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**STATISTICAL TREND ANALYSIS OF WEAR METAL
CONCENTRATION MEASUREMENTS —
CALCULATION OF SIGNIFICANT
WEAR METAL PRODUCTION RATES**

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DECEMBER 1979

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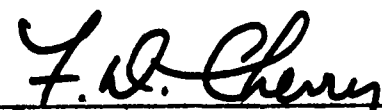
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analysis of concentration measurements are demonstrated to be greatly superior to both the foregoing methods for performing quantitative trend analysis. A "corrected" concentration concept is advanced and utilized to obtain a meaningful interpretation of concentration measurements on oil-consuming engines. Variance analysis techniques are described to deduce statistically valid regimes of constant wear rate from a set of concentration measurements. The use of this procedure to formulate guidelines for maintenance action is discussed.

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FOREWORD

This report was prepared for the Fluids, Lubricants and Elastomers Branch, Nonmetallic Materials Division, Air Force Materials Laboratory, under Job Order No. 2303Q201. The work was performed and reported by Dr. Karl Scheller, a Resident Scientist at the Air Force Materials Laboratory under contract to the Southeastern Center for Electrical Engineering Education. Dr. Kent J. Eisentraut was the Air Force Materials Laboratory Project Monitor for this task.

This report covers work conducted from February 1977 to October 1979 and was released for publication in October 1979. During the course of this effort, two papers were prepared for publication in appropriate technical publications, one paper was presented at an International Oil Analysis Symposium and two at national meetings.

The cooperation of Dr. Eisentraut and Capt. John M. Vice, formerly with MMETP at San Antonio Air Logistics Center (AFLC), in providing OAP records for this investigation, is gratefully acknowledged.

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SUMMARY

The JOAP methodology for the evaluation of wear metal concentration measurements has been investigated and its inadequacies arising from the lack of quantitative procedures to account for the effects of data scatter and oil consumption are elucidated. The probable error in the 10 hour trend, introduced by the allowable tolerance of ± 1 ppm in spectrometric concentration measurements, are defined in terms of the operating time between samples. Revised criteria for abnormal trend are advanced to compensate for the effect of random measurement errors. The effect of oil consumption and replenishment on wear metal concentrations is detailed and its influence on the validity of JOAP evaluation procedures is illustrated for a TF41 engine.

The calculation of wear metal production rates on a sample-to-sample basis, introduced to improve trend analysis of OAP concentration measurements is described. These are shown to be subject to the same type of error as the JOAP 10 hour trend, though they do take into consideration the effects of oil consumption.

Linear regression analysis of concentration measurements is shown to be greatly superior to both the foregoing methods for performing quantitative trend analysis. A "corrected" concentration concept is proposed for utilizing oil addition information to obtain a meaningful interpretation of concentration measurements on oil-consuming engines. The validity and utility of this concept is demonstrated for several TF-41 engines, two of which had reached steady-state wear metal concentrations.

A variance analysis and tracking technique is developed and described which is apparently capable of dividing a set of wear metal concentration measurements into statistically justifiable regimes with different wear rates. The use of this procedure to formulate guidelines for maintenance action is discussed.

SECTION I
INTRODUCTION

The military services monitor the health of their engines and other lubricated equipment through periodic spectrometric measurements of concentrations of significant wear metals in the lubricating oil. Their efforts in this behalf are governed by procedures set forth in complete detail in the Joint Oil Analysis Program Laboratory Manual (Air Force T033-1-37). Section VI of this manual prescribes a methodology for evaluating wear metal concentration measurements. Its techniques are largely qualitative in nature and place considerable weight upon the judgment and experience of the laboratory analyst in determining the trend of wear metal data. They are most deficient for high oil-consumption engines, since they lack quantitative methods to account for the effect of oil consumption and replenishment on the interpretation of wear metal concentration changes and a reliable determination of their trend. This task was established to evolve techniques to increase the utility of wear metal measurements in general and for oil-consuming engines in particular. Attention was to be focused on the TF41 engine, since its high oil consumption rate and small sump capacity make prediction of its wear, by JOAP evaluation procedures, one of the least accurate among the engines in its inventory.

In the present work, the TF41 engine is therefore used as an example to analyze the errors introduced into existing trending techniques by the permissible scatter in the concentration measurements, to delineate the masking effect of oil consumption and to illustrate superior statistical methods for trend analysis of wear metal concentration results. The concepts and the mathematical techniques described herein are, of course, applicable to any oil-wetted engine component or equipment wear.

SECTION II

NON-STATISTICAL TREND ANALYSIS

a. JOAP Evaluation Methodology

The JOAP Manual, cited previously, provides Wear Metal Tables for each item of monitored equipment, listing ranges of values for wear metal concentrations of significant elements and threshold limits for their trends. Maintenance actions are recommended upon the basis of the values of both these parameters. The tabulated trends specify the maximum normal increase in parts per million (ppm) of a given wear metal over an interval of 10 hours of operation. Trend values for comparison with these criteria are therefore calculated from the following simple relationship:

$$T \text{ (10 hour trend)} = \frac{(C_2 - C_1) \times 10}{\Delta T} \quad (1)$$

where C_2 = Present measured wear metal concentration
 C_1 = Previously measured wear metal concentration
 Δt = Operating hours between measurements

Concentration measurements are, of course, subject to experimental error and are required to satisfy a repeatability index of approximately ± 1 ppm (i.e. their standard deviation should not exceed 1 ppm). The repeatability criteria are approximately equal for all the important wear metal elements and are defined in such a manner that they increase with increasing concentration. They are not to exceed one-half of the maximum accuracy deviation allowable for laboratory certification. The latter values (accuracy index) are tabulated in the JOAP manual for each wear element and concentration. Those for iron, which are quite typical of the other wear elements, have been plotted in Figure 1. The repeatability index is seen to vary from about 1 ppm at a concentration of 10 ppm to slightly over 2.5 ppm at 50 ppm.

It will be taken as 1 ppm for the purposes of this report, in order to minimize the effect of random measurement error on trend evaluation. It is not unexpected that calculated trends may frequently be negative, as a result of the measurement scatter. These are to be taken as zero, to eliminate any possible confusion in their interpretation.

It is not intended that the laboratory analyst use the Tables on an absolute go-no go basis. Recommendations are to be tempered in the light of the past OAP and maintenance history of the equipment and its operating conditions, augmented by a healthy measure of the analyst's OAP experience and judgment in ascertaining the trend of the data. When this becomes difficult, it is suggested that a simple plot of wear metal concentration values against operating time be prepared as an aid, especially for engines such as the TF41. Such a plot is presented in Figure 2, showing representative OAP records for iron in oil from three TF41 engines. All are highly erratic in character and any trends deduced from them must be regarded with considerable scepticism. In a broad qualitative sense, one might assert that iron is increasing rapidly for Engine A, less rapidly for Engine B, and is substantially constant for Engine C. However one must have misgivings about the repeated increases and decreases in metal content, particularly marked in Engine A. The 10 hour trend calculation with its suppression of negative values, was introduced in an effort to mitigate confusion and aid interpretation of concentration measurements. However, as shown in Reference 1, this serves only to magnify the effect of the scatter in measured concentrations. The limitations of the JOAP evaluation methodology have been examined in detail in the cited report and touched on peripherally in another publication (2). For the sake of brevity, we summarize some of their findings here.

Application of the theory of propagation of errors to the trend relationship (Equation 1) leads to the expression

$$P_T = \left[\left(\frac{\partial T}{\partial C_2} \right)^2 (PC_2)^2 + \left(\frac{\partial T}{\partial C_1} \right)^2 (PC_1)^2 \right]^{1/2} \quad (2)$$

in which P_T is the precision of the trend calculation for precisions PC_2 and PC_1 in the measured concentrations, C_2 and C_1 respectively. Since the standard deviation (precisions) of the concentration are both approximately equal to 1 ppm. Equation 2 reduces to

$$P_T = \frac{10\sqrt{2}}{\Delta t} \quad (3)$$

For any given concentration measurement, the probable error will be approximately 0.7 times the standard deviation, hence the probable error in the calculated trend will be, to the same order of approximation

$$P_T = \frac{10}{\Delta t} \quad (4)$$

To illustrate the significance of this relationship, consider two consecutive samples taken 5 operating hours apart. The probable error in the trend calculated for these samples is approximately 2 ppm. Thus if the true trend of the concentration data is 4 ppm, there is a 50% chance that the calculated trend will lie in the range of 2-6 ppm. There is roughly only a 15% chance that it will be within ± 0.5 ppm of the actual trend (i.e. in the range 3.5 - 4.5 ppm) and a 50% chance that the calculated trend will be less than the true value. To increase the odds for detecting an abnormal trend of 4 ppm one might, for example, set the threshold limit at 6 ppm for this time interval. There is a 75% chance that samples exhibiting a calculated trend of 6 ppm or greater exceed an actual trend of 4 ppm. By similar reasoning, one can derive the following threshold trend

limits for detecting an abnormal trend of 4 ppm at a confidence level of 75% with the indicated operating hours between samples.

Sampling Interval, hrs.	<u>1</u>	<u>2</u>	<u>3</u>	<u>4-6</u>	<u>7-10</u>
10 Hour Trend, ppm	14	9	7	6	5

The threshold limits tabulated above allow a 25% chance of interpreting a true trend of 4 ppm or more as normal. If one wishes to reduce this possibility to 10%, the expected error in concentration (1.28 ppm) at the 80% confidence level is used to determine new threshold limits from the relationship

$$P_T = \frac{18}{\Delta t} \quad (5)$$

with the following results:

Sampling Interval, hrs.	<u>1</u>	<u>2</u>	<u>3</u>	<u>4-5</u>	<u>6-7</u>	<u>8-10</u>
10 Hour Trend, ppm	22	13	10	8	7	6

It was demonstrated in Reference 1 that the foregoing relationships for the error in trend calculations introduced by measurement scatter and sampling interval do, in fact, accurately describe the variations in trends calculated from OAP engine records. It was also shown that averaging of trend calculations compensates, in part, for the error introduced by the allowable tolerance (1 ppm) of the concentration measurements and, more importantly, that linear regression analysis is a far superior technique for extracting a valid trend from a series of concentration measurements. All of these contentions are illustrated in Figure 3 reproduced from that report, which presents and reduces OAP concentration data for the TF41 engine identified as A in Figure 2. This engine was removed 87 hours after an oil change for excessive iron content (in accordance with criteria in the

OAP Wear Metal Table for atomic absorption data). The raw measurements are plotted in the upper graph together with appropriate linear regression lines and their superimposed 95% confidence limits. To compare Figures 2 and 3, one should note that the time scales for the records in Figure 2 have been transposed, for the sake of graphical convenience, to provide a common origin for all three engines (i.e. 0 time in the case of Engine A corresponds to 59 operating hours after oil change.) One should further observe that the peak of 18 ppm at 4 hours in Figure 2 has been rejected as a "bad" point at 63 hours in Figure 3, on the basis that there is less than 1 chance in 1000 that it was due to random measurement error and not mishandling of the sample in some manner. Inspection of the modified plot offers little more insight into the condition of this engine over the original graph of wear metal concentration against operating time. One can only state that the engine appears to have experienced an abrupt increase in wear rate between 60 and 63 operating hours, worn at a low rate for the next 12 hours, and a more rapid rate from 76 operating hours until its removal. More detailed quantitative examination of the measurements greatly amplifies the qualitative impression. The data is best fitted with two regression lines, one of zero trend and the other with a trend of 3.1 ppm to a precision of 0.6 ppm. The concentrations of the samples at 59 and 60 operating hours (9 ppm) are so far removed from the mean value of 14 ppm that one can assert that the probability that they are both due to random measurement error is less than 1 in 1,000, 000 (i.e. they are, in all likelihood, real). The remaining data all fall comfortably within the 95% confidence band.

As anticipated by our error analysis, the point-to-point values of the projected trend, displayed in the lower graph, exhibit meaningless erratic fluctuations. The previous statistical treatment of the data indicated that

the first trend peak (25 ppm at 63 operating hours) is probably real, the others arise from random errors in the concentration measurements. During the interval the engine is experiencing essentially zero wear, trend values up to 10 ppm are obtained. Save for the spurious peak of 20 ppm at 77 operating hours, the trends calculated during the time the engine is undergoing appreciable wear, at a constant rate, are virtually identical to those derived for conditions of zero wear. The averaged value of all of the plotted trends is 5.5 ppm with a standard deviation 40% greater than the mean, certainly a result without significance. Suppression of the negative trend values (setting them equal to zero) impairs the ability of the averaging procedure to cope with the random error in the concentration measurements, by seriously inflating the average trend and minimizing the full extent of the chance variations in the individual calculated trends. It is clear that in the present instance, trends calculated in accordance with the JOAP manual are of little use and would be entirely misleading, if the measurements were not plotted. Even so, the graph of the test results lends itself only to imprecise qualitative interpretation and cannot yield quantitative trend estimates.

The problems encountered in using the JOAP evaluation methodology are compounded by its lack of a quantitative procedure to account for the effects of oil consumption and replenishment. Though the JOAP manual recognizes that the wear metal concentration will tend to level off in such circumstances, even for a constant wear metal production rate, it offers no prescription for interpreting them in these instances. The effect of oil consumption on the measured wear metal concentrations of an engine similar in characteristics to the TF41 engine is illustrated in Figure 4. Assuming the oil is absolutely clean initially and is sampled (and topped) at equal time intervals, the

concentration will build up smoothly to an asymptotic limit of 7 ppm for the given conditions. By contrast, if the engine were not consuming oil, the wear metal concentration would increase in linear fashion until the engine failed or the oil was changed. Thus after 100 hours of operating time, for example, the measured concentration would be 7 ppm, while the concentration indicative of actual engine wear would be 35 ppm, well above the guideline for removal. Of course, in a real case the sampling intervals are not equal and the precision of the measurements is ± 1 ppm; thus the concentration values would tend to fluctuate about their asymptotic limit much in the manner of that shown for Engine C in Figure 2, in which a limiting concentration appears to have been attained.

It may be instructive to delve a bit further into the attainment of a steady-state wear metal concentration in an engine lubrication system maintained at constant oil volume by continual replenishment of the oil as it is lost. Under this constraint, if both the wear and oil consumption rates remain constant, a material balance may be written as

$$VdC = qc dt + r dt \quad (6)$$

in which C = wear metal concentration at time t
(operating hours)
 q = oil consumption rate, qts/hr
 r = wear metal production rate, mg/hr

For steady-state conditions, $dC = 0$ and the limiting concentration, C_L is

$$C_L = \frac{r}{q}$$

Solving Equation 6, one finds that

$$C = C_L - (C_L - C_0) \exp\left(-\frac{qt}{V}\right) \quad (7)$$

which expresses the wear metal concentration, C , at any time, t , in terms of the initial concentration, C_0 , and the capacity of the oil system, V , in addition to the parameters C_L and q . For the engine illustrated in Figure 3, $r = 3.5$ mg/hr, $q = 0.5$ liters/hr (approximately 0.5 qts/hr)

and
$$C_L = 3.5/0.5 \approx 7 \text{ ppm}$$

Furthermore, since $C_0 = 0$ and $V = 10$ liters, the concentration, C is

$$C = 7(1 - e^{-0.05t})$$

In fifty operating hours, the concentration attains a value of 6.8 ppm, very close to its limiting value of 7 ppm.

In these circumstances, an analyst operating in accordance with JOAP procedures would note the approach to a steady-state concentration and correctly infer that the engine was probably using oil and categorize its condition as normal, though a recommendation for maintenance action would seem to be in order.

b. Wear Metal Production Rates

In an attempt to improve trend analysis of wear metal concentration measurements and account for the effect of oil consumption in a quantitative manner, the use of wear metal production rates, calculated on a sample-to-sample basis, has been advocated (3,4) as a criterion for engine serviceability. The wear metal production rate (WMPR) is essentially a concentration trend calculation corrected for oil consumption. It is derived from a material balance on the oil contaminant and may be expressed in a variety of ways, depending upon the particular assumptions made regarding the concentration of wear metal during the period of oil loss. Expressed in terms of a finite difference equation, one may write the material balance as

$$(V - \Delta V_2)C_2 + C_d \Delta V_2 = r\Delta t + (V - \Delta V_1)C_1 \quad (8)$$

in which

V = Volume of Oil System qts or liters

C_1 and C_2 = Concentration of Wear Metals Measured Before and After Engine Operation respectively, ppm.

Δt = Operating Hours Between Samples, Hrs.

