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LASER SYSTEM RELIABILITY

The BDM Corporation
Albuqueruqe, NM 87102

March 1977

Final Report

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AIR FORCE WEAPONS LABORATORY
Air Force Systems Command
Kirtland Air Force Base, NM 87117

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This technical report has been reviewed and is approved for publication.

Jerry D Wilcox

JERRY D. WILCOX
Major, USAF
Project Officer

FOR THE COMMANDER

Robert B. Mollo

ROBERT B. MOLLO
Lt Colonel, USAF
Chief, Laser Systems Analysis Branch

Louis H Bernasconi

LOUIS H. BERNASCONI
Colonel, USAF
Chief, LEAPS Division

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A study was conducted to develop techniques for considering reliability in the HEL (High Energy Laser) program at the Air Force Weapons Laboratory. This effort was specifically oriented toward developing reliability methodologies rather than toward performing complete reliability analyses with accurate numerical predictions. A reliability model was developed and a sufficiently detailed failure data base was established to allow application of the model to the three principal HEL systems under consideration: the GDL

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20. ABSTRACT

(gas dynamic laser), the CL (chemical laser), and the EDL (electrical discharge laser). Then methodologies were developed for treating the interactions of reliability with such closely related factors as life cycle cost and combat effectiveness. This pholosophy has resulted in a study with an extremely wide scope, but with a very low level of detail.

PREFACE

This report satisfies the requirements for Final Report (A004) under contract F29601-76-C-0145 (Laser System Reliability). The effort was performed by Mr. Paul W. Dueweke, Dr. Robert M. Smith, Dr. Roger K. Hoppe, Mr. Darrell E. Spreen, Mr. H. George Pringle, and Mr. David P. Vanarsdall at The BDM Corporation. The Project Officer was Major Jerry D. Wilcox (AFWL/PGA).

PROGRAM SUMMARY

To briefly summarize the Laser System Reliability Program, there are four areas of significant contributions to the reliability effort.

First, a HEL reliability prediction methodology was developed which includes some new concepts in the areas of statistical analysis as applied to reliability modeling. The methodology is general enough that it can be applied to any HEL system of interest. Both the physical description and the failure data base can be easily modified to accommodate both changes in physical configuration such as redundancies and deletion or addition of components, and changes in the failure data used. The mathematics of the reliabilty calculation are a modification of a classical statistics approach developed in 1972[1]. The technique synthesizes a pseudo-number of trials and successes for a conceptual HEL system based upon existing part failure rates. A synthesis technique is necessary because the HEL system under investigation is nonexistent. Thus, reliability inferences must be drawn based on existing data. In addition, the approach can be used to calculate confidence intervals for the reliability point estimates. The technique can handle a system of any complexity and requires no sophisticated computer programs for evaluation.

Second, some preliminary reliability point estimates and their associated lower 95 percent confidence bounds were calculated for three HEL systems based upon the physical system description and the failure data base acquired.

Third, methodologies were developed to consider cost and effectiveness as they apply to reliability of a conceptual system. An original marginal cost/marginal benefit analysis was developed to combine the costs and benefits due to LCC and availability producing a net marginal cost function from which the optimal reliability is derived (zero net marginal cost). A simplified combat effectiveness model was constructed to relate reliability to a measure of effectiveness such as exchange ratio. The effectiveness function along with the net marginal cost function are the inputs used in the reliability optimization.

---

[1] Easterling, R. G. "Approximate Confidence Limits for System Reliability," Journal of the American Statistical Association, March 1972.

Fourth, a reliability apportionment methodology was developed which considers both the criticality of parts and components and the budgeting of reliabilities among the subsystems and components to achieve the reliability objective defined in the optimal reliability decision process. A FMEA (Failure Modes and Effects Analysis) was developed which was a modification of one developed by the SAE (Society of Automotive Engineers)[2]. A criticality number was derived from the FMEA which identifies the importance of a subsystem or component assuming a failure has occurred (i.e., it separates criticality issues from reliability issues). Finally, a technique was developed to aid in the reliability apportionment among any combination of subsystems or components.

---

[2] Aerospace Recommended Practice (ARP 926), "Design Analysis for Failure Mode, Effects and Criticality Analysis," SAE, September 15, 1967 (see Appendix E, Proposal for LSR, BDM Corporation, BDM/A-75-458-PRP-0141, 26 April 1976).

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SECTION I
INTRODUCTION

A. LASER SYSTEM RELIABILITY PROBLEM

Reliability considerations are important inputs to the R&D stage of the weapon system acquisition cycle since they provide necessary inputs to comparative analyses, cost/benefit trade-offs, and system simulations. In addition, the DCP (Development Concept Paper) and the DSARC (Defense Systems Acquisition and Review Council) require reliability considerations to support the management decision at the Secretary of Defense level. The Air Force Weapons Laboratory has begun a reliability analysis effort which is designed to meet the above requirements and to provide insights into the strengths and weaknesses of the available failure data base and system description. The first phase of this reliability program was to develop a set of methodologies addressing reliability and such related factors as life cycle cost and combat effectiveness.

The remainder of this chapter addresses the following topics:

(1) Presentation of the relationship between reliability and the management of a HEL weapon system acquisition.

(2) Brief description of the LSR (Laser System Reliability) contract effort,

(3) Definition of terms used in the development of the technical approaches, and

(4) Roadmap to direct the reader through the rest of this report.

B. RELIABILITY INTERACTION WITH MANAGEMENT

This section describes how reliability, in a general sense, plays into the overall development of an optimally designed HEL weapon system. From this description a better appreciation will be developed for the ways in which reliability considerations impact costs, benefits, and management decisions. Figure 1 presents an overall flowchart of how the reliability, cost, and benefit considerations provide critical inputs to the HEL Management Program, and how the HEL Management Program in turn drives toward the goal of generating the Optimal HEL Weapon System Design.

The central horizontal thrust of figure 1 begins with the HEL Baseline Design and ends with the Optimal HEL Waapon System Design. This Baseline Design may be a conceptual design such as that derived from Cycle IV or from the SRAT (Short Range Applied Technology) Program. The HEL Management Program will probably reside in different places as the acquisition matures. The HEL Management Program provides the decision making force which drives the system from a baseline design to the optimal design. The LCC (Life Cycle Cost), Availability, and Combat Effectiveness considerations which flow into the HEL Management Program perform various analytical tasks which define the costs and the benefits of reliability changes. These three factors, when taken together, provide the HEL Management Program with a reliability goal which is optimized for combat effectiveness, availability, and LCC.

The Reliability Model provides the reliability inputs to these three cost/benefit analyses. The Reliability Model, in turn, is constructed by consideration of the next higher level factors. These factors include the generation of a Functional Block Diagram, a Reliability Block Diagram, a Failure Data Base, and the statistical Confidence Bounds on the reliability Point Estimates.

Thus the structure flowing into the HEL Management Program from above performs the function of analyzing the system trade-offs with respect to reliability to determine a reliability goal for achieving the optimal design.

The level flowing into the HEL management Program from below distirbutes the reliability changes among the HEL subsystems or components. If a system is composed of a number of subsystems, it would be desirable to put a reliability goal on each of those subsystems, or on the appropriate combination, to achieve the total system reliability goal.

The Subsystem Reliability Apportionment has two inputs. The first is a set of Design Guidelines which defines the engineering techniques which are available for achieving increased reliability. The second is a Criticality Ranking system which ranks each of the subsystems in terms of its criticality to the entire HEL system assuming that a failure of that subsystem has occurred. This acknowledges the fact that failures of some subsystems have much more serious repercussions on the HEL system than do failures of other subsystems.

Looking at the flowchart as a whole then, one can see three main thrusts. The first is the thrust towards an Optimal HEL Weapon System Design which is driven by the HEL Management Program. The second thrust from above determines the reliability goal which optimizes the Combat Effectiveness, Availability, and Life Cycle Cost of the system and serves as an input to the HEL Management Program. This reliability objective is the product of a system analyst whose task is to answer the question, "What is the reliability objective which optimizes the system?" The third thrust from below provides the necessary inputs for determining how the reliability goal can be accomplished. These inputs are developed by the system designer whose task is to answer the question, "How can I achieve this reliability objective?"

The entire process outlined in figure 1 is not a one-step process, but requires a considerable degree of iteration for it to function effectively. Thus, the system analyst may be required to perform a number of analyses based upon complicated options presented by the system designer. Even more significantly, the HEL program manager may require several iterations of the reliability goal and the reliability apportionment before he is satisfied that the overall HEL mission is best served.

Figure 1, then, demonstrates the importance of the reliability function in the design of a complete weapon system. It shows how the

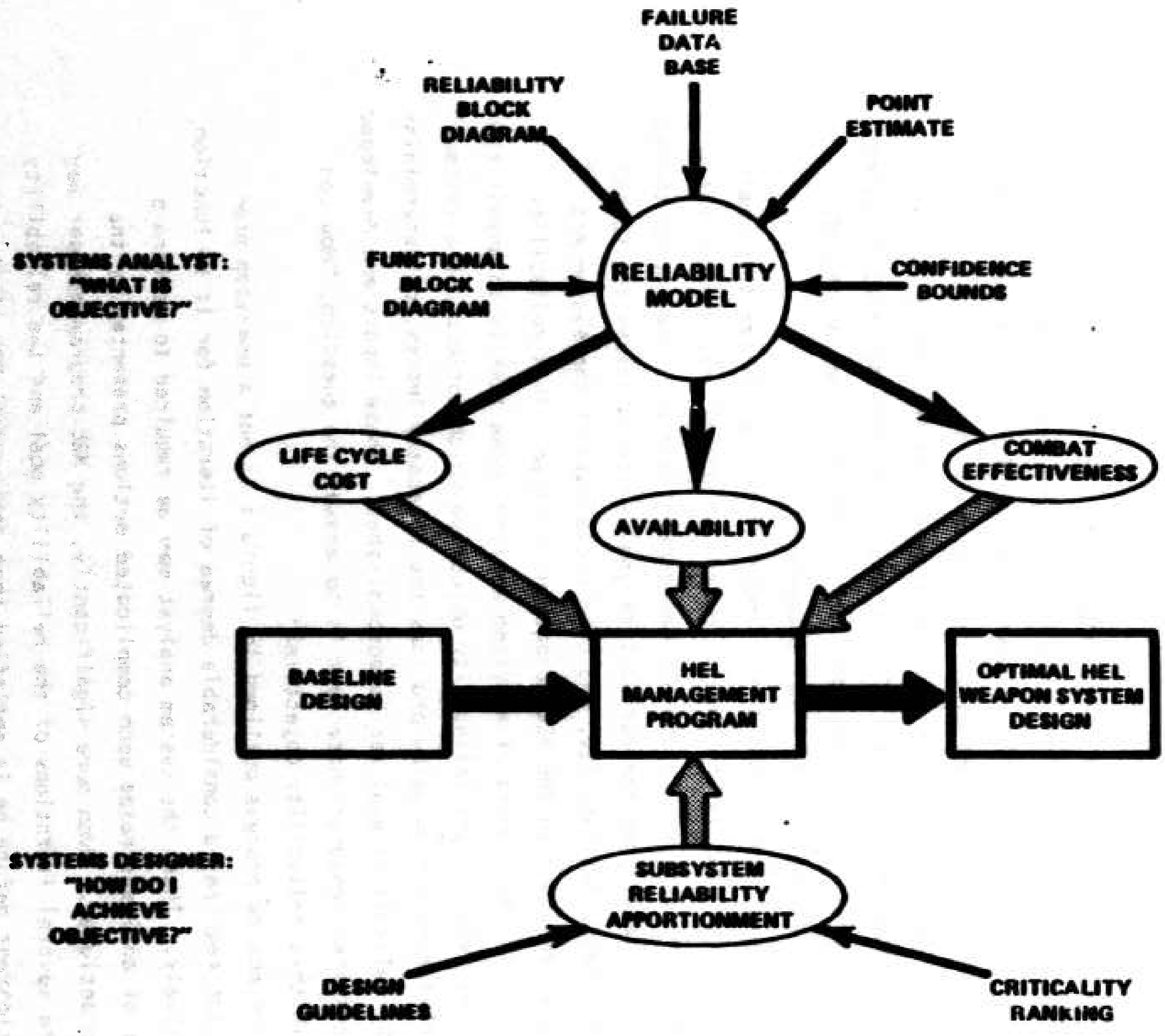

Figure 1. Reliability Interactions with HEL Management Program

reliability considerations directly influence the cost and benefit considerations, and how these in turn influence the HEL Management Program whose goal is to achieve an optimal weapon system design. Other studies have concluded that early consideration of reliability in a weapon system acquisition cycle can save significant amounts of LCC and also increase the effectiveness and the availability of the force.[3] This reliability awareness must begin very early in the acquisition cycle so that system design options will not be precluded.

C. LASER SYSTEM RELIABILITY PROGRAM

The LSR contract was a 4-month effort which addressed the major areas which both influence, and are influenced by, reliability. This initial effort was specifically oriented toward developing reliability methodologies rather than toward performing complete reliability analyses with accurate numerical predictions. Thus, the development of techniques for a wide range of reliability considerations was emphasized. A reliability model was developed and a sufficiently detailed failure data base was established to allow application of the model to the three principal HEL systems under consideration: the GDL (gas dynamic laser), the CL (chemical laser), and the EDL (electrical discharge laser). Then methodologies were developed for treating the interactions of reliability with such closely related factors as life cycle cost and combat effectiveness. This philosophy has resulted in a study with an extremely wide scope, but with a very low level of detail. Figure 2 shows a flowchart of the three tasks in this contract.

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[3] "Dormant Operations and Storage Effects on Electronic Equipment and Part Reliability," RADC-TR-66-348, Contract AF 30(602)-3772, October 1966.

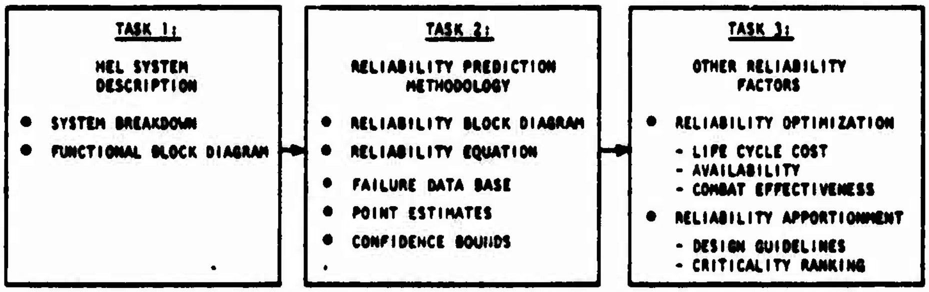

Figure 2. LSR Task Flowchart

Task 1 was to accurately describe a conceptual HEL system which would be used for a SRAT mission (i.e., the kind of system which more closely approximates a weapon system rather than a laboratory system). This physical system description decomposed the HEL system into three major subsystems: the laser device or beam generator, the BCO (beam control optics), and the FCS (fire control system). The laser devices described were the GDL, the CL, and the EDL. These five subsystems were then further decomposed into the components and finally to the part level of detail.[4] The resultant FBD (functional block diagram) shows how all of these subsystems and components interact with each other to generate the three HEL systems.

Task 2 was to develop a reliability prediction methodology. First, an RBD (reliability block diagram) was developed, the purpose of which was to show the failure relationships among subsystems and components so that the effects of a failure could be readily traced through the HEL system. Next, a failure data base was developed. The major sources of this data base were the GIDEP (Government Industry Data Exchange Program), the RADC (Rome Air Development Center) Non-Electronic Reliability Notebook, a variety of open literature publications and Government reports,

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[4] The definitions of the terms "system," "subsystem," "component," and "part" are very important to the understanding of the approaches developed later; so these terms are concisely defined in section D of this chapter.

and various internal documents at the ARTO (Advanced Radiation Technology Office).

Then the reliability equation was derived for each of the subsystems, thus defining five reliability functions in terms of the component reliabilities. This set of reliability equations, along with the technique for generating the reliability estimates from the failure data base, constitutes the actual reliability model. The reliability model was then applied to the three HEL systems to calculate reliability point estimates for each one. In addition, the statistical confidence bounds of the reliability estimates were calculated.

Task 3 was to develop methodologies for the consideration of a number of important reliability related factors. These factors were divided into two parts: (1) the reliability optimization, and (2) the reliability apportionment.

The reliability optimization was performed by analyzing the costs and the benefits attributed to changes in reliability of the HEL system. A methodology was developed to use historical LCC data for previous weapon systems in order to determine the sensitivity of LCC to reliability. The benefits were decomposed into two parts: the dollars saved from force reduction due to availability increase with an increase in reliability, and the exchange ratio increase due to increased combat effectiveness as a result of reliability increases.

The reliability apportionment was partitioned into three parts. The first was to identify specific design guidelines, that is, engineering methods for increasing system reliability. The second was to perform a criticality ranking which could be applied to the subsystems or components, thus ranking them in order of importance to the HEL system should a failure occur. Third, a reliability apportionment technique was developed which assists in budgeting the optimal reliability goal among the various subsystems and components.

D. DEFINITIONS

The hierarchy of levels of the physical system breakdown plays an important role in the total understanding of the methodologies to be described in detail in later chapters of this report. Thus, it is advisable here to define these levels for future reference.

LEVEL	EXAMPLE	NUMBER OF ELEMENTS IN THIS LEVEL
System	HEL	1
Subsystem	Beam Control System	5
Component	Servo-Hydraulic Drive	91
Part	Hydraulic Pump	273

In addition, the following acronyms will be used throughout the report:

ARTO	Advanced Radiation Technology Office
ALL	Airborne Laser Laboratory
BCS	Beam Control System
CL	Chemical Laser
EDL	Electrical Discharge Laser
ER	Exchange Ratio
FBD	Functional Block Diagram
FCS	Fire Control System
GDL	Gas Dynamic Laser
HEL	High Energy Laser
LSR	Laser System Reliability
LCC	Life Cycle Cost
LWS	Light Weight System
RBD	Reliability Block Diagram
SRAT	Short Range Applied Technology

E. ROADMAP

The remainder of this report will discuss the following items:

(1) Development of the methodologies including:
    a) Functional block diagrams
    b) Model and failure data base
    c) Reliability optimization
    d) Reliability apportionment
(2) Conclusions derived from the technical effort.
(3) Recommendations.

SECTION II

PHYSICAL SYSTEM DESCRIPTION

A. INTRODUCTION

This chapter presents the physical system description that was used to describe the HEL system prior to the construction of the reliability model itself. This chapter includes the results of task I as it was outlined in chapter I. The end product of the task was a functional block diagram of a HEL weapon system. These functional block diagrams are presented along with the reasons why certain choices were made in their development.

B. PHYSICAL SYSTEM DESCRIPTION

Although the functional reliability of a HEL system is the ultimate objective of this study, one must approach this objective by investigating the reliabilities of the individual subsystems, components, and parts. One cannot perform the analysis on functions because functions are abstract and can be performed by more than one set of physical components. The functional reliability depends upon which unique set of components is chosen to perform the function. For example, the threat acquisition function can be performed by the pilot with a pair of binoculars or by a phased array radar system. Obviously, the functional reliability (i.e., the reliability of the acquisition function) depends heavily on which of the two techniques is used. Thus, one must first transfer the function into a physical system description used to achieve that function and then consider the reliabilities of the individual subsystems, components, and parts. These reliabilities can then be combined to yield the reliability of the function.

The technique of physical component breakdown may be further complicated by the additional requirement of correspondence between the physical components and the functions. Ideally, one would desire that a particular function be uniquely related to a particular fixed set of physical components. If a single component contributes to more than one function, then the task of determining that component's reliability relative to each function becomes potentially very difficult.

In this study, this problem was avoided altogether by the judicious choice of functional and component breakdowns. One requirement was that there be a one-to-one correspondence between a function and a set of components. Secondly, the interfaces among components, either within the same function or between two different functions, were carefully chosen, both to be realistic and to minimize the practical and mathematical problems associated with both defining and operating with the reliabilities of intercomponent functions. Thus, the physical component description was constructed considering the constraint that this one-to-one correspondence between components and functions had to be maintained. For this reason, the functional block diagrams resulting from the study are identical to the physical system descriptions.

The structure of the physical system description is presented in figure 3. This figure shows the four levels of the breakdown that were defined in chapter I.

C. FUNCTIONAL BLOCK DIAGRAMS

Since the HEL system is quite complex, a separate FBD was prepared for each subsystem (i.e., there are five FBD's: GDL, CL, EDL, BCS, and FCS). These five FBD's are presented in figures 4 to 8. These FBD's represent a logical breakdown of the subsystems at the component level. Due to constraints in the failure data base, however, the complexity of each component is not reflected in the component breakdown of parts. The ultimate parts breakdown would probably show the BCS and the FCS to have a larger number of parts than any of the beam generating subsystems.

