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A FINITE ELEMENT MACH BOX PROCEDURE FOR FLUTTER PREDICTION OF PANELS IN THREE-DIMENSIONAL SUPERSONIC UNSTEADY POTENTIAL FLOW

DECEMBER 1976

TECHNICAL REPORT AFFDL-TR-76-161
FINAL REPORT FOR PERIOD JUNE 1976 - DECEMBER 1976

AIR FORCE FLIGHT DYNAMICS LABORATORY
AIR FORCE WRIGHT AERONAUTICAL LABORATORIES
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This technical report has been reviewed and is approved for publication.

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FOR THE COMMANDER

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A finite element formulation and solution procedure are developed for flutter predictions of rectangular panels with one surface exposed to three-dimensional supersonic unsteady potential flow. The effect of in-plane force is included. Each element is divided into several Mach boxes. The aerodynamic influence coefficients between each pair of sending and receiving boxes are computed by the method of Gaussian quadrature. The aerodynamic matrix is based on the numerically computed velocity potentials for all boxes.

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This development is particularly useful in the low supersonic range for panels with chord-span ratio less than about one, while the piston theory does not give satisfactory solution. Examples are demonstrated by using the 16 d.o.f. rectangular plate elements. Results for flutter boundaries for the unstressed panels agree well with an alternative Galerkin's modal solution. The examples demonstrate that flutter boundaries are dominated by higher modes for panels with higher chord-span ratio. They also demonstrate that the dominating flutter boundaries abruptly change modes as the Mach number is varied. The beneficial effect of in-plane tension is demonstrated.

FOREWORD

This report was prepared by Dr. T. Y. Yang, Visiting Scientist in the Optimization Group, Analysis and Optimization Branch, Structural Mechanics Division, Air Force Flight Dynamics Laboratory, Air Force Wright Aeronautical Laboratories, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio. This work was performed under Project Nr. 2307, "Research in Flight Vehicle Dynamics," Task Nr. 230705, "Basic Research in Structures and Dynamics."

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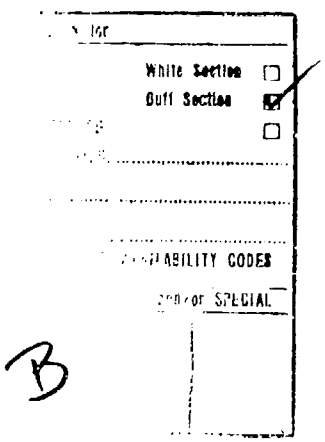

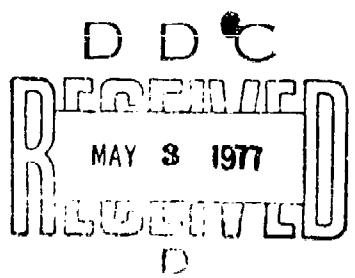

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SYMBOLS

a, b = length and width of the rectangular plate finite element as defined in Fig. 1.

$[A]$ = aerodynamic matrix of the panel system

B_x, B_{xs} = number of boxes in the stream and cross-stream directions, respectively

D = $Eh^3/12(1 - \gamma^2)$, bending rigidity with γ being Poisson's ratio

F = $N_x/\ell^2\omega_1{}^2\sigma h$, Initial in-plane tension parameter

$f_j(x,y)$ = shape function associated with degree of freedom "j"

g = structural damping coefficient

h = panel thickness

i, j = subscripts indicate d.o.f. number

$[K]$ = stiffness matrix of the panel system

k_ℓ, k_ε = $\omega\ell/V$, $\omega\varepsilon/V$, respectively, reduced frequency

ℓ = length of panel in stream direction

$[M]$ = mass matrix of the panel system

M = Mach number

m, n = receiving box index numbers in the stream and cross-stream directions, respectivley

$[N]$ = incremental stiffness matrix of the panel system

N_x, N_y, N_{xy} = panel in-plane line forces

r, s = $(m - \lambda)$, $(n - \nu)$, respectively

u, v = transformed variables of integration based on $\varepsilon/2$ as reference length, $u\beta/2 = x_m - \varepsilon$, $v/2 = y_n - \eta$

V = speed of undisturbed airstream

LIST OF SYMBOLS (Continued)

w	=	width of panel in cross-stream direction
w_j	=	downwash velocity at panel surface for d.o.f. "j"
x, y	=	panel coordinates as defined in Fig. 1
$\bar{x}$, $\bar{y}$	=	x/ℓ, y/w, respectively
x_m, y_n	=	values of x/ε and y/ε at center of box m,n
α_ϕ	=	aerodynamic influence coefficient relating the velocity potential at a box to unit downwash on another box (Eq. 12)
β	=	$(M^2 - 1)^{\frac{1}{2}}$
ε	=	w/B_{xs}, box width
ξ, η	=	values of x/ε and y/ε at any point in the sending box
μ	=	panel-air mass density ratio, $\sigma h/\rho\ell$
λ, ν	=	sending box index number in the stream and cross-stream directions, respectively
ρ	=	density of undisturbed airstream
σ	=	density of panel
τ	=	length-width ratio of box as defined in Fig. 1
$\phi_j(m,n)$	=	velocity potential at center of box m,n for d.o.f. "j"
$\bar{\phi}(m,n)$	=	velocity potential at center of box m,n due to unit downwash over box λ,ν
ω	=	frequency of flutter motion
ω_1	=	first natural frequency of the panel
Ω	=	$(\omega_1/\omega)^2(1 + ig)$
$\bar{\Omega}$	=	$M^2 k_\varepsilon/\beta^2$

SECTION I

INTRODUCTION

Ever since the earliest days of manned flight, panel flutter has been known as one of the most important problems in the design of aircraft, missiles, launched vehicles, and spacecraft. Extensive progress in wind tunnel tests and theoretical studies has been achieved. The basic theories and an account of the developments on panel flutter can be found in, among other books, a recent text by Dowell (Reference 1). A list of keyed bibliography and collection of some significant survey papers and original papers were prepared by Garrick (Reference 2).

The theoretical solution for a panel flutter problem usually requires an accurate aerodynamic and structural theory to formulate a set of complex eigenvalue equations of motion interacted between the panel and the flow. One of the most common theoretical methods is the modal method where the aerodynamic pressure and the inertial and elastic forces of the panel are obtained by assuming the displacements as composed of a number of natural modes and generalized coordinates. The natural frequencies and corresponding normal mode shapes are obtained either theoretically or experimentally, or both. Since the finite element method is powerful and practical in the free vibration analysis of panels with arbitrary geometrical and boundary conditions, it is commonly used in obtaining natural frequencies and mode shapes in the modal method.

As an alternative approach, the finite element workers have formulated the matrix of aerodynamic pressure by using the

displacement functions as composed of the nodal degrees of freedom and shape functions rather than the generalized coordinates and natural mode shapes. Such approach can directly solve for the flutter frequencies and corresponding normal mode shapes without having to seek the natural frequencies and modes and choose the number of modes before the eigensolution, and also without having to compute the flutter mode shapes after the eigensolution. Such approach permits expression of the equations of motion in an elegant and straightforward form. Such approach also permits generality in panel configurations and boundary conditions, and allows for flexibility to accurately include physical effect such as in-plane forces.

The finite element method was first extended to the panel flutter problems by Olson (Reference 3). He formulated the aerodynamic matrix explicitly for an infinite plate element. Olson (Reference 4) later formulated the aerodynamic matrices for two rectangular (12 and 16 d.o.f.) and an 18 d.o.f. triangular plate elements. Simultaneously, but independently, Appa and Somashekar (Reference 5) formulated the aerodynamic matrix for a 12 d.o.f. rectangular plate element. Appa, Somashekar and Shah (Reference 6) later extended their work by accounting for skew panels and yawed flow by means of coordinate transformation. Sander, Bon, and Geradin (Reference 7) employed the CQ conforming quadrilateral plate finite element for flutter analysis of rectangular panels with yawed flow and in-plane stresses.

In all the above finite element works (References 3 - 7), Lighthill's linearized piston theory was employed. The Mach numbers considered were above approximately 1.6. Recently, Yang (Reference 8) developed a finite element procedure using the exact linearized two-dimensional theory (strip theory) for unsteady supersonic flow to formulate an infinite plate finite element by means of numerical integration. The flutter speed considered thus could be in the lower supersonic range. Such formulation cannot, however, be adequately applied to the more general case of rectangular panels with finite aspect ratios.

In this report, the three-dimensional supersonic unsteady potential flow theory is employed to formulate the plate finite elements so that the flutter problems of the finite panels in low supersonic range can be treated.

The aerodynamic matrix is derived by using the principle of virtual work. The aerodynamic forces or velocity potentials that produce the work are obtained for each d.o.f. by the Mach box method. Each finite element is divided into several boxes. The aerodynamic influence coefficients for each pair of sending and receiving boxes are evaluated, for each d.o.f., by the method of Gaussian quadrature. The velocity potential at each receiving box is obtained, for each d.o.f., by summation of the product of corresponding downwash and influence coefficients for all sending boxes. It should be noted that a similar box method was used by Cunningham (Reference 9) in conjunction with a Galerkin's modal method for panel flutter analysis. Such 3-D aerodynamic theory was also employed by Dowell and Voss (Reference 10) in a theoretical and experimental correlation study of panel flutter.

The 16 d.o.f. conforming rectangular plate element (Reference 11) was used for example demonstrations. Flutter boundaries were found for clamped rectangular panels with various aspect ratios. The thickness ratios required to prevent flutter of an aluminum panel at sea level were plotted for Mach numbers ranging from 1.05 to 3. Results found are in favorable agreement with Cunninhgam's solution (Reference 9).

The initial in-plane tensile stresses were then included in finding the flutter boundaries for a clamped aluminum panel. The thickness ratios required to prevent flutter of the panel at 25,000 feet altitude were obtained for Mach numbers ranging from 1.05 to 2.0 for various values of tension. It was found that the dominating flutter mode changed abruptly as Mach number was varied.

SECTION II

FORMULATION OF EQUATIONS OF MOTION

The free vibration equations of motion for a finite element panel subjected to the effect of stiffness, in-plane force, inertia, and aerodynamic pressure may be written as

$$[K]\{q\} + [N]\{q\} + [M]\{\ddot{q}\} + [A]\{q\} = \{0\} \tag{1}$$

where [K], [N], [M], and [A] are, respectively, the stiffness, incremental stiffness, mass, and aerodynamic matrices assembled for the whole finite element system. The vector {q} contains the nodal degrees of freedom for the whole panel system. In this section, only the system aerodynamic matrix is formulated.

1. PRINCIPLE OF VIRTUAL WORK

For a system of plate finite elements, the deflection and aerodynamic pressure may be written by separating the time and space variables as,

$$\begin{aligned} \bar{z}(x,y,t) &= z(x,y)e^{i\omega t} \\ \bar{p}(x,y,t) &= p(x,y)e^{i\omega t} \end{aligned} \tag{2}$$

where the coordinates are defined in Fig. 1.

The strain energy for the panel system is equal to the work produced by the aerodynamic pressure

$$U = \frac{1}{2}\iint z(x,y)p(x,y)\,dxdy \tag{3}$$

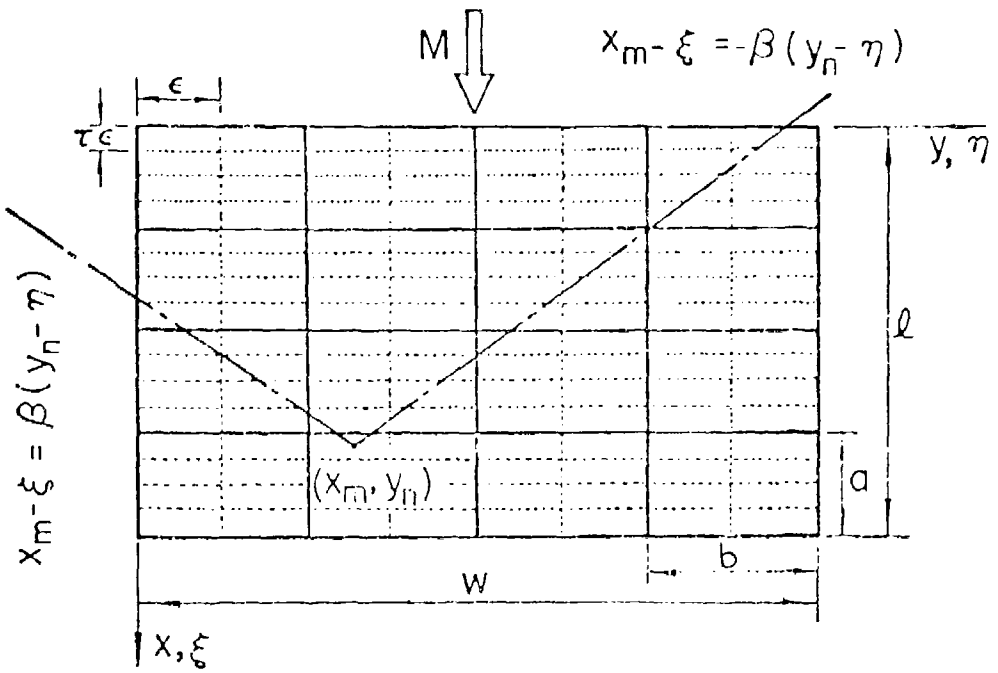

Figure 1. Panel divided into finite elements (solid lines) and boxes (dash lines) with coordinates, dimensions, and Mach cone

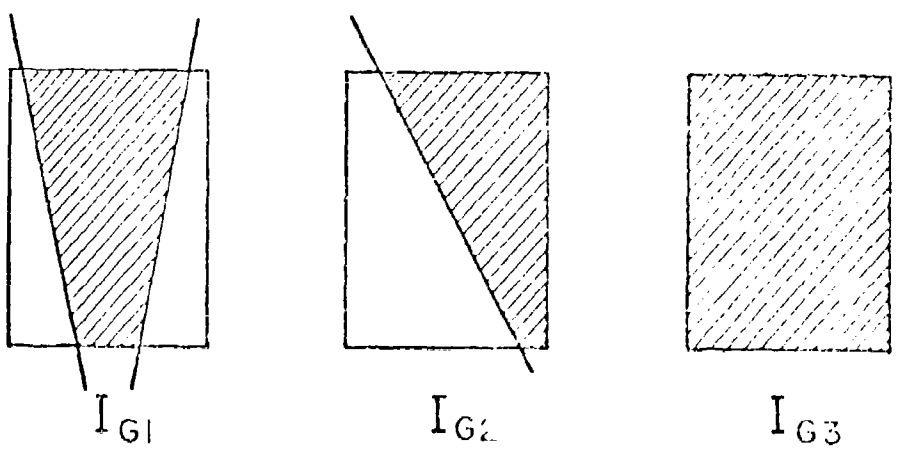

Figure 2. Three types of integrals for computing the aerodynamic influence coefficients

The deflection function for the total finite element system may be assumed as

$$z(x,y) = \sum_{j=1}^{N} f_j(x,y)q_j = \{f\}^T\{q\} \tag{4}$$

where $f_j(x,y)$ represents the assembled shape function corresponding to the nodal d.o.f. "q_j" and N is the number of degrees of freedom for the entire panel system. Usually, q_j represents deflection, slopes, twist, and curvatures of the plate at the nodal point j.

Similarly, the aerodynamic pressure may be assumed as

$$p(x,y) = \sum_{j=1}^{N} P_j(x,y)q_j = \{P\}^T\{q\} \tag{5}$$

where $P_j(x,y)$ is defined in Eq. (8) as the pressure function associated with the shape function $f_j(x,y)$ and d.o.f. "q_j."

Substituting Eqs. (4) and (5) into (3), the strain energy expression becomes

$$U = \frac{1}{2}\{q\}^T[A]\{q\} \tag{6}$$

with

$$[A] = \iint \{f\}\{P\}^T \, dxdy \tag{7}$$

Using the principle of virtual work, it can be interpreted that Eq. (7) yields the aerodynamic matrix.

2. AERODYNAMIC MATRIX AND AERODYNAMIC PRESSURE

The aerodynamic perturbation pressure is obtained from the linearized three-dimensional supersonic unsteady potential flow theory. When associated

with the d.o.f. "q_j" and the shape function $f_j(x,y)$, the perturbation pressure may be expressed in terms of the velocity potential through the relation (see, for example, Ref. 12),

$$P_j(\bar{x},\bar{y}) = -\frac{\rho V}{\ell}\left(\frac{\partial \phi_j}{\partial \bar{x}} + i\,\frac{\omega \ell}{V}\,\phi_j\right) \tag{8}$$

where ϕ_j is the velocity potential associated with d.o.f. "q_j".

The coefficient for the ith row and jth column of the aerodynamic matrix [A] is obtained by substituting Eq. (8) into Eq. (7).

$$A_{ij} = -\frac{\rho V}{\ell}\iint f_i(\bar{x},\bar{y})\left(\frac{\partial \phi_j}{\partial \bar{x}} + i\,\frac{\omega \ell}{V}\,\phi_j\right)d(\ell\bar{x})d(w\bar{y}) \tag{9}$$

Performing integration by parts and assuming restrained trailing edge, Eq. (9) becomes

$$A_{ij} = \frac{\rho V}{\ell}\iint \phi_j\left(\frac{\partial f_i}{\partial \bar{x}} - i\,\frac{\omega \ell}{V}\,f_i\right)d(\ell\bar{x})d(w\bar{y}) \tag{10}$$

where the quantity within the parentheses is the complex conjugate of the downwash ratio for unit d.o.f. "q_i" and unit panel length. The coefficient A_{ij} is evaluated numerically through the following simple summation

$$A_{ij} = \frac{\rho V}{\ell}\sum \phi_j\left(\frac{\partial f_i}{\partial \bar{x}} - i\,\frac{\omega \ell}{V}\,f_i\right)\cdot(\text{Box Area}) \tag{11}$$

where the summation is to be made for every box and the quantities related to ϕ_j and f_i are evaluated numerically at the center of each box. The method for numerically evaluating the velocity potential at the center of each box is given as follows.

3. VELOCITY POTENTIAL AND AERODYNAMIC INFLUENCE COEFFICIENT

The method of Mach box as employed by Cunningham in Reference 9 is used here to derive the aerodynamic influence coefficients between each pair of boxes and the resulting velocity potentials at each box.

In this method, each finite element is divided into several equal-size boxes as shown in Fig. 1. The numbers of boxes in the stream and cross-stream directions are defined as B_X and B_{XS}, respectively. The width and length of each box are defined as ε and $\tau\varepsilon$, respectively, where $\varepsilon = w/B_{XS}$ and $\tau = \lambda B_{XS}/wB_X$. The boxes are numbered in sequence from the origin with m (or λ) and n (or ν) as the box number in the stream and the cross-stream directions, respectively. The numbers (m,n) and (λ,ν) refer to the receiving and sending boxes, respectively.

The boxes are assumed as sufficiently small so that the downwash over any sending box is considered as uniformly distributed at any instant, and the resulting perturbation pressure at the center of each receiving box represents the average of the pressure distributed over the box.

The velocity potential at the center of a receiving box (m,n) due to a uniformly distributed but otherwise unspecified downwash $w(\lambda,\nu)$ over the sending box (λ,ν) can be expressed, for harmonic motion, as

$$\bar{\phi}(m,n) = \varepsilon\, w(\lambda,\nu)\alpha_{\phi}(r,s) \tag{12}$$

where the relative locations in stream and cross-stream directions, respectively, between the two boxes are defined as $r = m - \lambda$ and $s = n - \nu$. The aerodynamic influence coefficient from Reference 9 is,

$$\alpha_\phi(r,s) = \frac{1}{2\pi}\int_{u_1}^{u_2} e^{-i(\beta\bar{\Omega}/2)u}\left\{\cos^{-1}\frac{v_1}{u} - \cos^{-1}\frac{v_2}{u} + \int_{v_1}^{v_2}\frac{\cos[\frac{\beta\bar{\Omega}}{2M}(u^2-v^2)^{\frac{1}{2}}] - 1}{(u^2-v^2)^{\frac{1}{2}}}\,dv\right\}du \tag{13}$$

where all the parameters are defined in the List of Symbols.

In Eq. (13), the surface integration limits u_1, u_2, v_1, and v_2 result in only three different forms I_{G1}, I_{G2}, and I_{G3} as shown in Fig. 2.

The first form is for any portion of a sending box (λ, ν) cut by both sides of the Mach cone so that $v_2 = -v_1 = u$ and $s = 0$.

$$I_{G1} = \frac{1}{2}\int_{u_1}^{u_2} e^{-i(\beta\bar{\Omega}/2)u}\, J_0(\frac{\beta\bar{\Omega}}{2M}u)du \tag{14}$$

where J_0 is the Bessel function of the first kind of order zero.

The second form is for portions of a box cut by one side of the Mach cone so that the limits $v_2 = u$ and $v_1 = 2s - 1 \geq 1$.

$$I_{G2} = \frac{1}{2\pi}\int_{u_1}^{u_2} e^{-i(\beta\bar{\Omega}/2)u}\left\{\cos^{-1}\frac{v_1}{u} + \int_{v_1}^{u}\frac{\cos[\frac{\beta\bar{\Omega}}{2M}(u^2 - v^2)^{\frac{1}{2}}] - 1}{(u^2 - v^2)^{\frac{1}{2}}}\,dv\right\}du \tag{15}$$

The third form is for boxes that are completely within the Mach cone and also for portions of boxes ahead of the point where the Mach line cuts the side of the box.

$$I_{G3} = \frac{1}{2\pi}\int_{u_1}^{u_2} e^{-i(\beta\bar{\Omega}/2)u}\left\{\cos^{-1}\frac{v_1}{u} - \cos^{-1}\frac{v_2}{u} + \int_{v_1}^{v_2}\frac{\cos[\frac{\beta\bar{\Omega}}{2M}(u^2-v^2)^{\frac{1}{2}}] - 1}{(u^2 - v^2)^{\frac{1}{2}}}\,dv\right\}du \tag{16}$$

The complete $\alpha_\phi(r,s)$ for any one sending box (λ,ν) consists of I_{G1}, I_{G2}, or I_{G3}, or a combination of I_{G1} and I_{G2}, or of I_{G2} and I_{G3}. The types of integrals and limits of integration for computing $\alpha_\phi(r,s)$ for all possible relative locations of box (λ, ν) and the Mach cone from (m, n) are given in Reference 9.

The evaluation of the above three integrals is carried out through the method of Gaussian quadrature. In the subsequent numerical examples, three Gaussian points are used in both x and y directions for computing I_{G3}. Five Gaussian points in both x and y directions are used for computing I_{G1} and I_{G2}.

Once all possible values of the aerodynamic influence coefficient $\alpha_\phi(r,s)$ are obtained for a certain shape function $f_j(\bar{x},\bar{y})$, the total velocity potential at the center of a receiving box (m,n) for the downwash associated with the same shape function is a weighted sum of the $\bar{\phi}_j(m,n)$ defined in Eq. (12)

$$\phi_j(m,n) = V\varepsilon \sum_{\lambda} \sum_{\nu} \frac{w_j(\lambda,\nu)}{V} \alpha_\phi(r,s) \tag{17}$$

The summation is extended over all the sending boxes. The downwash ratios $w_j(\lambda,\nu)/V$ for a unit d.o.f. "q_j" are the total time derivatives of the shape function $f_j(\bar{x},\bar{y})e^{i\omega t}$

$$\frac{w_j}{V} = \frac{1}{\ell}\left(\frac{\partial f_j}{\partial \bar{x}} + i\,\frac{\omega \ell}{V} f_j\right) \tag{18}$$

Once the velocity potentials are obtained for each box for each shape function, they can readily be substituted into Eq. (11) for computing the aerodynamic matrix for the entire panel system.

SECTION III

FORMULATION FOR FLUTTER DETERMINANT IN TERMS OF FINITE ELEMENT SHAPE FUNCTIONS

In the present finite element flutter formulation, the parameters are so grouped and nondimensionalized that the solution is general enough to include every parameter. Assuming harmonic motion with natural frequency ω, the equations of motion (1) are rewritten in an element form as

$$\left\{\Omega\left[c_1[k] + Fc_2[n]\right] - \left[c_3[m] - c_4[A]\right]\right\}\{q\} = \{0\} \tag{19}$$

where $\Omega = \omega_1^2/\omega^2(1 + ig)$ is the flutter eigenvalue parameter, and

$$c_1 = \frac{\mu(\ell/a)^5 D}{\omega_1^2 \sigma h \ell^4} \quad ; \quad c_2 = \mu(\ell/a)^3 \quad ;$$

$$c_3 = \mu(\ell/a) \quad ; \quad c_4 = \left(\frac{\ell}{a}\right)^3\left(\frac{\epsilon}{\ell}\right)\left(\frac{1}{k_\ell}\right)^2\left(\frac{\tau\epsilon^2}{ab}\right) \tag{20}$$

The element matrix terms are obtained as

$$k_{ij} = \int_0^1\int_0^1 \left[\frac{\partial^2 f_i}{\partial\bar{x}^2}\frac{\partial^2 f_j}{\partial\bar{x}^2} + \left(\frac{a}{b}\right)^4 \frac{\partial^2 f_i}{\partial\bar{y}^2}\frac{\partial^2 f_j}{\partial\bar{y}^2} + \nu\left(\frac{a}{b}\right)^2 \frac{\partial^2 f_i}{\partial\bar{x}^2}\frac{\partial^2 f_j}{\partial\bar{y}^2}\right.$$

$$\left. + \nu\left(\frac{a}{b}\right)^2 \frac{\partial^2 f_j}{\partial\bar{x}^2}\frac{\partial^2 f_i}{\partial\bar{y}^2} + 2(1-\nu)\left(\frac{a}{b}\right)^2 \frac{\partial^2 f_i}{\partial\bar{x}\partial\bar{y}}\frac{\partial^2 f_j}{\partial\bar{x}\partial\bar{y}}\right] d\bar{x}d\bar{y} \tag{21}$$

$$n_{ij} = \int_0^1\int_0^1 \left[\frac{\partial f_i}{\partial\bar{x}}\frac{\partial f_j}{\partial\bar{x}} + \frac{N_y}{N_x}\frac{\partial f_i}{\partial\bar{y}}\frac{\partial f_j}{\partial\bar{y}} + \frac{N_{xy}}{N_x}\frac{\partial f_i}{\partial\bar{x}}\frac{\partial f_j}{\partial\bar{y}} + \frac{N_{xy}}{N_x}\frac{\partial f_j}{\partial\bar{x}}\frac{\partial f_i}{\partial\bar{y}}\right] d\bar{x}d\bar{y} \tag{22}$$

$$m_{ij} = \int_0^1\int_0^1 f_i f_j \, d\bar{x}d\bar{y} \tag{23}$$

$$A_{ij} = \sum \frac{\phi_j}{V\epsilon}\left(\frac{\partial f_i}{\partial\bar{x}} - i\,\frac{\omega a}{V} f_i\right) \tag{24}$$

The shape functions $f_i(\bar{x},\bar{y})$ are usually cubic or higher order functions of nondimensional coordinate variables $\bar{x}$ and $\bar{y}$. Their differentiations with respect to $\bar{x}$ and $\bar{y}$ are performed analytically. The subscript "i" indicates the degree of freedom number with which the shape function is associated. Eqs. (19-24) are suitable for any plate finite element so long as it is a displacement model with assumed shape functions. According to the state-of-the-art of the finite element development, the stiffness matrix [k], mass matrix [m], and incremental stiffness matrix [n] have been formulated analytically and explicitly for almost every plate and shell finite element. Only the aerodynamic matrix [A] remains to be formulated and it is to be obtained by numerical integration here.

Eq. (19) constitutes an eigenvalue problem. The flutter solution is obtained by first assuming a value of reduced frequency k_ℓ, then varying the air-panel mass ratio $1/\mu$ incrementally and solving for the corresponding eigenvalues Ω or $(\omega_1/\omega)^2(1 + ig)$. When the structural damping coefficient g changes its value from negative to positive, the panel goes from the stable region to unstable region, and vice versa. The values of k_ℓ and $1/\mu$ that correspond to zero g value define the flutter boundary and the corresponding mode shape defines the flutter mode. The complex eigenvalue problem is solved by using a subroutine in EISPACK provided by Argonne National Laboratory.

Before performing analysis for each class of problems, a convergence study by varying the meshes of Mach boxes and finite elements must be made in order to find the suitable meshes needed for obtaining converged results. Such meshes depend on the panel geometry, boundary conditions, flow speed, and flutter modes. However, due to the enormous computations needed for obtaining massive data in this study, not the finest meshes are used. Most of the present results are compared with those obtained by the modal method (Reference 9) and good results are obtained.

SECTION IV

RESULTS

The 16 d.o.f. conforming rectangular plate finite elements were employed to demonstrate the present formulation and procedure. The examples chosen were rectangular panels, stressed as well as unstressed, with clamped edges. For the unstressed panels, a sophisticated analytical solution by Cunningham (Reference 9) using the box method (400 boxes) and Galerkin's modal approach (6 to 16 modes) was available for comparison.

In all examples studied here, the flutter mode shapes were assumed as symmetrical about the center chord-line of the panel. Thus, only half of each panel was modeled by finite elements. In all cases, two elements in the cross stream direction were used for half of the panel. The number of elements in the stream direction varied from 4 to 10 dependent on the dominating flutter mode shapes. Each element was divided into 4 by 2 boxes in the stream and cross stream directions, respectively.

It is important to note that symmetry does not exist for the mesh of boxes unless the Mach cone apex is located at the center chord-line of the panel. The boxes on the other (disregarded) half of the panel can, however, still be accounted for since their deflection shapes are known by symmetry.

Case 1 Clamped Panels with Various Chord-span Ratios at M = 1.3

The rectangular panels with all edges clamped and chord-span ratios (ℓ/w) equal to 0, 1/4, 1/2, 1, 2, and 4 were studied for M = 1.3.

The resulting flutter boundaries are presented as plots of each dominating mode with the air-panel mass ratio $1/\mu$ as the vertical coordinate and the stiffness parameter $\omega_1 \ell/V$ as the horizontal coordinate. The values of reduced frequency k_ℓ or $\omega\ell/V$ are marked along each curve. The results for the infinite panel with $\ell/w = 0$ are in exact agreement with those obtained by Cunningham, therefore, they are not presented here.

Fig. 3 shows that for the wide panel with $\ell/w = 1/4$, the first mode flutter boundary crosses the second mode flutter boundary. The critical flutter boundaries are thus dominated by the first mode in the high mass ratio range and by the second mode in the low mass ratio range. A slight amount of structural damping was also included ($g = 0.01$). The results agree well with those obtained by Cunningham (Reference 9).

The first flutter mode shape (real part) corresponding to point A ($k_\ell = 0.635$) in Fig. 3 is shown in Fig. 4. It is seen that, due to the effect of the flow, the mode is not symmetrical about the cross stream centerline. The maximum deflection occurs at a point behind the center of the panel. The second flutter mode shape (real part) for the center chord-line of the panel corresponding to point B ($k_\ell = 1.59$) in Fig. 3 is shown in Fig. 5. The effect of the flow that pushes down the front part of the panel is seen.

Fig. 6 shows that, as the panel width is reduced ($\ell/w = 1/2$), the first mode boundary shifts to the left and the second mode boundary becomes the critical flutter boundary. Again, the results agree well with Cunningham's solution.

