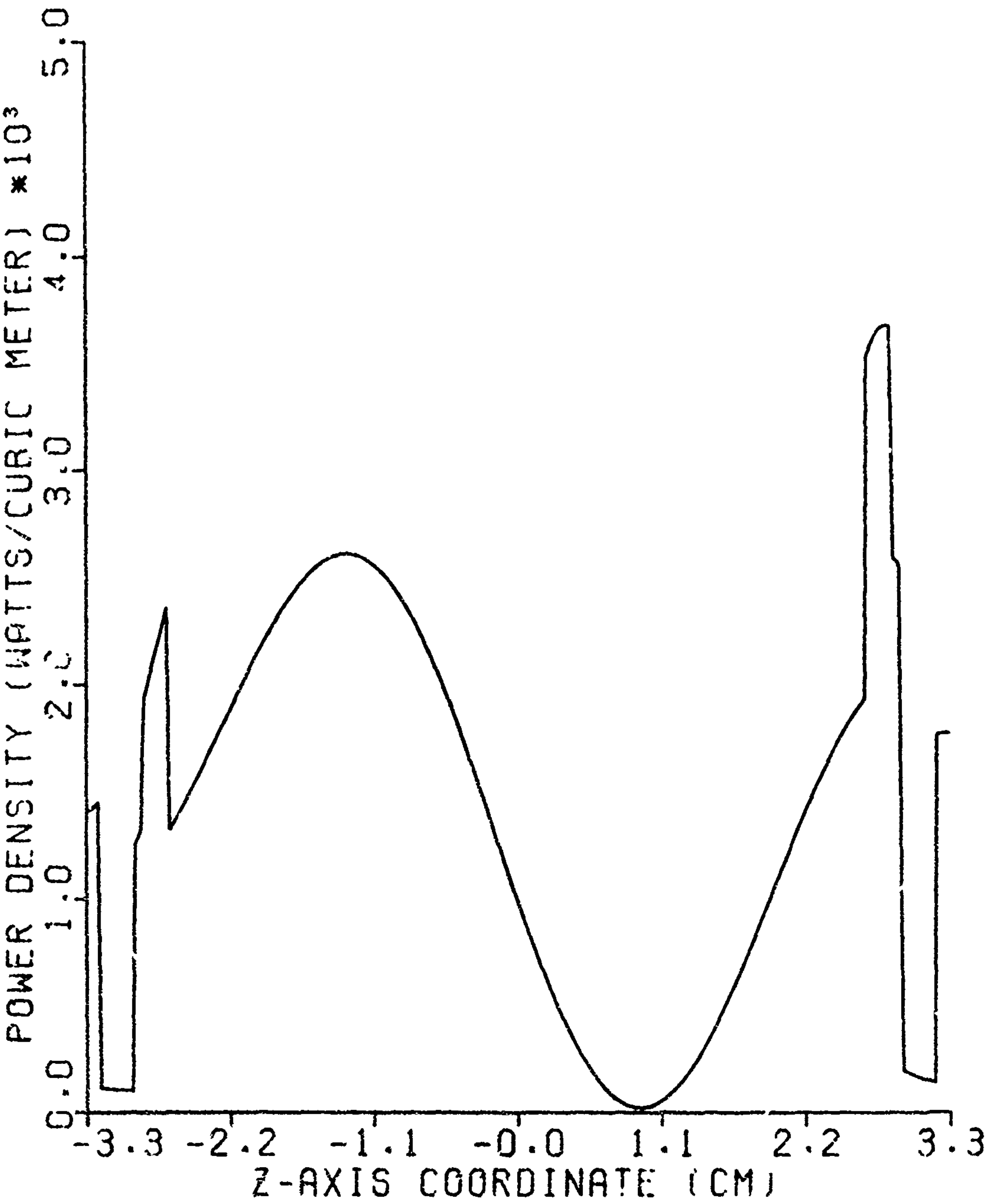

Figure 3.5.16. Power density along the z-axis of a six-layer simulated cranial structure exposed to 800-MHz radiation with a power of 10 mW/cm^2. The parameters defining this problem are given in Table 3.5.3.

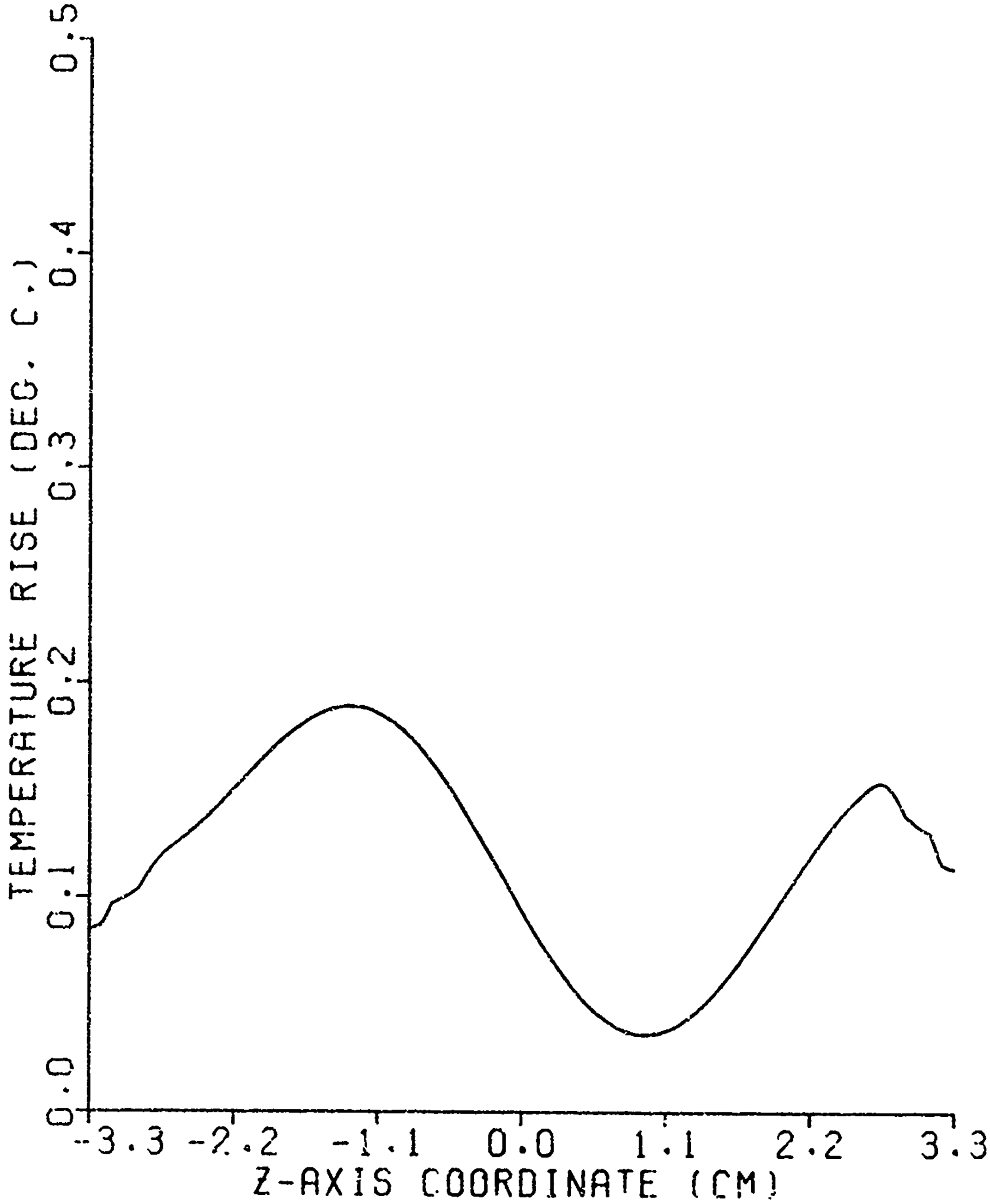

Figure 3.5.17. Thermal response of a six-layer simulated cranial structure exposed to 800-MHz radiation for 3 min. The parameters defining this problem are given in Table 3.5.3.

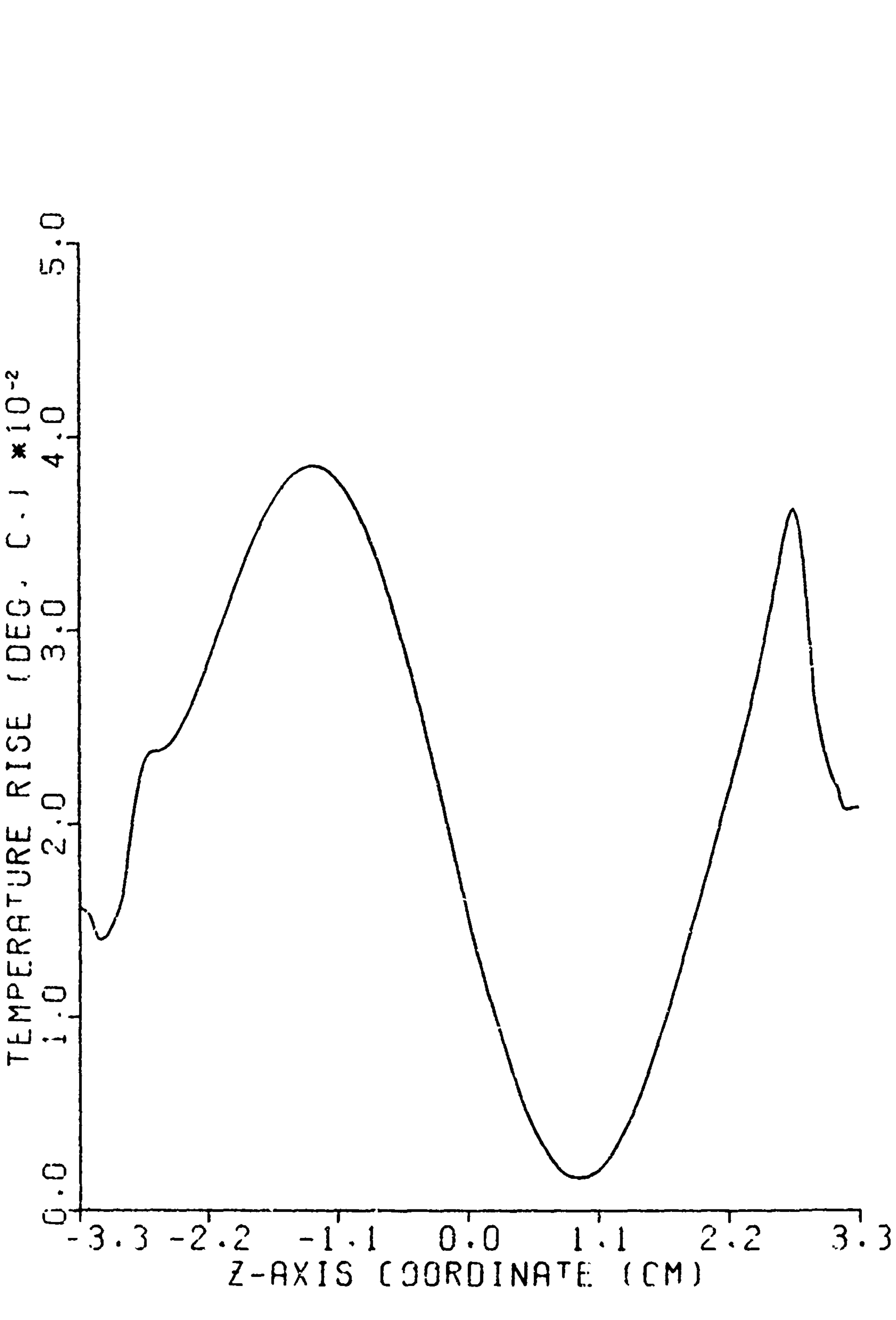

Figure 3.5.18. Thermal response of a six-layer simulated cranial structure exposed to 800-MHz radiation for 30 s. The parameters defining this problem are given in Table 3.5.3.

4. PROGRAM DESCRIPTION

4.1. Purpose of the Program

The computer program described in this report will predict the thermal response of an autothermally regulated, spherically symmetric, dielectric body with a finite microwave conductivity to a time-harmonic source of microwave radiation. The calculation can be carried out at any point in the interior of this body at any positive time. We also allow the source to be pulsed in the sense that the source of time harmonic radiation may be turned on and off in a complex manner such as that described in Figure 3.4.1. The scattering body consists of from one to six homogeneous regions bounded on the outside by a sphere of finite but positive radius; a description of a six-layer body is shown in Figure 3.5.1. The radiation source is given by an amplitude E_0 in volts per meter and a frequency FREQ in megahertz. The calculation is carried out by the evaluation of an analytical expression involving an infinite linear combination of spherical harmonics. The coefficients in this infinite linear combination are functions of time computed from a set of eigenvalues and a knowledge of the manner in which the microwave source has been turned on and off. We also permit a nonzero heat removal term BFRP in one or more of the layers.

4.2. Accessing the Program from the Library

The Job Control Language needed to access the program from the library is shown in Figure 4.2.1. The data deck, whose preparation will be explained in Section 4.2, goes between the card marked

//GO.SYSIN DD *

and the card

/*

The space requirement specified by the expression

REGION.GO=252K

will not change but if one desires temperature information at more points one needs to increase the running time parameter,

TIME.GO=4

and the effective time parameter,

EFFTIME=10

which represents elapsed time in our timesharing computing system. The parameter values listed in Figure 4.2.1 were adequate to determine the temperature excursions for a single fixed time at 13 different locations in space.

```
//HBR16TKS JOB(3H01,B020,,,,,,,00),'HBM379@@10R COHOON',CLASS=C,
// MSGLEVEL=(2,0)
/*JOBPARM RESTART,SINGLE, EFFTIME=10,LINES=1,CARDS=0
//STEP1 EXEC PLOTGO,PROGRAM=HBR16TRF,REGION.GO=252K,TIME.GO=4
//GO.SYSIN DD *
       *** DATA CARDS GO HERE ***
/*
```

Figure 4.2.1. Job control language for calling the microwave thermal response program from the library.

4.3. Glossary of Variables and Their Meaning

All FORTRAN variables used for input and output and important internal FORTRAN variables are listed here in alphabetical order. We, with each variable, give an explanation of its meaning. These variables are:

ALPNP(NNN) = the ALPHA SUB (L,P) coefficient used in formula (3.2.2) to expand the electromagnetic field with
NNN = (NREG-1)*NMIN+NN

and

ALPNP(NNN) = ALPHA SUB (NN,NREG) where NN is the spherical Bessel function order and NREG is the layer number.

ANP = the A SUB (L,P) coefficient used in expanding the electric field and which appears in formula (3.2.2) which is stored in a single array with ANP((NREG-1)*NMIN+NN) corresponding to the coefficient A SUB (NN,NREG) where NN indexes the spherical Bessel functions used in the expansion.

BETNP(NNN) = the BETA SUB (L,P) coefficient used in formula (3.2.2) to expand the electromagnetic field with
NNN = (NREG-1)*NMIN+NN

and

BETNP(NNN) = BETA SUB (NN,NREG)
where NN is the spherical Bessel function order and NREG is the layer number.

BFRP(I) = the blood flow radial perviousness term or the number of grams of blood per gram of tissue per second, with a typical value for brain tissue being .0122

BNP = the B SUB (L,P) coefficient used in formula (3.2.2) to expand the electromagnetic field and which is stored in a single array with BNP((NREG-1)*NMIN+NN) corresponding to the coefficient B SUB (NN,NREG) where NN indexes the spherical Bessel functions used in the expansion.

BP(I) = the product of BFRP(I), the number of grams of blood per gram of tissue per second, CBP = .98 = the specific heat of blood in calories per gram degree Centigrade, and the density RHOBP = 1.06 = the number of grams of tissue per cubic centimeter of tissue.

CALL BJYH(BJNP,BHNP,Q,NC,STOPR,MAX) = a call to a subroutine which determines the values of spherical Bessel functions of the first kind BJNP and spherical Hankel functions BHNP at the complex argument Q. We attempt to generate up to MAX such functions as we are limited by STOPR and we end up putting only NC such functions in the array.

CALL COEF = a call to a subroutine which produces the expansion coefficients
A SUB(L,P), ALPHA SUB(L,P),
B SUB(L,P) and BETA SUB(L,P)
used in expanding the electromagnetic field using equation (3.2.2)

CALL DRTMI(X,F,FNCAL,SL,SR,W,V,E,NITR) = the call to the bisection routine DRTMI which returns a value X such that FNCAL(X) = F =0 to within an accuracy of E with less than NITR iterations where FNCAL(SL)=W, FNCAL(SR)=V and W and V are on opposite sides of 0 on the real line.

CALL EPROP(FREQ,ITIS(I),EP,SIG) = a call to a subroutine which determines the relative permittivity EP and microwave conductivity SIG of tissue type ITIS(I) at the microwave frequency FREQ.

CALL PL = a call to a subroutine which computes the array P of associated Legendre polynomials of the first kind and order 1 and an array DP of their derivatives.

CALL TERM(NCK,T,KEY) = a call to a subroutine which computes the I**L multiplied by T appearing in formula (3.2.2) based on its preceding value, where the value of NCK ranges from 1 to 4 since I**1, I**2, I**3, I**4 ranges over all possible values of the square root of (-1) raised to a power, and where KEY takes on the value 1 or 0 depending on where in the process of summing the series we are computing I**L multiplied by the complex term T appearing in equation (3.2.2).

CP(I) = the tissue specific heat in calories per gram per degree centigrade.

DEN(NSBF,M2,NRT) = the integral of the square of the radial eigenfunction multiplied by the square of the radial coordinate, the density, and the specific heat from zero to the outer radius of the scatterer.

EO = the strength of the incident electric field vector which may be read in a certain number of milliwatts per square centimeter if IEO = 1 but must be expressed in volts per meter if IEO = 0, where we understand that if IEO = 1, then EO will be converted internally into volts per meter.

EPHI = the phi component of the electric field vector when the electric field is expressed in spherical coordinates and which consequently represents a tangential field component when the point at which the field is being computed is on a sphere defining a boundary of the body being heated by microwaves.

EPS = the relative error associated with the expansion of the electromagnetic field.

EPSP(I) = the relative dielectric constant of the Ith tissue layer at the frequency FREQ of the incoming radiation.

ERAD = the radial component of the electric vector in volts per meter where we assume that we have expressed the field vectors in the spherical coordinate system and which consequently represents the component of electric vector that is perpendicular to a boundary layer when the point at which the electric field is being computed is on a sphere defining a boundary of the body being heated.

ETHETA = the theta component of the electric field vector when the electric field is expressed in spherical coordinates and which consequently represents a tangential field component when the point at which the field is being computed is on a sphere defining a boundary of the body being heated by microwaves.

ETIME(NRT) = the time profile function, defined by dividing the right side of equation (3.4.7) by b-SUB-BAR-SUB-K-SUB-(M,N) or $b_k^{(m,n)}$, which describes the radar pulse emission patterns.

F = the factor in front of the integral on the right side of equation (3.3.11) in general equal to (2n+1)((n-m)!)/((n+m)!) multiplied by the factor in front of the integrals on the right side of equation (3.3.9) or (3.3.10) whichever is appropriate.

FKP(NREG) = the complex electromagnetic propagation constant associated with layer NREG which is defined by equation (3.2.7).

FREQ = the frequency of the incoming radiation in megahertz or millions of cycles per second.

FUNCTION ALP(N,M,X) = a function subroutine computing the associated Legendre function of the first kind of degree N and order M at the point X with the restriction that N and M are nonnegative integers and M does not exceed N.

FUNCTION FNCAL(EIGV) = a function subroutine whose output is the value of the Newton cooling function defined by equation (3.3.42) when LAMDA = EIGV.

IEO = a parameter for determining the way the input data EO is interpreted with IEO = 0 meaning that EO is a certain number of volts per meter and IEO = 1 meaning that EO is a certain number of milliwatts per square centimeter.

II = in the last print statement an index describing the number of the data card containing the point at which the temperature is being computed with II = 1 for the point on the first card and II = NOCR for the point on the last card.

ISAR = a parameter determining the way that the output data is expressed with ISAR = 0 if the predicted power density that is printed next to the predicted temperature is to be expressed in milliwatts per kilogram, and ISAR = 1 if it is to be expressed in watts per cubic meter.

ITIS(I) = the tissue type of the Ith tissue layer equal to 1,2,3,4,5,6, or 7 if the tissue type is (i) cerebrospinal fluid, (ii) blood, (iii) muscle, (iv) skin or dura, (v) brain, (vi) fat or bone, or (vii) yellow bone marrow, respectively.

KMAX = the number of radial eigenfunctions associated with a given order of Bessel function with the greatest accuracy being achieved by setting KMAX equal to its maximum value of 25.

MP = the number of points to be used in the Gauss quadrature integration scheme that executes the radial transform defined by equation (3.3.22) with this number being one of 32, 48, 64, or 80 and with the larger numbers giving the more accurate results.

MP1 = the number of points used in the Gauss quadrature scheme that performs the Legendre transform defined by equation (3.3.1) with the number being 32 or 48 and where the latter number gives the most accurate results.

NC = the maximum number of Bessel functions available based on the value of the particular point at which the field is being computed and the microwave electrical properties of the layer in which the point is located.

NMAX = the number of orders of spherical Bessel functions that will be used to describe the radial variation of the microwave radiation-induced temperature excursion with the greatest accuracy being achieved by setting NMAX equal to its maximum value of 12.

NMIN = the number of expansion coefficients available based on the radii of the spheres bounding the tissue layers and the microwave electrical properties of the material in these layers.

NNN = (NREG - 1)*NMIN + NN, where NN denotes the spherical Bessel function order.

NREG = the number of the layer in which the point at which the temperature is being computed is found.

NOCR = the number of spatial points at which the input data set is to be computed, the maximum value of the index II of the output temperature data for a particular exposure time, and the number of cards in the fourth input data set.

NORG = the number of layers in the model where NORG is 1 if the scatterer is a homogeneous ball and where NORG equals its maximum value of 6 if the body in which the microwave-induced temperature is being predicted is a ball surrounded by five outer layers.

NPOINT(I) = the Ith entry of a 5-element array containing allowable numbers of points that may be used in a Gauss quadrature scheme for evaluating expansion coefficients.