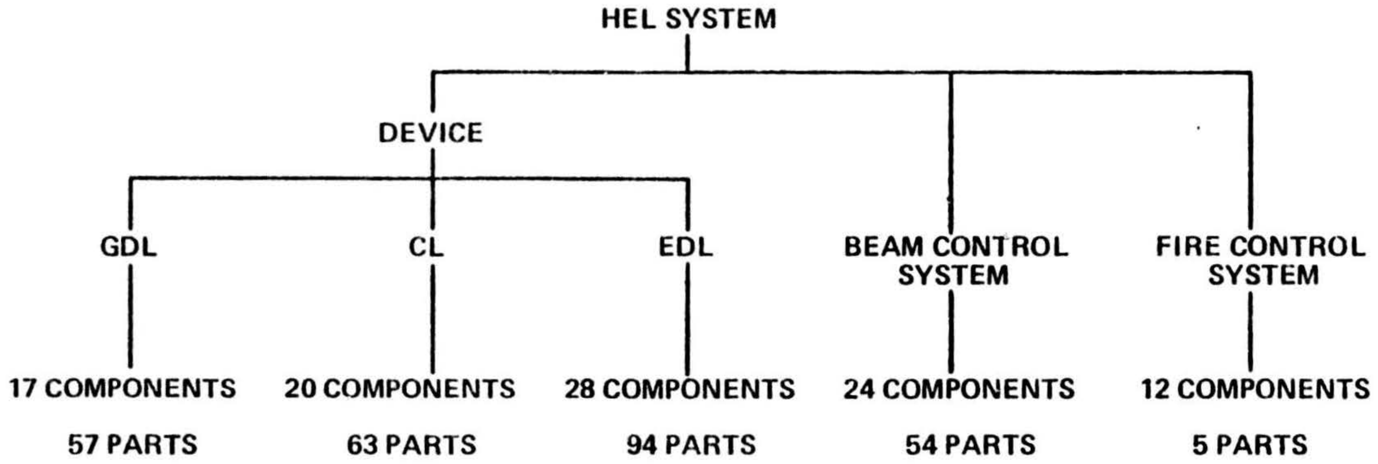


Figure 3. System Description STructure

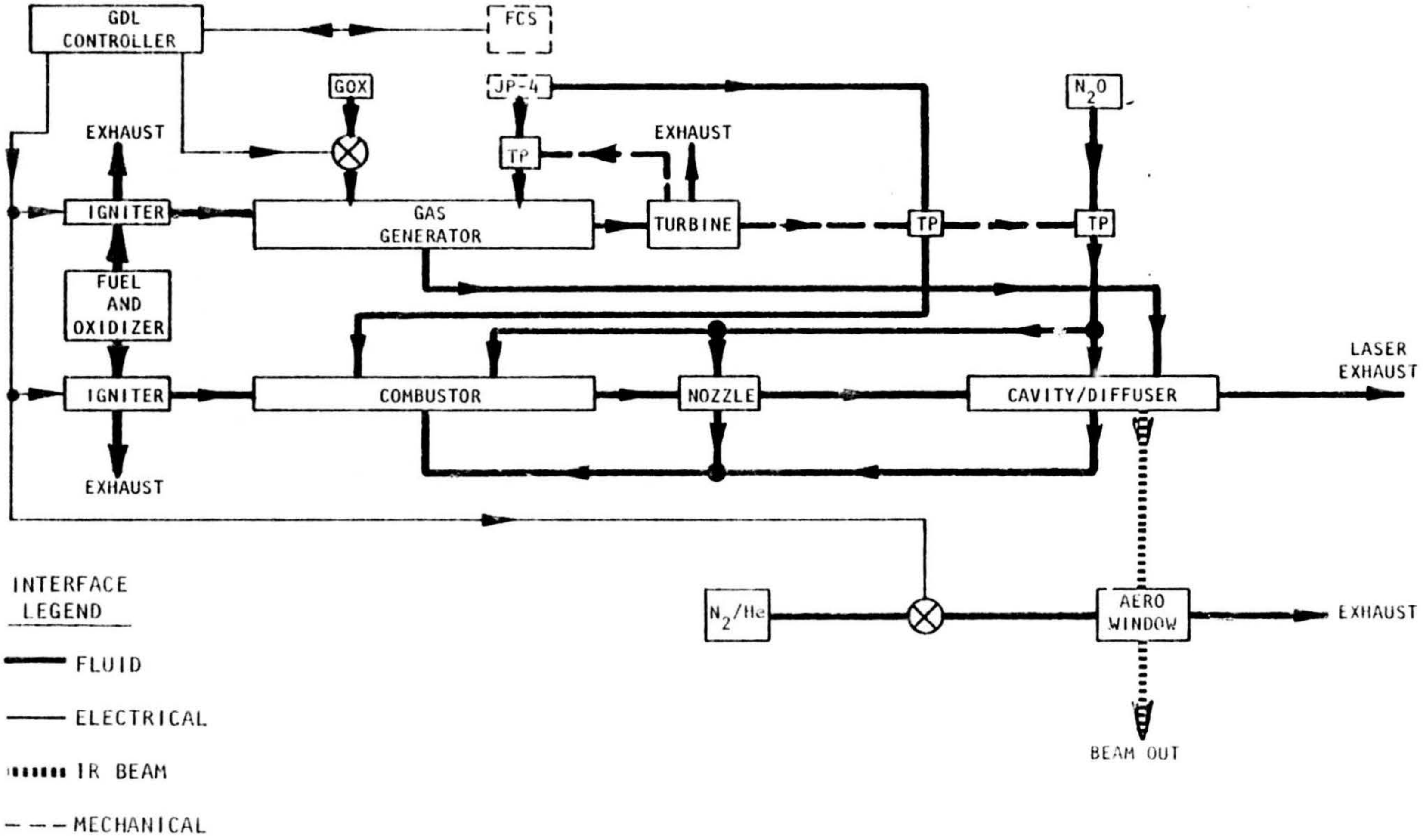


Figure 4. GDL Functional Block Diagram

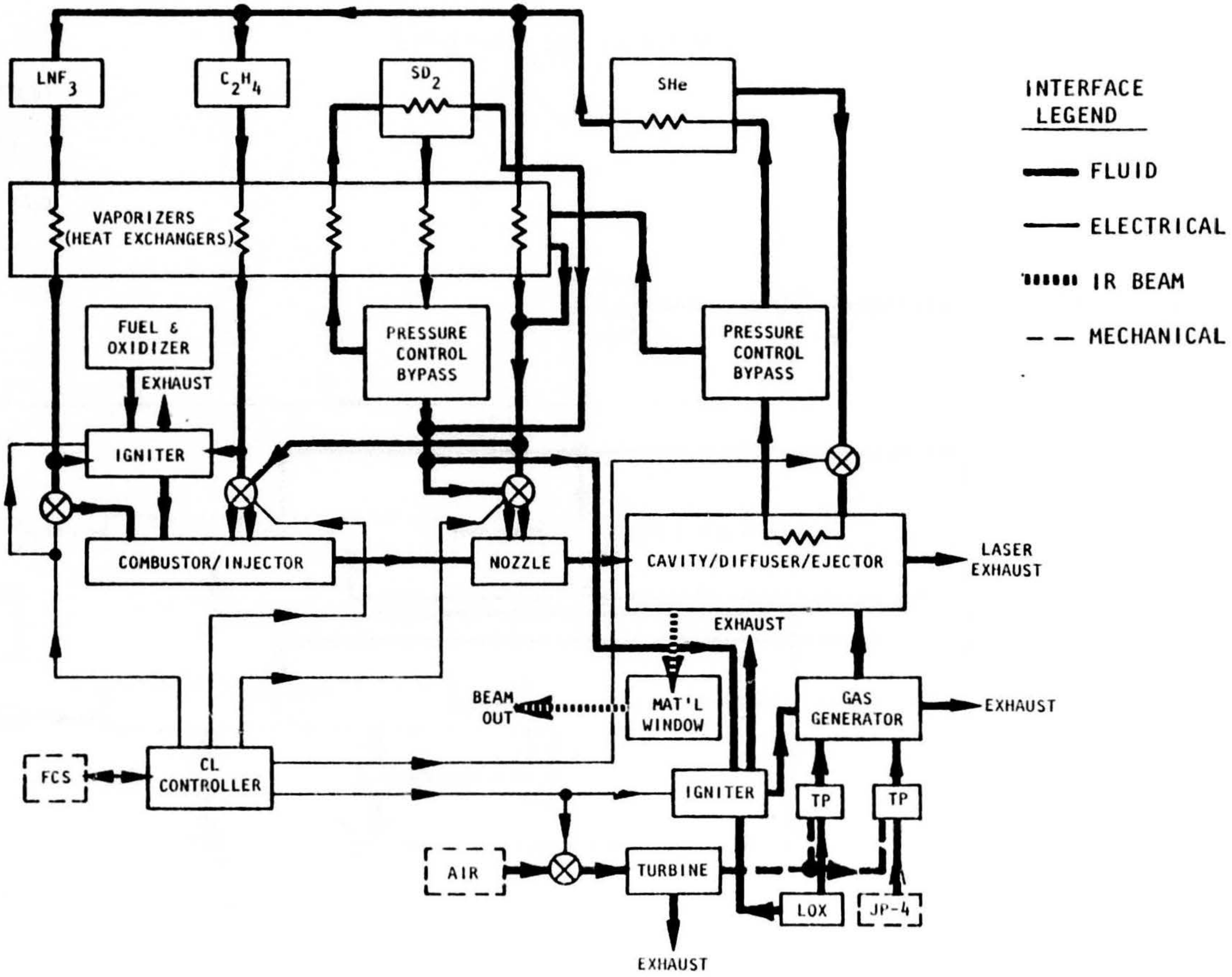
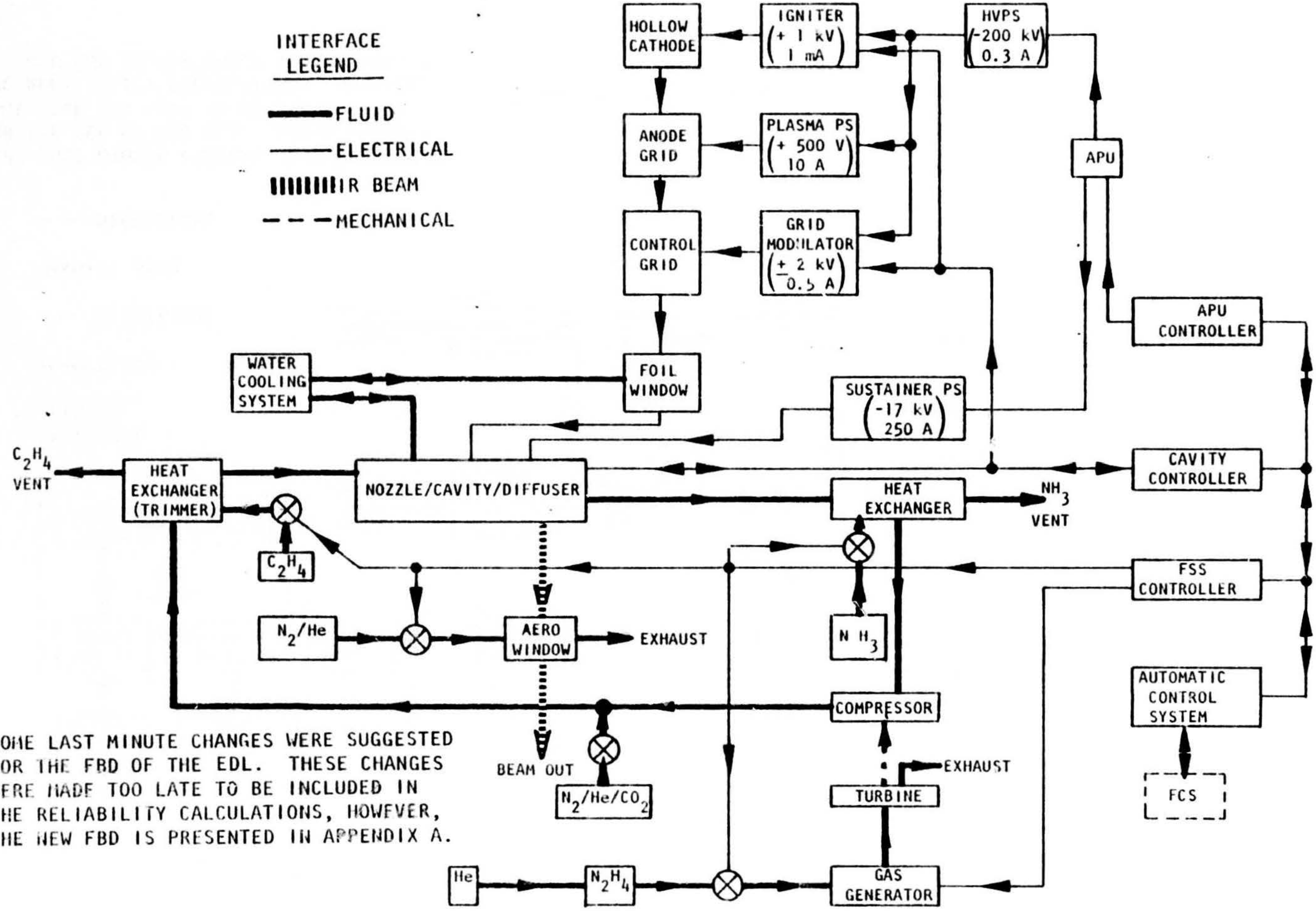
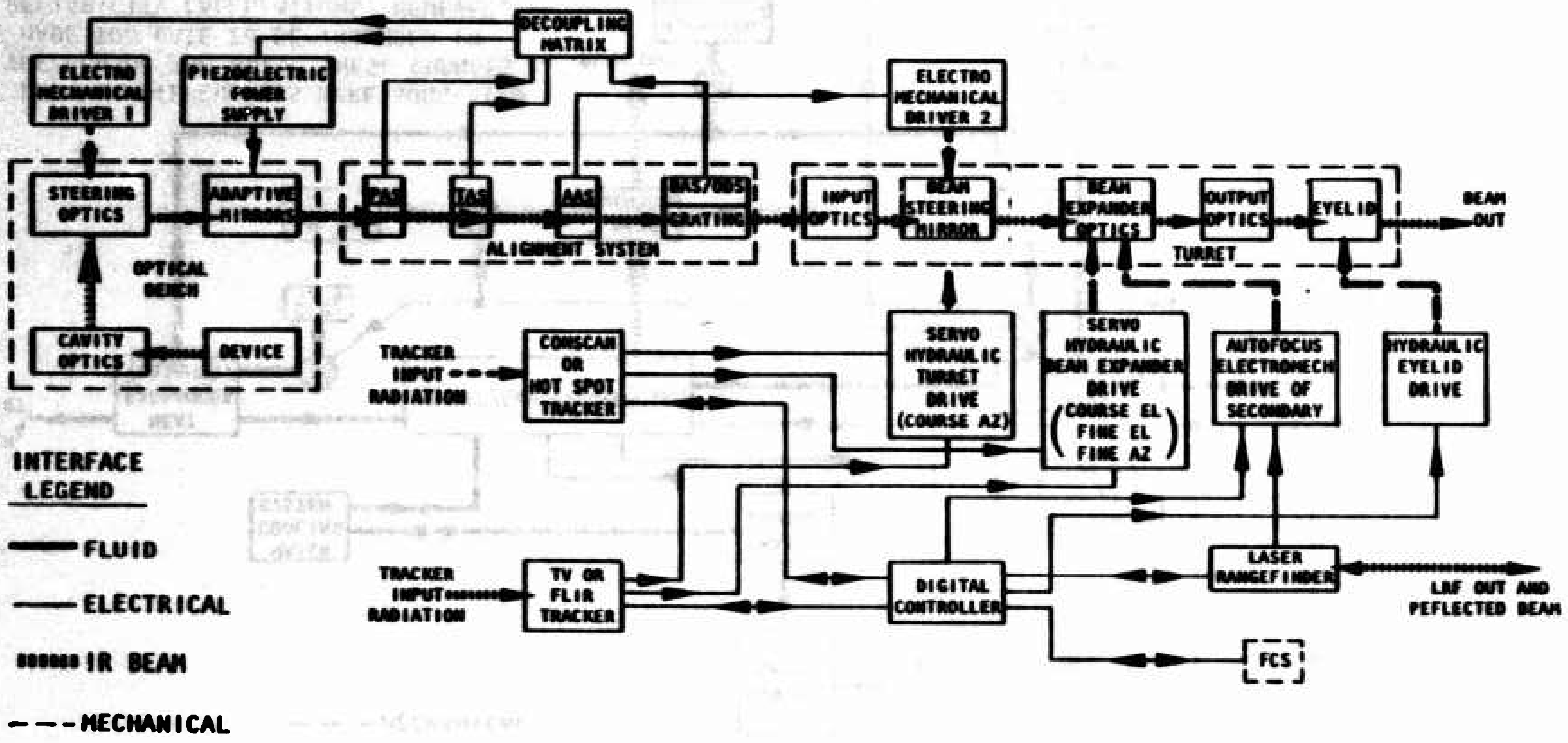


Figure 5. CL Functional Block Diagram



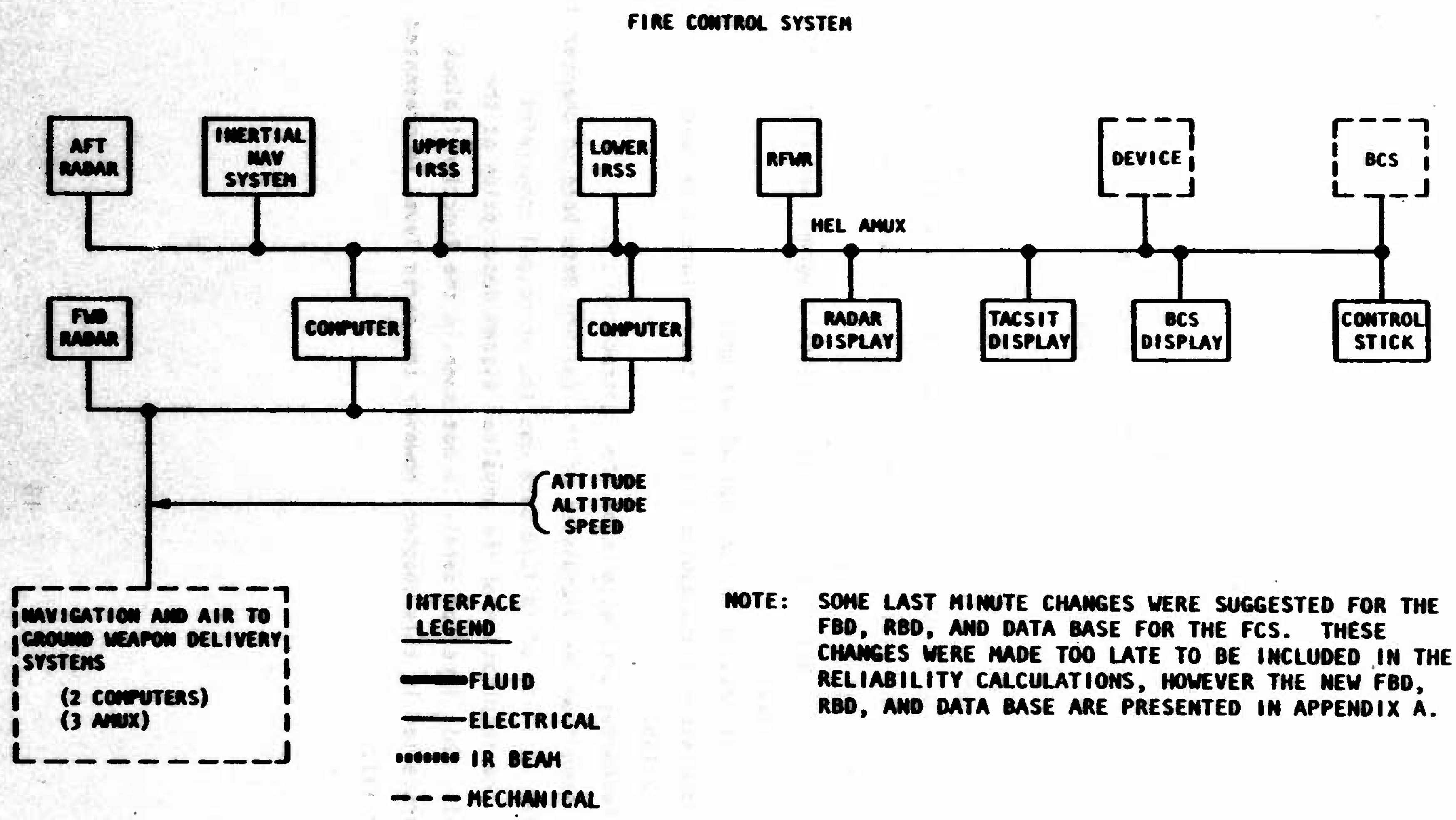
NOTE: SOME LAST MINUTE CHANGES WERE SUGGESTED FOR THE FBD OF THE EDL. THESE CHANGES WERE MADE TOO LATE TO BE INCLUDED IN THE RELIABILITY CALCULATIONS, HOWEVER, THE NEW FBD IS PRESENTED IN APPENDIX A.

Figure 6. EDL Functional Block Diagram



NOTE: SOME LAST MINUTE CHANGES WERE SUGGESTED FOR THE FBD OF THE BCS. THESE CHANGES WERE MADE TOO LATE TO BE INCLUDED IN THE RELIABILITY CALCULATIONS, HOWEVER, THE NEW FBD IS PRESENTED IN APPENDIX A.

Figure 7. BCS Functional Block Diagram



NOTE: SOME LAST MINUTE CHANGES WERE SUGGESTED FOR THE FBD, RBD, AND DATA BASE FOR THE FCS. THESE CHANGES WERE MADE TOO LATE TO BE INCLUDED IN THE RELIABILITY CALCULATIONS, HOWEVER THE NEW FBD, RBD, AND DATA BASE ARE PRESENTED IN APPENDIX A.

Figure 8. FCS Functional Block Diagram

The system configurations presented in these figures are the results of conversations with individuals in each of the subsystem areas. Table 1 presents the sources of the FBD data. In addition to these personal contacts, a number of AFWL and ASD reports were examined.

The configurations of the subsystems were chosen to be representative of a SRAT mission. Thus, it was desired that the system look more like a weapon system than like a laboratory system. Since the SRAT subsystems are very conceptual at this point, many of the components in the subsystems are very uncertain. In addition, many of the components are very mission dependent. Since this model is to have broad application, the attempt was made to include any components for any mission that might be performed by SRAT. Then certain components can be deleted from the FBD for application to a specific mission.

The level of detail of the functional block diagrams was dictated by three constraints:

(1) Completeness of the conceptual designs.

(2) Completeness and applicability of the failure data base available.

(3) Resources available under the contract effort.

It will be seen from the discussion of the failure data base in chapter III that the availability of failure data was the principal constraint leading to the termination of the physical system description at the parts level. This level of detail is not seen in the functional block diagrams presented in this chapter, however the parts level is presented in chapter III.

TABLE 1. SOURCE/CONTACTS FOR FBD DATA

GENERAL	MAJ. JERRY D. WILCOX
GDL	DR. STEVE G. HADLEY MR. JACK COLBERT (PRATT & WHITNEY) MR. MIKE McHALE (ROCKETDYNE)
EDL	LT. COL. PETER D. TANNEN MAJ. FRANK S. ZIMMERMAN MAJ. RICHARD C. OLIVER CAPT. GEORGE W. MAYES
CL	DR. LEROY E. WILSON CAPT. JOHN O'PRAY CAPT. LARRY D. BUELOW
BCS	LT. COL. RONALD F. PRATER LT. COL. R. DALE NEAL MAJ. KEN C. JUNGLING MAJ. OREST R. GOGOSHA
FCS	MAJ. DAVID G. NEALE CAPT. RANDALL D. GODFREY CAPT. JOHN E. ACTON MR. DAVE B. LEMMING (ASD)

SECTION III

RELIABILITY PREDICTION METHODOLOGY

A. INTRODUCTION

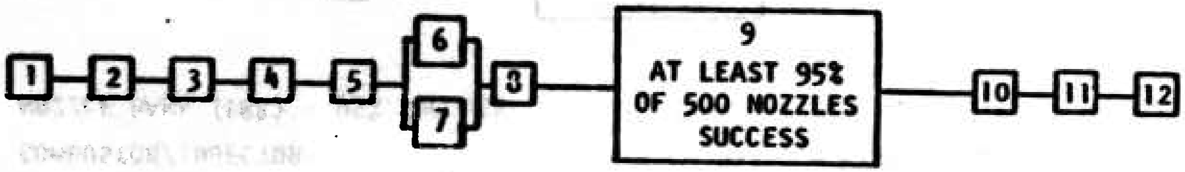
This chapter presents the reliability prediction methodology used to generate reliability point estimates from the FBD. The methodology includes the construction of the RBD, the assembly of a failure data base, and the calculation of lower confidence bounds on the point estimates. Results of the model application to the GDL, CL, and EDL systems are presented, and the failure criteria are discussed.