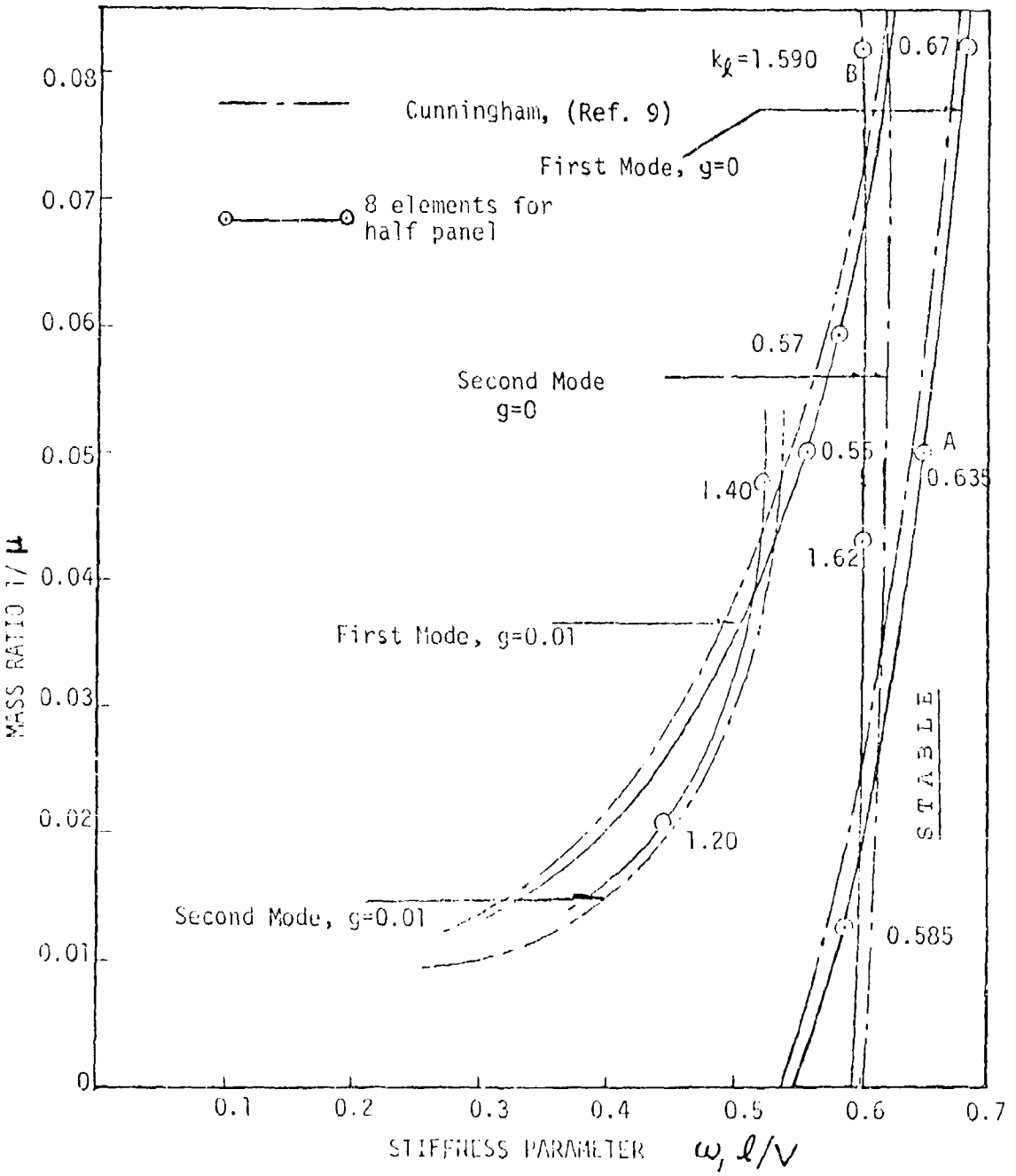

Figure 3 Flutter boundaries for clamped panel with $\ell/w = 1/4$ and $M = 1.3$

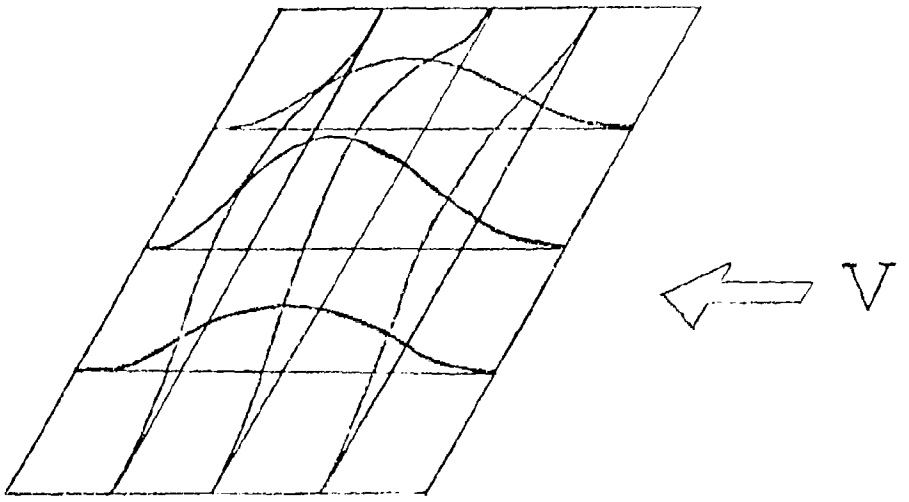

Figure 4. The first mode shape (real part) for panel corresponding to point A in Fig. 3

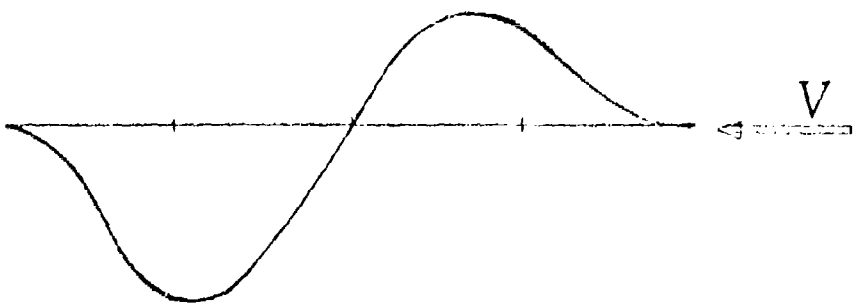

Figure 5 The second mode shape (real part) for the center line of panel corresponding to point B in Fig. 3

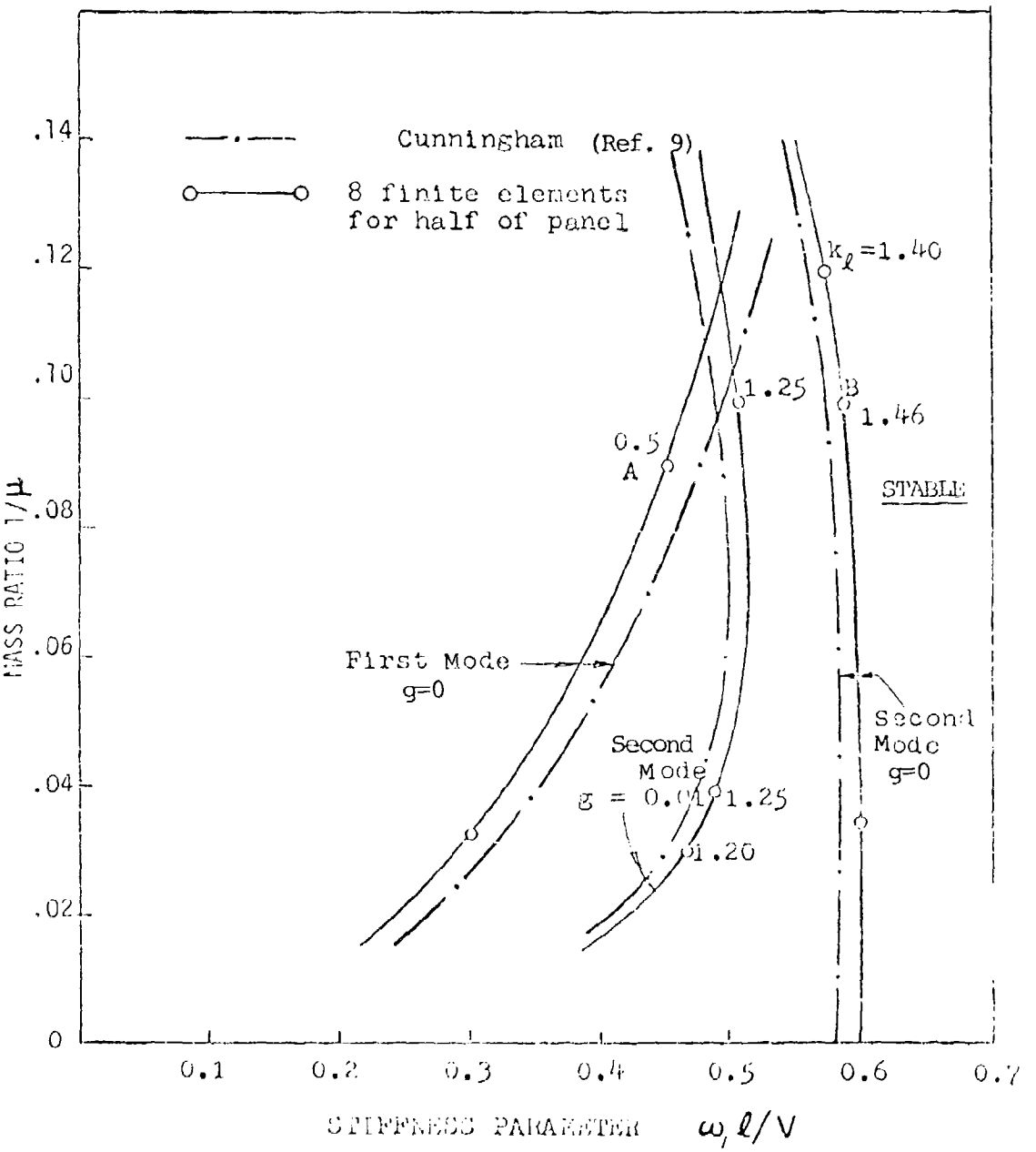

Figure 6 Flutter boundaries for clamped panel with $\ell/w = 1/2$ and $M = 1.3$

The first and second flutter mode shapes (real part) corresponding to points A and B in Fig. 6 are plotted in Figs. 7 and 8, respectively. They are quite similar to those shown in Figs. 4 and 5.

Fig. 9 shows that, as the panel becomes square, the third and fourth mode boundaries emerge and the third mode boundary becomes the critical flutter boundary. The results are in good agreement with Cunningham's solution. The results for the first mode boundary obtained by Houbolt using the piston theory are also shown in the figure.

The third and fourth flutter mode shapes (real part) corresponding to points A and B in Fig. 9 are shown in Figs. 10 and 11, respectively. The effect of the flow that tends to blow flat the front part of the panel is seen.

Fig. 12 shows that, for a long panel with $\ell/w = 2$, the fifth mode flutter boundary becomes dominant in the low mass ratio range and the first mode boundary dominates the high mass ratio range. It also shows that Houbolt's solution for first mode and this solution are very close. There are, however, some discrepancies between this solution and Cunningham's solution for the first mode boundary. This may be due to the fact that the stiffness coupling effect between assumed beam natural modes was uniformly neglected by Cunningham

Such coupling effect can become significant as the chord-span ratio increases. It should be noted that, due to the limited number of elements and boxes used, the present results are not the most accurate that this approach can produce.

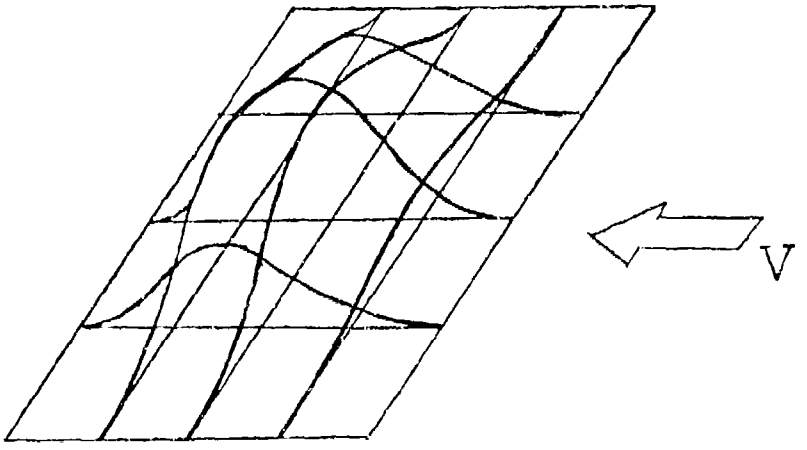

Figure 7 The first mode shape (real part) for panel corresponding to point A in Fig. 6

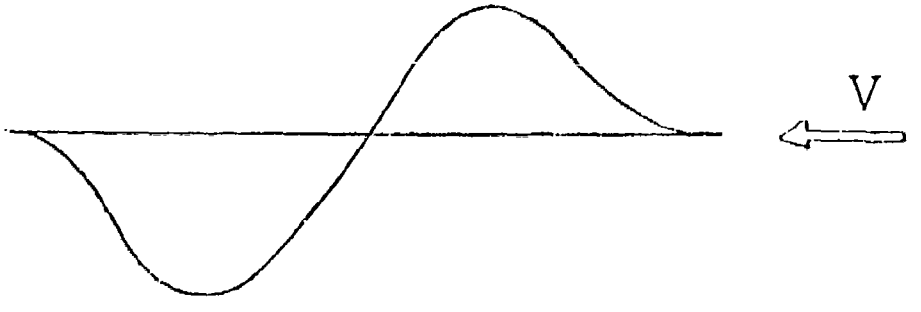

Figure 8 The second mode shape (real part) for the center line of panel corresponding to point B in Fig. 6

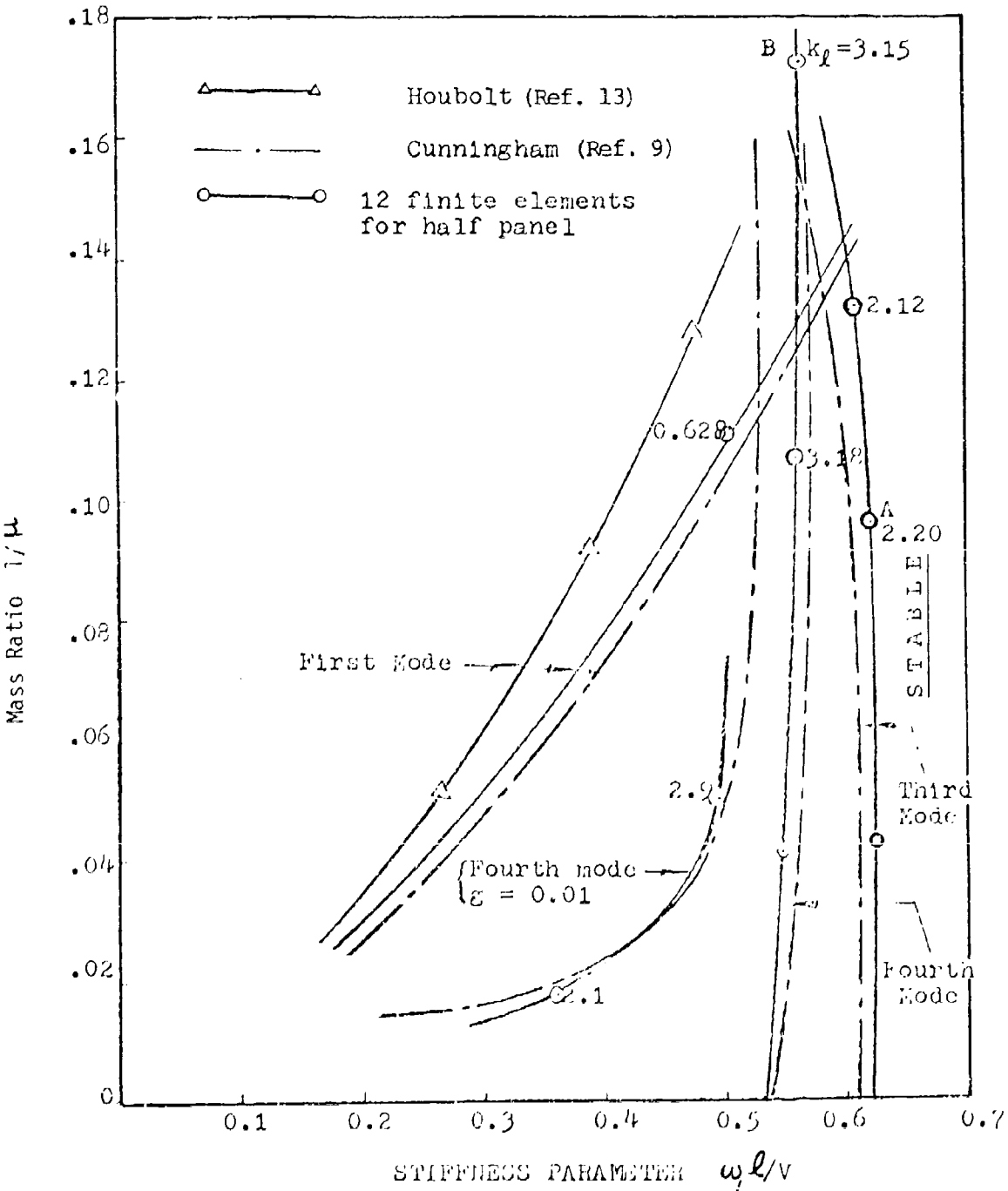

Figure 9 Flutter boundaries for clamped panel with $\ell/w = 1$ and $M = 1.3$

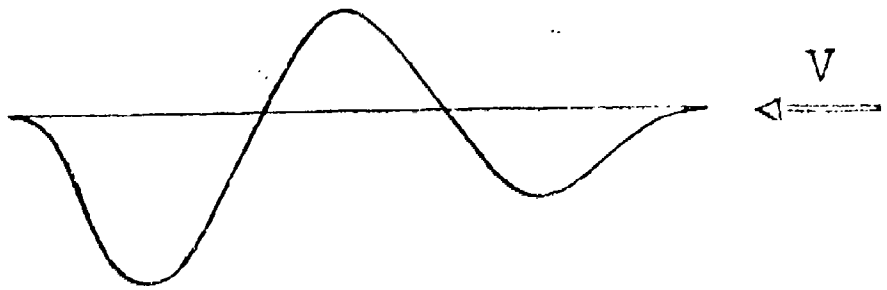

Figure 10 The third mode shape (real part) for the center line of panel corresponding to point A in Fig. 9

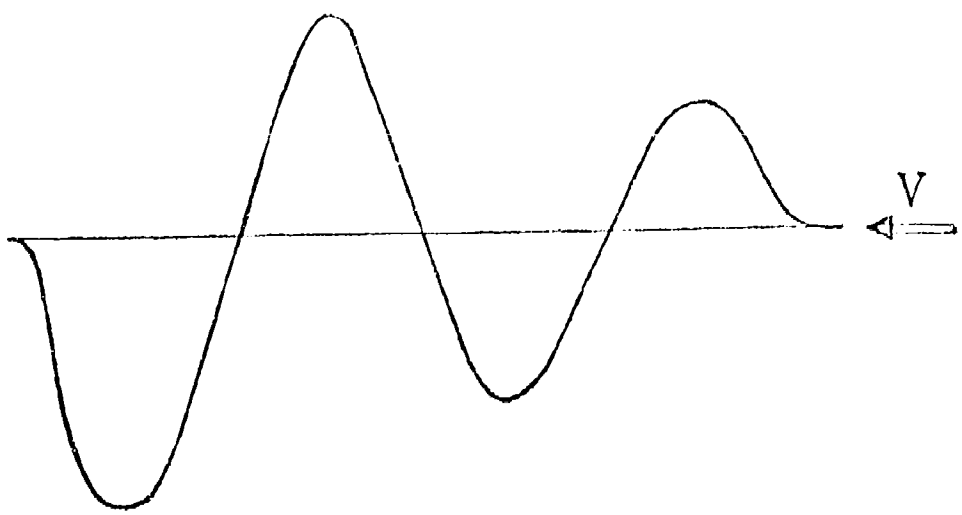

Figure 11 The fourth mode shape (real part) for the center line of panel corresponding to point B in Fig. 9

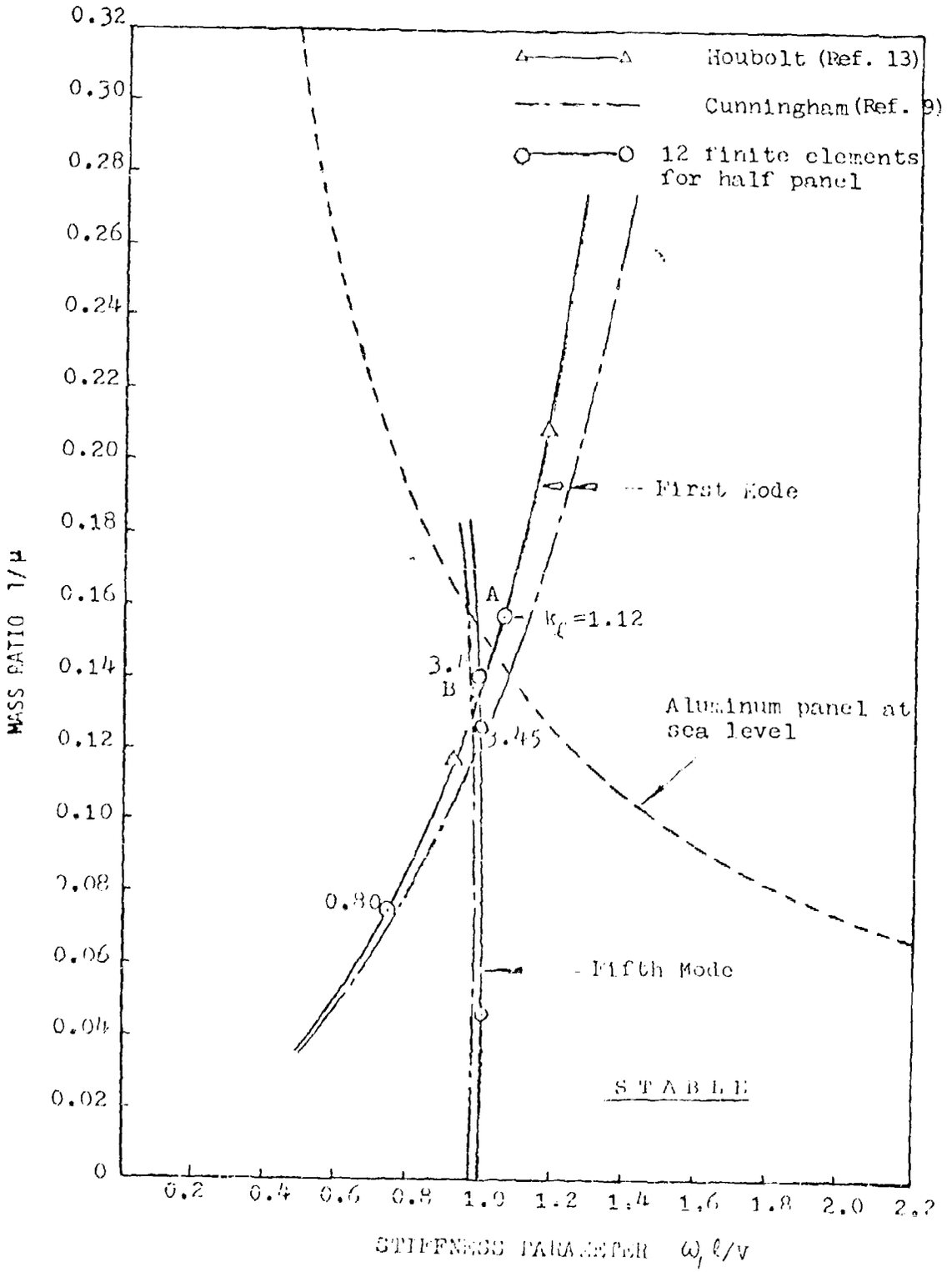

Figure 12 Flutter boundaries for clamped panel with $\ell/w = 2$ and $M = 1.3$

The first and third flutter mode shapes (real part) corresponding to points A and B in Fig. 12 are shown in Figs. 13 and 14, respectively. The difference between a symmetrical in vacuo first mode and the first flutter mode with front part blown flat is clearly shown in Fig. 13.

Fig. 15 shows that, for a very long panel with $\ell/w = 4$, the tenth mode flutter boundary dominates the lower mass ratio range while the first mode boundary dominates the upper mass ratio range. The discrepancy between the present solution and Cunningham's solution may be attributed to the same reasons as explained for Fig. 6. Houbolt's solution for first mode boundary is also shown.

The second and tenth flutter mode shapes (real part) corresponding to points A and B in Fig. 15 are plotted in Figs. 16 and 17, respectively.

Case 2 Clamped Panel with $\ell/w = 2$ at Various Mach Numbers

A clamped panel with $\ell/w = 2$ and with one surface exposed to air stream with various Mach numbers was studied. One of the main purposes was to establish a curve for the thickness ratios required to prevent flutter of panel at various air speeds and at sea level.

The first mode flutter boundaries for panel with $\ell/w = 2$ and $M = 1.05$, 1.1; 1.4, 1.5; 2.0, and 3.0 are shown in Figs. 18, 19, and 20, respectively. Corresponding to each curve, a dashed parabola is shown. Each parabola is plotted for the equation $xy = C$ or $(\rho\ell/\sigma h)(\omega_1 \ell/V) = C$. The constant C is dependent upon

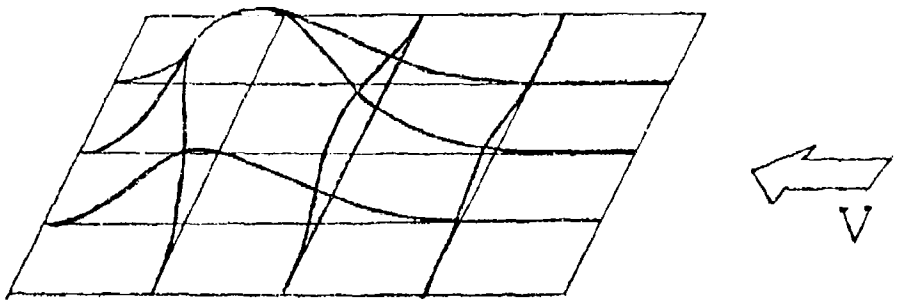

Figure 13 The first mode shape (real part) for panel corresponding to point A in Fig. 12.

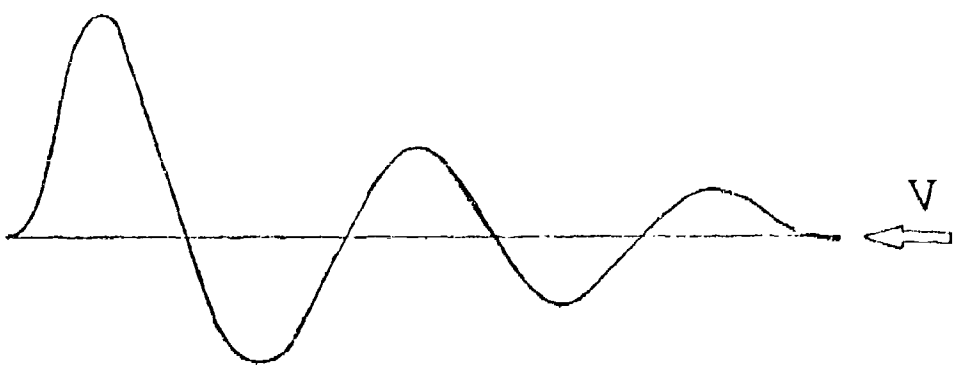

Figure 14 The fifth mode shape (real part) for the center line of panel corresponding to point B in Fig. 12.

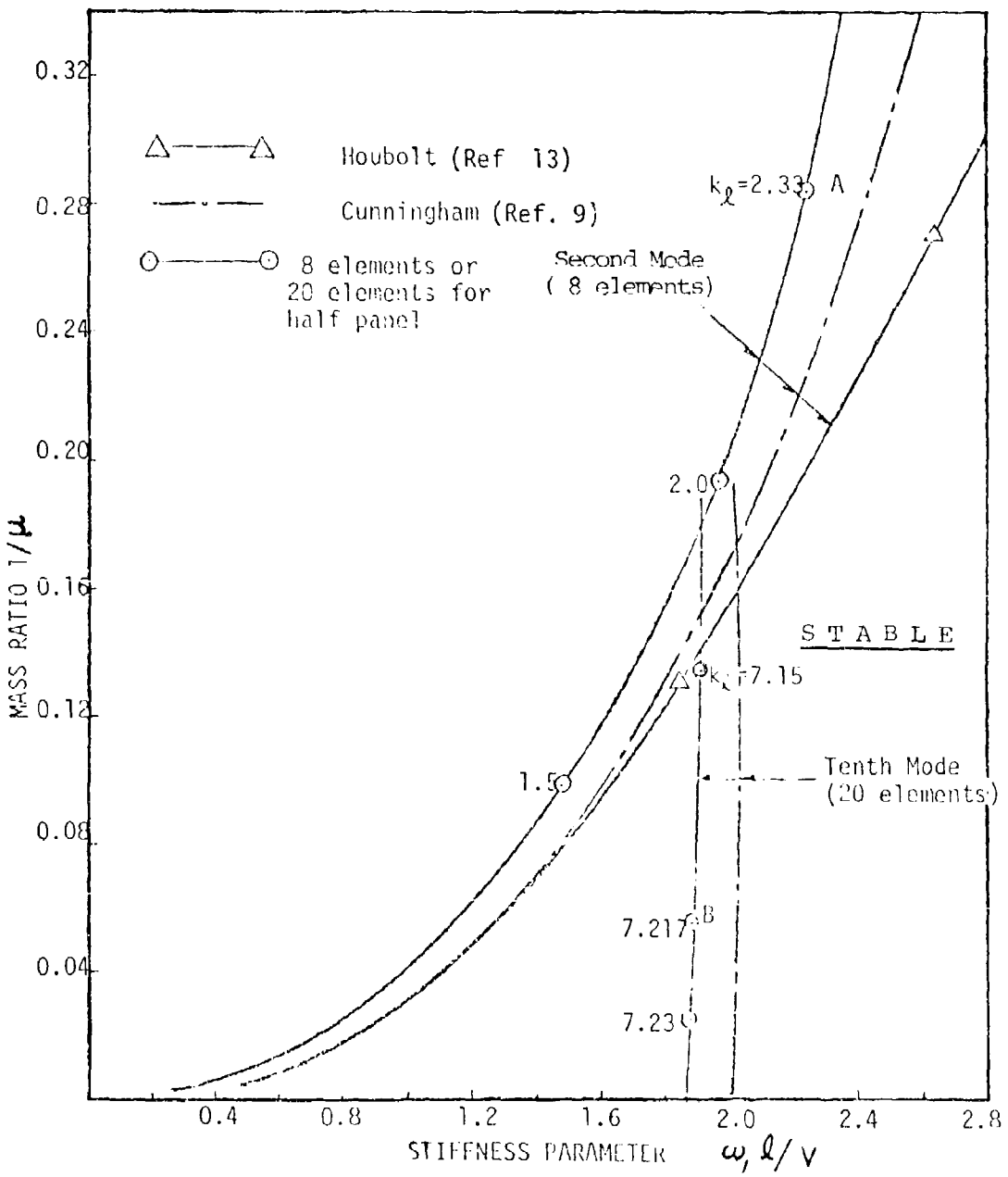

Figure 15 Flutter boundaries for clamped panel with $\ell/w = 4$ and $M = 1.3$

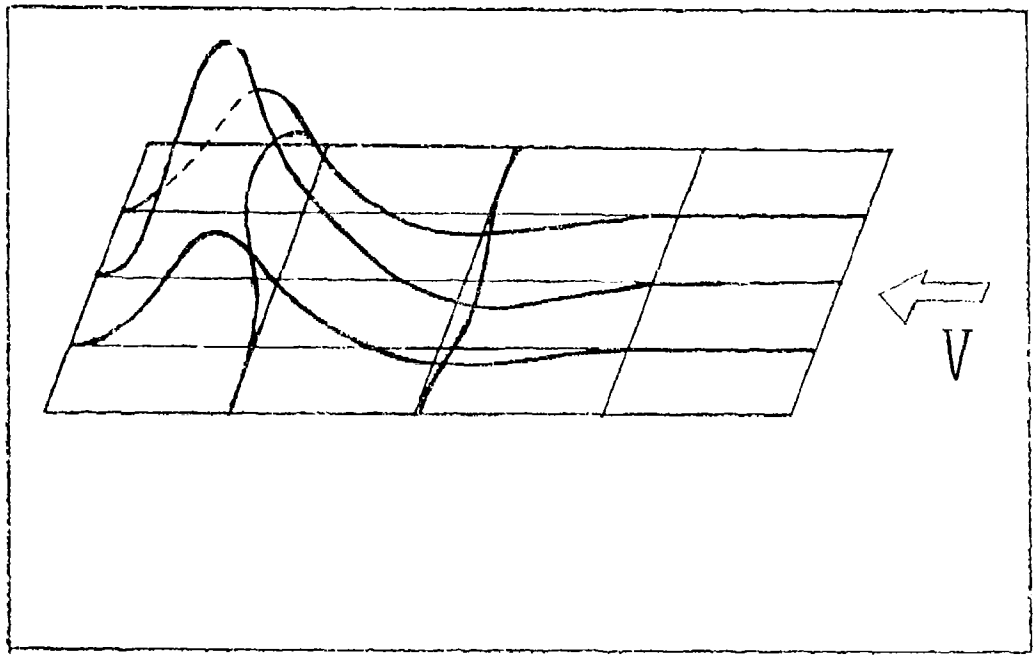

Figure 16 The second mode shape (real part) for panel corresponding to point A in Fig. 15.

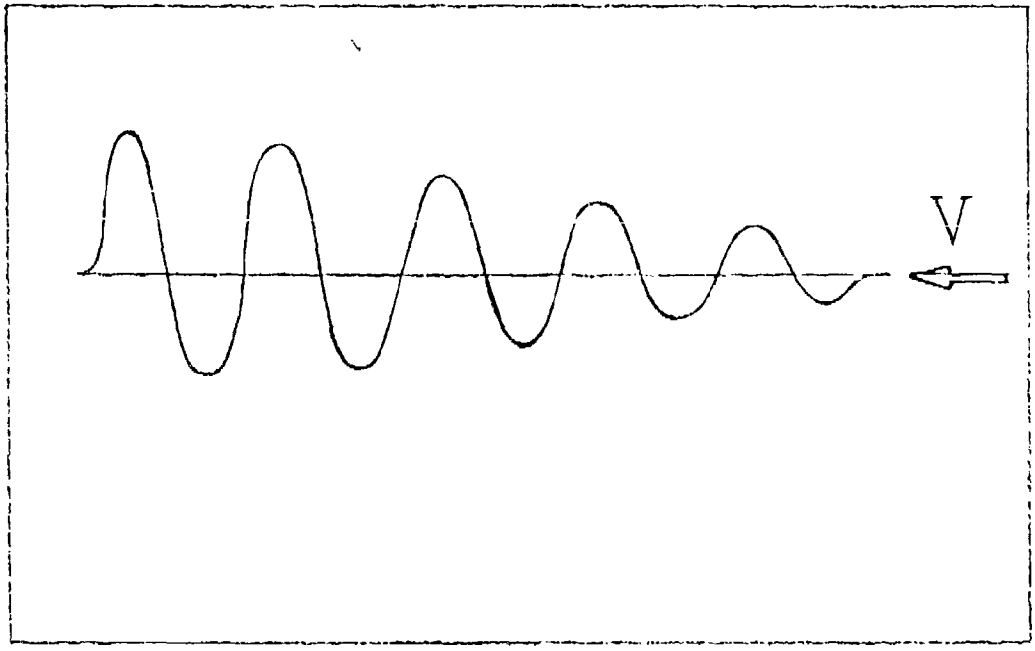

Figure 17 The tenth mode shape (real part) for the center line of panel corresponding to point B in Fig. 15.