NPUL = the number of pulses in a group, where for example NPUL = 3 if the radar emission pattern being modeled consists of 3 bursts of radiation followed by a quiet period, 3 bursts and a quiet period, et cetera.

NSBF = FORTRAN index equal to one plus the order of the Bessel function being considered in the computation of the microwave-induced temperature.

PAVG = the total absorbed power divided by the total volume of the region in which the temperature increase is being predicted expressed in watts per cubic meter.

PAVG1 = the total absorbed power divided by the total mass in kilograms of the body in which the temperature excursion is being predicted expressed in milliwatts per kilogram.

PCEBF = the relative error in temperature computation associated with using one less order of Bessel function but keeping the same number of eigenvalues for each Bessel function order which, for example, would mean computing the temperature SBFM1 using NMAX-1 Bessel functions and the full XMAX eigenvalues per Bessel function and defining PCEBF = (TRM-SBFM1)/TRM to be this relative error.

PCER = the relative error associated with leaving off the last eigenfunction or, for example, using 24 eigenfunctions instead of 25 eigenfunctions for each Bessel function order.

PD = the value of the divergence of the Poynting vector at the point whose spherical coordinates are (SAVER,THETAD,PHID) which value represents the number of milliwatts of power being deposited per cubic centimeter of tissue.

PHID = the phi coordinate of the point at which the microwave-induced temperature is to be computed, where phi is the spherical coordinate that ranges between zero and 360 degrees.

R = the radial coordinate of the point at which the temperature is being computed.

RHOP(I) = the tissue density of the Ith tissue layer in grams per cubic meter where typically RHOP(I)=1.

SBDP(I) = the radius in centimeters of the smallest sphere containing the Ith tissue layer where I ranges from 1 to NORG.

SBFM1 = the predicted temperature obtained by using KMAX roots per Bessel function order but only NMAX-1 Bessel function orders in approximating the infinite sum of equation (3.4.1).

SIGP(I) = the conductivity in mhos per meter of the Ith tissue layer where I ranges from 1 to NORG.

SRM1 = the estimated temperature using NMAX Bessel function orders and KMAX-1 eigenvalues per Bessel function order.

STOPR = a termination indicator for stopping the generation of Bessel functions based on the fact that STOPR exceeds the maximum absolute value of any of the spherical Bessel functions of the second kind used in describing the dependence of the induced and scattered electromagnetic fields on the radial variable with a typical value being 1.E35.

TBPER = the period of the pulse group envelope, where, for example, if there is a radar emission pattern consisting of a burst of three pulses of duration 3*TPER followed by a quiet period, a burst of three pulses followed by a quiet period, et cetera, then TBPER is equal to 3*TPER plus the major quiet period, where, of course, we define the major quiet period to be total quiet period minus the time between the individual pulses in the group or as the T-SUB-p in Figure 4.3.1.

TCP(I) = the thermal conductivity in calories per centimeter per degree centigrade per second of the Ith tissue layer where I ranges from 1 to NORG with a typical value being .0012.

TCUT = the time at which the source of pulsed microwave radiation is shut down or the T-SUB-R of Figure 4.4.1.

TDUR = the up time in seconds of an isolated pulse in a pulse group or the value of T-SUB-d in Figure 4.4.1.

THETAD = the theta coordinate of the point at which the temperature is being computed, where this is the spherical coordinate that ranges between zero and 180 degrees.

TIME = the time in seconds at which the microwave-induced temperature is to be computed.

TOTPOW = the total absorbed power in watts determined by carrying out an energy balance on the surface of the scatterer using the Poynting vector for the incident and reflected radiation.

TRM = the microwave-induced temperature obtained by adding up terms in an eigenfunction expansion at the point whose spherical coordinates are specified by the three-tuple, (SAVER, THETAD,PHID).

U(NSBF,M2,K) = the expansion coefficient which is defined by equation (3.4.16) at the observation time TIME and which is used in equation (3.4.1), where NSBF is one plus the order of the Bessel function, M2 is 1 or 2 depending on whether the index of the cosine transform defined by equations (3.3.9) and (3.3.10) is 0 or 2, and K is the index of the eigenvalue associated with a given Bessel function order.

VOL = the volume of the body in which the microwave-induced temperature is being predicted in cubic meters.

XLAMDA(K,NSBF) = an element of a KMAX by NMAX array (dimensioned as 25 by 12) which represents the Kth eigenvalue associated with the Bessel function of order NSBF, where each of these numbers is used to define a combination of spherical Bessel functions satisfying the Newton cooling law at the outer boundary with this combination of Bessel functions being the eigenfunction used in the eigenfunction expansion of the microwave-induced temperature.

XMASS = the mass of the scattering body in kilograms.

XNUM(NSBF,M2,NRT) = the transform of the source term with respect to its spatial variables.

ZLAB(1) = the first element of an alpha array containing the expression 'W/M**3'.

ZLAB(2) = the second element of an alpha array containing the expression 'MW/KG'.

4.4. Input Data Preparation

The purpose of this section is to tell a user how to prepare data to run the computer program to predict the thermal response of a spherically symmetric penetrable body to microwave radiation. The incoming radiation, the precision with which the response to this radiation will be calculated, the temporal envelope of the incoming radiation and the time at which the temperature response is to be computed, and the thermal and electrical properties of the body in which the temperature excursion is being predicted are described in the first three data sets. Data set three is a multi-card set with the number of cards being equal to the number of layers in the scattering body. The fourth data set is the collection of points at which one seeks to compute the temperature; each point is on a separate card.

The following paragraphs give details concerning the composition of the four data sets used by the computer program. Figures at the end give some data sets that direct the program to predict the microwave-radiation-induced temperature on the x, y, and z axes of the sphere.

Data set one consists of a single card containing FREQ, EO, STOPR, NORG, NMAX, KMAX, MP, MP1, IEO, and ISAR which is read in via the statements:

```
    READ 5,FREQ,EO,STOPR,NORG,NMAX,KMAX,MP,MP1,IEO,ISAR
  5 FORMAT(3D10.0,7I5)
```

In the above

FREQ = the frequency of the incoming radiation in megahertz,

EO = the strength of the incoming E-field in volts per meter (if IEO = 0) and in milliwatts per square centimeter (if IEO = 1),

STOPR = a termination indicator for stopping the generation of Bessel functions based on the fact that STOPR exceeds the absolute value of any of the spherical Bessel functions Y that will be used in describing the dependence of the induced and scattered fields on the radial variable with a typical value being 1.E35.

NORG = the number of layers in the model where NORG is 1 if the scatterer is a homogeneous ball and NORG equals its maximum value of 6 for a ball surrounded by 5 outer layers,

NMAX = the number of orders of spherical Bessel functions that will be used to help describe the microwave-radiation-induced temperature with the greatest accuracy being achieved by setting NMAX equal to its maximum value of 12,

KMAX = the number of radial eigenfunctions associated with a given order of Bessel function where the greatest accuracy is achieved by setting KMAX equal to its maximum value of 25,

MP = the number of points used in the Gauss quadrature scheme that carries out the radial transform defined by the formula(3.22) with this number being one of 32, 48, 64, or 80 and with the larger numbers giving the more accurate results,

MP1 = the number of points used in the Gauss quadrature scheme that carries out the Legendre transform defined by equation(3.11) with this number being 32 or 48,

IEO = a parameter for determining the way the input data EO is interpreted with IEO = 0 meaning that EO is a certain number of volts per meter and IEO = 1 meaning that EO is a certain number of milliwatts per square centimeter,

and

ISAR = a parameter determining the way that the output data is expressed with ISAR = 0 if the predicted power density is to be expressed in milliwatts per kilogram and ISAR = 1 if the predicted power density is to be written out and labeled as a certain number of watts per cubic meter.

Data set two consists also of a single card containing TDUR, TPER, TBPER, TCUT, TIME, NPUL, and NOCR, which is read in through the statements:

```
10 READ(5,15,END=350)TDUR,TPER,TBPER,TCUT,TIME,NPUL,NOCR,IPL1,IPL2
15 FORMAT(5D10.0,2I5)
```

In the above

TDUR = the duration of the pulse or the value of T-sub-d in Figure 4.4.1,

TPER = the period in the primary pulse group or the value of T-sub-p in Figure 4.4.1,

TBPER = the period of the pulse group envelope or the T-sub-P in Figure 4.4.1, possible time envelope function for incoming radiation.
This is similar to some radar emission patterns.

TCUT = the time at which the source is shut down or the T-sub-R in Figure 4.4.1,

TIME = the time at which the microwave-induced temperature is to be computed,

NPUL = the number of pulses per group, where we note that NPUL=2 in Figure 4.4.1 and NPUL=3 in Figure 3.4.1,

NOCR = the number of spatial points at which the temperature is to be computed.

IPL1 = an integer ranging from 0 to 7, which will indicate which of certain plots of temperature across the sphere diameters coinciding with the coordinate axes will be given. A value of IPL1 equal to

0 means no axis plots of temperature will be produced,
1 means a plot of temperature across the z-axis will be given,
2 means a plot of temperature across the x-axis will be given,
3 means a plot of temperature across the y-axis will be given,
4 means combined results of 1 and 2 are given,
5 means combined results of 1 and 3 are produced,
6 means combined results of 2 and 3 are produced, and
7 means combined results of 1, 2, and 3 are given,

and

IPL2 = an integer ranging from 0 to 7, which will indicate which of certain contour plots of isotherms will be produced on the plotting file FOR008.DAT. In the following description the x-z plane refers to the intersection of the plane containing the x-axis and the z-axis with the interior of the bounding sphere. The y-z plane will mean the plane containing the y and z axes or the plane x = 0. The x-y plane means the plane z = 0. A value of IPL2 equal to

0 means no axis plots of temperature will be produced,
1 means a contour plot in the x-z plane will be given,
2 means a contour plot in the y-z plane will be given,
3 means a contour plot in the x-y plane will be given,
4 gives the combined results of 1 and 2,
5 gives the combined results of 1 and 3,
6 gives the combined results of 2 and 3, and
7 gives the combined results of 1, 2, and 3.

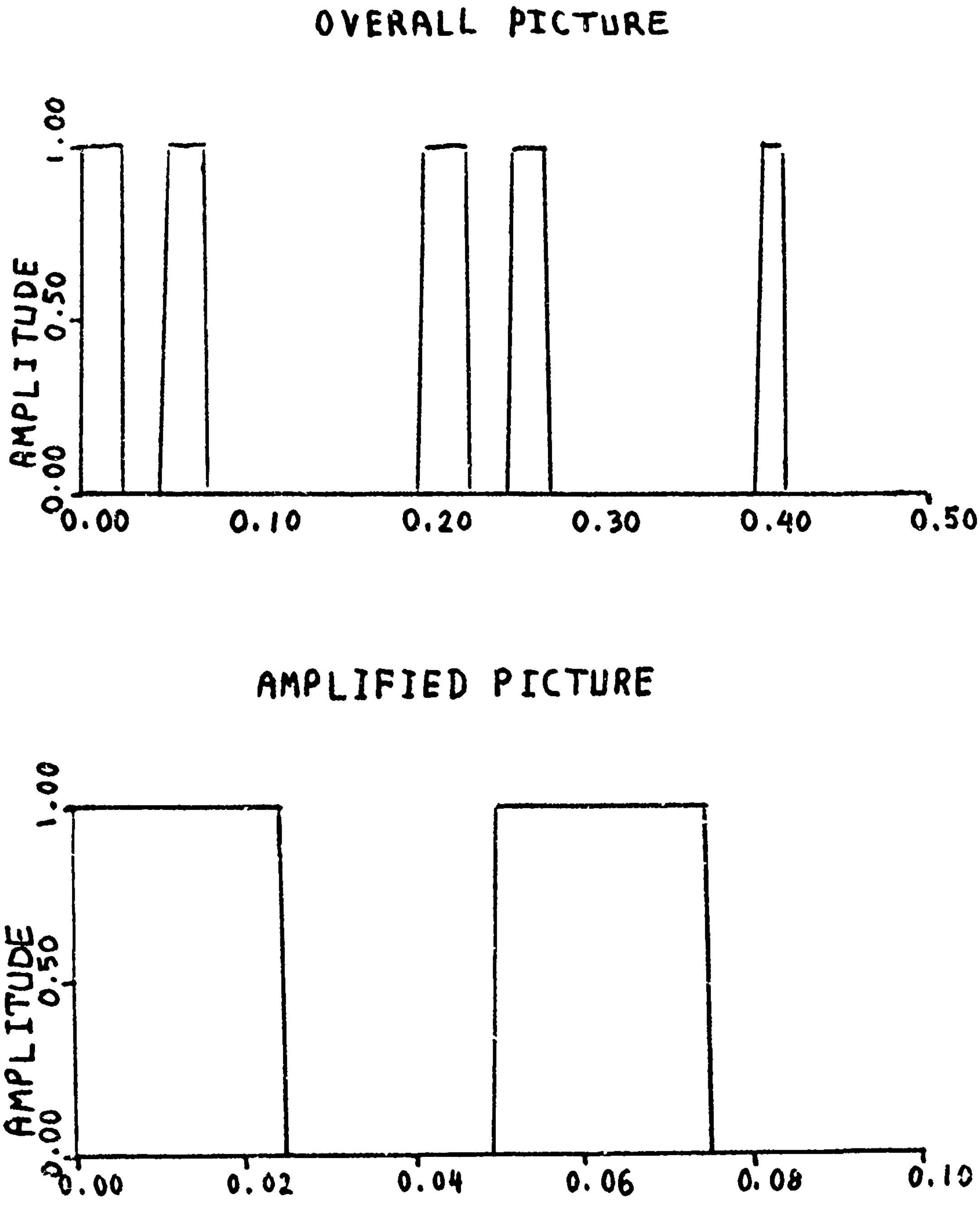

Figure 4.4.1. Typical time envelope function describing some radar emission patterns. In the above figure we have N_p = 2 pulses per group, T_d = .025 milliseconds (ms), T_p = .050 ms, $\underline{T_P}$ = 9 ms. and T_R = .4125 ms.

Data set three consists of NORG cards indexed by the parameter I augmented from 1 to NORG in a DO LOOP. The Ith card contains SBDP(I), EPSP(I), SIGP(I), TCP(I), RHOP(I), CP(I), BFRP(I), and ITIS(I) read in by means of the statements:

```
30 FORMAT(7F10.0,I5)
   DO 65 I = 1,NORG
   READ 30, SBDP(I), EPSP(I), SFGP(I), TCP(I), RHOP(I),
  1CP(I), BFRP(I), ITIS(I)
   ....
   ....
   ....
65 PRINT 70, I, SBDP(I), EPSP(I), SIGP(I),
  1TCP(I), RHOP(I), CP(I), BFRP(I),
  2TISSUE(ITIS(I)).
70 FORMAT(I4,F12.2,F12.2,F13.3,F15.6,F13.4,
  1F10.3,F12.5,3X,A8)
```

In the above, the properties of the Ith layer are specified by letting

SBDP(I) = its outer radius in centimeters,

EPSP(I) = its relative permittivity,

SIGP(I) = its conductivity in mhos per meter,

TCP(I) = its thermal conductivity in calories per centimeter per degree centigrade per second(typically TCP(I) = .0012),

RHOP(I) = tissue density in grams per cubic centimeter(typically RHOP(I) = 1),

CP(I) = tissue specific heat in calories per gram degree centigrade(typically CP(I) = .84),

BFRP(I) = the blood flow term that is equal to the product of the number of grams of blood per gram of tissue per second, the tissue density in grams of tissue per cubic centimeter of tissue, and the specfic heat of the blood (typically b = .0122),

and

ITIS(I) = the tissue type indicated by a positive integer where ITIS(I) = 1,2,3,4,5,6, or 7 denotes cerebrospinal fluid, blood, muscle, skin or dura, brain, fat or bone, or yellow bone marrow, respectively.

If EPSP(I) or SIGP(I) are read in as 0.D0, then the values of EPSP(I) or SIGP(I) or both are replaced by values determined from values stored in data tables by means of the commands:

```
   55 IF(EPSP(I).NE.0.D0.AND.SIGP(I).NE.0.D0)
     1GO TO 60
      CALL EPROP(FREQ,ITIS(I),EP,SIG)
      IF(EPSP(I).EQ.0.D0) EPSP(I) = EP
      IF(SIGP(I).EQ.0.D0) SIGP(I) = SIG
   60 FAC2 = EPSP(I)/2.D0
```

The fourth data set contains NOCR cards each containing the spherical coordinates (R,THETAD,PHID) of a point whose cartesian coordinates are (X,Y,Z) = (R*SIN(THETAD)*COS(PHID),R*SIN(THETAD)*SIN(PHID),R*COS(PHID)) at which the microwave-induced temperature rise is to be computed. The cards are read in via the statements:

```
   30 FORMAT(7F10.0)
      DO 345 II=1,NOCR
      READ 30,R,THETAD,PHID
  345 CONTINUE
```

Finally a typical problem is presented as it would be given to the user and the proper response is indicated. The user directions are to compute the thermal response of a one-layer spherically symmetric ball of brain tissue with a 2.804-cm radius, a permittivity of 31.09, a microwave conductivity of .0012, a density of 1.0, a specific heat of .84, and a blood flow perviousness term of 0.0 to a steady 30-s exposure to 2450-MHz radiation with a power of 70 mW/cm^2. The user is to compute the temperature along the x, y, and z-axes with x, y, and z-values being taken from the collection, -2.8, -2.45, -2.10,

-1.75, -1.40, -1.15, -.8, -.45, -.1, -1.E-4, +1.E-4, +.1, +.45, +.8, +1.15, +1.40, +1.75, +2.10, +2.45, and +2.8. The user is to obtain these results with maximum accuracy. The proper response is indicated in Figures 4.4.2 - 4.4.5.

```
  2804.D-3  3109.D-2  1414.D-3    12.D-4      1.D0     .84D+0      0.D0
    30.D0     30.D0     30.D0     30.D0     30.D0    1   60
  2450.D0     70.D0     1.D35    1   12   25   80   48    1    0
```

Figure 4.4.2. The first three data sets for the computation of the thermal response of a one-layer brain tissue structure exposed to 70 mW/cm^2 and 2450-MHz radiation for 30 s at 60 s[illegible] points.

```
280.E-2   180.E0    0.E0
245.E-2   180.E0    0.E0
210.E-2   180.E0    0.E0
175.E-2   180.E0    0.E0
140.E-2   180.E0    0.E0
115.E-2   180.E0    0.E0
 60.E-2   180.E0    0.E0
 45.E-2   180.E0    0.E0
 10.E-2   180.E0    0.E0
  1.E-4   180.E0    0.E0
  1.E-4     0.E0    0.E0
 10.E-2     0.E0    0.E0
 45.E-2     0.E0    0.E0
 80.E-2     0.E0    0.E0
115.E-2     0.E0    0.E0
140.E-2     0.E0    0.E0
175.E-2     0.E0    0.E0
210.E-2     0.E0    0.E0
245.E-2     0.E0    0.E0
280.E-2     0.E0    0.E0
```

Figure 4.4.3. Data set describing points on the z-axis in spherical coordinates. The points on the z-axis at which the temperature is to be computed are shown. The columns in which the data entries end are respectively 10, 20, and 30.

280.E-2	90.E0	0.E0
245.E-2	90.E0	0.E0
210.E-2	90.E0	0.E0
175.E-2	90.E0	0.E0
140.E-2	90.E0	0.E0
115.E-2	90.E0	0.E0
80.E-2	90.E0	0.E0
45.E-2	90.E0	0.E0
10.E-2	90.E0	0.E0
1.E-4	90.E0	0.E0
1.E-4	90.E0	180.E0
10.E-2	90.E0	180.E0
45.E-2	90.E0	180.E0
80.E-2	90.E0	180.E0
115.E-2	90.E0	180.E0
140.E-2	90.E0	180.E0
175.E-2	90.E0	180.E0
210.E-2	90.E0	180.E0
245.E-2	90.E0	180.E0
280.E-2	90.E0	180.E0

Figure 4.4.4. Data set describing points on the x-axis in spherical coordinates. The points on the x-axis at which the data entries end are shown. The columns in which the data entries end are respectively 10, 20, and 30.

280.E-2	90.E0	270.E0
245.E-2	90.E0	270.E0
210.E-2	90.E0	270.E0
175.E-2	90.E0	270.E0
140.E-2	90.E0	270.E0
115.E-2	90.E0	270.E0
80.E-2	90.E0	270.E0
45.E-2	90.E0	270.E0
10.E-2	90.E0	270.E0
1.E-4	90.E0	270.E0
1.E-4	90.E0	90.E0
10.E-2	90.E0	90.E0
45.E-2	90.E0	90.E0
80.E-2	90.E0	90.E0
115.E-2	90.E0	90.E0
140.E-2	90.E0	90.E0
175.E-2	90.E0	90.E0
210.E-2	90.E0	90.E0
245.E-2	90.E0	90.E0
280.E-2	90.E0	90.E0

Figure 4.4.5. Data set describing points on the y-axis in spherical coordinates. The points on the y-axis at which the temperature is to be computed are shown. The columns in which the data entries end are respectively 10, 20, and 30.

4.5. The Output and its Meaning

The output of our program to predict the thermal response of a spherically symmetric body to microwave radiation includes (i) the printing of the input data defining the scattering problem, (ii) the weight and volume of the scatterer, (iii) the average and total absorbed power, (iv) the eigenvalues associated with radial eigenfunctions, (v) the expansion coefficients used in expanding the temperature in spherical harmonics, and finally (vi) the predicted microwave-induced temperature increases and estimates of the theoretical error in our predictions.