ΔV_1 = Volume of Oil Added to Top System After Measurement of C_1

ΔV_2 = Volume of Oil Added to Top System After Measurement of C_2

C_d = Wear Metal Concentration During Oil Loss

r = Wear Metal Production Rate, mg/hr (noted previously)

To allay confusion at the outset, it should be noted that, for the sake of arithmetical convenience, quarts and liters have been used interchangeably and the density of oil as been taken as 1000 grams per liter so that parts per million are equivalent to milligrams (mg) per liter. If we denote the diluted wear metal concentration after oil replenishment as C_1' then

$$C_1' = \left(1 - \frac{\Delta V_1}{V}\right)C_1 \quad (9)$$

and the material balance becomes.

$$(V - \Delta V_2)C_2 + C_d \Delta V_2 = r\Delta t + \Delta V C_1' \quad (10)$$

If the oil loss occurs at the initial wear metal concentration after fill-up ($C_d = C_1'$) then

$$r = \frac{V - \Delta V_2}{\Delta t} \left[C_2 - C_1 \left(1 - \frac{\Delta V_1}{V}\right) \right] \quad (11)$$

For the case that the oil loss occurs at the concentration measured after engine operation ($C_d = C_2$)

$$r = \frac{V}{\Delta t} \left[C_2 - C_1 \left(1 - \frac{\Delta V_1}{V} \right) \right] \quad (12)$$

If one uses the average value of the concentration during the period of oil loss, $C_d = \frac{C_2 + C_1}{2}$ then

$$r = \frac{\left(V - \frac{\Delta V_2}{2} \right)}{\Delta t} \left[C_2 - C_1 \left(1 - \frac{\Delta V_1}{V} \right) \right] \quad (13)$$

Lotan (3) adopts a somewhat different line of reasoning in arriving at an expression for the wear metal production rate. Assuming constant oil leak and wear metal production rates and considering the process from time $t_0 = 0$, he writes a differential material balance as

$$(V_0 - qt) dC = r dt \quad (14)$$

where V_0 is now the initial amount of oil in the system and C is the variable wear metal concentration. Upon solution of Equation 14 in terms of our previous parameters, it is found that

$$r = \frac{\Delta V_2}{\Delta t} \left[C_2 - C_1 \left(1 - \frac{\Delta V_1}{V} \right) \right] / \ln \left(\frac{V}{V - \Delta V_2} \right) \quad (15)$$

A slight amount of algebraic manipulation reveals that this relationship is equivalent, to a very good order of approximation, to Equation (13) and unnecessarily complex. Application of all of these expressions for the wear metal production rate to the following OAP measurements on a TF41 engine

$$C_1 = C_2 = 13 \text{ ppm} \quad V_1 = 0.8 \text{ qts.}, \quad V_2 = 2.5 \text{ qts.}$$

$$\Delta t = 2 \text{ hrs} \quad V = 11 \text{ qts.}$$

shows that

$$r = 4.0 \text{ mg/hr for Equation 11}$$

$$r = 5.2 \text{ mg/hr for Equation 12}$$

$$r = 4.6 \text{ mg/hr for Equations 13 and 15}$$

As could have been anticipated, the assumption that the oil loss occurs at the initial wear metal concentration underestimates r , assuming that the loss occurs at the final concentration overestimates it, while the remaining two relationships yield a value intermediate between the low and high results. In view of the inherent errors to be demonstrated in such calculations, it makes little difference as to which specific assumption regarding oil loss is made. The simplest and most conservative expression is given by Equation 12.

Concentration-time data and derived wear metal production rates are depicted in Figure 5 for a fairly typical OAP iron record on a TF41 engine (SN 1670). The average oil consumption over the time period covered in the plot was 0.25 quarts/hr (moderate and representative of the engines for which oil consumption data was available). The wear metal production rate graph proves to be both disappointing and disturbing. At best, it mirrors the trend of the concentration curve in greatly magnified fashion, with some minor deviations due to the effect of oil addition (notably in the range from 23-29 hours where the concentration is constant). Its worst aspects are the numerous negative values of the WMPR and its highly erratic fluctuations from sample to sample. The physical interpretation of the negative WMPR values is that the engine is cleaning itself, an obvious impossibility. Hence they must be ascribed to the slight (and expected) scatter in the concentration measurements. Since the concentration

appears to be sensibly constant, one is led to determine the average concentration, \bar{C} , and the standard deviation of the measurements. The results indicate that the measured concentrations are probably constant over the entire operating time and that the precision of the measurements (± 0.8 ppm) is comfortably within the allowable tolerance of ± 1 ppm. Reasoning that since the concentration is constant, the WMPR might well be also, its average value, \bar{r} , and precision is determined with dismaying results. The standard deviation of the calculated WMPR's is twice the mean value. The relative difference in the precision of the concentration measurements as compared with the metal production rates calculated from them is strikingly demonstrated in Figure 6, where the 95% confidence limits on both values are plotted to the same scale. These results are not unique to this engine but representative of others analyzed. The variability of the WMPR's is approximately an order of magnitude greater than the variability of the concentration measurements, if one assumes that we are obtaining a very imprecise estimate of a constant value.

Since the conventional statistical method for determining the average wear metal production rate utilizing sample-to-sample calculations is so imprecise, other procedures for obtaining an average wear rate for a given operating interval might be worth examining. One could, for example, calculate an average rate from the total volume of oil added, the number of fillings, and the initial and final analyses for a specified elapsed operating time. A relationship for this purpose may be derived under rather restricted assumptions (e.g. Reference 3) and applied to more general situations. To accomplish our objective, it will be assumed that the oil is added in equal volumes at equal intervals, i.e.

$$\Delta V_1 = \Delta V_2 = \Delta V_n = \frac{V_T}{n}$$

$$\Delta T_1 = \Delta T_2 = \Delta T_n = \frac{T}{n}$$

where

V_T = Total Volume of Oil Added

T = Total Elapsed Time Between Analyses

n = Number of Fillings

Upon the premise that loss of wear metal in oil occurs at the concentration prevailing just prior to each filling, C_i , one may write a general overall material balance as

$$(V - \Delta V_n)C_{n+1} + \sum_1^n \Delta V_i C_{i+1} = \sum_1^n r_i \Delta T_i + V \left(1 - \frac{\Delta V_i}{V}\right) C_1 \quad (16)$$

in which C_1 and C_{n+1} are the initial and final concentrations, respectively, and V is the oil system volume. Assuming a constant WMPR, Equation 16 simplifies to

$$(V - \Delta V)C_{n+1} + \Delta V \sum_1^n C_{i+1} = rT + V \left(1 - \frac{\Delta V}{V}\right) C_1 \quad (17)$$

For any two consecutive samples with concentrations C_i and C_{i+1} , one may say that

$$C_{i+1} = \left(1 - \frac{\Delta V}{V}\right) C_i + \frac{r \Delta T}{V} = K_1 C_i + K_2 \quad (18)$$

If $\Delta V \ll V$, then $K_1 \approx 1$

and $C_{i+1} = C_i + K_2 = C_1 + iK_2$

$$\sum_1^n C_{i+1} = nC_1 + \frac{n(n+1)}{2} K_2$$

hence
$$rT = V(C_{n+1} - C_1) + \Delta V \left[(n+1)C_1 + n\frac{(n+1)K_2}{2} \right] - \Delta VC_2$$

or
$$r = \frac{V}{T} \left\{ C_{n+1} \left[1 + \frac{V_T}{2V} \left(\frac{n-1}{n} \right) \right] - C_1 \left[1 - \frac{V_T}{2V} \left(\frac{n+1}{n} \right) \right] \right\} \quad (20)$$

Applying this relationship to Engine SN1670 (Figure 5), the average value of the WMPR is found to be 2.4 for the recorded operating times and oil addition volumes. An estimate of this sort is somewhat reassuring, in that it masks negative values and sample-to-sample variation. However, its precision (theoretically of the same order as the concentration measurements) cannot be deduced from a single set of data and its reliability is indeterminate, since the calculated WMPR (r) is so highly dependent upon the final analysis, C_{n+1} . For a sufficiently large set of data points, the uncertainty of the estimate can be reduced by forming subsets of adequate number and obtaining the distribution of the rates calculated for each subset. In general, such a procedure is not very feasible, since an engine may well reach a critical condition before enough measurements are accumulated for a reliable analysis of its record.

In view of the fact that WMPRs will most likely be determined on a sample-to-sample basis, it is well to assess the probable error inherent in such a calculation. Noting that the WMPR is equivalent to a concentration trend, it would be expected to be subject to the same error, arising from the random scatter in the concentration measurements. It is readily apparent therefore, that the probable error, P_r , in the wear metal production rate will be given by

$$P_r = \frac{V}{\Delta T} P_c \sqrt{2} \quad (21)$$

where P_c is the probable error in the concentration measurement. For the TF41 engine, V is approximately equal to 11 quarts and the probable error in the concentration measurement is of order unity. Thus for an operating time of one hour, the probable error in the wear metal production rate is 15 mg/hr and its standard deviation is somewhat greater. It is not surprising then, to find the WMPRs for Engine SN1670 varying from -5 to +25 mg/hr with a precision of 9 mg/hr.

It is interesting to compare the relative trends in concentration and WMPR for an engine removed for teardown inspection upon the basis of an OAP laboratory recommendation. Such data are shown in Figure 7 for an engine used in Reference 4 to illustrate the utility of the WMPR concept. The concentration rises irregularly from 6 to 12 ppm in the interval between 5 and 38 operating hours and then shoots up abruptly to 16 ppm in the next hour of operation. The WMPR graph exhibits a magnified fluctuation but no discernible trend up to 38 hours, jumping from 6 to 50 mg/hr in the final hour before engine removal. It is contended that this is a much clearer indication of engine deterioration than the jump in concentration, even though the prior WMPR history gave no indication whatsoever of abnormal wear. In view of our previous discussion regarding the inherent errors in the WMPR calculation, one should treat large increases in their value with caution. The fact that the entire range of WMPR values prior to last hour of operation lies within approximately one standard deviation from the mean value suggests that the engine may have been wearing at a constant rate prior to the abrupt increase in the WMPR. As to this value, itself, the maximum error at the 95% confidence level for an operating interval of one hour could be as large as 38 mg/hr. Thus there is about a 5% probability of calculating a WMPR of

nearly 43 mg/hr on a random error basis, if the wear rate were actually constant at 4.7 mg/hr. The significance of the value of 50 mg/hr actually obtained is thus open to question. Consideration of the inevitable erratic nature of calculated WMPR's and the potential for large errors leads to the conclusion that they are not suitable indexes of engine wear and should not be used as criteria for maintenance action.

If one attempts to compensate for the random errors in the concentration measurements by the use of Equation 20, the average WMPR is found to be 5.4 mg/hr, another reason for regarding the sample-to-sample value of 50 mg/hr with scepticism.

SECTION III
STATISTICAL TREND ANALYSIS

a. Linear Regression of Concentration Measurements

The use of linear regression analysis to extract wear metal production rates from concentration measurements as a function of operating time has been discussed in a previous publication (2). It is based upon the assumption that the WMPR is sensibly constant over the operating interval under examination. The correlation coefficient of the regression line and the precision of the WMPR deduced from it afford a partial test of the validity of this assumption. Linear regression analysis will yield a trivial and incorrect result ($WMPR = 0$) when the concentration is ostensibly constant (as in Figure 5), due to the effects of oil consumption and replenishment. In most instances, however, the concentration measurements exhibit an increasing trend with time. Representative examples of such behavior are shown for the three TF41 engines covered in Figures 8-10. Sample-to-sample determinations of the WMPR have been plotted for each engine to demonstrate the glaring deficiencies of this method for assessing lubricated component wear. The concentration measurements for Engine SN1096 (Fig. 8) display the usual fluctuations in such records, attributable primarily to random instrumental error but perhaps due in part to the effects of oil consumption, which was not reported for this engine. Linear regression analysis produces a rather satisfactory correlation of the data, as measured by the correlation coefficient of 0.94 (compared with unity for an exact fit). The wear metal production rate and its precision (standard deviation), listed in this illustration, are deduced from the slope of the regression line. By contrast, the WMPR graph follows its usual erratic course, magnifying the fluctuations in the concentration. It exhibits no discernible trend, is nowhere alarmingly large and even tends to decrease as the concentration climbs to a value high

enough to signal maintenance action. Furthermore, the average of the plotted WMPR values (2.45) is only 70% of that found from the linear correlation and its standard deviation is 30 times as large. Figure 9, for Engine SN1126, illustrates the extreme fluctuation in sample-to-sample determinations of the WMPR that may result from concentration measurement scatter. The peak value of 50 mg/hr is produced by a large negative concentration excursion preceding a small positive one from the linear regression line, which gives a true representation of the steady increasing trend in the iron wear metal concentration at a WMPR (\bar{r}) of 4.75. The relatively low correlation coefficient of 0.8 is indicative of the scatter in the measurements. It should be noted that these data demonstrate the possibility of obtaining spurious values as high as that found for Engine SN1175 in Figure 7, used in Reference 4 to support the usefulness of the sample-to-sample WMPR concept. The average value of the WMPR obtained therefrom is far too high and its precision (21.1 mg/hr) is extremely poor, becoming ridiculous at the 95% confidence level. Further evidence for the unreliability of the sample-to-sample WMPR as an index of engine wear is afforded by Figure 10, which depicts the wild fluctuations in these values for Engine SN1644. Any attempted physical interpretation of such behavior would obviously be absurd. The dispersion of the concentration measurements about the linear regression lines, on the other hand, is in no way unusual, as evidenced by the correlation coefficient of nearly 0.9.

Having established the superiority of linear regression analysis as a tool for delineating meaningful trends of wear metal concentration measurements, it is necessary to incorporate the effect of oil consumption and dilution by addition into the procedure. This may be accomplished by calculating a "corrected" concentration, which is the fictitious value that

would be measured if all the wear metal stayed in the system and only the oil were consumed. The calculated "corrected" concentration will be dependent, of course, upon the particular assumption made regarding the wear metal concentration during oil loss. However, differences in the calculated values are not found to be significant for the three assumptions possible in the case of unequal filling intervals and oil addition volumes (concentration equal to that at the start of the operating interval, at the end of it, or an average between them). In the interests of consistency, the oil loss is assumed to occur at the wear metal concentration measured just after operation. Under these circumstances, for the Nth measured concentration the

$$\text{Corrected Concentration} = \frac{1}{V} \left[C_i (V - \Delta V_i) + \sum_1^n C_i \Delta V_i \right] \quad (22)$$

where as before

V = Oil System Volume

C_i = Concentration of Wear Metal
Before Oil Addition

ΔV_i = Oil Lost in the Interval Between
Two Concentration Measurements

This relationship yields the most conservative (i.e. highest) value for the corrected concentration. It is understandably a better indication of the actual wear undergone by the monitored equipment than the measured concentration. Evidence for this assertion is offered by Figures 11 and 12, which present measured and corrected concentrations for two TF41 engines which have been discussed previously. The first of these covers Engine SN1670, mentioned earlier in connection with Figure 5. The other deals with Engine SN1147, designated as Engine C in Figure 2. In both engines, the oil consumption effect has been magnified by the condition that the contaminant levels have apparently reached their limiting or steady-state concentrations, since the reported concentrations are substantially constant over the entire

operating interval. Linear regression of the as measured data for Engine SN1670, discloses a slight trend in the concentrations which may be disregarded, since the correlation coefficient is so low. The corrected concentrations, on the other hand exhibit a definite though modest wear rate and their correlation coefficient has been dramatically increased. For the sake of reducing clutter, the linear regression lines themselves have been omitted for the two sets of data. A similar situation prevails for Engine C, though in this instance the WMPR for the reported concentrations was found to be zero. For both engines the average as reported concentrations had precisions smaller than the allowable tolerance of ± 1 ppm. Their agreement with limiting concentration values (particularly in the case of Engine C), deduced from the slopes of the linear regression lines and the average oil consumptions, support the validity of the "corrected" concentration concept and indicate that it has real physical significance.

It may be of interest to present applications of the "corrected" concentration concept to other engines to illustrate its utility. Measured concentrations for Engine SN1644 were plotted in Figure 10 to support our discussion of the serious failings in the sample-to-sample determinations of the wear metal production rate. "Corrected" concentrations for this engine are shown in Figure 13. Comparison with Figure 10 reveals that a much better fit to the data has been secured with this approach. The correlation coefficient has been markedly increased and the derived WMPR is more than double its previous value. The excellent precision of this determination lends confidence to the acceptance of its validity. It is worthy of note to observe that the standard deviation of this value is 1/60 that obtained for the sample-to-sample WMPR, though the average values only differ by 20%.

Before proceeding further with examples of the use of the "corrected" concentration concept, it may be well to discuss the subject of confidence limits on regression lines, which were touched on briefly and without explanation in the discussion of the OAP record for Engine SN1218, displayed in Figure 3.

There are well-known statistical procedures for deducing the variance of the estimate of the magnitude of a dependent variable for a given value of the independent variable, based upon the regression line between them (see for example Reference 5 or similar texts). To summarize them briefly, we note that if a regression line is described by the equation.

$$Y_c = mX_o + b \quad (23)$$

in which

Y_c = Calculated Value of the Dependent Variable for a Given Value, X_o , of the Independent Variable

and if

Y_o = Observed Value of the Independent Variable at X_o

then the variance of the estimate, $S^2(Y_c)$ for N data points will be given by

$$S^2(Y_c) = \frac{n}{\sum_1} (Y_o - Y_c)^2 / (N-2) \quad (24)$$

The variance of the predicted value of the dependent variable for a given value (X_o) of the independent variable, $S^2(Y_c/X_o)$, will depend upon the variance of the estimate, $S^2(Y_c)$, in the following manner:

$$S^2(Y_c/X_o) = S^2(Y_c) \left[1 + \frac{1}{N} + \frac{(X_o - X_m)^2}{\sum_1 (X_i - X_m)^2} \right] \quad (25)$$

in which

X_m = Mean Value of the Independent Variable for the Set of N Observations

The square root of the variance of the estimate, $S(Y_c)$, is defined as the standard error of the estimate. Similarly, one might define $S(Y_c/X_0)$, the square root of the variance of the predicted value, as the standard error of the predicted value. In other words, within an error of one standard deviation, the observed value of the dependent variable will be $Y_c \pm S(Y_c/X_0)$. This corresponds to a confidence limit of approximately 68%. For other confidence limits and a finite set of data, one must make use of the t distribution, tabulated in most elementary statistics texts. (Tables of the t distribution list the number of standard deviations corresponding to a desired probability of occurrence for a specified number of degrees of freedom (N-2), for the case of linear regression of N data points). Thus for a specified confidence level, a given fraction of the observed values of the dependent variable are expected to lie within the band $Y_c \pm t.S(Y_c/X_0)$.