B. RELIABILITY BLOCK DIAGRAM AND MODEL DEVELOPMENT

A Reliability Block Diagram was generated directly from each FBD. The five RBD are shown in figures 9 through 13. A reliability equation is presented for each subsystem. The five subsystem reliabilities are denoted $g_G(R)$, $g_E(R)$, $g_C(R)$, $g_B(R)$, and $g_F(R)$ for the GDL, EDL, CL, BCS, and FCS, respectively, where R is a vector whose components are the component reliabilities which comprise the subsystems. The subsystem reliability is thus related to the point estimates, $\hat{R}_j$, of each of its components through this relationship. Each of the three HEL system reliabilities will be denoted $G_G(R)$, $G_E(R)$, and $G_C(R)$, respectively, for the GDL system, the EDL system, and the CL system. The G(R) are constructed as follows:

$$\begin{aligned} G_G(R) &= \left[g_G(R)\right]\left[g_B(R)\right]\left[g_F(R)\right] \\ G_C(R) &= \left[g_C(R)\right]\left[g_B(R)\right]\left[g_F(R)\right] \qquad \text{(Eq. 1)} \\ G_E(R) &= \left[g_E(R)\right]\left[g_B(R)\right]\left[g_F(R)\right] \end{aligned}$$

1) CRYOGENIC TASK
2) HIGH PRESSURE GAS VESSELS(4)
3) GAS GENERATOR - O_2/JP-4 DRIVEN
4) TURBINE - HOT GAS DRIVEN
5) TURBOPUMPS(3) - LIQUID
6),7) IGNITER (2 REDUNDANT SYSTEMS)
8) COMBUSTOR
9) NOZZLE BANK (500) - 95% SUCCESS
10) CAVITY/DIFFUSER
11) AERODYNAMIC WINDOW
12) DIGITAL CONTROLLER

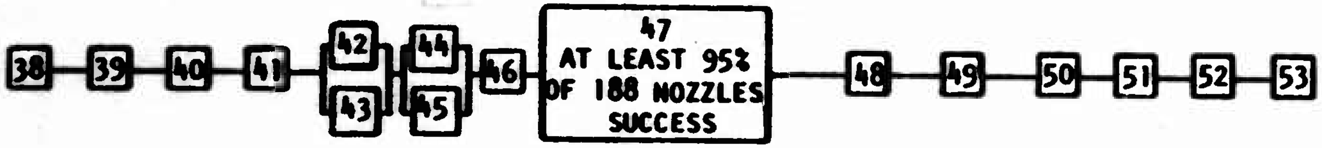


$$g_G(R) =$$

$$(R_1)(R_2)(R_3)(R_4)(R_5)\left[1-\left(1-R_6\right)\left(1-R_7\right)\right]R_8\left[\sum_{i=475}^{500}\binom{500}{i}R_9^i\left(1-R_9\right)^{500-i}\right](R_{10})(R_{11})(R_{12})$$

Figure 9. GDL Reliability Block Diagram

38) SUPER CRITICAL CRYOGENIC VESSELS (2)
39) CRYOGENIC VESSELS (3)
40) HIGH PRESSURE GAS VESSELS (2)
41) HEAT EXCHANGER (LIQUID/GAS)
42-45) IGNITERS (2 PR. OF REDUNDANT SYSTEMS)
46) COMBUSTOR/INJECTOR
47) NOZZLE BANK (188) - 95% SUCCESS
48) CAVITY/DIFFUSER/EJECTOR
49) SrF WINDOW
50) GAS GENERATOR - O_2/JP-4 DRIVEN
51) TURBO PUMPS (2) - LIQUID
52) TURBINE - AIR DRIVEN
53) DIGITAL CONTROLLER



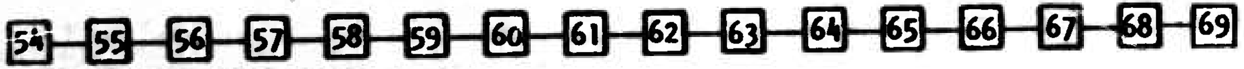
$$g_C(R) =$$

$$(R_{38})(R_{39})(R_{40})(R_{41})\left[1-\left(1-R_{42}\right)\left(1-R_{43}\right)\right]\left[1-\left(1-R_{44}\right)\left(1-R_{45}\right)\right]R_{46}\left[\sum_{i=179}^{188}\binom{188}{i}R_{47}^{i}\left(1-R_{47}\right)^{188-i}\right]$$

$$(R_{48})(R_{49})(R_{50})(R_{51})(R_{52})(R_{53})$$

Figure 10. CL Reliability Block Diagram

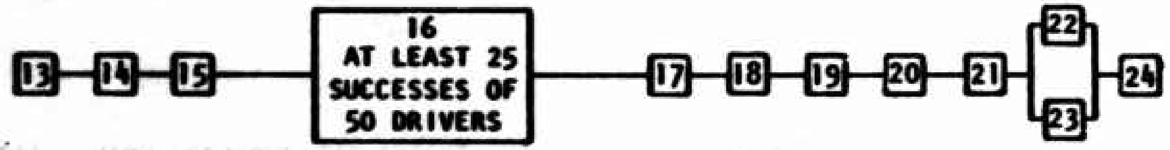
54) E-GUN
55) FOIL WINDOW
56) LOW POWER POWER SUPPLIES(4)
57) HIGH POWER POWER SUPPLY
58) APU
59) NOZZLE/CAVITY/DIFFUSER
60) HEAT EXCHANGERS(2) (LIQUID/GAS)
61) HIGH PRESSURE GAS VESSELS (3)
62) CRYOGENIC TANK
63) LIQUID TANKS(2)
64) COMPRESSOR
65) TURBINE - HOT GAS DRIVEN
66) GAS GENERATOR $-N_2H_4$ DRIVEN
67) AERODYNAMIC WINDOW
68) WATER COOLING SYSTEM (HEAT EXCHANGER)
69) DIGITAL CONTROLLERS(4)



$$g_E(R) = (R_{54})\ (R_{55})\ (R_{56})\ (R_{57})\ (R_{58})\ (R_{59})\ (R_{60})\ (R_{61})\ (R_{62})\ (R_{63})\ (R_{64})\ (R_{65})\ (R_{66})\ (R_{67})\ (R_{68})\ (R_{69})$$

Figure 11. EDL Reliability Block Diagram

13) MIRRORS (15)
14) GRATINGS (2)
15) APT WINDOWS (2)
16) PIEZOELECTRIC DRIVER (25 OF 50 SUCCESS)
17) ELECTROMECANNICAL MIRROR DRIVERS (3)
18) ALIGNMENT SYSTEMS (4)
19) SERVO/HYDRAULIC DRIVE SYSTEMS (4)
20) ON/OFF HYDRAULIC DRIVE SYSTEM (1)
21) LASER RANGEFINDER
22) CONSCAN OR HOT SPOT TRACKER
23) FLIR OR TV TRACKER
24) DIGITAL CONTROLLER

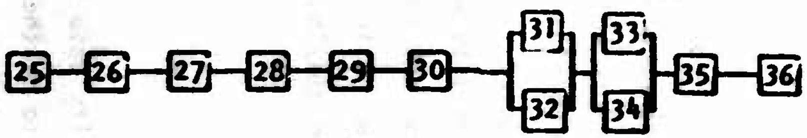


$g_B(R) =$

$$(R_{13})\ (R_{14})\ (R_{15}) \left[\sum_{i=25}^{50} \binom{50}{i}\ R_{16}^{i} \left(1-R_{16}\right)^{50-i} \right] (R_{17})(R_{18})(R_{19})(R_{20})(R_{21}) \left[1-\left(1-R_{22}\right)\left(1-R_{23}\right) \right] (R_{24})$$

Figure 12. BCS Reliability Block Diagram

25) AFT RADAR
26) INERTIAL NAVIGATION SYSTEM
27) UPPER IRSS
28) LOWER IRSS
29) RFWR
30) FORWARD RADAR
31, 32) COMPUTER (2 REDUNDANT SYSTEMS)
33, 34) AMUX (2 REDUNDANT SYSTEMS) - BCS & DEVICE INTERFACE
35) CONTROL STICK
36) DISPLAYS (RADAR, TACSIT, BEAM CONTROL)



$$g_F(R) =$$

$$(R_{25})(R_{26})(R_{27})(R_{28})(R_{29})(R_{30})\left[1-(1-R_{31})(1-R_{32})\right]\left[1-(1-R_{33})(1-R_{34})\right](R_{35})(R_{36})$$

NOTE: SOME LAST MINUTE CHANGES WERE SUGGESTED FOR THE FBD, RBD, AND DATA BASE FOR THE FCS. THESE CHANGES WERE MADE TOO LATE TO BE INCLUDED IN THE RELIABILITY CALCULATIONS, HOWEVER THE NEW FBD, RBD, AND DATA BASE ARE PRESENTED IN APPENDIX A.

Figure 13. FCS Reliability Block Diagram

Since the failure data base, which was assembled to determine these $\hat{R}_j$, was composed only of failure rates (i.e., number of failures per 10^6 part-hours of testing), it was necessary to assume an exponential failure distribution to calculate $\hat{R}_j$ as follows:

$$\hat{R}_j = e^{-\lambda_j t} \qquad \text{(Eq. 2)}$$

where λ is the failure rate and t is the time period over which survival is desired. The $\hat{R}_j$ is the probability of survival of the j-th component for the time period t. If the failure history (i.e., actual times-to-failure) were recorded in addition to the failure rate in the failure data base, a more realistic distribution could be used to describe failures of the component. However, the exponential is the only distribution which can accommodate a constant failure rate with no additional assumptions. Therefore, the use of the exponential distribution was dictated by the type of failure data available.

As indicated above, the complete reliability model is simply the combination of the g(R)'s for each subsystem and the $\hat{R}_j$'s for each component in that subsystem. The HEL system reliability estimate, G(R), is the product of the appropriate subsystem g(R)'s, since the subsystems are serially connected.

C. DATA BASE ASSEMBLY

A general search of hardware reliability data banks led to several sources of data. The most useful appears to be the NRN (Nonelectronic Reliability Notebook), which resulted from efforts funded by Rome Air Development Center, and the GIDEP (Government Industry Data Exchange Program) failure rate data bank. In addition, some data have been obtained from Hughes, Rocketdyne, Garrett, and the AFWL's APT Failure Log. These sources present failure rate data in different formats, and much of the data analysis effort was centered on screening the data for applicability and on applying statistical methods to reduce the various types of data to a common format (i.e., failures per million part-hours). The SOR was also investigated as a possible source of data, however, the SOR records would not allow transformation to the required format.

A number of telephone contacts were made with such organizations as ASD, AFAL, AFLC, and SPO's; however, no useful information or data resulted from these inquiries. Other sources of data explored were government documents and open literature. These were examined first by scanning the NASA Abstracts as far back as 1961. A few documents of general interest in reliability and cost analysis were uncovered, however none of them had any significant applicability to the Laser System Reliability problem.

In addition, a Work Unit Summary/Report Bibliography at the Secret level covering 10 years was requested from DDC. The request was for reliability data, OR failure data, OR acceptance test data for each item of a list of 50 HEL components. Unfortunately, reliability, failure, or acceptance test data was found for only a few of the items; however none of the documents contain information useful to the present scope of the effort.

The data base assembled for the HEL system is presented in appendix B. Since the NRN and GIDEP were the principal sources of data, the decision was made to represent the components of the RBD by as many of the NRN or GIDEP parts as seemed applicable based on engineering judgment. This approach was selected because the data available at the component level was very sparse, was of suspect applicability, and did not begin to adequately reflect the complexity of each component. The approach taken puts the burden of the ultimate system description on the available data base rather than allowing the system description to dictate the data requirements. This approach generates a much more realistic and satisfying model application in the short run, since it at least provides representative values of reliability for analysis. In the long run, however, it is much more beneficial to generate the successive increases in model detail based upon engineering considerations rather than on data availability considerations. The major advantages of the engineering approach are: (1) it points out data deficiencies so that test data generation can be directed more efficiently, and (2) the accuracy of the model application will ultimately be greater.

D. FAILURE CRITERIA

The definition of failure is ideally performed independently of data base constraints. In this case, however, the failure criteria identification required a degree of definition and understanding of the physical system which could not be achieved because the system is not sufficiently well defined to allow an accurate identification of the relationships of part and component performance parameters to HEL system output parameters. For example, ideally one would determine the relation of the performance characteristics of a turbo pump to the output characteristics of the HEL system. Then turbo pump failure would be defined as the point where the variation of turbo pump performance caused failure of the HEL system. Since this degree of analysis was precluded by the conceptual nature of the HEL system, we were forced to make two important assumptions concerning failure definition: (1) the failure criteria of a part were taken to be those assumed by the failure test designer, (2) the failure of any component was assumed to cause failure of the subsystem and, consequently, the system.

The consequence of the first assumption is that the assembled data base defines HEL system failure independent of engineering considerations of performance effects. An alternative approach to the second assumption might be to isolate certain components whose individual failure would not cause system failure but whose cumulative failure would ultimately cause system failure. Then one could construct the reliability block diagram such that a failure of any three of six components, for example, would produce subsystem failure. This approach was not taken becuuse it required more insight into the engineering and performance relationships among the components than was immediately available.

In summary, then, the failure criteria are as follows. A part fails if the failure data base indicates failure. A component fails if any part fails. A subsystem fails if any component fails (with the few exceptions shown in the reliability block diagrams). The HEL system fails if any subsystem fails.

E. MODEL APPLICATION

The failure rate data in appendix B was applied to the reliability model by using equation 2 to generate the point estimates of $\hat{R}_j$ to be used in the g(R)'s and the G(R)'s. Since each of the subsystems (e.g., GDL, BCS, and FCS) are in series, the g(R)'s in that HEL system are simply multiplied together as shown in equation 1. The component reliability estimates are tabulated in table 2, the subsystem and system reliability estimates in table 3 and the composites for the three HEL systems are plotted in figure 14.

Table 3 shows the equivalent MTBF's assuming the systems and subsystems actually experienced the pseudo-number of trials and successes calculated according to the reliability synthesis technique and assuming the systems and subsystems obey an exponential failure distribution. These equivalent MTBF's are presented only to give a feeling for the magnitude of the reliabilities and are not a legitimate product of the analysis.

The lower 95 percent confidence bounds are calculated according to techniques developed in section F of this chapter. Since the confidence bounds are statistical measures of the confidence in the reliability calculations, they should not be interpreted as implying anything about the confidence in the HEL system itself. The major sources of the wide variation in the lower confidence bounds are: (1) the amount of failure

TABLE 2. COMPOSITE RELIABILITY ESTIMATES BY SUBSYSTEM

(page 1 of 2)

	30 SEC	1 HOUR
GDL		
1	0.9999670	0.9960408
2	0.9999957	0.9994894
3	0.9999946	0.9993506
4	0.9999947	0.9993677
5	0.9999994	0.9999280
6	0.9999424	0.9931033
7	0.9999977	0.9997273
8	0.9999918	0.9741471
9	0.9999991	0.9998938
10	0.9999977	0.9997235
11	0.9999801	0.9976129
CL		
38	0.9999658	0.9959030
39	0.9999670	0.9960408
40	0.9999957	0.9994893
41	0.9999987	0.9998480
42-45	0.9999424	0.9931033
46	0.9999977	0.9997273
47	0.9997818	0.9741471
48	0.9999991	0.9998938
49		
50	0.9999946	0.9993504
51	0.9999994	0.9999280
52	0.9999956	0.9994737
53	0.9999301	0.9976129
EDL		
54		
55		
56	0.9999977	0.9997246
57	0.9999977	0.9997246
58	0.9999843	0.9981144
59	0.9997818	0.9741471
60	0.9999987	0.9998480
61	0.9999957	0.9994894
62	0.9999670	0.9960408
63	0.9999983	0.9997912

TABLE 2. COMPOSITE RELIABILITY ESTIMATES BY SUBSYSTEM

(page 2 of 2)

	30 SEC	1 HOUR
64	0.9999979	0.9997514
65	0.9999947	0.9993677
66	0.9999942	0.9993084
67	0.9999977	0.9997235
68	0.9999975	0.9996970
69	0.9999801	0.9976129
BCS		
13		
14		
15		
16	0.9999943	0.9993143
17	0.9999964	0.9995690
18	0.9999774	0.9972970
19	0.9999889	0.9986707
20	0.9999906	0.9988676
21		
22	0.9998856	0.9863594
23		
24	0.9999801	0.9976129
FCS		
Helrats	0.9997860	0.9746382

TABLE 3. RELIABILITY ESTIMATES AND LOWER 95 PERCENT CONFIDENCE BOUNDS

		30 SECONDS		1 HOUR	
	MTBF	$\hat{R}$	LOWER 95 PERCENT CONFIDENCE BOUND	$\hat{R}$	LOWER 95 PERCENT CONFIDENCE BOUND
HEL SYSTEMS					
GDL	11 hr	0.99925	0.9985	0.914	0.827
CL	11 hr	0.99922	0.9986	0.910	0.246
EDL	12 hr	0.99928	0.9987	0.917	0.567
SUBSYSTEMS					
GDL	24 hr	0.99965		0.959	
CL	22 hr	0.99962		0.955	
EDL	26 hr	0.99968		0.962	
BCS	45 hr	0.99981		0.978	
FCS	39 hr	0.99979		0.975	

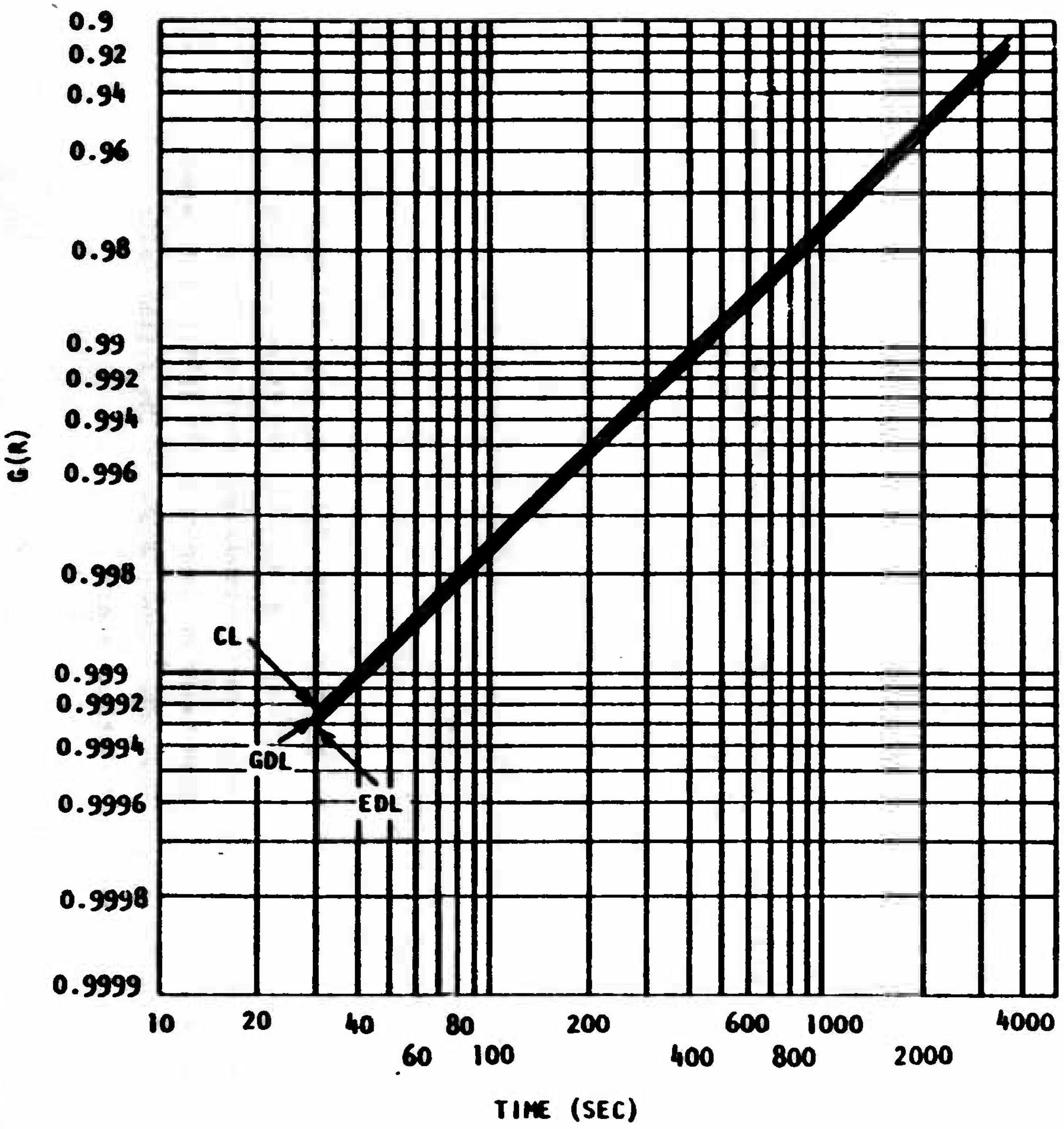


Figure 14. Plot of Reliability Estimates for Three HEL Systems

data available for the parts making up the system, and (2) the reliability constraints involving the number of individual nozzle failures required to cause failure of the entire nozzle bank.

These reliability estimates are demonstrations of the reliability prediction methodology and should not be used as engineering measures of HEL system effectiveness for two reasons:

(1) Mission success or effectiveness relates key system parameters, such as intensity on target and jitter, back to the operational characteristics of each component. Since the failure data base and the system description preclude such a sophisticated analysis, the reliability estimates simply describe the probability of success of hardware based upon a composite of many unknown failure criteria of the failure test designers.

(2) The accuracy of many hardware reliability estimates is compromised by the incompleteness and the wide variation in the applicability of the failure data base. Thus, the actual complexity of many of the components is not adequately reflected by component breakdowns which are based upon available data.

F. LOWER CONFIDENCE BOUNDS

The methodology used to determine confidence bounds on the HEL system reliability is a modification of a recently developed analysis technique.[5] The technique determines a pseudo number of trials, n, and successes, x, of the system based on component reliability estimates. Then, for these numbers, a lower confidence bound on the system reliability is determined. The relations are:

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[5] Easterling, R. G. "Approximate Confidence Limits for System Reliability," Journal of the American Statistical Association, March 1972.

$$\sum_{j=1}^{m} \left[\frac{\partial\, G(\underline{R})}{\partial\, R_j}\right]^2_{R_j = \hat{R}_j} \mathrm{Var}\ (\hat{R}_j) = G(\hat{\underline{R}})\ [1-G(\hat{R})]/\hat{n} \qquad \text{(Eq. 3)}$$

$$\hat{x} = \hat{n}G(\hat{\underline{R}}) \qquad \text{(Eq. 4)}$$

$$I(R_L,\ \hat{x},\ \hat{n}-\hat{x}+1) = 1-\gamma \qquad \text{(Eq. 5)}$$

where there are m components in the system, $G(\underline{R})$ is the reliability function, $\underline{R} = (R_1, R_2, ---, R_m)$, a hat (^) denotes a point estimate, I is the incomplete beta distribution, and R_L is the approximate lower confidence bound (100 γ percent) on the true system reliability.

As explained earlier, point estimates of component reliabilities were obtained from the exponential distribution due to failure data base constraints. Under this distribution the variance of all the $\hat{R}_j$'s is given by

$$\mathrm{Var}\left(\hat{R}_j\right) = R_j^{\,2}\left[\ln R_j\right]^2/n_j \qquad \text{(Eq. 6)}$$

where n_j denotes the number of failures of the j-th component.

From this relation and equation 3 above, it can be seen that the number of failures, not just the failure rate, is required for all components of the HEL system. However, this number was not reported for every component. In those cases where only failure rates and the 90 percent confidence bounds were reported, the following relationship was derived and utilized to determine the n:

$$n = 2 \Bigg/ \left[9\left\{2\left[\left(c_2/\lambda\right)^{1/3} - 1\right] - \left(1.645\right)^2\right\} \pm 9 \sqrt{2\left[\left(c_2/\lambda\right)^{1/3} - 1\right] - (1.645)^{2} + 4\left[\left(c_2/\lambda\right)^{2/3} + 2\left(c_2/\lambda\right)^{1/3}\right] - 1}\,\right] \qquad \text{(Eq. 7)}$$

where c_2 denotes the upper 90 percent confidence bound on the failure rate λ. This relation is derived in appendix C.

The FCS was handled somewhat differently since failure rate and confidence bound data were not available. Instead, engineering estimates were used, and a value of $n = 1$ was assumed since this represented the most conservative estimate of the number of failures which could be generated. In the case of the nozzle component of the GDL and CL there were no observed failures and only 18 failures were identified for the EDL nozzle/cavity/diffuser. Relative to the number of failures identified for most of the other components, these numbers are particularly small. The small amount of information on the nozzle is reflected analytically by a corresponding large variance in comparison with the other components.

Another important factor in the wide variation of the lower confidence bounds of the HEL systems is the requirement placed on the respective nozzle systems. In the case of the CL system, at least 95 percent of only 188 nozzles are required to operate. This implies that no more than 9 nozzles can fail and the subsystem still perform adequately. By comparison, the GDL system allows no more than 25 nozzle failures for a similar 95 percent restriction. Data upon which these conclusions are based is given in appendix D.

The lower 95 percent confidence bounds presented in table 3 were calculated based on the system reliability estimates (for $t = 30$ sec and $t = 1$ hour) and equation 5. These lower bounds could not be obtained directly from tables of the incomplete beta function so an approximation[6] was utilized as follows:

$$R_L = \frac{a}{a + be^{2\omega}}$$

where

$$\omega = \frac{Z_\gamma (h + \lambda)^{1/2}}{h} - \left[\frac{1}{2b - 1} - \frac{1}{2a - 1}\right]\left[\lambda + \frac{5}{6} - \frac{2}{3h}\right],$$

$$h = 2\left[\frac{1}{2a - 1} + \frac{1}{2b - 1}\right]^{-1}, \quad \lambda = \left(Z_\gamma^{\,2} - 2\right)/6,$$

$a = \hat{x}$, $b = \hat{n} - \hat{x} + 1$, and Z_γ is the lower confidence bound percentage point ($100\ \gamma$) of the normal distribution function.

Calculations that produced the lower bounds are given in appendix D. Examination of these calculations indicates that, for an initial short period of continuous operation, the FCS and BCS control the estimate of system reliability for all three HEL systems. For a longer period of operation, the nozzle configuration and the requirements imposed on it become the largest contributors to overall system reliability. It should be noted that the FCS also exerts an influence here, but to a much smaller extent in the CL and EDL systems.

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[6] "Handbook of Mathematical Functions", Milton Abromowitz and Irène A. Stegun, National Bureau of Standards, AMS55, 1964.

SECTION IV

RELIABILITY OPTIMIZATION METHODOLOGY

A. INTRODUCTION

This chapter presents the analysis of costs and benefits which will be used to determine the optimal HEL system reliability under the constraints of minimizing costs and maximizing benefits. This is the second major part of the LSR methodology. This optimal reliability then becomes the goal of the system designers.

The costs and benefits are actually composed of three parts:

(1) LCC (Life Cycle Cost) which is a cost element coupled with some benefits due to reduced maintenance requirements with increasing reliability.

(2) Availability, which is a benefit that allows smaller force requirements as reliability increases.

(3) Combat effectiveness, a benefit which is not readily measurable in dollars but which can be expressed in terms of exchange ratio for many tactical situations.

The LCC and the availability analyses will be presented together since their figure-of-merit is dollars. This will be followed by the combat effectiveness model.