The input data which defines the input radiation is printed out and is identified by labels. This data includes the frequency, the field strength, STOPR which is defined in Section 4.3, the up time of a single pulse or TDUR, its period or TPER, the number NPUL of pulses in a single pulse train, the time TBPER that a single pulse train lasts which includes the quiet period after the initial burst of NPUL pulses, the time TCUT after which the incident wave is cut off and the TIME at which one observes the temperature; this output data set is printed by the commands:

```
  PRINT 20, FREQ,E01,UNIT(IEO+1),STOPR,TDUR,TPER,
  1NPUL,TBPER,TCUT,TIME
20 FORMAT('1THERMAL RESPONSE OF CONCENTRIC SPHERICAL',
  1'HEAD MODEL TO RFR'/'-FREQUENCY =',F9.2,
  1'MHZ-------FIELD STRENGTH =',F9.2,1X,A8,
  16X,'STOPR ='1PD12.4/
  1'0 FOR ONE PULSE, UP TIME IS',D12.4,'SEC'
  1' AND PERIOD IS',D12.4,'SEC.'/
  1'0 ONE PULSE TRAIN CONTAINS',I4,'PULSES AND LASTS',
  1D12.4,'SEC.'/'0 THE INCIDENT WAVE IS CUT OFF AFTER',
  1D12.4,'SEC. AND THE TEMPERATURE',
  1 'IS OBSERVED AFTER',D12.4,'SEC.')
```

The next set of output data defines the body in which the microwave-induced temperature is to be predicted. We print out NORG lines of data, each of which defines the outermost bounding sphere of radius SBDP(I), the relative permittivity EPSP(I), the microwave conductivity SIGP(I), the thermal conductivity TCP(I), the density RHOP(I), the specific heat CP(I), the blood flow radial perviousness term BFRP(I), and the tissue type ITIS(I) for the Ith tissue layer. This output data set is defined by the following lines:

```
   PRINT 25
25 FORMAT('-REGION',3X,'SURFACE',4X,'RELATIVE',5X,
  1'ELECTRIC',7X,'THERMAL',6X,'DENSITY',3X,
  1'SPECIFIC',3X,'BLOOD FLOW',4X,'TISSUE'/9X,
  1'BOUNDARY',3X,'DIELECTRIC',2X,'CONDUCTIVITY',
  12X,'CONDUCTIVITY',16X,'HEAT',8X,'RATE',7X,
  1'TYPE'/21X,'CONSTANT'/11X,'(CM)',20X,'(MHO/M)',3X,
  1'(CAL/CM-SEC-C)',3X,'(G/CM3)',2X,'(CAL/G/S)',4X,
  1'(CC/SEC)'/)
   ...
   ...
   ...
   DO 65 I = 1,NORG
65 PRINT 70,I,SBDP(I),EPSP(I),SIGP(I),TCP(I),RHOP(I),
  1CP(I),BFRP(I),TISSUE(ITIS(I))
70 FORMAT(I4,F12.2,F13.3,F15.6,F13.4,F10.3,
  1F12.5,3X,A8)
```

The next set of output data describes intermediate output resulting from defining the microwave heat source term for the heat transfer equation. The data printed out includes the mass XMASS, in kilograms, of the scattering body, its volume VOL, in cubic meters, the average absorbed power PAVG per unit volume expressed in watts per cubic meter, and the average absorbed power PAVG1 per unit mass expressed in milliwatts per kilogram. This mode by which output is printed is described by the statements:

```
      PRINT 80,XMASS,VOL,PAVG,ZLAB(2),PAVG1,ZLAB(1),TOTPOW
   80 FORMAT('-WEIGHT=',1PD12.4,'KG'/'0 VOLUME=',D12.4,
     1'M**3'/'0FOR A CONTINUOUS WAVE THE AVERAGE',
     1'ABSORBED POWER IS',D13.5,A7,'OR',D13.5,A7/
     1'0TOTAL ABSORBED POWER=',D13.5,'WATTS'/
     1'-ROOTS OF THE EIGENFUNCTION'/)
```

The last phrase '-ROOTS OF THE EIGENFUNCTION' is a heading for the next output data set which is the set of eigenvalues needed to define the eigenfunctions used in expressing the microwave-induced temperature excursion.

The next set of output data provide us with the array XLAMDA of eigenvalues defined by equations (3.3.42) and (3.3.44), and which are used to define the radial eigenfunctions used in expanding the microwave-induced temperature excursion. The eigenvalucs are printed using the statements:

```
      DO 90 NSBF = 1,NMAX
      ....
      N1 = NSBF - 1
   90 PRINT 95,N1,(XLAMDA(K,NSBF),K = 1,kMAX)
   95 FORMAT(1H ,I5,1PD12.4,9PD12.4/(7X,10D12.4))
```

Each row of printing in this output data set displays the sequence defined by equation (3.3.43) where the row index N1 denotes the actual Bessel function order and K is the index of the sequence in equation (3.3.43).

Once the eigenvalues defined as the solution of equation (3.3.44) are determined, we can compute the radial transform, defined by equation (3.3.22), of the source term and subsequently obtain the expansion coefficients U(NSBF,M2,K), defined by equation (3.4.2), that are used in representing the microwave-induced temperature TRM. The expansion coefficients are indexed by

NSBF, which is one plus the order of the Bessel function under consideration, M2 which is 1 if the order of the cosine transform is 0 and is 2 if this is the order of the associated cosine transform, and finally K which is the index of the eigenvalue associated with a given Bessel function order. The expansion coefficients are printed out through the instructions:

```
      DO 270 NSBF = 1,NMAX
      ...
      ...
      ...
      DO 270 M = 1,NSBF
      ...
      ...
      ...
      DO 260 NRT = 1,KMAX
      ...
      ...
      ...
  260 U(NSBF,M2,NRT) = ETIME(NRT)*F*XNUM(NSBF,M2,NRT)/
     1DEN(NSBF,M2,NRT)
      PRINT 265,N1,M1,(U(NSBF,M2,K),K = 1,KMAX)
  265 FORMAT(2I3,1P10D12.4/(8X,1CD12.4))
  270 CONTINUE
```

The final and most important output describes the location of the point at which the temperature is sought, the predicted microwave-induced power density, the temperature excursion, and an estimate of the error associated with approximation of the infinite sum, defined by equation (3.4.1), by only a finite sum. This output is described by the statements:

```
      PRINT 275,ZLAB(ISAR+1)
  275 FORMAT('-',29X'INTERNAL POINT',11X,'ABSORBED',
     1'POWER',7X,'TEMPERATURE',8X,'APPROXIMATE',
     1/11X/'POINT',2X,'REGION',2X,'RADIUS',3X,
     1'THETA',4X,'PHI',12X,'DENSITY',14X,
     1'RISE',14X,'ERROR'/28X,'CM',6X,'DEG',12X,
     1A7,13X,'DEG C',13X,'PER CENT'/)
```

```
      DO 345 II = 1,NOCR
      ...
      ...
      ...
      PRINT 340,I  NREG,SAVR,THETAD,PHID,PD,
     1TRM,PCEBF,PC...
  340 FORMAT(I14,I8,F10.3,2F8.2,F19.8,1PD20.4,2P2F12.. ,
  345 CONTINUE
```

In the above ZLAB(ISA..+1) is an alpha array containing the label 'MW/KG' which stands for milliwatts per kilogram or 'W/M**3' which stands for watts per cubic meter. The parameters II and NREG denote, respectively, the index, ranging from 1 to NOCR, of the point at which the temperature is to be computed and the layer number, ranging from 1 to NORG, of the layer in which the point is found. The three-tuple (SAVER, THETAD, PHID) is the spherical coordinate representation of the point at which the temperature is to be computed. The variables PD and TRM denote, respectively, the microwave power per unit volume, and the microwave-induced temperature excursion at the point (SAVER, THETAD, PHID). The error estimation parameter PCEBF denotes the relative error in temperature prediction associated with using, NMAX - 1 orders of Bessel functions and KMAX eigenvalues per Bessel function instead of using NMAX and KMAX to get a temperature estimate SBFM1. On the other hand, we see whether or not we have used enough eigenvalues per Bessel function order by using NMAX orders of Bessel functions and KMAX - 1 eigenvalues per Bessel functions order obtaining a temperature estimate SRM1 and computing the relative error PCER by the statement,

```
      PCER = (TRM - SRM1)/TRM
```

4.6. Program Size and Running Time

The program requires 252K on the 'GO' step for an IBM 360 and has a running time that is dependent on the accuracy demanded and the number of layers in the model. For a one-layer model demanding maximum accuracy and computing the temperature at 60 points for one exposure time, the time on the 'GO' step was 2.93 minutes.

Gaussian quadrature is used to compute cosine, Legendre, and radial transforms of the source term divided by the product of the density and specific heat. We do these computations in an optimal way by precomputing needed values and taking care not to compute any complex function more than once at the same argument.

4.7. Error Messages

In this section we explain the error messages that the program provides to the user when he has inadvertently provided unsuitable input data. Some of the errors are fatal and some merely provide a warning to the user regarding the accuracy of their results.

An example of the latter occurs often when one attempts to compute the thermal response at points on the positive z-axis in that the series expansion of the electric field vector may not have enough terms in it to guarantee eight significant digits of accuracy in the answer. The coding which prints out this error message is given by the statements:

```
    PRINT 30,NMIN,NC,STOPR,EPS
 30 FORMAT(15X,'NMIN =',I3,' NC =',I3,2X,
   1'STOPR =',1PD14.4,'IS TOO SMALL',
   1'FOR ACCURACY OF',D14.4)
```

where NC is the number of spherical Bessel functions available to estimate the field at a given point, NMIN is the number of expansion coefficients available based on the location of the layer electrical properties, and EPS is the relative error demanded in the solution.

The error messages are described next in the order in which they are found in the main program. The first message in the main program gives an obvious constraint on the parameters defining the time profile of the beam. This is printed out when appropriate by the statements:

```
    IF (NPUL.GT.0.AND.TDUR.GT.0.DO.
   1AND.TCUT.GT.0.DO.AND.TIME.GT.0.DO)GO TO 24
 21 PRINT 22
 22 FORMAT('****ERROR IN TIMES****')
```

```
      STOP
   24 IF(TPER.LT.TDUR.OR.TBPER.LT.NPUL*TPER)GO TO 21
```

The next fatal error messages deal with the fact that the radii of the layers should be in ascending order and that there are only 7 possible tissue types, recognized by the program, assignable to a layer. These messages, when appropriate, are printed by the commands:

```
      IF(I.EQ.1.OR.SBDP(I).GT.SBDP(I-1))GO TO 45
      PRINT 40
   40 FORMAT('****LAYER RADII MUST BE',
     1' IN ASCENDING ORDER****)
      STOP
   45 IF(ITIS(I).GT.0.AND.ITIS(I).LT.8)GO TO 55
      PRINT 50,I,ITIS(I)
```

The next control on the input is based on the fact that the number of points used in the Gauss guadrature integration scheme for determining the expansion coefficients can only assume certain discrete values contained in a five-element array NPOINT. These error messages controlling the number of points requested to be used in evaluating the radial transform are, when appropriate, printed by means of the commands:

```
      DO 100 I = 1,5
      IF(MP.EQ.NPOINT(I))GO TO 110
  100 CONTINUE
      PRINT 105,MP
  105 FORMAT('0INTEGRATION CONTROL =',I9,2X,
     1'IS NOT AVAILABLE')
      STOP
```

The error message controlling the number of points requested to be used in evaluating the Legendre transform is, when appropriate, printed by means of the commands:

```
      DO 115 I = 1,5
      IF(MP1.EQ.NPOINT(I))GO TO 120
  115 CONTINUE
      PRINT 105,MP1
      STOP
```

The above errors are fatal in the sense that the program stops execution as soon as the errors are recognized.

The next error message is another check on the accuracy with which the electric vectors are computed. This check deals with the computation of fields at the Gauss quadrature points for the purpose of eliminating the temperature excursion induced by the microwave radiation. The message described at the beginning of this section dealt with the computation of power density at the points at which the temperature is to be computed or, said differently, with the accuracy of column 6 in the last output data table.

These error messages are described by the following statements:

```
      IR = 0
      DO 165 N = 1,NC
      FAC1 = 2*N + 1
      IF(IR.EQ.1)GO TO 155
      T = P(N)*TR(N)
      ERAD = ERAD + T
      IF(CDABS(T).LT.CDABS(ERAD)*EPS) IR = 1
  155 IF(ITP.EQ.1)GO TO 160
```

```
      NP1 = N + 1
      RATIO = FAC1/(N*NP1)
      A = RATIO*P(N)/SINTH
      B = -RATIO*DP(N)
      C = A*TEI(N) + B*TE(N)
      ETHETA = ETHETA + C
      T = A*TEI(N) + B*TE(N)
      EPHI = EPHI + T
      IF(CABS(C).LT.CDABS(ETHETA)*EPS.
     1AND.CDABS(T).LT.CDABS(EPHI)*EPS) ITP = 1
  160 IF(IR + ITP.EQ.2)GO TO 175
  165 CONTINUE
      PRINT 170,NMIN,NC,THETA,R,STOPR,EPS
  170 FORMAT(15X,'NMIN='I3,'NC=',I3,
     1'THETA=',F9.6,'R=',2PF9.6,
     1'STOPR='1PD9.2,'ISTOO SMALL FOR',1X,
     1'ACCURACY OF',D9.2)
  175 ERAD = ERAD/Q
```

We note that in the above code C represents an amount to be added to the series representation of ETHETA. Thus, if CDABS(C) is small in comparison to CDABS(ETHETA)*EPS, then we say that adding the term C affected the value of ETHETA in the decimal place equal to the integer value of 1/EPS or, said differently, that EPS is the relative error associated with using one less term in the series representation of ETHETA. It is also clear from the above coding that the term T is used in the same way to describe the accuracy with which ERAD, the component of the electric field vector in the radial direction, and EPHI, the component of the electric field vector in phi direction, are computed.

The final error message of the control program is a nonfatal error message that warns the user when he attempts to predict the microwave heating in the free space outside the body that is being irradiated. This message, when appropriate, is printed by means of the commands:

```
      DO 285 NREG = 1,NORG
      IF(R.LE.SBDP(NREG))GO TO 300
  285 CONTINUE
  290 NREG = 1000000000
      PRINT 295,U,NREG,SAVR,THETAD,PHID
  295 FORMAT(I14,I8,F10.3,3X,'**THE RADIUS',1X,
     1'IS OUTSIDE THE SPHERE***')
      GO TO 345
  300 IREG = NREG
```

The statement

```
      GO TO 345
```

directs the program around the temperature computation part of the program when this error message is printed. In other words, the computer program will not let the user waste his time by attempting to compute microwave-induced temperature excursions at points outside the body being irradiated.

4.8. Program and Subprogram Description

In this section we give an executive description of the overall program, list the subroutines called, and give their purpose.

The main program is divided into five parts. These parts carry out (i) the scattering problem definition by reading in the data and using subroutine EPROP, (ii) the electromagnetic field expansion coefficient determination and surface energy balance from the results of the COEF subroutine, (iii) the determination of the eigenvalues of the elliptic part of the heat transfer operator using the RFNDR subroutine, (iv) the microwave heat source expansion and thermal expansion coefficient using the subroutines BJYH, TERM, PL, ALP, and SRBF, and finally (v) the power density, temperature and error estimation portion using only the subroutines SRBF and ALP. The beginning and ending of the above five sections are marked by comment cards in the listing of the program in Appendix A.

In the next part of this section we describe all of these subroutines in the order in which they occur in the main program.

The subroutine EPROP determines, by interpolating tabulated data, the relative permittivity or microwave conductivity of any of the seven tissue types from tabulated data. The subroutine is called by the statement,

```
CALL EPROP(FREQ,ITIS(I),EP,SIG)
```

The user must supply FREQ, the frequency of the incoming radiation in megahertz, and the tissue type ITIS(I) of the Ith layer of the scatterer, where ITIS(I) = 1, 2, 3, 4, 5, 6, or 7 depending on whether the tissue type is cerebrospinal fluid, blood, muscle, skin or dura, brain, fat or bone, or yellow bone marrow, respectively.

The subroutine COEF generates expansion coefficients ANP, BNP, ALPNP, and BETNP used in expanding the electromagnetic field. It is called by the statement,

CALL COEF

The subroutine RFNDR is used to determine the eigenvalues of the elliptic part of the heat transfer operator by a shooting method. Basically we start out with a trial value of the eigenvalue, an assumption about the asymptotic behavior of the radial eigenvalue at the origin so that with this assumption the solution of the singular ordinary differential equation with which the eigenvalue is associated is unique. We then check and see if the Newton cooling condition on the boundary is satisfied. If it is, we know that we have an eigenvalue. If the Newton cooling condition or equation (3.3.44) is not satisfied, we increase the trial value slightly and try again.

When we find two trial values at which the Newton cooling function, the output of the function subroutine FNCAL, differ in sign, we then use a bisection routine DRTMI to get the value of the eigenvalue to as many decimal places as is desired.

The subroutine BJYH generates arrays BJNP and BHNP of spherical Bessel functions and Hankel functions, respectively, used in the determination of the functions defined in equations (3.2.2) - (3.2.6). It is called by the statement,

CAL BJYH(BJNP,BHNP,Q,NC,STOPR,MAX)

where Q is equal to a sphere radius multiplied by the complex propagation constant FKP(NREG) defined by equation (3.2.7). We compute up to MAX values limited by the size constraint STOPR, but we fill the arrays with only NC values since we have the same number of spherical Bessel functions in all regions.

The subroutine TERM computes the product of the square root of (-1) raised to the power NCK and the factor of I**L appearing in equation (3.2.2) where here "I" denotes the square root of -1.

The subroutine PL computes an array P of associated Legendre polynomials of the first kind and order 1 and an array DP of their derivatives.

The function subroutine ALP computes the associated Legendre function of the first kind, degree N and order M with M and N being nonnegative integers and with M not exceeding N. It is a function subroutine returning a single value at a single point.

The function subroutine SRBF computes the spherical Bessel functions XJ and XY of the first and second kind, respectively, and their respective derivatives DJ and DY at the value equal to the square root of S1 multiplied by SAVR or the arguments appearing in the discussion in Section 3.3.