The wear metal concentration data for Engine SN1126, shown previously in Figure 9, are replotted in the upper graph of Figure 14 together with their regression line and superimposed 95% confidence limits. The low value, at 10 operating hours, is barely within the confidence band and is certainly responsible for the misleading abnormally high WMPR of 50 mg/hr calculated at 11 operating hours. Corrected concentrations for this engine are depicted in the lower half of Figure 14. The wear metal production rate determined for these data has increased to nearly 6 mg/hr, though the precision of this value is identical to that obtained for the uncorrected concentrations. It is interesting to note that averaged sample-to-sample WMPR of 6.5 mg/hr (see Figure 9) is only about 10% larger than the regression value, indicating that it does compensate, though not satisfactorily, for the masking effect of oil consumption. However, its precision (21 mg/hr) is abysmal, being more than 140 times larger than the regression line value of 0.14 mg/hr.

Measured iron concentrations for Engine 1175, first presented in Figure 7 and discussed at some length, are plotted again in Figure 15 with the addition of their regression line and 95% confidence band. All of the data lie comfortably within the confidence limits, with the exception of the last point, upon the basis of which a T code was issued. The regression line WMPR for the observed concentrations is much lower than the average sample-to-sample value (4.7 mg/hr) as expected, since the former does not correct for the effect of oil consumption.

Corrected concentrations for this engine are shown in Figure 16. The WMPR appropriate to their magnitudes has increased by more than 50%; however, the precision of the determination has not changed significantly. It is seen that the final data point still lies outside the 95% confidence band. Though the averaged sample-to-sample WMPR is within 20% of the more valid regression line value, its precision is still quite poor, exceeding the latter by more than a factor of eight. We may repeat again, that though the averaged sample-to-sample WMPR is a rough approximation to the true (regression line) value, the individual determinations of the former quantity are misleading and erroneous.

b. Analysis of Variance

Wear metal concentration data for Engine SN1096 (see Figure 8) are depicted again in Figure 17, supplemented by their regression line and 95% confidence limits. The expectation that at least 95% of the measured concentrations will lie within this band is well satisfied. The width of the confidence band is a measure of the scatter of the data and it appears to be quite large, despite the high correlation coefficient, R , and small standard deviation of the WMPR derived from the regression line (ca 10%, see

Figure 8). One is led to wonder whether the scatter can be attributed entirely to random errors of measurement or is due, in part, at least, to intrinsic changes in the nature of the data, i.e. systematic variations in the wear metal production rate. In the latter event, the attempt to correlate the entire data set with a single regression line will be invalid and give rise to an overly large error in the estimate. This possibility can be checked by breaking down the sample set into subsets characterized by constant wear metal production rates.

Since the variance of the estimate is the best simple measure of the "goodness of fit" of observed data to a regression line, a variance tracking procedure was devised which calculated a new regression line and its error variance as each successive data point was added to the previously analyzed ensemble of measurements. An abrupt increase in the variance signaled a change in the wear metal production rate and initiated calculations on a new wear regime. Since these calculations are highly repetitive and lengthy, a computer program capable of execution on a quite unsophisticated desk computer (the Commodore PET) was written for this purpose (6). Results for Engine SN1096 are shown in Figure 18. The tracking procedure has broken down the concentration measurements into five separate wear regimes. An initial rather high rate of wear is followed by a more moderate, though still high wear rate, succeeded by two periods of essentially zero wear and terminated by a final period of exceedingly high and abnormal wear. The latter indicates a need for maintenance action. None was recommended by the OAP laboratory reporting the data, based upon their qualitative interpretative procedures. The regression lines in each wear regime have been bounded by lines indicating the standard deviation of the predicted value of the concentration (ca 68% confidence limits). It is

instructive to compare the variance of the estimate for each of the regression lines with the overall regression line shown in Figure 17. In every instance the variances, $S^2(Y_c)$, are much smaller than that for the single line. For the second wear regime (WMPR = 3.75 ± 1.38) two data points were so grossly in error that the regression line was recalculated after their elimination.

Though casual visual inspection indicates that variance tracking has produced a much better fit to the experimental measurements than a single regression line, it would be reassuring to confirm this quantitatively. A statistical test, Gauss's Criterion, may be employed for this purpose. Where several regression lines are used to correlate an ensemble of sample points, Gauss's Criterion states that the best fit to the data is that for which the sum, V , defined below is a minimum

$$V = \sum_1^i (n_i - 2) S_i^2(Y_c) / (N - 2i)$$

in which

i = Number of Individual Regression Lines

n_i = Number of Data Points Included in Each Line

$$N = \sum_1^i n_i$$

Gauss's Criterion is a necessary condition for testing the "goodness of fit", however, it is not a sufficient one. The possibility exists that the decomposition of the data into several sub-populations is merely a statistical artifact arising from a paucity of measurements in the different wear regimes. We may test the likelihood of this by an analysis of the covariance of the specific subsets and the total sample universe. The test most pertinent to our current interests is one to determine whether or not a

single regression line is statistically valid for the data. This is decided by means of an F test, a ratio of the particular variances involved in the hypothesis under study. For the case at hand, if

N = Total Number of Data Points

$S_n^2(Y_c)$ = Variance of the Estimate for a Single Regression Line Including all Data Points

$$W = \sum_1^i (n_i - 2) S_i^2(Y_c)$$

$$F = \left[\frac{(N-2) S_n^2(Y_c) - W}{2i-2} \right] \cdot \frac{1}{V}$$

To appreciate the physical meaning of the F test, one may note that V is in reality the pooled variance of the different subsets of data. If all the sample points came from the same population, V would be an estimate of the variance of the total population and the F ratio would be quite small (unity, in fact, if the data were free from random error fluctuations). F values may be calculated for assigned probabilities that their departure from unity is due to random errors in measurement. They are listed in most texts on statistics (e.g. Reference 7) at the 5% and 1% points (i.e., 95% and 99% confidence levels).

It was convenient and simple to incorporate both tests into the variance tracking program. Results for Engine SN1096 are noted in the legend for Figure 18. The multifit variance is significantly less than the variance of the single line and more importantly, the F value exceeds the 99% confidence level by a very appreciable margin. One may therefore conclude that there is very little chance that the grouping of the concentration data into five wear regimes is the result of random measurement errors and the use of a

single regression line to correlate the data is inappropriate, despite its high correlation coefficient.

It is interesting to subject the corrected concentration values for Engine SN1147 (shown in Figure 12) to the same type of analysis. Results are displayed in Figure 19. The data may be segregated into four wear regimes, two of them with abnormally high wear rates. The multifit variance is slightly less than the single line variance, indicating a somewhat better fit to the sample points, though both are acceptably small. The F test, however, indicates that the reverse might be true. At the 95% level of confidence, it would appear that a single regression line is perhaps more appropriate to the data than four separate lines. Caution would dictate acceptance of the latter hypothesis, with a single WMPR as the best indication of the wear metal trend. It is worthy of note that the correction for oil consumption coupled with regression and variance analysis have revealed evidence of appreciable and possibly abnormal wear in an engine which was experiencing zero wear upon the basis of the measured wear metal concentrations.

The illustration designated as Figure 20 involves an engine removed for high copper content upon the recommendation of an OAP Laboratory. The action was based upon the measured concentrations plotted here, presumably upon the sharp rise in concentration to a threshold value as it approached the 90 hour operating mark. Its OAP history just prior to the concentration increase was uneventful, the engine exhibiting an apparent zero wear rate for the forty previous hours. The initial statistical analysis of the concentration and oil consumption measurements for the engine are presented in terms of the calculated regression line for the corrected concentrations, bounded by the standard deviation for predicted concentrations. In this

frame of reference, the apparent sharp rise in measured concentration as the 90 hour operating time is approached appears to be a mere statistical fluctuation due to random measurement error. All corrected concentration values are within a standard deviation of those predicted from the regression line. Even the final two abnormally high values at 90 hours (presumably measured after ground runs) fall within or just at the 95% confidence limits. These circumstances, combined with the rather modest wear rate derived from the linear regression line, give rise to some doubt regarding the necessity for removal of this engine. Despite the high correlation coefficient of the regression line, the variance of the estimate, $S^2(Y_c)$, is large enough to suggest a strong possibility of multiple wear regimes and indicate the desirability of variance tracking over this segment of the engine's history. The results are shown in Figure 21. The corrected concentration data fall into five wear regimes characterized by either zero or very modest rates of wear. It is interesting to note that the rate of wear was virtually zero, when the engine was subjected to ground runs just prior to removal. That the segregation of the data in this manner is no mere statistical artifact is attested to by both the multifit variance and F values. The former gives evidence of a far better fit to the data than a single regression line while the F ratio (exceeding the 99% level by a factor of ten) indicates that the possibility that the observed grouping is due to random measurement errors is vanishingly small. A confidence limit calculation on the final wear regime (80-90 hours) suggests that the high concentrations measured after the groundrun were probably real, with a 1% or less chance that they resulted from random error. Lacking evidence to the contrary, our analysis leads to the conclusion that the engine began to wear abnormally during ground runs of unreported duration. If this is indeed

the case, it appears that designating this engine for maintenance action was a lucky OAP hit.

For our final example of the knowledge regarding engine wear that can be garnered by variance tracking, we return once more to Engine SN1175 removed for high iron content and discussed at some length in connection with Figures 7, 15 and 16. Results of variance tracking of the corrected concentrations are presented in Figure 22. The data fall into three wear regimes, the first exhibiting a modest and the remaining two rather high wear rates. Both the multifit variance and the F values indicate that this breakdown of the data is statistically justifiable. The F ratio, five times as large as the 99% level value, suggests, as in the case of the previous engine, that the probability that the observed grouping is due to random error is negligible. Consideration of the deviation of final corrected concentration (at 39 operating hours) from the regression line indicates that the chance that it resulted from a random measurement error is much less than 1%, and that the rise in concentration at this point was real. In this instance, variance tracking has enabled us to distinguish a real concentration increase from a chance fluctuation. This could not be done with assurance upon the basis of the information presented in Figures 7, 15, and 16. A confidence limit calculation based upon the regression line in Figure 16 would permit at least a 2% probability of random error.

SECTION IV

DISCUSSION

The qualitative nature of the JOAP evaluation methodology introduces an element of ambiguity into the interpretation of wear metal concentration measurements. In an effort to supplement bare concentration limits with some quantitative measure of the variation of wear metal concentration with operating time, 10 hour trend threshold values have been incorporated into the most recent JOAP laboratory manual. Our analysis has indicated that the utility of this criterion in evaluating the condition of an engine is open to question. It has been demonstrated that the trend between two consecutive samples, calculated in accordance with the JOAP prescription, is subject to large errors as a result of the tolerance of ± 1 ppm in the concentration measurements. This type of error varies inversely with the operating time between samples, for engines using negligible quantities of oil, and has a 40% chance of exceeding 4 ppm for a sampling interval of one hour. Since sampling times are frequently quite brief for single engine aircraft and the threshold limits for acceptable trends are rarely less than 4 ppm, abnormal trend numbers must be regarded with a great deal of scepticism in such cases. Threshold limits may be increased to compensate for their lack of precision. Tables have been prepared to indicate acceptable maximum trend levels as a function of operating time intervals between samples for specified confidence levels.

Doubts regarding trend numbers are compounded for oil-consuming engines. Superimposed upon the error introduced by scatter is the masking effect of oil addition, which dilutes the wear metal concentration. As a result, the trend calculated for long sampling intervals may be a less reliable index of

engine wear than those determined for shorter ones. Wear metal concentrations may attain a steady state value due to the effects of oil-consumption and replenishment. JOAP offers no quantitative procedure to account for this in interpreting their spectrometric measurements. In these circumstances, calculated trends merely reflect and magnify the imprecision in the concentration measurements and have no physical significance. They can serve their purpose best by being ignored.

In an attempt to circumvent the inadequacies of the JOAP evaluation approach, the use of wear metal production rates, calculated on a sample-to-sample basis, has been advocated as a criterion for engine serviceability. The WMPR is intended to account for the effects of both engine operating time and oil consumption in a quantitative manner. It is, in essence, merely a concentration trend calculation corrected for oil consumption and subject to the same type of imprecision, magnification of the random error of the concentration measurements. Though it does compensate, in part, for the effects of oil consumption, its erratic fluctuations from sample-to-sample are devoid of significance and grossly misleading. Averaging, to cancel out the effect of random variations, does not overcome the inherently low precision of this type of calculation and allow it to serve as a valid index of engine wear.

Linear regression analysis is shown to be a greatly superior technique for extracting wear metal production rates from concentration data. The WMPR is equal to the product of the slope of the regression line and the oil system volume. It is generally more precise than the concentration measurements themselves. Reliable WMPR's are considered to be a better index of engine wear than concentration trends, since they are normalized

to a common basis (i.e. for a given WMPR, an engine with a large oil system volume will exhibit a smaller trend than one with a smaller volume). To handle the effects of oil consumption and replenishment on wear metal concentration trends, a "corrected" concentration concept is introduced and demonstrated. It is proved to be a better indication of engine wear than the measured concentration, particularly when the latter attains a steady-state value. Regression analysis also offers the capability of establishing confidence bands to assess the probability of random error in any specific concentration measurement. Variance analysis further increases the usefulness of regression techniques, by permitting the decomposition of concentration measurements into distinct wear regimes. These can then be tested for their statistical validity.

The techniques that have been described here seem readily applicable to the formulation of guidelines for maintenance action. Linear regression analysis and variance tracking of the OAP history on an engine or other monitored equipment can establish the normal rates of wear for that particular piece of equipment. Variance tracking of current OAP data on a real-time basis can then be utilized to signal appropriate maintenance action when the wear rate begins to exceed normal values within certain preassigned limits. For earlier warning of possible abnormal wear, a signal can be generated when the latest measured concentration exceeds the value predicted from the regression trend line by an amount greater than a certain prescribed confidence level (say 95%). This will alert maintenance personnel to the need for close watching of the monitored equipment. It is apparent that criteria defined in this manner are specific to a given piece of equipment and superior to absolute guidelines for each equipment type.

Our discussion of the effect of oil consumption on measured wear metal concentrations indicates quite clearly that the analytical techniques that have been described will not be useful if the oil consumption in lubricated equipment is not carefully observed and recorded. Virtually nothing can be done to assess wear trends if the wear metal content has reached its limiting concentration and oil consumption has not been reported.

The computer program devised for this work was tailored for use on an inexpensive desk top instrument with the thought that it might be readily obtained by OAP laboratories in the field. Requiring concentration measurements, oil consumption, and operating time as input data, it calculates the corrected concentrations and performs a progressive regression analysis upon them, tracking the variance as each successive data point is added. The program can determine break points in the wear metal production rates in accordance with a predetermined criterion, but optimum results have not been obtained, to date, using this procedure. For the purpose of the present work, the program was modified to allow the operator to determine the break points upon query by the computer. Once these have been decided, the program calculates confidence limits for each wear regime and the multifit variance, single line variance, and F ratio for the entire data ensemble. In its present form, where interaction with the operator is desirable, the program is not yet suitable for use by untutored field personnel. Further efforts will be required to reduce the knowledge required for its execution. Unfortunately, this can only be accomplished at the expense of greater program complexity and redundancy.

SECTION V

CONCLUSIONS

JOAP evaluation procedures do not make full use of concentration measurements to monitor the condition of oil-wetted components in engine and other lubricated equipment. They lack quantitative methods to account for the effects of oil consumption and random errors in spectrometric measurements. These deficiencies assume increasing importance for oil-consuming engines and single-engine aircraft sampled at frequent intervals. Their methodology is of doubtful utility when the wear metal concentration attains a steady-state value.

Sample-to-sample wear metal production rates do take into consideration the effects of oil consumption and replenishment on wear metal concentrations. However, their inherently poor precision, arising from their magnification of the imprecision in concentration measurements, render them unsuitable as an index of engine wear.

Linear regression and variance analysis techniques applied in this report appear to be the best current method of performing quantitative trend analysis on wear metal concentration data.

In oil-consuming equipment, consumption information is indispensable for the accomplishment of meaningful analysis.

Variance tracking is capable of delineating different rates of wear in a set of wear metal concentration measurements.

The techniques that have been described in this work have been incorporated into a simple computer program which with slight modifications and extensions seems suitable for the generation of guidelines for maintenance action.