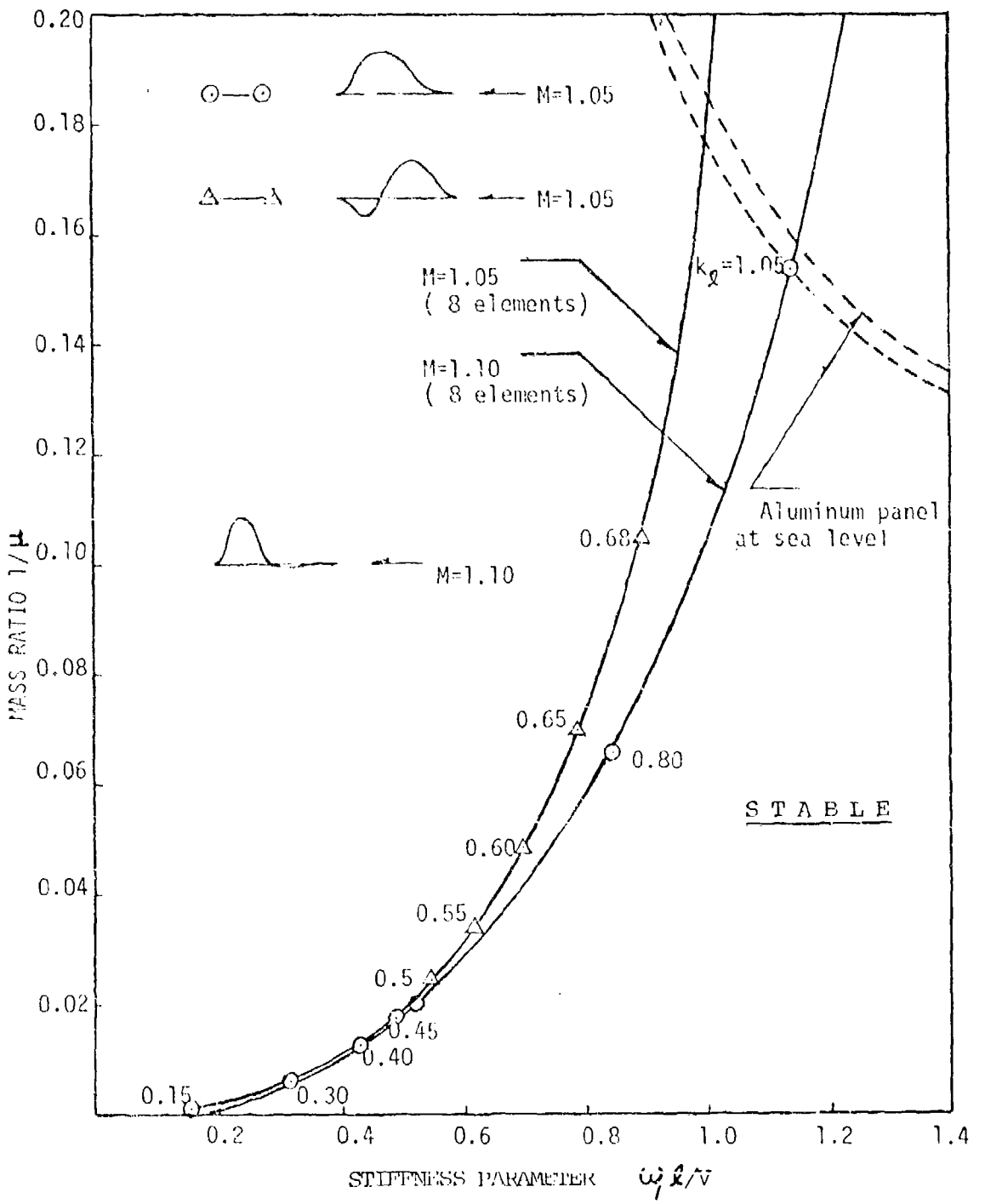

Figure 18 Flutter boundaries for clamped panel with $\ell/w = 2$ and $M = 1.05, 1.10$.

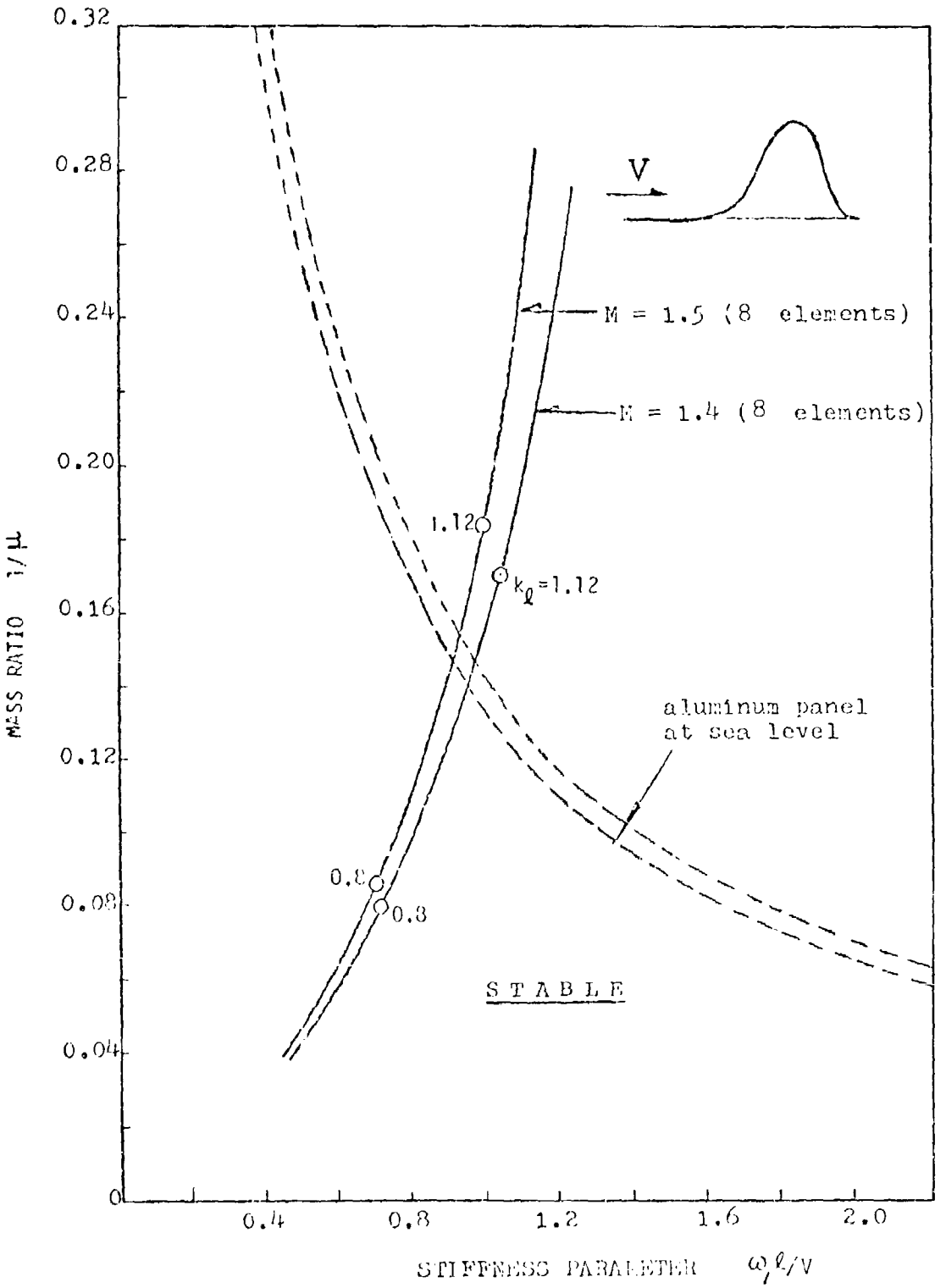

Figure 19 Flutter boundaries for clamped panel with $\ell/w = 2$ and M = 1.40, 1.50.

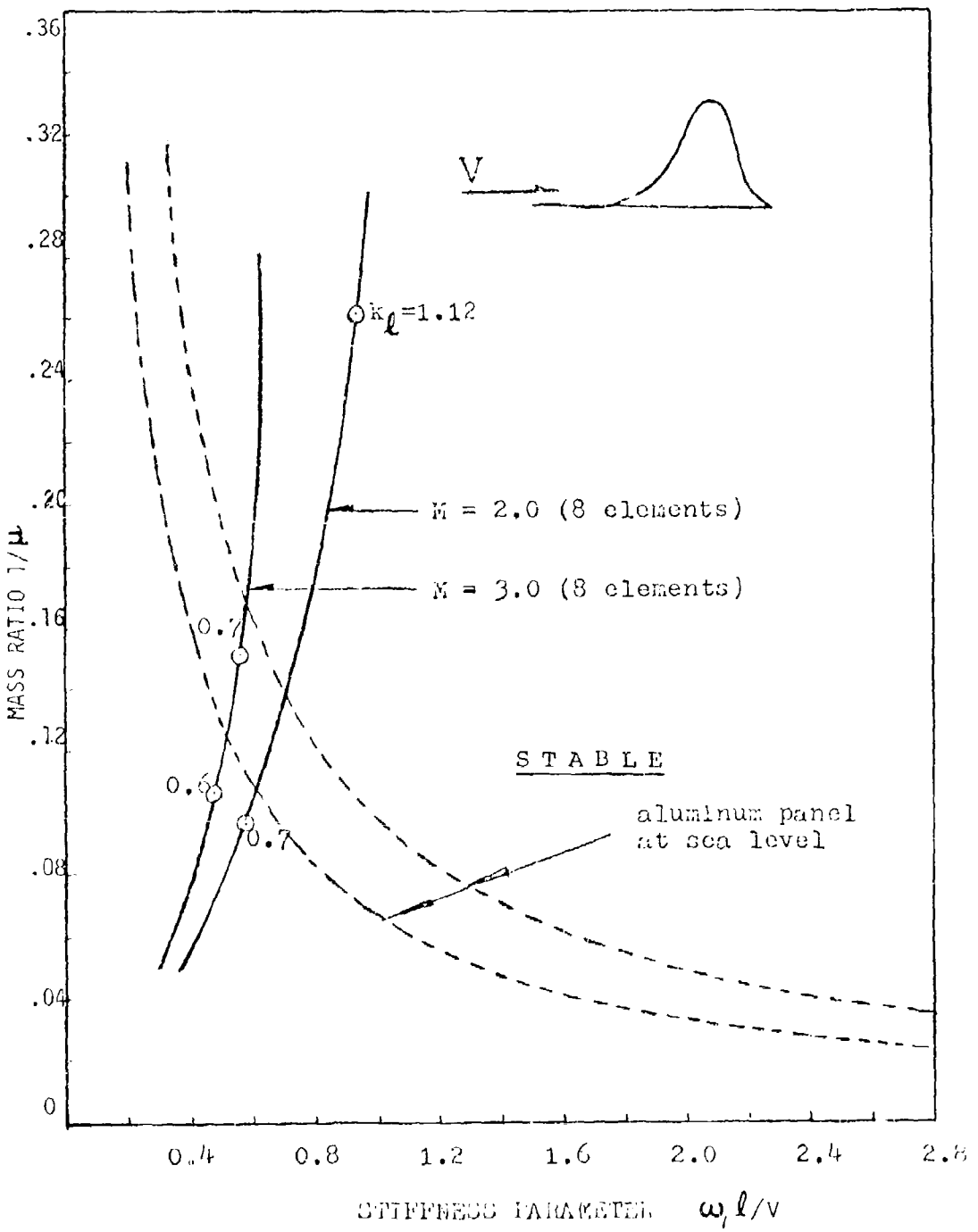

Figure 20 Flutter boundaries for clamped panel with $\ell/w = 2$ and M = 2.0 and 3.0.

the densities of the aluminum and the air at sea level, the dimensions of the panel, the air speed, and the bending rigidity of the panel. The intersecting point between a pair of dashed and solid curves gives the thickness ratio h/ℓ required to prevent flutter of the aluminum panel at sea level. For all Mach numbers considered here, the first mode flutter boundaries are critical for the dashed curves for the sea level altitude.

The results are shown in Fig. 21. The present results are slightly on the unconservative side as compared to the results of Cunningham.

Case 3 Pinned-Edge Panels with Various Chord-Span Ratios at M = 1.3

The panels with simply-supported edges and various chord length-span width ratios were studied. The flow speed considered was M = 1.3. The dominating flutter boundaries for panels with ℓ/w = 0.25, 0.5, 1.0, and 2.0 are shown in Figs. 22, 24, 26 and 28, respectively. They are all in good agreement with those found by Cunningham (Reference 9).

Fig. 22 shows that the first mode flutter boundary is the critical boundary for panels with ℓ/w = 0.25. The mode shape (real part) is shown in Fig. 23.

Fig. 24 shows that the second mode flutter boundary is the critical boundary for panels with ℓ/w = 0.5. The first and second flutter mode shapes (real part) are shown in Fig. 25.

Fig. 26 shows that the third mode flutter boundary is the critical boundary for panel with ℓ/w = 1.0. The first mode boundary found by Hedgepeth (Reference 14) using the piston theory is also shown in the figure. The corresponding first and third flutter mode shapes (real part) are shown in Fig. 27.

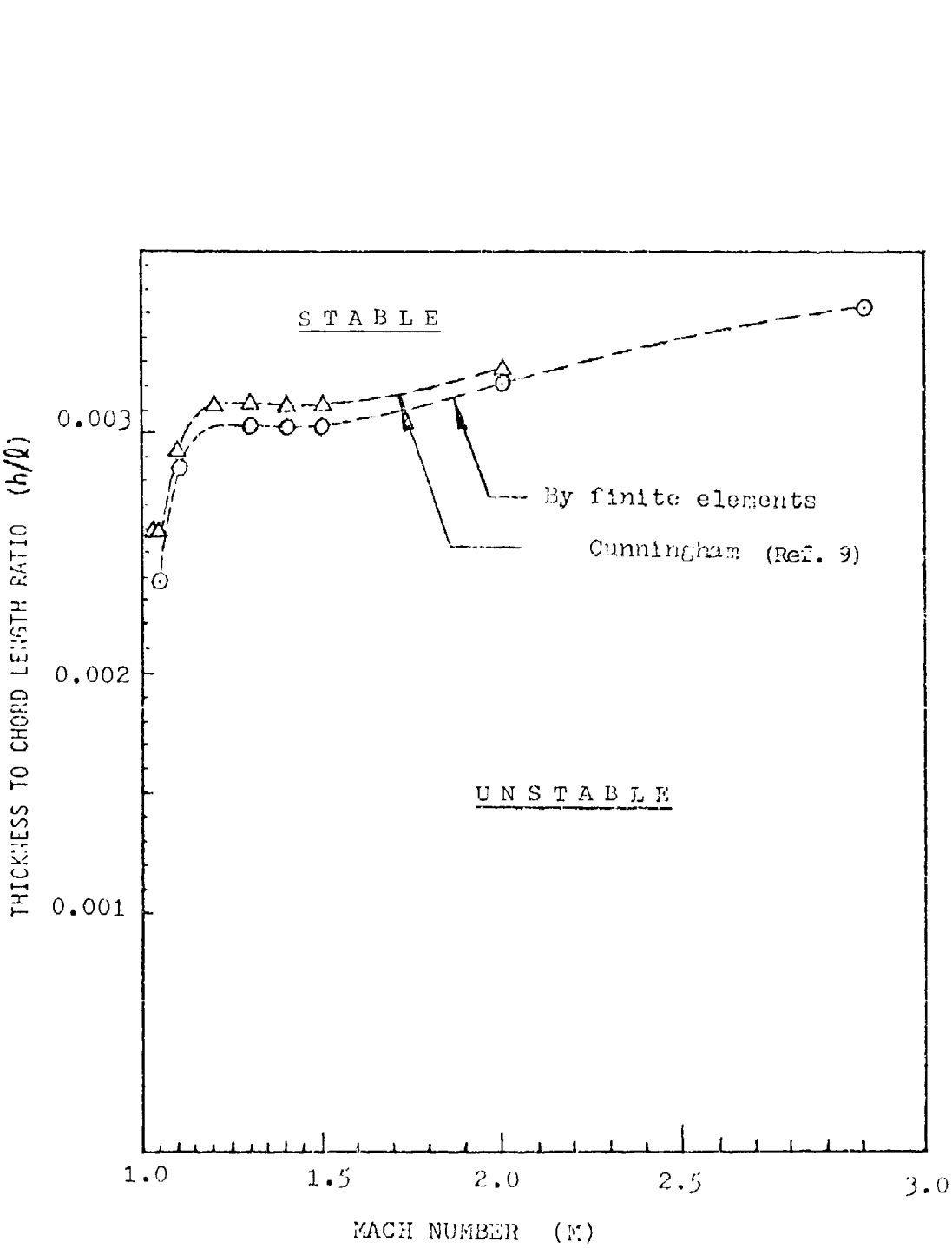

Figure 21 Thickness required to prevent flutter for clamped aluminum panel with $\ell/w = 2$ at sea level.

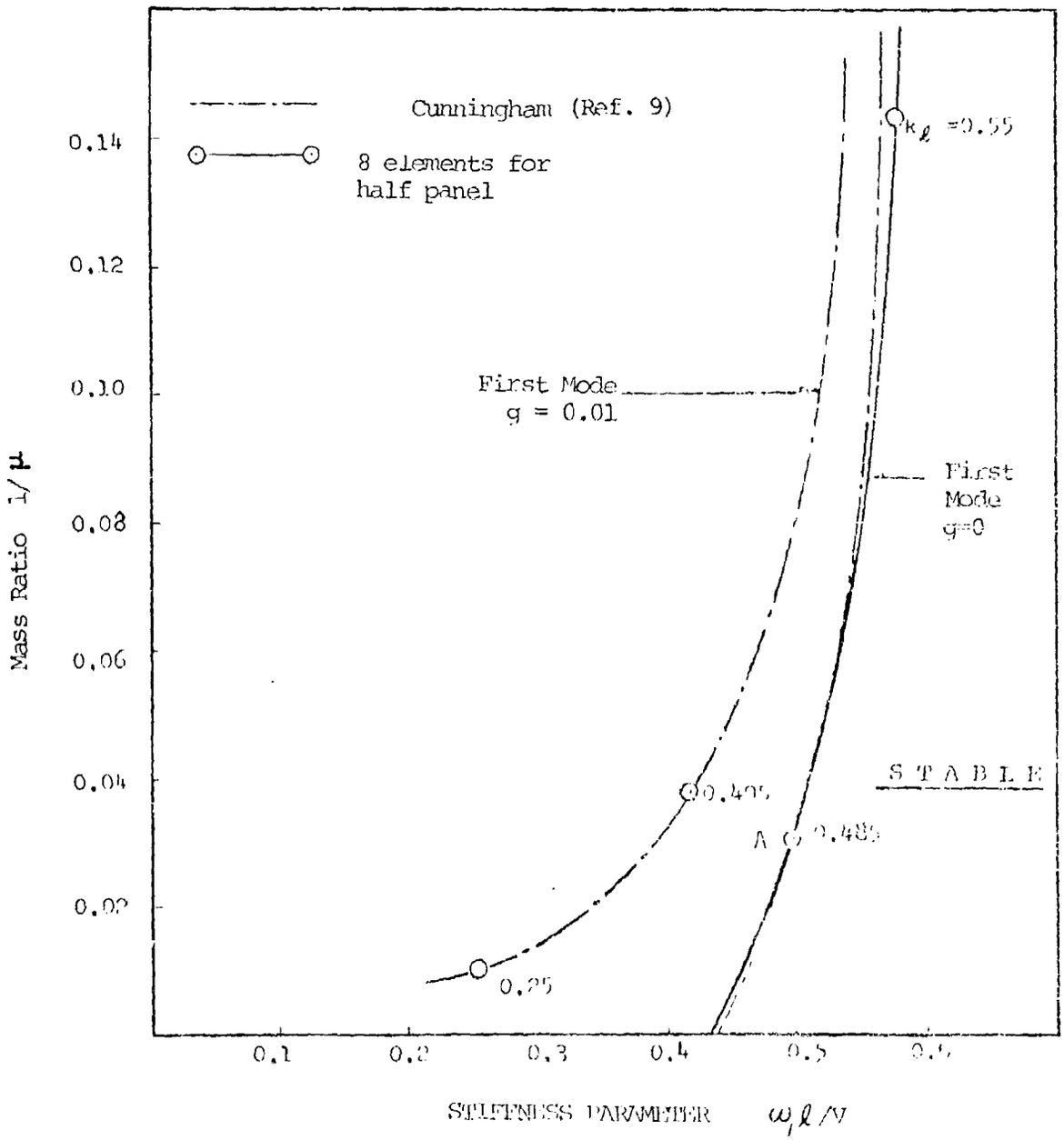

Figure 22 Flutter boundaries for pinned edge panel with $\ell/w = 1/4$ and $M = 1.3$.

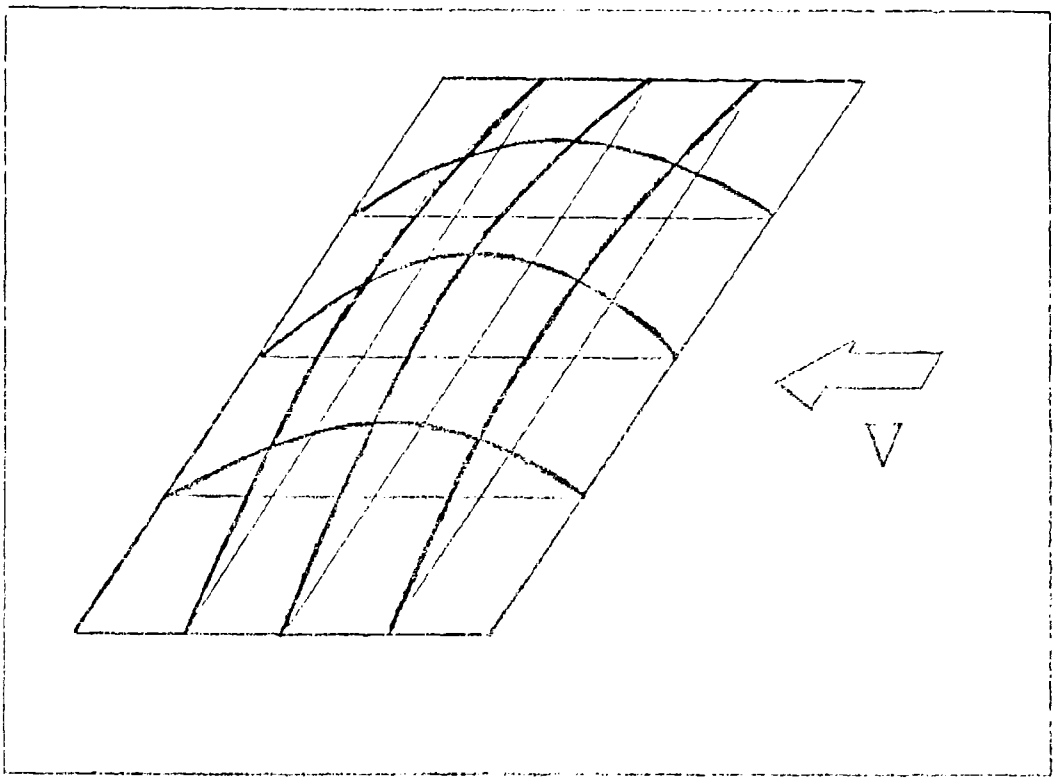

Figure 23 The First Mode Shape (Real Part) for Panel Corresponding to Point A in Fig. 22.

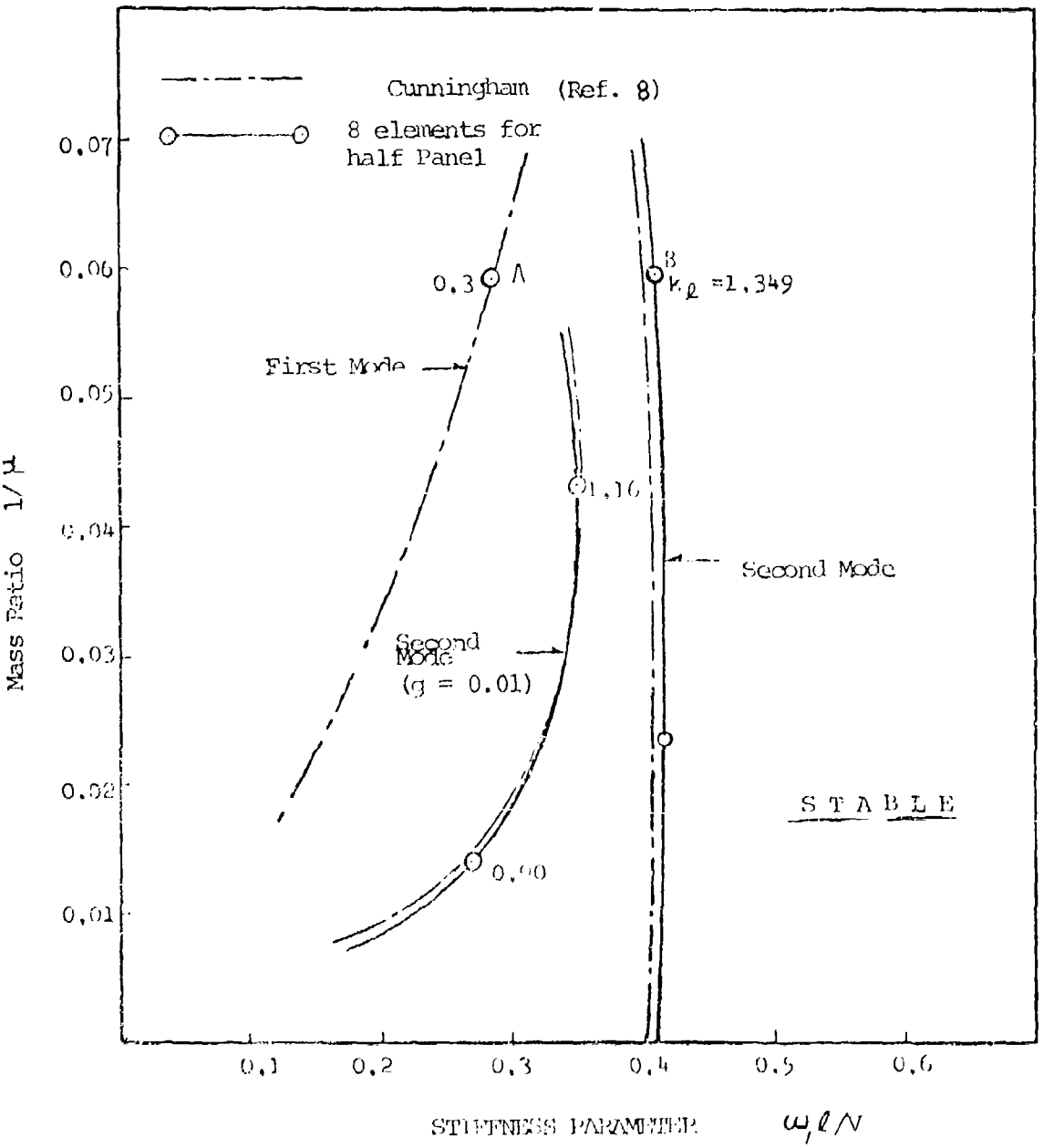

Figure 24 Flutter boundaries for pinned-edge panel with $\ell/w = 1/2$ and M = 1.3.

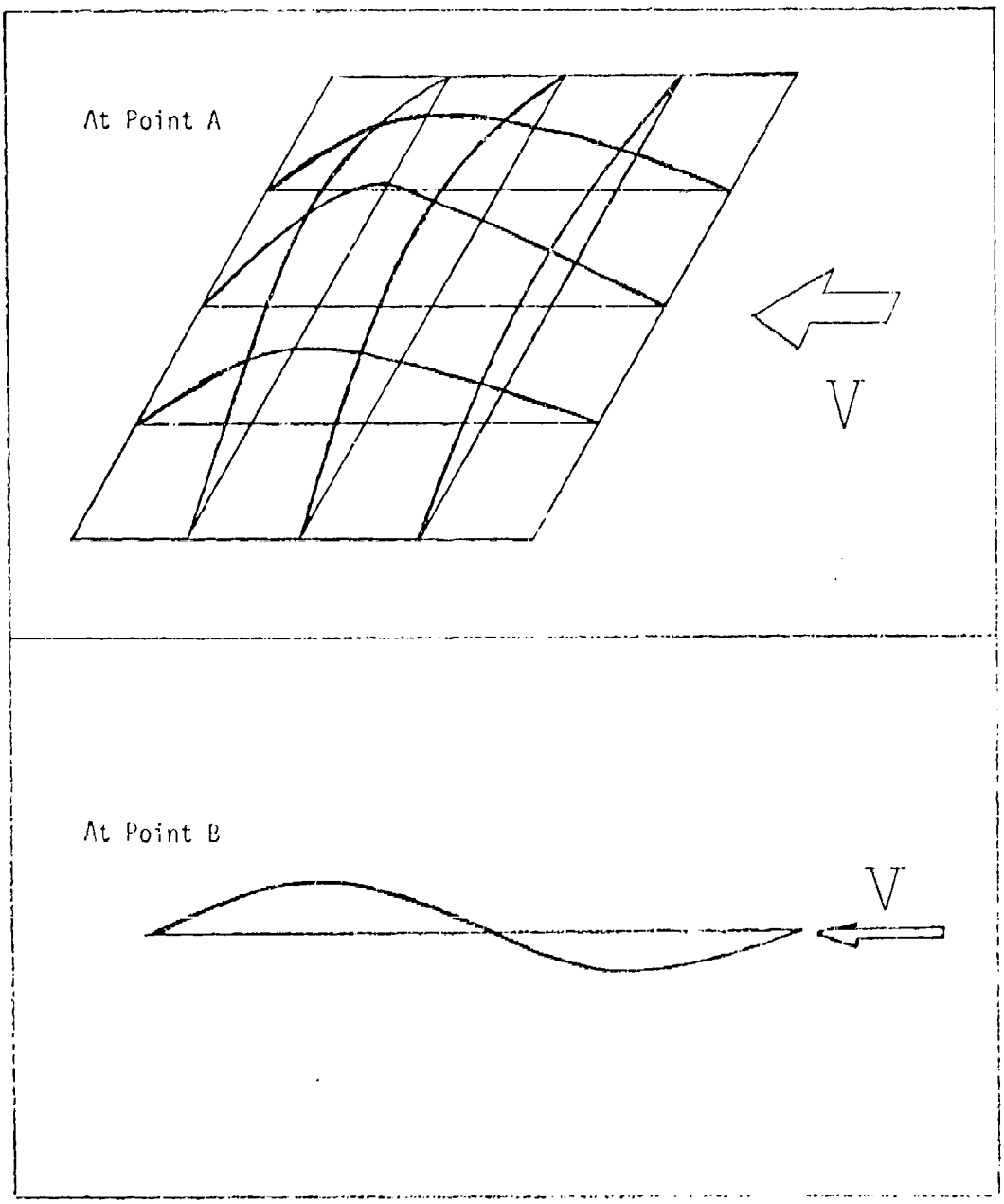

Figure 25 The First Mode Shape (Real Part) for Panel Corresponding to Point A in Fig. 24 and the Second Mode Shape (Real Part) for the Center Line of Panel Corresponding to Point B in Fig. 24.

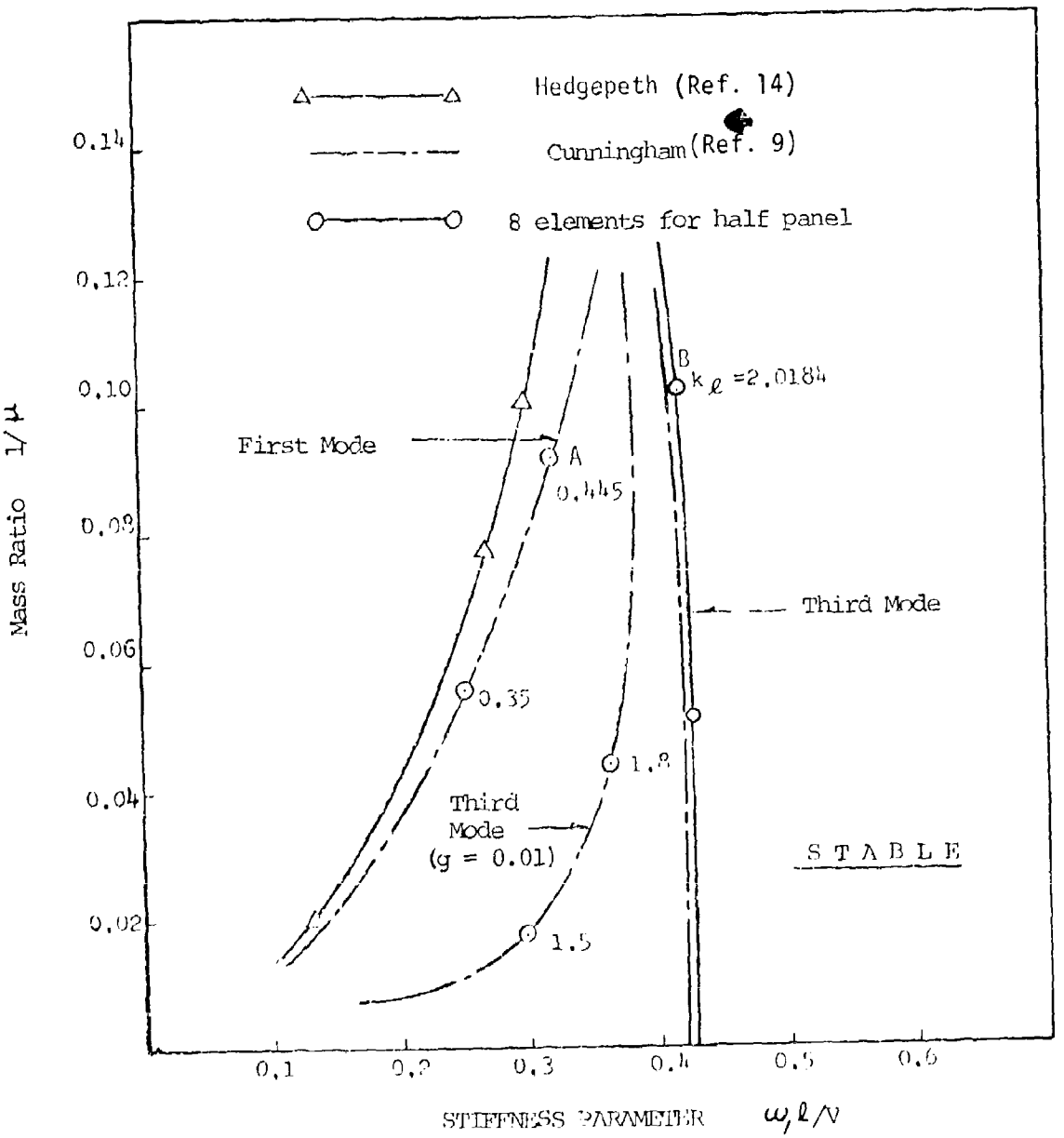

Figure 26 Flutter boundaries for pinned-edge panel with $\ell/w = 1$ and $M = 1.3$.