B. LIFE CYCLE COST AND AVAILABILITY

1. Introduction

This section relates the LCC of a HEL system to its reliability. The purpose of the analysis is to provide the capability for assuring the lowest LCC of the HEL system in terms of the system reliability. It is important to perform this kind of analysis early in the development of the system since other studies have indicated that significant reductions of LCC can be accomplished by expending resources during the design and development of the system.[7]

The analysis used here decomposes the LCC into its three principal components and then performs a parametric analysis to determine the relationship between reliability (MTBF for an exponential distribution) and LCC. This parametric analysis uses historical data taken from similar weapon systems. The methodology presented in this analysis is, of course, no better than the data base from which the historical failure rate versus LCC data is drawn. Since insufficient resources were available in this study to adequately generate the appropriate data base, sample data will be presented to show how the analysis can be done when the methodology is applied to a HEL system.

2. Approach

a. Overview

The LCC is decomposed into three components: R&D, acquisition, and O&S (operating and support). This decomposition is performed so that a higher degree of accuracy can be introduced into the parametric analysis and since the decomposition will aid in the later analysis.

It is necessary here to define each of these three components since the definitions for them vary depending upon the user and his particular needs. For the HEL system, the R&D costs are those incurred before the prototype weapon system is developed. This definition is necessary since the HEL invokes an entirely new principal for a weapon system and so requires a considerable amount of R&D effort before the weapon system can even be defined. Thus, the R&D costs would include all of the ARTO expenditures at least through SRAT. The second element

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[7] "Dormant Operating and Storage Effects on Electronic Equipment and Part Reliability," Martin Co., Orlando, FL., RADC-TR-66-348, Contract AF30(602)-3772, October 1966.

of the LCC breakdown, acquisition costs, includes the cost of designing and developing the weapon after the system definition has been completed. It also includes the cost of the fabrication of the hardware itself. The O&S costs include all those costs required to operate and maintain the system for a period of 10 years. When these three cost components are added together, the result is the total LCC. Experience has shown that the O&S costs dominate the other two components (> 70 percent of total LCC), and so particular attention must be maintained for the O&S cost in generating an accurate data base.

Once these three LCC cost curves have been generated from historical data, the curves are differentiated to produce marginal cost curves. Marginal cost is the added cost of one more increment in reliability. The three marginal cost curves are then summed to produce a composite marginal cost curve for the entire LCC. The point at which this composite marginal LCC curve intersects the reliability axis is the optimum reliability for the system in terms of LCC.

The LCC versus reliability analysis is not, however, complete without consideration of another factor. In addition to the costs (both positive and negative) of the LCC, certain benefits are also derived from increased reliability. Thus, a benefit analysis must be performed to address the benefits for an entire force of HEL systems derived from increasing the reliability of the system. The total HEL force benefits are divided into two parts: those due to an increase in the availability of the force and those due to an increase in its combat effectiveness. The availability benefit is expressed in terms of dollars, however, the combat effectiveness is expressed in terms of an exchange ratio, since there is no accurate way of translating effectiveness into dollars. A marginal benefit curve due to availability is produced which is combined with the composite marginal LCC curve to describe the net marginal cost/ benefit due to LCC and availability considerations.

b. Parametric Analysis of LCC

In this subsection, the LCC is decomposed into three components and representative examples of the shapes of these curves are presented. Henceforth, in this chapter, MTBF will be used synonymously with reliability for two reasons: (1) all costs versus reliability data in the literature are referenced to MTBF, and (2) the two are equivalent if the exponential distribution assumption is made. It is undesirable to assume the exponential failure distribution, however, since it is not representative of real world failures. The left half of figure 15 presents an example of the cost of R&D, acquisition, and O&S versus MTBF. The shapes of the first two curves have been assumed to be of second order equations, since both R&D and acquisition costs increase faster as each of the systems being developed or fabricated approach the state of the art for that kind of a system. The O&S cost is also second order due to the economy of scale of a larger force, but that economy is assumed to decrease at a constant rate. The composite LCC (i.e., the sum of R&D, acquisition, and O&S) will have a minimum cost as shown in figure 15. It is important to remember two things in interpreting the total LCC curves:

(1) Each curve is for a fixed number of HEL systems (e.g., LCC could be normalized to one system).

(2) The O&S component dominates the composite curve.

The right half of figure 15 shows the MC (marginal cost) curves derived by differentiating the appropriate total LCC curve on the left. The point at which the composite MC is equal to zero is the optimal MTBF.

The upper half of figure 16 shows the actual LCC as a function of MTBF for recent military aircraft systems, and was derived from data presented in a symposium addressing design to cost and life cycle cost considerations.[8] It is important to remember that this curve does not accurately represent the actual cost of a HEL system, however,

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[8] "Total Life Cycle Costing," Proceedings and Related Papers of the Fall Technical Symposium, 14-15 November, 1974, American Defense Preparedness Association, Washington, D.C., Section 6, Design to Cost and Life Cycle Cost.

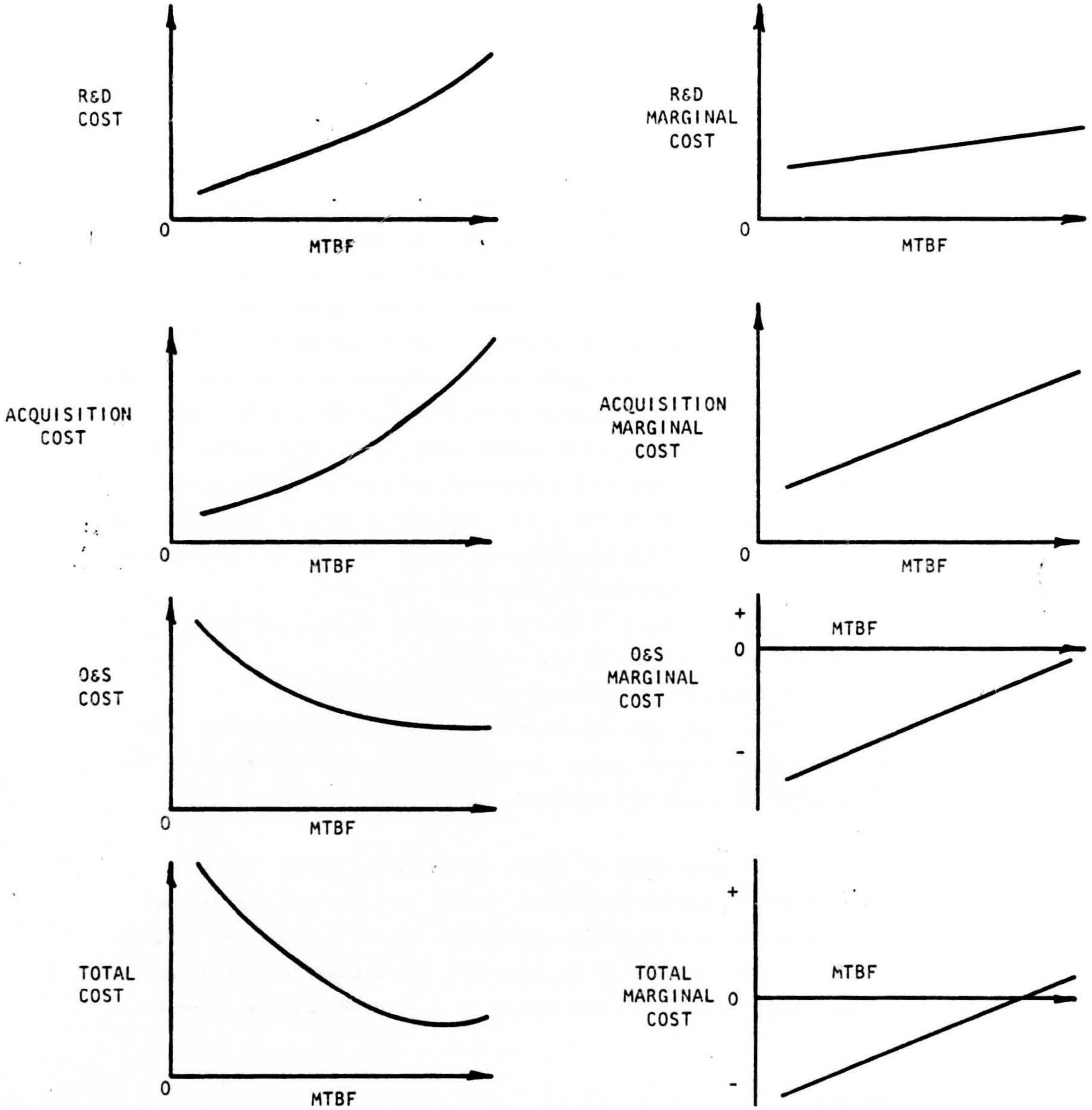

Figure 15. Representative Curves of Total LCC and Marginal LCC

It is assumed that the shape of the curve does represent the shape of the curve for a HEL system. If the shape is representative, one can differentiate the curve with respect to MTBF in order to determine an MC curve. The optimum MTBF, then, is the point on the total LCC curve at which the curve crosses the MTBF axis (i.e., the point at which MC equals 0). The MTBF at MC equals 0 is the point at which total LCC is minimized since any MTBF to the right of this point will incur additional cost (positive MC), while any MTBF to the left of this point will incur additional savings (negative MC).

Figure 16 is an example of the kind of data which has been uncovered in the literature search which has been conducted. In this case, the available data were not broken down into the three components discussed above, but were in the form of composite LCC. If the data could be found for the three components and summed, that method would be superior to the kind of data in figure 16 for two reasons:

(1) The data summary leaves no room for analysis to determine where the costs breaks specifically exist or to investigate the reasonableness of the conclusions.

(2) The data summary does not define exactly what costs are included in the LCC.

If the decomposed data cannot be found when the reliability optimization methodology is applied in the future as desired, then composite data of the kind displayed in figure 16 will have to be used. If that situation exists, the subsequent analysis will remain unchanged. Only the credibility of the conclusions and the depth of the analysis will suffer.

c. Marginal Benefit From Availability

The analysis so far has considered only the costs from the three components of LCC. Although some of these costs are negative (i.e., benefits as indicated by the O&S cost curve in figure 15), the total benefits of the reliability program have not yet been considered. In order to identify the overall benefits of increasing MTBF, one must consider the benefits of an entire force of HEL's, rather than merely

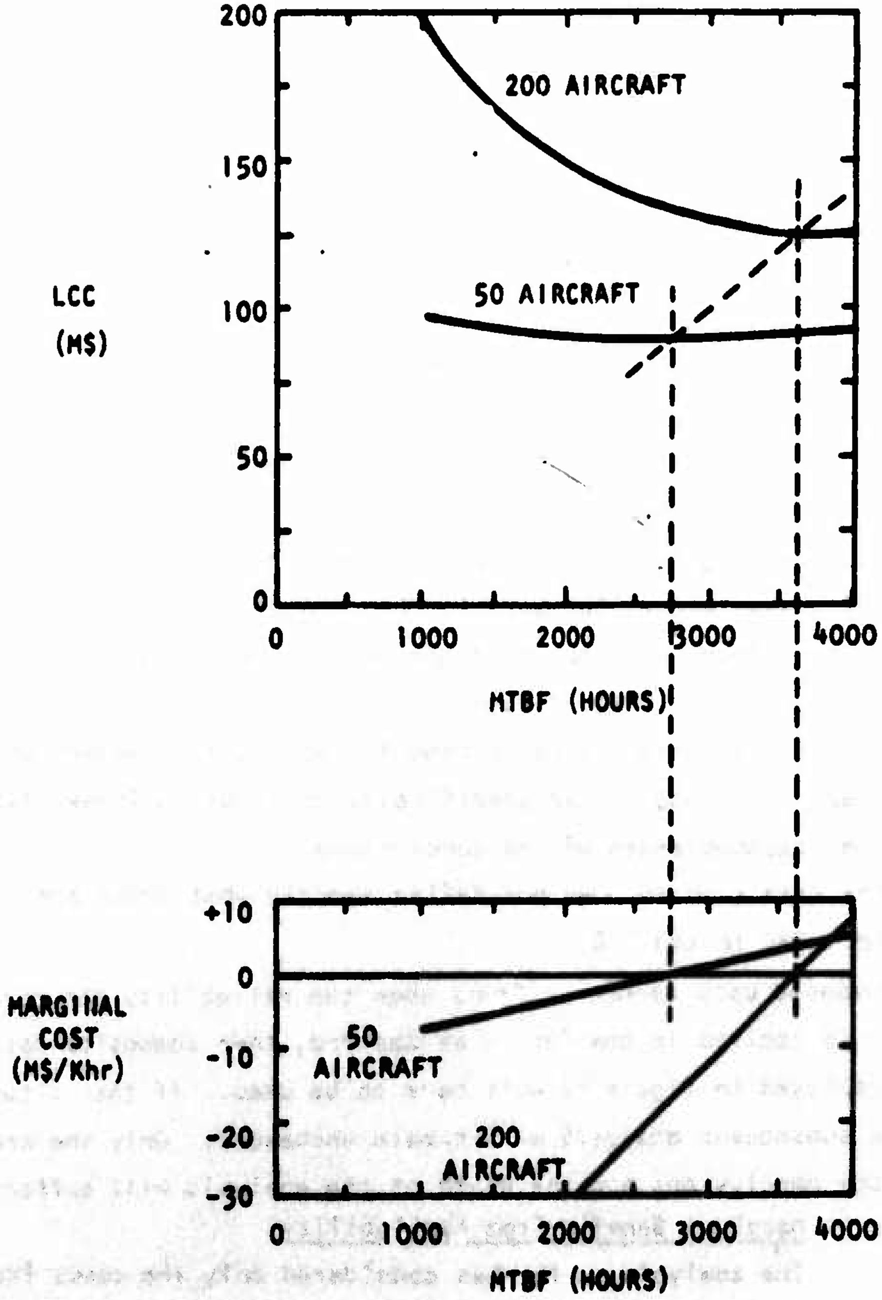

Figure 16. LCC Data for Recent Military Aircraft Acquisitions

analyzing the cost of one HEL system. This section addresses those benefits derived from the availability. Combat effectiveness benefits are treated in a later section of this chapter.

Since an increased MTBF increases the availability of a force, fewer numbers of HEL systems are required to perform the same function than at a lower MTBF. This decrease in the force size can be expressed as a savings of dollars. For example, if there is a requirement for 100 available HEL systems, increasing the availability of the force from 0.9 to 0.91 will result in the savings of one HEL system.

The change in availability of a force due to a change in reliability can be calculated by using one of a number of existing models. One such model is called ARIES (Aggregate Recoverable Item Evaluation System) which calculates the cost of unscheduled maintenance in the life cycle of a weapon system. ARIES, or some similar model, can be used to quantify the resultant change in availability due to a given increase in the reliability of the HEL system. The ARIES model computes the 10-year LCC for unscheduled maintenance for the aggregate of all the recoverable items in the system as the system is deployed throughout a multiechelon, base-depot, maintenance concept configuration.

The marginal benefit would be derived from the ARIES model as follows. ARIES would calculate force availability as a function of HEL system reliability. Once the availability is known, the force size, and thus its cost, would be calculated, again as a function of reliability. Finally, the cost function would be differentiated with respect to reliability to generate the marginal benefit.

To illustrate the technique, a simplified model has been used, and each of the three steps to find the marginal benefit relation has been demonstrated. First, let us assume that availability is related to MTBF as

$$A = \frac{MTBF}{MTBF + MTTR} \qquad \text{(Eq. 8)}$$

where MTTR = mean time to repair. This relation is shown in figure 17.

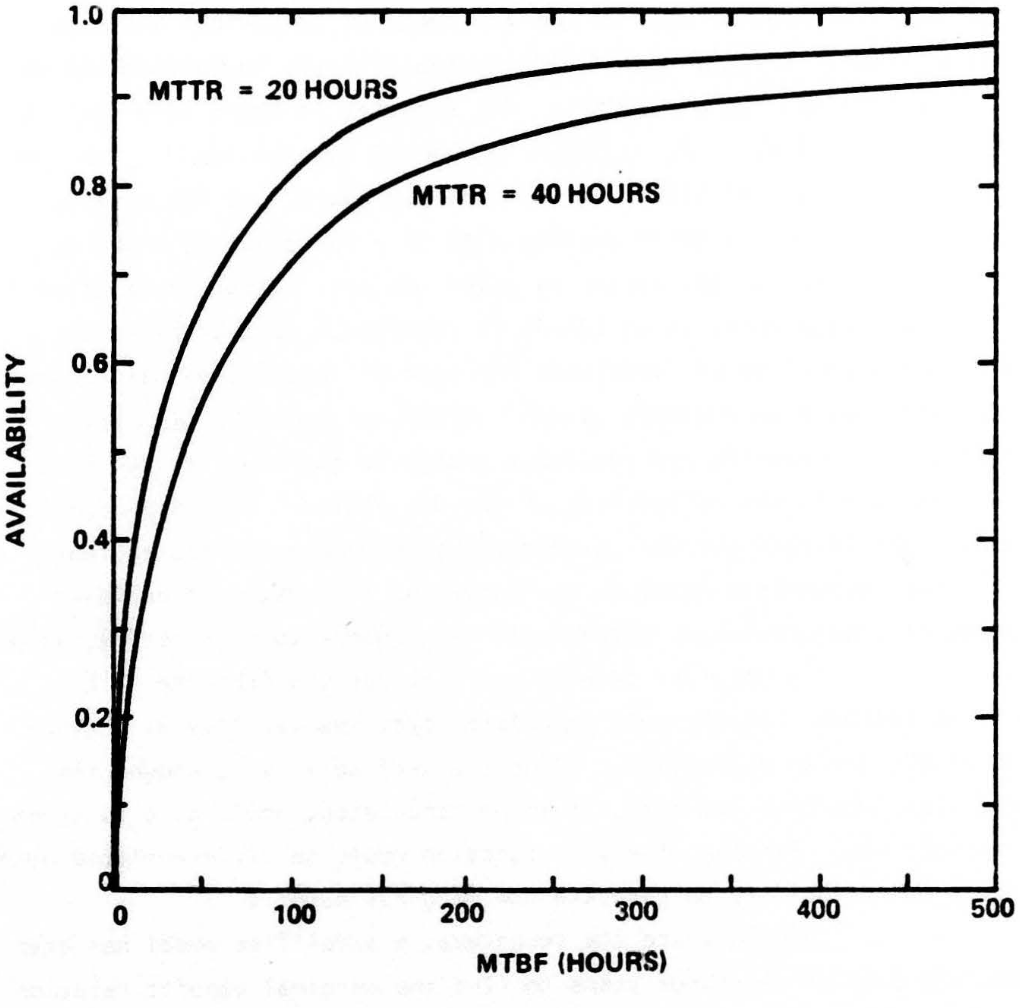

Figure 17. Availability Versus MTBF

Assume now that the cost is simply that due to the acquisition of a sufficient number of weapon systems to keep the available force fixed at F, i.e.,

$$N = F/A \qquad \text{(Eq. 9)}$$

where N is the number of HEL systems required at a given MTBF according to equation 8. Combining equations 8 and 9 yields

$$N = F\,\frac{\text{MTBF} + \text{MTTR}}{\text{MTBF}} \qquad \text{(Eq. 10)}$$

If N is directly proportional to force cost and F is set to unity to normalize the relation, equation 10 can be plotted as shown in figure 18.

The marginal cost can be found by differentiating equation 10 with respect to MTBF to get (with F = 1)

$$\text{MC} = -\,\frac{\text{MTTR}}{\text{MTBF}^2}$$

or

$$\begin{aligned}\text{MB} &= -\ \text{MC}\\ &= -\,\frac{\text{MTTR}}{\text{MTBF}^2}\end{aligned} \qquad \text{(Eq. 11)}$$

Equation 11 is plotted in figure 19.

The marginal benefit curve of figure 19 is of the proper form (except for denormalizing to get absolute dollars) to combine with the composite marginal cost of figure 15 to yield NMC (Net Marginal Cost):

$$\text{NMC} = \text{CMC} - \text{MB} \qquad \text{(Eq. 12)}$$

where CMC = composite marginal cost from the LCC analysis, and MB = marginal benefit from the availability analysis. When NMC = 0, the optimal system MTBF has been achieved (with the exception of combat effectiveness considerations).

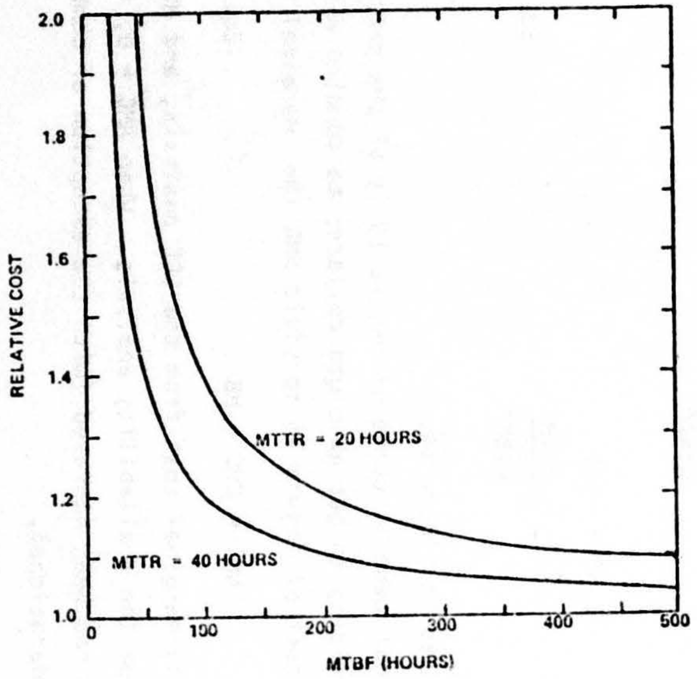

Figure 18. Relative Cost of HEL Force

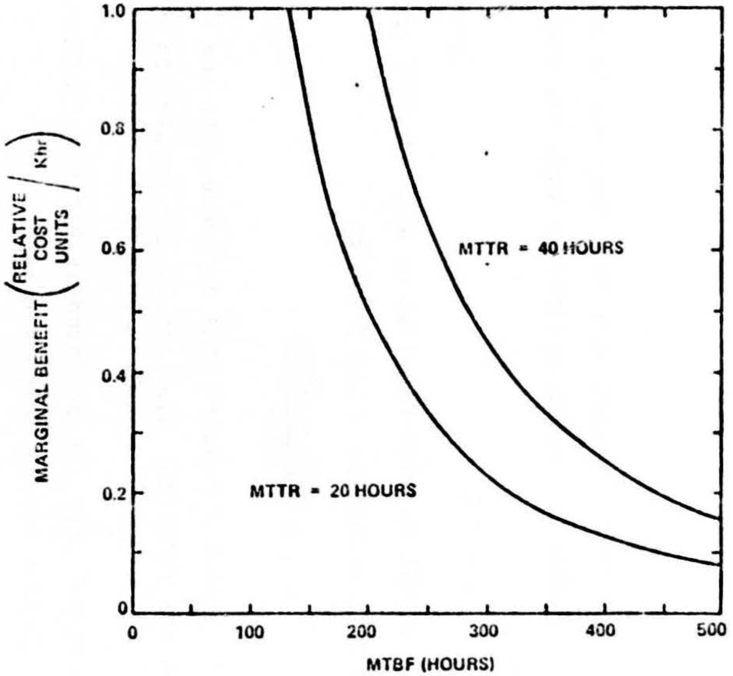

Figure 19. Marginal Benefit from Availability

Needless to say, this entire procedure can be performed either analytically or graphically or in a mixed mode depending upon the format and quality of the data and upon the analyst's preference.

d. Discounted Cash Flow

One additional factor needs to be considered here. Any time a flow of money over a long period of time is considered, one encounters the problem of comparing present dollars with future dollars. In this analysis, we are trading off costs expended initially (or at least early in the life cycle) with savings or benefits to be realized at some future date. This time-value-of-money problem becomes even more acute during inflationary periods such as the present. To solve this problem, a complete analysis would discount the future flow of costs and benefits back to the present. Instead of merely summing the costs and benefits of mixed value, a present value of each flow component would be calculated using an appropriate discount rate or series of rates. This is a very straightforward exercise using the discounted cash flow technique.

The technique is demonstrated by Equation 13.

$$PV = \frac{C_1}{(1 + i)^1} + \frac{C_2}{(1 + i)^2} + \cdots + \frac{C_n}{(1 + i)^n} \qquad \text{(Eq. 13)}$$

where

PV = present value of cash flow

C_n = cash flow (net cost) in year n

i = discount rate

The technique can be generalized to accommodate a varying discount rate from year to year.

$$PV = \frac{C_1}{(1 + i_1)} + \frac{C_2}{(1 + i_1)(1 + i_2)} + \cdots + \frac{C_n}{(1 + i_1)(1 + i_2)\ldots(1 + i_n)} \qquad \text{(Eq. 14)}$$

To illustrate the importance of the time value of money, the present value of a flow of $1/year for 10 years discounted at a 10 percent rate is $6.14. The present value of a $1 payment in year 10 is $0.39. Thus, if a comparison of costs and benefits over a 10 year period is to be made, it is very important to express the resulting flows in terms of a common value of a dollar.