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APPENDIX A

LISTING OF THE PROGRAM

```
                          PROGRAM TRP
          THERMAL RESPONSE OF CONCENTRIC SPHERICAL
                    HEAD MODEL TO RFR

 IMPLICIT REAL*8 (A-H,O-Z)
 COMPLEX*16 FKP,ANP,BNP,ALPNP,BETNP,BJNP,BHNP,ERAD,ETHETA,EPHI,T,C,
1W,X,Y1,Z,Q,TE(50),TE1(50),TR(50)
 COMMON FKP(7),ANP(300),BNP(300),ALPNP(300),BETNP(300),BJNP(100),BH
1NP(100),BDP(6),P(51),DP(50),R,THETA,COSTH,PHI,SINTH,STOPR,EO
 COMMON /A/NORG,NREG,NRT,NSBF,NMIN,NC,ICODE
 COMMON /B/FACT(6,25,18),AJ(6,25,18),BY(6,25,18),XLAMDA(25,18),SBDP
1(6),RHOP(6),CP(6),BP(6),TCP(6),H
 COMMON /C/AJ1,S1,F,R1,IREG
 INTEGER*2 IFL(102,102)
 REAL*4 R3(304),TR3(304),X2(102),DAR(102,102),CLAB(3,3),ANG,AX1,AY
 DIMENSION U(18, 2,25),EPSP(6),SIGP(6),BFRP(6),
1          S(80,64,2),XNUM(18,2,25),DEN(18,2,25),RR(80),ETIME(25)
1                         ,                           THET1(64),COSTH1(
164),SINTH1(64),WTTH(64),ALPOL(64)        ,
1          NPOINT(5),KEY(6),Y(116),       WT(116),        ARG3(2)
1,UNIT(2),ZLAB(2),
1          SUM2(80,2) ,
1          TISSUE(7),ITIS(6)
1     ,BLAB(3),AX(3),                     DLAB(4)
 DATA TISSUE/'CS FLUID','BLOOD','MUSCLE','SKIN-DUR','BRAIN','FAT-BO
1NE','Y.B.M.'/
 DATA                                                       UNIT/'V/
1M'    ,'MW/CM**2'/,ZLAB/' MW/KG',' W/M**3'/
1     ,EPS/1.D-8/
 DATA CLAB/'E PL','ANE',' ','H PL','ANE',' ','X-Y','PLAN','E'/
1,BLAB/'Z-AXIS C','OORDINAT'
1     ,'E (CM)'/,AX/'Z-AXIS C','X-AXIS C','Y-AXIS C'/,
1                                   DLAB/'TEMPERAT','URE RISE',' (D
1EG. C','.)'/
 DATA NPOINT/32,48,64,80,8/,KEY/1,17,41,73,113,117/
 DATA Y/.04830766568800,.1444719615800,.2392873622500,.33186860228D
10,.4213512761300,.5068999089300,.5877157572400,.6630442669300,.732
11821187400,.7944837959700,.8493676137300,.8963211557700,.934906075
19400,.9647622555900,.9856115115500,.9972638618500,.0323801709630O,
1.0970046992090O,.1612223560700,.2247637903900,.2873624873600,.3487
15588629DO,.40868648199DO,.46690290475DO,.52316097472DO,.5772247260
18DO,.62886739678DO,.67787237963DO,.72403413092DO,.76715903252DO,.8
10706620403DO,.84358826162DO,.87657202027DO,.90587913672DO,.9313866
19071DO,.95298770316DO,.97059159255DO,.98412458372DO,.99353017227DO
1,.99877100725DO,.02435029266300,.07299312178800,.121462819300,.169
164442042DO,.21742364374DO,.26468716221DO,.31132287199DO,.357220158
134DO,.40227015796DO,.44636601725DO,.48940314571DO,.53127946402DO,.
15718956462DO,.61115535517DO,.64896547125DO,.68523631305DO,.7198818
15017DO,.75281990726DO,.78397235894DO,.81326531512DO,.84062929625DO
1,.86599939815DO,.889315446DO,.91052213708DO,.92956917213DO,.946411
137486DO,.96100879965DO,.97332682779DO,.98333625388DO,.99101337148D
10,.99634011677DO,.99930504174DO,
1        .019511383257DO,.058504437152DO,.097408398442DO,.136164022
```

```
 1809D0,.1747122918 3D0,.21299450286D0,.25095235839D0,.28852805488D0,
 1.32566437075D0,.3623047535D0,.39839340588D0,.43387537083D0,.468696
 161517D0,.50280411189D0,.536145920 9D0,.56867126812D0,.60033062283D0
 1,.63107577305D0,.66085989899D0,.68963764434D0,.71736518536D0,.7440
 10029758D0,.76950242014D0,.7938327175D0,.81695413868D0,.83883147358
 1D0,.8594314066 6D0,.87872256768D0,.89667557944D0,.91326310257D0,.92
 1845987717D0,.94224276131D0,.95459076634D0,.96548508904D0,.97490914
 1059D0,.98284857274D0,.989291302 5D0,.99422754097D0,.99764986440D0,.9
 19955382265D0,.1834346425D0,.52553240992D0,.79666647741D0,.96028985
 165D0/
  DATA WT/.096540088515D0,.095638720079D0,.093844399081D0,.091173878
 1696D0,.087652093004D0,.083311924227D0,.078193895787D0,.07234579410
 19D0,.065822227 76D0,.058684093479D0,.050998059262D0,.042835898022D
 10,.034273862913D0,.025392065309D0,.016274394731D0,.0070186100095D0
 1,.064737696813D0,.064466164436D0,.063924238585D0,.063114192286D0,.
 106203942316D0,.060704439166D0,.059114839698D0,.0572772921D0,.05519
 195037D0,.052890189485D0,.050359035554D0,.047616658492D0,.044674560
 1857D0,.041545082943D0,.038241351066D0,.034777222565D0,.03116722783
 13D0,.027426509708D0,.023570760839D0,.019616160457D0,.015579315723D
 10,.011477234579D0,.0073275539013D0,.0031533460523D0,.048690957009D
 10,.048575467442D0,.048344762235D0,.047999388596D0,.047540165715D0,
 1.046968182816D0,.046284796581D0,.045491627927D0,.044590558164D0,.0
 143583724529D0,.042473515124D0,.041262563243D0,.039953741133D0,.038
 1550153179D0,.037055128540D0,.035472213257D0,.033805161837D0,.032057
 1928355D0,.030234657072D0,.028339672614D0,.026377469715D0,.02435270
 12569D0,.022270173808D0,.020134823154D0,.017951715776D0,.015726030 4
 176D0,.013463047897D0,.011168139460D0,.008846759826 4D0,.006504457969
 1D0,.0041470332606D0,.0017832807217D0,
 1          .039017813656D0,.038958395963D0,.038839651059D0,.03866175
 19774D0,.038424993007D0,.038129711314D0,.037776364362D0,.0373654902
 139D0,.036897714638D0,.036373749906D0,.035794333953D0,.035160529045
 1D0,.034473120452D0,.033733214985D0,.032941939398D0,.032100498673D0
 1,.031210174188D0,.03027231176D0,.029288369583D0,.028259816057D0,.0
 1271882275D0,.026075235768D0,.024922535764D0,.023731882866D0,.02250
 15090246D0,.021244026116D0,.019950610878D0,.018626814208D0,.0172746
 152056D0,.015896183584D0,.014493508041D0,.013068761592D0,.011624114
 1121D0,.010161766041D0,.008339452693D0,.007192904768 1D0,.005690922
 14514D0,.0041803131247D0,.0026635335895D0,.0011449500032D0,.3626837
 18338D0,.31370664588D0,.22238103445D0,.10122853629D0/
  IPLSW=0
  PIE=3.141592653589793D0
  RAD=180.D0/PIE
  EPS0=8.85416D-12
  VEL=2.997924562D8
  RHOBP=1.06D0
  CBP=0.98D0
  H=5.72D-5
  ITME=0
**********FIRST DATA CARD - CONTROL PARAMETERS
  READ (5,5)FREQ,E0,STOPR,NORG,NMAX,KMAX,MP,MP1,IE0,ISAR
5 FORMAT (3D10.0,7I5)
         FREQ   FREQUENCY IN MEGAHERTZ
         E0     STRENGTH OF INCIDENT E-FIELD
         STOPR  CUTOFF FOR SBF COMPUTATIONS
         NORG   NUMBER OF LAYERS IN SPHERE
                IS DESIRED. A CARD WILL BE READ FOR EACH POINT.
         NMAX   NUMBER OF ORDERS OF BESSEL FUNCTIONS USED. MAX=12
```

```
C         KMAX    NUMBER OF ROOTS FOR EACH ORDER. MAX=25
C         MP      NUMBER OF POINTS FOR INTEGRATION FOR RADIUS. 32, 48,
C                   64 OR 80. (, IS AVAILABLE FOR TEST RUNS)
C         MP1     NUMBER OF POINTS FOR INTEGRATION FOR THETA.  32 OR 48
C         IEO     INPUT EO UNITS
C                   0 - VOLTS/METER
C                   1 - MILLIWATTS/SQUARE CENTIMETER
C         ISAR    OUTPUT POWER DENSITY UNITS
C                   0 - MILLIWATTS/KG
C                   1 - WATTS/CUBIC METER
      E01=EO
      IF (IEO.EQ.0) GO TO 10
      IEO=1
      EO=DSQRT(3767.DO*EO)
C**********SECOND DATA CARD. TIMES IN SECONDS FOR INCIDENT WAVE PULSES.
C         FIRST PULSE TURNS ON AT ZERO SECONDS.
   10 READ (5,15,END=495) TDUR,TPER,TBPER,TCUT,TIME,NPUL,NOCR,IPL1,IPL2,
     1INTP
   15 FORMAT (5D10.0,5I5)
C         TDUR    TIME DURATION OF A PULSE
C         TPER    PERIOD FROM START OF A PULSE TO START OF NEXT PULSE.
C         TBPER   PERIOD FOR A GROUP OF PULSES.
C         TCUT    TIME AT WHICH WAVE IS CUT OFF
C         TIME    TIME WHEN TEMPERATURE RISE IS OBSERVED.
C         NPUL    NUMBER OF PULSES IN A GROUP
C         NOCR    NUMBER OF POINTS IN SPHERE AT WHICH TEMPERATURE RISE
C***      PRINT OUT TITLE AND BASIC INPUT DATA
      PRINT 20,FREQ,E01,UNIT(IEO+1),STOPR,TDUR,TPER,NPUL,TBPER,TCUT,TIME
   20 FORMAT ('1THERMAL RESPONSE OF CONCENTRIC SPHERICAL HEAD MODEL TO R
     1FR'/'-FREQUENCY =',F9.2,' MHZ       FIELD STRENGTH =',F9.2,1X,A8,6
     1X,'STOPR =',1PD12.4/
     1'OFOR ONE PULSE, UP TIME IS',D12.4,' SEC. AND PERIOD IS',D12.4,' S
     2EC.'/'OONE PULSE TRAIN CONTAINS',I4,' PULSES AND LASTS',D12.4,' SE
     3C.'/'OTHE INCIDENT WAVE IS CUT OFF AFTER',D12.4,' SEC. AND THE TEM
     4PERATURE IS OBSERVED AFTER',D12.4,' SEC.')
      IF (NPUL.GT.0.AND.TDUR.GT.0.DO.AND.TCUT.GT.0.DO.AND.TIME.GT.0.DO)
     1GO TO 24
   21 PRINT 22
   22 FORMAT ('**** ERROR IN TIMES ****')
      STOP
   24 IF (TPER.LT.TDUR.OR.TBPER.LT.NPUL*TPER) GO TO 21
      ITME=ITME+1
      IF (ITME.GT.1) GO TO 215
      PRINT 25
   25 FORMAT ('OREGION   SURFACE    RELATIVE     ELECTRIC         THERMAL
     1     DENSITY   SPECIFIC   BLOOD FLOW    TISSUE'/
     1                                          9X,'BOUNDARY   DIELECTRIC  C
     1ONDUCTIVITY  CONDUCTIVITY',16X,'HEAT',8X,'RATE         TYPE'/
     1                                                  21X,'CONSTANT'/11X
     1,'(CM)',20X,'(MHO/M)   (CAL/CM-SEC-C)   (G/CM3)  (CAL/G/S)    (CC/
     1SEC)'/)
      OMEGA=2.D6*PIE*FREQ
      FAC1=OMEGA/VEL
      START=1.D38
      XMASS=0.DO
      OLDVOL=0.DO
C**********READ LAYER PROPERTIES AND COMPUTE PROPAGATION CONSTANTS
```

```
      DO 65 I=1,NORG
      READ (5,30)SBDP(I),EPSP(I),SIGP(I),TCP(I),RHOP(I),CP(I),BFRP(I),
     1 ITIS(I)
   30 FORMAT (7F10.0,I5)
             SBDP   OUTER RADIUS OF LAYER IN CENTIMETERS
             BDP    LAYER OUTER RADIUS IN METERS
             EPSP   PERMITTIVITY(RELATIVE)
             SIGP   CONDUCTIVITY (MHOS PER METER)
             TCP    THERMAL CONDUCTIVITY
             RHOP   DENSITY
             CP     SPECIFIC HEAT
             BFRP   BLOOD FLOW RADIAL PERVICIVITY
             ITIS   CODE FOR LAYER TISSUE TYPE
               1 DENOTES CEREBROSPINAL FLUID
               2 DENOTES BLOOD
               3 DENOTES MUSCLE
               4 DENOTES SKIN OR DURA
               5 DENOTES BRAIN
               6 DENOTES FAT OR BONE
               7 DENOTES YELLOW BONE MARROW
      BDP(I)=SBDP(I)/1.D2
      VOL=4.DO*PIE*BDP(I)**3/3.DO
      RVOL=VOL-OLDVOL
      XMASS=XMASS+RVOL*RHOP(I)*1.D3
      OLDVOL=VOL
      BP(I)=RHOBP*CBP*BFRP(I)
      A=BP(I)/(RHOP(I)*CP(I))
      IF(A.LT.START) START=A
      IF (I.EQ.1.OR.SBDP(I).GT.SBDP(I-1)) GO TO 45
      PRINT 40
   40 FORMAT ('0**** LAYER RADII MUST BE IN ASCENDING ORDER ****')
      STOP
   45 IF (ITIS(I).GT.0.AND.ITIS(I).LT.8) GO TO 55
      PRINT 50,I,ITIS(I)
   50 FORMAT ('0****TISSUE TYPE CODE FOR LAYER',I2,' IS',I5,', OUTSIDE T
     1HE RANGE 1-7****')
      STOP
   55 IF (EPSP(I).NE.0.DO.AND.SIGP(I).NE.0.DO) GO TO 60
      CALL EPROP(FREQ,ITIS(I),EP,SIG)
      IF (EPSP(I).EQ.0.DO) EPSP(I)=EP
      IF (SIGP(I).EQ.0.DO) SIGP(I)=SIG
   60 FAC2=EPSP(I)/2.DO
      FAC3=DSQRT(1.DO+(1.DO/(EPSO*OMEGA)**2)*(SIGP(I)/EPSP(I))**2)
      FKP(I)=DCMPLX(FAC1*DSQRT(FAC2*(FAC3+1.DO)),FAC1*DSQRT(FAC2*(FAC3-1
     1.DO)))
   65 PRINT 70,I,SBDP(I),EPSP(I),SIGP(I),TCP(I),RHOP(I),CP(I),BFRP(I),TI
     1SSUE(ITIS(I))
   70 FORMAT (I4,F12.2,F12.2,F13.3,F15.6,F13.4,F10.3,F12.5,3X,A8)
      FKP(NORG+1)=DCMPLX(FAC1,0.DO)
      IF (START.EQ.0.DO) START=1.D-9
            COMPUTE EXPANSION COEFFICIENTS AND TOTAL ABSORBED POWER
      CALL COEF
      NN=NORG*NMIN
      QS=0.DO
      QT=0.DO
      DO 75 N=1,NMIN
      FACN=2*N+1
```

```
      NNN=NN+N
      X3=FACN*DREAL(ALPNP(NNN)+BETNP(NNN))
      Y3=FACN*(CDABS(ALPNP(NNN))**2+CDABS(BETNP(NNN))**2)
      QT=QT-X3
      QS=QS+Y3
      IF (DABS(X3).LT.DABS(QT)*1.D-6.AND.Y3.LT.QS*1.D-6) GO TO 242
   75 CONTINUE
      PRINT 241
  241 FORMAT('0**** TOO FEW EXPANSION COEFFICIENTS ****')
  242 TOTPOW=2.65441D-3*E0**2*2.D0*PIE*(QT-QS)/(2.D0*FAC1*FAC1)
      PAVG=TOTPOW/VOL
      PAVG1=1.D3*TOTPOW/XMASS
***        PRINT OUT AVERAGE ABSORBED POWER DENSITY AND TOTAL ABSORBED
***        POWER
      PRINT 80,XMASS,VOL,PAVG,ZLAB(2),PAVG1,ZLAB(1),TOTPOW
   80 FORMAT ('OWEIGHT =',1PD12.4,' KG'/'OVOLUME =',D12.4,' M**3'/
     1'OFOR A CONTINUOUS WAVE THE AVERAGE ABSORBED POWER IS',D13.5,A7,'
     1OR',D13.5,A7/
     1'OTOTAL ABSORBED POWER =',D13.5,' WATTS'/
     1        'OROOTS OF THE EIGENFUNCTION'/)
            GET ROOTS OF EQUATION
      STEP=1.D-8
      ICODE=0
      AJ1=1.D0
        PRINT 543,START,STEP
  543   FORMAT ('OSTART =',1PD15.7,'  STEP =',D15.7)
      DO 90 NSBF=1,NMAX
      AJ1=AJ1*(2*NSBF-1)
      CALL RFNDR(START,STEP,1.D-6,KMAX,1000,10000)
      START=XLAMDA(1,NSBF)
      STEP=(XLAMDA(2,NSBF)-XLAMDA(1,NSBF))/10.D0
      N1=NSBF-1
      PRINT 95,N1,(XLAMDA(K,NSBF),K=1,KMAX)
   95 FORMAT(1H ,I5,1PD12.4,9D12.4/(7X,10D12.4))
      IF (NSBF.EQ.1) GO TO 90
      DO 89 K=1,KMAX
      IF (XLAMDA(K,NSBF).LE.XLAMDA(K,NSBF-1))GO TO 500
   89 CONTINUE
   90 CONTINUE
      ICODE=1
            DEVELOP U(N,M,K) ARRAY
      DO 100 I=1,5
      IF (MP.EQ.NPOINT(I)) GO TO 110
  100 CONTINUE
      PRINT 105,MP
  105 FORMAT ('OINTEGRATION CONTROL =',I9,' IS NOT AVAILABLE')
      STOP
  110 JF=KEY(I)
      JL=KEY(I+1)-1
      DO 115 I=1,5
      IF (MP1.EQ.NPOINT(I)) GO TO 120
  115 CONTINUE
      PRINT 105,MP1
      STOP
  120 JF1=KEY(I)
      JL1=KEY(I+1)-1
      PD2=PIE/2.D0
```

```
      K=1
      DO 130 J=JF1,JL1
      IF (Y(J).NE.0.D0) GO TO 125
      THET1(K)=PD2
      WTTH(K)=WT(J)
      SINTH1(K)=1.D0
      COSTH1(K)=0.D0
      GO TO 130
  125 PDY=PD2*Y(J)
      THET1(K)=PD2+PDY
      WTTH(K)=WT(J)
      SINTH1(K)=DSIN(THET1(K))
      COSTH1(K)=DCOS(THET1(K))
      K=K+1
      THET1(K)=PD2-PDY
      WTTH(K)=WT(J)
      SINTH1(K)=DSIN(THET1(K))
      COSTH1(K)=DCOS(THET1(K))
  130 K=K+1
      EOSQ=EO*EO
      MAX=NMIN+15
      IF (MAX.GT.100) MAX=100
            CLEAR STORAGE FOR INTEGRALS
      DO 135 NSBF=1,NMAX
      DO 135 M=1,2
      DO 135 NRT=1,KMAX
      DEN(NSBF,M,NRT)=0.D0
  135 XNUM(NSBF,M,NRT)=0.D0
            SUM OVER REGIONS
      DO 210 NREG=1,NORG
      IREG=NREG
            PRECALCULATE THE POWER DENSITY TIMES PHI INTEGRAL
            FIRST CALCULATE RADIUS DEPENDENT PART OF SOURCE TERM
      NN=(NREG-1)*NMIN
      R1=SBDP(NREG)
      R2=0.D0
      IF (NREG.GT.1) R2=SBDP(NREG-1)
      R11=R1+R2
      R13=R1-R2
      RAVG=R13/2.D0