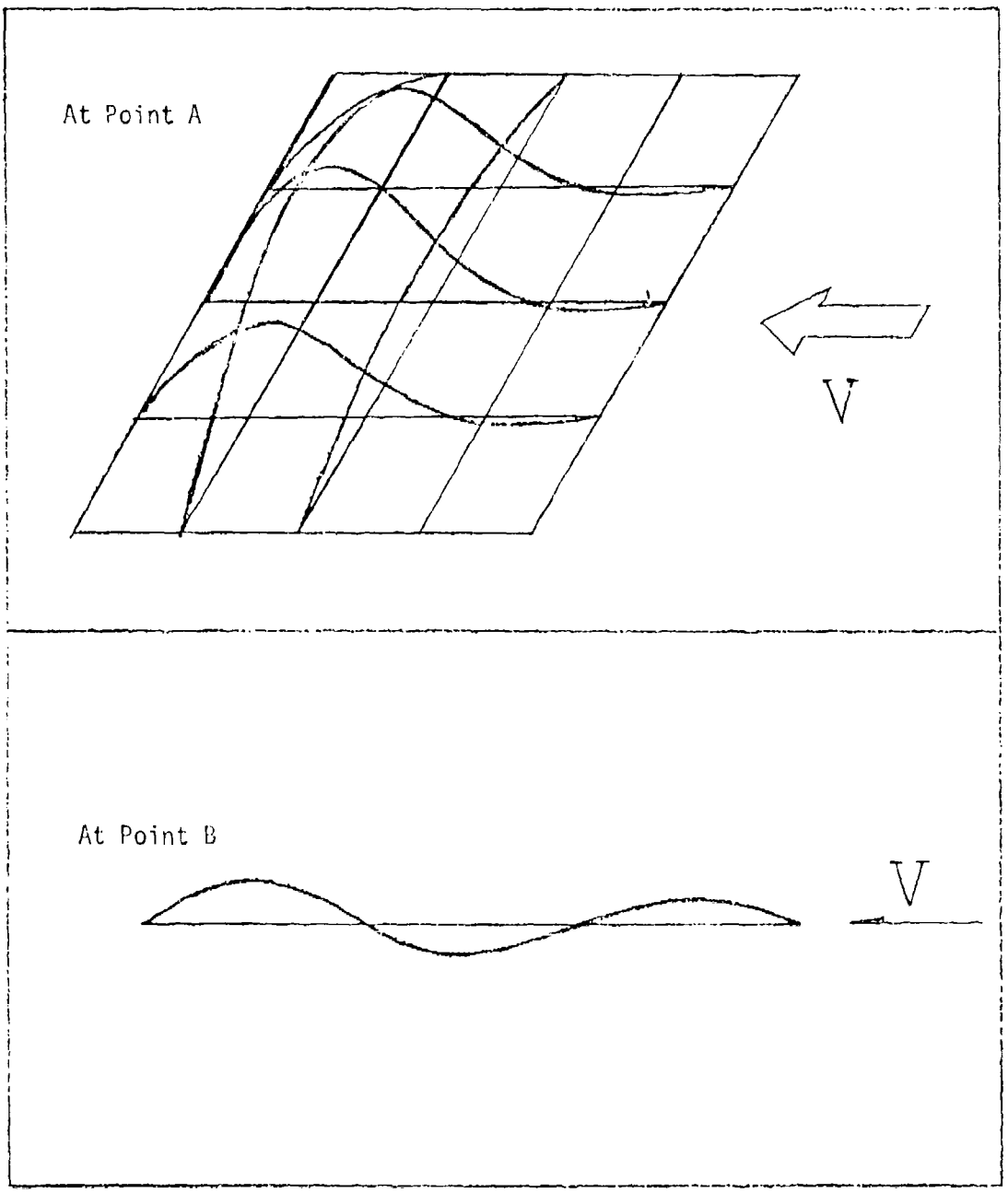

Figure 27 The First Mode Shape (Real Part) for Panel Corresponding to Point A in Fig. 26 and the Third Mode Shape (Real Part) for the Center Line of Panel Corresponding to Point B in Fig. 26.

Fig. 28 shows that the fifth mode flutter boundary is the critical boundary for panels with $\ell/w = 2.0$. The results by Hedgepeth and Cunningham are seen to be in good agreement with the present results. The corresponding first and fifth flutter mode shapes (real part) are shown in Fig. 29.

Case 4 Clamped Square Panels with Various Tension Parameters and Mach Numbers

One of the advantages of the finite element method is that the effect of in-plane forces can be included in a direct and accurate fashion. To demonstrate this, a square clamped panel was chosen and four different tension parameters $F = 0.01$, 0.1, 0.5, and 1 were considered. The flutter boundaries were obtained for various Mach numbers.

Fig. 30 shows the dominating first mode flutter boundaries and the hyperbola for the square, aluminum panel at 25,000 feet above sea level and at $M = 1.1$. Four values of tension parameters were included.

The mode shapes (real part) corresponding to points A ($F = 0.01$) and point B ($F = 0.1$) of the flutter boundaries in Fig. 30 are plotted in Figs. 31 and 32. The mode shapes for the case $F = 0.5$ and 1.0 are similar to those shown in Figs. 31 and 32. They are thus not shown here.

Fig. 33 shows the first and the third mode flutter boundaries for $M = 1.2$ and various tension parameters. For aluminum panels at 25,000 feet above sea level (dashed parabola), the third mode boundaries are the critical flutter boundaries. The first flutter mode shapes (real part) corresponding to points A and B in Fig. 33 are presented in Figs. 34 and 35, respectively.

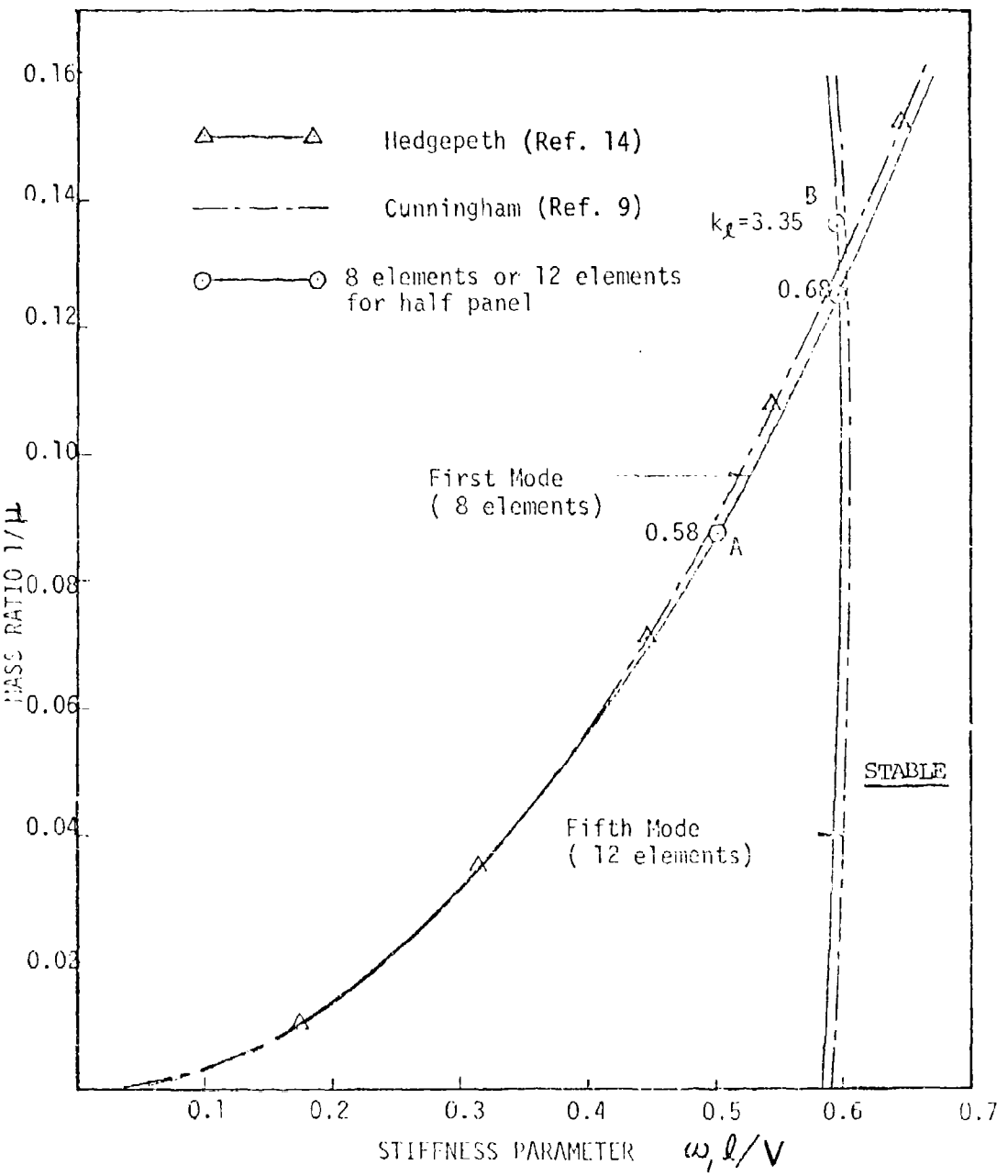

Figure 28 Flutter boundaries for pinned-edge panel with $\ell/w = 2$ and $M = 1.3$.

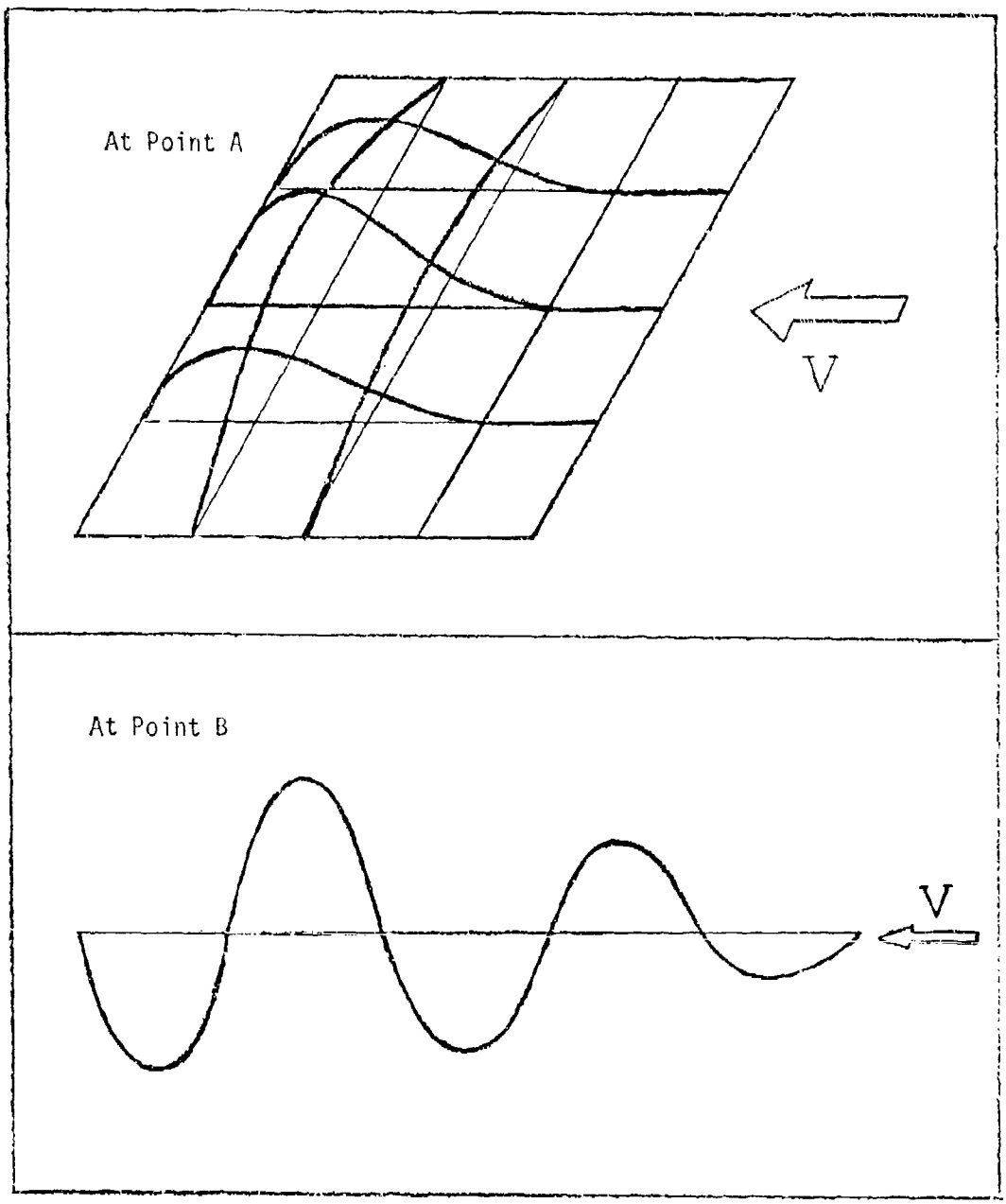

Figure 29 The First Mode Shape (Real Part) for Panel Corresponding to Point A in Fig. 28 and the Fifth Mode Shape (Real Part) for the Center Line of Panel Corresponding to Point B in Fig. 28.

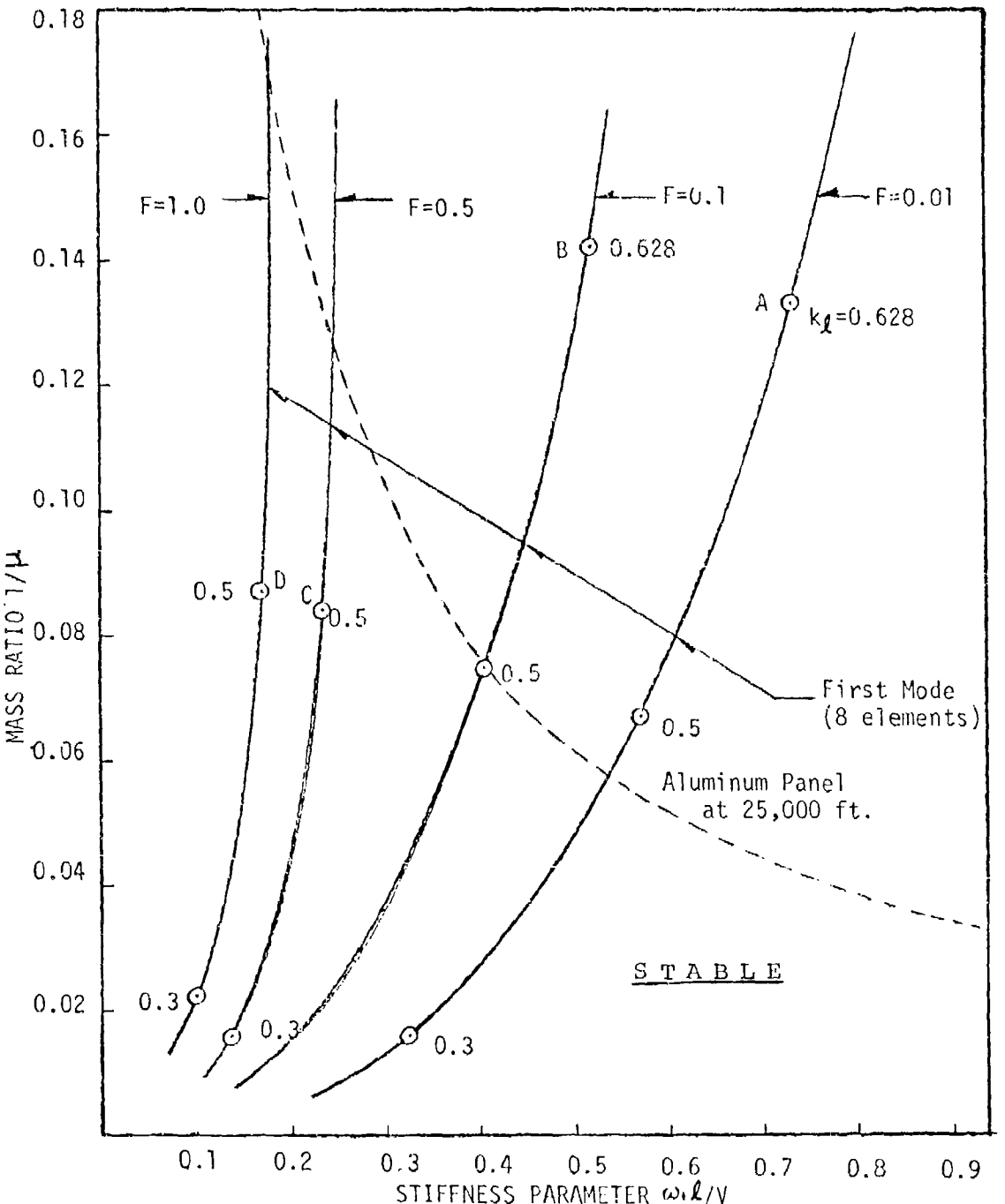

Figure 30. Flutter boundaries for clamped panels of ℓ/w =1.0 with M= 1.1 and various tension parameters F.

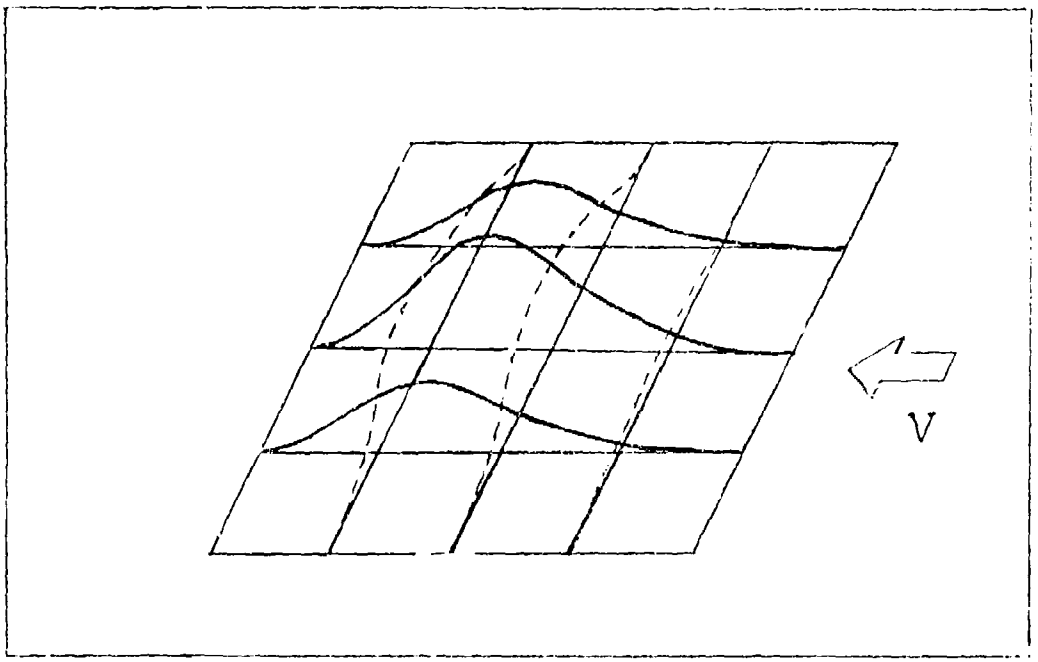

Figure 31 The First Mode Shape (Real Part) for Panel Corresponding to Point A in Fig. 30.

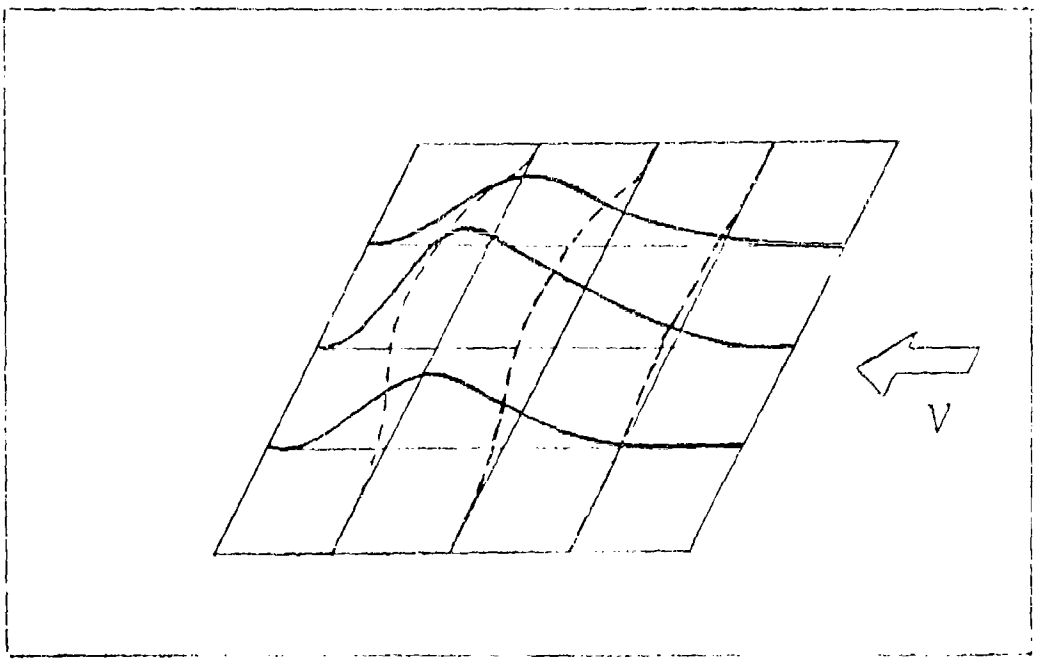

Figure 32 The First Mode Shape(Real Part) for Panel Corresponding to Point B in Fig. 30.

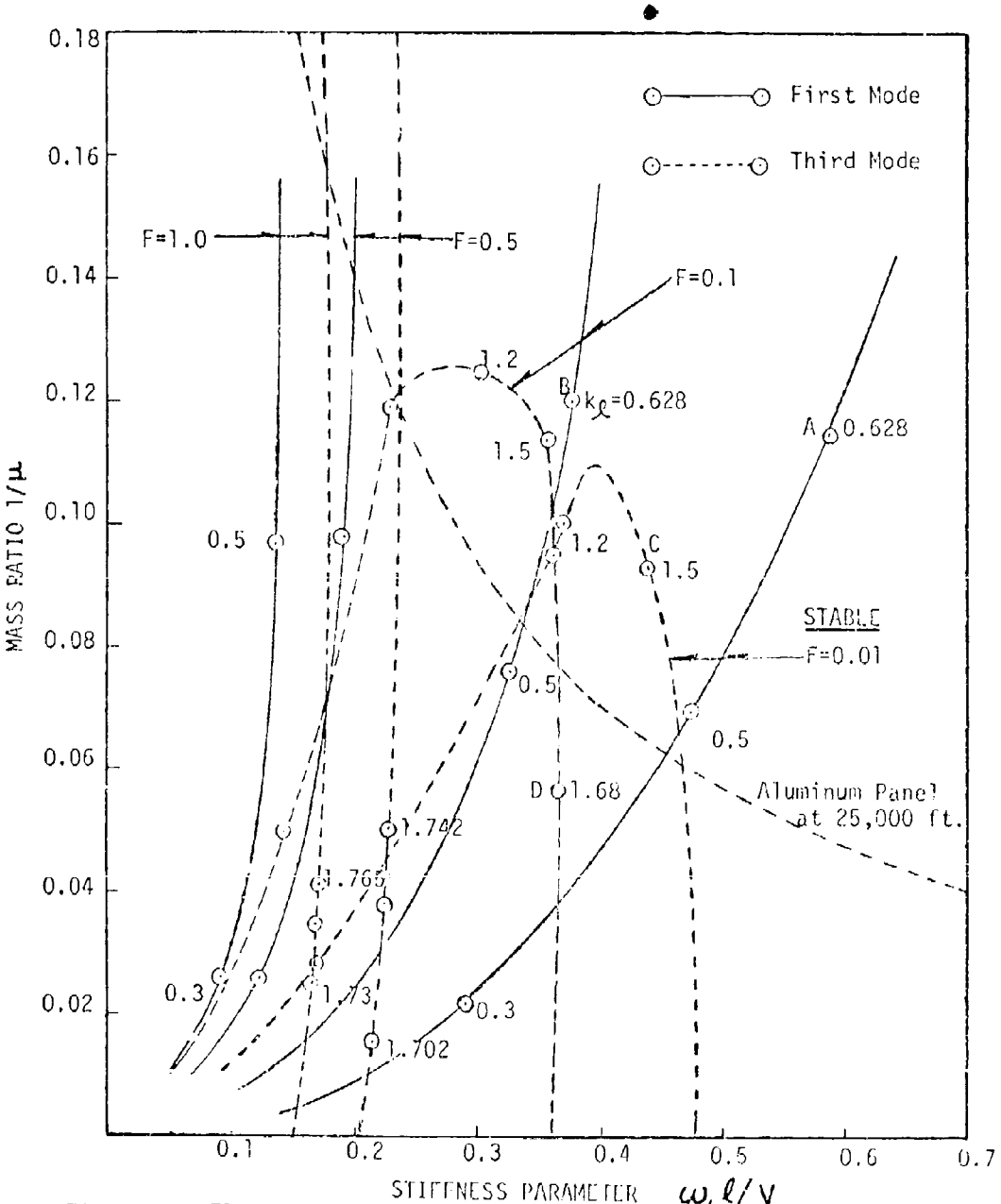

Figure 33 Flutter boundaries for clamped panels of ℓ/w=1.0 with M=1.2 and various tension parameters $\bar{F}$

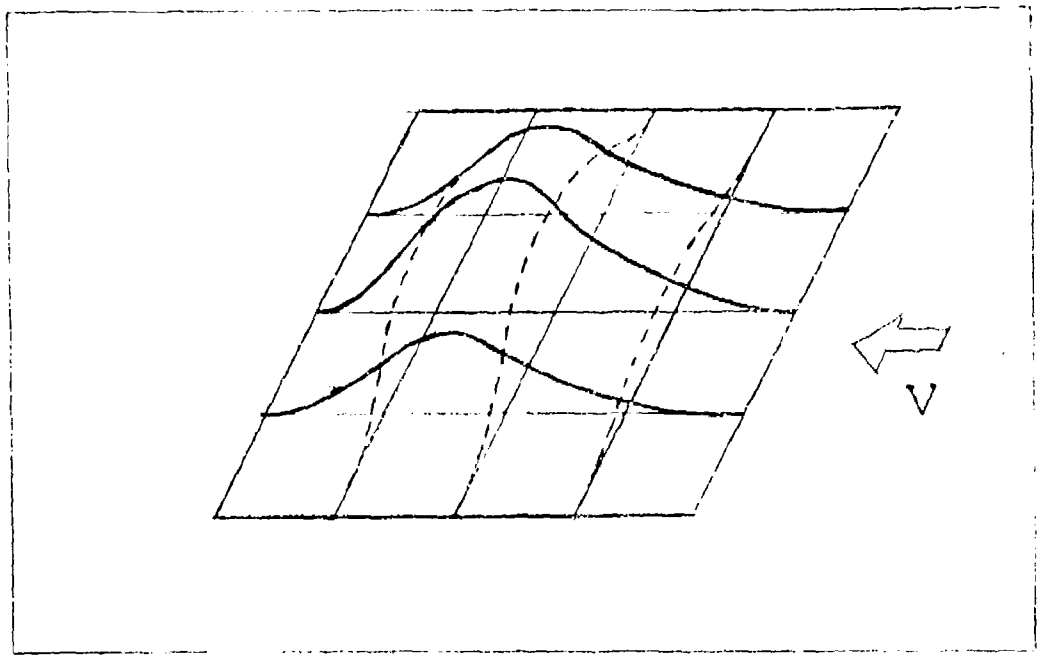

Figure 34 The First Mode Shape(Real Part) for Panel Corresponding to Point A in Fig. 33.

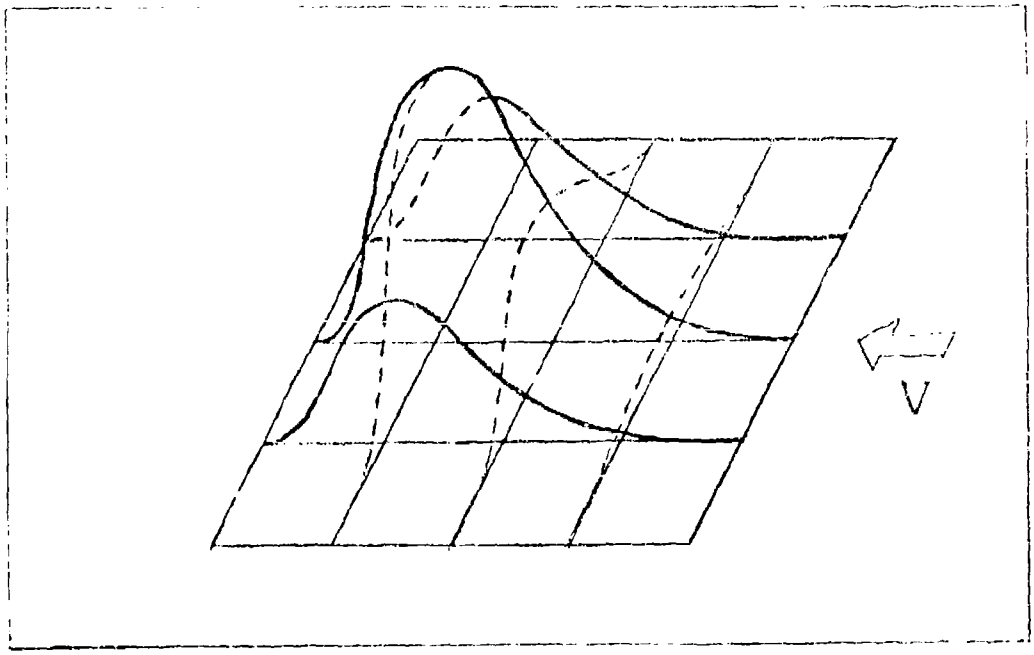

Figure 35 The First Mode Shape (Real Part) for Panel Corresponding to Point B in Fig. 33.

The third flutter mode shapes (real part) for the center chord lines corresponding to points C, D, E, and F are shown in Fig. 36.

Fig. 37 shows the first mode flutter boundaries for M = 1.3 and the third mode flutter boundaries for M = 1.3, 1.32, 1.35 for various values of tension. The flutter boundaries are dominated by the third mode. Since the flutter mode shapes are similar to those shown in the previous figures, they are not presented here.

Fig. 38 shows the flutter boundaries for M = 1.4, 1.45, 1.48 and various tension parameters. The third mode flutter boundaries shift to the right as the Mach number increases. It is interesting to see that the top portions of the third mode flutter boundaries begin to bend down to the left as the Mach number increases.

Fig. 39 shows that for M = 1.5 and various values of tension, the third mode flutter boundaries continue to bend down rapidly with a slight increase in Mach number.

Figs. 40 and 41 show that for M = 1.52 and 1.54 and various values of tension, the third mode flutter boundaries bend down substantially to be small loops. They only dominate the lower range of the mass ratio.

Fig 42 shows that for M = 1.6, all the third mode flutter boundaries disappear and the first mode flutter boundaries once again become the critical boundaries. Such a phenomenon is also the case for M = 2.0 as shown in Fig. 43.

Also shown in Figs. 37-43 are the dashed parabolas for an aluminum panel at 25,000 feet above sea level.

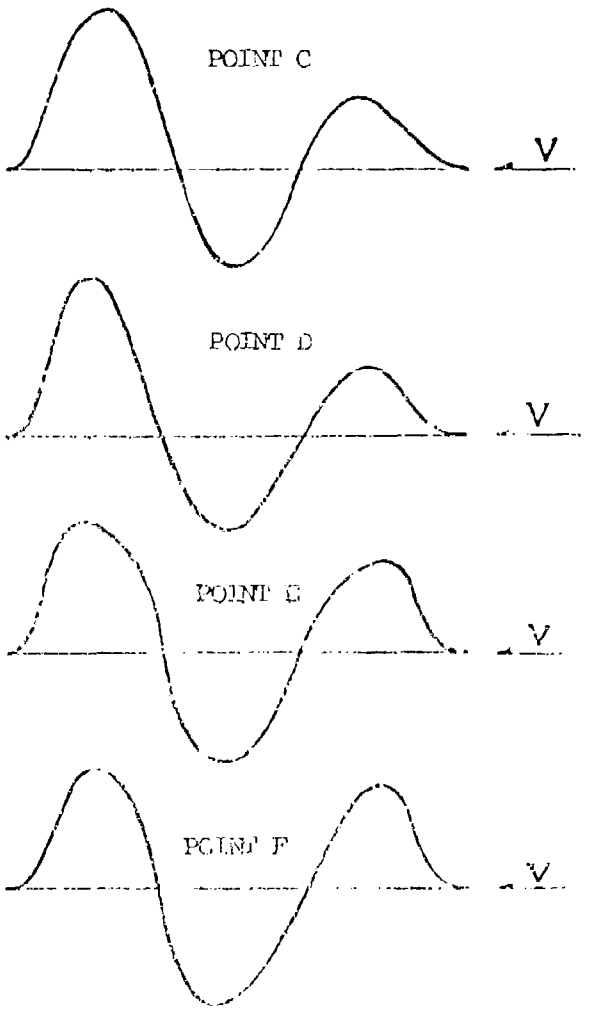

Figure 36 The Third Mode Shape (Real Part) for the Center Line of Panel with various tension parameter Corresponding to Point C, D, E, and F respectively in Fig. 35.