The only problem with this method is that of arriving at an appropriate discount rate. The discount rate must include consideration of both the inflation factor and cost-of-capital factor for the DOD. The inflation factor is easily obtained by using the Price Escalator Indices from the Comptroller, Assistant Secretary of Defense. The same office could probably also contribute a value for the cost-of-capital factor. In any event, a period as long as 10 years can cause seriously erroneous conclusions if the time value of the funds flow is ignored.

C. COMBAT EFFECTIVENESS

This section provides an algorithm for first order estimates of the effects of HEL reliability on HEL airborne combat effectiveness. A discussion is also presented on the impact of laser reliability. The model is then used to generate the exchange ratio versus reliability data which becomes an input to the cost/benefit analysis to optimize reliability. This model uses exchange ratio as the MOE (measure of effectiveness). No single MOE can be used for all possible missions, so the ER was chosen as an example.

1. Combat Effectiveness Algorithm

The algorithm presented here enables first order estimates of laser reliability on outcomes of aerial combat between a laser armed aircraft and an attacking aircraft employing any feasible number of AAM's or gun firing attempts. Enemy SAM attacks on the laser armed aircraft are also accommodated, as well as any reasonable number of successive encounters. Friendly and enemy aircraft kill probabilities are directly computed, enabling direct determination of values of such MOE's as the ER.

We start with the following definitions of terms:

(1) Defensive hassle = an encounter between a laser armed aircraft and any feasible number of successive attacking SAM's or AAM's, or gun firing passes. A defensive hassle terminates prior to the next offensive laser shot (e.g., a laser shot at the enemy aircraft).

(2) Offensive hassle = one or more laser shots at the enemy aircraft. The offensive hassle terminates prior to the next defensive hassle.

Figure 20 presents the generic probability tree diagram for the kill calculations algorithm. Note that the order or number of offensive and defensive hassles is arbitrary, since we may set certain input parameters to values such that nonoccurring hassles indicated on figure 20 are accounted for, retaining algorithm validity.

Let

R_i = laser reliability[9] for the i^{th} laser firing,

P_i = probability that the laser kills the threat, on i^{th} laser firing

$\overline{P}_i = 1-P_i$

For the j^{th} offensive hassle (see figure 20), we define:

θ_j = probability that laser does not kill the enemy aircraft, and laser is operable at end of j^{th} offensive hassle (given surviving and operable laser and laser armed aircraft at start of j^{th} offensive hassle);

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[9] The i^{th} firing is of time duration t_i, and R_i is determined from expressions for G(R) described in chapter III.

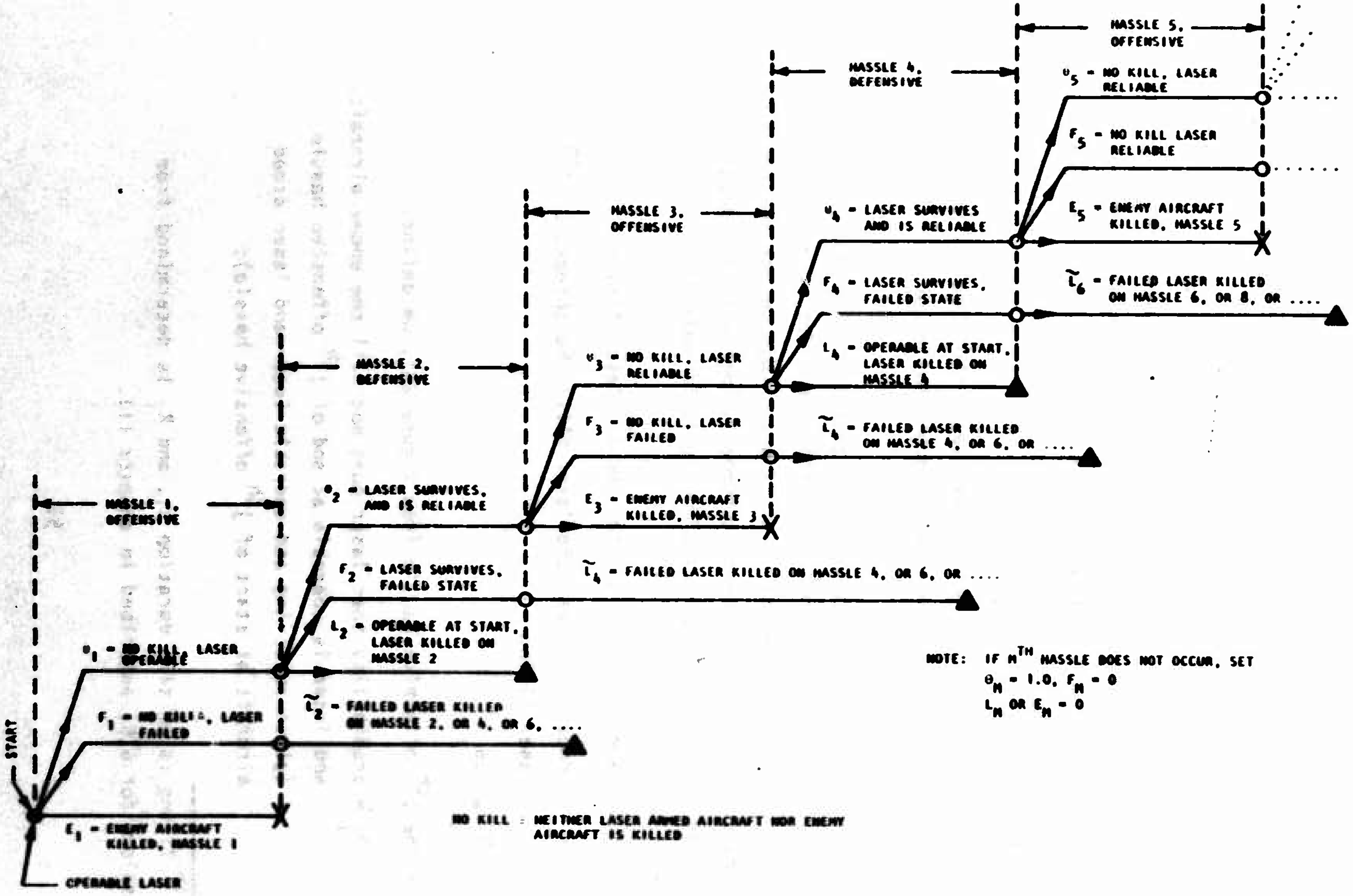

Figure 20. Generic Probability Tree Diagram for Kill Calculations

F_j = probability that laser does not kill the enemy aircraft, and laser is inoperable (failed) at end of j^{th} offensive hassle (given surviving and operable laser and laser armed aircraft at start of j^{th} offensive hassle);

E_j = probability that laser kills the enemy aircraft (given surviving and operable laser and laser armed aircraft at start of the j^{th} offensive hassle);

and

$$\theta_j = R_j \overline{P}_j$$

$$F_j = \overline{R}_j$$

$$E_j = R_j P_j$$

and

$$\theta_j + F_j + E_j = 1.0, \text{ as required.}$$

Note that if the first hassle, for example, is a defensive hassle, we set $\theta_1 = 1.0$, $F_1 = E_1 = 0.0$.

The model calculates ER as a function of the parameters presented in figure 20 which use reliability as the starting point. The details of the derivation of the model are presented in appendix E.

2. Impact of Reliability on HEL Combat Effectiveness

The algorithm developed in appendix E and the worked examples of appendix F indicate the potential for significant impact of HEL reliability on HEL combat performance. In any scenario involving several laser firings for survival against attacking threats, HEL reliability less than 1.0 can significantly detract from the advantage of laser SSPK's close to 1.0 in values.

Table 4 shows how ER varies with reliability from the examples in appendix F with the following constraints.

(1) Laser SSPK's much superior to (greater than) attacker's AAM SSPK's.

(2) A maximum of only five laser firings.

TABLE 4. RELIABILITY EFFECTS ON COMBAT EFFECTIVENESS

EXAMPLE PROBLEM	HEL RELIABILITY					EXCHANGE RATIO (ER)
	HASSLE 2		HASSLE 3	HASSLE 4		
	R_1	R_2	R_3	R_1	R_2	
2	1.0	1.0	1.0	1.0	1.0	84.3
1	0.90	0.93	0.90	0.92	0.92	7.0

One implication, among several that will be discussed below, is that payoff (higher ER) might be obtained, for example, by an increase in the maneuverability of laser armed aircraft maneuverability increase which could enable laser first shot(s), by altering the hassle sequence, provided laser SSPK's did not thereby suffer excessive degradation.

Since the effect of laser reliability less than 1.0 is cumulative with each successive defensive firing requirement, there is a distinct payoff for retention of sufficient laser armed aircraft maneuverability, radar/ECM performance, etc., so that the laser armed aircraft may have increased chance of first shot(s) against an enemy aircraft.

The specific degree of the above mentioned payoff would depend upon the specific values of hassle input parameters. However, it is not difficult to envision the advantage of laser first shot(s) attainment. Referring to the appendix F examples, with all individual hassle input parameters held constant, increased payoff (higher ER values) is realized as one progresses in order from first to last of the following improvements.

(1) Shift of hassle sequence to offensive, defensive, defensive.

(2) Shift of hassle sequence and avoidance of 2nd defensive hassle, resulting in an offensive, defensive sequence.

(3) Change of hassle sequence to one offensive hassle only.

(4) Change of hassle sequence to several offensive hassles prior to any defensive hassle.

The above improvements might be attained by such means as increase in laser armed aircraft speed, maneuverability, and detection and/or acquisition capabilities.[10] Specific payoff (increased ER) magnitude would of course depend upon specific input parameter values, including laser firing times and associated reliabilities.

D. SUMMARY OF OPTIMIZATION RELIABILITY METHODOLOGY

The three considerations which provide the inputs to the reliability optimization methodology are life cycle cost, availability, and combat effectiveness. The LCC and availability costs and benefits were expressed in terms of dollars as functions of reliability. Then these relations were differentiated to yield marginal costs and marginal benefits. The combat effectiveness model was developed to express exchange ratio as a function of reliability. Finally, the net marginal cost from LCC and availability are combined with the ER benefit to provide the inputs for making the optimal reliability decision. This final combination is necessarily a qualitative one relying on engineering judgment since the combat effectiveness benefit is not expressed in dollars as the other costs and benefits are. This entire methodology is illustrated in figure 21.

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[10] Without sacrifice of superior laser SSPK values.

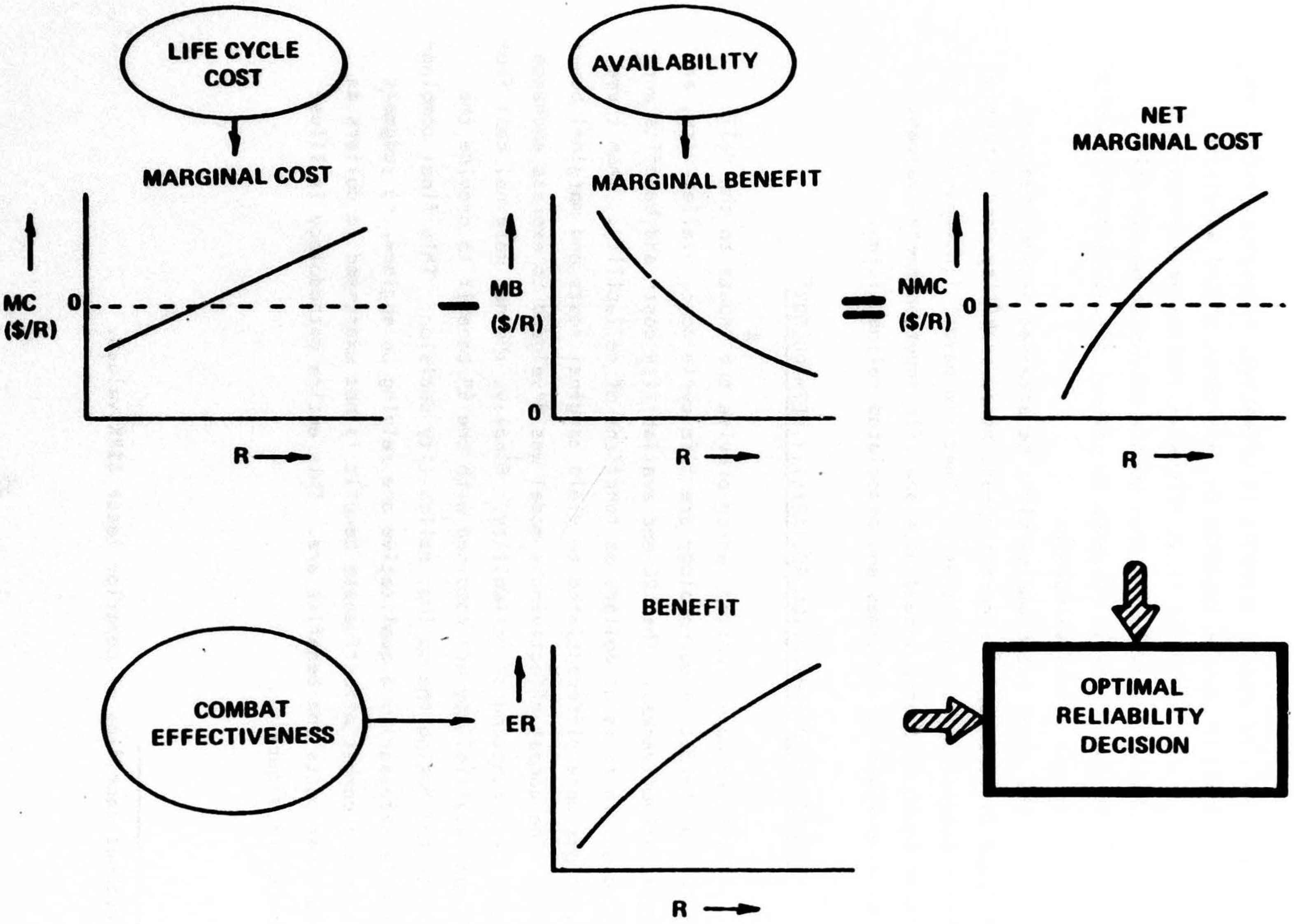

Figure 21. Reliability Optimization Methodology

SECTION V

RELIABILITY APPORTIONMENT METHODOLOGY

A. INTRODUCTION

This chapter describes the third part of the LSR methodology which is designed to apportion the reliability goal among the subsystems and components. The first section discusses the available methods for improving the system reliability (engineering design guidelines) since this is the responsibility of the system designer. The next sections present the analytical procedures for allocating the reliability improvements among the subsystems and components.

B. DESIGN GUIDELINES

System reliability can be upgraded by the application of six techniques: (1) redundancy, (2) increase in inherent reliability, (3) quality assurance, (4) burn-in testing, (5) maintenance procedures, and (6) built in test equipment.

Redundancy can be included in several ways, at both the component and the part level. Both hardware redundancy and functional redundancy are possible. There are three types of hardware redundancy.

(1) Non-decision redundancy (active) is most commonly characterized by two parallel (often identical) components which always operate simultaneously to increase the success probability of the function which they perform. Non-decision redundancy is usually easiest to incorporate into a system and introduces minimal complexity, but has several disadvantages.

   (a) Increases the weight and volume of the system.
   (b) Does not increase the lifetime of the components.
   (c) All components are active and consume power.

(2) Decision redundancy without switching (active) is used primarily in digital applications, to assure that errors in decision logic are minimized. As with nondecision redundancy, component lifetime is not increased; and all components are active and consume power.

(3) Decision redundancy with switching is usually characterized by the use of "stand-by" components and sensors or command links to provide switching when one component fails. Weight and volume penalties must be paid; but, with this type of redundancy, hardware life may be extended since only one component is active at a time (reducing power consumption.)

The inherent reliability can be increased by derating the parts, by using conservative design procedures, and by tightening the specifications on each part.

A quality assurance program can be established to upgrade the quality of parts during manufacture (i.e., to prevent the delivery of parts which do not meet specifications).

Burn-in testing and weeding out the failed parts before assembly is an effective way of significantly reducing the high failure rate early in the life of a system. A RADC study has indicated that burn-in testing can decrease early failure rate by a factor of seven for electronic components.[11]

Specific maintenance procedures can be implemented, such as the removal of limited-life parts on a routine schedule or the use of a very conservative maintenance schedule.

Built-in test equipment can be used to improve fault detection and fault isolation.

C. SUBSYSTEM CRITICALITY RANKING

In this section, a quantitative methodology is presented for ranking subsystems in terms of the criticality of those subsystems to the overall

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[11] Dormant Operating and Storage Effects on Electronic Equipment and Part Reliability," Martin Co., Orlando, FL., RADC-TR-66-348, Contract AF 30(602)-3772, October 1966.

functioning of the HEL system. The objective of the criticality ranking methodology is to quantify the importance of a particular subsystem to the HEL system by examining the effects of all possible failure modes of the subsystems. The first step in this criticality ranking methodology is to perform a FMEA (Failure Mode and Effects Analysis). The second step is to perform a CA (Critical Analysis).

1. Failure Mode and Effects Analysis

The purpose of the FMEA is to identify the failure modes associated with each subsystem and to analyze the effects of such a failure on the component which has failed, on the next higher level function, and finally on the HEL system. The FMEA procedure is a modification of that presented by the Society of Automotive Engineers.[12]

The FMEA procedure is as follows:

(1) Make a concise statement of the function performed

(2) Consider at least the four typical failure modes:

  (a) Premature operation

  (b) Failure to operate at a prescribed time

  (c) Failure to cease operations at a prescribed time

  (d) Failure during operation

(3) Define the mode of operation in which the failure occurs (e.g., threat detection, threat acquisition, threat tracking, firing, or kill assessment).

(4) Make a brief statement describing the ultimate effect of the failure on the component being analyzed.

(5) Make a brief description of the effect of the failure on the next higher level of the functional block diagram.

(6) Describe the effect of the component failure on the HEL system.

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[12] "Design Analysis Procedure for Failure Mode, Effects and Criticality Analysis," Society of Automotive Engineers, Aerospace Recommended Practice, ARP 926, September 1967.

(7) Define α = fraction of failures in that failure mode.

(8) Define β = conditional probability that described effect actually occurs.

(9) Define γ = failure effect consequence

0 - no effect
1 - minor degradation
2 - moderate degradation
3 - serious degradation
4 - total loss
5 - catastrophic failure, possible loss of life.

These nine factors, when taken together, define the failure mode and the effects of that failure mode on each of the subsystems under investigation. These factors can be summarized in a table, and as an example table 5 is presented for the parts of a GDL cryogenic storage component.

2. Procedure for Criticality Analysis

The criticality number, C, for the component under investigation can be calculated according to the following expression:

$$C = \sum_{n=1}^{j} \alpha_n \beta_n \gamma_n$$

where j = total number of failure modes of that component; and α, β, and γ are as they were defined above.

This criticality number does not include any of the reliability factors normally included in the calculation of a criticality number. This has been done because these reliability effects have already been included in the reliability model and the subsequent point estimates of reliability. The criticality number defined above addresses only those factors which are directly affected by the failure mode or the effects of that failure mode on the HEL system. Thus, the criticality number addresses only criticality factors, while the reliability number addresses

TABLE 5. FAILURE MODE AND EFFECTS ANALYSIS

SYSTEM: GDL

SUBSYSTEM: CRYOGENIC VESSEL

COMPONENT	FUNCTION	FAILURE MODE	MODE OF OPERATION	EFFECT ON			α	β	γ
				COMPONENT	SUBSYSTEM	GDL			
GENERAL FITTINGS	COUPLING	LEAK	CONT.	DEGRADE	DEGRADE	DEGRADE (MINOR)	0.15	0.1	1
GASKETS	COUPLING	LEAK	CONT.	DEGRADE	DEGRADE	DEGRADE (MINOR)	0.15	0.1	1
METAL TUBING	FLUID TRANSFER	LEAK	CONT.	DEGRADE	DEGRADE	DEGRADE (MINOR)	0.01	0.1	1
		FRACTURE	CONT. CONT.	TOTAL LOSS	TOTAL LOSS	TOTAL LOSS	0.01	0.5	
LIQUID LEVEL INDICATORS	FLUID REMAINING INDICATION	NOISY	ENGAGEMENT	DEGRADE	DEGRADE	NO "OUT OF FUEL" SIGNAL	0.1	0.1	1
RATE OF FLOW INDICATOR	FLUID FLOW RATE INDICATION	NOISY	FIRE	DEGRADE	DEGRADE	NO "FLOW RATE" SIGNAL	0.1	0.1	1
PRESSURE REGULATOR	PRESSURE AND FLOW CONTROL	LEAK	FIRE	DEGRADE	DEGRADE	DEGRADE (MINOR)	0.1	0.1	1
		DRIFT	FIRE	ACCURACY LOSS	DEGRADE	DEGRADE (MINOR)	0.05	0.1	1
LOX TANK	LOX STORAGE	CONTAMINATE	CONT.	NONE	NONE	DEGRADE	0.03	0.2	2
		LEAK	CONT.	DEGRADE	DEGRADE	DEGRADE (MINOR)	0.07	0.1	1
		FRACTURE	CONT.	TOTAL LOSS	TOTAL LOSS	TOTAL LOSS	0.01	0.5	4
SOLENOID VALVE	OPEN/ CLOSE FLUID LINE	LEAK	CONT.	DEGRADE	DEGRADE	DEGRADE (MINOR)	0.1	0.1	1
		BIND	FIRE	TOTAL LOSS	TOTAL LOSS	TOTAL LOSS	0.02	0.5	4
		OPEN	FIRE	TOTAL LOSS	TOTAL LOSS	TOTAL LOSS	0.05	1.0	4
		SHORT	FIRE	TOTAL LOSS	TOTAL LOSS	TOTAL LOSS	0.05	1.0	4

only reliability factors, so there is no chance of double counting of reliability considerations.

An example of the criticality number calculation for the table 5 data is as follows:

$$C = (0.15 \times 0.1 \times 1) + (0.15 \times 0.1 \times 1) + \ldots + (0.05 \times 1.0 \times 4)$$
$$= 0.56$$

D. RELIABILITY APPORTIONMENT

1. General Methodology

Once the engineering methods for reliability improvement are understood as they can be applied to the HEL system, and the subsystem criticalities have been calculated and ranked, the apportionment or budgeting of the reliability upgrade must be determined. The methodology for this function follows, and is a modification of, that presented in Lloyd and Lipow.[13]

First the reliability of the system before reliability upgrade is calculated and compared to the reliability required for optimizing Life Cycle Cost, Availability, and Combat Effectiveness. If the required reliability is larger than the expected reliability, then a new reliability is calculated for each subsystem which would produce the required system reliability (i.e., assume that the total change in system reliability is to be accomplished by a change in the reliability of one of its subsystems). If this is done for each subsystem, then the subsystems can be ranked according to their reliability increases needed to achieve the system reliability goal. One can carry this further by attributing the reliability upgrade to two or more of the subsystems which is a far more realistic, but more complicated, approach to the problem.

2. Single Parameter Apportionment

To illustrate the reliability apportionment technique, let us assume that a reliability goal has been established for the GDL subsystem, and the objective is to budget that goal among the components of the GDL subsystem. The variables are defined as follows:

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[13] "Reliability: Management, Methods, and Mathematics," David K. Lloyd and Myron Lipow, Prentice-Hall, 1962, Appendix 9A.

R = subsystem reliability before upgrade

R' = subsystem reliability goal

r_n = reliability of n-th component before upgrade

r'_n = reliability of n-th component after upgrade

Assume that the components are all serially connected for simplicity in the illustration (i.e., $R = r_1 \times r_2 \times \ldots \times r_n$). Now the $r'_1, r'_2, \ldots, r'_n$ can each be calculated assuming that each of them is sufficient to generate the required R'.

$$r'_1 = \frac{R'}{r_2 \times r_3 \times \ldots \times r_n} \longrightarrow \Delta r_1$$

$$r'_2 = \frac{R'}{r_1 \times r_3 \times \ldots \times r_n} \longrightarrow \Delta r_2$$

$$\vdots$$

$$r'_n = \frac{R'}{r_1 \times r_2 \times \ldots \times r_{n-1}} \longrightarrow \Delta r_n$$

If the cost of each of the Δr's can be satisfactorally determined, then the choice of which component or components should benefit from a reliability upgrade program can be based upon such a ranking. However, it is more likely that the cost estimates will be very imprecise since the Life Cycle Cost analysis cannot begin to differentiate among costs at this level of detail. In this situation, the components may be ranked in order of decreasing Δr, and engineering judgment may then be used. For example, one may assume that the cost per Δr (i.e., the marginal cost) for each component is the same; thus the Δr ranking is also the ranking of cost of implementation.