ACCURACY INDEX FOR IRON, JOAP LABORATORY MANUAL

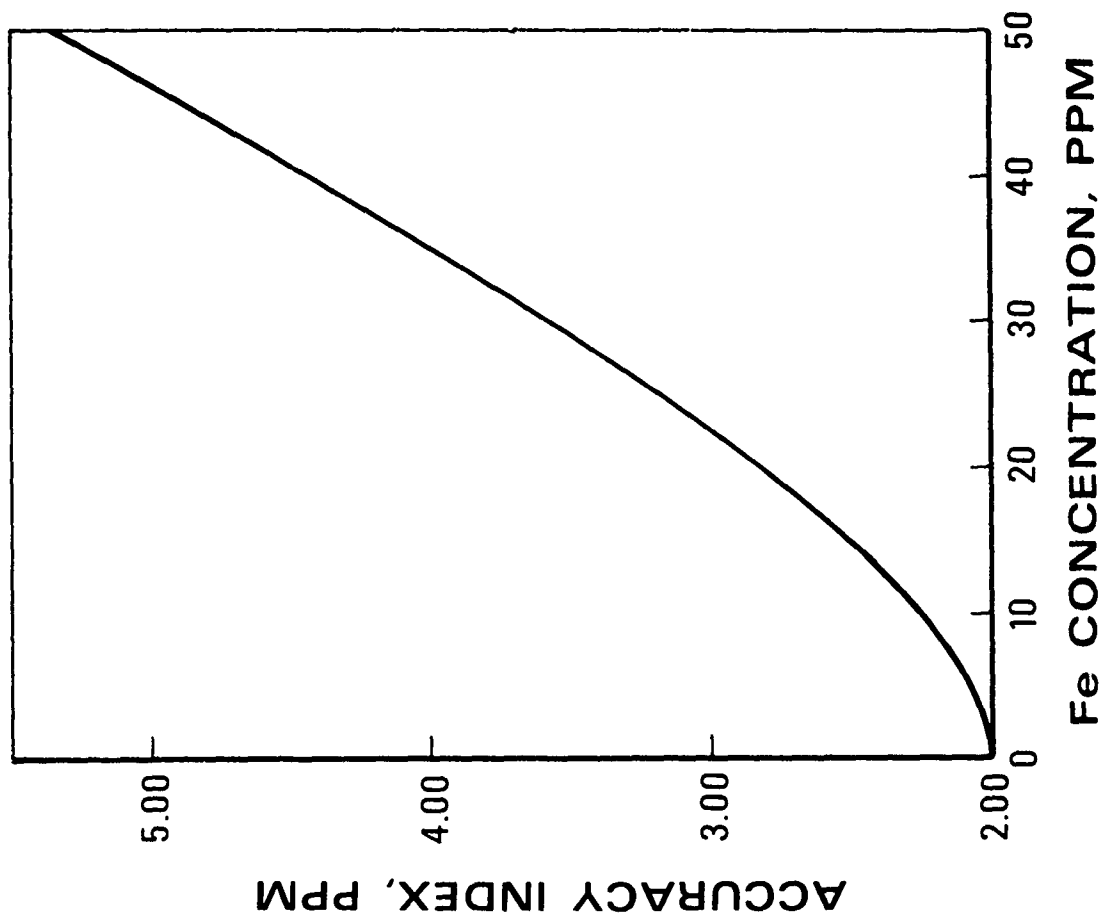


Figure 1

Accuracy Index for Iron

TYPICAL SOAP RECORDS FOR IRON IN OIL FOR TF-41 ENGINES (HIGH OIL CONSUMPTION, SMALL TANK CAPACITY)

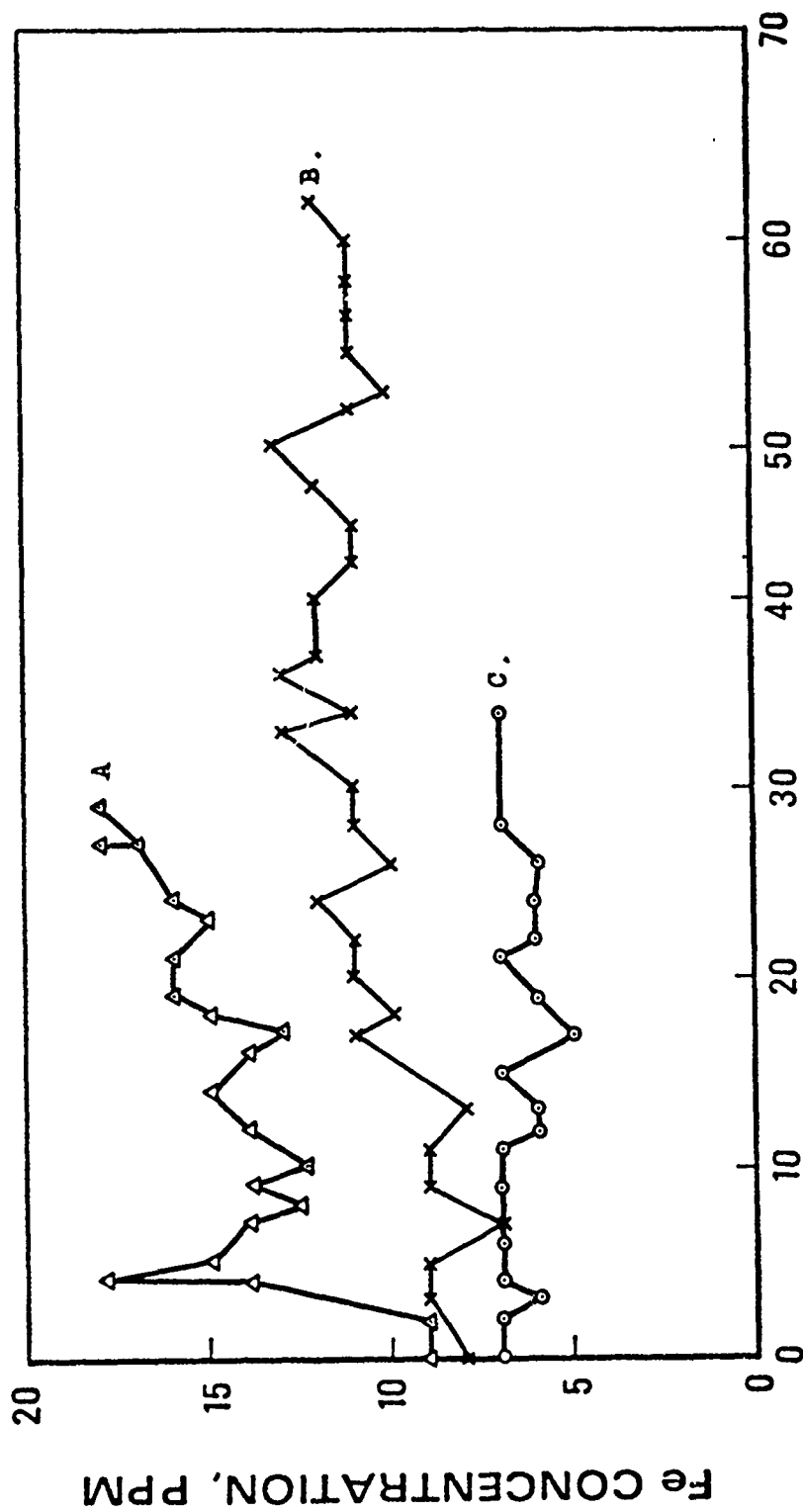


Figure 2

Typical Iron Wear Metal Records, TF41 Engines

QAP CONCENTRATION DATA ON TF41 ENGINE: SN1218
 ENGINE REMOVED FOR INSPECTION AT TSO = 87 HRS.
 (HIGH IRON)

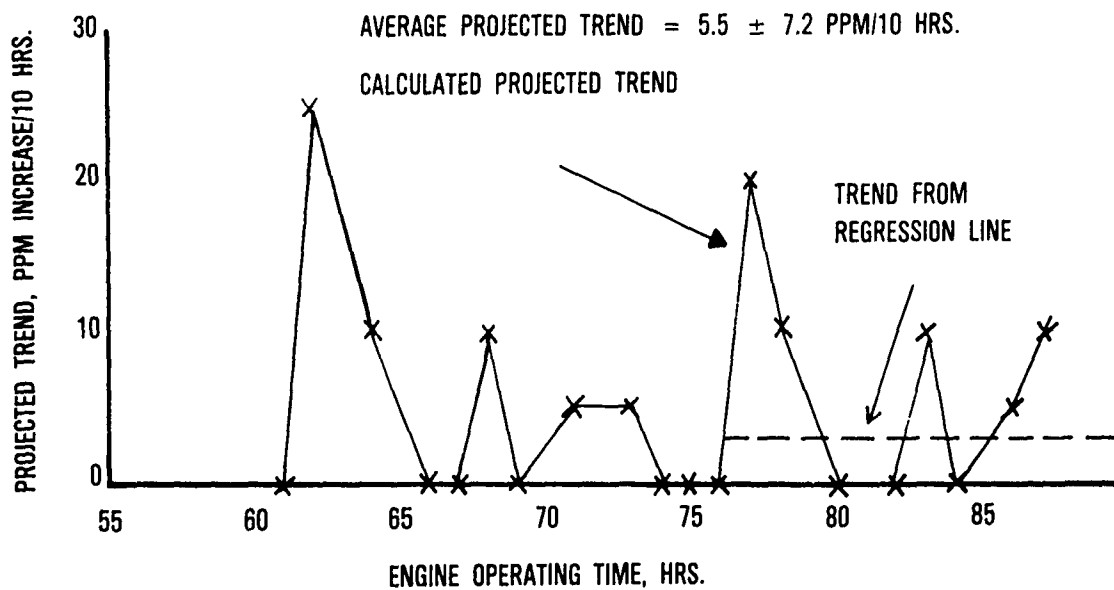
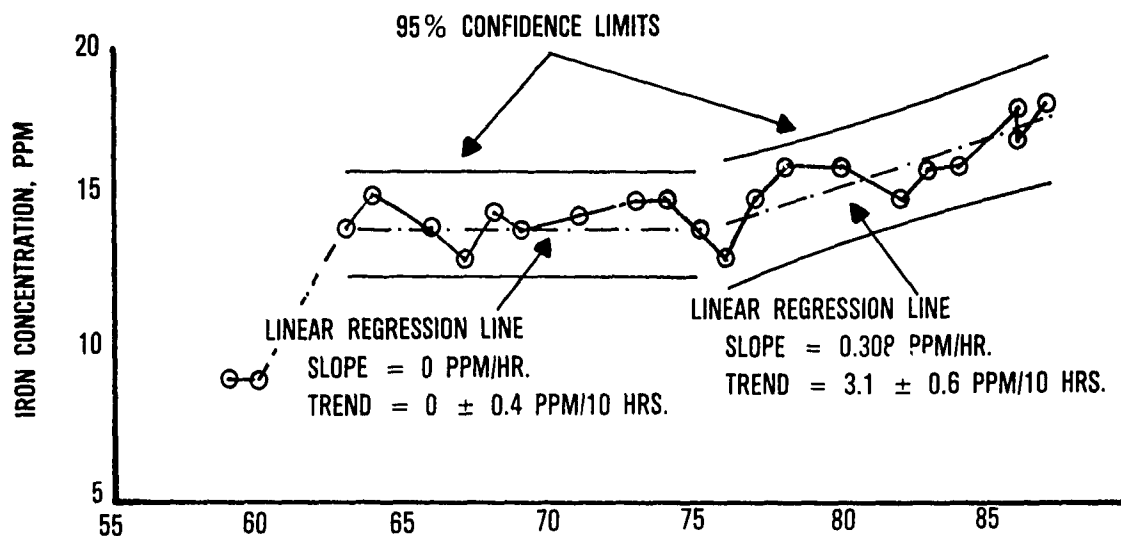