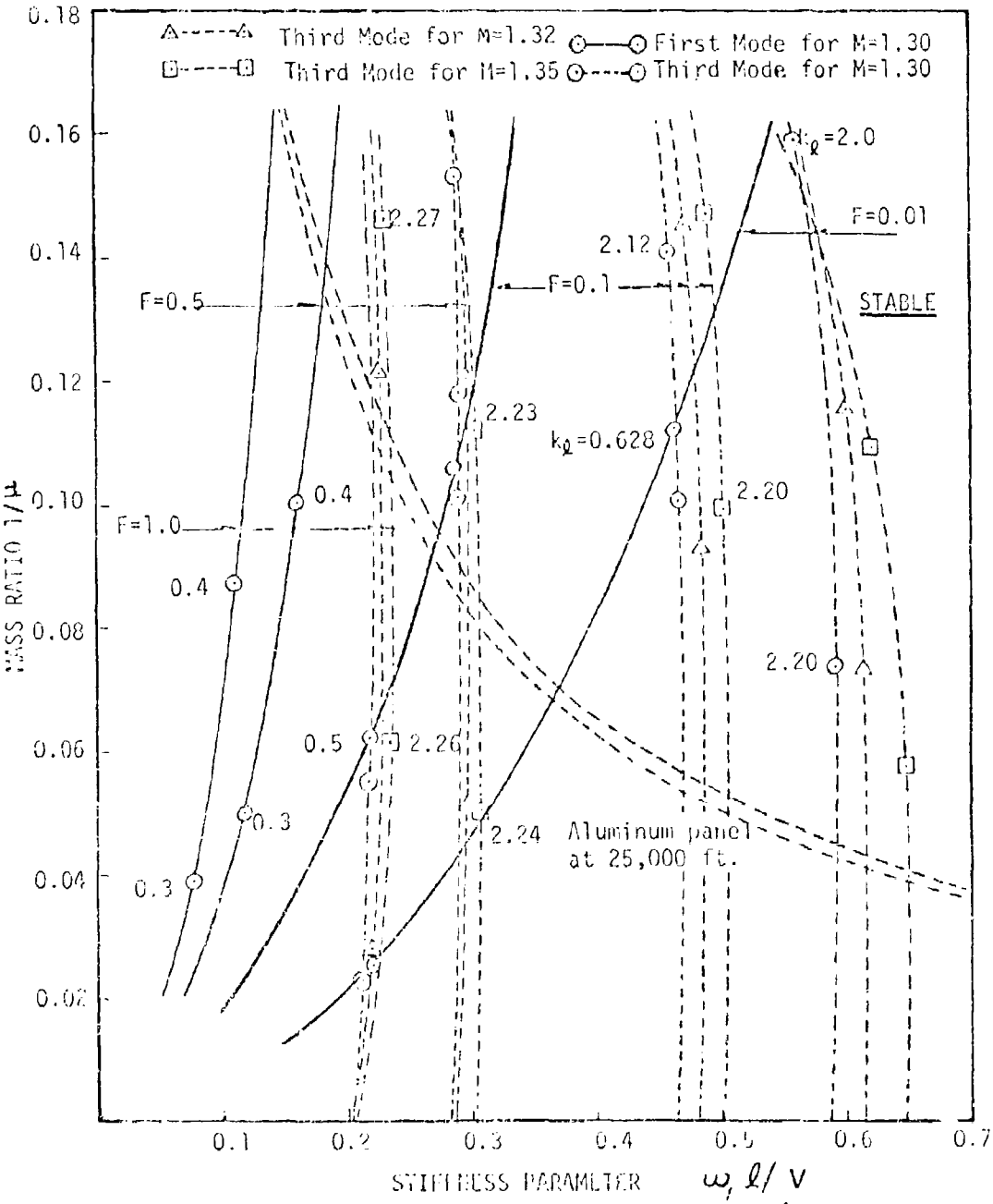

Figure 37 Flutter boundaries for clamped panels of $\ell/w=1.0$ with M 1.3, M=1.32, M=1.35 and various values of tension parameters F.

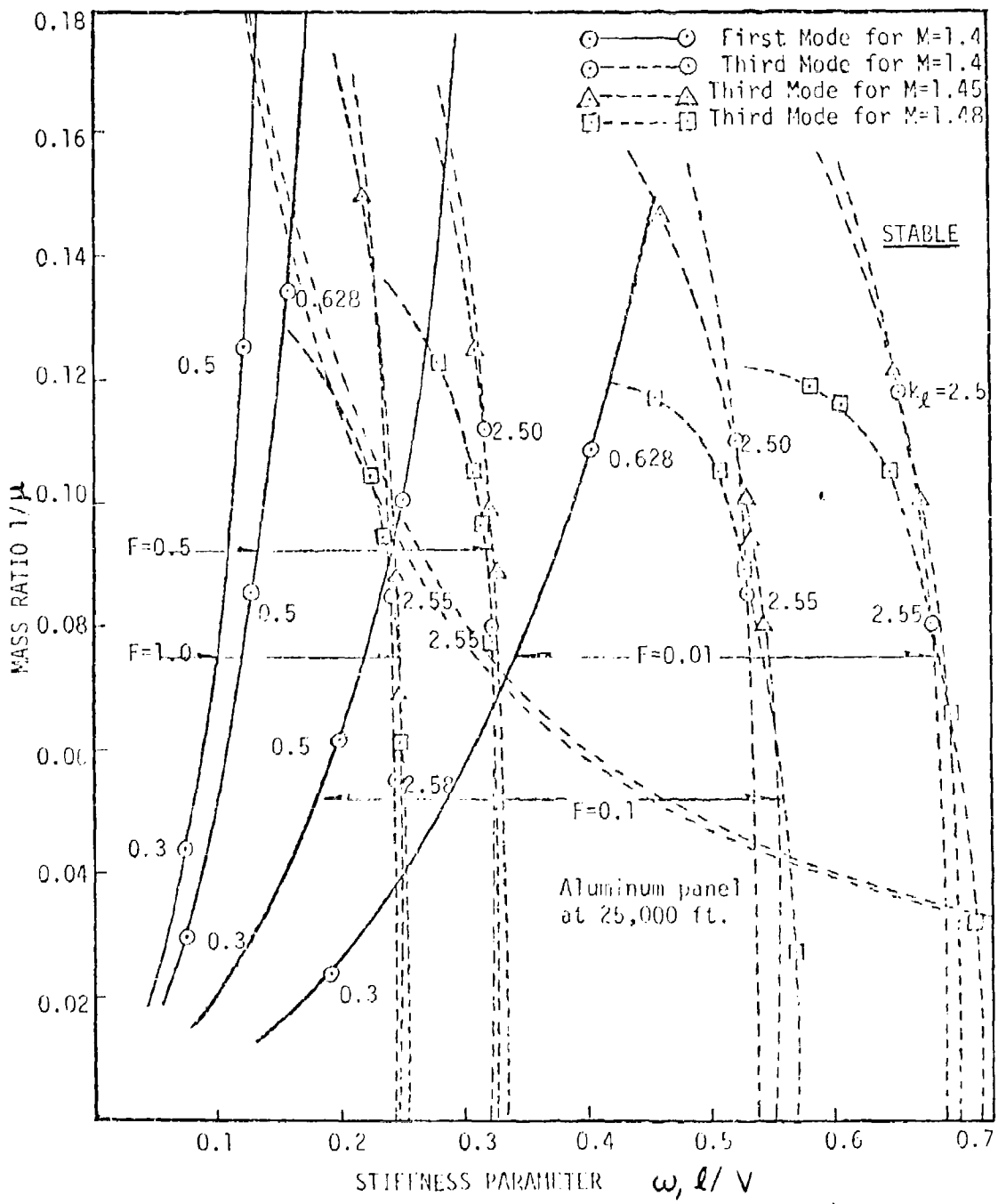

Figure 38 Flutter boundaries for clamped panels with ℓ/w=1.0 with M=1.4, M=1.45, M=1.48 and various values of tension parameters F.

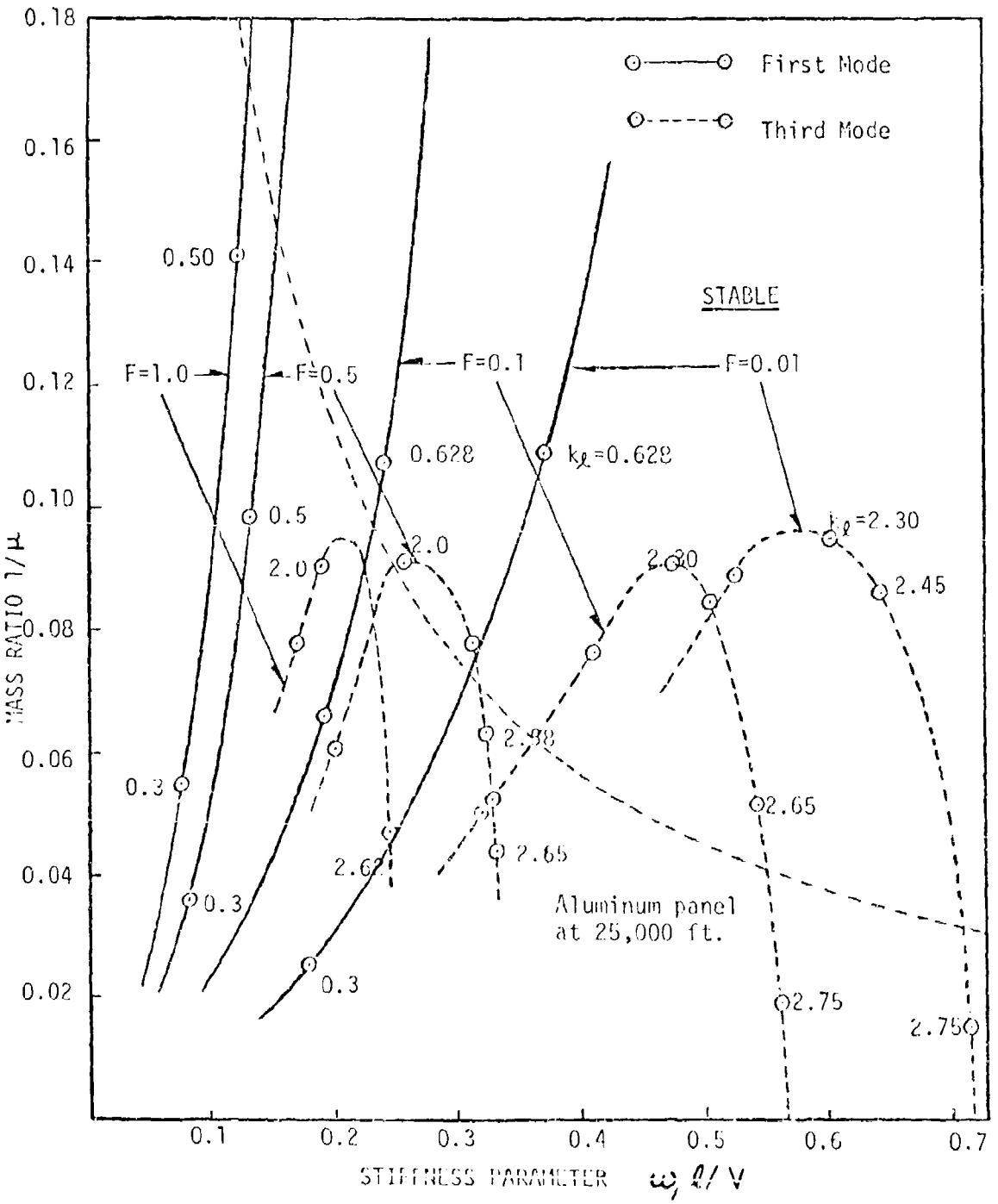

Figure 39 Flutter boundaries for clamped panels of ℓ/w 1.0 with M=1.5 and various values of tension parameters F.

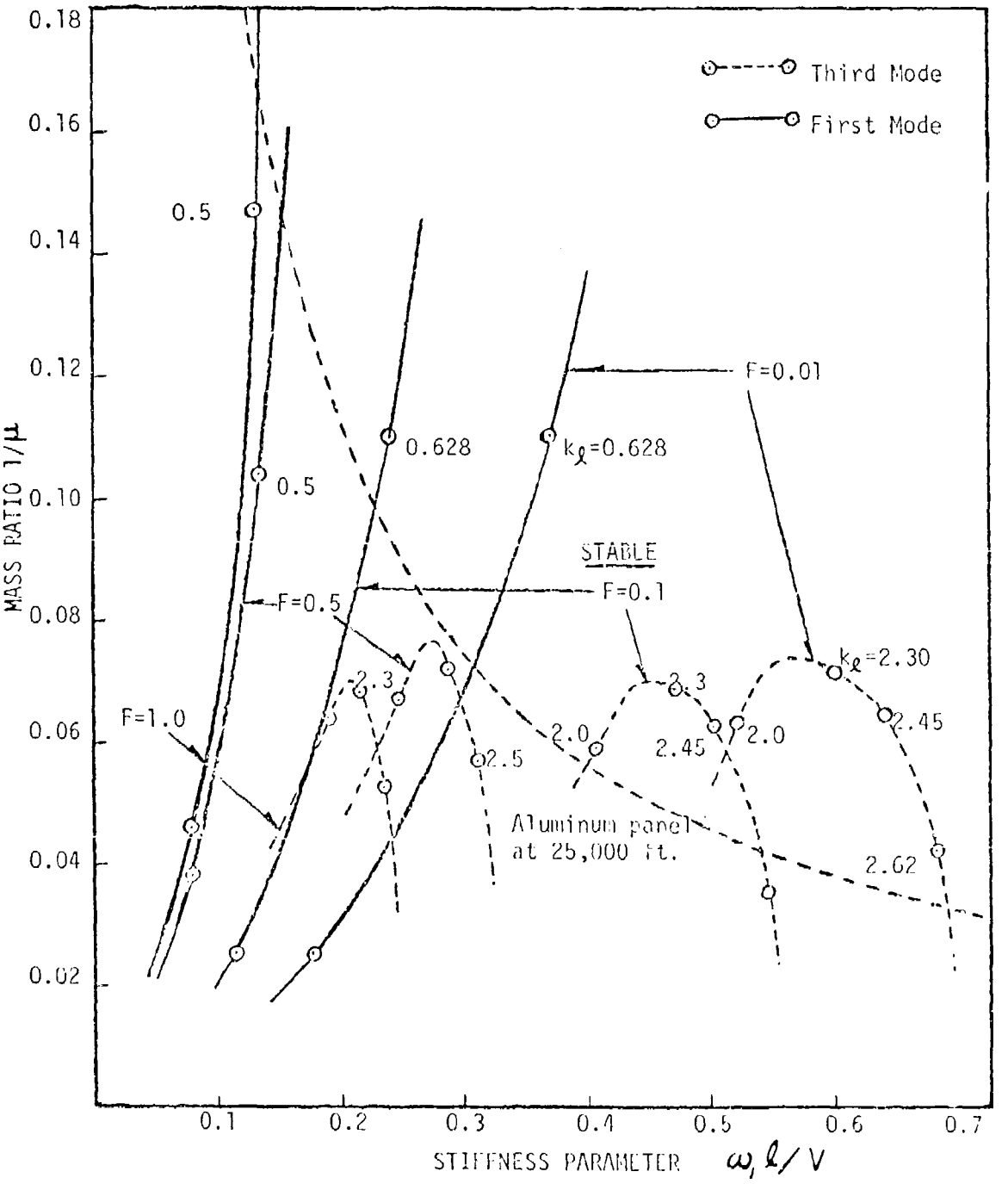

Figure 40. Flutter boundaries for clamped panels of ℓ/w=1.0 with M=1.52 and various values of tension parameters F.

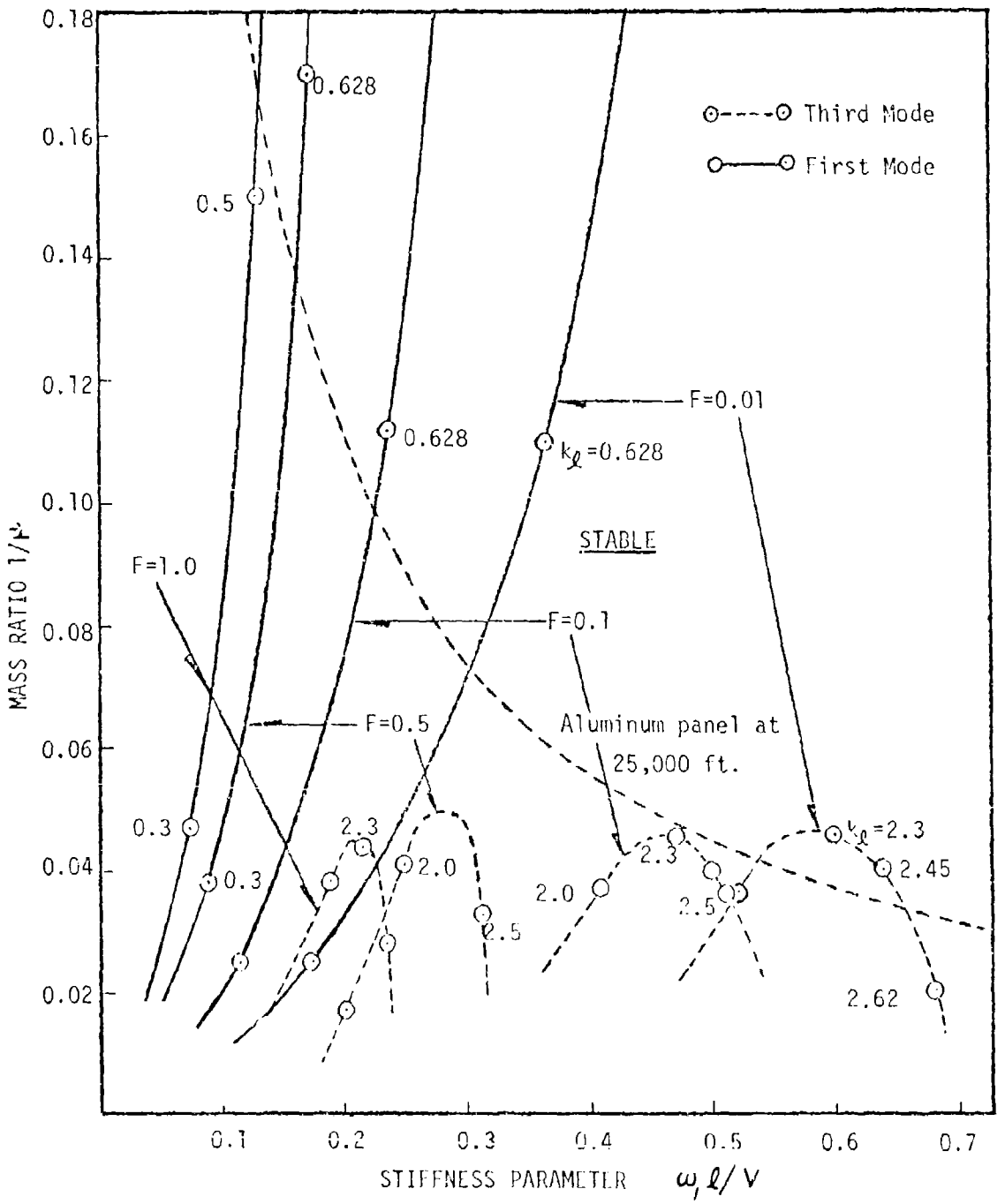

Figure 41 Flutter boundaries for clamped panels of ℓ/w=1.0 with M=1.54 and various values of tension parameters F.

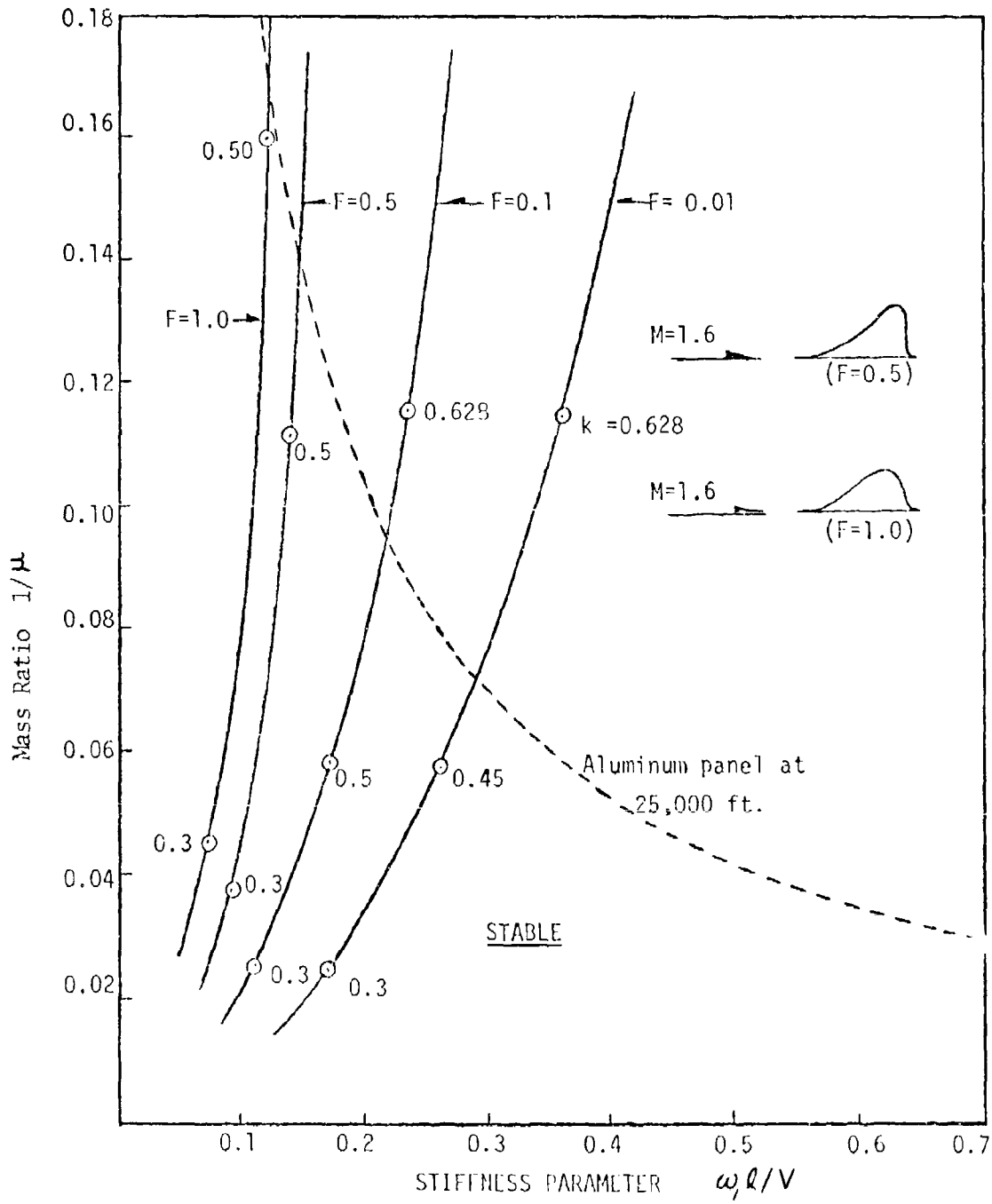

Figure 42 Flutter boundaries for clamped panels of ℓ/w=1.0 with M=1.6 and various values of tension parameters F.

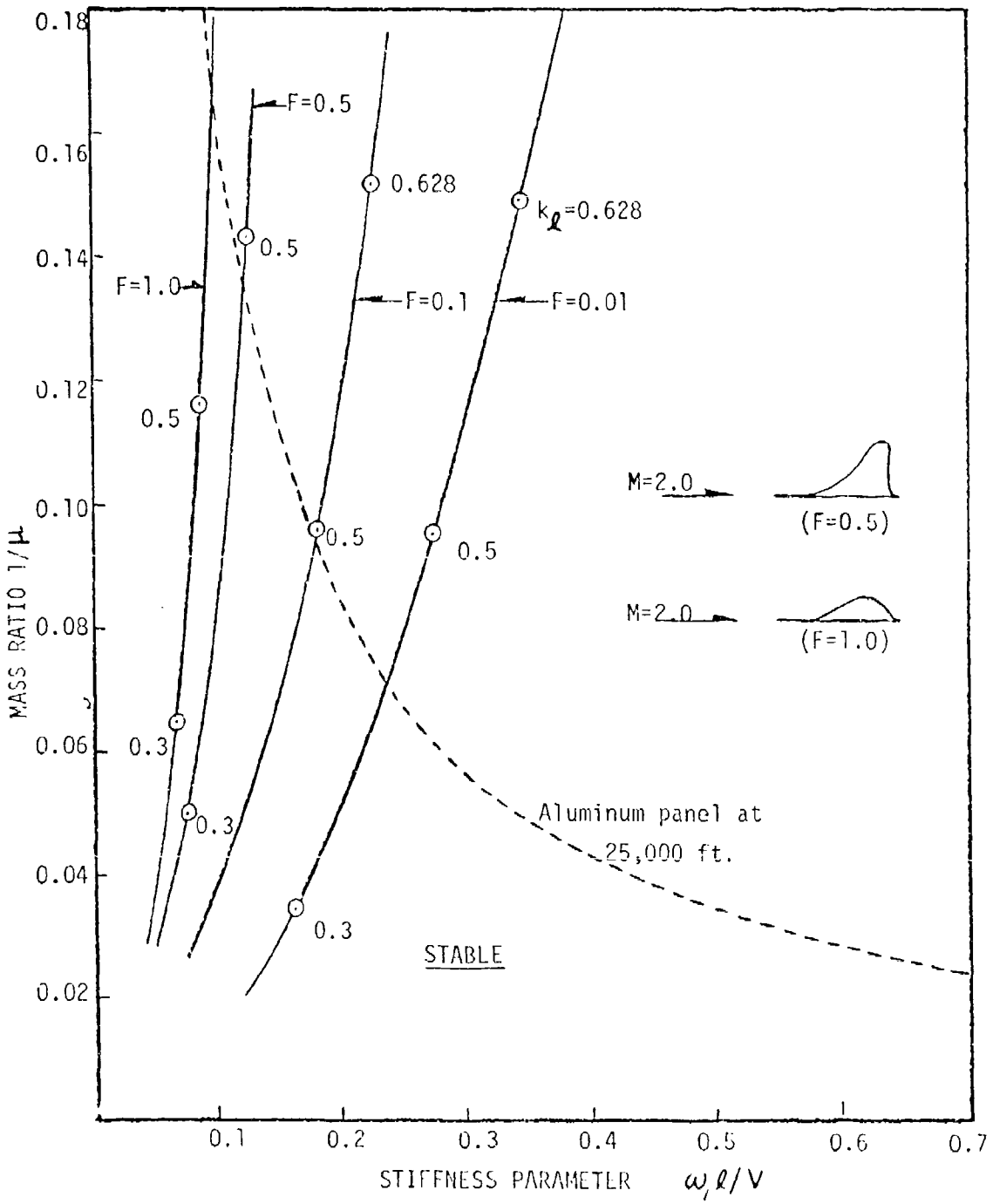

Figure 43 Flutter boundaries for clamped panel of ℓ/w=1.0 with M=2.0 and various values of tension parameters F.

By collecting all the intersecting points between the dashed parabolas and the critical flutter boundaries in Figs. 30, 33, and 37-43, the results for thickness ratios required to prevent flutter of the panel are shown in Fig. 44 for various values of Mach numbers and tension.

In Fig. 44, the critical flutter boundaries were dominated by the third mode in the Mach region between approximately 1.2 and 1.5. For other lower and higher Mach regions, the first mode flutter boundaries [illegible]ate. The sharp drops in the curves are due to the abrupt changes in [illegible]l flutter modes.

The beneficial effect of introducing in-plane tensions to reduce the required panel thickness to avoid flutter is clearly demonstrated. Fig. 44 should be of value to panel flutter analysts and designers.

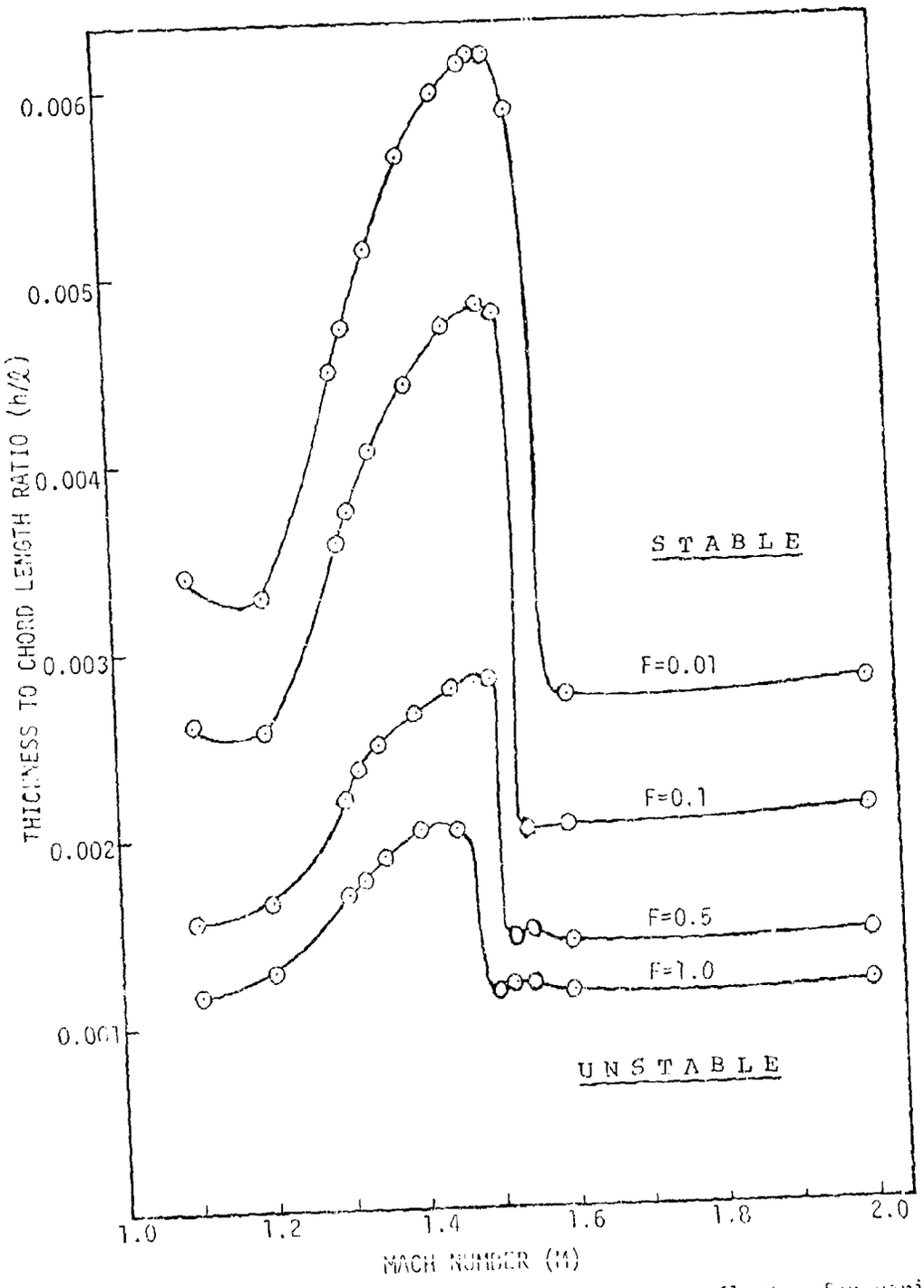

Figure 44 Thickness ratio required to prevent flutter for various values of tension parameter F for a square clamped aluminum panel at 25,000 ft. above sea level

SECTION V

CONCLUDING REMARKS

A basic finite element procedure for panel flutter analysis has been developed and performed with examples. The following concluding remarks may be made.

(1) The three-dimensional supersonic unsteady potential flow theory was employed. This theory allows the treatment of panels with finite aspect ratio. It is particularly advantageous at low supersonic range ($1 < M < \sqrt{2}$) for panels with chord-span ratio less than one, when the piston theory does not give satisfactory results.

(2) The finite element method offers a set of elegant and straightforward eigenvalue equations. It can be used directly to solve for flutter frequencies and mode shapes without requiring the natural vibration frequencies and mode shapes before the eigensolution, and also without requiring the computation of mode shapes after the eigensolution.

(3) If the modal method is used in panel flutter analysis, one could use the natural frequencies and modes found by the finite element method. Such procedure is, however, basically different from the present finite element meth[illegible]

(4) The formulations here (Eqs. 19-24) are general. They are readily applicable to any plate finite element displacement model whose shape functions are known.

(5) The present results agree well with Galerkin's modal solution by Cunningham.

(6) The critical flutter boundaries for a square clamped panel change modes abruptly as the Mach number is varied.

(7) Fig. 44 clearly demonstrates the beneficial effect of in-plane tensions.

(8) The present development may provide a basic finite element procedure for panel flutter analysis. The present results may provide useful data to the flutter analysts and designers.

(9) A logical next step appears to be the development of a method using triangular plate finite elements combined with triangular Mach boxes. Such development will be of great value to the flutter predictions of wing panels with arbitrary configurations. It is suggested that the triangular element developed by Bell (Reference 15) be used.

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APPENDIX

THE COMPUTER PROGRAM

The description and listing of the computer program are provided in this appendix. The flow chart is given on pages 69 and 70. The procedure for input is given on pages 62, 67, and 68. The use of the program is demonstrated by considering an example of a square clamped panel with $k_\ell = 0.5$, $M = 1.2$, and $F = 0.1$. Both input and output data are provided.

The stiffness, mass, and incremental stiffness matrices are based on the known explicit coefficients. These coefficients are read in as input data given on pages 63-66. The program uses the subroutine EISPACK for solving the general complex eigenvalue equations.

INPUT DATA

1. Control Card (2F5.2, 7I3, 4F7.4)

Columns	1-5	Panel chordwise length (XL)
	6-10	Panel spanwise width (YL)
	11-13	Number of elements in chordwise direction (NX)
	14-16	Half number of elements in spanwise direction (NY)
	17-19	Number of boxes in streamwise (or chordwise) direction for one element (IBX)
	20-22	Number of boxes in cross-streamwise (or spanwise) direction for one element (IBY)
	23-25	Total number of degrees of freedom for the panel (NXD)
	26-28	Number of nodes in streamwise direction (NDX)
	29-31	Number of nodes in cross-streamwise direction (NDY)
	32-38	Structural damping coefficient (SG)
	39-45	Mach number (MACH)
	46-52	Starting value for the mass ratio range (MUO)
	53-59	Increment for the values of mass ratios (XINCR)

2. Element matrix cards (8I4, 2I2, 2I4, 2I6)

Read in the data given on pages 63-66.