      RCP=RHOP(NREG)*CP(NREG)
      FAC=EOSQ*.5D0*SIGP(NREG)/4186.D3
      J=0
      DO 180 J3=JF,JL
      IF (Y(J3).NE.0.D0) GO TO 140
      I3=1
      ARG3(1)=R11
      GO TO 145
  140 I3=2
      R12=R13*Y(J3)
      ARG3(1)=R11+R12
      ARG3(2)=R11-R12
  145 DO 180 L=1,I3
      R=ARG3(L)/2.D0
      J=J+1
      RR(J)=R
      R=R/100.D0
```

```
      Q=R*FKP(NREG)
      CALL BJYH(BJNP,BHNP,Q,NC,STOPR,NMIN+2)
      NC=MIN0(NC-2,NMIN)
      NCK=0
      DO 150 N=1,NC
      FAC1=2*N+1
      NNN=NN+N
      W=BNP(NNN)
      X=BJNP(N+1)
      Y1=BETNP(NNN)
      Z=BHNP(N+1)
      NCK=NCK+1
      T=FAC1*(W*X+Y1*Z)
      CALL TERM(NCK,T,1)
      TR(N)=T
      T=ANP(NNN)*X+ALPNP(NNN)*Z
      CALL TERM(NCK,T,0)
      TE(N)=T
      A=N+1
      B=N
      T=(W*(A*BJNP(N)-B*BJNP(N+2))+Y1*(A*BHNP(N)-B*BHNP(N+2)))/FAC1
      CALL TERM(NCK,T,1)
      TE1(N)=T
  150 IF (NCK.EQ.4)NCK=0
              THEN CALCULATE THETA DEPENDENT PART OF SOURCE TERM
      DO 180 J2=1,MP1
      THETA=THET1(J2)
      SINTH=SINTH1(J2)
      COSTH=COSTH1(J2)
      CALL PL
      ERAD=DCMPLX(0.D0,0.D0)
      ETHETA=DCMPLX(0.D0,0.D0)
      EPHI=DCMPLX(0.D0,0.D0)
      ITP=0
      IR=0
      DO 165 N=1,NC
      FAC1=2*N+1
      IF (IR.EQ.1) GO TO 155
      T=P(N)*TR(N)
      ERAD=ERAD+T
      IF (CDABS(T).LT.CDABS(ERAD)*EPS)IR=1
  155 IF (ITP.EQ.1) GO TO 160
      NP1=N+1
      RATIO=FAC1/(N*NP1)
      A=RATIO*P(N)/SINTH
      B=-RATIO*DP(N)
      C=A*TE(N)+B*TE1(N)
      ETHETA=ETHETA+C
      T=A*TE1(N)+B*TE(N)
      EPHI=EPHI+T
      IF (CDABS(C).LT.CDABS(ETHETA)*EPS.AND.CDABS(T).LT.CDABS(EPHI)*EPS)
     1ITP=1
  160 IF (IR+ITP.EQ.2) GO TO 175
  165 CONTINUE
      PRINT 170,NMIN,NC,THETA,R,STOPR,EPS
  170 FORMAT (15X,'NMIN =',I3,' NC =',I3,' THETA =',F9.6,' R =',2PF9.6,'
     1 STOPR =',1PD9.2,' IS TOO SMALL FOR ACCURACY OF',D9.2)
```

```
175 ERAD=ERAD/Q
          STORE SOURCE TERM TIMES PHI INTEGRAL
    ERT=DREAL(ERAD*DCONJG(ERAD)+ETHETA*DCONJG(ETHETA))
    EP1=DREAL(EPHI*DCONJG(EPHI))
    S(J,J2,1    )=FAC*PIE*(ERT+EP1)
180 S( J,J2,2   )=FAC*PD2*(ERT-EP1)
          CALCULATE NUMERATOR AND DENOMINATOR INTEGRALS
    DO 210 NSBF=1,NMAX
    N1=NSBF-1
    DO 210 M=1,NSBF
    IF (M.NE.1.AND.M.NE.3) GO TO 210
    M2=M/2+1
    M1=M-1
    DO 185 J=1,MP1
185 ALPOL(J)=ALP(N1,M1,COSTH1(J))*SINTH1(J)
    DO 195  J3 = 1,MP
    SUMJ2= 0.D0
    DO 190  J2 = 1,MP1
190 SUMJ2 = SUMJ2 + WTTH(J2)*S(J3,J2,M2)*ALPOL(J2)
195 SUM2(J3,M2) = SUMJ2
          SUM2 IS THE INTEGRAL OF THE SOURCE TERM TIMES ALPOL
          ALPOL IS THE PRODUCT OF THE LEGENDRE POLYNOMIAL TIMES THE
            SINE OF THETA
          J2 IS AN INDEX FOR THE THETA COORDINATE ASSOCIATED WITH
            GAUSSIAN INTEGRATION
    DO 205 NRT=1,KMAX
          INTEGRATE OVER RADIUS
    F=FACT(IREG,NRT,NSBF)
    S1=XLAMDA(NRT,NSBF)*RCP-BP(IREG)
    SUM=0.D0
    SUM3=0.D0
    DO 200 J3=1,MP
    R=RR(J3)
    R1=R
    CALL SRBF(XJ,XY,DJ,DY)
    ZZ=AJ(NREG,NRT,NSBF)*XJ        +BY(NREG,NRT,NSBF)*XY
    IF (DABS(XY).GT.1.D34) PRINT 666,NSBF,M,NRT,AJ(NREG,NRT,NSBF),
   1  XJ,BY(NREG,NRT,NSBF),XY,ZZ
666  FORMAT (3I5,1P7D15.7)
    RSQ=R*R
          INTEGRATE OVER THETA
    J=JF+(J3-1)/2
    WTJ=WT(J)*ZZ*RSQ
    SUM=SUM+WTJ*ZZ
200 SUM3 = SUM3 + WTJ*SUM2(J3,M2)
    DEN(NSBF,M2,NRT)=DEN(NSBF,M2,NRT)+RCP*SUM*RAVG
205 XNUM(NSBF,M2,NRT)=XNUM(NSBF,M2,NRT)+SUM3*RAVG
210 CONTINUE
          CALCULATE COEFFICIENTS U(N,M,K)
215 IF (TCUT.GT.TIME) TCUT=TIME
    IA=TCUT/TBPER
    IC=(TCUT-IA*TBPER)/TPER
    IB=MIN0(NPUL,IC)
    XL=IA*TBPER+IB*TPER
    D=0.D0
    IF (IC.LT.NPUL) D=1.
    XU=DMIN1(TCUT,XL+D*TDUR)
```

```
      TA=TIME-TDUR-(IA-1)*TBPER-(NPUL-1)*TPER
      TB=TIME-TDUR-IA*TBPER-(IB-1)*TPER
      PRINT 220
  220 FORMAT ('OU COEFFICIENTS')
      DO 270 NSBF=1,NMAX
      PRINT 225
  225 FORMAT (' ')
      N1=NSBF-1
      DO 270 M=1,NSBF
      IF (M.NE.1.AND.M.NE.3) GO TO 270
      M1=M-1
      M2=M/2+1
      NMM=N1-M1
      NPM=N1+M1
      F=1.D0
      IF (M1.EQ.0) GO TO 255
      IF (N1.NE.M1) GO TO 240
      DO 235 I=2,NPM
  235 F=F*I
      GO TO 250
  240 II=2*M1
      F1=NMM+1
      DO 245 I=1,II
      F=F*F1
  245 F1=F1+1.D0
  250 F=1.D0/F
  255 F=(2.D0*N1+1.D0)/(2.D0*PIE)*F*PD2
      DO 260 NRT=1,KMAX
      IF (M.NE.1) GO TO 260
      XR=XLAMDA(NRT,NSBF)
      D=0.D0
      IF (IA+IB.EQ.0) GO TO 258
      X1=XR*TPER
      X3=1.D0
      IF (X1.LE.40.D0) X3=1.D0-DEXP(-X1)
      IF (IA.LE.0) GO TO 256
      X4=XR*TA
      IF (X4.GT.87.D0) GO TO 256
      X1=XR*NPUL*TPER
      X5=1.D0
      IF (X1.LE.40.D0) X5=1.D0-DEXP(-X1)
      X1=XR*TBPER
      X6=1.D0
      X7=1.D0
      IF (X1.GT.40.D0) GO TO 261
      X6=1.D0-DEXP(-X1)
      X1=X1*IA
      IF (X1.LE.40.D0) X7=1.D0-DEXP(-X1)
  261 D=D+DEXP(-X4)*X5*X7/(X3*X6)
  256 IF (IB.LE.0) GO TO 257
      X4=XR*TB
      IF (X4.GT.87.D0) GO TO 257
      X1=XR*IB*TPER
      X5=1.D0
      IF (X1.LE.40.D0) X5=1.D0-DEXP(-X1)
      D=D+DEXP(-X4)*X5/X3
  257 X1=XR*TDUR
```

```
      X4=1.D0
      IF (X1.LE.40.D0) X4=1.D0-DEXP(-X1)
      D=D*X4
  258 IF (XU.LE.XL) GO TO 259
      X1=XR*(TIME-XU)
      IF (X1.GE.87.D0) GO TO 259
      X3=XR*(XU-XL)
      X4=1.D0
      IF (X3.LE.40.D0) X4=1.D0-DEXP(-X3)
      D=D+DEXP(-X1)*X4
  259 ETIME(NRT)=D/XR
  260 U(NSBF,M2,NRT)=ETIME(NRT)*F*XNUM(NSBF,M2,NRT)/DEN(NSBF,M2,NRT)
      PRINT 265,N1,M1,(U(NSBF,M2,K),K=1,KMAX)
  265 FORMAT (2I3,1P10D12.4/(8X,10D12.4))
  270 CONTINUE
***         ABSORBED-POWER DENSITY AND TEMPERATURE RISE AT
***         GIVEN POINTS INTERIOR TO P-TH REGION
      IF (ISAR.NE.0) ISAR=1
      PRINT 275,ZLAB(ISAR+1)
  275 FORMAT ('0',29X,'INTERNAL POINT',11X,'ABSORBED POWER',7X,'TEMPERAT
     1URE',8X,'APPROXIMATE'
     1    /11X,'POINT  REGION  RADIUS   THETA    PHI',12X,'DENSITY',14X,
     1'RISE',14X,'ERROR'
     1/28X,'CM      DEG      DEG',12X,A7,13X,'DEG C',13X,'PER CENT'/)
      DO 345 II=1,NOCR
      READ (5,30) R,THETAD,PHID
C             R         R-COORDINATE OF PT
C             THETAD    THETA COORDINATE(DEGREES)
C             PHID      PHI-COORDINATE(IN EQUATORIAL PLANE)(DEGREES)
      IF (R.LE.0.D0) GO TO 290
      DO 285 NREG=1,NORG
      IF (R.LE.SBDP(NREG)) GO TO 300
  285 CONTINUE
  290 NREG=1000000000
      PRINT 295,II,NREG,R,THETAD,PHID
  295 FORMAT (I14,I8,3F10.3,'  ** THE RADIUS IS OUTSIDE THE SPHERE **')
      GO TO 345
C             NREG = NUMBER OF THE REGION IN WHICH TEMP IS TO BE COMPUTED
  300 IREG=NREG
      R1=R
      R=R/1.D2
      THETA=THETAD/RAD
      PHI=PHID/RAD
      CALL BJYH(BJNP,BHNP,FKP(NREG)*R,NC,STOPR,NMIN+2)
      NC=MIN0(NC-2,NMIN)
      SINTH=DSIN(THETA)
      COSTH=DCOS(THETA)
      CALL PL
      CALL EVEC(PD)
      PD=.5D0*SIGP(NREG)*PD
      KMAX1=KMAX
      K1=KMAX-1
      DO 315 KMAX=K1,KMAX1
      TRM=0.D0
      DO 315 NSBF=1,NMAX
      N1=NSBF-1
      DO 315 M=1,NSBF
```

```
      IF (M.NE.1.AND.M.NE.3) GO TO 315
      M1=M-1
      M2=M/2+1
      ALPNM=ALP(N1,M1,COSTH)*DCOS(M1*PHI)
      IF (ALPNM.EQ.0.D0) GO TO 310
      SUM=0.D0
      DO 305 NRT=1,KMAX
      S1=XLAMDA(NRT,NSBF)*RHOP(IREG)*CP(IREG)-BP(IREG)
      F=FACT(IREG,NRT,NSBF)
      CALL SRBF(XJ,XY,DJ,DY)
  305 SUM=SUM+U(NSBF,M2,NRT)*(AJ(NREG,NRT,NSBF)*XJ+BY(NREG,NRT,NSBF)*XY)
  310 TRM=TRM+SUM*ALPNM
      IF (M.NE.3) GO TO 315
      IF (KMAX.EQ.K1.AND.NSBF.EQ.NMAX) SRM1=TRM
      IF (KMAX.EQ.KMAX1.AND.NSBF.EQ.NMAX-1) SBFM1=TRM
  315 CONTINUE
      KMAX=KMAX1
      PCER=(TRM-SRM1)/TRM
      PCEBF=(TRM-SBFM1)/TRM
      IF (ISAR.EQ.0)PD=PD/RHOP(NREG)
***         PRINT PARTICULARS OF INTERIOR POINT OF REGION P
      PRINT 340,II,NREG,R1,THETAD,PHID,PD,TRM,PCEBF,PCER
  340 FORMAT (I14,I8,F10.3,2F8.2,F19.8,1PD20.4,2P2F14.7)
  345 CONTINUE
      IF (IPL1.EQ.0.AND.IPL2.EQ.0) GO TO 10
      IF (IPLSW.EQ.1) GO TO 350
      IPLSW=1
      CALL PLOTS(0,0,8)
      CALL PLOT(0.,-11.,-3)
      CALL PLOT(0.,2.,-3)
      NTR=MAX0(NTR,1)
      NTR=MIN0(NTR,5)
  350 IF (IPL1.EQ.0) GO TO 405
*** PLOT POWER DENSITIES ALONG Z, X AND/OR Y AXIS
      NPTS=300
      NPTD2=NPTS/2
      NP2=NPTD2+1
      DX=SBDP(NORG)/NPTD2
      DO 400 KJI=1,3
      IF (KJI.EQ.1.AND.(IPL1.EQ.2.OR.IPL1.EQ.3.OR.IPL1.EQ.6)) GO TO 400
      IF (KJI.EQ.2.AND.(IPL1.EQ.1.OR.IPL1.EQ.3.OR.IPL1.EQ.5)) GO TO 400
      IF (KJI.EQ.3.AND.(IPL1.EQ.1.OR.IPL1.EQ.2.OR.IPL1.EQ.4)) GO TO 400
      PRINT 355
  355 FORMAT ('0')
      TRMAX=0.
      COSTH=0.D0
      IF (KJI.EQ.1) COSTH=1.D0
      IREG=NORG
      R1=SBDP(NORG)
*** CALCULATE POWER DENSITIES ALONG SPHERE DIAMETER
      DO 370 I=1,NP2
      RC=RHOP(IREG)*CP(IREG)
      BP1=BP(IREG)
      TRM=0.D0
      TRM1=0.D0
      DO 365 NSBF=1,NMAX
      N1=NSBF-1
```

```
      COSMP=1.D0
      DO 365 M=1,NSBF
      IF (M.NE.1.AND.M.NE.3) GO TO 365
      IF (KJI.EQ.3.AND.M.EQ.3) COSMP=-1.D0
      M1=M-1
      M2=M/2+1
      SUM=0.D0
      DO 360 NRT=1,KMAX
      S1=XLAMDA(NRT,NSBF)*RC-BP1
      F=FACT(IREG,NRT,NSBF)
      CALL SRBF(XJ,XY,DJ,DY)
  360 SUM=SUM+U(NSBF,M2,NRT)*(AJ(IREG,NRT,NSBF)*XJ+BY(IREG,NRT,NSBF)*XY)
      TRM=TRM+SUM*ALP(N1,M1,COSTH)*COSMP
      TRM1=TRM1+SUM*ALP(N1,M1,-COSTH)*COSMP
  365 CONTINUE
      R3(I)=R1
      TR3(I)=TRM
      R3(NPTS-I+3)=-R1
      TR3(NPTS-I+3)=TRM1
      TRMAX=DMAX1(TRM,TRM1,TRMAX)
      R1=R1-DX
      IF (IREG.GT.1.AND.R1.LT.SBDP(IREG-1))IREG=IREG-1
  370 IF (R1.LT..0001)R1=.0001
  *** DETERMINE PLOT SCALE FOR POWER DENSITIES
      PD3=.0001
      DO 375 I=1,10
      PD3=5.*PD3
      IF (TRMAX.LT.PD3) GO TO 380
      PD3=PD3*2.
      IF (TRMAX.LT.PD3) GO TO 380
  375 CONTINUE
  380 TRMAX=PD3
  *** PLOT POWER DENSITY ALONG DIAMETER ON Z, X OR Y AXIS
      BLAB(1)=AX(KJI)
      DO 390 I=1,NTR
      ANG=2*(I-1)*PIE/NTR
      AX1=.01*COS(ANG)
      AY=.01*SIN(ANG)
      IF (NTR.EQ.1) AX1=0.
      CALL PLOT(AX1,AY,-3)
  390 CALL PLTCV1(R3,TR3,5.,6.,BLAB,DLAB,22,26,NPTS+2,0,1,1,-R3(1),
     1R3(1),0.,TRMAX,0,0,.14,R3(1)/3.,TRMAX/5.,1)
      CALL PLOT(7.,0.,-3)
  400 CONTINUE
  405 IF (IPL2.EQ.0) GO TO 10
  *** PLOT POWER DENSITY CONTOURS IN E PLANE, H PLANE AND/OR X-Y PLANE
      NPTS=100
      NPTD2=NPTS/2
      NPTP2=NPTS+2
      X1=SBDP(NORG)
      XF=10./(2.*X1)
      DX=X1/NPTD2
      X3=X1
      DO 410 I=1,NPTD2
      X2(I)=X3
      X2(NPTS+3-I)=-X3
  410 X3=X3-DX
```

```
      X2(NPTD2+1)=.0001
      X2(NPTD2+2)=-.0001
*** CALCULATE POWER DENSITIES AT POINTS IN PLANE
      N12=NPTP2/2
      DO 465 KJI=1,3
      IF (KJI.EQ.1.AND.(IPL2.EQ.2.OR.IPL2.EQ.3.OR.IPL2.EQ.6)) GO TO 465
      IF (KJI.EQ.2.AND.(IPL2.EQ.1.OR.IPL2.EQ.3.OR.IPL2.EQ.5)) GO TO 465
      IF (KJI.EQ.3.AND.(IPL2.EQ.1.OR.IPL2.EQ.2.OR.IPL2.EQ.4)) GO TO 465
      Y3=0.
      X3=0.
      Z3=0.
      DO 455 I=1,N12
      I1=NPTS+3-I
      DO 455 J=1,N12
      J1=NPTS+3-J
      IF (KJI.GT.1) GO TO 415
      X3=X2(I)
      Z3=X2(J)
      GO TO 425
415   IF (KJI.GT.2) GO TO 420
      Y3=X2(I)
      Z3=X2(J)
      GO TO 425
420   X3=X2(I)
      Y3=X2(J)
425   R1=DSQRT(X3*X3+Y3*Y3+Z3*Z3)
      IF (R1.LE.X1) GO TO 430
      DAR(I,J)=-1.
      DAR(I,J1)=-1.
      GO TO 453
430   DO 435 IREG=1,NORG
      IF (R1.LE.SBDP(IREG)) GO TO 440
435   CONTINUE
*** CALCULATE TEMPERATURE RISE AT POINTS IN PLANE
440   COSTH=Z3/R1
      PHI=DATAN2(Y3,X3)
      RC=RHOP(IREG)*CP(IREG)
      BP1=BP(IREG)
      TRM=0.D0
      TRM1=0.D0
      DO 450 NSBF=1,NMAX
      N1=NSBF-1
      DO 450 M=1,NSBF
      IF (M.NE.1.AND.M.NE.3) GO TO 450
      M1=M-1
      M2=M/2+1
      SUM=0.D0
      DO 445 NRT=1,KMAX
      S1=XLAMDA(NRT,NSBF)*RC-BP1
      F=FACT(IREG,NRT,NSBF)
      CALL SRBF(XJ,XY,DJ,DY)
445   SUM=SUM+U(NSBF,M2,NRT)*(AJ(IREG,NRT,NSBF)*XJ+BY(IREG,NRT,NSBF)*XY)
      TRM=TRM+SUM*ALP(N1,M1,COSTH)*DCOS(M1*PHI)
      TRM1=TRM1+SUM*ALP(N1,M1,-COSTH)*DCOS(M1*PHI)
450   CONTINUE
      DAR(I,J)=TRM
      DAR(I,J1)=TRM1
```

```
453 DAR(I1,J)=DAR(I,J)
455 DAR(I1,J1)=DAR(I,J1)
*** PLOT CONTOURS
    DO 460 I=1,NTR
    ANG=2*(I-1)*PIE/NTR
    AX1=.01*COS(ANG)
    AY=.01*SIN(ANG)
    IF (NTR.EQ.1) AX1=0.
    CALL PLOT(AX1,AY,-3)
    CALL SYMBOL(-.5,6.,.21,CLAB(1,KJI),0.,9)
460 CALL CNTRP1(X2,NPTP2,X2,NPTP2,DAR,10,0,IFL)
    CALL PLOT(10.,0.,-3)
465 CONTINUE
    GO TO 10
495 IF (IPLSW.NE.0) CALL PLOT(0.,0.,999)
500 STOP
    END


    SUBROUTINE COEF
    IMPLICIT REAL*8 (A-H,O-Z)
         GENERATES EXPANSION COEFFICIENTS
    COMPLEX*16 FKP,ANP,BNP,ALPNP,BETNP,BJNP,BHNP,BJHP1(500),BJHP2(500)
   1,SJNP1(100),SHNP1(100),DELNP,SNT11,SNT12,SNT21,SNT22,TNT11,TNT12,T
   1NT21,TNT22,ETAP1,ETAP2,ZEP1,ZEP2,SNP11,SNP12,SNP21,SNP22,TNP11,TNP
   112,TNP21,TNP22,DEL1,DEL2,RATIO,Z
    COMMON FKP(7),ANP(300),BNP(300),ALPNP(300),BETNP(300),BJNP(100),BH
   1NP(100),BDP(6),P(51),DP(50),R,THETA,COSTH,PHI,SINTH,STOPR,EO
    COMMON /A/NORG,NREG,NRT,NSBF,NMIN,NC,ICODE
    DIMENSION NTER(6)
         COMPUTE EXPANSION COEFFICIENTS AN1,BN1,ANN,BNN,ALPN1,BETN1,
         ALPNN,BETNN
    N1=1
    NMIN=100
    DO 10 NR=1,NORG
    CALL BJYH(SJNP1,SHNP1,FKP(NR)*BDP(NR),N,STOPR,NMIN)
    CALL BJYH(BJNP,BHNP,FKP(NR+1)*BDP(NR),NN,STOPR,NMIN)
    NMIN=MIN0(N,NN,NMIN)
    N2=N1+NMIN
    DO 5 I=1,NMIN
    BJHP1(N1)=SJNP1(I)
    BJHP1(N2)=SHNP1(I)
    BJHP2(N1)=BJNP(I)
    BJHP2(N2)=BHNP(I)
    N1=N1+1
  5 N2=N2+1
    N1=N1+NMIN
 10 NTER(NR)=NMIN
    NMIN=NMIN-2
    IF (NMIN.LE.50.AND.N2.LE.301) GO TO 20
    PRINT 15,N2,NMIN
 15 FORMAT ('OCOEF ERROR: N2 =',I3,'  NMIN =',I3,'  DIMENSIONS ARE TOO
   1 SMALL')
    STOP
 20 DO 25 I=1,NMIN
    ALPNP(I)=DCMPLX(0.D0,0.D0)
 25 BETNP(I)=DCMPLX(0.D0,0.D0)
```

```
      NSUM=NORG*NMIN
      DO 35 I=1,NMIN
      JJ=0
      KK=0
      XI=I
      XI1=I+1
      XI2=2*I+1
      SNT11=DCMPLX(1.D0,0.D0)
      SNT12=DCMPLX(0.D0,0.D0)
      SNT21=SNT12
      SNT22=SNT11
      TNT11=SNT11
      TNT12=SNT12
      TNT21=SNT12
      TNT22=SNT11
      DO 30 J=1,NORG
      KK=KK+NTER(J)