Figure 3

Measured Concentrations for TF41 Engine: SN1218

EFFECT OF OIL CONSUMPTION (AND ADDITION) ON MEASURED WEAR METAL CONCENTRATIONS

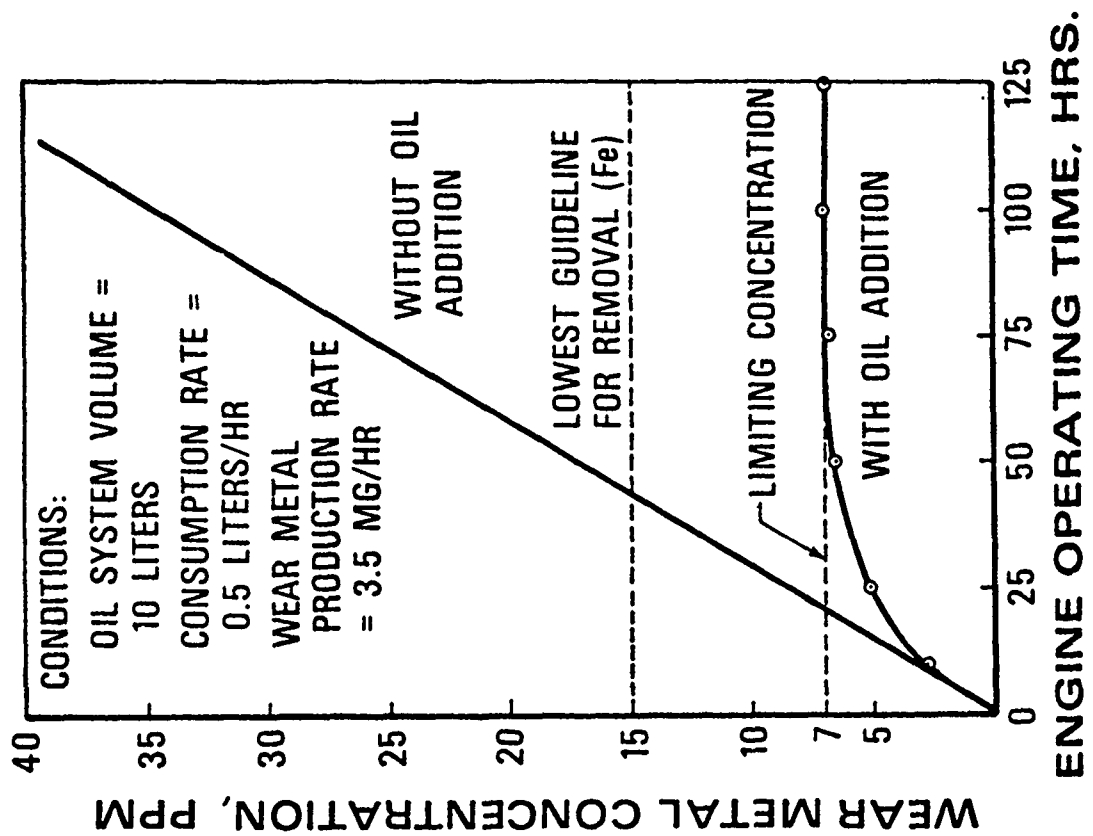


Figure 4

Effect of Oil Consumption on Apparent Concentration

ENGINE SN 1670 (TF-41)
TYPICAL IRON DATA AND OIL CONSUMPTION

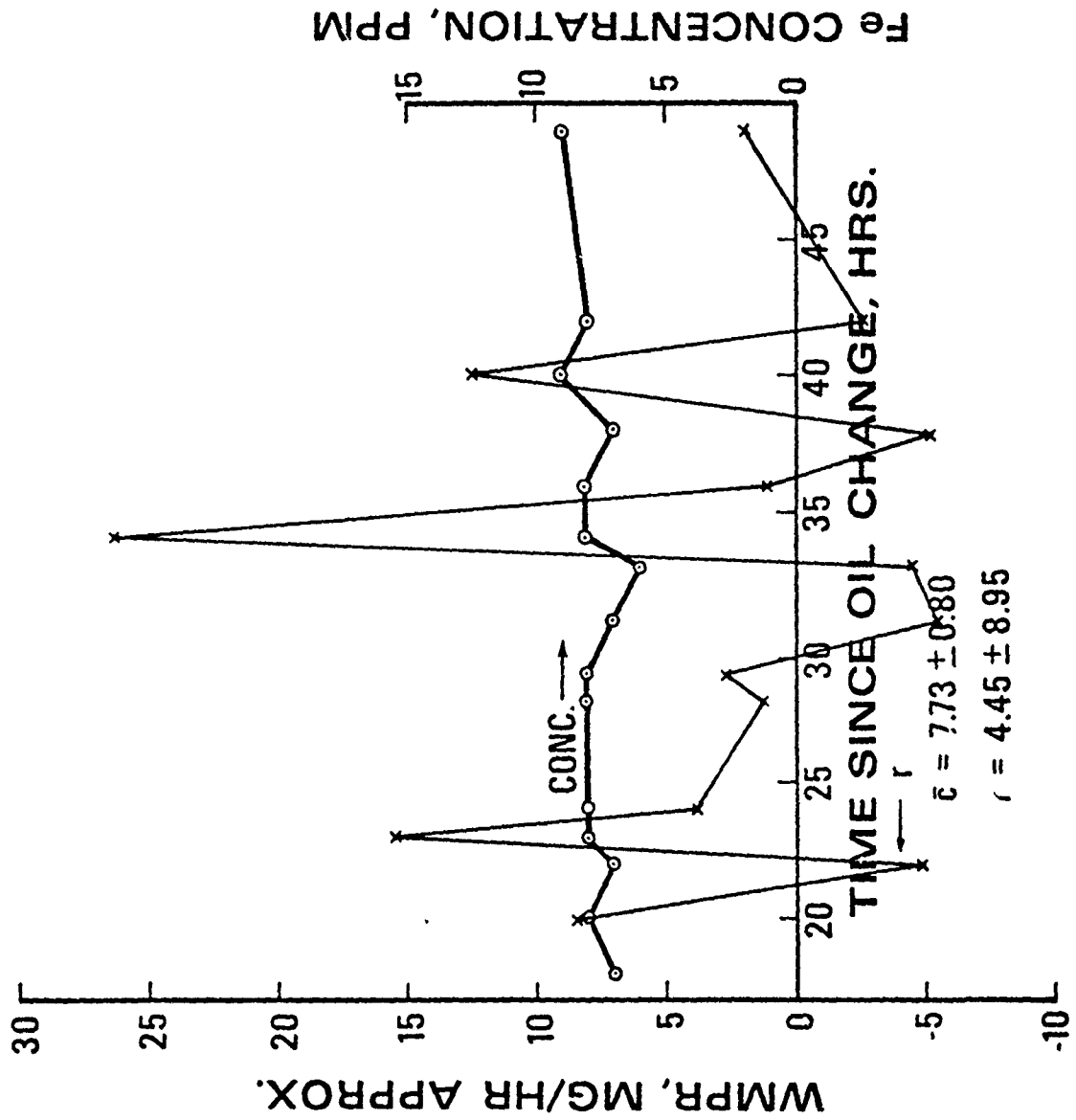


Figure 5

Concentration Measurements and WMPR for TF41 Engine: SN1670

ENGINE SN 1670

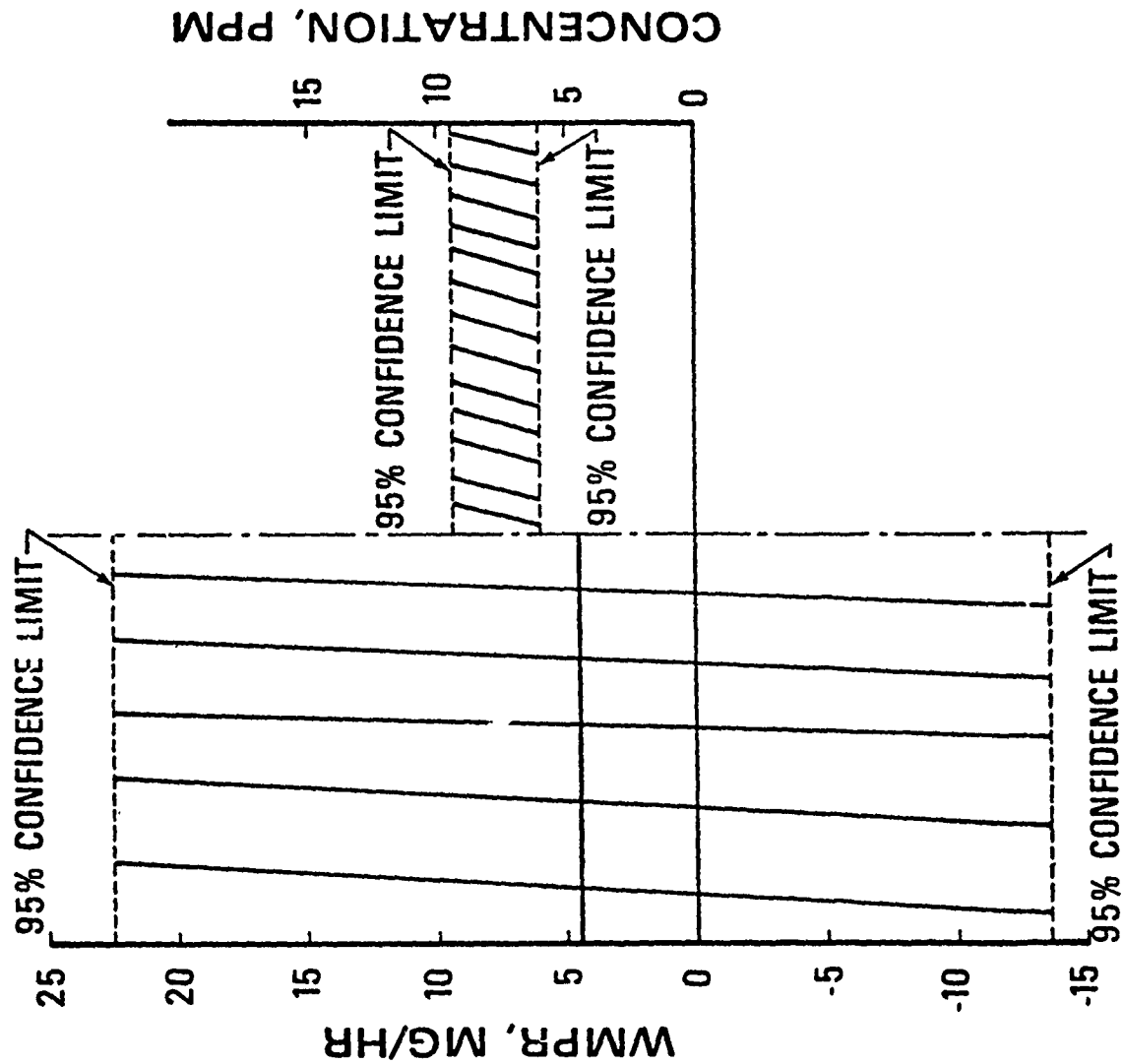


Figure 6

Confidence Limits on Concentration Measurements
and WMPR, Engine SN1670

ENGINE SN 1175

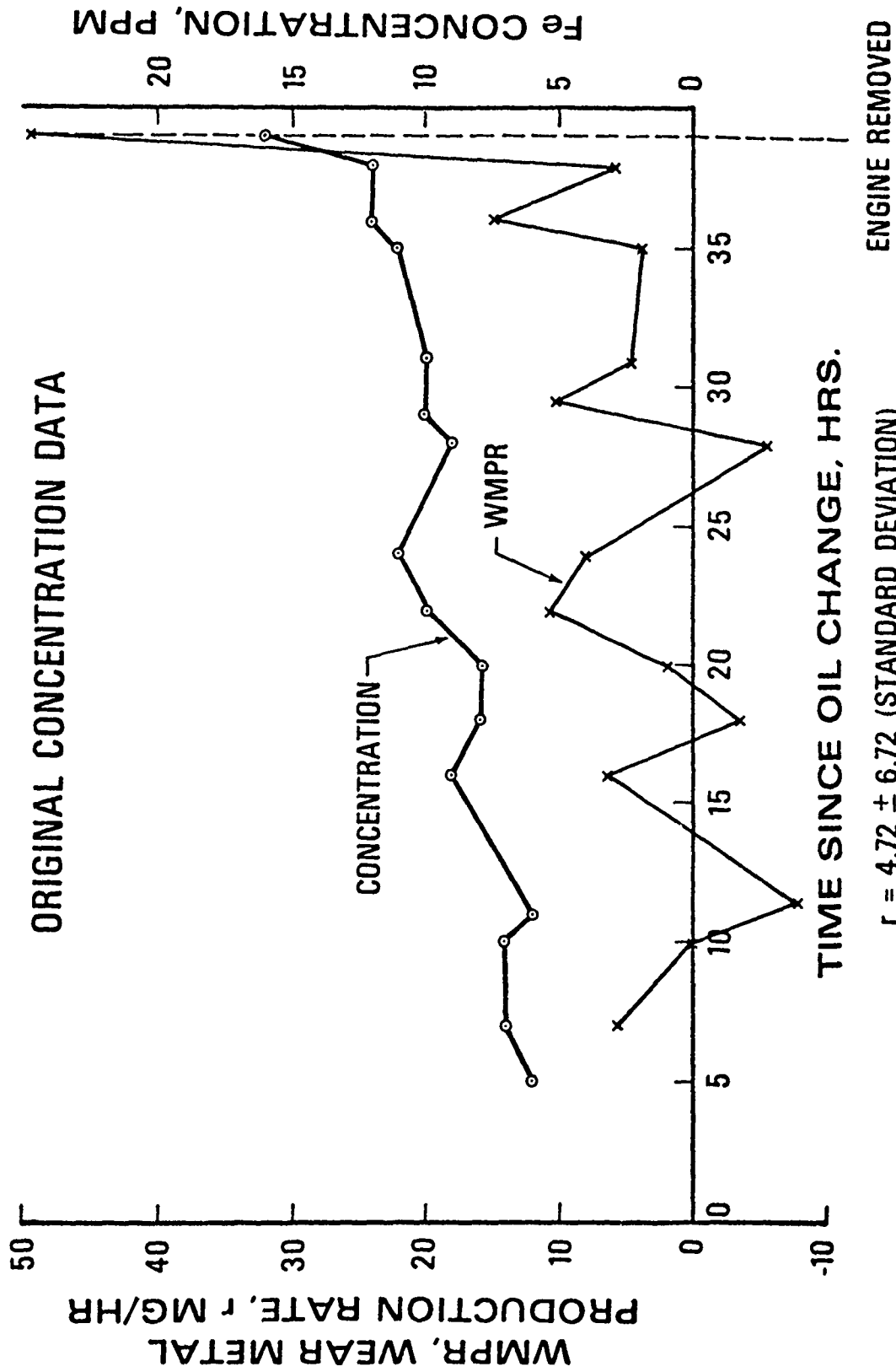


Figure 7. Concentration Measurements and WMPR for TF 41 Engine: SN1175, Removed for High Iron

ENGINE SN 1096

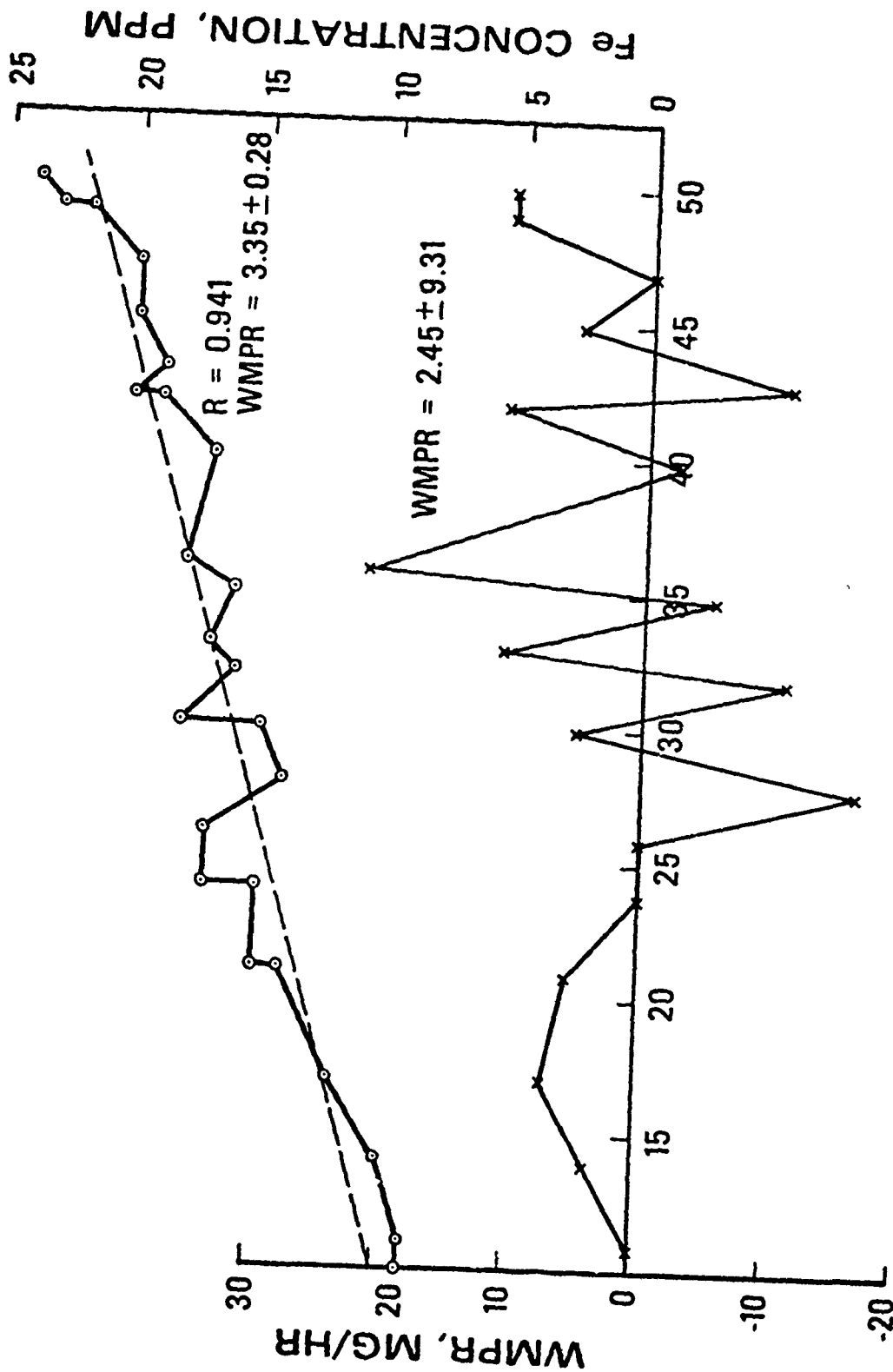


Figure 8
Concentration Measurements and WMPR for TF47 Engine: SN1096

ENGINE SN 1126
JUMP IN IRON CONTENT (NEGLECTING OIL ADDITIONS)

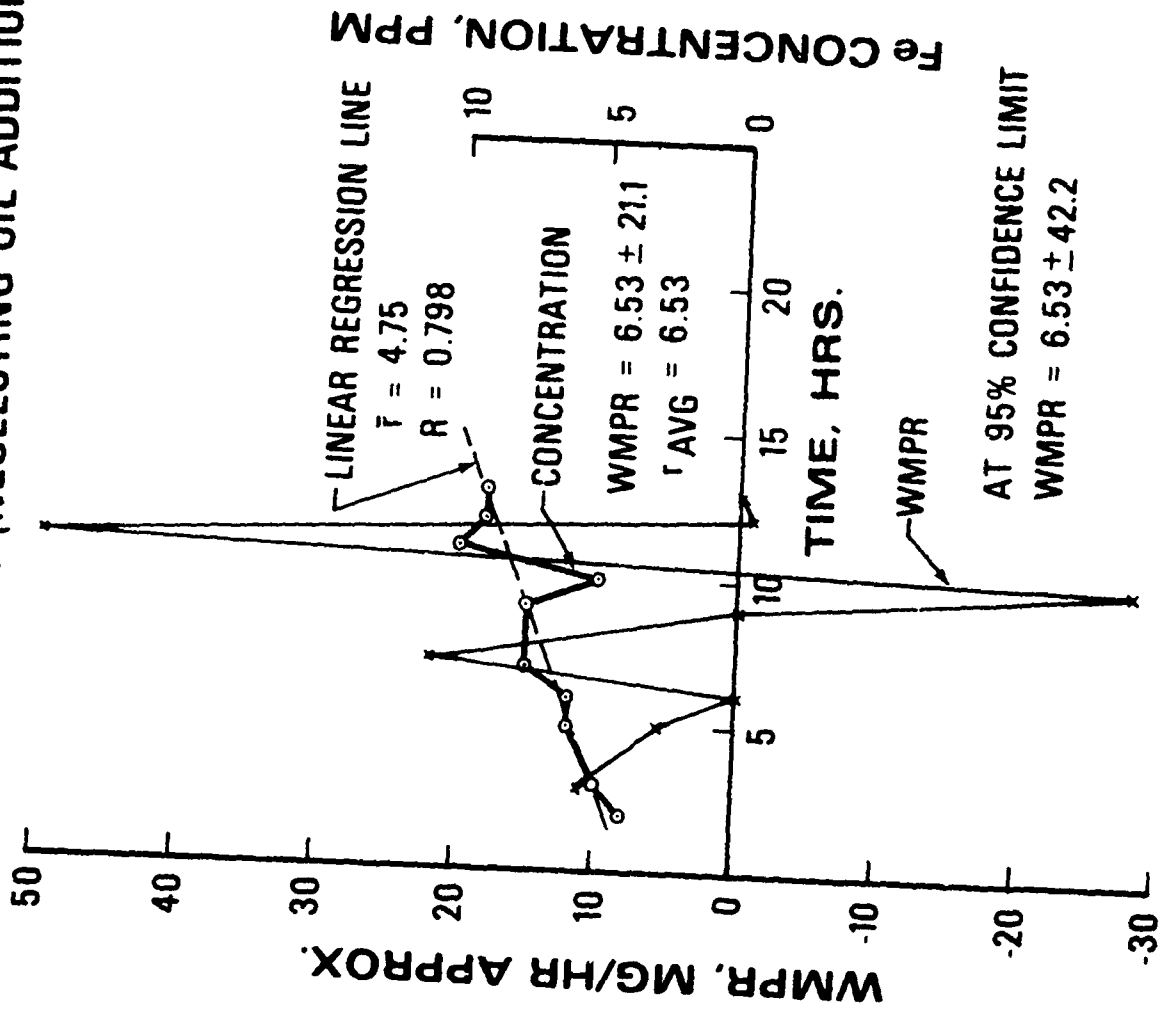


Figure 9

Concentration Measurements and WMPR for TF41 Engine: SN1126

ENGINE SN 1644
FAIRLY TYPICAL Fe DATA
(NEGLECTING OIL ADDITIONS)