TABLE I DATA INFØRMATIØN FØR QQ(I,J), QM(I,J) and QN(I,J)

QQ(I,J)-ELEMENT STIFFNESS MATRIX
QM(I,J)-ELEMENT MASS MATRIX
QN(I,J)-ELEMENT INCREMENTAL STIFFNESS MATRIX

MATRIX LOCATION	CARD NUMBER	COLUMNS	QQ(I,J) DATA JS(1) 1-4	JS(2) 5-8	JS(3) 9-12	JS(4) 13-16	JS(5) 17-20	JS(6) 21-24	JS(7) 25-28	JS(8) 29-32	INDEX IZ,IV 33-34	35-36	QM(I,J) DATA JM(1) 37-40	JM(2) 41-44	QN(I,J) DATA JN(1) 45-50	JN(2) 51-56
(1, 1)	1		156	35	156	35	72	25	-0	1	-0	-0	169	1	78	175
(6, 1)	2		78	35	22	35	6	25	6	5	1	-0	143	6	13	350
(5, 2)	3		52	35	4	35	0	25	-0	1	2	-0	13	3	26	525
(3, 1)	4		22	35	78	35	6	25	6	5	-0	1	143	6	33	525
(3, 2)	5		11	35	11	35	1	50	6	5	1	1	121	33	11	2100
(3, 3)	6		4	35	52	35	0	25	-0	1	-0	2	13	3	2	175
(4, 1)	7		11	35	11	35	1	50	1	5	1	1	121	33	11	2100
(4, 2)	8		22	105	2	35	2	75	2	15	2	1	11	18	11	1575
(4, 3)	9		2	35	22	105	2	75	2	15	1	2	11	18	1	1050
(4, 4)	10		4	105	4	105	0	225	-0	1	2	2	1	9	2	1575
(5, 1)	11		54	35	-156	35	-72	25	-0	1	-0	-0	117	2	27	175
(5, 2)	12		27	35	-100	35	-6	25	-6	5	1	-0	33	4	9	700
(5, 3)	13		13	35	-78	35	-6	25	-0	1	-0	1	169	12	39	1050
(5, 4)	14		13	70	-11	35	-1	50	-1	10	1	1	143	72	13	4200
(5, 5)	15		156	35	156	35	72	25	-0	1	-0	-0	169	1	78	175
(6, 1)	16		27	35	-22	35	-6	25	-6	5	1	-0	33	4	9	700
(6, 2)	17		18	35	-4	35	-6	25	-0	1	2	-0	3	2	9	525
(6, 3)	18		13	70	-11	35	-1	50	-1	10	1	1	143	72	13	4200
(6, 4)	19		13	105	-2	35	-2	75	-0	1	2	1	13	36	13	3150
(6, 5)	20		78	35	22	35	6	25	6	5	1	-0	143	6	13	350
(6, 6)	21		52	35	4	35	0	25	-0	1	2	-0	13	3	26	525
(7, 1)	22		-13	35	78	35	6	25	-0	1	-0	1	-169	12	-13	350
(7, 2)	23		-13	70	11	35	1	50	1	10	1	1	-143	72	-13	4200
(7, 3)	24		-8	35	26	35	-2	25	-0	1	-0	2	-13	4	-3	350
(7, 4)	25		-3	70	11	105	-1	150	-1	30	1	2	-11	24	-1	1400
(7, 5)	26		-22	35	-78	35	-6	25	-6	5	-0	1	-143	6	-11	175
(7, 6)	27		-11	35	-11	35	-1	50	-6	5	1	1	-121	33	-11	2100
(7, 7)	28		4	35	52	35	0	25	-0	1	-0	2	13	3	2	175
(8, 1)	29		-13	70	11	35	1	50	1	10	1	1	-143	72	-13	4200
(8, 2)	30		-13	105	2	35	2	75	-0	1	2	1	-13	36	-13	3150
(8, 3)	31		-8	70	11	105	-1	150	-1	30	1	2	-11	24	-1	1400
(8, 4)	32		-1	35	2	105	-2	225	-0	1	2	2	-1	12	-1	1050
(9, 5)	33		-11	35	-11	35	-1	50	-1	5	1	1	-121	33	-11	2100
(8, 6)	34		-20	105	-2	35	-2	75	-2	15	2	1	-11	18	-11	1575
(9, 7)	35		2	35	22	105	2	75	2	15	1	2	11	18	1	1050

TABLE I DATA INFØRMATIØN FØR QQ(I,J), QM(I,J) and QN(I,J), continued ...

TABLE I DATA INFØRMATIØN FØR QQ(I,J), QM(I,J) and QN(I,J), continued ...

(13, 3)	81	-22	35	27	35	-6	25	-6	5	-0	1	33	4	-33	525
(13, 4)	82	-11	35	13	70	-1	50	-1	10	1	1	143	72	-11	2100
(13, 5)	83	-54	35	-54	35	72	25	-0	1	-0	-0	81	4	-27	175
(13, 6)	84	-27	35	-13	35	6	25	-0	1	1	-0	39	8	-9	700
(13, 7)	85	13	35	27	35	-6	25	-0	1	-0	1	-39	8	13	350
(13, 8)	86	13	70	13	70	-1	50	-0	1	1	1	-169	144	13	4200
(13, 9)	87	54	35	-135	35	-72	25	-0	1	-0	-0	117	2	27	175
(13,10)	88	-27	35	22	35	6	25	-6	5	1	-0	-33	4	-9	700
(13,11)	89	-13	35	78	35	6	25	-0	1	-0	1	-169	12	-13	350
(13,12)	90	13	70	-11	35	-1	50	-1	10	1	1	143	72	13	4200
(13,13)	91	156	35	156	35	72	25	-0	1	-0	-0	169	1	78	175
(14, 1)	92	78	35	-13	35	6	25	-0	1	1	-0	-169	12	13	350
(14, 2)	93	26	35	-3	35	-2	25	-0	1	2	-0	-13	4	-13	1050
(14, 3)	94	11	35	-13	70	1	50	1	10	1	1	-143	72	11	2100
(14, 4)	95	11	105	-3	70	-1	150	-1	30	2	1	-11	24	-11	6300
(14, 5)	96	27	35	15	35	-6	25	-0	1	1	-0	-39	8	9	700
(14, 6)	97	9	35	3	35	2	25	-0	1	2	-0	-9	8	-9	2100
(14, 7)	98	-13	70	-13	70	1	50	-0	1	1	1	169	144	-13	4200
(14, 8)	99	-13	210	-3	70	-1	150	-0	1	2	1	13	48	13	12600
(14, 9)	100	-27	35	22	35	6	25	6	5	1	-0	-33	4	-9	700
(14,10)	101	18	35	-4	35	-8	25	-0	1	2	-0	3	2	9	525
(14,11)	102	13	70	-11	35	-1	50	-1	10	1	1	143	72	13	4200
(14,12)	103	-13	105	2	35	2	75	-0	1	2	1	-13	36	-13	3150
(14,13)	104	-78	35	-22	35	-6	25	-6	5	1	-0	-143	6	-13	350
(14,14)	105	52	35	4	35	8	25	-0	1	2	-0	13	3	26	525
(15, 1)	106	-22	35	27	35	-6	25	-6	5	-0	1	33	4	-22	350
(15, 2)	107	-11	35	13	70	-1	50	-1	10	1	1	143	72	-11	2100
(15, 3)	108	-4	35	18	35	-8	25	-0	1	-0	2	3	2	-2	175
(15, 4)	109	-2	35	13	105	-2	75	-0	1	1	2	12	36	-1	1050
(15, 5)	110	-13	35	-27	35	6	25	-0	1	-0	1	39	8	-13	350
(15, 6)	111	-13	70	-13	70	1	50	-0	1	1	1	169	144	-13	4200
(15, 7)	112	3	35	9	35	2	25	-0	1	-0	2	-9	8	3	350
(15, 8)	113	3	70	13	210	1	150	-0	1	1	2	-13	48	1	1400
(15, 9)	114	13	35	-78	35	-6	25	-0	1	-0	1	169	12	13	350
(15,10)	115	-13	70	11	35	1	50	1	10	1	1	-143	72	-13	4200
(15,11)	116	-3	35	26	35	-2	25	-0	1	-0	2	-13	4	-3	350
(15,12)	117	3	70	-11	105	1	150	1	30	1	2	11	24	1	1400
(15,13)	118	22	35	78	35	6	25	6	5	-0	1	143	6	22	350
(15,14)	119	-11	35	-11	35	-1	50	-6	5	1	1	-121	36	-11	2100
(15,15)	120	4	35	52	35	8	25	-0	1	-0	1	13	3	2	175
(16, 1)	121	11	35	-13	70	1	50	1	10	1	1	-143	72	11	2100
(16, 2)	122	11	105	-3	70	-1	150	-1	30	2	1	-11	24	-11	6300
(16, 3)	123	2	35	-13	105	2	75	-0	1	1	2	-13	36	1	1050
(16, 4)	124	2	105	-1	35	-2	225	-0	1	2	2	-1	12	-1	3150
(16, 5)	125	13	70	13	70	-1	50	-0	1	1	1	-169	144	13	4200

TABLE I DATA INFØRMATIØN FØR QQ(I,J), QM(I,J) and QH(I,J), continued ...

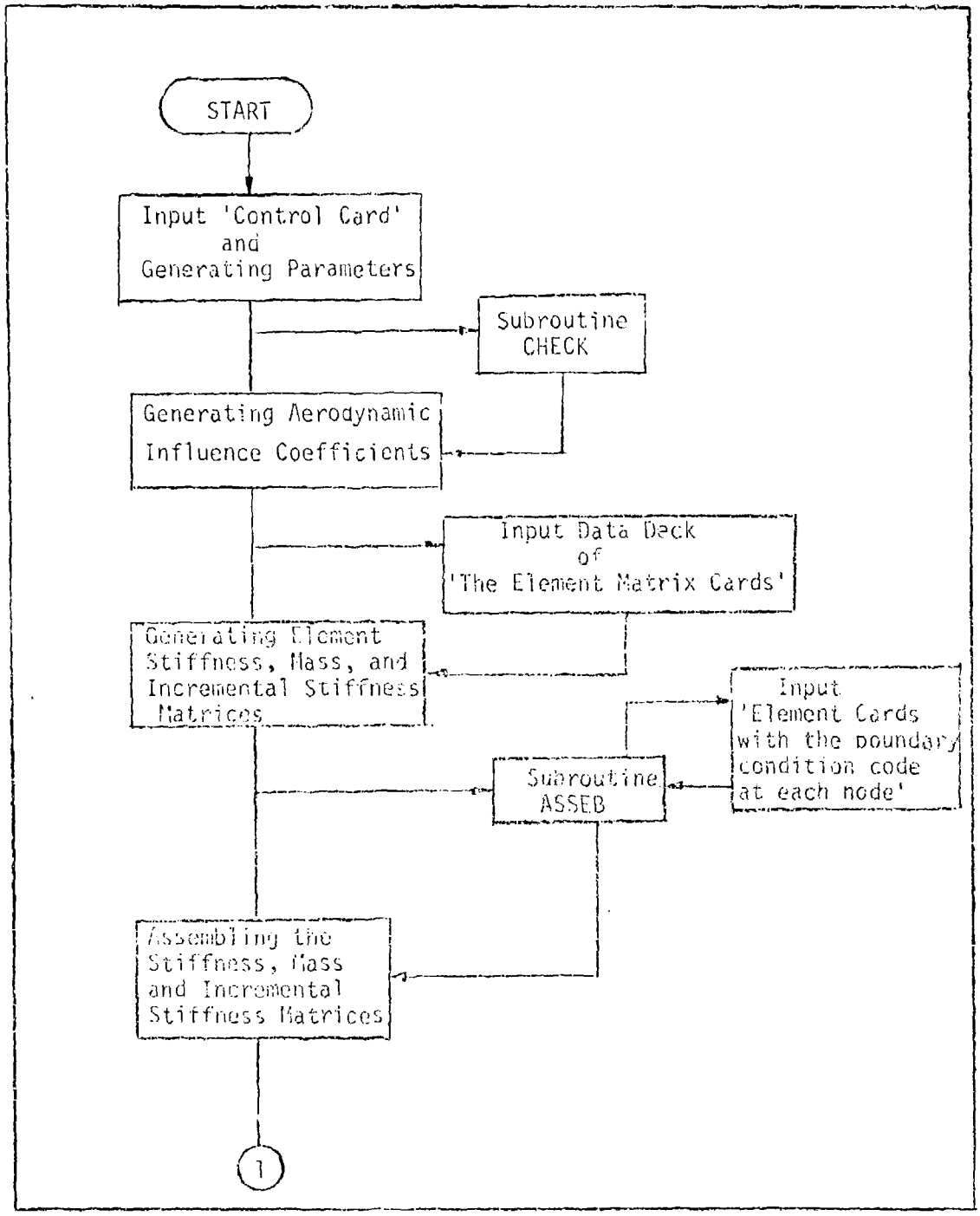
3. Element Cards with the boundary condition code at each node (2213).

Columns	1-3	The sequence number for the element	
	4-6	The starting matrix row position for the degrees of freedom at nodal point I	(ID(1))
	7-9	The starting matrix row position for the degrees of freedom at nodal point J	(ID(2))
	10-12	The starting matrix row position for the degrees of freedom at nodal point K	(ID(3))
	13-15	The starting matrix row position for the degrees of freedom at nodal point L	(ID(4))
	16-18	Boundary condition code for deflection Z at nodal point I	(w(1))
	19-21	Boundary condition code for $Z_{,x}$ at nodal point I	(w(2))
	22-24	Boundary condition code for $Z_{,y}$ at nodal point I	(w(3))
	25-27	Boundary condition code for $Z_{,xy}$ at nodal point I	(w(4))
	28-30	Boundary condition code for Z at nodal point J	(w(5))
	31-33	Boundary condition code for $Z_{,x}$ at nodal point J	(w(6))
	34-36	Boundary condition code for $Z_{,y}$ at nodal point J	(w(7))
	37-39	Boundary condition code for $Z_{,xy}$ at nodal point J	(w(8))
	40-42	Boundary condition code for Z at nodal point K	(w(9))
	43-45	Boundary condition code for $Z_{,x}$ at nodal point K	(w(10))
	46-48	Boundary condition code for $Z_{,y}$ at nodal point K	(w(11))
	49-51	Boundary condition code for $Z_{,xy}$ at nodal point K	(w(12))

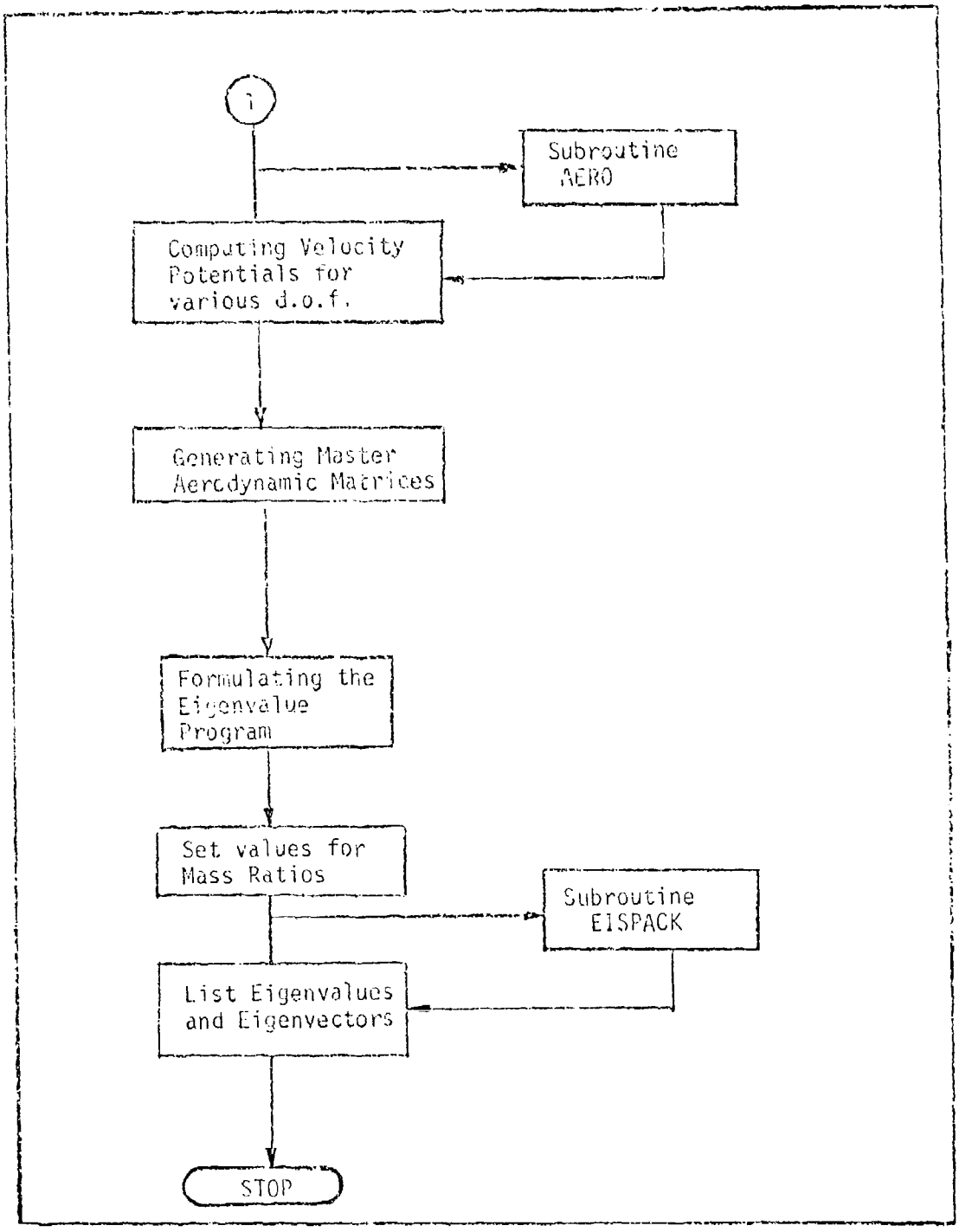
52-54	Boundary condition code for Z at nodal point L	(w(13))
55-57	Boundary condition code for $Z_{,x}$ at nodal point L	(w(14))
58-60	Boundary condition code for $Z_{,y}$ at nodal point L	(w(15))
61-63	Boundary condition code for $Z_{,xy}$ at nodal point L	(w(16))

The 'ID' array at I, J, K and L is assigned for each element for a specific mesh of the elements of the panel. The permutation for I, J, K and L is clockwise and I is corresponding to the origin of each local element as shown in Fig. 2. If ID(I) = 0, all the degrees of freedom at node I are zero. Same situation is applied for other nodes for the element.

The 'w' array defines the boundary condition codes for the degrees of freedom at each node. If w(I) = 0, the associated degree of freedom is zero and if $w(I) \neq 0$, the associated degree of freedom is taken into account.



Program Flow Chart



Program Flow Chart continued .....