      KKI=KK+I
      JJI=JJ+I
      ETAP1=(XI1*BJHP1(JJI)-XI*BJHP1(JJI+2))/XI2
      ETAP2=(XI1*BJHP2(JJI)-XI*BJHP2(JJI+2))/XI2
      ZEP1=(XI1*BJHP1(KKI)-XI*BJHP1(KKI+2))/XI2
      ZEP2=(XI1*BJHP2(KKI)-XI*BJHP2(KKI+2))/XI2
      DELNP=BJHP1(JJI+1)*ZEP1-BJHP1(KKI+1)*ETAP1
      RATIO=FKP(J+1)/FKP(J)
      Z=RATIO*ETAP2
      SNP11=(ZEP1*BJHP2(JJI+1)-Z*BJHP1(KKI+1))/DELNP
      SNP21=(Z*BJHP1(JJI+1)-ETAP1*BJHP2(JJI+1))/DELNP
      Z=RATIO*ZEP2
      SNP12=(ZEP1*BJHP2(KKI+1)-Z*BJHP1(KKI+1))/DELNP
      SNP22=(Z*BJHP1(JJI+1)-ETAP1*BJHP2(KKI+1))/DELNP
      Z=SNT11
      SNT11=SNT11*SNP11+SNT12*SNP21
      SNT12=Z*SNP12+SNT12*SNP22
      Z=SNT21
      SNT21=SNT21*SNP11+SNT22*SNP21
      SNT22=Z*SNP12+SNT22*SNP22
      Z=RATIO*ZEP1
      TNP11=(Z*BJHP2(JJI+1)-BJHP1(KKI+1)*ETAP2)/DELNP
      TNP12=(Z*BJHP2(KKI+1)-BJHP1(KKI+1)*ZEP2)/DELNP
      Z=RATIO*ETAP1
      TNP21=(BJHP1(JJI+1)*ETAP2-Z*BJHP2(JJI+1))/DELNP
      TNP22=(BJHP1(JJI+1)*ZEP2-Z*BJHP2(KKI+1))/DELNP
      Z=TNT11
      TNT11=TNT11*TNP11+TNT12*TNP21
      TNT12=Z*TNP12+TNT12*TNP22
      Z=TNT21
      TNT21=TNT21*TNP11+TNT22*TNP21
      TNT22=Z*TNP12+TNT22*TNP22
      JJ=JJ+2*NTER(J)
   30 KK=KK+NTER(J)
      ANP(I)=SNT11-(SNT12*SNT21)/SNT22
      BNP(I)=TNT11-(TNT12*TNT21)/TNT22
      LL=NSUM+I
      ANP(LL)=DCMPLX(1.D0,0.D0)
      BNP(LL)=DCMPLX(1.D0,0.D0)
      ALPNP(LL)=-SNT21/SNT22
```

```
35 BETNP(LL)=-TNT21/TNT22
   IF (NORG.EQ.1) RETURN
        COMPUTE EXPANSION COEFFICIENTS AN2,...,AN(N-1);BN2,...,BN(N-1
        );ALPN2,...,ALPN(N-1);BETN2,...,BETN(N-1)
   JJ=0
   KK=0
   MM1=0
   MM2=NMIN
   NRGM1=NORG-1
   DO 45 J=1,NRGM1
   KK=KK+NTER(J)
   DO 40 I=1,NMIN
   KKI=KK+I
   JJI=JJ+I
   XI=I
   XI1=I+1
   XI2=2*I+1
   ETAP1=(XI1*BJHP1(JJI)-XI*BJHP1(JJI+2))/XI2
   ETAP2=(XI1*BJHP2(JJI)-XI*BJHP2(JJI+2))/XI2
   ZEP1=(XI1*BJHP1(KKI)-XI*BJHP1(KKI+2))/XI2
   ZEP2=(XI1*BJHP2(KKI)-XI*BJHP2(KKI+2))/XI2
   DELNP=BJHP1(JJI+1)*ZEP1-BJHP1(KKI+1)*ETAP1
   RATIO=FKP(J+1)/FKP(J)
   Z=RATIO*ETAP2
   SNP11=(ZEP1*BJHP2(JJI+1)-Z*BJHP1(KKI+1))/DELNP
   SNP21=(Z*BJHP1(JJI+1)-ETAP1*BJHP2(JJI+1))/DELNP
   Z=RATIO*ZEP2
   SNP12=(ZEP1*BJHP2(KKI+1)-Z*BJHP1(KKI+1))/DELNP
   SNP22=(Z*BJHP1(JJI+1)-ETAP1*BJHP2(KKI+1))/DELNP
   DEL1=SNP11*SNP22-SNP12*SNP21
   Z=RATIO*ZEP1
   TNP11=(Z*BJHP2(JJI+1)-BJHP1(KKI+1)*ETAP2)/DELNP
   TNP12=(Z*BJHP2(KKI+1)-BJHP1(KKI+1)*ZEP2)/DELNP
   Z=RATIO*ETAP1
   TNP21=(BJHP1(JJI+1)*ETAP2-Z*BJHP2(JJI+1))/DELNP
   TNP22=(BJHP1(JJI+1)*ZEP2-Z*BJHP2(KKI+1))/DELNP
   DEL2=TNP11*TNP22-TNP12*TNP21
   NN1=MM1+I
   NN2=MM2+I
   ANP(NN2)=(ANP(NN1)*SNP22-ALPNP(NN1)*SNP12)/DEL1
   BNP(NN2)=(BNP(NN1)*TNP22-BETNP(NN1)*TNP12)/DEL2
   ALPNP(NN2)=(-ANP(NN1)*SNP21+ALPNP(NN1)*SNP11)/DEL1
40 BETNP(NN2)=(-BNP(NN1)*TNP21+BETNP(NN1)*TNP11)/DEL2
   JJ=JJ+2*NTER(J)
   KK=KK+NTER(J)
   MM1=MM1+NMIN
45 MM2=MM2+NMIN
   RETURN
   END


   SUBROUTINE EVEC(PD)
   IMPLICIT REAL*8 (A-H,O-Z)
        COMPUTES THE RADIAL,COLATITUDE,AND AZIMUTHAL
        COMPONENTS OF ELECTRIC FIELD VECTOR E FOR
   REGION P AND SCALAR PRODUCT  E.E*
   COMPLEX*16 FKP,ANP,BNP,ALPNP,BETNP,BJNP,BHNP,ERAD,ETHETA,EPHI,T,T1
```

```
     1,C,W,X,Y,Z
      COMMON FKP(7),ANP(300),BNP(300),ALPNP(300),BETNP(300),BJNP(100),BH
     1NP(100),BDP(6),P(51),DP(50),R,THETA,COSTH,PHI,SINTH,STOPR,EO
      COMMON /A/NORG,NREG,NRT,NSBF,NMIN,NC,ICODE
      DATA EPS/1.D-8/
      ERAD=DCMPLX(0.D0,0.D0)
      ETHETA=DCMPLX(0.D0,0.D0)
      EPHI=DCMPLX(0.D0,0.D0)
      NCK=0
      NN=(NREG-1)*NMIN
      IR=0
      IF (THETA.EQ.0.D0) IR=1
      ITP=0
      DO 25 N=1,NC
      FAC1=2*N+1
      NNN=NN+N
      W=BNP(NNN)
      X=BJNP(N+1)
      Y=BETNP(NNN)
      Z=BHNP(N+1)
      NCK=NCK+1
      IF (IR.EQ.1) GO TO 5
      T=FAC1*P(N)*(W*X+Y*Z)
      CALL TERM(NCK,T,1)
      ERAD=ERAD+T
      IF (CDABS(T).LT.CDABS(ERAD)*EPS)IR=1
    5 IF (ITP.EQ.1) GO TO 20
      T=ANP(NNN)*X+ALPNP(NNN)*Z
      CALL TERM(NCK,T,0)
      NP1=N+1
      RATIO=FAC1/(N*NP1)
      A=NP1
      B=N
      T1=(W*(A*BJNP(N)-B*BJNP(N+2))+Y*(A*BHNP(N)-B*BHNP(N+2)))/FAC1
      CALL TERM(NCK,T1,1)
      IF (SINTH.GT.1.D-6) GO TO 10
      A=FAC1/2.D0
      IF (THETA.GE.3.14159D0)A=A*(-1.D0)**NP1
      GO TO 15
   10 A=RATIO*P(N)/SINTH
   15 B=-RATIO*DP(N)
      C=A*T+B*T1
      ETHETA=ETHETA+C
      T=A*T1+B*T
      EPHI=EPHI+T
      IF (CDABS(C).LT.CDABS(ETHETA)*EPS.AND.CDABS(T).LT.CDABS(EPHI)*EPS)
     1ITP=1
   20 IF (IR+ITP.EQ.2) GO TO 35
   25 IF (NCK.EQ.4)NCK=0
      PRINT 30,NMIN,NC,STOPR,EPS
   30 FORMAT (15X,'NMIN =',I3,' NC =',I3,'  STOPR =',1PD14.4,' IS TOO SM
     1ALL FOR ACCURACY OF',D14.4)
   35 ECOSPH=EO*DCOS(PHI)
      ERAD=-ECOSPH/(FKP(NREG)*R)*ERAD
      ETHETA=ECOSPH*ETHETA
      EPHI=EO*DSIN(PHI)*EPHI
      PD=DREAL(ERAD*DCONJG(ERAD))+DREAL(ETHETA*DCONJG(ETHETA))+DREAL(EPH
```

```
 1I*DCONJG(EPHI))
  RETURN
  END


  SUBROUTINE TERM(NCK,T,KEY)
  IMPLICIT REAL*8 (A-H,O-Z)
       COMPUTES I**NCK*(N-TH TERM IN SERIES)
  COMPLEX*16 T
  IF (KEY.EQ.1) GO TO 5
  GO TO (10,15,20,25),NCK
5 GO TO (15,20,25,10),NCK
10 T=DCMPLX(-DIMAG(T),DREAL(T))
  GO TO 25
15 T=-T
  GO TO 25
20 T=DCMPLX(DIMAG(T),-DREAL(T))
25 RETURN
  END


  SUBROUTINE PL
  IMPLICIT REAL*8 (A-H,O-Z)
       ASSOCIATED LEGENDRE FUNCTIONS OF THE FIRST
       KIND, DEGREE K AND ORDER 1 AND THEIR FIRST
       DERIVATIVES
  COMPLEX*16 FKP,ANP,BNP,ALPNP,BETNP,BJNP,BHNP
  COMMON FKP(7),ANP(300),BNP(300),ALPNP(300),BETNP(300),BJNP(100),BH
 1NP(100),BDP(6),P(51),DP(50),R,THETA,COSTH,PHI,SINTH,STOPR,EO
  COMMON /A/NORG,NREG,NRT,NSBF,NMIN,NC,ICODE
  P(1)=SINTH
  P(2)=3.DO*SINTH*COSTH
  DP(1)=COSTH
  DO 10 M=2,NC
  A=M
  MP1=M+1
  P(MP1)=(2.DO*A+1.DO)/A*COSTH*P(M)-(A+1.DO)/A*P(M-1)
  IF (SINTH.GT.1.D-6) GO TO 5
  DP(M)=M*MP1/2
  IF (THETA.GE.3.141591DO)DP(M)=(-1.DO)**M*DP(M)
  GO TO 10
5 DP(M)=(A*COSTH*P(M)-(A+1.DO)*P(M-1))/SINTH
10 CONTINUE
  RETURN
  END


  FUNCTION ALP(N,M,X)
  IMPLICIT REAL*8 (A-H,O-Z)
        ASSOCIATED LEGENDRE FUNCTIONS OF THE FIRST KIND,
       DEGREE N AND ORDER M.  N AND M GTE 0,  N GTE M
  FM=M
  IF (M.GT.0) GO TO 5
  P1=1.DO
  GO TO 25
5 IF (M.GT.1) GO TO 10
  SUM=2.DO
```

```
      GO TO 20
   10 J=2*M
      SUM=J
      IS=M-1
      DO 15 I=1,IS
   15 SUM=SUM*(J-I)
   20 P1=SUM*((1.D0-X*X)**(FM/2.D0))/(2.D0**M)
   25 IF (N.NE.M) GO TO 30
      ALP=P1
      GO TO 40
   30 ALP=(2.D0*FM+1.D0)*X*P1
      IF (N.EQ.M+1) GO TO 40
      IS=N-M
      DO 35 I=2,IS
      P2=ALP
      C1=2*(M+I)-1
      ALP=(C1*X*P2-(C1-I)*P1)/I
   35 P1=P2
   40 RETURN
      END


      SUBROUTINE RFNDR(RSTART,STEP1,E,NRTS,M1,NITR)
      IMPLICIT REAL*8 (A-H,O-Z)
            ROOT FINDER
      COMMON /A/NORG,NREG,NRT,NSBF,NMIN,NC,ICODE
      COMMON /B/FACT(6,25,18),AJ(6,25,18),[illegible](6,25,18),XLAMDA(25,18),SBDP
     1(6),RHOP(6),CP(6),BP(6),TCP(6),H
      EXTERNAL FNCAL
      STEP=STEP1
      M=M1-3
      I=1
      SL=RSTART
    5 X=SL
      NRT=I
      W=FNCAL(X)
   10 IF (W)  15,55,25
   15 DO 20 J=1,M
      X=X+STEP
      V=FNCAL(X)
      IF (V)  20,55,50
   20 W=V
      GO TO 35
   25 DO 30 J=1,M
      X=X+STEP
      V=FNCAL(X)
      IF (V)  50,55,30
   30 W=V
   35 IF(M.GT.1000) GO TO 40
      M=M+1
      STEP=STEP*1.D1
      GO TO 10
   40 PRINT 45,SL,X
   45 FORMAT ('ORFNDR ERROR: NO ROOTS FROM',1PE14.4,' TO',E14.4)
      STOP
   50 SL=X-STEP
      SR=X
```

```
      CALL DRTMI(X,F,FNCAL,SL,SR,W,V,E,NITR)
   55 XLAMDA(I,NSBF)=X
      SL=X+STEP
      IF(I+NSBF.EQ.2) STEP =DMAX1(X/10.D0,STEP)
      IF (I.GT.1)STEP=(X-XLAMDA(I-1,NSBF))/10.D0
      I=I+1
      IF (I.LE.NRTS) GO TO 5
      RETURN
      END


      SUBROUTINE DRTMI(X,F,FCT,XLI,XRI,FLI,FRI,EPS,IEND)
      IMPLICIT REAL*8 (A-H,O-Z)
            BISECTION METHOD
      XL=XLI
      XR=XRI
      FL=FLI
      FR=FRI
      I=0
      TOLF=100.D0*EPS
    5 I=I+1
      DO 30 K=1,IEND
      X=.5D0*(XL+XR)
      F=FCT(X)
      IF (F.EQ.0.D0) GO TO 45
      IF (DSIGN(1.D0,F)+DSIGN(1.D0,FR).NE.0.D0) GO TO 10
      TOL=XL
      XL=XR
      XR=TOL
      TOL=FL
      FL=FR
      FR=TOL
   10 TOL=F-FL
      A=F*TOL
      A=A+A
      IF (A.GE.FR*(FR-FL)) GO TO 25
      IF (I.GT.IEND) GO TO 25
      A=FR-F
      DX=(X-XL)*FL*(1.D0+F*(A-TOL)/(A*(FR-FL)))/TOL
      XM=X
      FM=F
      X=XL-DX
      F=FCT(X)
      IF (F.EQ.0.D0) GO TO 45
      TOL=EPS
      A=DABS(X)
      IF (A.GT.1.D0)TOL=TOL*A
      IF (DABS(DX).GT.TOL) GO TO 15
      IF (DABS(F).LE.TOLF) GO TO 45
   15 IF (DSIGN(1.D0,F)+DSIGN(1.D0,FL).NE.0.D0) GO TO 20
      XR=X
      FR=F
      GO TO 5
   20 XL=X
      FL=F
      XR=XM
      FR=FM
```

```
      GO TO 5
   25 XR=X
      FR=F
      TOL=EPS
      A=DABS(XR)
      IF (A.GT.1.D0)TOL=TOL*A
      IF (DABS(XL-XR).GT.TOL) GO TO 30
      IF (DABS(FR-FL).LE.TOLF) GO TO 40
   30 CONTINUE
      PRINT 35,XL,XR
   35 FORMAT ('ODRTMI ERROR: ROOT BETWEEN',1PD15.7,' AND',D15.7,' MAY BE
     1 INACCURATE')
   40 IF (DABS(FR).LE.DABS(FL)) GO TO 45
      X=XL
      F=FL
   45 RETURN
      END


      FUNCTION FNCAL(EIGV)
C           FUNCTION EVALUATOR USED IN THE DETERMINATION
C           OF THE EIGENVALUES LAMBDANK
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /B/FACT(6,25,18),AJ(6,25,18),BY(6,25,18),XLAMDA(25,18),SBDP
     1(6),RHOP(6),CP(6),BP(6),TCP(6),H
      COMMON /C/AJ1,S,F,R,I
      COMMON /A/NORG,NREG,NRT,NSBF,NMIN,NC,ICODE
      BY(1,NRT,NSBF) = 0.D0
      DO 35 I = 1,NORG
      S  =(EIGV*RHOP(I)*CP(I)-BP(I))/TCP(I)
      F=DSQRT(DABS(S))
      FACT(I,NRT,NSBF)=F
      IF (I.NE.1) GO TO 27
      IF (F.NE.0.D0) GO TO 5
      AJ(1,NRT,NSBF)=AJ1
      GO TO 30
    5 AJ(1,NRT,NSBF)=AJ1/F**(NSBF-1)
      IF (S.LT.0.D0) AJ(1,NRT,NSBF)=AJ(1,NRT,NSBF)/((-1)**((NSBF-1)/2))
      GO TO 30
   27 R=SBDP(I-1)
      CALL SRBF(AM,BM,ATM,BTM)
      DELTA = AM*BTM-ATM*BM
      T1 = AJ(I-1,NRT,NSBF)*A + BY(I-1,NRT,NSBF)*BE
      T2 = AJ(I-1,NRT,NSBF)*AT + BY(I-1,NRT,NSBF)*BT
      AJ(I,NRT,NSBF) = (T1*BTM-T2*BM)/DELTA
      BY(I,NRT,NSBF) = (T2*AM-T1*ATM)/DELTA
   30 R=SBDP(I)
   35 CALL SRBF(A,BE,AT,BT)
      FNCAL=AJ(NORG,NRT,NSBF)*AT+BY(NORG,NRT,NSBF)*BT
     1 +H*(AJ(NORG,NRT,NSBF)*A+BY(NORG,NRT,NSBF)*BE)
      RETURN
      END


      SUBROUTINE BJYH(BJNP,BHNP,Z,N,STOPR,NBF)
      IMPLICIT COMPLEX*16(A-H,O-Z)
      DIMENSION BJNP(62),BHNP(62)
```

```
      REAL*8 STOPR,X,XNPH,DREAL,DIMAG
      BJNP(1)=CDSIN(Z)/Z
      BJNP(2)=(BJNP(1)-CDCOS(Z))/Z
      ZTI=DCMPLX(-DIMAG(Z),DREAL(Z))
      T1=CDEXP(ZTI)/Z
      T1=DCMPLX(DIMAG(T1),-DREAL(T1))
      BHNP(1)=T1
      BHNP(2)=DCMPLX(DIMAG(T1),-DREAL(T1))*(1.D0-1.D0/ZTI)
      ZSQ=Z*Z
      TDZ=2.D0/Z
      X=1.D0/STOPR
      DO 15 N=3,NBF
      XNPH=DFLOAT(N)-.5D0
      XNU=-(XNPH+1.D0)*TDZ
      A1=XNPH*TDZ
      DEN=XNU+1.D0/A1
      F=XNU/(DEN*A1)
      CF=-TDZ
      DO 5 I=2,200
      CF=-CF
      A1=CF*(XNPH+I)
      XNU=A1+1.D0/XNU
      DEN=A1+1.D0/DEN
      F1=XNU/DEN
      F=F*F1
      IF (DABS(CDABS(F1)-1.D0).LT.1.D-14) GO TO 10
    5 CONTINUE
   10 BJNP(N)=F*BJNP(N-1)
      Q=1.D0/(ZSQ*BJNP(N-1))
      BHNP(N)=F*BHNP(N-1)-DCMPLX(-DIMAG(Q),DREAL(Q))
      IF (CDABS(BJNP(N)).LT.X.OR.CDABS(BHNP(N)).GT.STOPR) GO TO 20
   15 CONTINUE
      N=N-1
   20 IF (N.LT.5) PRINT 25,N,Z
   25 FORMAT (25X,'ONLY',I3,' BESSEL FUNCTIONS FOR Z =',1P2D12.4)
      RETURN
      END


      SUBROUTINE SRBF (A,Y,AD,YD)
            GET J, J', Y AND Y' FOR NEWTON'S COOLING FUNCTION AND RETURN
            THE APPROPRIATE PART OF COMPLEX VALUES ADJUSTED FOR REAL
            VALUE CALCULATIONS
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /A/NORG,NREG,NRT,NSBF,NMIN,NC,ICODE
      COMMON /B/FACT(6,25,18),AJ(6,25,18),BY(6,25,18),XLAMDA(25,18),SBDP
     1(6),RHOP(6),CP(6),BP(6),TCP(6),H
      COMMON /C/AJ1,S,F,R,I
      COMPLEX*16 BJ,YF,BJD,BYD
      COMMON /BES/BJ,YF,BJD,BYD,RJ,RY,RJD,RYD
      IF (S) 5,15,20
    5 CALL CSBFD(DCMPLX(0.D0,R*F))
            FOR S<0.
      IF (NSBF.EQ.2*(NSBF/2)) GO TO 10
            FOR S<0. AND EVEN ORDER BESSEL FUNCTIONS
      A=DREAL(BJ)
      Y=DIMAG(YF)
```

```
      IF (ICODE.EQ.1) GO TO 25
      C=TCP(I)*F
      AD=-C*DIMAG(BJD)
      YD=C*DREAL(BYD)
      GO TO 25
            FOR S<0. AND ODD ORDER BESSEL FUNCTIONS
   10 A=DIMAG(BJ)
      Y=DREAL(YF)
      IF (ICODE.EQ.1) GO TO 25
      C=TCP(I)*F
      AD=C*DREAL(BJD)
      YD=-C*DIMAG(BYD)
      GO TO 25
            FOR S=0.
   15 A=R**(NSBF-1)
      Y=1.D0/R**(NSBF)
      IF (ICODE.EQ.1) GO TO 25
      AD=TCP(I)*(NSBF-1)*R**(NSBF-2)
      YD=-TCP(I)*(NSBF)/R**(NSBF+1)
      GO TO 25
            FOR S>0.
   20 CALL SBFAD(R*F)
      A=RJ
      Y=RY
      IF (ICODE.EQ.1) GO TO 25
      C=TCP(I)*F
      AD=C*RJD
      YD=C*RYD
   25 RETURN
      END


      SUBROUTINE SBFAD(Z)
            SPHERICAL BESSEL FUNCTIONS OF THE FIRST
            AND SECOND KINDS AND THEIR FIRST DERIVATIVES
            FOR REAL ARGUMENT
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /A/NORG,NREG,NRT,NSBF,NMIN,NC,ICODE
      COMPLEX*16 BJ,YF,BJD,BYD
      COMMON /BES/BJ,YF,BJD,BYD,RJ,RY,RJD,RYD
      COMMON /C/AJ1,S,F,R,I
      SINZ = DSIN(Z)/Z
      COSZ =  DCOS(Z)/Z
      Y1 = -COSZ
      RY = Y1/Z - SINZ
      IF(NSBF.GE.3) GO TO 12
      IF(NSBF.GT.1) GO TO 25
      RJ = SINZ
      RY=-COSZ
      IF(ICODE.EQ.1) GO TO 55
      RJD= COSZ - SINZ/Z
      RYD = SINZ + COSZ/Z
      GO TO 55
   12 IF (I.EQ.1) GO TO 25
      DO 15 M = 3,NSBF
      Y0=Y1
      Y1=RY
```

```
      IF (DABS(Y1).GT.1.D34) PRINT 500,NRT,M,NSBF,Z,Y1
500   FORMAT (' SBFAD: ROOT',I3,' FOR BF',I3,' OF',I3,'  Z =',1PD12.4,
     1'  Y =',D12.4)
   15 RY = (2*M-3)*Y1/Z  - Y0
   25 C = DABS(Z)
      IF(C.GE.3.D0) GO TO 30
      RJ = BES1(NSBF-1,Z)
      GO TO 35
   30 RJ = SBFJ(NSBF-1,Z)
   35 IF(ICODE.EQ.1) GO TO 55
      IF(NSBF.GT.2) GO TO 40
      BJ1= SINZ
      GO TO 50
   40 IF(C.GE.3.D0) GO TO 45