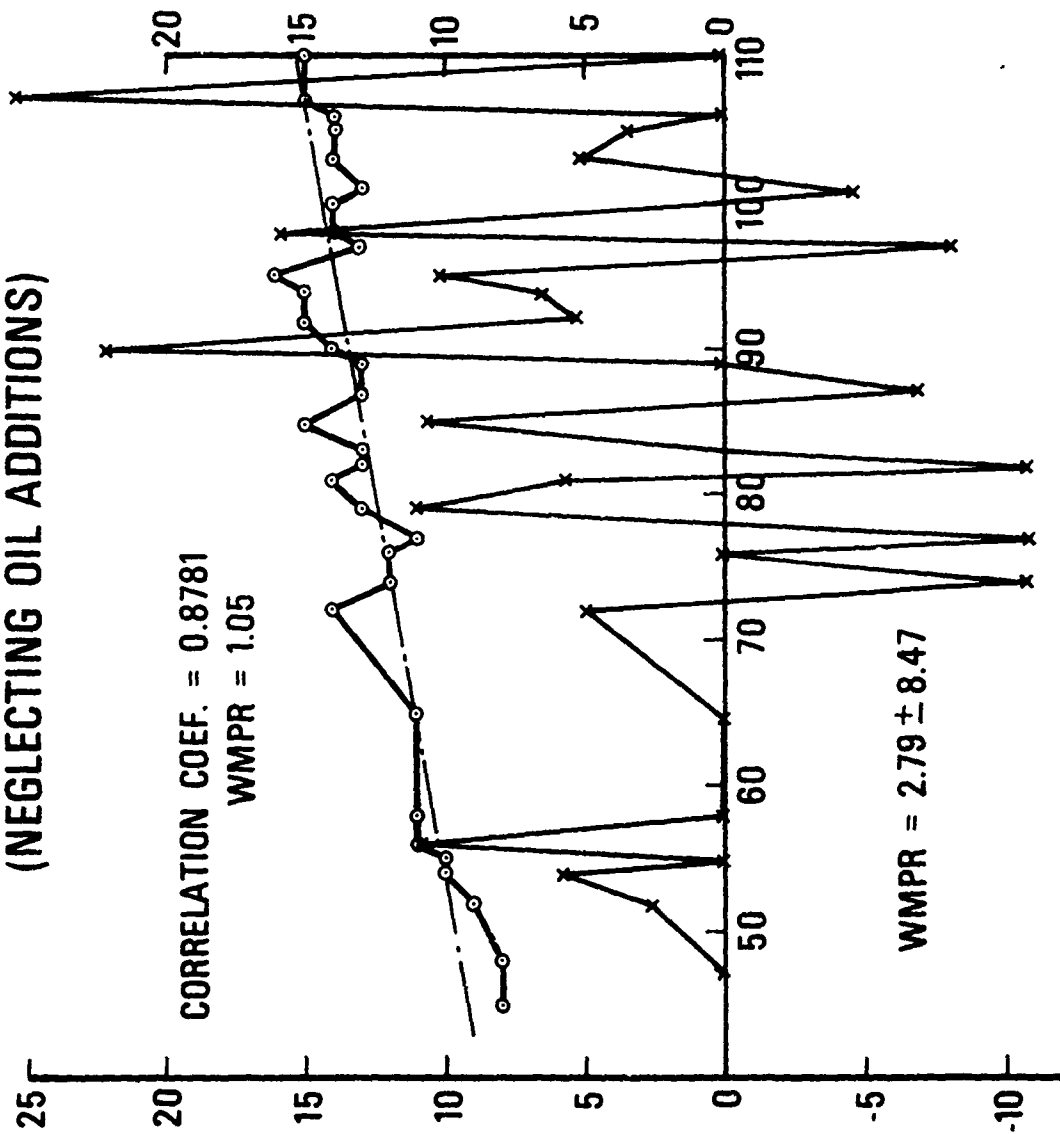


Figure 10

Concentration Measurements and WMPR for TF41 Engine: SN1644

**ENGINE SN 1670
HIGH OIL CONSUMPTION
COMPARISON OF MEASURED AND CORRECTED CONCENTRATION**

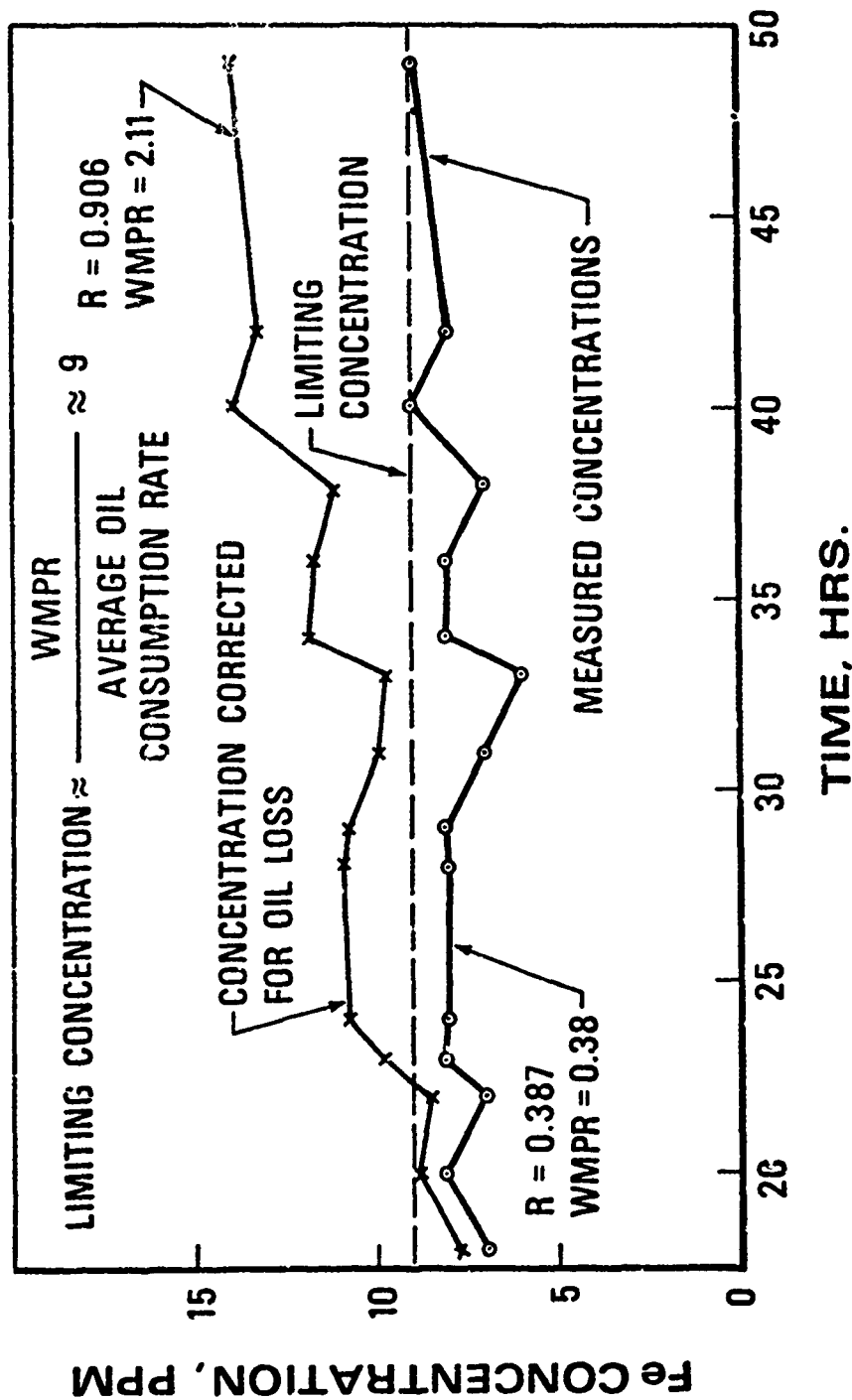


Figure 11
Measured and Corrected Iron Concentrations
for TF41 Engine: SN1670

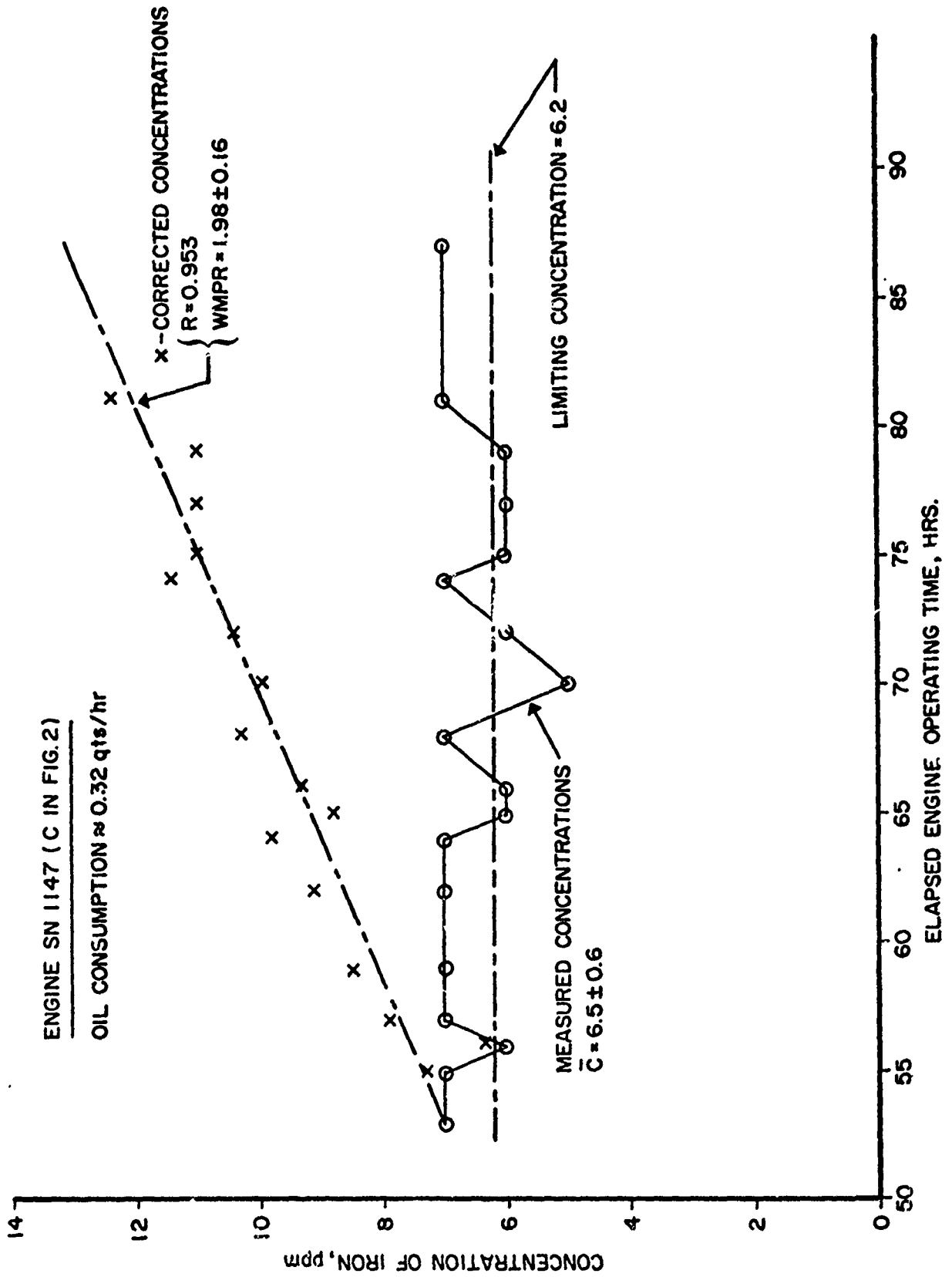


Figure 12. Concentration Data for TF41 Engine: SN1147

ENGINE SN 1644

Fe CONCENTRATION CORRECTED FOR OIL ADDITION

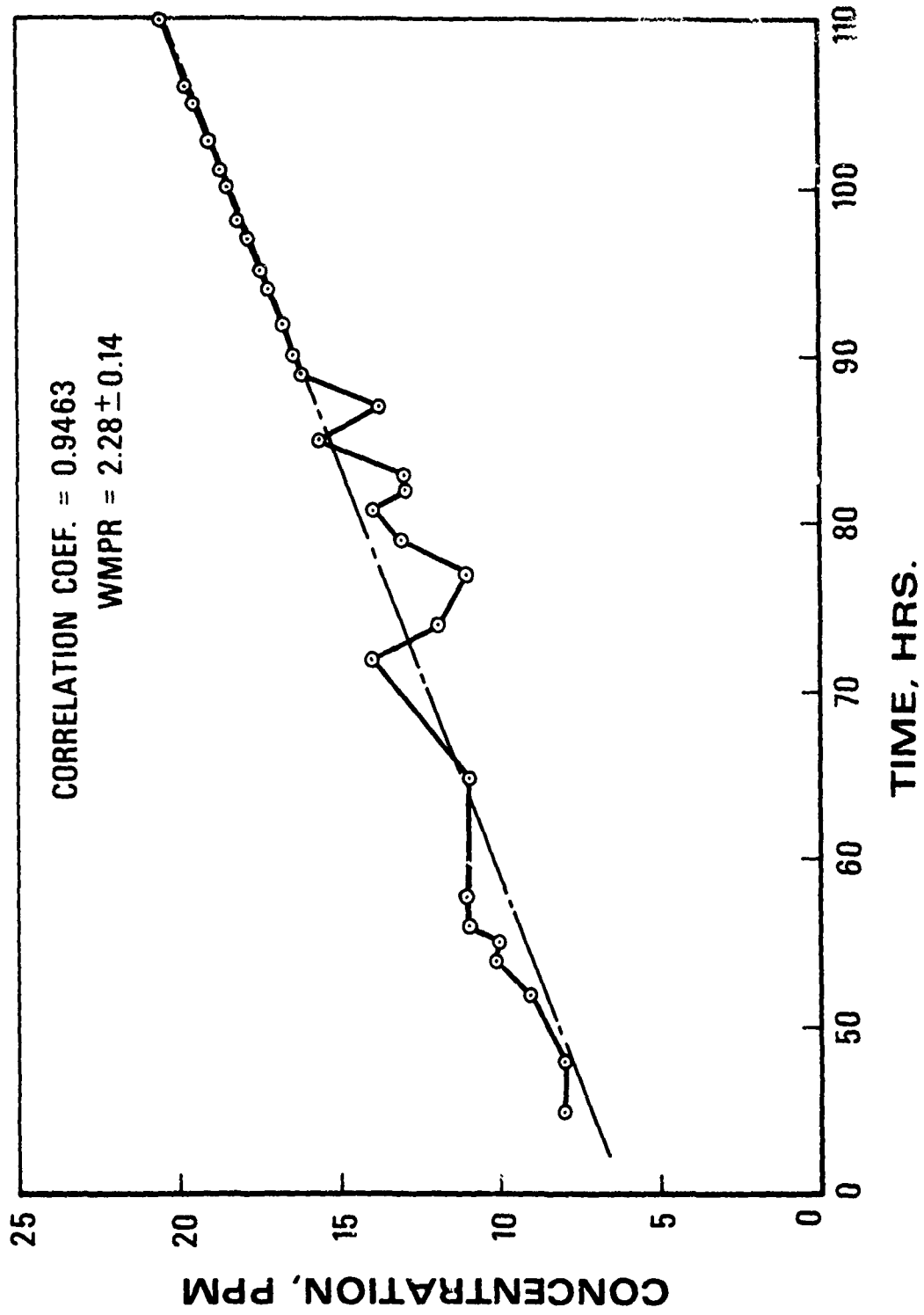


Figure 13. Corrected Iron Concentrations for TF41 Engine: SN1644

TF41 ENGINE SN1126

MEASURED AND CORRECTED CONCENTRATIONS
AVERAGE OIL CONSUMPTION = 0.23qts/hr.
CONFIDENCE LIMITS AT 95% LEVEL

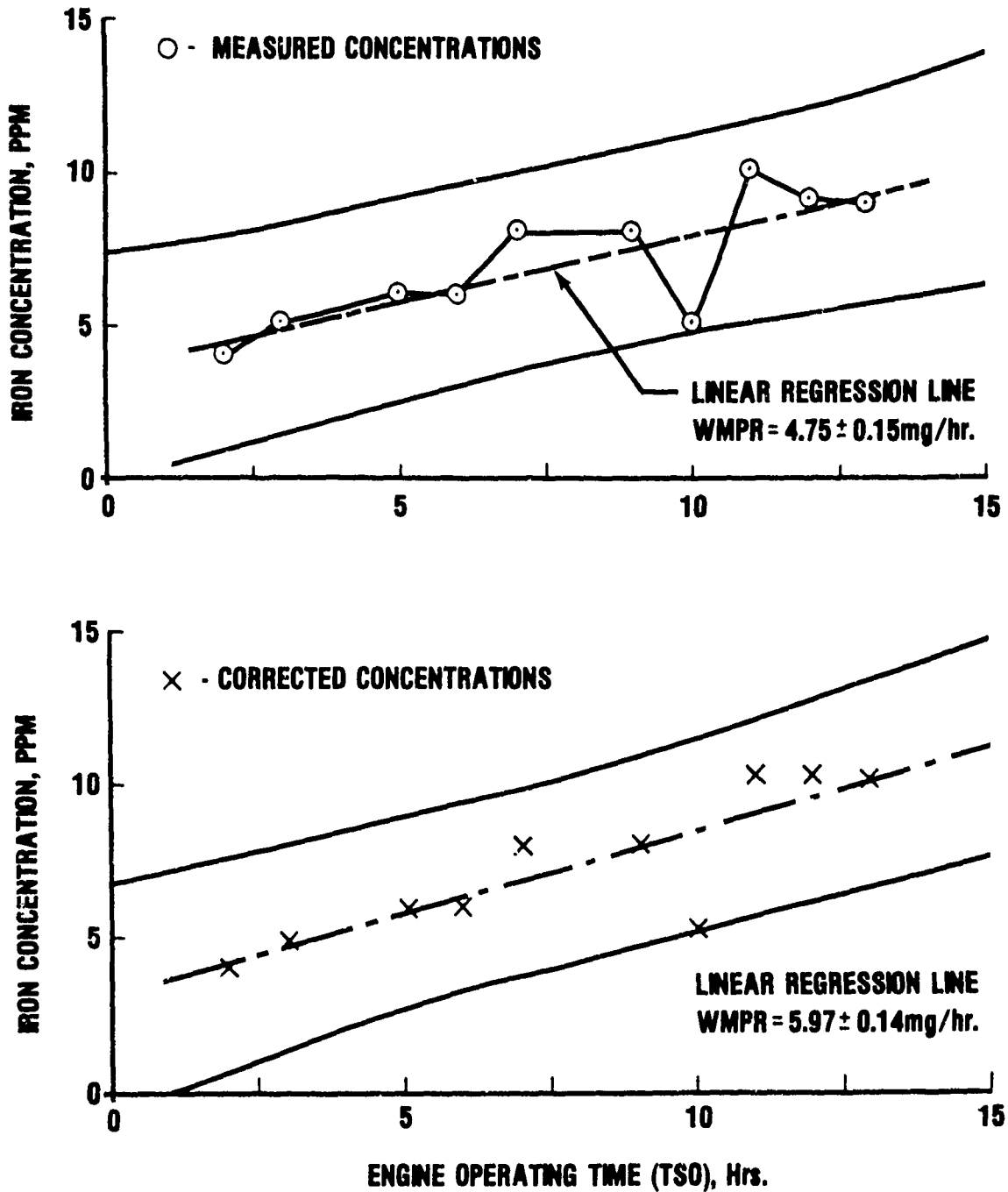


Figure 14. Measured and Corrected Concentrations for TF41 Engine: SN1126

TF41 ENGINE SN1175

MEASURED CONCENTRATIONS
AVERAGE OIL CONSUMPTION = 0.18qts/hr.