An additional piece of data to be considered in the reliability apportionment is the criticality number, C, derived in the previous section. As long as the procedure for criticality ranking of each subsystem is identical, the internal consistency of the C's is maintained. It is quite difficult, however, to attempt to quantitatively link a C_n to a Δr_n since the relative weighting of the two quantities is unknown. A much better use of the C ranking is to merely put the criticality ranking next to the Δr ranking and then make a qualitative engineering/management judgement about the relative importance of the two scales and how they should impact the apportionment decision.

The important point is to use the criticality ranking <u>and</u> the reliability ranking with other available engineering and/or management considerations to make the best apportionment decision possible.

3. Multi Parameter Apportionment

If engineering considerations indicate that the required R' may be difficult or undesireable to achieve by means of a single component upgrade, then the component with the next higher Δr is also considered, and some combination of the two component Δr's is necessary. This expansion to increasing numbers of components can be carried as far as desired, however the complexity of the problem increases with the number of possible combinations of Δr. This solution can be symbolically written as:

$$r_1' \times r_2' = \frac{R'}{r_3 \times r_4 \times \ldots \times r_n} \rightarrow \Delta r_1 \times \Delta r_2$$

$$r_1' \times r_2' \times r_3' = \frac{R'}{r_4 \times r_5 \times \ldots \times r_n} \rightarrow \Delta r_1 \times \Delta r_2 \times \Delta r_3$$

$$\vdots$$

Thus, as can be seen from the second equation, any possible combination of Δr_1, Δr_2, ... which satisfies the equation, is a possible target for the reliability upgrade process.

In the case of the apportionment to two or more components, the required component reliability changes are not uniquely defined since there are, in general, an infinite number of combinations of component reliabilities which can achieve the subsytem reliabilty goal. This is an optimization problem and can be solved by the use of linear programming techniques. Such factors as criticality number, costs, time, weight, and volume can be used as constraints in the linear programming problem.

E. SUMMARY OF RELIABILITY APPORTIONMENT METHODOLOGY

The reliabilty apportionment methodology can be summarized as follows. First, evaluate the design guidelines (i.e., the engineering techniques) which are available to the system designer and applicable to the HEL system. Second, perform a failure modes and effects and criticality analysis to construct a ranking of the components according to their criticality to the HEL system if component failure occurs. Finally, the criticality ranking can be combined with either the single or multiple parameter apportionment analysis; and the results can be used in conjunction with engineering/management considerations to allocate the reliability goal among the subsystems and components. Keep in mind that the techniques described in this chapter should be used to provide an order to the various inputs used in conjunction with engineering/management judgment to make the apportionment decision.

SECTION VI

CONCLUSIONS

This chapter deals with the conclusions and observations made during the study and is divided into three parts: (1) reliability prediction methodology, (2) reliability optimization; and (3) reliability apportionment.

A. RELIABILITY PREDICTION METHODOLOGY

1. Physical Description

The component level of the system description is the lowest level at which a high degree of confidence exists. Examples of this level are the Servo Hydraulic Drive of the BCS, the Sustainer Power Supply of the EDL, the Aerodynamic Window of the GDL, the Nozzle Bank of the CL, and the Computer of the FCS. Unfortunately, at this level of detail there was insufficient data available in the failure data base to be able to calculate any reliability point estimates. Thus, it was decided to carry the physical description one level further; that is, to the parts level (e.g., hydraulic pump for the Servo Hydraulic Drive). The criteria used for determining which parts were used to describe each component were twofold. First, the part had to belong to the existing failure data base; and secondly, the part had to belong to the description of the component according to general engineering considerations. Thus, only parts for which data were available were listed under each component.

This technique for building the parts level of the system description was based upon two assumptions (both expedient but open to question). First, it was assumed that all of the parts were connected in series. That is, no one part was more or less important than any other part, and the failure of any one part would lead to a failure of that component. Second, it was assumed that the list of parts for each component completely described that component. Neither of the assumptions is completely true, so the parts level of the physical description is technically inaccurate.

2. Failure Data Base

Times-to-failure data were completely absent from the available data base. That is, in every case the failure data were summarized such that the times-to-failure data were not recorded. Thus, when the test was recorded, only the number of failures and the number of part-hours of testing were recorded. As will be discussed later, this constrained the model to the exponential failure distribution.

Many of the parts that were judged necessary to complete the component description had no failure data available. In addition, several of the components could not be described in terms of any parts for which failure data were available. Thus, these components could not be represented by parts, so their reliabilities were assumed to be near unity.

Third, the parts that were represented in the failure data base were not well identified. For example, from the identification of the parts, the capacity of a pump, the size of a valve, or the rating of a power supply could not be determined. Thus, it was impossible to determine whether or not the part chosen from the data base was actually applicable to the HEL component under consideration.

Finally, the performance and test environment specification were absent or inadequate on all of the parts. Thus, one could not determine what performance specifications were used to define failure.

3. Effects on the Model

The principal effect of the above limitations was that an exponential distribution had to be assumed since failure history data were not available to determine what distribution best described the failures. The exponential distribution is a single parameter distribution and thus is used universally because of its simplicity. It does not, however, very accurately represent real world failures. It does not, for example, describe such real world effects as infant mortality (burn-in failures), or wearout. However, since only failure rate data are available in all of the data banks investigated, one does not have the option of determining which distribution best fits the failure data.

The failure criteria of the failure test designer was assumed to match the failure criteria of the HEL system since these criteria were not identified in any of the data bases. Since the failure criteria of the data base were completely undefined, it was assumed that they were reasonable in their application to the HEL system. In addition, the concept of degraded performance at the part level was completely precluded.

Since the parts in the data base were not adequately identified nor were the performance or environmental specifications identified, it was assumed that the data applied completely to the HEL system. Thus, the accuracy of the part level of the system description was compromised severely.

The absence or limited availability of failure data contributed to the large variation in the confidence bounds calculations since the number of observed failures in the testing sequence appears as a divisor in the confidence bounds expression. Thus, the lower the number of observed failures, the larger the confidence bounds of the reliability point estimates. Since some of the failure data were the result of testing programs in which a large number of failures were reported, and other failure data were the result of programs in which few or no failures were reported, the value of the confidence bounds is diminished in an engineering sense.

It must be remembered in interpreting the calculated reliability point estimates that they should not be used to compare one HEL or subsystem to another. The limitations imposed by the failure data base would render such comparisons completely inappropriate.

4. Operating Time

Reliability is a time dependent parameter; that is, it is the probability of successful operation for a given period of time. Thus, when reliability point estimates are calculated, the operating time of the system must be chosen. A representative operating time of the HEL system was difficult to choose since some components of the system operate continuously (e.g., threat detection and acquisition components), some components operate during the engagement (e.g., beam control alignment systems and digital controllers), and still others operate only during the firing sequence (e.g., nozzles and turbo pumps). In addition, all components are subject to loads exerted during flight and maintenance-generated stresses. Thus the reliability calculations were made for two representative times -- the 30-second firing time of a mission, and a typical tactical mission time of 1 hour. Therefore, there is room for interpretation in the definition of operating time for future applications.

B. RELIABILITY OPTIMIZATION METHODOLOGY

The costs and benefits which impact the reliability decision can be analyzed in a mutually consistent and logical manner. The marginal costs/marginal benefits approach is well suited to reliability optimization since optimization is an extremum problem, and the marginal approach defines the extremum.

Three problems need to be addressed in the application of the reliability optimization methodology to the HEL system. First, the LCC data will be difficult to collect. However, one must remember that, since this is an extremum problem, the similar system (historical) cost versus reliability data need not be accurate, but rather the shapes of the curves, and thus the derivatives, must be accurate. Thus, so long as the cost data behaves similarly with respect to reliability for both the similar system and for the HEL system, the minimum cost will be experienced at approximately the same value of reliability for both systems; and this is the ultimate objective. This methodology cannot be used to decide whether or not to develop the HEL weapon system since that decision requires a knowledge of the absolute cost, not just the marginal cost.

Second, the appropriate availability model must be chosen to accurately reflect the kinds of costs and maintenance which would be representative of a force of HEL systems. A number of such models are available which offer a wide range of features; however, the actual decision on a specific model would be made prior to its application.

Third, the combat effectiveness model which was developed to relate effectiveness to reliability is quite simple; however, since it is just a single variable model, it should be reasonably accurate in demonstrating the effect of reliability changes on the chosen measure of effectiveness.

C. RELIABILITY APPORTIONMENT METHODOLOGY

The reliability apportionment technique and its associated criticality ranking rationale are very useful methodologies to aid the reliability budgeting problem. The Failure Modes and Effects Analysis, which provides the inputs to the criticality ranking, is an essential step in the system design for both reliability and system safety considerations. Although the emphasis of a fault tree analysis for system safety is different from that for a FMEA, the technique and the required understanding of the system are nearly identical.

The criticality number derived from the FMEA reflects the true importance of a subsystem or component once it has failed. Thus the criticality number addresses criticality issues only (i.e., portion of total failures in a particular failure mode, probability of the identified effect actually occurring, weighing of the importance of the effect on the HEL system). These parameters have meaning only <u>after</u> failure has occurred, whereas reliability predicts the probability of failure <u>before</u> it occurs. The separation of these two kinds of parameters is essential to keep one of them from being inadvertently considered twice in the apportionment analysis.

Finally, the apportionment technique itself provides a consistent and comprehensive method for evaluating the entire set of possible ways of achieving the system reliability objective. Since both the reliabilities and the criticalities can be ranked independently, they can be evaluated and weighed separately using engineering judgment or can be quantitatively combined using some preconceived weighing scheme. The apportionment methodology does not presently include a means of optimizing the combination of reliability changes; however, the technique suggests the use of such optimization methods as linear programming to choose the best combination.

SECTION VII

RECOMMENDATIONS

This chapter presents recommendations for the future direction of Laser System Reliability analysis. They are based upon the experience achieved in the Laser System Reliability effort and incorporate the conclusions described in chapter II. The recommendations have been broken down into four separate parts, each of which will be described individually. These four parts are:

(1) Model construction for ALL (Airborne Laser Laboratory) or LWS (Light Weight System) at North Oscura Peak.

(2) Data base expansion.

(3) Sensitivity analysis.

(4) Model validation.

Now that the overall methodology for Laser System Reliability has been developed, it should be applied to a well defined and characterizable situation; for example, the ALL or the LWS. The application of the physical system description and the modeling methodology presented in chapters III and IV to the LWS will result in a model application which will present an accurate and realistic test case of the methodologies. To construct such a model of the LWS, for example, would require the description of the complete LWS system down to the parts level. Care would have to be taken that this parts level description is internally consistent and complete.

Following the model construction, the data base expansion would require two prime tasks. First, an examination of the source documents for the data base would be performed. This examination would be used to determine whether or not the data are applicable to a HEL system in terms of both performance specifications and testing environment, and whether or not sufficient data are available to use failure distributions other than the exponential. The second task would be to investigate the failure data or reliability estimates of the subsystem and component fabricators. This effort would both uncover existing data and determine the resources required to obtain additional data experimentally. This data base expansion would cover all of the parts required by the LWS system description performed in the first of these recommendations.

The third recommendation centers on an analysis of the sensitivity of system reliability to both the physical system description and the failure data base which has been constructed. In the area of the physical system description, an analysis of the sensitivity of reliability to various system configurations would be performed. For example, one would add or subtract redundancies or reduce the number of parts in a component and then trace the effects of these changes to the reliability estimate for the subsystem or the entire HEL system. In the area of failure data, an analysis would be performed of the sensitivity of reliability to either the failure rate or the sample size. The failure rate influences the reliability estimates while sample size influences the confidence bounds of those estimates. One could, for example, change the failure data sample size of critical components in the system description to determine the effect of that sample size change on the confidence of the reliability estimates for the HEL system.

The objectives of the sensitivity analysis have both short-term and long-term implications. The short-term implication is that it should allow the analyst to optimize the data collection and data reduction techniques for the LWS failure data. In the long run, the sensitivity analysis could be used to determine where resources could best be spent for further physical system descriptions and failure data bases. This long range objective would be to reduce the amount of wasted effort in future physical system descriptions and failure data base constructions by directing those programs only in the directions of maximum payoff.

The final recommendation involves the validation of the model against LWS experience. This validation phase draws very heavily upon the sensitivity analysis phase of the recommendations since the success or failure of this validation depends, to an important extent, upon the availability of the right kinds of failure data from the LWS experiments.

The overall benefits derived from these four recommendations can be summarized as follows. The application of the reliability methodologies in an accurate and complete way to an existing HEL system would provide a considerable amount of credibility to the modeling procedure and would increase its value in the long run in terms of modeling actual HEL weapon systems. Since the ultimate purpose of this reliability effort is to provide accurate inputs to the HEL Management Program, it is important to apply the reliability methodologies to an existing system to validate those developed techniques. The ALL and the LWS are systems which reflect the complexity and the general engineering considerations of HEL weapon systems. The sensitivity analysis performed immediately before the validation phase will provide significant insight into the relationships among the physical system description, the failure data base, and the reliability model which would be important in future reliability analyses. In addition, the application of LSR techniques to the ALL or LWS will infuse those programs with very significant insights into their data management tasks so that important data will be easily accessible and will reflect the requirements of both the experimenter and the high level manager.

APPENDIX A

FBD MODIFICATIONS

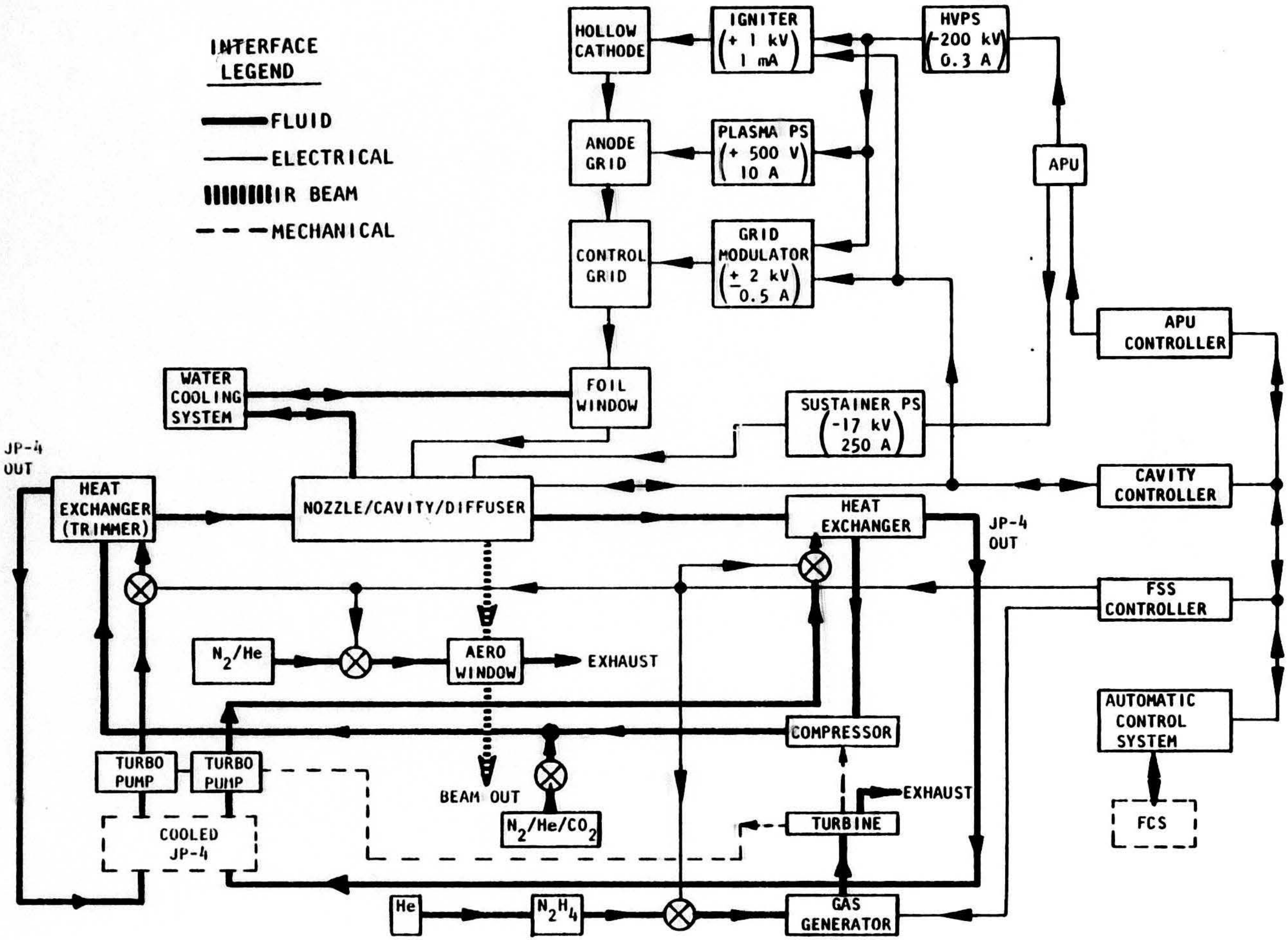

Figure A-1. EDL Functional Block Diagram

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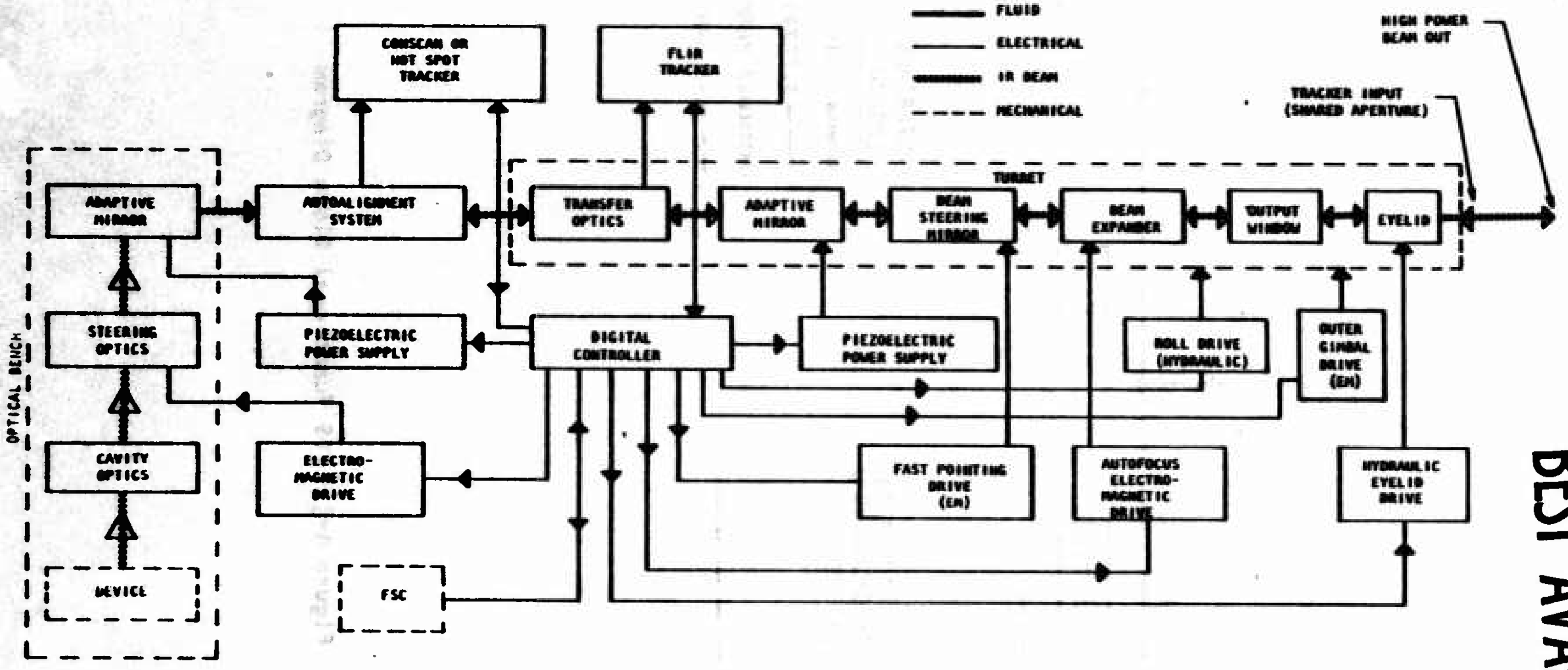


Figure A-2. BCS Functional Block Diagram

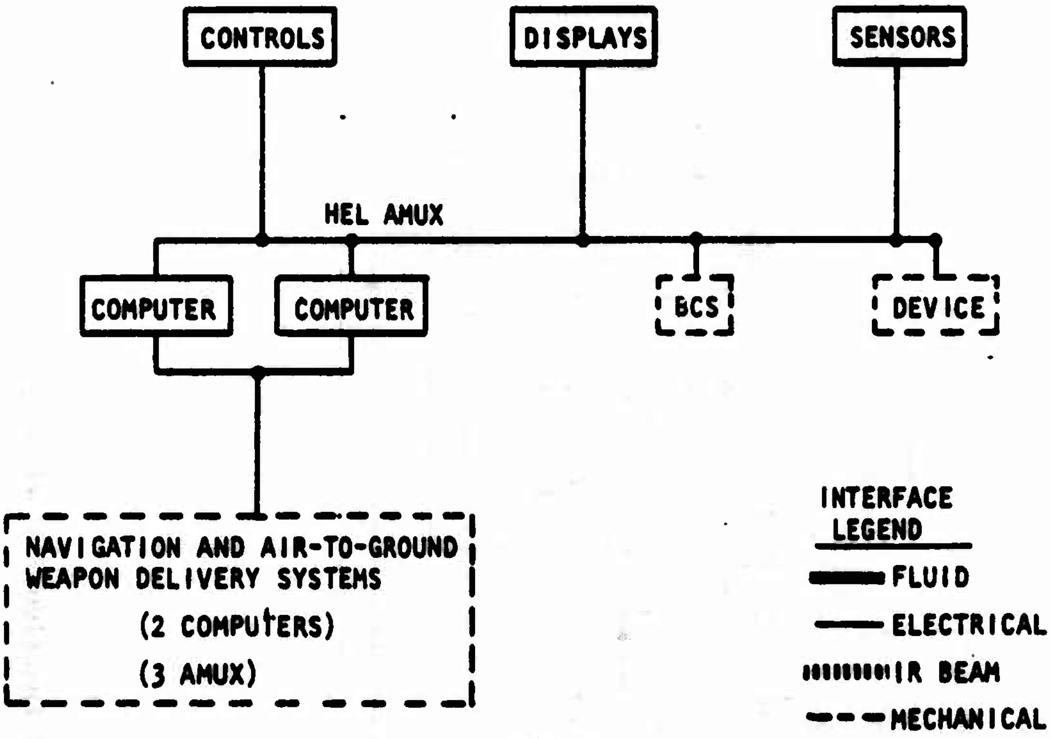
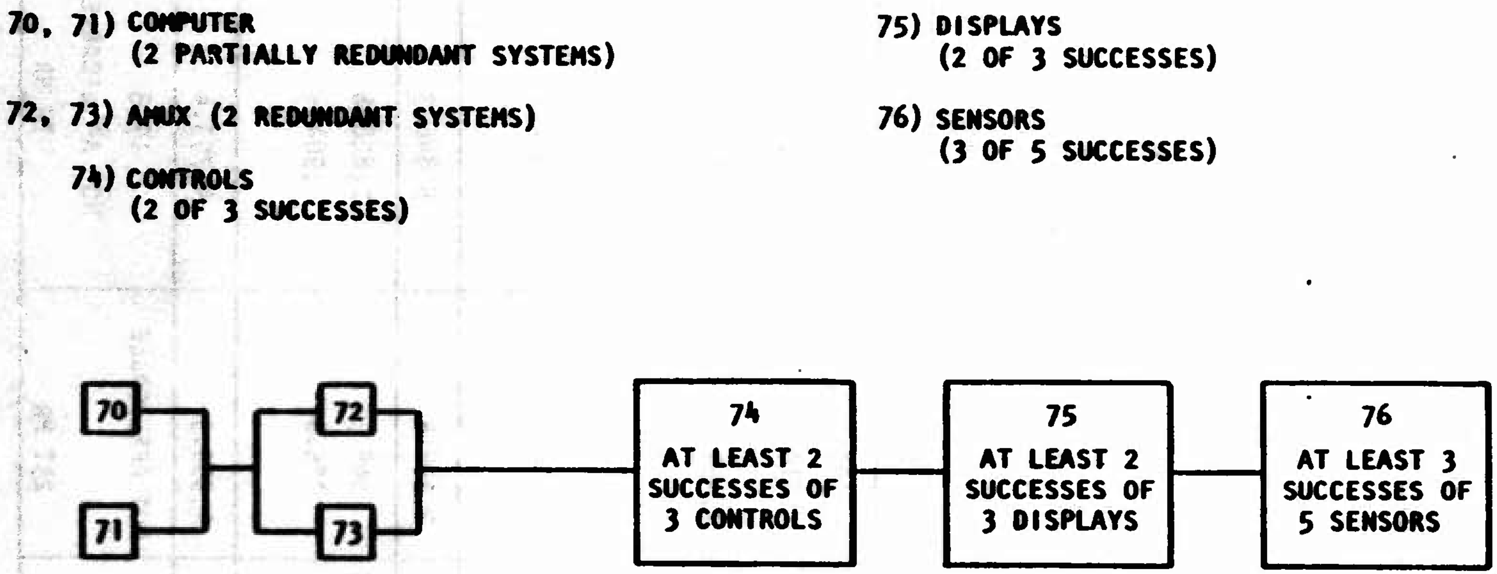