PROGRAM LISTING AND SAMPLE PROBLEM

```
      PROGRAM FLUTTER (INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)                A   10
      COMPLEX CS(10,18),W(18),RT,CF                                          A   20
      REAL DV(16,16),DM(16,16),GR(2),GRR(16,8),GRI(16,8),IM(18,18),CK(18     A   30
     1,18),DN(16,16),SN(18,18),MACH,MU,KL,MU0,V(40,16),VS(40,16),AK(18,1     A   40
     28),W1(10,18)                                                           A   50
      INTEGER P,S,PMAX,P1,I1,IMAX,JS(8),JM(2),JG(16),JB(2)                   A   60
      COMMON /BLOCK0/ BM,UB,C,CB                                             A   70
      COMMON /BLOCK1/ IBX,IBY,GRR,GRI,IBY,KBY                                A   80
      COMMON /BLOCK2/ DD,DM,DN                                               A   90
      COMMON /BLOCK3/ CK,SM,SN                                               A  100
C                                                                            A  110
C     PANEL FLUTTER FOR CLAMPED PANEL                                        A  120
C                                                                            A  130
C                                                                            A  150
C     THE D.O.F.#S FOR EACH NODAL POINT ON THE ELEMENT                       A  160
C                                                                            A  170
      IX1(NC)=5*NC                                                           A  180
      IX2(NC)=12*NC                                                          A  190
      IX3(NC)=NC                                                             A  200
      IX4(NC)=4*NC                                                           A  210
      PRINT 133                                                              A  140
      READ 134, XL,YL,NX,NY,IBX,IBY,NXD,NDX,NDY,SG,MACH,MU0,XINCR            A  220
C                                                                            A  230
C     PANEL PARAMETERS                                                       A  240
C                                                                            A  250
      POISN=0.32                                                             A  260
      AD=XL/FLOAT(NX)                                                        A  270
      BD=YL/2.0/FLOAT(NY)                                                    A  280
      AB=AD/BD                                                               A  290
      BA=BD/AD                                                               A  300
      XLW=XL/YL                                                              A  310
      PRINT 135, XLW,AD,BD                                                   A  320
C                                                                            A  330
C     THE FIRST FREQUENCY PARAMETER FOR CLAMPED PANEL                        A  340
C                                                                            A  350
      CK0=4.73                                                               A  360
      CK2=0.9825022                                                          A  370
      CK3=2.464754                                                           A  380
      CK1=(CK0**4+((XLW*CK0)**4)+2.*((XLW*CK0*CK2*CK3)**2)                   A  390
C                                                                            A  400
C     BOX PARAMETERS                                                         A  410
C                                                                            A  420
      IBT=NX*NY*IBX*IBY                                                      A  430
      PBX=1./FLOAT(IBX)                                                      A  440
      PBY=1./FLOAT(IBY)                                                      A  450
      EPISN=BD/FLOAT(IBY)                                                    A  460
      SIGMA=P**2/EPISN                                                       A  470
      EL=EPISN/XL                                                            A  480
      PRINT 136, EPISN,SIGMA                                                 A  490
C                                                                            A  500
C     REFERENCE REDUCED FREQUENCY PARAMETER                                  A  510
C                                                                            A  520
      KL=0.50                                                                A  530
      PRINT 137, KL                                                          A  540
C                                                                            A  550
C     INITIAL IN-PLANE FORCE PARAMETER                                       A  560
C                                                                            A  570
```

```
      F=0.1                                                              A  590
      PRINT 138, F                                                       A  600
C                                                                        A  610
CC    FLOW PARAMETERS                                                    A  620
C                                                                        A  630
      PRINT 139, MACH                                                    A  640
      BETA=SQRT(MACH**2-1.)                                              A  650
      OMEGA=((MACH**2)*KL*EL)/(BETA**2)                                  A  660
      BM=BETA*OMEGA/(2.*MACH)                                            A  670
      UB=BM*MACH                                                         A  680
      C=1./(2.*3.14159)                                                  A  690
      CB=BO                                                              A  700
C                                                                        A  710
CC    GENERATING AERODYNAMIC INFLUENCE COEFFICIENTS                      A  720
C                                                                        A  730
      RMAX=NX*IBX                                                        A  740
      SMAX=2*NY*IBY                                                      A  750
      DO 101 I=1,RMAX                                                    A  760
      DO 101 J=1,SMAX                                                    A  770
         GRR(I,J)=0.                                                     A  780
  101 GRI(I,J)=0.                                                        A  790
      DO 102 IS=1,RMAX                                                   A  800
         R=IS-1                                                          A  810
         DS=SIGMA/BETA*(EPSN/2.+EPSN*FLOAT(R))                           A  820
         JF=IFIX((DS-(EPSN/2.))/EPSN)+2                                  A  830
         IF (JF.GT.SMAX) JF=SMAX                                         A  840
         IF (DS.LT.(EPSN/2.)) JF=1                                       A  850
         JG(IS)=JF                                                       A  860
      DO 102 J=1,JF                                                      A  870
         S=J-1                                                           A  880
         CALL CHECK (R,S,BETA,SIGMA,GR)                                  A  890
         R1=R+1                                                          A  900
         S1=S+1                                                          A  910
         GRR(R1,S1)=GR(1)                                                A  920
         GRI(R1,S1)=GR(2)                                                A  930
  102 CONTINUE                                                           A  940
C                                                                        A  950
CC    GENERATING ELEMENT STIFFNESS MATRIX, ELEMENT INCREMENTAL           A  960
C     STIFFNESS MATRIX, AND ELEMENT MASS MATRIX                          A  970
C                                                                        A  980
      DO 103 I=1,16                                                      A  990
      DO 103 J=1,I                                                       A 1000
         READ 140, (JS(L),L=1,8),IZ,IY,(JM(L),L=1,2),(JB(L),L=1,2)       A 1010
         QQ(I,J)=(BA**IY)*(FLOAT(JS(1))/FLOAT(JS(2))+(AB**4)*FLOAT(JS(3)  A 1020
     1   )/FLOAT(JS(4))+(AB**2)*FLOAT(JS(5))/FLOAT(JS(6))+(AB**2)*POISN*  A 1030
     2   FLOAT(JS(7))/FLOAT(JS(8)))                                      A 1040
         QM(I,J)=(BA**IY)*(FLOAT(JM(1))/FLOAT(JM(2)))                    A 1050
         QN(I,J)=(BA**IY)*(FLOAT(JB(1))/FLOAT(JB(2)))*(BA)               A 1060
  103 CONTINUE                                                           A 1070
      DO 104 I=1,16                                                      A 1080
      DO 104 J=I,16                                                      A 1090
         QQ(I,J)=QQ(J,I)                                                 A 1100
         QN(I,J)=QN(J,I)                                                 A 1110
  104 QM(I,J)=QM(J,I)                                                    A 1120
C                                                                        A 1130
CC    ASSEMBLE ELEMENT STIFFNESS MATRIX, ELEMENT INCREMENTAL STIFFNESS   A 1140
C     MATRIX, AND ELEMENT MASS MATRIX                                    A 1150
C                                                                        A 1160
```

```
      CALL ASSEB (NX,NY,NXD)                                                A 1160
C                                                                           A 1170
C     GENERATING VELOCITY POTENTIAL FUNCTIONS                               A 1180
C                                                                           A 1190
      KL=KL/FLOAT(NX)                                                       A 1200
      CALL AERO (NX,NY,NXD,IBT,KL,V,VS,EPISN,SIGMA,BETA,AO,BA,JG,NDX,NDY    A 1210
     1)                                                                     A 1220
C                                                                           A 1230
C     GENERATING MASTER AERODYNAMIC MATRIX                                  A 1240
C                                                                           A 1250
      DO 123 IIX=1,NDX                                                      A 1260
         DO 122 IIY=1,NDY                                                   A 1270
            ND14=4                                                          A 1280
            IF (IIY.EQ.NDY) ND14=2                                          A 1290
            DO 121 ND1=1,ND14                                               A 1300
               II=(IIX-1)*4*NDX+(IIX-1)*4+ND1                               A 1310
               IF (IIY.EQ.NDY) II=(IIY-1)*4*NDX+(IIX-1)*2+ND1               A 1320
               DO 120 JJX=1,NDX                                             A 1330
               DO 120 JJY=1,NDY                                             A 1340
                  ND24=4                                                    A 1350
                  IF (JJY.EQ.NDY) ND24=2                                    A 1360
               DO 120 ND2=1,ND24                                            A 1370
                  JJ=(JJY-1)*4*NDX+(JJX-1)*4+ND2                            A 1380
                  IF (JJY.EQ.NDY) JJ=(JJY-1)*4*NDX+(JJX-1)*2+ND2            A 1390
                  Q=0.                                                      A 1400
                  QC=0.                                                     A 1410
                  DO 118 NL1=1,NX                                           A 1420
                     N1=NL1-IIX                                             A 1430
                     IF (N1.LT.0) GO TO 118                                 A 1440
                     IF (N1.GT.1) GO TO 119                                 A 1450
                     DO 117 NL2=1,NY                                        A 1460
                        N2=NL2-IIY                                          A 1470
                        IF (N2.LT.0) GO TO 117                              A 1480
                        IF (N2.GT.1) GO TO 119                              A 1490
                        IF (N1.EQ.0) GO TO 105                              A 1500
                        IF (N2.EQ.0) GO TO 106                              A 1510
                        IKL=IX3(ND1)                                        A 1520
                        GO TO 108                                           A 1530
  105                   IF (N2.EQ.0) GO TO 107                              A 1540
                        IKL=IX2(ND1)                                        A 1550
                        GO TO 108                                           A 1560
  106                   IKL=IX4(ND1)                                        A 1570
                        GO TO 108                                           A 1580
  107                   IKL=IX1(ND1)                                        A 1590
  108                   DO 116 M=1,IBX                                      A 1600
                           DO 115 MC=1,IBY                                  A 1610
                              ML=(NL2-1)*IBY+MC                             A 1620
                              MK=(ML-1)*IBX+M                               A 1630
                              XM=FLOAT(M-1)*PBX+PBX/2.                      A 1640
                              YM=FLOAT(MC-1)*PBY+PBY/2.                     A 1650
                              CALL SHAPE (F,IFA,IFL,XM,YM,BA)               A 1660
                              IF (JJX-NL1) 109,102,110                      A 1670
  109                         JJ=(JJY-1)*4*(NDX+1)+(JJX-1)*4+ND2+(NX-NL1    A 1680
     1                        )*4                                           A 1690
                              IF (JJY.EQ.NDY) JJ=(JJY-1)*4*(NDX+1)+(JJX-    A 1700
     1                        1)*2+(NX-NL1)*2+ND2                           A 1710
                              GO TO 111                                     A 1720
  110                         VK=0.                                         A 1730
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                              VKS=0.                                                A 1740
                              GO TO 114                                             A 1750
  111                         IF (JJY.EQ.NDY) GO TO 113                             A 1760
                              JJ7=JJ+2*(NDX+1)*((2*NDY-2)-JJY)*4*(NDX+1)            A 1770
                              IF (ND2.EQ.3.OR.ND2.EQ.4) GO TO 112                   A 1780
                              VK=V(JJ,MM)+V(JJ7,MM)                                 A 1790
                              VKS=VS(JJ,MM)+VS(JJ7,MM)                              A 1800
                              GO TO 114                                             A 1810
  112                         VK=V(JJ,MM)-V(JJ7,MM)                                 A 1820
                              VKS=VS(JJ,MM)-VS(JJ7,MM)                              A 1830
                              GO TO 114                                             A 1840
  113                         VK=V(JJ,MM)                                           A 1850
                              VKS=VS(JJ,MM)                                         A 1860
  114                         O=O+(-FA*VK+DFA/KL*VKS)*(SIGMA*(EPISN**2))            A 1870
     1                        /(AO*BO)                                              A 1880
                              OS=OS+(-FA*VKS-DFA/KL*VK)*(SIGMA*(EPISN**2            A 1890
     1                        ))/(AO*BO)                                            A 1900
  115                       CONTINUE                                                A 1910
  116                     CONTINUE                                                  A 1920
  117                   CONTINUE                                                    A 1930
  118                 CONTINUE                                                      A 1940
  119                 AK(II,JO)=O                                                   A 1950
                      AKI(II,JO)=OS                                                 A 1960
  120               CONTINUE                                                        A 1970
  121             CONTINUE                                                          A 1980
  122           CONTINUE                                                          A 1990
  123 CONTINUE                                                                    A 2000
C                                                                                 A 2010
C     FINDING EIGENVALUES AND EIGENVECTORS FOR VARIOUS RATIOS OF AIR              A 2020
C     MASS TO PANEL MASS                                                          A 2030
C                                                                                 A 2040
      YN=NY*2                                                                     A 2050
      XN=NX                                                                       A 2060
      DO 124 I=1,NXD                                                              A 2070
      DO 124 J=1,NXD                                                              A 2080
  124 SK(I,J)=0.01*((XN**5)/CK1*SK(I,J)+F*(XN**2)*(YN)*SM(I,J))                   A 2090
      CALL GIVV2 (SK,NXD,NXD,NXD,KZ)                                              A 2100
      IF (KZ.EQ.0) GO TO 131                                                      A 2110
      PRINT 141, SG                                                               A 2120
      KL=EL*SH                                                                    A 2130
      C10=SH*MU0/1225.                                                            A 2140
      FACTOR=1./MU0                                                               A 2150
      FACTOR=0.01*FACTOR                                                          A 2160
      DO 125 I=1,NXD                                                              A 2170
      DO 125 J=1,NXD                                                              A 2180
         SM(I,J)=C10*SM(I,J)                                                      A 2190
  125 CONTINUE                                                                    A 2200
      DO 130 ITER=1,9                                                             A 2210
         INT=1                                                                    A 2220
         MU=MU0+XINCR                                                             A 2230
         PRINT 144, MU                                                            A 2240
         RATIO=MU/MU0                                                             A 2250
         FACTOR=FACTOR*RATIO                                                      A 2260
         DO 126 I=1,NXD                                                           A 2270
         DO 126 J=1,NXD                                                           A 2280
            SM(I,J)=1./RATIO*SM(I,J)                                              A 2290
  126    CONTINUE                                                                 A 2300
         DO 128 I=1,NXD                                                           A 2310
```

```
      DO 128 J=1,NXD                                                    A 2320
         AC=0.                                                          A 2330
         BC=0.                                                          A 2340
         DO 127 KI=1,NXD                                                A 2350
            AC1=SK(I,KI)*(EM(KI,J)-AK(KI,J))                            A 2360
            BC1=SK(I,KI)*(-AK1(KI,J))                                   A 2370
            AC=AC+(AC1+(SG*BC1))                                        A 2380
            BC=BC+(BC1-(SG*AC1))                                        A 2390
  127    CONTINUE                                                       A 2400
         CS(I,J)=CMPLX(AC,BC)                                           A 2410
  128 CONTINUE                                                          A 2420
      CALL CGEIG (NXD,NXD,CS,IND,WS)                                    A 2430
      DO 129 I=1,5                                                      A 2440
         CF=CMPLX(FACTOR,0.)                                            A 2450
         WC(I)=WC(I)*CF                                                 A 2460
         AC=REAL(WC(I))                                                 A 2470
         BC=AIMAG(WC(I))                                                A 2480
         G=BC/AC                                                        A 2490
         RT=CABS(WC(I))                                                 A 2500
         SR=REAL(RT)                                                    A 2510
         TK=SR*KL                                                       A 2520
         PRINT 142, WS(I),G,SR,TK                                       A 2530
  129 PRINT 143, (CS(J,I),J=1,NXD)                                      A 2540
      MUO=MU                                                            A 2550
  130 CONTINUE                                                          A 2560
      GO TO 132                                                         A 2570
  131 PRINT 145                                                         A 2580
  132 STOP                                                              A 2590
C                                                                       A 2600
  133 FORMAT (  34H1  PANEL FLUTTER FOR CLAMPED PANEL,///)              A 2610
  134 FORMAT (2F5.2,7I3,4F7.4)                                          A 2620
  135 FORMAT (2X, 20HLENGTH-WIDTH RATIO =,1F6.3/2X, 26HCHORDWISE ELEMENT  H 2630
     1 LENGTH =,1F6.3,3X, 21HSPANWISE ELEMENT WIDTH =,1F6.3/)           H 2640
  136 FORMAT (2X, 11HBOX WIDTH =,1F6.3,3X, 34HRATIO OF BOX LENGTH TO BOX  A 2650
     1 WIDTH =,1F6.3/)                                                  A 2660
  137 FORMAT (2X, 17HREDUCED FREQUENCY =,1F6.3/)                        A 2670
  138 FORMAT (2X, 23HLIMITING IN-PLANE FORCE PARAMETER=,1F6.3/)         A 2680
  139 FORMAT (2X, 13HMACH NUMBER =,1F5.2/)                              A 2690
  140 FORMAT (8I4,2I3,2I4,2I6/)                                         A 2700
  141 FORMAT (2X, 32HSTRUCTURAL DAMPING COEFFICIENT =,1F8.3///)         A 2710
  142 FORMAT ( 15H0  EIGENVALUE= ,1E12.5,1X,1E12.5,3X,  5H G = ,1F8.5,2X  A 2720
     1, 17HFREQUENCY RATIO =,1F8.5,2X, 22HSTIFFNESS PARAMETER = ,1F8.5)  H 2730
  143 FORMAT ( 14H   EIGENVECTOR,2X,10E12.5/2X,10E12.5/2X,10E12.5/2X,10  A 2740
     1E12.5/2X,10E12.5/2X,10E12.5/2X,10E12.5/2X,10E12.5)                A 2750
  144 FORMAT ( 20H0  DENSITY RATIO MU=,1F9.5)                           A 2760
  145 FORMAT ( 17H  SINGULAR MATRIX)                                    A 2770
C                                                                       A 2780
      END                                                               H 2790

      SUBROUTINE ASSEB (NX,NY,NXD)                                      B   10
      REAL SK(18,18),SM(18,18),OO(16,16),OM(16,16),ON(16,16),SN(18,18)  B   20
      INTEGER W(16),ID(4)                                               B   30
      COMMON /BLOCK2/ OO,OM,ON                                          B   40
      COMMON /BLOCK3/ SK,SM,SN                                          B   50
```

```
      DO 101 I=1,NXD                                                    B   60
      DO 101 J=1,NXD                                                    B   70
         SK(I,J)=0.                                                     B   80
         SM(I,J)=0.                                                     B   90
         SN(I,J)=0.                                                     B  100
  101 CONTINUE                                                          B  110
      NTX=NX*NY                                                         B  120
      DO 106 NE=1,NTX                                                   B  130
         READ 107, ID,(ID(I),I=1,4),(W(I),I=1,16)                       B  140
         DO 105 L=1,4                                                   B  150
            IF (ID(L).EQ.0) GO TO 105                                   B  160
            II=ID(L)                                                    B  170
            DO 104 M=1,4                                                B  180
               IE=4*(L-1)+M                                             B  190
               IF (W(IE).EQ.0) GO TO 104                                B  200
               DO 103 LS=1,4                                            B  210
                  IF (ID(LS).EQ.0) GO TO 103                            B  220
                  JJ=ID(LS)                                             B  230
                  DO 102 MS=1,4                                         B  240
                     IES=4*(LS-1)+MS                                    B  250
                     IF (W(IES).EQ.0) GO TO 102                         B  260
                     SK(II,JJ)=SK(II,JJ)+QQ(IE,IES)                     B  270
                     SM(II,JJ)=SM(II,JJ)+QM(IE,IES)                     B  280
                     SN(II,JJ)=SN(II,JJ)+QN(IE,IES)                     B  290
                     JJ=JJ+1                                            B  300
  102             CONTINUE                                              B  310
  103          CONTINUE                                                 B  320
               II=II+1                                                  B  330
  104       CONTINUE                                                    B  340
  105    CONTINUE                                                       B  350
  106 CONTINUE                                                          B  360
      RETURN                                                            B  370
C                                                                       B  380
  107 FORMAT (2213)                                                     B  390
C                                                                       B  400
      END                                                               B  410

      SUBROUTINE HERO (NX,NY,NXD,IBT,KA,V,VS,EPISN,SIGMA,BETA,AO,BA,JG,N  C   10
     1DX,NDY)                                                           C   20
      REAL V(40,16),VS(40,16),GRR(16,8),GRI(16,8),KA                    C   30
      INTEGER JG(16)                                                    C   40
      COMMON /BLOCK1/ IBX,RBX,GRR,GRI,IBY,RBY                           C   50
      IX1(NC)=8+NC                                                      C   60
      IX2(NC)=12+NC                                                     C   70
      IX3(NC)=NC                                                        C   80
      IX4(NC)=4+NC                                                      C   90
      NY1=NY*2                                                          C  100
      NDX1=NDX+1                                                        C  110
      NDY1=2*NDY-1                                                      C  120
      NC=KA                                                             C  130
      DO 116 NC=1,NY                                                    C  140
         DO 115 M=1,IBX                                                 C  150
         DO 115 N=1,IBY                                                 C  160
            MN=(NE-1)*IBX+M                                             C  170
```

```
      MT=(NCS-1)*IBY+N                                                  C  180
      MT=(N1-1)*IBX+M                                                   C  190
      DO 114 JJ=1,NDY1                                                  C  200
         DO 113 II=1,NDY1                                               C  210
            N4=4                                                        C  220
            IF (II.EQ.NDY) N4=2                                         C  230
            DO 112 ND=1,N4                                              C  240
               JI=(II-1)*4*NDX1+(JJ-1)*4+ND                             C  250
               IF (II.EQ.NDY) JI=(II-1)*4*NDX1+(JJ-1)*2+ND              C  260
               IF (II.GT.NDY) JI=(II-2)*4*NDX1+2*NDX1+(JJ-1)*4+ND       C  270
               G=0.                                                     C  280
               GS=0.                                                    C  290
               DO 110 NK=1,NC                                           C  300
                  N1=MK-JJ                                              C  310
                  IF (N1.GT.1) GO TO 111                                C  320
                  IF (N1.LT.0) GO TO 110                                C  330
                  DO 109 NC2=1,NY1                                      C  340
                     N2=NCS-II                                          C  350
                     IF (N2.GT.1) GO TO 110                             C  360
                     IF (N2.LT.0) GO TO 109                             C  370
                     IF (N1.EQ.0) GO TO 101                             C  380
                     IF (N2.EQ.0) GO TO 102                             C  390
                     IKL=IX3(ND)                                        C  400
                     GO TO 104                                          C  410
101                  IF (N2.EQ.0) GO TO 103                             C  420
                     IKL=IX2(ND)                                        C  430
                     GO TO 104                                          C  440
102                  IKL=IX4(ND)                                        C  450
                     GO TO 104                                          C  460
103                  IKL=IX1(ND)                                        C  470
104                  IBF=IBX                                            C  480
                     IF (NK.EQ.NC) IBF=M                                C  490
                     DO 108 MK=1,IBF                                    C  500
                        MM=(NK-1)*IBX+MK                                C  510
                        NC=MM-MN+1                                      C  520
                        JX=JG(NC)                                       C  530
                        DO 107 MS=1,IBY                                 C  540
                           MSL=(NCS-1)*IBY+MS                           C  550
                           IF (NT.GT.MSL) GO TO 105                     C  560
                           ISL=MSL-NT+1                                 C  570
                           IF (ISL.GT.JX) GO TO 108                     C  580
                           GO TO 106                                    C  590
105                        ISL=NT-MSL+1                                 C  600
                           IF (ISL.GT.JX) GO TO 107                     C  610
106                        XX=FLOAT(MK-1)*PBX+PBX/2.                    C  620
                           YY=FLOAT(MS-1)*PBY+PBY/2.                    C  630
                           CALL SHAPE (FAL,DFAL,IFL,XX,YY,BA)           C  640
                           G=G+(FAL*GPR(NC,ISL)+DFAL/KA*GRI(NC,ISL      C  650
     1                     ))*(EPIEN)/AO                                C  660
                           GS=GS+(FAL*GRI(NC,ISL)-DFAL/KA*GRR(NC,I      C  670
     1                     SL))*(EPIEN)/AO                              C  680
107                     CONTINUE                                        C  690
108                  CONTINUE                                           C  700
109               CONTINUE                                              C  710
110            CONTINUE                                                 C  720
111            V(JI,MT)=G                                               C  730
               VS(JI,MT)=GS                                             C  740
112         CONTINUE                                                    C  750
```

```
  113          CONTINUE                                                  C  760
  114        CONTINUE                                                    C  770
  115      CONTINUE                                                      C  780
  116 CONTINUE                                                          C  790
      RETURN                                                            C  800
C                                                                       C  810
C                                                                       C  820
      END                                                               C  830

      SUBROUTINE CHECK (R,S,BETA,SIGMA,GR)                              D   10
      DIMENSION GR(2), DX(4), DY(4)                                     D   20
      INTEGER R,S                                                       D   30
      COMMON /BLOCK0/ BM,UB,C,CB                                        D   40
      A1=2.*FLOAT(S)-1.                                                 D   50
      A2=2.*FLOAT(S)+1.                                                 D   60
      A3=FLOAT(2*R-1)*SIGMA/BETA                                        D   70
      A4=FLOAT(2*R+1)*SIGMA/BETA                                        D   80
      DO 101 I=1,2                                                      D   90
         DX(I)=SIGMA*(FLOAT(R)+0.5)                                     D  100
         I2=I+2                                                         D  110
  101 DX(I2)=SIGMA*(FLOAT(R)-0.5)                                       D  120
      DO 102 I=1,4,3                                                    D  130
  102 DY(I)=FLOAT(S)+0.5                                                D  140
      DO 103 I=2,3                                                      D  150
  103 DY(I)=FLOAT(S)-0.5                                                D  160
      IF (S.EQ.0) GO TO 113                                             D  170
      IF (DY(1).GT.(DX(1)/BETA)) GO TO 105                              D  180
      IF (DY(4).GT.(DX(4)/BETA)) GO TO 109                              D  190
      DO 104 JK=1,2                                                     D  200
         IK=JK                                                          D  210
         CALL GIR (IK,A1,A2,A3,A4,YG3)                                  D  220
         GR(JK)=YG3                                                     D  230
  104 CONTINUE                                                          D  240
      RETURN                                                            D  250
  105 IF (DY(3).GT.(DX(3)/BETA)) GO TO 107                              D  260
      DO 106 IK=1,2                                                     D  270
         CALL GIPS (IK,A1,A3,A4,YG2)                                    D  280
         GR(IK)=YG2                                                     D  290
  106 CONTINUE                                                          D  300
      RETURN                                                            D  310
  107 DO 108 IK=1,2                                                     D  320
         CALL GIPS (IK,A1,A1,A4,YG2)                                    D  330
         GR(IK)=YG2                                                     D  340
  108 CONTINUE                                                          D  350
      RETURN                                                            D  360
  109 IF (DY(3).GT.(DX(3)/BETA)) GO TO 111                              D  370
      DO 110 IK=1,2                                                     D  380
         CALL GIR (IK,A1,A2,A2,A4,YG3)                                  D  390
         CALL GIPS (IK,A1,A3,A2,YG2)                                    D  400
         GR(IK)=YG3+YG2                                                 D  410
  110 CONTINUE                                                          D  420
      RETURN                                                            D  430
  111 DO 112 IK=1,2                                                     D  440
         CALL GIR (IK,A1,A2,A3,A4,YG3)                                  D  450
```

```
      CALL GIP3 (IK,A1,A2,YG2)                                          D  460
      GP(IK)=YG3+YG2                                                    D  470
  112 CONTINUE                                                          D  480
      RETURN                                                            D  490
  113 IF (P.EQ.0) GO TO 120                                             D  500
      IF (DY(1).GT.(DX(1)/BETA)) GO TO 114                              D  510
      GO TO 116                                                         D  520
  114 DO 115 IK=1,2                                                     D  530
      CALL OG3N (IK,A2,A4,YG)                                           D  540
      GP(IK)=YG                                                         D  550
  115 CONTINUE                                                          D  560
      RETURN                                                            D  570
  116 IF (DY(4).GT.(DX(4)/BETA)) GO TO 118                              D  580
      DO 117 IK=1,2                                                     D  590
      CALL GIP (IK,0.,1.,A3,A4,YG3)                                     D  600
      GP(IK)=2.*YG3                                                     D  610
  117 CONTINUE                                                          D  620
      RETURN                                                            D  630
  118 DO 119 IK=1,2                                                     D  640
      CALL OG3N (IK,A3,1.,YG)                                           D  650
      CALL GIP (IK,0.,1.,1.,A4,YG3)                                     D  660
      GP(IK)=2.*YG3+YG                                                  D  670
  119 CONTINUE                                                          D  680
      RETURN                                                            D  690
  120 IF (DY(1).LE.(DX(1)/BETA)) GO TO 122                              D  700
      DO 121 IK=1,2                                                     D  710
      CALL OG3N (IK,0.,A4,YG)                                           D  720
      GP(IK)=YG                                                         D  730
  121 CONTINUE                                                          D  740
      RETURN                                                            D  750
  122 DO 123 IK=1,2                                                     D  760
      CALL OG3N (IK,0.,1.,YG)                                           D  770
      CALL GIP (IK,0.,1.,1.,A4,YG3)                                     D  780
      GP(IK)=2.*YG3+YG                                                  D  790
  123 CONTINUE                                                          D  800
      RETURN                                                            D  810
C                                                                       D  820
C                                                                       D  830
      END                                                               D  840

      SUBROUTINE GIPS (IK,YS,XL,XU,YG2)                                 E   10
      COMMON /BLOCK0/ IM,UB,C,CB                                        E   20
      A=0.5*(XU+XL)                                                     E   30
      S=XU-XL                                                           E   40
      Y1=0.2[illegible]*S*B                                             E   50
      Y2=0.[illegible]*B                                                E   60
      DELTA0=A                                                          E   70
      DELTA1=A-Y1                                                       E   80
      DELTA3=A+Y1                                                       E   90
      DELTA2=A+Y2                                                       E  100
      DELTA4=A-Y2                                                       E  110
      YG2=(Y*B/2.)*(0.568[illegible]*FY(IK,DELTA0,YS,DELTA0)+0.47862967*(FY(I  E  120
     1K,DELTA1,YS,DELTA1)+FY(IK,DELTA3,YS,DELTA3))+0.23692688*(FY(IK,DE  E  130
     2LTA2,YS,DELTA2)+FY(IK,DELTA4,YS,DELTA4)))                         E  140

      RETURN                                                            E  150
C                                                                       E  160
      END                                                               E  190
```

```
      FUNCTION FY(IK,DELTA,YL,YU)                                          F   10
      COMMON /BLOCK0/ BM,UB,C,CB                                           F   20
      FK(BM,DELTA,Z)=(COS(BM*SQRT(DELTA**2-Z**2))-1.)/SQRT(DELTA**2-Z**2   F   30
     1)
      A=0.5*(YU+YL)                                                        F   40
      B=YU-YL                                                              F   50
      Y1=0.269234605*B                                                     F   60
      Y2=0.453089923*B                                                     F   70
      DELTA0=A                                                             F   80
      DELTA1=A+Y1                                                          F   90
      DELTA3=A-Y1                                                          F  100
      DELTA2=A+Y2                                                          F  110
      DELTA4=A-Y2                                                          F  120
      IF (IK.EQ.1) GO TO 101                                               F  130
      FY=-SIN(UB*DELTA)*(ACOS(YL/DELTA)+(B/2.)*(0.568888889*FK(BM,DELTA,   F  140
     1DELTA0)+0.478628687*(FK(BM,DELTA,DELTA1)+FK(BM,DELTA,DELTA3))+0.236  F  150
     2926885*(FK(BM,DELTA,DELTA2)+FK(BM,DELTA,DELTA4))))                  F  160
      RETURN                                                               F  170
  101 FY=COS(UB*DELTA)*(ACOS(YL/DELTA)+(B/2.)*(0.568888889*FK(BM,DELTA,D   F  180
     1ELTA0)+0.478628687*(FK(BM,DELTA,DELTA1)+FK(BM,DELTA,DELTA3))+0.2369  F  190
     226885*(FK(BM,DELTA,DELTA2)+FK(BM,DELTA,DELTA4))))                    F  200
      RETURN                                                               F  210
C                                                                          F  220
      END                                                                  F  230
                                                                           F  240
      SUBROUTINE GIP (IK,YL,YU,XL,XU,YG3)                                  G   10
      COMMON /BLOCK0/ BM,UB,C,CB                                           G   20
      A=0.5*(XU+XL)                                                        G   30
      B=XU-XL                                                              G   40
      Y1=0.387298346*B                                                     G   50
      DELTA0=A                                                             G   60
      DELTA1=A+Y1                                                          G   70
      DELTA2=A-Y1                                                          G   80
      YG3=C*(B/2.)*(0.888888889*FX(IK,DELTA0,YL,YU)+0.555555556*(FX(IK,D   G   90
     1ELTA1,YL,YU)+FX(IK,DELTA2,YL,YU)))                                   G  100
      RETURN                                                               G  110
C                                                                          G  120
      END                                                                  G  130
                                                                           H   10
      FUNCTION FX(IK,DELTA,YL,YU)                                          H   20
      COMMON /BLOCK0/ BM,UB,C,CB                                           H   30
      FS(BM,DELTA,Z)=(COS(BM*SQRT(DELTA**2-Z**2))-1.)/SQRT(DELTA**2-Z**2   H   40
     1)
      A=0.5*(YU+YL)                                                        H   50
      B=YU-YL                                                              H   60
      Y1=0.387298346*B                                                     H   70
      IF (IK.EQ.1) GO TO 101                                               H   80
      FX=-SIN(UB*DELTA)*(ACOS(YL/DELTA)-ACOS(YU/DELTA)+(B/2.)*(0.8888888   H   90
     188*FS(BM,DELTA,A)+0.555555556*(FS(BM,DELTA,A+Y1)+FS(BM,DELTA,A-Y1)   H  100
     2))                                                                   H  110
      RETURN                                                               H  120
  101 FX=COS(UB*DELTA)*(ACOS(YL/DELTA)-ACOS(YU/DELTA)+(B/2.)*(0.8888888   H  130
     188*FS(BM,DELTA,A)+0.555555556*(FS(BM,DELTA,A+Y1)+FS(BM,DELTA,A-Y1))  H  140
     2)                                                                    H  150
      RETURN                                                               H  160
C                                                                          H  170
      END                                                                  H  180
```

```
      SUBROUTINE QG3A (IK,XL,XU,YG)                                     I   10
      COMMON /BLOCK0/ BM,UB,C,CB                                        I   20
      H=0.5*(XU+XL)                                                     I   30
      B=XU-XL                                                           I   40
      Y1=0.26923465*B                                                   I   50
      Y2=0.46308923*B                                                   I   60
      Y=BM*(H+Y1)                                                       I   70
      YY=BM*(H-Y1)                                                      I   80
      Z=BM*(H+Y2)                                                       I   90
      ZZ=BM*(H-Y2)                                                      I  100
      X=BM*H                                                            I  110
      CALL BESJ (Y,0,BJ1,0.0001,IER)                                    I  120
      CALL BESJ (YY,0,BJ2,0.0001,IER)                                   I  130
      CALL BESJ (Z,0,BJ3,0.0001,IER)                                    I  140
      CALL BESJ (ZZ,0,BJ4,0.0001,IER)                                   I  150
      CALL BESJ (X,0,BJ0,0.0001,IER)                                    I  160
      Y=UB*(H+Y1)                                                       I  170
      YY=UB*(H-Y1)                                                      I  180
      Z=UB*(H+Y2)                                                       I  190
      ZZ=UB*(H-Y2)                                                      I  200
      X=UB*H                                                            I  210
      IF (IK.EQ.1) GO TO 101                                            I  220
      YG=-0.5*(B/2.)*(0.56888889*SIN(X)*BJ0+0.47862867*(SIN(Y)*BJ1+SIN( I  230
     1YY)*BJ2)+0.23692689*(SIN(Z)*BJ3+SIN(ZZ)*BJ4))                     I  240
      RETURN                                                            I  250
  101 YG=0.5*(B/2.)*(0.56888889*COS(X)*BJ0+0.47862867*(COS(Y)*BJ1+COS(Y I  260
     1Y)*BJ2)+0.23692689*(COS(Z)*BJ3+COS(ZZ)*BJ4))                      I  270
      RETURN                                                            I  280
C                                                                       I  290
      END                                                               I  300
```

```
      SUBROUTINE BESJ (X,N,BJ,D,IER)                                    10
      BJ=.0                                                             20
      IF (N) 101,102,102                                                30

  101 IER=1                                                             40
      RETURN                                                            50
  102 IF (X) 103,103,104                                                60
  103 IER=2                                                             70
      RETURN                                                            80
  104 IF (X-15.) 105,105,106                                            90
  105 NTEST=20.+10.*X-X**2/3.                                           100
      GO TO 107                                                         110
  106 NTEST=90.+X/2.                                                    120
  107 IF (N-NTEST) 109,108,108                                          130
  108 IER=4                                                             140
      RETURN                                                            150
  109 ILP=0                                                             160
      N1=N+1                                                            170
      BPREV=.0                                                          180
C                                                                       190
C     COMPUTE STARTING VALUE OF M                                       200
C                                                                       210
      IF (X-5.) 110,111,111                                             220
  110 MA=X+6.                                                           230
      GO TO 112                                                         240
  111 MA=1.1*X+60./X                                                    250
  112 MB=N+IFIX(X)/4+2                                                  260
      MZERO=MAX0(MA,MB)                                                 270
C                                                                       280
C     SET UPPER LIMIT OF M                                              290
C                                                                       300
      MMAX=NTEST                                                        310
      DO 121 M=MZERO,MMAX,3                                             320
C                                                                       330
C     SET F(M),F(M-1)                                                   340
C                                                                       350
         FM1=1.0E-28                                                    360
         FM=.0                                                          370
         ALPHA=.0                                                       380
         IF (M-(M/2)*2) 114,113,114                                     390
  113    JT=-1                                                          400
         GO TO 115                                                      410
  114    JT=1                                                           420
  115    M2=M-2                                                         430
         DO 118 K=1,M2                                                  440
            MK=M-K                                                      450
            BMK=2.*FLOAT(MK)*FM1/X-FM                                   460
            FM=FM1                                                      470
            FM1=BMK                                                     480
            IF (MK-N-1) 117,116,117                                     490
  116       BJ=BMK                                                      500
  117       JT=-JT                                                      510
            S=1+JT                                                      520
  118    ALPHA=ALPHA+BMK*S                                              530
         BMK=2.*FM1/X-FM                                                540
         IF (N) 120,119,120                                             550
  119    BJ=BMK                                                         560
  120    ALPHA=ALPHA+BMK                                                570
         BJ=BJ/ALPHA                                                    580
         IF (ABS(BJ-BPREV)-ABS(D*BJ)) 122,122,121                       590
  121 BPREV=BJ                                                          600
      IER=3                                                             610

  122 RETURN                                                            620
C                                                                       630
      END                                                               640
```

```
      SUBROUTINE SHAPE (FA,DFA,IK,X,Y,BA)                                K   10
      F1(Z)=1.+2.*(Z**3)-3.*(Z**2)                                       K   20
      F2(Z)=Z*(Z-1.)**2)                                                 K   30
      F4(Z)=(Z**3)-(Z**2)                                                K   40
      DF1(Z)=6.*(Z**2)-6.*Z                                              K   50
      DF2(Z)=3.*(Z**2)-4.*Z+1.                                           K   60
      DF4(Z)=3.*(Z**2)-2.*Z                                              K   70
      IF (IK.EQ.0) GO TO 106                                             K   80
      IF (IK.EQ.1.OR.IK.EQ.3.OR.IK.EQ.5.OR.IK.EQ.7) GO TO 101            K   90
      IF (IK.EQ.2.OR.IK.EQ.4.OR.IK.EQ.6.OR.IK.EQ.8) GO TO 102            K  100
      IF (IK.EQ.9.OR.IK.EQ.11.OR.IK.EQ.13.OR.IK.EQ.15) GO TO 103         K  110
      IF (IK.EQ.10.OR.IK.EQ.12.OR.IK.EQ.14.OR.IK.EQ.16) GO TO 104        K  120
  101 AA=F1(X)                                                           K  130
      CC=DF1(X)                                                          K  140
      GO TO 105                                                          K  150
  102 AA=F2(X)                                                           K  160
      CC=DF2(X)                                                          K  170
      GO TO 105                                                          K  180
  103 AA=1.-F1(X)                                                        K  190
      CC=-DF1(X)                                                         K  200
      GO TO 105                                                          K  210
  104 AA=F4(X)                                                           K  220
      CC=DF4(X)                                                          K  230
  105 IF (IK.EQ.1.OR.IK.EQ.2.OR.IK.EQ.13.OR.IK.EQ.14) BB=F1(Y)           K  240
      IF (IK.EQ.3.OR.IK.EQ.4.OR.IK.EQ.15.OR.IK.EQ.16) BB=F2(Y)*BA        K  250
      IF (IK.EQ.5.OR.IK.EQ.6.OR.IK.EQ.9.OR.IK.EQ.10) BB=1.-F1(Y)         K  260
      IF (IK.EQ.7.OR.IK.EQ.8.OR.IK.EQ.11.OR.IK.EQ.12) BB=F4(Y)*BA        K  270
      FA=AA*BB                                                           K  280
      DFA=CC*BB                                                          K  290
      RETURN                                                             K  300
  106 FA=0.                                                              K  310
      DFA=0.                                                             K  320
      RETURN                                                             K  330
C                                                                        K  340
      END                                                                K  350
```

PANEL FLUTTER FOR CLAMPED PANEL

LENGTH-WIDTH RATIO = 1.000
CHORDWISE ELEMENT LENGTH = 1.000 SPANWISE ELEMENT WIDTH = 1.000

BOX WIDTH = .500 RATIO OF BOX LENGTH TO BOX WIDTH = .500

REDUCED FREQUENCY = .500

INITIAL IN-PLANE FORCE PARAMETER= .100

MACH NUMBER = 1.20

STRUCTURAL DAMPING COEFFICIENT = 0.000

DENSITY RATIO MU= .01500

EIGENVALUE= 5.09752E-01 -1.44227E-02 G = -.02829 FREQUENCY RATIO = .71404 STIFFNESS PARAMETER = .35702
EIGENVECTOR
2.71207E-01 1.73227E-03 3.49503E-01-6.74931E-04 3.81089E-01-1.28203E-03 4.54907E-01-5.78048E-03 5.15697E-01-5.75751E-03
7.94788E-02-1.14777E-02 6.93768E-01-1.03775E-02-2.26185E-02-9.97564E-03 3.60810E-01-1.24987E-02-3.70766E-01 3.94762E-03
4.70692E-01-1.05057E-02-5.14060E-01 1.27013E-02 5.00000E-01 0. 6.26550E-01-4.88439E-03 9.23917E-01-1.81921E-02
1.12643E-01-1.46563E-02 6.32644E-01-1.74853E-02-6.59218E-01 9.42035E-03

EIGENVALUE= 1.31424E-01 5.67010E-04 G = .00431 FREQUENCY RATIO = .36252 STIFFNESS PARAMETER = .18126
EIGENVECTOR
-1.60359E-01-1.27951E-02-1.36713E-01-5.06522E-03-1.94742E-01-1.56964E-02-1.16354E-01-8.20403E-03-9.96963E-03 4.70427E-03
4.39146E-01 2.77076E-02 7.02061E-02-1.60734E-02 6.51616E-01-2.97707E-03 3.58316E-01 8.31471E-03 6.65135E-03-1.87187E-02
5.29156E-01-3.09380E-02-1.84926E-01-1.33035E-02-2.62591E-01-2.38017E-02-1.78307E-01-1.45153E-02 6.40818E-02-1.49576E-01
8.15929E-01 1.43044E-02 6.92169E-01-2.27189E-02-5.56703E-02-1.20074E-02

EIGENVALUE= 7.18589E-02 -7.13719E-04 G = -.00993 FREQUENCY RATIO = .26807 STIFFNESS PARAMETER = .13403
EIGENVECTOR
3.10569E-01-7.85265E-03 4.15737E-01 6.45275E-04-3.41736E-01-7.91461E-03-4.82366E-01-7.69072E-03 5.69337E-01 2.40738E-02
-2.52618E-02 5.61690E-02-6.72746E-01-2.88038E-02-2.65645E-01-1.18713E-02 2.97272E-01 6.31795E-02-4.00658E-01-9.24970E-03
-5.70016E-01 4.70239E-03 4.19108E-01 2.84316E-02-2.93596E-01-1.34572E-02-3.40682E-01-2.06160E-02-5.51195E-01-2.77996E-02
-1.63270E-01 1.98235E-02-5.31106E-01 2.46954E-02 3.20200E-01 4.10527E-02

EIGENVALUE= 5.16318E-02 1.92179E-03 G = .03722 FREQUENCY RATIO = .22727 STIFFNESS PARAMETER = .11363
EIGENVECTOR
-1.17736E-01 5.32891E-02-1.65443E-02 8.54458E-02 3.03037E-02-1.34324E-01 1.72478E-01-5.95052E-02 9.16944E-02 7.18449E-02
2.09495E-01-1.70090E-01 2.28483E-01 5.52993E-02-1.56286E-01 2.06447E-01-1.70349E-01-2.79402E-01-4.32421E-01-2.13038E-01
-6.96704E-01-2.76921E-02-8.82933E-01-1.71284E-01-5.09649E-02-1.27769E-01 1.55512E-01 3.37926E-03 2.50693E-01 1.03309E-01
-1.05555E-01 1.80141E-01-7.06758E-01-1.45885E-01-9.38594E-01-3.63306E-01

EIGENVALUE= 4.99324E-02 -2.70008E-03 G = -.05500 FREQUENCY RATIO = .22165 STIFFNESS PARAMETER = .11083
EIGENVECTOR
-1.63754E-01 2.58604E-01-1.30407E-01 1.10518E-01 2.57932E-01-2.15057E-01 1.84878E-01-3.37519E-01-3.54089E-02-3.32093E-02
4.35613E-01-5.33176E-01-5.87049E-02-2.04364E-01-5.63612E-01 5.35764E-01 4.60904E-01-9.16208E-02 1.67570E-01 4.03084E-01
-1.89898E-01 6.52879E-01 2.36540E-01 6.31718E-01 2.19078E-01-1.39609E-01 7.43524E-02-2.13390E-01-1.06027E-01-2.17721E-01
-4.58719E-01 3.40671E-01-3.08150E-02 6.19561E-01 4.61583E-01 6.10616E-01

DENSITY RATIO MU= .02500

EIGENVALUE= 5.38925E-01 -2.16467E-02 G = -.04062 FREQUENCY RATIO = .73017 STIFFNESS PARAMETER = .36508

EIGENVECTOR
2.50000E-01 0. 3.41448E-01-3.51760E-03 3.61033E-01-5.53808E-03 4.56446E-01-1.23192E-02 5.20538E-01-1.24326E-02
1.25275E-01-1.74406E-02 7.07136E-01-2.16722E-02 2.88874E-02-1.38357E-02 3.96015E-01-2.21435E-02-3.76765E-01 7.79675E-03
5.10147E-01-2.08298E-02-5.41673E-01 2.31675E-02 4.69379E-01-4.82755E-03 6.22640E-01-1.21634E-02 9.38408E-01-2.59552E-02
1.84722E-01-2.23647E-02 6.89759E-01-3.27375E-02-6.68778E-01 1.85861E-02

EIGENVALUE= 1.38812E-01 1.04935E-03 G = .00756 FREQUENCY RATIO = .37258 STIFFNESS PARAMETER = .18629
EIGENVECTOR
-1.04603E-01-8.68992E-03-1.16775E-01-2.49426E-03-1.23101E-01-1.31556E-02-8.71043E-02-9.96728E-03-3.76974E-03 8.18587E-03
3.75485E-01 2.41706E-02 1.10173E-01-2.36345E-02 6.18405E-01-2.38567E-02 4.03026E-01 7.76814E-05 5.77830E-02-3.10660E-02
6.36505E-01-6.61053E-02-9.69061E-02-2.79283E-02-1.64549E-01-1.93819E-02-1.35186E-01-1.59207E-02 1.06455E-01-2.17587E-02
7.49558E-01-6.43066E-03 8.15534E-01-5.92423E-02 5.58570E-02-2.93602E-02

EIGENVALUE= 7.65897E-02 -2.53252E-03 G = -.03307 FREQUENCY RATIO = .27679 STIFFNESS PARAMETER = .13839
EIGENVECTOR
4.12027E-01-1.95277E-01 6.65281E-01-2.99352E-01-4.14071E-01 1.57604E-01-6.30450E-01 2.42194E-01 9.68503E-01-3.81281E-01
7.40915E-02 1.04675E-01-9.69439E-01 3.72827E-01-7.93337E-01 4.36014E-01 2.24850E-01 1.43230E-01-1.06373E+00 5.65499E-01
-1.59347E+00 9.96205E-01 6.75880E-02 2.77023E-01-3.81301E-01 1.32272E-01-4.26051E-01 1.25336E-01-7.43764E-01 2.73503E-01
-5.60018E-01 4.05967E-01-1.61428E+00 1.07460E+00-2.28128E-01 4.43503E-01