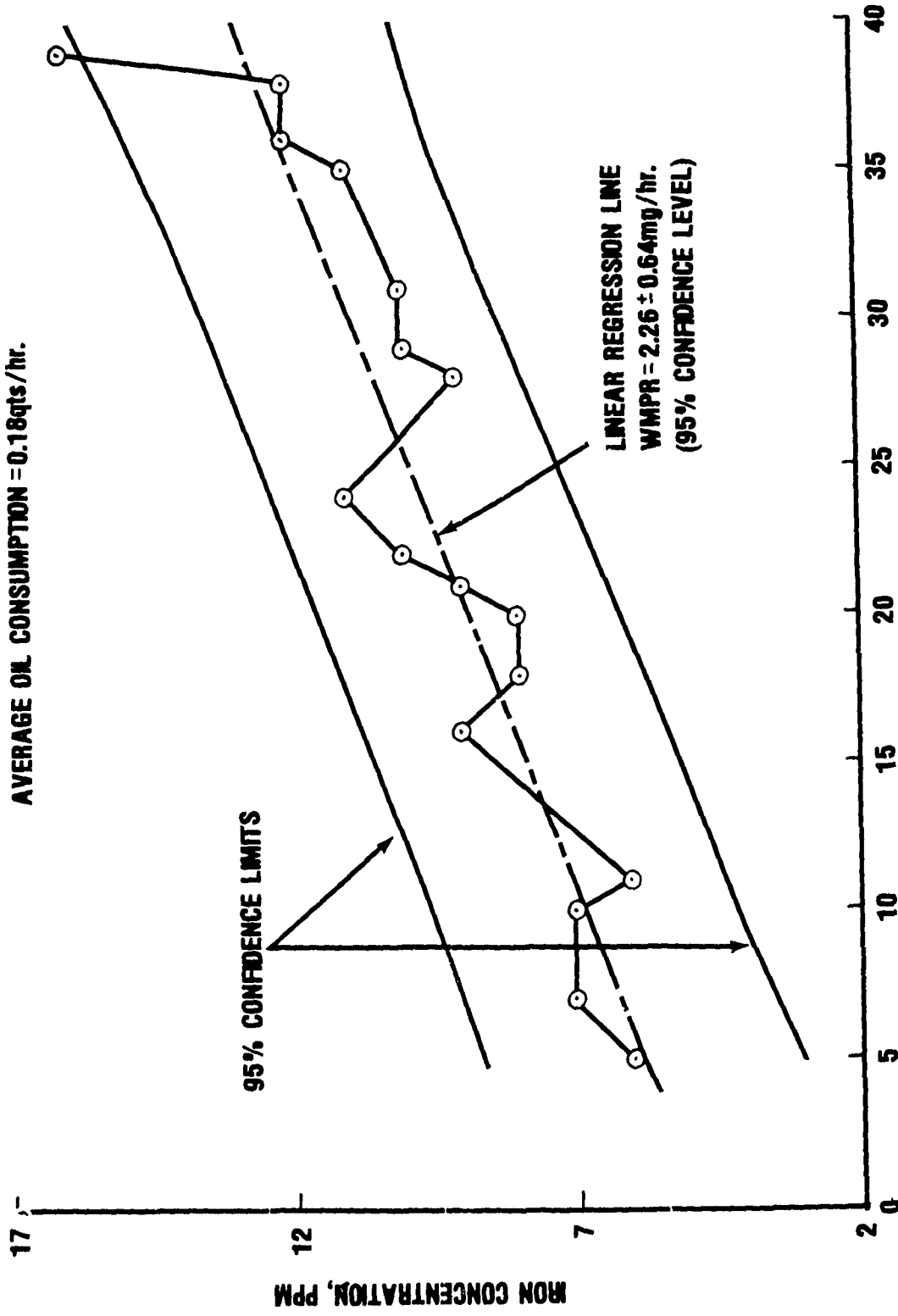


Figure 15. Measured Concentrations for TF41 Engine: SN1175

TF41 ENGINE SN1175

CORRECTED CONCENTRATIONS
AVERAGE OIL CONSUMPTION = 0.18qts./hr.

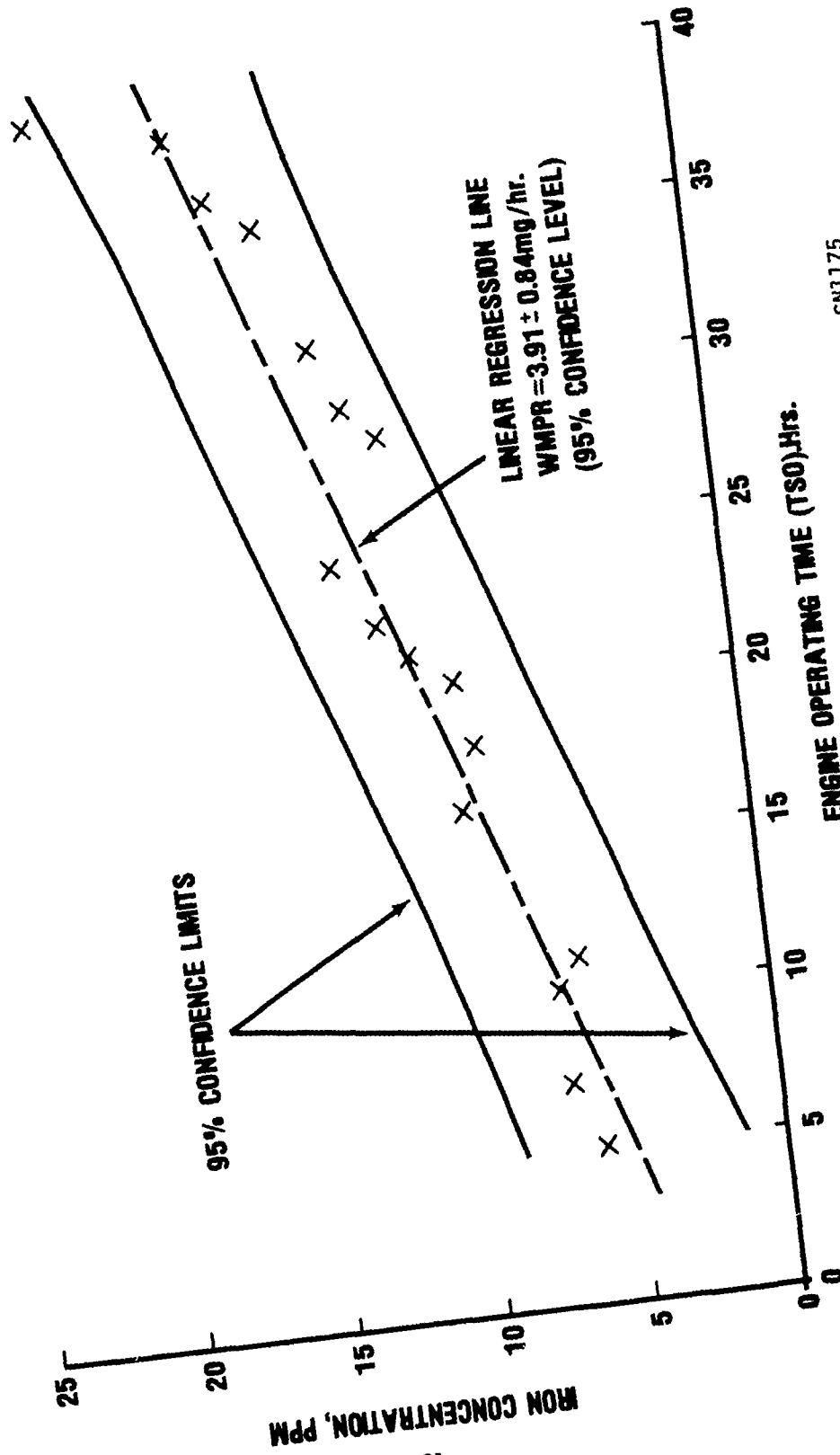


Figure 16. Corrected Concentrations for TF41 Engine: SN1175

ENGINE SN 1096

CONFIDENCE LIMITS AND VARIANCES

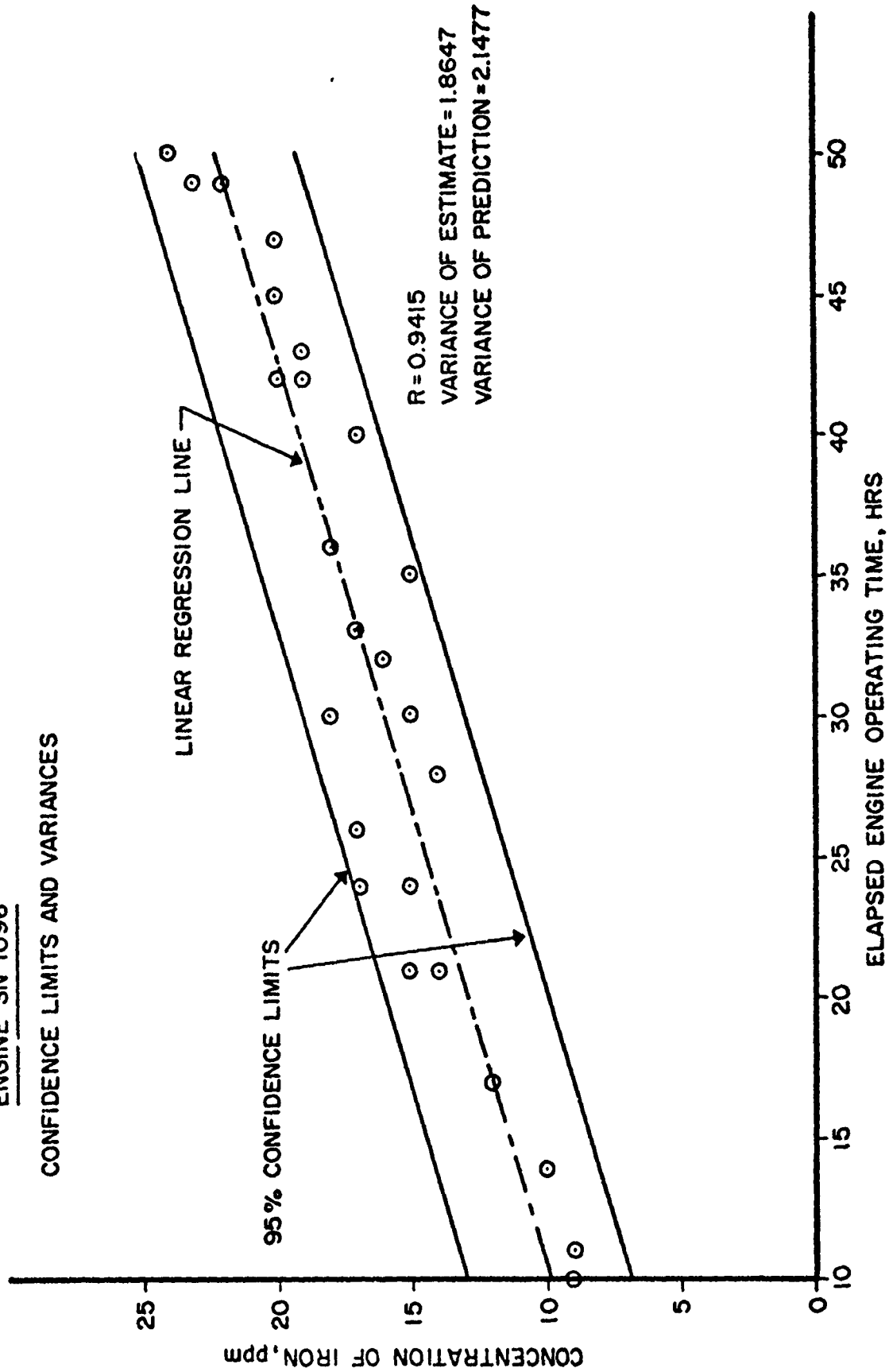


Figure 17

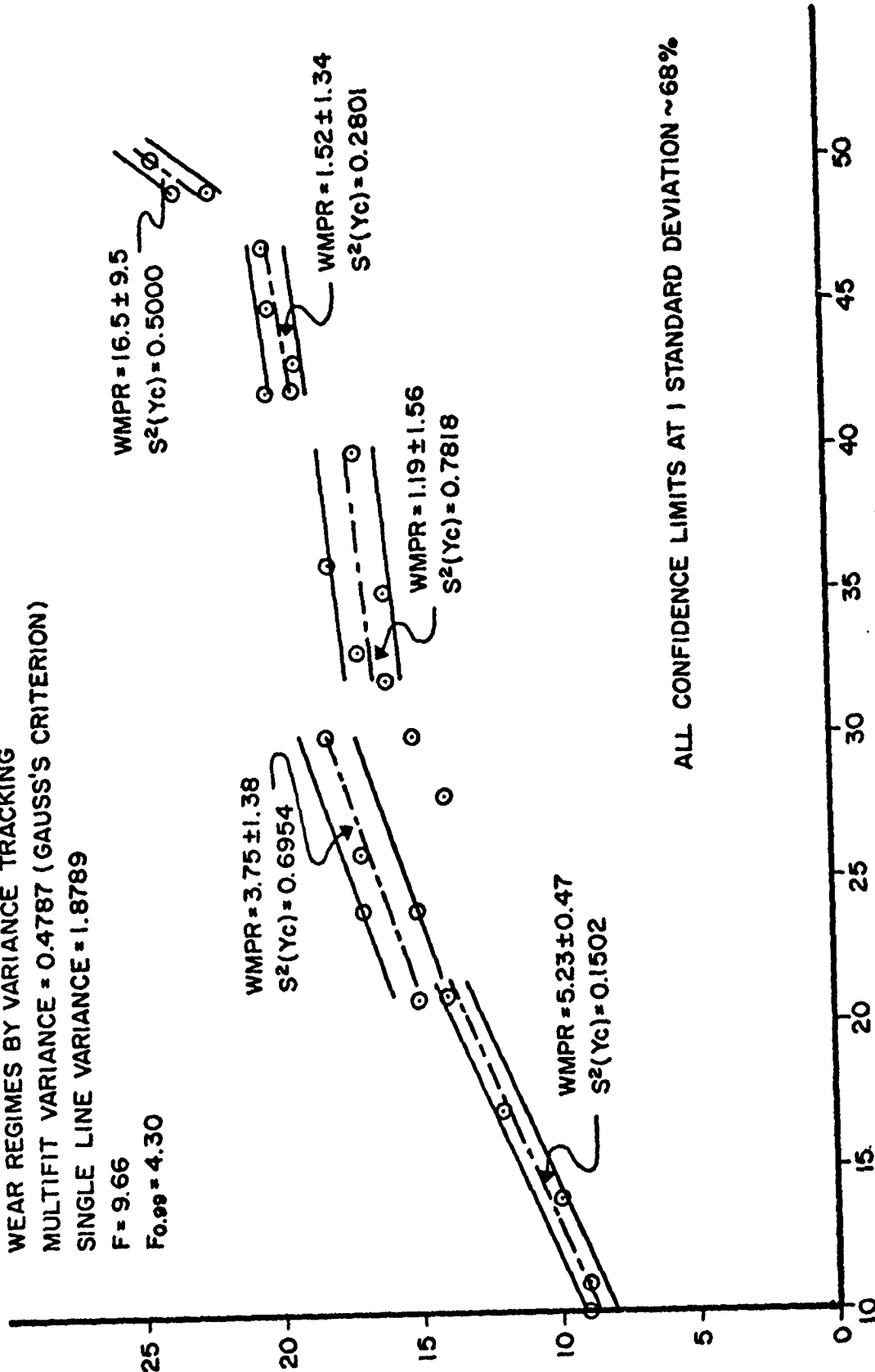
Confidence Limits and Variances, TF41 Engine: SN1096

ENGINE SN 1096

DATA BROKEN DOWN INTO FIVE (5)
WEAR REGIMES BY VARIANCE TRACKING
MULTIFIT VARIANCE = 0.4787 (GAUSS'S CRITERION)
SINGLE LINE VARIANCE = 1.8789

F = 9.66
F_{0.99} = 4.30

CONCENTRATION OF IRON, ppm



ALL CONFIDENCE LIMITS AT 1 STANDARD DEVIATION ~68%

ELAPSED ENGINE OPERATING TIME, HRS.