      BJ1 = BES1 (NSBF-2,Z)
      GO TO 50
   45 BJ1 = SBFJ (NSBF-2,Z)
   50 RJD = BJ1 - NSBF*RJ/Z
      RYD = Y1 - NSBF*RY/Z
   55 RETURN
      END


      FUNCTION BES1(N,Z)
      IMPLICIT REAL*8 (A-H,O-Z)
      BES1=DSIN(Z)/Z
      IF (N.EQ.0) GO TO 15
      TDZ=2.D0/Z
      I1=0
      DO 10 M=1,N
      XNUPH=DFLOAT(M)+.5D0
      A0=XNUPH*TDZ
      A1=-(XNUPH+1.D0)*TDZ
      RNUM=A1+1.D0/A0
      RDEN=A1
      COLD=A0*RNUM/RDEN
      CFAC=-TDZ
      DO 5 I=2,200
      CFAC=-CFAC
      A=CFAC*(XNUPH+I)
      RNUM=A+1.D0/RNUM
      RDEN=A+1.D0/RDEN
      C=RNUM/RDEN
      COLD=COLD*C
      IF (DABS(DABS(C)-1.D0).LT.1.D-8) GO  TO 10
    5 CONTINUE
   10 BES1=BES1/COLD
   15 RETURN
      END


      FUNCTION SBFJ(N,Z)
      IMPLICIT REAL*8 (A-H,O-Z)
      Q=0.D0
      P=1.D0
      IF (N.EQ.0) GO TO 10
      XN1=N+1
```

```
      XN2=N
      F=1.D0
      Z2=2.D0*Z
      T=1.D0
    5 T=T*((XN1*XN2)/(F*Z2))
      Q=Q+T
      IF (XN2.EQ.1.D0) GO TO 10
      XN1=XN1+1.D0
      XN2=XN2-1.D0
      F=F+1.D0
      T=-T*((XN1*XN2)/(F*Z2))
      P=P+T
      IF (XN2.EQ.1.D0) GO TO 10
      XN1=XN1+1.D0
      XN2=XN2-1.D0
      F=F+1.D0
      GO TO 5
   10 A=Z-N*1.5707963267948965D0
      SBFJ=(P*DSIN(A)+Q*DCOS(A))/Z
      RETURN
      END


      SUBROUTINE CSBFD(Z)
      IMPLICIT COMPLEX*16 (A-H,O-Z)
      COMMON /BES/BJ,YF,BJD,BYD
      COMMON /A/NORG,NREG,NRT,NSBF,NMIN,NC,ICODE
      COMMON /C/AJ1,S,F,R,I
      REAL*8 C,AJ1,S,F,R
      C = CDABS(Z)
      SINZ = CDSIN(Z)/Z
      COSZ = CDCOS(Z)/Z
      Y1 = -COSZ
      YF = Y1/Z - SINZ
      IF(NSBF.GE.3) GO TO 12
      IF(NSBF.GT.1) GO TO 25
      BJ = SINZ
      YF=-COSZ
      IF(ICODE.EQ.1) GO TO 55
      BJD= COSZ - SINZ/Z
      BYD = SINZ + COSZ/Z
      GO TO 55
   12 IF (I.EQ.1) GO TO 25
      DO 15 M = 3,NSBF
      Y0=Y1
      Y1=YF
   15 YF = (2*M-3)*Y1/Z  - Y0
   25 IF(C.GE.15.D0) GO TO 30
      BJ = BES1C(NSBF-1,Z)
      GO TO 35
   30 BJ = SBFJC(NSBF-1,Z)
   35 IF(ICODE.EQ.1) GO TO 55
      IF(NSBF.GT.2) GO TO 40
      BJ1= SINZ
      GO TO 50
   40 IF(C.GE.15.D0) GO TO 45
      BJ1 = BES1C(NSBF-2,Z)
```

```
      GO TO 50
   45 BJ1 = SBFJC(NSBF-2,Z)
   50 BJD = BJ1 - NSBF*BJ/Z
      BYD = Y1 - NSBF*YF/Z
   55 RETURN
      END


      FUNCTION BES1C(N,Z)
      IMPLICIT COMPLEX*16 (A-H,O-Z)
      BES1C = CDSIN(Z)/Z
      IF(N.EQ.0) GO TO 15
      BES1C=(BES1C-CDCOS(Z))/Z
      IF (N.EQ.1) GO TO 15
      TDZ = 2.D0/Z
      DO 10 M = 2,N
      CM = DCMPLX(DFLOAT(M),0.D0)
      XNUPH = CM + .5D0
      A0 = XNUPH*TDZ
      A1 = -(XNUPH + 1.D0)*TDZ
      RNUM = A1 + 1.D0/A0
      RDEN = A1
      COLD = A0*RNUM/RDEN
      CFAC = -TDZ
      DO 5 I = 2,200
      CI = DCMPLX(DFLOAT(I),0.D0)
      CFAC = -CFAC
      A = CFAC*(XNUPH + CI)
      RNUM = A + 1.D0/RNUM
      RDEN = A + 1.D0/RDEN
      C = RNUM/RDEN
      COLD = COLD*C
      IF(DABS(CDABS(C)-1.D0).LT.1.D-8) GO TO 10
    5 CONTINUE
   10 BES1C = BES1C/COLD
   15 RETURN
      END


      FUNCTION SBFJC(N,Z)
      IMPLICIT COMPLEX*16 (A-H,O-Z)
      REAL*8 XN1,XN2,F,DREAL,DIMAG
      Q=0.D0
      P=1.D0
      IF (N.EQ.0) GO TO 10
      XN1=N+1
      XN2=N
      F=1.D0
      Z2=2.D0*Z
      T=1.D0
    5 T=T*((XN1*XN2)/(F*Z2))
      Q=Q+T
      IF (XN2.EQ.1.D0) GO TO 10
      XN1=XN1+1.D0
      XN2=XN2-1.D0
      F=F+1.D0
      T=-T*((XN1*XN2)/(F*Z2))
```

```
      P=P+T
      IF (XN2.EQ.1.DO) GO TO 10
      XN1=XN1+1.DO
      XN2=XN2-1.DO
      F=F+1.DO
      GO TO 5
   10 A = Z - DCMPLX(DFLOAT(N)*1.5707963267948965D0,0.D0)
      T = (P*CDSIN(A)+Q*CDCOS(A))/Z
      IF (DREAL(Z).EQ.0.D0) GO TO 17
      SBFJC=T
      GO TO 20
   17 IF (N.NE.2*(N/2)) GO TO 15
      SBFJC=DCMPLX(DREAL(T),0.D0)
      GO TO 20
   15 SBFJC=DCMPLX(0.D0,DIMAG(T))
   20 RETURN
      END


      SUBROUTINE EPROP(F ,ITIS,EPS,SIG)
      IMPLICIT REAL*8 (A-H,O-Z)
            INTERPOLATE EPS AND SIGMA FROM TABLES
            F       FREQUENCY IN MEGAHERTZ
            ITIS    TISSUE TYPE
              1 DENOTES CEREBROSPINAL FLUID
              2 DENOTES BLOOD
              3 DENOTES MUSCLE
              4 DENOTES SKIN OR DURA
              5 DENOTES BRAIN
              6 DENOTES FAT OR BONE
              7 DENOTES YELLOW BONE MARROW
            EPS     REAL PART OF DIELECTRIC CONSTANT
            SIG     CONDUCTIVITY
      DIMENSION FR(32),EA(32,7),SA(32,7),SA1(128),SA5(96),EA1(128),EA5(9
     16)
      EQUIVALENCE (SA1,SA),(SA5,SA(1,5)),(EA1,EA),(EA5,EA(1,5))
      DATA FR/0.1D8,.1259D8,.1585D8,.1995D8,.2512D8,.3162D8,.3981D8,.501
     12D8,.631D8,.7943D8,.1D9,.1259D9,.1585D9,.1995D9,.2512D9,.3162D9,.3
     1981D9,.5012D9,.631D9,.7943D9,.1D10,.1259D10,.1585D10,.1995D10,.251
     12D10,.3162D10,.3981D10,.5012D10,.631D10,.7943D10,.8913D10,.1D11/
      DATA SA1/.75D0,.762D0,.78D0,.798D0,.816D0,.84D0,.876D0,.9D0,.96D0,
     1.102D1,.114D1,.1224D1,.1308D1,.1392D1,.1452D1,.1524D1,.1572D1,.160
     18D1,.1644D1,.174D1,.1812D1,.1932D1,.2064D1,.2292D1,.2616D1,.3084D1
     1,.3744D1,.4716D1,.642D1,.918D1,.1076D2,.1236D2,.6875D0,.6985D0,.71
     15D0,.7315D0,.748D0,.77D0,.803D0,.825D0,.88D0,.935D0,.1045D1,.1122D
     11,.1199D1,.1276D1,.1331D1,.1397D1,.1441D1,.1474D1,.1507D1,.1595D1,
     1.1661D1,.1771D1,.1892D1,.2101D1,.2398D1,.2827D1,.3432D1,.4323D1,.5
     1885D1,.8415D1,.9867D1,.1133D2,.625D0,.635D0,.65D0,.665D0,.68D0,.7D
     10,.73D0,.75D0,.8D0,.85D0,.95D0,.102D1,.109D1,.116D1,.121D1,.127D1,
     1.131D1,.134D1,.137D1,.145D1,.151D1,.161D1,.172D1,.191D1,.218D1,.25
     17D1,.312D1,.393D1,.535D1,.765D1,.897D1,.103D2,.5313D0,.5393D0,0.55
     125D0,.5653D0,0.578D0,.595D0,.6205D0,.6375D0,.68D0,.7225D0,.8075D0,
     1.867D0,.9265D0,.986D0,.1029D1,.108D1,.1114D1,.1139D1,.1165D1,.1233
     1D1,.1284D1,.1369D1,.1462D1,.1624D1,.1853D1,.2185D1,.2652D1,.3341D1
     1,.4548D1,.6503D1,.7625D1,.8755D1/
      DATA SA5/.4116D0,.4181D0,0.4280D0,.4379D0,.4478D0,.461D0,.4807D0,.
     14939D0,.5268D0,.5597D0,.6256D0,.6717D0,.7178D0,.7639D0,.7968D0,.83
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163D0,.8626D0,.8824D0,.9021D0,.9548D0,.9943D0,.106D1,.1133D1,.1258D
11,.1436D1,.1692D1,.2055D1,.2588D1,.3523D1,.5038D1,.5907D1,.6783D1,
1.22D-1,.228D-1,.235D-1,.25D-1,.26D-1,.28D-1,.32D-1,.348D-1,.38D-1,
1.4D-1,.45D-1,.47D-1,.52D-1,.57D-1,.628D-1,.69D-1,.74D-1,.81D-1,.8
18D-1,.96D-1,.103D0,.113D0,.124D0,.138D0,.154D0,.176D0,.201D0,.236D
10,.274D0,.342D0,.384D0,.4365D0,.198D-1,.2052D-1,.2115D-1,.225D-1,.
1234D-1,.252D-1,.288D-1,.3132D-1,.342D-1,.36D-1,.3825D-1,.423D-1,.4
168D-1,.513D-1,.5652D-1,.621D-1,.666D-1,.729D-1,.792D-1,.864D-1,.92
17D-1,.1017D0,.1116D0,.1242D0,.1386D0,.1584D0,.1809D0,.2124D0,.2466
1D0,.3078D0,.3456D0,.3929D0/
 DATA EA1/.192D3,.1782D3,.165D3,.1517D3,.1393D3,.1277D3,.1171D3,.10
172D3,.9876D2,.9144D2,.8412D2,.7716D2,.7116D2,.6744D2,.6588D2,.648D
12,.6396D2,.6324D2,.624D2,.6156D2,.6072D2,.5976D2,.5868D2,.576D2,.5
1652D2,.5532D2,.5412D2,.5268D2,.5136D2,.498D2,.4896D2,.4788D2,.192D
13,.1782D3,.165D3,.1517D3,.1393D3,.1277D3,.1171D3,.1072D3,.9876D2,.
19144D2,.8412D2,.7716D2,.7116D2,.6744D2,.6588D2,.648D2,.6396D2,.632
14D2,.624D2,.6156D2,.6072D2,.5976D2,.5868D2,.576D2,.5652D2,.5532D2,
1.5412D2,.5268D2,.5136D2,.498D2,.4896D2,.4788D2,.16D3,.1485D3,.1375
1D3,.1264D3,.1161D3,.1064D3,.976D2,.893D2,.823D2,.762D2,.701D2,.643
1D2,.593D2,.562D2,.549D2,.54D2,.533D2,.527D2,.52D2,.513D2,.506D2,.4
198D2,.489D2,.48D2,.471D2,.461D2,.451D2,.439D2,.428D2,.415D2,.408D2
1,.399D2,.1424D3,.1322D3,.1224D3,.1125D3,.1033D3,.947D2,.8686D2,.79
148D2,.7325D2,.6782D2,.6239D2,.5723D2,.5278D2,.5002D2,.4886D2,.4806
1D2,.4744D2,.469D2,.4628D2,.4566D2,.4503D2,.4432D2,0.4352D2,.4272D2
1,.4192D2,.4103D2,.4014D2,.3907D2,.3809D2,.3694D2,.3631D2,.3551D2/
 DATA EA5/.1054D3,.9779D2,.9054D2,.8323D2,0.7645D2,.7006D2,.6427D2,
1.588D2,.5419D2,.5018D2,0.4616D2,.4234D2,.3905D2,.3701D2,.3615D2,.3
1556D2,.351D2,.3470D2,.3424D2,.3378D2,.3332D2,.3279D2,.322D2,.3161D
12,.3102D2,.3036D2,.297D2,.2891D2,.2818D2,.2733D2,.2687D2,.2627D2,.
136D2,.318D2,.279D2,.243D2,0.208D2,.178D2,.148D2,.123D2,.102D2,.86D
11,.745D1,.68D1,.63D1,.6D1,.58D1,.57D1,.565D1,.563D1,.562D1,.561D1,
1.56D1,.559D1,.557D1,.556D1,.554D1,.552D1,.55D1,.548D1,.52D1,.49D1,
1.473D1,.45D1,.324D2,.2862D2,.2511D2,.2187D2,.1872D2,.1602D2,.1332D
12,.1107D2,.918D1,.774D1,.6705D1,.612D1,.567D1,.54D1,.522D1,.513D1,
1.5085D1,.5067D1,.5058D1,.5049D1,.504D1,.5031D1,.5013D1,.5004D1,.49
186D1,.4968D1,.495D1,.4932D1,.468D1,.441D1,.4257D1,.405D1/
 FREQ=F *1.D6
 NDX=ITIS
 IF (FREQ.GE.FR(1).AND.FREQ.LE.FR(32)) GO TO 10
 PRINT 5
5 FORMAT ('0**** FREQUENCY LIES OUTSIDE THE RANGE 10 - 10000 MHZ ***
1*')
 STOP
10 DO 15 IJ=1,31
 IF (FREQ.LE.FR(IJ+1)) GO TO 20
15 CONTINUE
20 X=(FREQ-FR(IJ))/(FR(IJ+1)-FR(IJ))
 EPS=EA(IJ,NDX)+(EA(IJ+1,NDX)-EA(IJ,NDX))*X
 SIG=SA(IJ,NDX)+(SA(IJ+1,NDX)-SA(IJ,NDX))*X
 RETURN
 END


 SUBROUTINE PLTCV1(X,Y,XLEN,YLEN,XTL,YTL,NXTL,NYTL,NP,ICRCT,ISYM,
1 IMM,XMIN,XMAX,YMIN,YMAX,INPLT,LINTYP,SOCH,DELX,DELY,
2 NDEC)
 WE ARE PLOTTING Y AS A FUNCTION OF X
```

```
**** THIS IS A VARIATION OF PLTCRV TO PERMIT SPECIFYING THE BLIP
     INTERVAL AND THE NUMBER OF DECIMAL PLACES AND CHARACTER SIZE FOR
     SCALE NUMBERS AND LABELS.
       X       ARRAY TO PLOT ON X (HORIZONTAL) AXIS - DIMENSION (NP+2)
       Y       ARRAY TO PLOT ON Y (VERTICAL) AXIS - DIMENSION (NP+2)
       XLEN    LENGTH IN INCHES OF X AXIS
       YLEN    LENGTH IN INCHES OF Y AXIS
       XTTL    ARRAY CONTAINING X AXIS TITLE
       YTTL    ARRAY CONTAINING Y AXIS TITLE
       NX      NUMBER OF CHARACTERS IN XTTL
       NY      NUMBER OF CHARACTERS IN YTTL
       NP      NUMBER OF POINTS TO PLOT IN ARRAYS X AND Y
       ICRCT   0 - PLOT AXES AND LINE PLOT
               1 - PLOT LINE ON EXISTING AXES
       ISYM    CODE (0-13) TO SELECT SYMBOL TO MARK PLOTTED POINTS
       IMM     0 - GET SCALE END VALUES BY SCANNING X AND Y ARRAYS
               1 - GET SCALE END VALUES FROM INPUT ARGUMENTS
       XMIN    MINIMUM VALUE ON X AXIS
       XMAX    MAXIMUM VALUE ON X AXIS
       YMIN    MINIMUM VALUE ON Y AXIS
       YMAX    MAXIMUM VALUE ON Y AXIS
       INPLT   0 - DRAW SCALES AND LINE
               1 - GET MAXIMA AND MINIMA OF X AND Y ARRAYS, NO PLOT
       LINTYP MAGNITUDE GIVES FREQUENCY OF SYMBOLS - EVERY LINTYP PTS.
               =0 - LINE PLOT, NO SYMBOLS
               >0 - LINE PLOT WITH SYMBOLS
               <0 - NO LINE, SYMBOLS ONLY
       SOCH    CHARACTER HEIGHT FOR TITLE AND SCALE (INCHES)
       DELX    FOR X AXIS, POSITIVE VALUE TO DEFINE UNITS BETWEEN TIC
                 MARKS (USER UNITS).   IF DELX = 0., TIC MARKS WILL BE
                 ONE INCH APART.
       DELY    FOR Y AXIS, POSITIVE VALUE TO DEFINE UNITS BETWEEN TIC
                 MARKS (USER UNITS).   IF DELY = 0., TIC MARKS WILL BE
                 ONE INCH APART.
       NDEC    NUMBER OF DECIMAL PLACES IN SCALE NUMBERS
                 >=0 - SPECIFIES NUMBER OF DECIMAL PLACES AFTER
                         DECIMAL POINT
                 -1  - ROUNDED INTEGER DRAWN
    DIMENSION X(NP),Y(NP),XTL(1),YTL(1)
    IF(ICRCT.EQ.1) GO TO 20
    IF(IMM.EQ.1) GO TO 10
    XMIN = 1.E35
    XMAX = -1.E35
    YMIN =  1.E35
    YMAX =  -1.E35
    DO  5 I = 1,NP
    XMIN=AMIN1(X(I),XMIN)
    YMIN=AMIN1(Y(I),YMIN)
    XMAX=AMAX1(X(I),XMAX)
  5 YMAX=AMAX1(Y(I),YMAX)
    IF (INPLT.EQ.1) RETURN
 10 DELVX = (XMAX-XMIN)/XLEN
    DELVY = (YMAX-YMIN)/YLEN
    CALL BAXIS (0.,0.,XTL,-NXTL,XLEN,0.,XMIN,DELVX,DELX,SOCH,NDEC)
    CALL BAXIS (0.,0.,YTL,NYTL,YLEN,90.,YMIN,DELVY,DELY,SOCH,NDEC)
 20 IF(ISYM.LT.0.OR.ISYM.GT.13) ISYM = 1
    X(NP+1) = XMIN
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```
      Y(NP+1) = YMIN
      X(NP+2) = (XMAX-XMIN)/XLEN
      Y(NP+2) = (YMAX-YMIN)/YLEN
      CALL LINE(X,Y,NP,1,LINTYP,ISYM)
      RETURN
      END


      SUBROUTINE BAXIS (XPAGE,YPAGE,IBCD,NCHAR,AXLEN,ANGLE,FIRSTV,DELTAV
     1,DELTIC,SOCH,NDEC)
         THIS SUBROUTINE IS AN EXTENSION OF THE CALCOMP 'AXIS' ROUTINE
         TO ALLOW THE USER TO SPECIFY THE SIZE OF CHARACTERS, THE
         DISTANCE BETWEEN TIC MARKS AND THE NUMBER OF DECIMAL PLACES IN
         THE SCALE NUMBERS.
            XPAGE  - X COORDINATE OF AXIS STARTING POINT (INCHES)
            YPAGE  - Y COORDINATE OF AXIS STARTING POINT (INCHES)
            IBCD   - ARRAY WITH AXIS TITLE
            NCHAR  - NUMBER OF CHARACTERS IN AXIS TITLE
                       <0 - ALL NOTATION ON CLOCKWISE SIDE OF AXIS
                       >0 - ALL NOTATION ON COUNTERCLOCKWISE SIDE
            AXLEN  - AXIS LENGTH (INCHES)  (MUST BE POSITIVE)
            ANGLE  - ANGLE (POSITIVE OR NEGATIVE) AT WHICH AXIS IS DRAWN
                       (DEGREES)
            FIRSTV - STARTING VALUE (MAX OR MIN) OF AXIS AT FIRST TIC
                       (USER UNITS)
            DELTAV - INCREMENT OR DECREMENT VALUE ASSOCIATED WITH ONE
                       INCH ON AXIS (USER UNITS)
            DELTIC - POSITIVE VALUE TO DEFINE UNITS BETWEEN TIC MARKS
                       (USER UNITS)   IF DELTIC = 0., TIC MARKS WILL BE
                       ONE INCH APART.
            SOCH   - CHARACTER HEIGHT FOR TITLE AND SCALE (INCHES)
            NDEC   - NUMBER OF DECIMAL PLACES IN SCALE NUMBERS
                       >=0 - SPECIFIES NUMBER OF DECIMAL PLACES AFTER
                               DECIMAL POINT
                       -1  - ROUNDED INTEGER DRAWN
      DIMENSION IBCD(1)
      IF (AXLEN.GT.0..AND.DELTIC.GE.0..AND.NDEC.LE.9) GO TO 10
      PRINT 5,AXLEN,DELTIC,NDEC
    5 FORMAT ('0**** BAXIS ERROR: AXLEN =',1PD15.7,'   DELTIC =',D15.7,'
     1   NDEC =',I5,' ****')
      STOP
   10 IF (NDEC.LT.-1)NDEC=-1
      AIR=3.1415927*ANGLE/180.
      CA=COS(AIR)
      SA=SIN(AIR)
            DRAW AXIS LINE