EIGENVALUE= 6.29265E-02 2.92551E-03 G = .04649 FREQUENCY RATIO = .25092 STIFFNESS PARAMETER = .12546
EIGENVECTOR
-1.06237E-02 2.48302E-02-3.42093E-03 2.35291E-02-2.76617E-03-2.01850E-02 1.77626E-02-3.92211E-02 5.77140E-03 3.09169E-03
-1.29227E-03-5.63873E-02 4.29342E-02-4.21253E-02 2.70701E-03 3.83219E-02-1.00462E-01 1.50242E-02-1.14810E-01 6.88837E-02
-1.19270E-01 1.46426E-01-1.87306E-01 2.20100E-01-5.45281E-03-1.51556E-02 1.89388E-02-2.47087E-02 5.00192E-02-3.71381E-02
1.54123E-03 3.17613E-02-1.59034E-01 1.47531E-01-2.39997E-01 2.00764E-01

EIGENVALUE= 5.60590E-02 -5.09036E-03 G = -.09080 FREQUENCY RATIO = .23701 STIFFNESS PARAMETER = .11851
EIGENVECTOR
-1.55215E-01-8.35883E-02-1.24132E-01-7.18035E-02 1.34613E-01 1.19970E-01 2.43910E-01 1.35685E-01 3.72302E-02 2.84460E-02
4.04032E-01 3.31513E-01 1.89025E-01-2.16774E-02-3.12504E-01-3.62939E-01 1.07862E-02 4.01859E-01-4.18115E-01 1.12980E-01
-6.83619E-01-2.30812E-01-8.84097E-01 3.72806E-02 9.51827E-02 9.89955E-02 1.36316E-01 6.32020E-02 1.78155E-01-5.39036E-02
-1.97373E-01-2.93916E-01-6.62807E-01-9.57312E-02-8.41283E-01 2.48088E-01

DENSITY RATIO MU= .03500

EIGENVALUE= 5.48314E-01 -2.74249E-02 G = -.05002 FREQUENCY RATIO = .74071 STIFFNESS PARAMETER = .37036
EIGENVECTOR
1.30468E-01 1.85881E-03 1.68868E-01 6.22261E-04 1.94158E-01-1.26495E-03 2.59137E-01-4.89550E-03 2.97623E-01-4.65082E-03
9.75207E-02-1.08678E-02 4.08458E-01-9.65306E-03 4.76952E-02-9.85904E-03 2.45424E-01-1.25471E-02-2.15790E-01 1.42145E-03
3.11886E-01-1.11887E-02-3.23800E-01 1.16760E-02 2.50000E-01 0. 3.50007E-01-3.26934E-03 5.41070E-01-1.04682E-02
1.47464E-01-1.44768E-02 4.27525E-01-1.80855E-02-3.86904E-01 6.75952E-03

EIGENVALUE= 1.46696E-01 9.67724E-04 G = .00660 FREQUENCY RATIO = .38301 STIFFNESS PARAMETER = .19151
EIGENVECTOR
-1.00681E-01-4.54888E-03-1.37433E-01 4.11746E-03-1.04070E-01-1.61930E-02-7.07381E-02-1.99360E-02 1.70118E-02 1.59962E-02
5.04615E-01 1.65580E-02 2.40483E-01-5.84057E-02 9.08978E-01-9.15125E-02 7.03397E-01-3.74541E-02 2.57288E-01-7.86166E-02
1.15589E+00 1.95723E-01-5.16128E-03-7.69797E-02-1.39487E-01-2.04390E-02-1.20644E-01-2.58449E-02 2.45044E-01-5.47607E-02
1.07955E+00-7.06030E-02 1.46038E+00-1.98238E-01 2.66987E-01-8.98851E-02

EIGENVALUE= 8.67425E-02 -5.55596E-03 G = -.06405 FREQUENCY RATIO = .29467 STIFFNESS PARAMETER = .14734
EIGENVECTOR
-2.05544E-02 7.17684E-03-6.20731E-02 7.39889E-03 2.85026E-02 1.56747E-04 4.16231E-02-6.01985E-03-1.01876E-01-7.78799E-04
-2.22429E-02-1.74389E-02 9.45134E-02-1.98665E-03 1.89510E-01-2.90839E-03 1.02499E-01-8.84133E-04 2.81119E-01 1.47955E-02
4.72562E-01-3.35314E-02 2.81982E-01 5.18324E-03 3.02093E-02 2.13243E-03 2.87098E-02-2.07017E-03 6.98916E-02-2.00229E-02
1.63850E-01-3.47974E-02 5.19389E-01-2.77407E-02 3.54760E-01 2.27551E-02

EIGENVALUE= 7.20165E-02 3.63030E-03 G = .05041 FREQUENCY RATIO = .26844 STIFFNESS PARAMETER = .13422
EIGENVECTOR
2.07282E-01 7.79365E-03 2.86757E-01 2.15528E-02-8.09929E-02 2.52052E-02-2.66363E-01-9.98767E-03 2.06329E-01 1.49953E-01

-2.14335E-01 3.67896E-01-5.01886E-01-1.56715E-01-7.61090E-02-3.00804E-01 9.21890E-01 6.08058E-01 1.10741E+00 9.23564E-02
1.49016E+00-5.37328E-01 2.78452E+00-4.78647E-01-5.44011E-02 2.01384E-02-1.62698E-01-4.64047E-02-4.52407E-01-1.94180E-01
-2.30232E-02-2.30086E-01 1.80859E+00-2.79209E-01 2.97406E+00 4.91414E-04

EIGENVALUE= 6.19034E-02 -6.25723E-03 G = -.10108 FREQUENCY RATIO = .24912 STIFFNESS PARAMETER = .12456
EIGENVECTOR
-2.06653E-01 9.02497E-02-2.19588E-01 1.31835E-01 2.36622E-01-2.01705E-02 4.06739E-01-9.22814E-02 1.19068E-01 1.07294E-01
9.49457E-01-2.85686E-02 1.81907E-01-2.51758E-01-7.65737E-01-2.18981E-01 7.27124E-01 6.21490E-01-3.33501E-01 7.76834E-01
-1.54335E+00 4.31544E-01-1.89053E+00 1.22374E+00 1.88221E-01-7.25742E-03 2.04225E-01-3.94087E-02 8.80376E-02-2.53965E-01
-6.17539E-01-2.50016E-01-1.25672E+00 6.21582E-01-1.09660E+00 1.52778E+00

DENSITY RATIO MU= .04500

EIGENVALUE= 5.54946E-01 -3.18754E-02 G = -.05744 FREQUENCY RATIO = .74525 STIFFNESS PARAMETER = .37263
EIGENVECTOR
1.27393E-01 2.36980E-03 1.95585E-01 1.52735E-03 1.96141E-01-1.54294E-03 2.76377E-01-5.49729E-03 3.20769E-01-3.72462E-03
1.34285E-01-1.25434E-02 4.47582E-01-1.12455E-02 8.84213E-02-1.24174E-02 2.89749E-01-1.50521E-02-2.31148E-01-8.35919E-05
3.58810E-01-1.92102E-02-3.61675E-01 1.33098E-02 2.50000E-01 0. 3.69389E-01-3.11467E-03 5.88394E-01-1.15392E-02
2.07382E-01-1.71204E-02 5.01989E-01-2.21087E-02-4.20157E-01 6.27684E-03

DENSITY RATIO MU= .03500

EIGENVALUE= 5.48314E-01 -2.74249E-02 G = -.05002 FREQUENCY RATIO = .74071 STIFFNESS PARAMETER = .37036
EIGENVECTOR
1.30468E-01 1.85881E-03 1.88868E-01 6.22261E-04 1.94158E-01-1.26495E-03 2.59137E-01-4.89550E-03 2.97823E-01-4.65282E-03
9.75207E-02-1.08678E-02 4.09453E-01-9.65306E-03 4.76952E-02-9.85994E-03 2.46424E-01-1.25471E-02-2.15790E-01 1.42145E-03
3.11886E-01-1.11887E-02-3.23870E-01 1.16760E-02 2.50000E-01 0. 3.50007E-01-3.26934E-03 5.41070E-01-1.04682E-02
1.47464E-01-1.44768E-02 4.27525E-01-1.80850E-02-3.86984E-01 6.75952E-03

EIGENVALUE= 1.46696E-01 9.67724E-04 G = .00660 FREQUENCY RATIO = .38301 STIFFNESS PARAMETER = .19151
EIGENVECTOR
-1.00681E-01-4.54888E-03-1.37483E-01 4.11746E-03-1.04070E-01-1.61980E-02-7.07381E-02-1.99360E-02 1.70119E-02 1.59662E-02
5.01615E-01 1.65530E-02 2.40483E-01-5.84057E-02 9.08978E-01-9.15125E-02 7.03397E-01-3.74541E-02 2.57288E-01-7.86166E-02
1.15589E+00 1.95723E-01-5.16128E-03-7.69797E-02-1.39487E-01-2.04390E-02-1.20644E-01-2.58449E-02 8.45044E-01-5.47607E-02
1.07955E+00-7.06030E-02 1.46308E+00-1.98238E-01 2.66987E-01-8.98861E-02

EIGENVALUE= 8.67425E-02 -5.55596E-03 G = -.06405 FREQUENCY RATIO = .29467 STIFFNESS PARAMETER = .14734
EIGENVECTOR
-2.05344E-02 7.17694E-03-6.30715E-02 7.39889E-03 2.85026E-02 1.56747E-04 4.16231E-02-6.01985E-03-1.01876E-01-7.78799E-01
-2.22429E-02-1.74379E-02 9.45134E-02-1.98645E-02 1.89510E-01-2.90898E-03 1.02499E-01-8.84133E-04 2.81119E-01 1.47933E-02
4.72562E-01-3.35114E-02 2.81982E-01 6.18324E-03 3.02009E-02 2.13243E-03 2.87092E-02-2.07017E-03 6.98916E-02-2.00229E-02
1.68850E-01-3.47974E-02 5.19389E-01-2.77407E-02 3.54760E-01 2.27551E-02

EIGENVALUE= 7.20165E-02 3.63030E-03 G = .05041 FREQUENCY RATIO = .26844 STIFFNESS PARAMETER = .13422
EIGENVECTOR
2.07282E-01 7.79365E-03 2.86757E-01 2.15528E-02-8.09929E-02 2.52052E-02-2.66363E-01-9.99767E-03 2.06329E-01 1.49963E-01

-2.14335E-01 3.67896E-01-5.01886E-01-1.56515E-01-7.61090E-02-3.00904E-01 9.21890E-01 6.08053E-01 1.10741E+00 9.23564E-02
1.49016E+00-5.37328E-01 2.72452E+00-4.78647E-01-5.44011E-02 2.01324E-02-1.62693E-01-4.64047E-02-4.52907E-01-1.94180E-01
-2.30232E-02-2.30086E-01 1.80859E+00-2.79209E-01 2.97406E+00 4.91414E-04

EIGENVALUE= 6.19034E-02 -8.25723E-03 G = -.10109 FREQUENCY RATIO = .24912 STIFFNESS PARAMETER = .12456
EIGENVECTOR
-2.06553E-01 9.03497E-02-2.19588E-01 1.31835E-01 2.36228E-01-2.01705E-02 4.06739E-01-9.22814E-02 1.19682E-01 1.07294E-01
9.49457E-01-2.85586E-02 1.81307E-01-2.51753E-01-7.65737E-01-2.18981E-01 7.27124E-01 6.81490E-01-3.38301E-01 7.76834E-01
-1.54335E+00 4.31544E-01-1.89453E+00 1.22874E+00 1.88221E-01-7.25742E-03 2.04225E-01-3.94087E-02 8.80376E-02-2.53965E-01
-6.17539E-01-2.59016E-01-1.25672E+00 6.21582E-01-1.09660E+00 1.52778E+00

DENSITY RATIO MU= .04500

EIGENVALUE= 5.54946E-01 -3.18754E-02 G = -.05744 FREQUENCY RATIO = .74525 STIFFNESS PARAMETER = .37263
EIGENVECTOR
1.27393E-01 2.36980E-03 1.95585E-01 1.52735E-03 1.96111E-01-1.54294E-03 2.76377E-01-5.49729E-03 3.20769E-01-3.72462E-03
1.34285E-01-1.25434E-02 4.47582E-01-1.12455E-02 8.84213E-02-1.24174E-02 2.89749E-01-1.50521E-02-2.31148E-01-8.35919E-05
3.68810E-01-1.32102E-02-3.61675E-01 1.33098E-02 2.50000E-01 0. 3.69389E-01-3.11457E-03 5.88394E-01-1.15392E-02
2.07382E-01-1.71204E-02 5.01989E-01-2.21087E-02-4.20157E-01 6.27684E-03

EIGENVALUE= 8.48193E-02 2.74160E-02 G = .32912 FREQUENCY RATIO = .29505 STIFFNESS PARAMETER = .14753
EIGENVECTOR
2.73838E-02 3.08450E-02 6.87357E-02 1.86055E-02 5.64523E-02-1.08934E-02 9.36030E-02-7.34521E-02 9.79577E-03 5.52837E-02
-1.06542E-01 1.49245E-01 4.53610E-02-5.25170E-02-4.51729E-02 1.06927E-01 3.61157E-01 1.96636E-01 6.09093E-01-1.26205E-01
6.08063E-01-1.34523E-01 8.09825E-01-5.47811E-01 6.39462E-02-3.19843E-03 1.22536E-01-5.98964E-02 4.81070E-02-5.04236E-02
-8.19423E-02 1.33127E-01 7.37270E-01-3.91096E-02 1.03565E+00-3.98229E-01

EIGENVALUE= 9.44164E-02 -1.12974E-02 G = -.11966 FREQUENCY RATIO = .30782 STIFFNESS PARAMETER = .15391
EIGENVECTOR
-6.50417E-02 2.67698E-03-1.47682E-01-3.19602E-02 1.28629E-01 1.02468E-01 2.78463E-01 2.37084E-01-2.94163E-01-3.24131E-01
-1.45058E-01-5.69548E-01 5.22790E-01 4.38277E-01 6.32014E-01 5.71665E-01 3.14075E-01 1.76762E-01 9.66599E-01 1.44018E+00
1.72623E+00 2.38668E+00 1.10025E+00 2.37102E+00 1.46164E-01 1.40600E-01 2.62813E-01 3.03762E-01 4.65054E-01 4.38576E-01
5.07297E-01 3.96929E-01 1.81207E+00 2.46367E+00 1.22857E+00 2.43933E+00

EIGENVALUE= 7.07209E-02 -2.57349E-02 G = -.36399 FREQUENCY RATIO = .27017 STIFFNESS PARAMETER = .13508
EIGENVECTOR
-2.16520E-02-7.39054E-02 5.42405E-02-1.29479E-01 9.87012E-02-3.02939E-02 2.35089E-01 2.51194E-02-2.24012E-02-1.03209E-01
-2.62906E-01 4.87816E-03 7.57333E-02 8.53733E-02-3.49689E-01-3.16834E-02 2.04445E-01-1.68266E-01 8.66318E-01-3.53159E-01
9.34454E-01-4.67458E-01 2.05383E+00-4.36256E-01 9.20349E-02-3.59456E-02 2.24342E-01-1.80033E-02 8.40527E-02 6.32851E-02
-3.45990E-01-4.83319E-02 9.99820E-01-6.11273E-01 2.05250E+00-5.98348E-01

DENSITY RATIO MU= .06500

EIGENVALUE= 5.27070E-01 -3.58407E-02 G = -.06792 FREQUENCY RATIO = .72683 STIFFNESS PARAMETER = .36341
EIGENVECTOR
1.17261E-01 3.74064E-03 2.09554E-01 4.78854E-03 2.01613E-01-2.15845E-03 3.23816E-01-6.11582E-03 3.87713E-01 1.67754E-04
2.55066E-01-1.28055E-02 5.67428E-01-1.30575E-02 2.34814E-01-1.71986E-02 4.39353E-01-1.77942E-02-2.64790E-01-7.61677E-03
5.62942E-01-1.86250E-02-4.75691E-01 1.34288E-02 2.50000E-01 0. 4.20779E-01-1.46708E-03 7.34520E-01-1.05166E-02
4.12510E-01-1.91731E-02 7.65900E-01-2.81871E-02-5.06411E-01 2.99491E-04

EIGENVALUE= 2.64673E-01 -1.71390E-02 G = -.06476 FREQUENCY RATIO = .51473 STIFFNESS PARAMETER = .25737
EIGENVECTOR
4.37906E-03-1.74077E-03 5.37378E-02-3.68016E-03 7.65742E-02-1.81183E-02 2.11679E-01-3.34171E-02 2.99809E-01-7.69874E-03
5.56061E-01-4.77464E-03 6.17164E-01-6.15874E-02 8.24343E-01-4.75925E-02 8.71307E-01-2.55735E-02 4.16254E-02-1.58685E-02
1.28022E+00-8.57642E-02-3.32435E-01 1.78597E-02 8.07261E-02-1.97224E-02 2.39815E-01-3.43112E-02 7.36850E-01-6.13420E-03
1.07183E+00-3.93471E-02 1.66740E+00-9.01435E-02-1.18517E-01 9.10428E-03

EIGENVALUE= 9.43066E-02 4.74660E-02 G = .50332 FREQUENCY RATIO = .31614 STIFFNESS PARAMETER = .15807
EIGENVECTOR
3.75718E-02 3.20938E-02 8.54917E-02-2.39622E-03 7.13685E-02-6.41912E-03 1.11777E-01-8.49976E-02 1.11181E-02 4.19844E-02
-1.19614E-01 1.83143E-01 6.38045E-02-2.17715E-02-3.13859E-02 2.07451E-01 3.60966E-01 1.63741E-01 5.67700E-01-2.30429E-01
5.86131E-01-8.42902E-02 6.32171E-01-6.69970E-01 8.33220E-02 6.13906E-03 1.53548E-01-7.86543E-02 6.65757E-02-1.90454E-02
-8.03509E-02 2.44447E-01 7.18405E-01 5.82309E-03 8.75560E-01-5.69090E-01

EIGENVALUE= 8.23575E-02 -4.36470E-02 G = -.52997 FREQUENCY RATIO = .29628 STIFFNESS PARAMETER = .14814
EIGENVECTOR
-6.53250E-02-3.29323E-02-3.08282E-02-1.23946E-01 1.23157E-02-1.53127E-01 1.67850E-01-2.68841E-01-9.31677E-02-8.61446E-03
-2.67302E-01 2.10478E-01 8.41301E-02-1.22756E-01-3.57774E-01 1.98189E-01-2.49966E-01-5.54812E-01 4.23183E-01-9.79671E-01
6.07352E-02-1.12514E+00 1.03143E+00-1.61394E+00 1.84738E-03-1.60736E-01 1.43845E-01-2.90617E-01 6.91950E-02-1.28770E-01
-4.18761E-01 2.20633E-01-1.53191E-02-1.31363E+00 9.67508E-01-1.83985E+00

EIGENVALUE= 9.39977E-02 -2.09312E-02 G = -.92270 FREQUENCY RATIO = .30845 STIFFNESS PARAMETER = .15422
EIGENVECTOR
1.13344E-01 4.51403E-02 1.73698E-01 1.76758E-01-1.92134E-01-7.48691E-02-3.89458E-01-3.08710E-01 9.36626E-03 5.83171E-01
-5.99415E-01 7.17481E-01-3.51915E-01-3.24207E-01 1.84633E-01-1.05730E+00-8.97665E-01 2.37370E-02-3.64748E-01-1.78309E+00
5.34947E-01-2.16185E+00 1.87897E+00-1.15457E+00-1.77927E-01-1.36908E-01-2.57576E-01-3.98383E-01-8.28149E-01-7.26884E-01

2.12438E-01-5.99033E-01 1.80346E-01-2.09634E+00 4.10725E-01-1.45633E+00

DENSITY RATIO MU= .07500

EIGENVALUE= 4.28697E-01 -1.53065E-02 G = -.03570 FREQUENCY RATIO = .65495 STIFFNESS PARAMETER = .32743
EIGENVECTOR
9.91578E-02 6.90697E-03 2.19630E-01 1.14132E-02 2.10383E-01-4.01199E-03 4.01410E-01-1.56619E-02 5.03521E-01-1.61673E-03
5.14823E-01-3.91934E-02 8.11365E-01-4.56170E-02 5.28036E-01-7.34697E-02 7.91200E-01-6.28837E-02-2.78021E-01-2.03095E-02
1.04783E+00-9.65312E-02-6.64743E-01 2.42660E-02 2.50000E-01 0. 5.00525E-01-7.07869E-03 1.02425E+00-4.03991E-02
8.81877E-01-7.85979E-02 1.41101E+00-1.23976E-01-6.07372E-01-1.20143E-02

EIGENVALUE= 4.06318E-01 -4.62001E-02 G = -.11356 FREQUENCY RATIO = .63885 STIFFNESS PARAMETER = .31942
EIGENVECTOR
1.56612E-01-3.31363E-01 3.90745E-01-7.49090E-01 3.05106E-01-7.53539E-01 7.05247E-01-1.47245E+00 9.97612E-01-1.78519E+00
1.15010E+00-1.87362E+00 1.56239E+00-2.95974E+00 1.36144E+00-2.29837E+00 1.70938E+00-2.89987E+00-5.03444E-01 9.05852E-01
2.29079E+00-3.90012E+00-1.24198E+00 2.37998E+00 3.99185E-01-8.85854E-01 8.92579E-01-1.78411E+00 1.99059E+00-3.69477E+00
2.90456E+00-3.25579E+00 3.07201E+00-5.22387E+00-1.08923E+00 2.10410E+00

EIGENVALUE= 1.01304E-01 6.52020E-02 G = .64363 FREQUENCY RATIO = .33300 STIFFNESS PARAMETER = .16650
EIGENVECTOR
4.70299E-02 3.69425E-02 1.03130E-01-1.48487E-02 8.30204E-02-2.57378E-03 1.84279E-01-9.69425E-02 1.57537E-02 3.75457E-02
-1.23006E-01 2.07605E-01 6.87356E-02-6.04737E-03-3.40678E-02 2.65498E-01 3.58183E-01 1.45538E-01 5.96127E-01-2.98579E-01
5.60940E-01-8.71078E-12 5.13956E-01-7.43003E-01 9.83163E-02 1.45918E-02 1.77531E-01-8.51452E-02 7.61216E-02-3.19837E-03
-9.02798E-02 3.09101E-01 6.94834E-01 1.96797E-02 7.63207E-01-6.73596E-01

EIGENVALUE= 8.93778E-02 -6.13055E-02 G = -.69297 FREQUENCY RATIO = .31474 STIFFNESS PARAMETER = .15737
EIGENVECTOR
-6.05321E-02-3.77777E-02-1.21006E-02-1.31507E-01-2.20041E-02-1.46934E-01 1.12686E-01-2.66281E-01-5.93163E-02-7.02856E-01
-2.43573E-01 2.31092E-01 1.33665E-02-1.21737E-01-3.81043E-01 2.09726E-01-2.71748E-01-4.42783E-01 3.14103E-01-8.20631E-01
-1.19456E-01-9.07480E-01 8.21764E-01-1.26643E+00-3.84838E-02-1.57825E-01 9.14718E-02-3.03496E-01 3.83139E-03-1.27535E-01
-4.31208E-01 2.61967E-01-2.32374E-01-1.05828E+00 7.46103E-01-1.47328E+00

EIGENVALUE= 1.00812E-01 -2.73917E-02 G = -.27334 FREQUENCY RATIO = .31945 STIFFNESS PARAMETER = .15973
EIGENVECTOR
1.17849E-01 5.19427E-02 1.51466E-01 2.04656E-01-1.89757E-01-3.48161E-02-3.87223E-01-2.38992E-01-1.05452E-01 4.61638E-01
-7.26005E-01 3.59766E-01-1.67419E-01-7.51564E-01 5.97041E-01-9.30731E-01-7.74589E-01-1.85602E-01 9.29656E-02-1.33174E+00
1.02014E+00-8.18109E-01 1.15032E+00 7.55451E-01-1.60316E-01-8.90751E-02-2.17433E-01-3.00128E-01-5.77106E-02-6.04771E-01
4.95298E-01-4.40683E-01 6.59070E-01-8.33513E-01 4.69059E-01 1.65958E-02

DENSITY RATIO MU= .08500

EIGENVALUE= 4.53302E-01 1.12032E-01 G = .24715 FREQUENCY RATIO = .67832 STIFFNESS PARAMETER = .33916
EIGENVECTOR
-5.14835E-02 2.64032E-02-8.94650E-02 7.87637E-02-7.29098E-02 7.40772E-02-1.07526E-01 1.63074E-01-1.31170E-01 2.14023E-01
-2.85796E-02 2.59027E-01-1.48123E-01 3.50530E-01 3.14786E-02 2.98216E-01-5.40799E-02 3.70376E-01 1.37772E-01-1.15778E-01
-4.15239E-02 4.77348E-01 1.55138E-01-3.07168E-01-9.35313E-02 8.33370E-02-1.49746E-01 1.95971E-01-2.09430E-01 4.37918E-01
-1.60196E-02 4.36990E-01-6.93967E-02 6.49781E-01 2.14071E-01-2.63845E-01

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EIGENVALUE=  4.20653E-01 -1.91739E-01     G =  -.42596  FREQUENCY RATIO =  .60724  STIFFNESS PARAMETER =   .33362
EIGENVECTOR
7.06038E-02-1.33011E-01 2.04912E-01-2.26934E-01 1.65846E-01-2.17265E-01 4.10337E-01-3.[illegible] 5.31832E-01-3.47691E-01
6.19911E-01-1.04004E-01 8.[illegible]-01-1.75691E-01 7.43647E-01 6.84663E-03 8.97485E-01-2.16[illegible]-01-2.87[illegible]E-01 3.13239E-01
1.20682E+00-2.21[illegible]E-01-7.43219E-01 4.61932E-01 2.13[illegible]-01-3.71261E-01 [illegible]-01-4.25[illegible]-01 1.09737E+00-5.24705E-01
1.07623E+00-1.07558E-01 1.61432E+00-3.29700E-01-6.55098E-01 5.59429E-01

EIGENVALUE=  1.07486E-01  8.16771E-02     G =   .75989  FREQUENCY RATIO =  .34820  STIFFNESS PARAMETER =   .17410
EIGENVECTOR
-8.32747E-02 3.10637E-02-1.37933E-01 7.31643E-02-1.15187E-01 3.08830E-02-1.27010E-01 1.89958E-01-3.[illegible]E-02-3.65371E-02
7.92718E-02-2.27[illegible]E-01-6.5[illegible]E-02 2.12[illegible]E-02-5.48459E-02-3.91[illegible]-01-4.[illegible]E-01-3.52[illegible]E-02-4.[illegible]E-01 6.[illegible]E-01
-6.51139E-01 2.72490E-01-2.43301E-01 1.13551E+00-1.45368E-01 1.30483E-02-2.09733E-01 1.91831E-01-1.[illegible]E-01 9.11206E-02
-6.40624E-03-4.74705E-01-8.50311E-01 2.17735E-01-5.72095E-01 1.16927E+00

EIGENVALUE=  9.47[illegible]E-02 -7.87859E-02     G =  -.83124  FREQUENCY RATIO =  .23018  STIFFNESS PARAMETER =   .16509
EIGENVECTOR
1.96334E-02 5.03[illegible]E-02-5.44[illegible]E-02 8.60028E-02-3.61831E-02 9.84935E-02-1.61350E-01 1.13900E-01 8.50698E-02 2.13[illegible]
2.29797E-01-4.68[illegible]E-02-3.54[illegible]E-02 6.82558E-02 3.80006E-01 2.30[illegible]E-02 9.63[illegible]E-0[illegible] 3.28[illegible]E-01-4.55[illegible]E-01 3.0[illegible]
-2.05[illegible]E-01 5.21105E-01-8.7[illegible]E-01 3.50283E-01-2.82737E-02 1.19443E-01-1.[illegible]E-01 1.59401E-01-3.40[illegible]E-02 7.62531E-02
3.68[illegible]E-01 6.07[illegible]E-03-1.91[illegible]E-01 6.42041E-01-9.03970E-01 4.59441E-01

EIGENVALUE=  1.03068E-01 -3.347[illegible]E-02     G =  -.30571  FREQUENCY RATIO =  .33257  STIFFNESS PARAMETER =   .16623
EIGENVECTOR
-6.46117E-02-4.75364E-02-5.66690E-02-1.46457E-01 1.10670E-01 3.88331E-02 2.04610E-01 1.83853E-01 1.47415E-01-2.33763E-01
4.72[illegible]E-01-1.21[illegible]E-02-5.21144E-02 4.33[illegible]E-01-5.95553E-01 3.33784E-01 3.40824E-01 2.64[illegible]E-01-3.5[illegible]E-01 6.87[illegible]E-01
-5.54[illegible]E-01 1.46[illegible]E-02-2.35[illegible]E-01-9.76[illegible]E-01 8.15427E-02 6.[illegible]E-02 8.44427E-02 1.84914E-01-6.[illegible]E-02 3.20[illegible]E-01
-1.03[illegible]E-01 1.27[illegible]E-0[illegible]-4.84[illegible]E-01 1.08187E-01-9.04558E-02-2.59[illegible]E-01

DENSITY RATIO MU=   .09500

EIGENVALUE=  4.43391E-01 -2.50[illegible]E-01     G =  -.57592  FREQUENCY RATIO =  .69103  STIFFNESS PARAMETER =   .34552
EIGENVECTOR
9.54315E-03 9.70070E-02-3.68[illegible]E-02 1.96538E-01-2.62177E-02 1.72520E-01-1.19867E-01 2.85416E-01-1.76700E-01 3.220[illegible]E-01
-2.06[illegible]E-01 1.75[illegible]E-01-2.34[illegible]E-01 4.5[illegible]E-01-4.01970E-01 1.16775E-01-4.[illegible]E-01 2.80711E-01 7.11[illegible]E-02-2.6[illegible]E-01
-5.57190E-01 3.39[illegible]E-01 2.79[illegible]E-01-4.35484E-01-2.12576E-02 2.10[illegible]E-01-1.34056E-01 3.693[illegible]E-01-3.95000E-01 5.96985E-01
-5.46[illegible]E-01 2.52[illegible]E-01-7.39[illegible]E-01 4.77546E-01 2.04350E-01-4.89404E-01

EIGENVALUE=  4.81690E-01  1.77629E-01     G =   .36876  FREQUENCY RATIO =  .7[illegible]537  STIFFNESS PARAMETER =   .35268
EIGENVECTOR
8.66629E-02 1.03748E-02 1.69329E-01-3.14827E-02 1.41098E-01-3.23003E-02 2.39667E-01-1.22312E-01 2.90172E-01-1.74502E-01
1.54195E-01-3.13321E-01 3.75730E-01-3.24466E-01 7.48056E-02-3.92909E-01 2.33506E-01-4.17000E-01-2.64314E-01 6.42402E-02
2.54600E-01-5.40324E-01-3.73[illegible]E-01 2.73[illegible]E-01 1.74680E-01-2.57566E-02 3.12811E-01-1.30480E-01 5.04[illegible]E-01-3.90477E-01
2.06356E-01-5.33[illegible]E-01 3.48[illegible]E-01-7.35029E-01-4.47745E-01 2.01730E-01
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EIGENVALUE= 1.13320E-01 9.73798E-02 G = .85933 FREQUENCY RATIO = .36245 STIFFNESS PARAMETER = .18122
EIGENVECTOR
8.46356E-02 5.48869E-03 1.08699E-01-1.08206E-01 9.55197E-02-5.69047E-02 6.11954E-02-1.94183E-01 4.20102E-02 1.83424E-02
1.79628E-02 3.01013E-01 7.42469E-02-3.36914E-02 1.63030E-01 3.33672E-01 4.02712E-01-9.70787E-02 2.01375E-01-6.20915E-01
4.56965E-01-3.69276E-01-1.53614E-01-9.77852E-01 1.29402E-01-4.59769E-02 1.34629E-01-3.37412E-01 8.07385E-02-3.78187E-02
1.33086E-01 4.21366E-01 6.33038E-01-3.75547E-01 9.62209E-02-1.10091E+00

EIGENVALUE= 9.96124E-02 -9.47821E-02 G = -.95131 FREQUENCY RATIO = .34431 STIFFNESS PARAMETER = .17215
EIGENVECTOR
7.18246E-03 9.84635E-02-1.44047E-01 1.19257E-01-9.94265E-02 1.51773E-01-3.14180E-01 1.12531E-01 2.20328E-02 4.47269E-02
3.97766E-01 2.37798E-02-7.14356E-02 9.89583E-02 5.31431E-01 1.85096E-01-1.30668E-01 5.05516E-01-9.81354E-01 2.43348E-01
-4.02630E-01 7.10066E-01-1.44482E+00 2.73233E-02-9.11390E-02 1.93772E-01-8.49033E-01 1.78914E-01-7.52334E-02 1.14095E-01
6.19214E-01 1.74487E-01-5.17358E-01 9.06008E-01-1.55768E+00 2.35240E-01

EIGENVALUE= 1.16445E-01 -3.94212E-02 G = -.33854 FREQUENCY RATIO = .34596 STIFFNESS PARAMETER = .17298
EIGENVECTOR
1.07258E-01 4.47055E-02 1.24316E-01 1.89934E-01-1.60449E-01-1.82903E-02-3.51617E-01-1.87345E-01-1.49195E-01 3.43489E-01
-6.50410E-01 8.60978E-02-2.01115E-02-5.96301E-01 7.30495E-01-6.78343E-01-4.72423E-01-3.07315E-01 4.34327E-01-5.79693E-01
8.59934E-01 1.08947E-02 3.22782E-01 1.55176E+00-1.32897E-01-5.68395E-02-1.78236E-01-2.15110E-01 4.53045E-02-4.39453E-01
5.67783E-01-2.55322E-01 5.83367E-01-8.94748E-02 8.05739E-02 5.65670E-01