Figure A-3. FCS Functional Block Diagram

TABLE A-1. FCS FAILURE DATA BASE*

LOCATION	UNIT	FAILURE RATE ($\times 10^{-6}$)	
		VAN INSTALLATION	ALL INSTALLATION
RADAR FAIRING INSTALLATION	ANTENNA/RANDOME	22.9	114.2
	TRANSMITTER	118.6	593.3
	MICROWAVE	121.5	607.4
	BEAM STEERING CONTROLLER	72.2	360.8
	LPRF	211.2	1,056.2
	POWER SUPPLIES	35.0	175.4
	INS	NOT AVAILABLE	NOT AVAILBLE
	SUPPORT STRUCTURE		
	COOLING	33.4	33.4
	SUBTOTAL	618.8	2,940.7
RACK	DISC MEMORY	192.3	480.7
	TUTERFACE ADAPTER UNIT	210.5	526.2
	DIGITAL ASSY	1,451.7	3,629.3
	A/D CONVERTER	16.4	41.0
	PRE/POST SIGNAL PROCESSOR	321.6	804.0
	POWER SUPPLIES	146.1	365.1
	RACK CABINET	-	-
	SUBTOTAL	2,338.6	5,846.3
CONSOLE	DISPLAYS & CONTROLS	565.7	2,828.8
	VIDEO RECORDER	785.8	1,964.6
	CONSOLE CABINET		
	SUBTOTAL	1,351.5	4,793.4
OTHER	APT BUFFER	NOT APPLICABLE	17.8
	INS		NOT APPLICABLE
	MTBF	232 HR	73 HR

*Westinghouse HELRATS Phase 0 Report (Draft), August 1976.



$$g_F(R) = \left[1-(1-R_{70})(1-R_{71})\right]\left[1-(1-R_{72})\ (1-R_{73})\right]\left[\sum_{i=2}^{3}\binom{3}{i}R_{74}^{i}\ (1-R_{74})^{3-i}\right]$$

$$\left[\sum_{i=2}^{3}\binom{3}{i}R_{75}^{i}\ (1-R_{75})^{3-i}\right]\left[\sum_{i=3}^{5}\binom{5}{i}R_{76}^{i}\ (1-R_{75})^{5-i}\right]$$

Figure A-4. FCS Reliability Block Diagram

APPENDIX B

FAILURE RATE DATA BASE

Abbreviations:

Source ID

N - Nonelectronic Reliability Notebook[*]

G - GIDEP[**]

Environment

G - Ground

S - Ship or submarine

H - Helicopter

A - Aircraft

L - Laboratory

NOTE: Asterisk (*) on data means that failure rate has been recalculated correctly from GIDEP data.

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[*] Nonelectronic Reliability Notebook, RADC-TR-75-22, 1975.

[**] Summaries of Failure Rate Data, volume 1, GIDEP, August 1975.

GDL

A. CRYOGENIC VESSELS (SAME AS CL, EDL)

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
N-77	GENERAL FITTINGS	H	29.7		
N-81	GASKETS	H	20.3		
N-101	METAL TUBING	G	2.0		
N-130	LIQUID LEVEL INDICATOR	G	11.9		
N-133	RATE OF FLOW INDICATOR	A	1061.9		
N-203	PRESSURE REGULATOR	A	125.0		
N-250	LIQUID OXYGEN TANK	A	2615.4		
N-277	SOLENOID VALUE	A	100.9		
			3967.1		

GDL

B. HIGH PRESSURE GAS VESSELS (SAME AS EDL)

SOURCE	COMPONENT	ENVIR	FR (/10⁶ HRS)	L90% (/10⁶ HRS)	U90% (/10⁶ HRS)
M-77	GENERAL FITTING	H	29.7	22.6	38.2
M-81	GASKET	H	20.3	15.0	27.0
M-101	METAL TUBING	G	2.0	1.0	2.9
M-203	PRESSURE REGULATOR	A	125.0	120.0	130.2
M-247	COMPRESSED GAS TANK	A	138.3	131.5	145.3
M-259	PRESSURE TRANSDUCER	A	94.5	89.4	99.8
M-277	SOLENOID VALUE	A	100.9	91.2	111.3
			510.7		

GDL

C. GAS GENERATOR - O_2/JP-4 DRIVEN

SOURCE	COMPONENT	ENVIR	FR ($/10^6$ HRS)	L90% ($/10^6$ HRS)	U90% ($/10^6$ HRS)
N-77	GENERAL FITTINGS	H	29.7	22.6	38.2
N-81	GASKETS	H	20.3	15.0	27.0
N-96	DUCT HARDWARE	A	59.9	50.3	70.7
N-101	TUBING, METAL HARDWARE	G	2.0	1.4	2.9
N-103	HEAT EXCHANGERS	A	2.8	2.6	2.9
N-136	TEMPERATURE INDICATORS	A	103.2	94.7	112.3
N-140	MANIFOLDS	A	33.0	30.8	35.2
N-201	FUEL REGULATORS	A	178.8	169.7	188.2
N-261	TEMPERATURE TRANSDUCERS	A	97.2	91.1	103.5
N-283	PRESSURE REGULATOR, FUEL	A	65.0	54.1	77.4
N-286	SOLENOID, FUEL	A	57.7	51.0	64.9
			649.6		

GDL

D. TURBINE - HOT GAS DRIVEN (SAME AS EDL)

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
N-38	BALL BEARINGS	A	3.4	3.0	3.8
N-77	GENERAL FITTINGS	H	29.7	22.6	38.2
N-84	GENERAL SEALS, (GASKETS)	A	28.1	27.6	28.6
N-96	DUCT - HARDWARE	A	60.0	50.3	70.7
N-103	HEAT EXCHANGERS	A	2.8	2.6	2.9
N-136	TEMPERATURE INDICATORS	A	103.2	94.7	112.3
N-140	MANIFOLDS	A	33.0	30.8	35.2
N-153	GEAR - POWER TRANSMITTALS	A	12.5	12.1	12.8
N-156	SHAFT - POWER TRANSMITTALS	A	7.1	6.3	7.9
N-177	RESILIENT MOUNTS	A	6.5	2.8	12.8
N-261	TEMPERATURE TRANSDUCERS	A	97.2	91.1	103.5
N-267	TURBINES, ROCKET ENGINE *(K-1000 ASSUMED)	D*	$\frac{249}{632.5}$		

GDL

E. TURBO PUMPS (LIQUID), SAME AS GDL)

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
N-198	TURBINE DRIVEN	A	71.970 72.0	56.862	89.555

GDL

F. IGNITERS (SAME AS C1)

SOURCE	COMPONENT	ENVIR	ER (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
G-415.40	IGNITION PORTS (LIQ. ROCK ENG.)	A	$\frac{6920.6}{6920.6}$		

GDL

G. COMBUSTOR (SAME AS CL)

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
N-77	GENERAL FITTINGS	H	29.655	22.621	38.241
N-81	GASKETS	H	20.349	15.029	26.977
N-84	GENERAL SEALS	A	28.075	27.597	28.559
N-103	HEAT EXCHANGERS	A	2.761	2.637	2.889
N-108	ELECTRIC IGNITER	D	0.019	0.011	0.033
N-259	PRESSURE TRANSDUCER	A	94.520	89.388	99.848
N-261	TEMPERATURE TRANSDUCER	A	97.193	91.132	103.521
			272.6		

GDL
H. NOZZLE BANK (SAME AS CL)

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
ROCKETDYNE	NOZZLE	L	26193		
			26193		

GDL

I. CAVITY/DIFFUSER (SAME AS CL)

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/106 HRS)	U90% (/10^6 HRS)
G-563.35	LIQUID FUEL ENGINE MANIFOLD	H	106.2		
			106.2		

GDL

J. AERODYNAMIC WINDOW (SAME AS EDL)

SOURCE	COMPONENT	ENVIR	ER (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
N-77	GENERAL FITTINGS	H	29.655	22.621	38.241
N-81	GASKETS	H	20.349	15.029	26.977
N-86	PRESSURE SEALS	A	87.073	83.468	90.783
N-101	METAL TUBING (HARDWARE)	G	2.047	1.411	2.878
N-303	PNEUMATIC, PRESSURE REGULATOR	A	115.526	109.990	121.250
N-307	PNEUMATIC, SOLENOID VALVE	A	19.888	16.444	23.786
			276.5		

GDL

K. DIGITAL CONTROLLER (SAME AS BCS, CL, EDL)

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
G-190.00	JET AC COMPUTER	A	2336.0*		
G-230.10	ANALOG/DIGITAL CONVERTER	A	54.0*		
			2390.0		

CL

A. SUPER CRITICAL CRYOGENIC VESSELS

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
N-77	GENERAL FITTINGS	H	29.655	22.621	38.241
N-81	GASKETS	H	20.349	15.029	26.977
N-101	METAL TUBING (HARDWARE)	G	2.047	1.411	2.878
N-130	LIQUID TEST INSTRUMENTS	G	11.905	2.116	37.500
N-135	RATE OF FLOW "	A	1061.905	947.025	1185.871
N-203	PRESSURE REGULATORS	A	125.010	119.951	130.212
N-247	COMPRESSED GAS TANKS	A	138.299	131.537	145.295
N-250	LIQUID OXYGEN TANKS	A	2615.385	2373.184	2873.825
N-277	SOLENOIA VALUES	A	100.921	91.242	111.273
			4105.4		

CL

B. CRYOGENIC VESSELS

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
N-77	GENERAL FITTINGS	H	29.655	22.621	38.241
N-81	GASKETS	H	20.349	15.029	26.977
N-101	METAL TUBING (HARDWARE)	G	2.047	1.411	2.878
N-130	LIQUID LEVEL INDICATOR	G	11.905	2.116	37.500
N-133	RATE OF FLOW "	A	1061.905	947.025	1185.871
N-203	PRESSURE REGULATORS	A	125.010	119.951	130.212
N-250	LIQUID OXYGEN TANKS	A	2615.285	2373.184	2873.825
N-277	SOLENOID VALUES	A	100.921	91.242	111.273
			3967.1		

CL

C. HIGH PRESSURE GAS VESSELS

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
N-77	GENERAL FITTINGS	H	29.655	22.621	38.241
N-81	GASKETS	H	20.349	15.024	26.977
N-101	METAL TUBINS (HARDWARE)	G	2.047	1.411	2.878
N-203	PRESSURE REGULATORS	A	125.010	119.951	130.212
N-247	COMPRESSED GAS TANKS	A	138.299	131.537	145.295
N-259	PRESSURE TRANSDUCERS	A	94.520	89.388	99.848
N-277	SOLENOID VALUES	A	100.921	91.242	111.273
			510.8		

CL

D. HEAT EXCHANGER (LIQUID/GAS)

SOURCE	COMPONENT	ENVIR	FR ($/10^6$ HRS)	L90% ($/10^6$ HRS)	U90% ($/10^6$ HRS)
N-77	GENERAL FITTINGS	H	29.655	22.621	38.241
N-81	GASKETS	H	20.349	15.029	26.977
N-101	METAL TUBING (HARDWARE)	G	2.047	1.411	2.878
N-103	HEAT EXCHANGERS	A	2.761	2.637	2.889
N-261	TEMPERATURE TRANSDUCERS	A	97.193	91.132	103.521
			152.0		

CL

E. IGNITERS (SAME AS GDL

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
G-415.40	IGNITION PARTS (LIQ. ROCK. ENG.)	A	6920.6		
			6920.6		

CL

F. COMBUSTOR (SAME AS GDL)

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
N-77	GENERAL FITTINGS	H	29.7		
N-81	GASKETS	H	20.3		
N-84	GENERAL SEALS	A	28.1		
N-103	HEAT EXCHANGER	A	2.8		
N-108	ELECTRIC IGNITOR	D	0.019		
N-259	PRESSURE TRANSDUCER	A	94.5		
N-261	TEMPERATURE TRANSDUCER	A	97.2		
			272.6		

CL

G. NOZZLE BANK (SAME AS GDL)

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
ROCKETDYNE	NOZZLE	L	26193		
			26193		

CL

H. CAVITY/DIFFUSER/EJECTOR (SAME AS GDL)

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
G-563.35	LIQUID FUEL ENGINE MANIFOLD	H	106.2		
			106.2		

CL

J. GAS GENERATORS - O_2/JP-4 DRIVEN

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
N-77	GENERAL FITTINGS	H	29.655	22.621	38.241
N-81	GASKETS	H	20.349	15.029	26.977
N-96	DUCT (HARDWARE)	A	59.891	50.260	70.693
N-101	METAL TUBING (HARDWARE)	G	2.047	1.411	2.878
N-103	HEAT EXCHANGERS	A	2.761	2.637	2.889
N-136	TEMPERATURE INDICATORS	A	103.217	94.681	112.261
N-140	MANIFOLDS	A	32.964	30.808	35.221
N-201	FUEL REGULATORS	A	178.807	169.721	188.218
N-261	TEMPERATURE TRANSDUCERS	A	97.193	91.132	103.521
N-283	FUEL, PRESSURE REGULATOR	A	65.029	54.077	77.382
N-286	FUEL, SOLENOID	A	57.670	51.040	64.859
			649.8		

CL

K. TURBO PUMP (SAME AS GDL, CL)

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
N-198	TURBINE DRIVEN PUMP	A	72.0	56.9	89.6
			72.0		

CL

L. TURBINE - AIR DRIVEN

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
N-38	BALL BEARINGS	A	3.420	3.030	3.842
N-77	GENERAL FITTINGS	H	29.655	22.621	38.241
N-84	GENERAL SEALS	A	28.075	27.597	28.559
N-96	DUCT (HARDWARE)	A	59.891	50.260	70.693
N-140	MANIFOLDS	A	32.964	30.808	35.221
N-153	GEAR-POWER TRANSMITTER	A	12.458	12.101	12.823
N-156	SHAFT- " "	A	7.069	6.297	7.902
N-177	RESILIENT MOUNTS	A	6.472	2.821	12.783
N-261	TEMPERATURE TRANSDUCERS	A	97.193	91.132	103.521
N-267	ROCKET ENGINE TURBINES *(k = 1000 ASSUMED)	D*	249	---	---
			526.4		

CL

H. DIGITAL CONTROLLER (SAME AS BCS, GDL, EDL)

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
G-190.00	JET AC COMPUTER	A	2336.0*		
G-230.10	ANALOG/DIGITAL CONVERTER	A	54.0*		
			2390.0		

EDL

C. LOW-POWER POWER SUPPLIES

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
N-62	SOLDER CONNECTIONS	A	0.023	0.019	0.026
N-69	CONNECTOR PINS	G	0.000604		
G-561.00	ELECTRICAL POWER SUPPLY	A	225.5*		
N-67	COAXIAL CONNECTORS	A	1.03	0.85	1.24
G-515.50	AMPLIFIER	G	23.8	16.4	37.1
G-530.50	MICROELECTRONIC MODULE	S	25.1	22.4	28.3
			275.4		

EDL

D. HIGH POWER POWER SUPPLY (SAME AS CL)

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
N-62	SOLDER CONNECTIONS	A	0.023	0.019	0.026
N-69	CONNECTOR PINS	G	0.000604		
G-561.00	ELECTRICAL POWER SUPPLY	A	225.5*		
N-67	COAXIAL CONNECTORS	A	1.03	0.85	1.24
G-515.50	AMPLIFIER	G	23.8	16.4	37.1
G-530.50	MICROELECTRONIC MODULE	S	25.1	22.4	28.3
			275.4		

EDL

E. AIRBORNE POWER UNIT

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
N-54	CIRCUIT BREAKER PROTECTION DEVICE	A	1.965	1.759	2.188
N-63	WELDED CONNECTIONS	A	0.0446	0.0209	0.0837
N-69	CONNECTOR PINS	G	0.000604		
N-81	GASKETS	H	20.349	15.029	26.977
N-101	METAL TUBING (HARDWARE)	G	2.047	1.411	2.878
N-135	TACHOMETER	A	86.570	79.269	94.315
N-154	GEARBOX - POWER TRANSMITTAL	A	8.799	7.317	10.471
N-156	SHAFT - POWER TRANSMITTAL	A	7.069	6.297	7.902
N-174	AC GENERATOR	A	1105.253	1081.893	1128.956
N-182	FUEL PUMP	A	45.081	41.966	48.350
N-191	OIL PUMP	A	59.459	43.514	79.550
N-201	FUEL REGULATOR	A	178.807	169.721	188.218
N-203	PRESSURE REGULATOR	A	125.010	119.951	130.212
N-247	COMPRESSED GAS TANK	A	138.299	131.537	145.295
N-248	FUEL TANK	GM	7.745	6.380	9.293
N-277	SOLENOID VALVE	A	100.921	91.242	111.273
			1887.4		

EDL

F. NOZZLE/CAVITY/DIFFUSER (SAME AS CL, GDL)

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
G-563.35	LIQUID FUEL ENGINE MANIFOLD	H	106.2		
			106.2		

EDL

G. HEAT EXCHANGERS (LIQUID/GAS)

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
N-77	GENERAL FITTINGS	H	29.655	22.621	38.241
N-81	GASKETS	H	20.349	15.029	26.977
N-101	METAL TUBING (HARDWARE)	G	2.047	1.411	2.878
N-103	HEAT EXCHANGERS	A	2.761	2.637	2.889
N-261	TEMPERATURE TRANSDUCERS	A	97.193	91.132	103.521
			152.0		

EDL

H. HIGH PRESSURE GAS VESSELS (SAME AS GDL)

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
N-77	GENERAL FITTINGS	H	29.655	22.621	38.241
N-81	GASKETS	H	20.349	15.029	26.977
N-101	METAL TUBING (HARDWARE)	G	2.047	1.411	2.878
N-203	PRESSURE REGULATORS	A	125.010	119.951	130.212
N-247	COMPRESSED GAS TANKS	A	138.299	131.537	145.295
N-259	PRESSURE TRANSDUCERS	A	94.520	89.388	99.848
N-277	SOLENOID VALVE	A	100.921	91.242	111.273
			510.8		

EDL

I. CRYOGENIC TANK (SAME AS CL, GDL)

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
N-77	GENERAL FITTINGS	H	29.655	22.621	38.241
N-81	GASKETS	H	20.349	15.029	26.977
N-101	METAL TUBING (HARDWARE)	G	2.047	1.411	2.878
N-130	LIQUID LEVEL INSTRUMENTS	G	11.905	2.116	37.500
N-133	RATE OF FLOW INSTRUMENTS	A	1061.905	947.025	1185.871
N-203	PRESSURE REGULATORS	A	125.010	119.951	130.212
N-250	LIQUID OXYGEN TANKS	A	2615.385	2373.184	2873.825
N-277	SOLENOID VALVES	A	100.921	91.242	111.273
			4105.4		

EDL

J. LIQUID TANKS

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
N-81	GASKETS	H	20.349	15.029	26.977
N-101	METAL TUBING (HARDWARE)	G	2.047	1.411	2.878
N-201	FUEL REGULATORS	A	178.807	169.721	188.218
N-248	FUEL TANKS	G	7.745	6.380	9.293
			208.8		

EDL

K. COMPRESSOR

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
N-86	PRESSURE SEALS	A	87.073	83.468	90.783
N-198	TURBINE DRIVEN PUMPS	A	71.970	56.862	89.555
N-77	GENERAL FITTINGS	H	29.7	22.6	38.2
N-96	DUCT	A	59.9	50.3	70.7
			248.6		

EDL

L. TURBINE - HOT GAS DRIVEN (SAME AS GDL)

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
N-38	BALL BEARINGS	A	3.420	3.030	3.842
N-77	GENERAL FITTINGS	H	29.655	22.621	38.241
N-84	GENERAL SEALS	A	28.075	27.597	28.559
N-96	DUCT (HARDWARE)	A	59.891	50.260	70.693
N-103	HEAT EXCHANGERS	A	2.761	2.637	2.889
N-136	TEMPERATURE INDICATOR	A	103.217	94.681	112.261
N-140	MANIFOLDS	A	32.964	30.808	35.221
N-153	GEAR - POWER TRANSMITTAL	A	12.458	12.101	12.823
N-156	SHAFT - POWER TRANSMITTAL	A	7.069	6.297	7.902
N-177	RESILIENT MOUNTS	A	6.472	2.821	12.783
N-261	TEMPERATURE TRANSDUCERS	A	97.193	91.132	103.521
N-267	ROCKET ENGINE TURBINES	D*	249.		
	* (k = 1000 ASSUMED)		632.3		

EDL

M. GAS GENERATOR - $N_2 H_4$ DRIVEN

SOURCE	COMPONENT	ENVIR	FR ($/10^6$ HRS)	L90% ($/10^6$ HRS)	U90% ($/10^6$ HRS)
N-77	GENERAL FITTINGS	H	29.655	22.621	38.241
N-81	GASKETS	H	20.349	15.029	26.977
N-96	DUCT (HARDWARE)	A	59.891	50.260	70.693
N-101	METAL TUBING (HARDWARE)	G	2.047	1.411	2.878
N-103	HEAT EXCHANGERS	A	2.761	2.637	2.889
N-107	GENERAL IGNITOR	A	42.025	34.870	50.107
N-136	TEMPERATURE INDICATOR	A	103.217	94.681	112.261
N-140	MANIFOLD	A	32.964	30.808	35.221
N-201	FUEL REGULATOR	A	178.807	169.721	188.218
N-261	TEMPERATURE TRANSDUCER	A	97.193	91.132	103.521
N-283	FUEL, PRESSURE REGULATOR	A	65.029	54.077	77.382
N-286	FUEL, SOLENOID	A	57.670	51.040	64.859
			691.8		

EDL

N. AERODYNAMIC WINDOW

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
N-77	GENERAL FITTINGS	H	29.655	22.621	38.241
N-81	GASKETS	H	20.349	15.029	26.977
N-86	PRESSURE SEALS	A	87.073	83.468	90.783
N-101	METAL TUBING (HARDWARE)	G	2.047	1.411	2.878
N-303	PNEUMATIC PRESSURE REGULATOR	A	115.526	109.990	121.250
N-307	PNEUMATIC, SOLENOID	A	19.888	16.444	23.786
			276.5		

EDL

O. WATER COOLING SYSTEM (HEAT EXCHANGERS)

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
N-77	GENERAL FITTINGS	H	29.655	22.621	38.241
N-81	GASKETS	H	20.349	15.029	26.977
N-101	METAL TUBING (HARDWARE)	G	2.047	1.411	2.878
G-386.97	HEAT EXCHANGERS (WATER)	S	153.8*		
N-261	TEMPERATURE TRANSDUCERS	A	97.193	91.132	103.521
			303.0		

EDL

P. DIGITAL CONTROLLER (SAME AS BCS, GOL, CL)

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
G-190.00	JET AC COMPUTER	A	2336.0*		
G-230.10	ANALOG/DIGITAL CONVERTER	A	54.0*		
			2390.0		

BCS

D. PIEZOELECTRIC DRIVERS

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L30% (/10^6 HRS)	U90% (/10^6 HRS)
G-852.80	VIBRATION TRANSDUCERS	A	459.4*		
G-561.00	ELECTRICAL POWER SUPPLY	A	225.5*		
N-67	COAXIAL CONNECTORS	A	1.03	0.85	1.24
			685.9		