      CALL PLOT(XPAGE,YPAGE,3)
      CALL PLOT(XPAGE+AXLEN*CA,YPAGE+AXLEN*SA,2)
      FSTV=FIRSTV
      DELV=DELTAV
      A=AMAX1(ABS(FSTV),ABS(FSTV+DELV*AXLEN))
      M=ALOG10(A)
      IF (A.LT..1)M=M-1
      TM=10.**M
      DTIC=ABS(DELTIC/DELV)
      IF (DELTIC.EQ.0)DTIC=1.0
      DELV=DELV/TM
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```
      FSTV=FSTV/TM
      X1=SOCH/2.
      TICH=X1
      IF (NCHAR.LT.0)TICH=-TICH
      XT=-TICH*SA
      YT=TICH*CA
            COMPUTE POSITION OF AXIS SCALE NUMBERS RELATIVE TO TIC
            MARKS AND ADJUST FOR NUMBER OF DECIMAL POINTS
      FN=X1
      IF (NDEC.GE.0)FN=FN*(2+NDEC)
      FN=FN-.429*X1
      XN=1.4*XT-FN*CA
      YN=1.4*YT-FN*SA
      IF (NCHAR.GT.0) GO TO 20
            FOR TICS ON CLOCKWISE SIDE OF AXIS, NUMBERS MUST BE MOVED
            AWAY FROM AXIS BY ONE CHARACTER WIDTH
      XN=XN+2.*XT
      YN=YN+2.*YT
   20 XTIC=XPAGE
      YTIC=YPAGE
      DX=DTIC*CA
      DY=DTIC*SA
      FPN=FSTV
      DTIC=DTIC*DELV
      X=.571*SOCH-FN
      IL=0
            LOOP TO DRAW TICS AND SCALE NUMBERS
   25 CALL PLOT(XTIC,YTIC,3)
      CALL PLOT(XTIC+XT,YTIC+YT,2)
      IF (IL.EQ.0.AND.NDEC.GE.0) GO TO 30
      X=0.
      IF (FPN.LT.0.)X=X1
   30 CALL NUMBER(XTIC+XN-X*CA,YTIC+YN-X*SA,SOCH,FPN,ANGLE,NDEC)
      XTIC=XTIC+DX
      YTIC=YTIC+DY
      FPN=FPN+DTIC
      ALEN=(XTIC-XPAGE-DX*.5)/CA
      IF (ALEN.GT.AXLEN) GO TO 45
      IL=IL+1
      IF (IL.LE.100) GO TO 25
      PRINT 40
   40 FORMAT ('0**** BAXIS ERROR: MORE THAN 100 TIC MARKS ****')
      STOP
            CENTER AXIS TITLE AND PLOT IT
   45 IL=IABS(NCHAR)
      IF (M.NE.0)IL=IL+4
      X=IL*SOCH
      HTL=(AXLEN-X)/2.
      XN=XPAGE+4.6*XT+HTL*CA
      YN=YPAGE+4.6*YT+HTL*SA
      IF (NCHAR.GT.0) GO TO 50
            LEAVE ROOM FOR TITLE CHARACTERS ON CLOCKWISE SIDE OF AXIS
      XN=XN+2.*XT
      YN=YN+2.*YT
   50 CALL SYMBOL(XN,YN,SOCH,IBCD,ANGLE,IABS(NCHAR))
      IF (M.EQ.0) GO TO 55
            ADD SCALE FACTOR
```

```
      CALL SYMBOL(999.,999.,SOCH,' *10',ANGLE,4)
      XN=XN+X*CA-X1*SA
      YN=YN+X*SA+1.5*X1*CA
      CALL NUMBER(XN,YN,X1,FLOAT(M),ANGLE,-1)
   55 RETURN
      END


      SUBROUTINE CNTRP1(X,NROW,Y,NCOL,D,NLEV1,NSYM1,IFL)
      DIMENSION X(NROW),Y(NCOL),D(NROW,NCOL),FLEV(10),XST(60),YST(60)
      INTEGER*2 IFL(NROW,NCOL),IST(60),JST(60)
      NLEV=NLEV1
      IF (NLEV.LT.1)NLEV=1
      IF (NLEV.GT.10)NLEV=10
      NSYM=NSYM1
      IF (NSYM.LE.0)NSYM=NROW*NCOL
      AXLEN=6.
             SCALE THE DATA FOR THE COORDINATE AXES
      ZMAX=-1.E38
      ZMIN=1.E38
      XMAX=-1.E38
      XMIN=1.E38
      DO 5 I=1,NROW
      IF (X(I).GT.XMAX)XMAX=X(I)
      IF (X(I).LT.XMIN)XMIN=X(I)
      DO 5 J=1,NCOL
      IF (D(I,J).GT.ZMAX)ZMAX=D(I,J)
    5 IF (D(I,J).GT.0..AND.D(I,J).LT.ZMIN)ZMIN=D(I,J)
      YMAX=-1.E38
      YMIN=1.E+38
      DO 10 J=1,NCOL
      IF (Y(J).GT.YMAX)YMAX=Y(J)
   10 IF (Y(J).LT.YMIN)YMIN=Y(J)
      PRINT 15,XMIN,XMAX,YMIN,YMAX,ZMIN,ZMAX
   15 FORMAT ('0X RANGE',1P2E12.4,'     Y RANGE',2E12.4,'     Z RANGE',2
     1E12.4)
      XFAC=AXLEN/(XMAX-XMIN)
      YFAC=AXLEN/(YMAX-YMIN)
      CDIF1=(ZMAX-ZMIN)/(2*NLEV)
      IL=-ALOG10(CDIF1)+1.
   20 T=10.**IL
      ICDIF=5*((IFIX(CDIF1*T)+2)/5)
      S=(ZMIN+ZMAX-FLOAT(2*NLEV*ICDIF)/T)/2.
      IS=0
      IF (S.NE.0.)IS=5*(IFIX(S*T+2.5*S/ABS(S))/5)
      T1=FLOAT(IS+ICDIF)/T
      CDIF=FLOAT(2*ICDIF)/T
      S=T1+CDIF*(NLEV-1)
      S1=CDIF*.1
      IF (ZMIN.LT.T1-S1.AND.ZMAX.GT.S+S1.AND.ZMAX.LT.S+CDIF) GO TO 25
      IL=IL+1
      GO TO 20
   25 FLEV(1)=T1
      IF (NLEV.EQ.1) GO TO 35
      DO 30 K=2,NLEV
   30 FLEV(K)=T1+FLOAT(2*ICDIF*(K-1))/T
   35 AXLP1=AXLEN+.5
```

```
      AXLP2=AXLEN+.75
      RSQ=X(1)**2
      AX2=AXLEN/2.
      AX2S=(.985*AX2)**2
      NROWM1=NROW-1
      NCOLM1=NCOL-1
      DO 40 K=1,NLEV
      S=FLEV(K)+.001*CDIF
      T=FLEV(K)-.001*CDIF
      DO 40 I=1,NROW
      DO 40 J=1,NCOL
   40 IF (D(I,J).LT.S.AND.D(I,J).GT.T)D(I,J)=S
      DO 380 K=1,NLEV
      F=FLEV(K)
      IEND=0
      DO 150 I=1,NROWM1
      DO 150 J=1,NCOLM1
      IFL(I,J)=0
      DIJ=D(I,J)
      DI1J=D(I+1,J)
      DIJ1=D(I,J+1)
      DI1J1=D(I+1,J+1)
      IF (DIJ.GT.F.OR.DIJ1.LT.F) GO TO 85
      T=DI1J1-F
      A=DIJ
      B=DI1J1
   45 IF (I.GT.1) GO TO 60
      IF (A.GT.0.) GO TO 50
      YC=SQRT(RSQ-X(I)**2)
      IF (Y(J+1).LT.0.)YC=-YC
      GO TO 55
   50 S=(F-DIJ)/(DIJ1-DIJ)
      YC=Y(J)+S*(Y(J+1)-Y(J))
   55 IEND=IEND+1
      YST(IEND)=YC
      XST(IEND)=X(I)
      IST(IEND)=0
      JST(IEND)=J
   60 IF (T.GT.0.) GO TO 80
   65 IF (J.LT.NCOLM1) GO TO 80
      IF (B.GT.0.) GO TO 70
      XC=SQRT(RSQ-Y(J+1)**2)
      IF (X(I+1).LT.0.)XC=-XC
      GO TO 75
   70 S=(F-DIJ1)/(DI1J1-DIJ1)
      XC=X(I)+S*(X(I+1)-X(I))
   75 IEND=IEND+1
      XST(IEND)=XC
      YST(IEND)=Y(J+1)
      IST(IEND)=I
      JST(IEND)=NCOL
   80 IFL(I,J)=1
      GO TO 95
   85 IF (DIJ.LT.F.OR.DIJ1.GT.F) GO TO 90
      T=F-DI1J1
      A=DIJ1
      B=A
```

```
      GO TO 45
   90 B=DIJ1
      IF (DIJ.LT.F.AND.DI1J1.GT.F) GO TO 65
      B=DI1J1
      IF (DIJ.GT.F.AND.DI1J1.LT.F) GO TO 65
   95 IF (DIJ.GT.F.OR.DI1J.LT.F) GO TO 140
      T=DI1J1-F
      A=DIJ
      B=DI1J1
  100 IF (J.GT.1) GO TO 115
      IF (A.GT.0.) GO TO 105
      XC=SQRT(RSQ-Y(J)**2)
      IF (X(I+1).LT.0.)XC=-XC
      GO TO 110
  105 S=(F-DIJ)/(DI1J-DIJ)
      XC=X(I)+S*(X(I+1)-X(I))
  110 IEND=IEND+1
      XST(IEND)=XC
      YST(IEND)=Y(J)
      IST(IEND)=I
      JST(IEND)=0
  115 IF (T.GT.0.) GO TO 135
      IFL(I,J)=IFL(I,J)+2
  120 IF (I.LT.NROWM1) GO TO 135
      IF (B.GT.0.) GO TO 125
      YC=SQRT(RSQ-X(I+1)**2)
      IF (Y(J+1).LT.0.)YC=-YC
      GO TO 130
  125 S=(F-DI1J)/(DI1J1-DI1J)
      YC=Y(J)+S*(Y(J+1)-Y(J))
  130 IEND=IEND+1
      YST(IEND)=YC
      XST(IEND)=X(I+1)
      IST(IEND)=NROW
      JST(IEND)=J
  135 IF (IFL(I,J).EQ.0)IFL(I,J)=1
      GO TO 150
  140 IF (DIJ.LT.F.OR.DI1J.GT.F) GO TO 145
      T=F-DI1J1
      A=DI1J
      B=A
      GO TO 100
  145 B=DI1J
      IF (DIJ.LT.F.AND.DI1J1.GT.F) GO TO 120
      B=DI1J1
      IF (DIJ.GT.F.AND.DI1J1.LT.F) GO TO 120
  150 CONTINUE
  155 IF (IEND.EQ.0) GO TO 160
             SET UP TO PLOT NEXT CONTOUR FROM EDGE OF GRID
      I=IST(1)
      J=JST(1)
      IOLD=I
      JOLD=J
      CALL PLOT((XST(1)-XMIN)*XFAC,(YST(1)-YMIN)*YFAC,3)
      IF (I.EQ.0)I=1
      IF (J.EQ.0)J=1
      IF (I.EQ.NROW)I=NROWM1
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      IF (J.EQ.NCOL)J=NCOLM1
      ISTC=1
      GO TO 180
            ALL CONTOURS THAT LEAVE GRID HAVE BEEN DRAWN
            SET UP TO PLOT NEXT CONTOUR THAT DOES NOT LEAVE GRID
  160 I1=1
  165 J1=1
  170 IF (IFL(I1,J1).NE.0) GO TO 175
      J1=J1+1
      IF (J1.LT.NCOL) GO TO 170
      I1=I1+1
      IF (I1.LT.NROW) GO TO 165
      GO TO 375
  175 ISTC=0
      I=I1
      J=J1
  180 ISYM=NSYM-1
            FIND ENDS OF LINES IN UPPER LEFT TRIANGLE
  185 DIJ=D(I,J)
      DI1J=D(I+1,J)
      DIJ1=D(I,J+1)
      DI1J1=D(I+1,J+1)
      IF (DIJ.GT.F.OR.DIJ1.LT.F) GO TO 220
      T=DI1J1-F
      A=DIJ
      B=DI1J1
  190 IF (A.GT.0.) GO TO 195
      YC=SQRT(RSQ-X(I)**2)
      IF (Y(J+1).LT.0.)YC=-YC
      GO TO 200
  195 S=(F-DIJ)/(DIJ1-DIJ)
      YC=Y(J)+S*(Y(J+1)-Y(J))
  200 XC=X(I)
      IB=I-1
      JB=J
      IF (T.GT.0.) GO TO 250
      IF (IFL(I,J).EQ.2) GO TO 250
      IF (B.GT.0) GO TO 205
      XC1=SQRT(RSQ-Y(J+1)**2)
      IF (X(I+1).LT.0.)XC1=-XC1
      GO TO 210
  205 S=(F-DIJ1)/(DI1J1-DIJ1)
      XC1=X(I)+S*(X(I+1)-X(I))
  210 YC1=Y(J+1)
      IE=I
      JE=J+1
      IF (IOLD.EQ.IB.AND.JOLD.EQ.JB) GO TO 215
      IF (IOLD.NE.IE.OR.JOLD.NE.JE) GO TO 250
  215 IFL(I,J)=IFL(I,J)-1
      GO TO 310
  220 IF (DIJ.LT.F.OR.DIJ1.GT.F) GO TO 225
      T=F-DI1J1
      A=DIJ1
      B=A
      GO TO 190
  225 IF (DIJ.GT.F.OR.DI1J1.LT.F) GO TO 245
      IF (DIJ1.GT.0.) GO TO 235
```

```
230 XC=SQRT(RSQ-Y(J+1)**2)
    IF (X(I+1).LT.0.)XC=-XC
    GO TO 240
235 S=(F-DIJ1)/(DI1J1-DIJ1)
    XC=X(I)+S*(X(I+1)-X(I))
240 YC=Y(J+1)
    IB=I
    JB=J+1
    GO TO 250
245 IF (DIJ.LT.F.OR.DI1J1.GT.F) GO TO 250
    IF (DI1J1.GT.0.) GO TO 235
    GO TO 230
          FIND ENDS OF LINES IN LOWER RIGHT TRIANGLE
250 IF (DIJ.GT.F.OR.DI1J.LT.F) GO TO 290
    T=DI1J1-F
    A=DIJ
    B=DI1J1
255 IF (A.GT.0.) GO TO 260
    XC1=SQRT(RSQ-Y(J)**2)
    IF (X(I+1).LT.0.)XC1=-XC1
    GO TO 265
260 S=(F-DIJ)/(DI1J-DIJ)
    XC1=X(I)+S*(X(I+1)-X(I))
265 IF (T.GT.0) GO TO 285
    IF (IFL(I,J).LT.2) GO TO 310
    XC=XC1
    YC=Y(J)
    IB=I
    JB=J-1
    IFL(I,J)=IFL(I,J)-2
270 IF (B.GT.0.) GO TO 275
    YC1=SQRT(RSQ-X(I+1)**2)
    IF (Y(J+1).LT.0)YC1=-YC1
    GO TO 280
275 S=(F-DI1J)/(DI1J1-DI1J)
    YC1=Y(J)+S*(Y(J+1)-Y(J))
280 XC1=X(I+1)
    IE=I+1
    JE=J
    GO TO 310
285 YC1=Y(J)
    IE=I
    JE=J-1
    IFL(I,J)=0
    GO TO 310
290 IF (DIJ.LT.F.OR.DI1J.GT.F) GO TO 295
    T=F-DI1J1
    A=DI1J
    B=A
    GO TO 255
295 IF (DIJ.GT.F.OR.DI1J1.LT.F) GO TO 305
    B=DI1J
300 IFL(I,J)=0
    GO TO 270
305 B=DI1J1
    IF (DIJ.GT.F.AND.DI1J1.LT.F) GO TO 300
310 IF (ISTC.NE.0) GO TO 320
```

```
          PLOT FIRST SEGMENT OF NEW CONTOUR
      CALL PLOT((XC1-XMIN)*XFAC,(YC1-YMIN)*YFAC,3)
      ISTC=1
  315 PX=(XC-XMIN)*XFAC
      PY=(YC-YMIN)*YFAC
      IOLD=I
      JOLD=J
      I=IB
      J=JB
      GO TO 340
          MATCH CURRENT PEN POSITION TO ONE END OF NEW LINE SEGMENT
  320 IF (IOLD.EQ.IB.AND.JOLD.EQ.JB) GO TO 335
      IF (IOLD.EQ.IE.AND.JOLD.EQ.JE) GO TO 315
      PRINT 330,I,J,IOLD,JOLD,IB,JB,IE,JE,DIJ,DIJ1,DI1J,DI1J1
  330 FORMAT ('-LOGIC ERROR:  AT',2I3,'  FROM',2I3,' TO',2I3,' OR',2I3,4
     1F8.4)
      STOP
  335 PX=(XC1-XMIN)*XFAC
      PY=(YC1-YMIN)*YFAC
      IOLD=I
      JOLD=J
      I=IE
      J=JE
          PLOT LINE SEGMENT
  340 R1SQ=(PX-AX2)**2+(PY-AX2)**2
      IF (R1SQ.GT.AX2S) GO TO 345
      ISYM=ISYM+1
      IF (ISYM.LT.NSYM) GO TO 345
      CALL SYMBOL(PX,PY,.07,K,0.,-2)
      ISYM=0
      GO TO 350
  345 CALL PLOT(PX,PY,2)
          DETERMINE WHETHER CONTOUR HAS ENDED
  350 IF (I.EQ.0.OR.I.EQ.NROW.OR.J.EQ.0.OR.J.EQ.NCOL) GO TO 355
      IF (IFL(I,J).NE.0) GO TO 185
      IF (IEND.EQ.0) GO TO 170
          REMOVE END POINTS OF LAST CONTOUR FROM TABLE
  355 I1=0
      IF (IEND.EQ.1) GO TO 370
      DO 365 L=2,IEND
      IF (I.EQ.IST(L).AND.J.EQ.JST(L)) GO TO 365
      I1=I1+1
      XST(I1)=XST(L)
      YST(I1)=YST(L)
      IST(I1)=IST(L)
      JST(I1)=JST(L)
  365 CONTINUE
  370 IEND=I1
      GO TO 155
          PUT SYMBOL AND LEVEL ON PLOT
          ALL CONTOURS AT THIS LEVEL HAVE BEEN DRAWN
  375 FLK=FLOAT(K-1)*.6
      CALL SYMBOL(AXLP1,FLK+.06,.14,K,0.,-1)
      CALL FNUM(AXLP2,FLK,FLEV(K),2,0.,.14)
  380 CONTINUE
  *** PLOT AXES
      CALL PLOT(AX2,AXLEN,3)
```

```
      CALL PLOT(AX2,0.,2)
      CALL PLOT(0.,AX2,3)
      CALL PLOT(AXLEN,AX2,2)
***   PLOT CIRCLE AROUND CONTOURS
      DTH=2.*3.1415927/288
      THETA=DTH
      DO 385 K=1,288
      R=AX2*(1.+COS(THETA))
      P=AX2*(1.+SIN(THETA))
      THETA=THETA+DTH
  385 CALL PLOT(R,P,2)
      RETURN
      END


      SUBROUTINE FNUM(XPAGE,YPAGE,FPN,ND,ANGLE,HEIGHT)
           EDIT A FLOATING POINT NUMBER FOR THE PLOTTER
              XPAGE  X COORDINATE OF STARTING POINT (INCHES)
              YPAGE  Y COORDINATE OF STARTING POINT (INCHES)
              FPN    NUMBER TO BE PLOTTED
              ND     IF 10.**-ND <= FPN < 10.**ND, THE NUMBER IS PLOTTED
                     WITHOUT EXPONENT
              ANGLE  ANGLE AT WHICH NUMBER IS PLOTTED (DEGREES)
              HEIGHT CHARACTER SIZE (INCHES)
      DIMENSION B(6)
      DATA B/.999999,.99999,.9999,.999,.99,.9/
      X=ABS(FPN)
      M=ND
      IF (X.NE.0.) GO TO 5
              PLOT ZERO
      CALL NUMBER(XPAGE,YPAGE,HEIGHT,X,ANGLE,1)
      RETURN
    5 N=ALOG10(X)
      IF (X.LT.1.)N=N-1
      X=X*10.**(-N)
      T=X/10.-X*1.E-7
      DO 10 J=1,6
      T1=T-INT(T)
      IF (T1.LE.0) GO TO 15
      IF (T1.GE.B(J)) GO TO 15
   10 T=T*10.
   15 J=J-1
      T=ABS(FPN)
      IF (T.GE.10.**M) GO TO 20
      IF (T+.5*10.**(N-6).LT.10.**(-M)) GO TO 20
             PLOT NUMBERS WHICH DO NOT NEED EXPONENTS
      M=J-N-1
      IF (M.LT.1)M=1
      IF (M.GT.9)M=9
      CALL NUMBER(XPAGE,YPAGE,HEIGHT,FPN,ANGLE,M)
      RETURN
             PLOT NUMBERS WITH EXPONENTS
   20 IF (J.GT.1) GO TO 25
      X=1.
      N=N+1
      J=2
   25 IF (FPN.LT.0.)X=-X
```

```
      CALL NUMBER(XPAGE,YPAGE,HEIGHT,X,ANGLE,J-1)
      CALL SYMBOL(999.,999.,HEIGHT,3H*10,ANGLE,3)
      X1=HEIGHT/2
      A=3.1415927*ANGLE/180.
      SA=SIN(A)
      CA=COS(A)
      NC=J+4
      IF (FPN.LT.0.)NC=NC+1
      S=NC*HEIGHT
      PX=XPAGE+S*CA-X1*SA
      PY=YPAGE+S*SA+1.5*X1*CA
      CALL NUMBER(PX,PY,X1,FLOAT(N),ANGLE,-1)
      RETURN
      END
```

DEPARTMENT OF THE AIR FORCE
AIR FORCE INSTITUTE FOR OPERATIONAL HEALTH (AFMC)
BROOKS CITY-BASE TEXAS

31 August 2007

MEMORANDUM FOR DTIC-OCQ
ATTN: LARRY DOWNING
8725 JOHN J. KINGMAN ROAD, SUITE 0944
FORT BELVOIR, VA 22060-6218

FROM: AFIOH/DOBP (STINFO)
2513 Kennedy Circle
Brooks City-Base TX 78235-5116

SUBJECT: Changing the Distribution Statement on a Technical Report

This letter documents the requirement for DTIC to change the distribution statement from "B" to "A" (Approved for public release; distribution is unlimited.) on the following technical report: AD Number ADB071126, SAM-TR-82-22, A Computer Model Predicting the Thermal Response to Microwave Radiation.

If additional information or a corrected cover page and SF Form 298 are required please let me know. You can reach me at DSN 240-6019 or my e-mail address is sherry.mathews@brooks.af.mil.

Thank you for your assistance in making this change.

Sherry Y. Mathews
SHERRY Y. MATHEWS
AFIOH STINFO Officer