Figure 18. Variance Tracking, TF41 Engine: SN1096

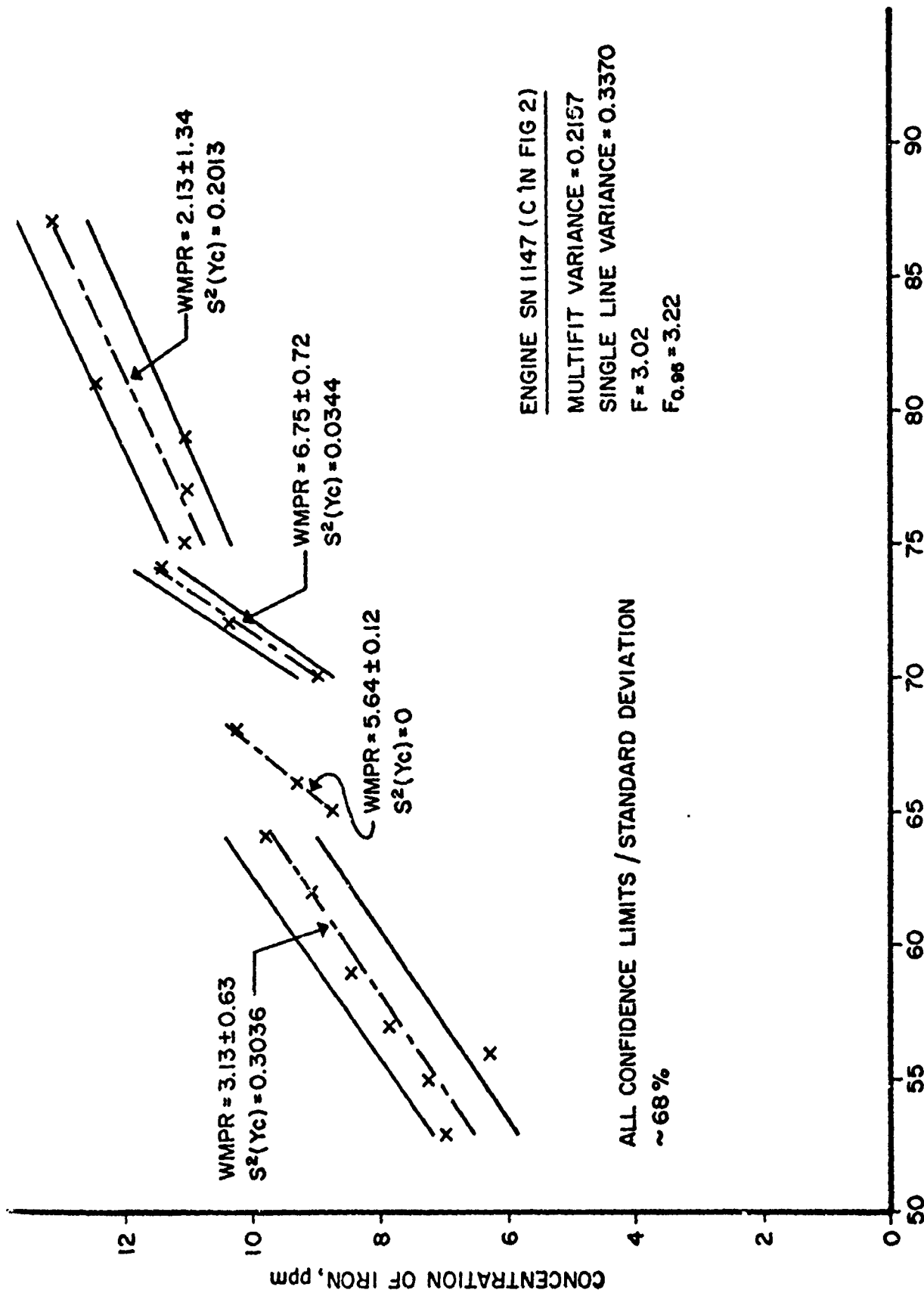


Figure 19. Variance Tracking, TF41 Engine: SN1147

ENGINE SN 1112

GROUNDING FOR HIGH COPPER
PLOTS OF MEASURED AND CORRECTED CONCENTRATIONS
CALCULATED REGRESSION LINE AND CONFIDENCE LIMITS

x=Corrected Concentrations

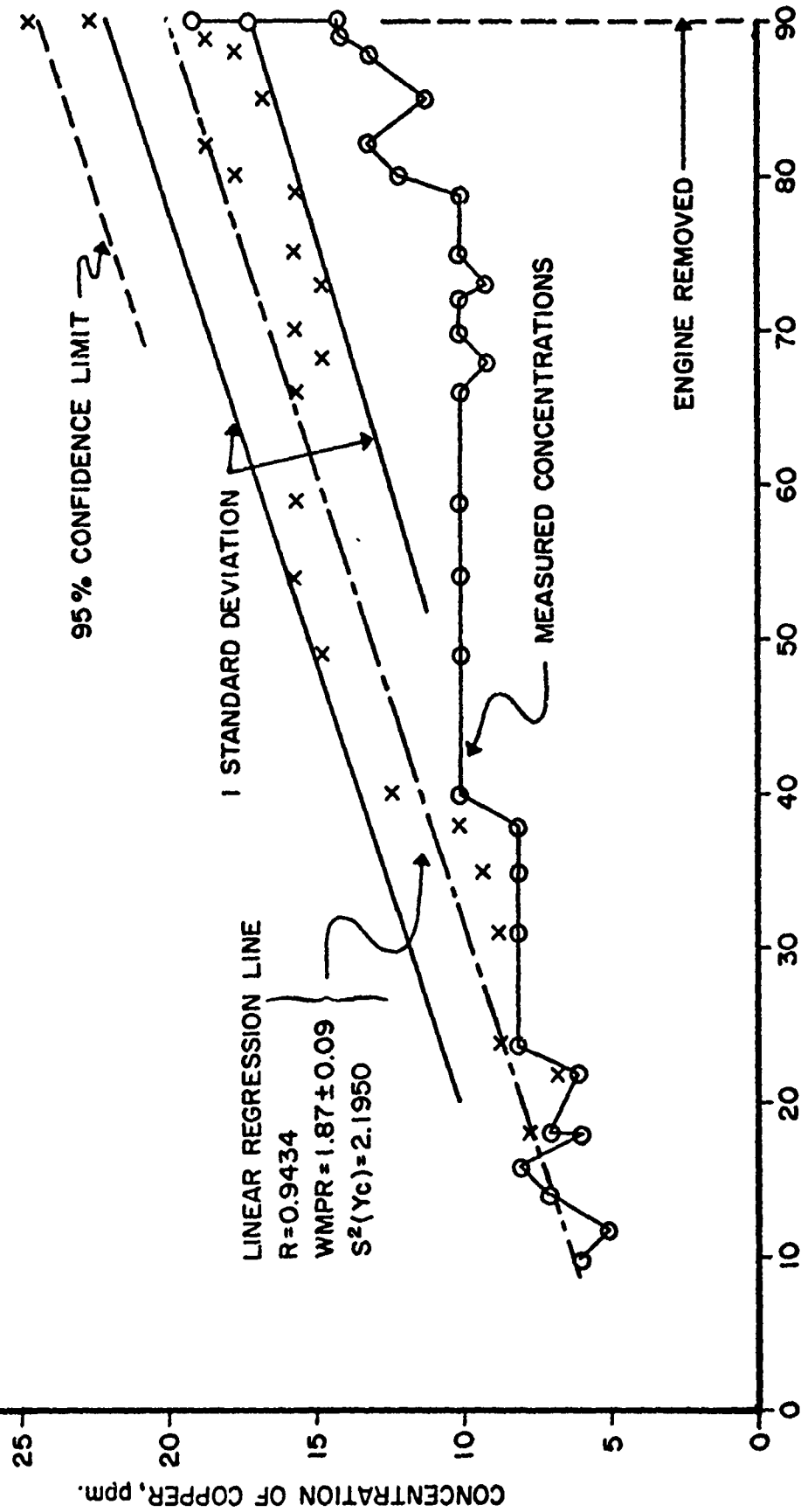


Figure 20. Measured and Corrected Concentrations for TF41 Engine: SN1112

ENGINE SN 1112

GROUNDING FOR HIGH COPPER

DATA BROKEN DOWN INTO FIVE (5)

WEAR REGIMES BY VARIANCE TRACKING

MULTIFIT VARIANCE = 0.3418

SINGLE LINE VARIANCE = 2.1950

F = 32.4

F_{0.99} = 3.17

ALL CONFIDENCE LIMITS AT 1 STANDARD
DEVIATION EXCEPT AS OTHERWISE NOTED.

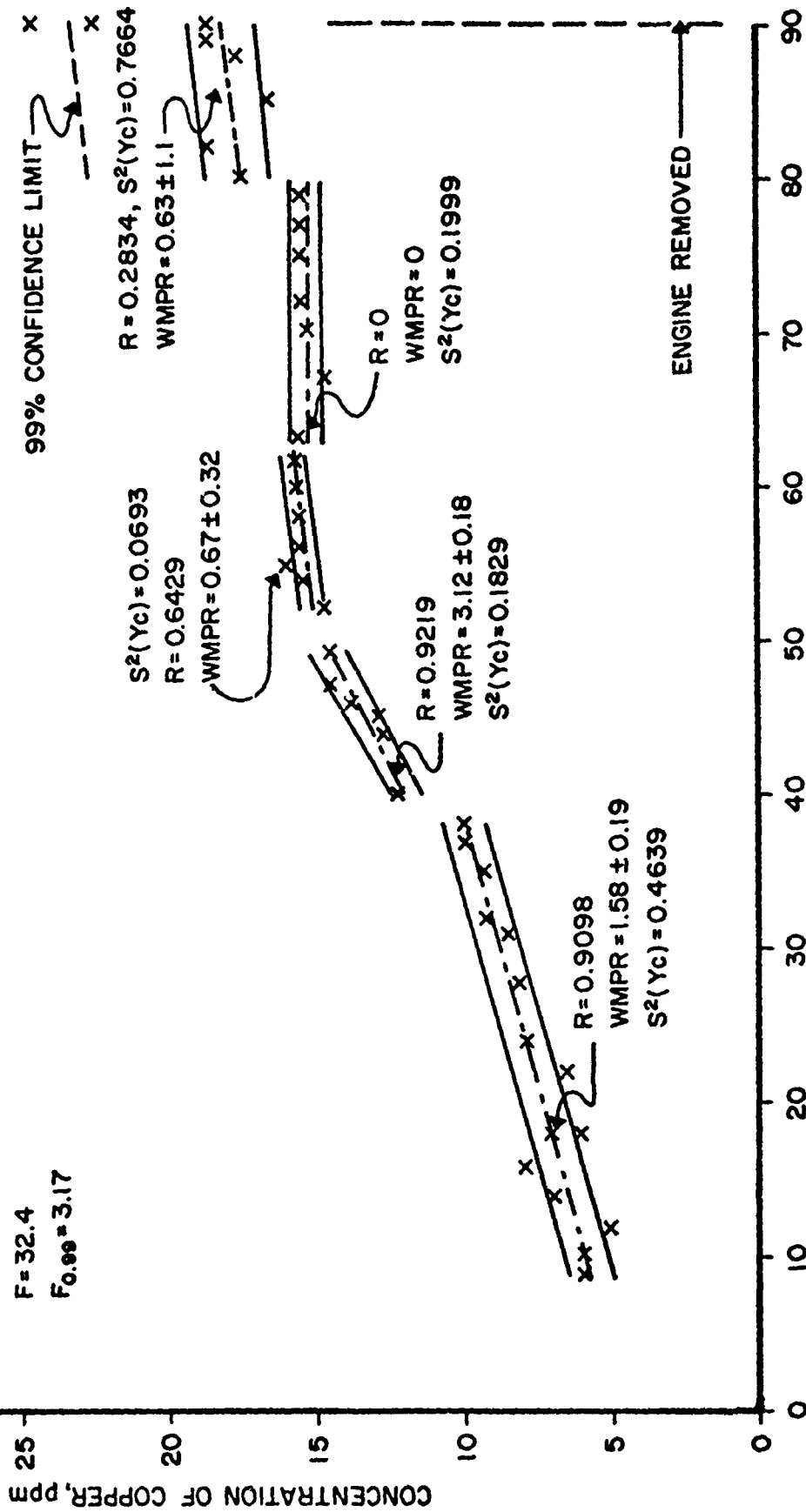


Figure 21. Variance Tracking, TF41 Engine: SN1112

TF41 ENGINE SN1175

CORRECTED CONCENTRATIONS DATA BROKEN DOWN
 INTO THREE (3) WEAR BY VARIANCE TRACKING

MULTIPLY VARIANCE = 0.2928

SINGLE LINE VARIANCE = 2.4536

F = 28.68

F_{0.99} = 5.67

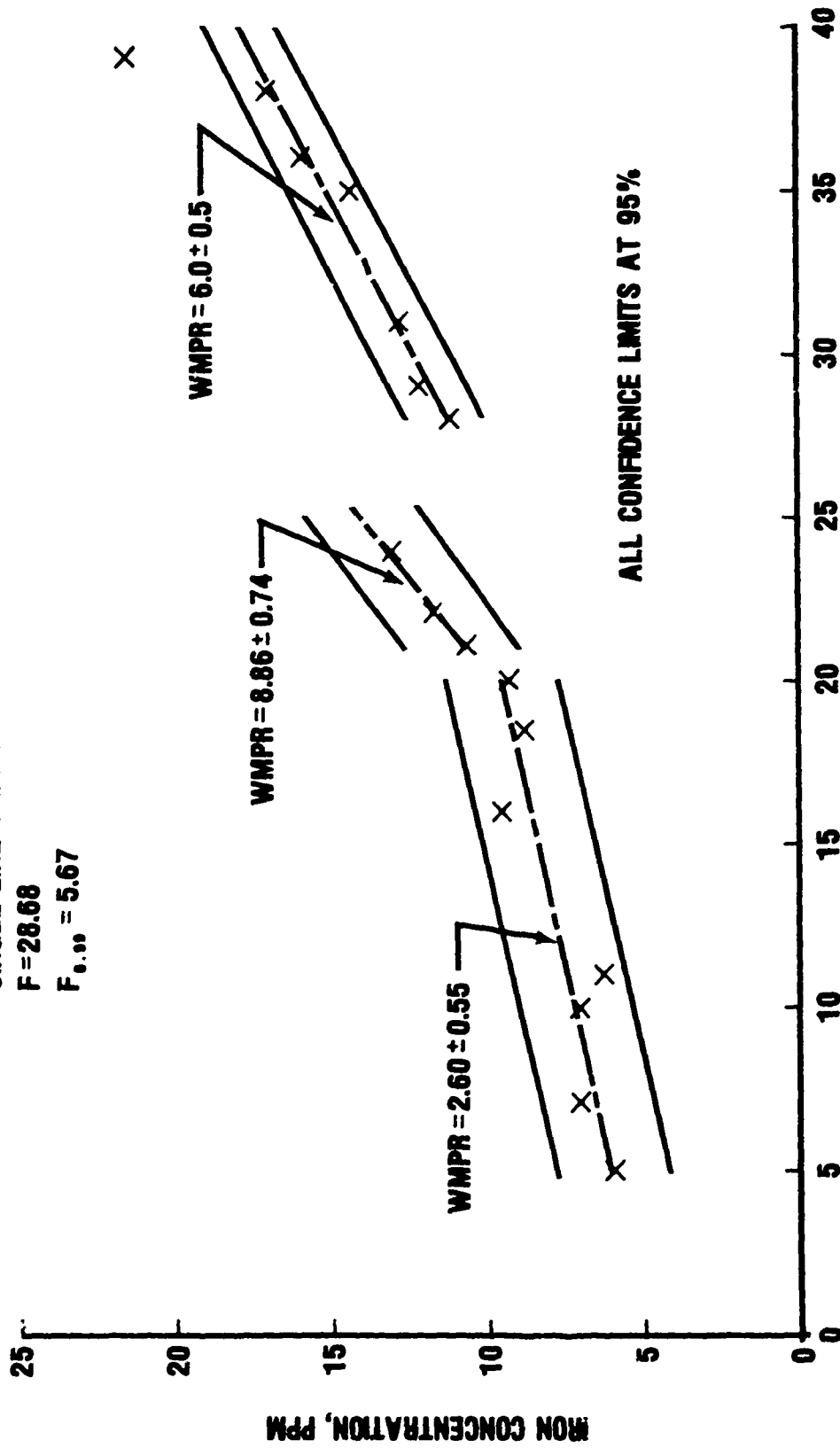


Figure 22. Variance Tracking, TF41 Engine: SN1175

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