BCS

E. ELECTROMECHANICAL MIRROR DRIVERS

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
N-22	ELECTRICAL LINEAR ACTUATORS	A	66.947	62.957	71.107
N-38	BALL BEARINGS	A	3.420	3.030	3.842
N-62	SOLDER CONNECTIONS	A	0.023	0.019	0.026
N-69	CONNECTOR PINS	G	0.000604		
N-87	GIMBALS	A	7.829		
N-235	SENSITIVE SWITCHES	A	14.650	12.908	16.543
N-245	RESOLVER SYNCHROS AND RESOLVERS	A	87.097	78.063	96.813
N-258	MOTIONAL TRANSDUCERS	A	254.237	156.780	391.525
			431.1		

BCS

F. ALIGNMENT SYSTEMS

SOURCE	COMPONENT	ENVIR	FR (/10^6 HR)	L90% (/10^6 HR)	U90% (/10^6 HR)
N-72	CONSTANT SPEED DRIVES	A	321	293	350
N-82	GASKETS, O-RINGS	A	2.39	1.83	3.07
N-117	AMMETER	A	198	124	300
N-142	AXLE	A	6.42	1.76	16.6
N-148	MECHANICAL COUPLING	A	119	115	124
N-153	GEAR	A	12.5	12.1	12.8
N-170	dc SERVO MOTOR	A	15.6	7.32	29.3
N-205	RELAYS	A	6.74	5.34	8.37
N-224	SOLENOIDS	A	62.2	58.1	66.4
N-229	LIMIT SWITCHES	A	28.0	25.5	30.6
N-254	THERMOCOUPLES	A	31.9	21.2	46.4
N-266	VARIABLE TRANSFORMER	S	89.7	60.0	129
RADC-TR -75-210	HeNe LASER	L	815.0	-	-
G-104.10	LINEAR BEARINGS	A	12.1*	-	-
G-951.00	CABLE HARNESS	A	600.2*	-	-
G-545.70	OPTICAL DEVICES-TELESCOPE	A	< 59.3	-	-
G-561.00	POWER SUPPLY-NONROTATING	A	225.5*	-	-
G-515.50	AMPLIFIER	G	23.21*	-	-
G-230.10	ANALOG/DIGITAL CONVERTER	A	54.0*	19.4	356.8
G-530.50	MICROELECTRONIC MODULE	A	25.11	22.43	28.29
			2707.7		

BCS

G. SERVOL HYDRAULIC DRIVE SYSTEMS

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
N-21	HYDRAULIC ACCUMULATORS	A	118.275	113.104	123.605
N-24	LINEAR, HYDRAULIC SERVO ACTUATOR	A	130.423	124.737	136.282
N-38	NEEDLE BEARINGS	A	3.420	3.030	3.842
N-51	ELECTRIC BRAKES	G	11.976†	4.087	27.395
N-62	SOLDER CONNECTIONS	A	0.023	0.019	0.026
N-67	COAXIAL CONNECTORS	A	1.031	0.847	1.240
N-69	CONNECTOR PINS	G	0.000604		
N-70	FIXED ROTATIONAL COUNTERS	A	228.010	318.175	358.670
N-70	HYDRAULIC FITTINGS	H	3.898	2.838	5.238
N-81	GASKETS	H	20.349	15.029	26.977
N-84	GENERAL SEALS	A	28.075	27.597	28.559
N-105	GENERAL HOSES	A	115.830	83.398	157.143
N-153	GEAR - POWER TRANSMITTAL	A	12.458	12.101	12.823
N-168	HYDRAULIC, DC, ELECTRIC MOTOR	A	148.472	119.839	181.367
N-189	VARIABLE DELIVERY HYDRAULIC PUMP	A	17.570	10.102	28.477
N-205	GENERAL RELAY	A	6.743	5.339	8.374
N-231	HYDRAULIC PRESSURE SWITCHES	A	137.968	126.529	150.091
N-245	RESOLVER, SYNCHROS AND RESOLVERS	A	87.097	78.063	96.813
N-253	HYDRAULIC RESERVOIR TANK	A	50.575	47.773	53.487
G-515.50	AMPLIFIER	G	223.825	16.439	37.138
G-230.10	ANALOG/DIGITAL CONVERTER	A	53.501	19.412	356.849
G-530.50	MICROELECTRONIC MODULE	S	25.112	22.427	28.287
N-88	GYRO	A	7.829		
			1330.2		

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† FAILURE, PER 10^6 PART - CYCLES

BCS

H. ON/OFF HYDRAULIC DRIVE SYSTEM

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
N-21	HYDRAULIC ACCUMULATORS	A	118.275	113.104	123.605
N-24	LINEAR, HYDRAULIC SERVO ACTUATOR	A	130.423	124.737	136.282
N-38	BALL BEARINGS	A	3.420	3.030	3.842
N-62	SOLDER CONNECTION	A	0.023	0.019	0.026
N-67	COAXIAL CONNECTOR	A	1.031	0.847	1.240
N-70	FIXED ROTATIONAL COUNTERS	A	338.010	318.175	358.670
N-78	HYDRAULIC FITTINGS	A	3.898	2.838	5.238
N-81	GASKETS	H	20.349	15.029	26.977
N-84	GENERAL SEALS	A	28.075	27.597	28.559
N-105	GENERAL HOSES	A	115.830	83.398	157.143
N-153	GEAR - POWER TRANMITTAL	A	12.458	12.101	12.823
N-168	HYDRAULIC DC ELEC. MOTOR	A	148.472	119.839	181.367
N-189	VARIABLE DELIVERY HYDRAULIC PUMP	A	17.570	10.102	28.477
N-205	GENERAL RELAYS	A	6.743	5.339	8.374
N-231	HYDRAULIC PRESSURE SWITCHES	A	137.968	126.529	150.091
N-253	HYDRAULIC RESERVOIR TANK	A	50.575	47.773	53.487
			1133.0		

BCS

J. CONSCAN OR HOT SPOT TRACKER

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
APT LOG	T/I	A	7406		
APT LOG	CRYOGENIC SYSTEM	A	6334		
			13740		

BCS

L. DIGITAL CONTROLLER (SAME AS CL, GDL, EDL)

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
G-190.00	JET AC COMPUTER	A	2336.0*		
G-230.10	ANALOG/DIGITAL CONVERTER	A	54.0*		
			2390.0		

FCS

SOURCE	COMPONENT	ENVIR	FR (/10^6 HRS)	L90% (/10^6 HRS)	U90% (/10^6 HRS)
HUGHES	HELRATS	ESTIMATES			
	APG-63		18389		
	ELECTRONICALLY SCANNED ARRAY		2000		
	COMPUTER		1000		
	DISPLAY SYSTEM		2000		
	INERTIAL SYSTEM		2300		
			25689		

NOTE: Some last minute changes were suggested for the FBD, RBD, and data base for the FCS. These changes were made too late to be included in the reliability calculations, however the new FBD, RBD, and data base are presented in appendix A.

APPENDIX C

DERIVATION OF PSEUDONUMBER OF TRIALS APPROXIMATION

For the exponential distribution

$$t \sim \frac{1}{\theta} e^{-t/\theta} \quad , \; t \geq 0$$

and

$$R(t) = e^{-t/\theta}$$

where

t = time

$\theta = 1/\lambda$ = MTBF

λ = failure rate

A point estimate of reliability is given by

$$\hat{R}(t) = e^{-t/\hat{\theta}}$$

where

$$\hat{\theta} = \sum_{t=1}^{n} t_i/n = T/n$$

It is known that $2n\hat{\theta}/\theta \sim \chi^2(2n)$. Thus, $P\left[\theta > \frac{2n\hat{\theta}}{\chi^2_{\alpha}(2n)}\right] = 1-\alpha$, is a one-sided confidence interval for θ, where $P[a>b]$=probability that $a>b$.

A two-sided confidence interval on $1/\theta$ is given by

$$P\left[\frac{\chi^2_{1-\alpha/2}(2n)}{2n\hat{\theta}} < 1/\theta < \frac{\chi^2_{\alpha/2}(2n)}{2n\hat{\theta}}\right] = 1-\alpha.$$

Let $c_2 = \chi^2_{\alpha/2}(2n) \big/ 2n\hat{\theta}$.

Now, $\hat{\theta}$ and c_2 are known from the data base;

But n and $\chi^2_{\alpha/2}(2n)$ are not known. To circumvent this difficulty, apply the Wilson-Hilferty approximation

$$\chi^2_{\alpha/2}(2n)/2n = \left[1 - \frac{2}{9(n)} + z_{\alpha/2}\sqrt{2/9(2n)}\right]^3$$

where $z_{\alpha/2}$ is the 100 $\alpha/2$ percentage point of the normal distribution function.

$$\text{thus, } c_2 = \frac{1}{\hat{\theta}}\left[1-2/9(2n) + z_{\alpha/2}\sqrt{2/9(2n)}\right]^3$$

$$= \hat{\lambda}\left[1-2/9(2n) + z_{\alpha/2}\sqrt{2/9(2n)}\right]^3$$

Since all the confidence intervals on λ are 90 percent intervals, $z_{\alpha/2} =$ $z_{\alpha/2} = z_{.05} = 1.645$.

$$\text{Then, } \left(c_2/\lambda\right)^{1/3} = 1 - \frac{2}{9(2n)} + 1.645\sqrt{2/9(2n)}$$

$$\text{and } \left[\left(c_2/\lambda\right)^{1/3} - 1 + \frac{1}{9n}\right]^2 = \left[1.645\sqrt{\frac{1}{9n}}\right]^2$$

Squaring and combining terms, gives

$$n = 2/\left[-9\left\{2\left[\left(c_2/\lambda\right)^{1/3}-1\right] - (1.645)^2\right\} \pm 9\sqrt{\left\{2\left[\left(c_2/\lambda\right)^{1/3}-1\right]-(1.645)^2\right\}}\right.$$

$$\left. +4\left[\left(c_2/\lambda\right)^{2/3}+2\left(c_2/\lambda\right)^{1/3}-1\right]\right]$$

APPENDIX D

LOWER CONFIDENCE BOUNDS CALCULATIONS

The fundamental reliability equations for the respective kinds of HEL systems are:

$$G_G(\underline{R})=g_G(\underline{R})g_B(\underline{R})g_F(\underline{R})$$

$$G_C(\underline{R})=g_C(\underline{R})g_B(\underline{R})g_F(\underline{R})$$

$$G_E(\underline{R})=g_E(\underline{R})g_B(\underline{R})g_F(\underline{R})$$

where the $g(\underline{R})$'s are the reliability functions of each of the major systems and are given by $g_G(\underline{R})=R_1R_2R_3R_4R_5[1-(1-R_6)(1-R_7)]R_8[\sum_{475}^{500}\binom{500}{i}R_9{}^i(1-R_9)^{500-i}]R_{10}R_{11}$

$$g_B(\underline{R})=R_{13}R_{14}R_{15}[\sum_{25}^{50}\binom{50}{i}R_{16}^i(1-R_{16})^{50-i}]R_{17}R_{18}R_{19}R_{20}R_{21}[1-(1-R_{22})(1-R_{23})]R_{24}$$

$$g_F(\underline{R})=R_{25}R_{26}R_{27}R_{28}R_{29}R_{30}R_{31}[1-(1-R_{32})(1-R_{33})][1-(1-R_{34})(1-R_{35})]R_{36}R_{37}$$

$$G_C(\underline{R})=R_{38}R_{39}R_{40}R_{41}[1-(1-R_{42})(1-R_{43})][1-(1-R_{44})(1-R_{45})]R_{46}$$
$$[\sum_{179}^{188}\binom{188}{i}R_{47}^i(1-R_{47})^{188-i}]R_{48}R_{49}R_{50}R_{51}R_{52}R_{53}$$

$$g_E(\underline{R})=R_{54}R_{55}R_{56}R_{57}R_{58}R_{59}R_{60}R_{61}R_{62}R_{63}R_{64}R_{65}R_{66}R_{67}R_{68}R_{69}$$

The following relations hold for the GDL, in general, except for terms involving R_6, R_7, R_{16}, R_{22}, R_{23}, R_{32}, R_{33}, R_{34}, and R_{35}.

$$\left[\frac{\partial G_G(\underline{R})}{\partial R_1}\right]^2 \mathrm{Var}(\hat{R}_1)=\prod_{\substack{j=1\\ j\neq 6,7,9,16,22,23,32,33,34,35}}^{37}\left[R_j{}^2\right](a)(b)(c)(d)(e)(f)\left[\ln R_1\right]^2/n_1$$

$$\vdots$$

$$\left[\frac{\partial G_G(\underline{R})}{\partial R_{37}}\right]^2 \mathrm{Var}(\hat{R}_{37})=\prod_{\substack{j=1\\ j\neq 6,7,9,16,22,23,32,33,34,35}}^{37}\left[R_j^2\right](a)(b)(c)(d)(e)(f)\left[\ln R_{37}\right]^2/n_{37}$$

where

$$a = \left[1-\left(1-R_6\right)\left(1-R_7\right)\right]^2$$

$$b = \left[\sum_{475}^{500} \binom{500}{i} R_q^{\,i}\left(1-R_q\right)^{500-i}\right]^2$$

$$c = \left[\sum_{25}^{50} \binom{50}{i} R_{16}^{\,i}\left(1-R_{16}\right)^{50-i}\right]^2$$

$$d = \left[1-\left(1-R_{22}\right)\left(1-R_{23}\right)\right]^2$$

$$e = \left[1-\left(1-R_{32}\right)\left(1-R_{33}\right)\right]^2$$

$$f = \left[1-\left(1-R_{34}\right)\left(1-R_{35}\right)\right]^2$$

thus,

$$\sum_{\substack{j=1 \\ j\neq 6,7,9,16,22,23, \\ 32,33,34,35}}^{37} \left[\frac{\partial G_G(\underline{R})}{\partial R_j}\right]^2 \mathrm{Var}\left(\hat{R}_j\right) = \sum_{\substack{j=1 \\ j\neq 6,7,9,16,22,23,32,33,34,35}}^{37} \left[\prod_{j=1}^{37} R_j^2\right]\left[\ln R_j\right]^2 (a)(b)(c)(d)(e)(f)/n_j$$

The remaining partial derivatives must be found separately.

$$\left[\frac{\partial G_G(\underline{R})}{\partial R_6}\right]^2 \mathrm{Var}\left(\hat{R}_6\right) = \left[\prod_{\substack{j=1 \\ j\neq 7,9,16,22,23,32,33,34,35}}^{37} R_j^2\right]\left[\ln R_6\right]^2\left[1-R_7\right]^2 (b)(c)(d)(e)(f)/n_6$$

$$\left[\frac{\partial G_G(\underline{R})}{\partial R_7}\right]^2 \mathrm{Var}\left(\hat{R}_7\right) = \left[\prod_{\substack{j=1 \\ j\neq 6,9,16,22,23,32,33,34,35}}^{37} R_j^2\right]\left[\ln R_7\right]^2\left[1-R_6\right]^2 (b)(c)(d)(e)(f)/n_7$$

$$\left[\frac{\partial G_G(\underline{R})}{\partial R_9}\right]^2 \mathrm{Var}\left(\hat{R}_9\right) = \left[\overset{37}{\underset{j=1}{\pi}} R_j^2\right]\left[\ln R_9\right]^2 \left\{\sum_{475}^{500}\binom{500}{i}\left[-R_9^i\left(500-i\right)\left(1-R_9\right)^{499-i} + \left(1-R_9\right)^{500-i} iR_9^{\,i-1}\right]\right\}^2 (a)(c)(d)(e)(f)/n_9$$

$$j \neq 6,7,16,22,23,32,33,34,35$$

$$\left[\frac{\partial G_G(\underline{R})}{\partial R_{16}}\right]^2 \mathrm{Var}\left(\hat{R}_{16}\right) = \left[\overset{37}{\underset{j=1}{\pi}} R_j^2\right]\left[\ln R_{16}\right]^2 \left\{\sum_{25}^{50}\binom{50}{i}\left[-R_{16}^i\left(50-i\right)\left(1-R_{16}\right)^{49-i} + \left(1-R_{16}\right)^{50-i} iR_{16}^{\,i-1}\right]\right\} \left[(a)(b)(d)(e)(f)/n_{16}\right]$$

$$j \neq 6,7,9,22,23,32,33,34,35$$

$$\left[\frac{\partial G_G(\underline{R})}{\partial R_{22}}\right]^2 \mathrm{Var}\left(\hat{R}_{22}\right) = \left[\overset{37}{\underset{j=1}{\pi}} R_j^2\right]\left[\ln R_{22}\right]^2\left[1-R_{23}\right]^2 (a)(b)(c)(e)(f)/n_{22}$$

$$j \neq 6,7,9,23,32,33,34,35$$

$$\left[\frac{\partial G_G(\underline{R})}{\partial R_{23}}\right]^2 \mathrm{Var}\left(\hat{R}_{23}\right) = \left[\overset{37}{\underset{j=1}{\pi}} R_j^2\right]\left[\ln R_{23}\right]^2\left[1-R_{22}\right]^2 (a)(b)(c)(e)(f)/n_{23}$$

$$j \neq 6,7,9,16,22,32,33,34,35$$

$$\left[\frac{\partial G_G(\underline{R})}{\partial R_{32}}\right]^2 \mathrm{Var}\left(\hat{R}_{32}\right) = \left[\overset{37}{\underset{j=1}{\pi}} R_j^2\right]\left[\ln R_{32}\right]^2\left[1-R_{33}\right]^2 (a)(b)(c)(d)(f)/n_{32}$$

$$j \neq 6,7,9,16,22,23,33,34,35$$

$$\left[\frac{\partial G_G(\underline{R})}{\partial R_{33}}\right]^2 \mathrm{Var}\left(\hat{R}_{33}\right) = \left[\overset{37}{\underset{j=1}{\pi}} R_j^2\right]\left[\ln R_{33}\right]^2\left[1-R_{32}\right]^2 (a)(b)(c)(d)(f)/n_{33}$$

$$j \neq 6,7,9,16,22,23,32,34,35$$

$$\left[\frac{\partial G_G(\underline{R})}{\partial R_{34}}\right]^2 \mathrm{Var}\left(\hat{R}_{34}\right) = \left[\prod_{\substack{j=1\\ j\neq 6,7,9,16,22,23,32,33,35}}^{37} R_j^2\right]\left[\ln R_{34}\right]^2\left[1-R_{35}\right]^2 (a)(b)(c)(d)(e)/n_{34}$$

$$\left[\frac{\partial G_G(\underline{R})}{\partial R_{35}}\right]^2 \mathrm{Var}\left(\hat{R}_{35}\right) = \left[\prod_{\substack{j=1\\ j\neq 6,7,9,16,22,23,32,33,34}}^{37} R_j^2\right]\left[\ln R_{35}\right]^2\left[1-R_{34}\right]^2 (a)(b)(c)(d)(e)/n_{35}$$

Similar relations were obtained for the CL and the EDL systems.

These relations were then added together to form the left hand side of

$$\sum_j \left[\frac{\partial G(\underline{R})}{\partial R_j}\right]^2 \mathrm{Var}\ \hat{R}_j = G(\hat{\underline{R}})\left[1-G(\hat{R})\right]/\hat{n}.$$

$G(\hat{R})$ and $1-G(\hat{R})$ were obtained and $\hat{n}$ was then calculated for the three HEL systems for each of two operating time periods; t=30 sec and t=3600 sec.

For calculation purposes it was decided to break down each system into subsets which were common to all systems and had similar computational forms. For example, g_B and g_F are terms common to all three systems. Additionally, quantities of the general form

(1) $(R_1)(R_2)(R_3)\ldots(R_n)$

(2) $[1-(1-R_a)(1-R_b)]$

(3) $\sum_i \binom{n}{i} R^i (1-R)^{n-i}$

are common to all the five major subsystems, g_G, g_C, g_E, g_B, and g_F. Thus, the calculations were obtained in these forms and subsets, then summed for each HEL system. To be specific, For t = 30 sec.

$$L_G = \sum_j \left[\frac{\partial G_G \; (\underline{R})}{\partial R_j} \right]^2 \text{Var}(\hat{R}_j)$$

$$= L(A_1) + L(A_2) + L(A_3) + L(B_1) + L(B_2) + L(B_3) + L(F)$$

$$= 6.0481136 \times 10^{-13} + 8.6325898 \times 10^{-11} + 0 + 4.0066856 \times 10^{-8}$$
$$+ 1.0289781 \times 10^{-16} + 0 + 4.573672 \times 10^{-8}$$

$$= 8.5890507 \times 10^{-8}$$

where $L(A_1)$, $L(A_2)$, $L(A_3)$ are terms of the form (1), (2), (3) respectively in the GDL; $L(B_1)$, $L(B_2)$, $L(B_3)$ are terms of the form (1), (2), (3) respectively in the BCO subsystem and L9F) represents the FCS.

$$L_C = \sum_j \left[\frac{\partial G_C \; (\underline{R})}{\partial R_j} \right]^2 \text{Var}(\hat{R}_j)$$

$$= L(C_1) + L(C_2) + L(C_3) + L(B_1) + L(B_2) + L(B_3) + L(F)$$

$$= 9.986852 \times 10^{-9} + 1.726518 \times 10^{-10} + 1.0 \times 10^{-14}$$
$$+ 4.0066856 \times 10^{-8} + 1.028978 \times 10^{-16} + 0 +$$
$$4.573672 \times 10^{-8}$$

$$= 9.596308 \times 10^{-8}$$

where $L(C_1)$, $L(C_2)$, $L(C_3)$ are terms of the form (1), (2), (3) respectively in the CL. $L(B_1)$, $L(B_2)$, $L(B_3)$, and $L(F)$ are as above.

$$L_E = \sum_j \left[\frac{\partial G_E \; (\underline{R})}{\partial R_j} \right]^2 \text{Var}\ (\hat{R}_j)$$

$$= L(E_1) + L(E_2) + L(E_3) + L(B_1) + L(B_2) + L(B_3) + L(F)$$

$$= 2.263538 \times 10^{-8} + 0 + 0 + 4.0066856 \times 10^{-8} + 1.028978 \times 10^{-16}$$
$$+ 0 + 4.573672 \times 10^{-8}$$

$$= 1.0843889 \times 10^{-7}$$

Where $L(E_1)$, $L(E_2)$, $L(E_3)$ are terms of the form (1), (2), (3) respectively for the EDL.

For $t = 3600$ sec.

$$L_G = \sum_j \left[\frac{\partial G_G \; (\underline{R})}{\partial \cdot R_j} \right]^2 \quad \text{Var } (\hat{R}_j)$$

$$= L(A_1) + L(A_2) + L(A_3) + L(B_1) + L(B_2) + L(B_3) + L(F)$$

$$= 7.999183 \times 10^{-9} + 1.136424 \times 10^{-6} + 1.020510 \times 10^{-4} + 1.661103 \times 10^{-6} + 1.722335 \times 10^{-12} + 0 + 6.050557 \times 10^{-4}$$

$$= 7.099100 \times 10^{-4}$$

$$L_C = \sum_j \left[\frac{\partial G_C \; (\underline{R})}{\partial R_j} \right]^2 \quad \text{Var } (\hat{R}_j)$$

$$= L(C_1) + L(C_2) + L(C_3) + L(B_1) + L(B_2) + L(B_3) + L(F)$$

$$= 2.032451 \times 10^{-8} + 5.133400 \times 10^{-11} + 1.897151 \times 10^{-2} + 1.661103 \times 10^{-6} + 1.722335 \times 10^{-12} + 0 + 6.050557 \times 10^{-4}$$

$$= 1.957825 \times 10^{-2}$$

$$L_E = \sum_j \left[\frac{\partial G_E \; (\underline{R})}{\partial R_j} \right]^2 \quad \text{Var } (\hat{R}_j)$$

$$= L(E_1) + L(E_2) + L(E_3) + L(B_1) + L(B_2) + L(B_3) + L(F)$$

$$= 6.872963 \times 10^{-3} + 0 + 0 + 1.661103 \times 10^{-6} + 1.722335 \times 10^{-12} + 0 + 6.050557 \times 10^{-4}$$

$$= 7.479680 \times 10^{-3}$$

Since the equations are large and cumbersome, a programmable HP-67 was used to obtain the $\hat{n}$'s for each of the six situations.

RESULTS OF THE CALCULATION WERE:

	GDL		CL		EDL	
	$\hat{n}$	$\hat{x}$	$\hat{n}$	$\hat{x}$	$\hat{n}$	$\hat{x}$
t = 30 sec	8776	8769	9206	9199	11022	11014
t = 3600 sec	58	53	4	3	9	8

These results were then used to determine the 95 percent lower confidence bounds on the HEL systems given in chapter IV.

APPENDIX E
COMBAT EFFECTIVENESS ALGORITHM

A. INTRODUCTION

This appendix provides an algorithm for first order estimates of the effects of HEL reliability on HEL airborne combat effectiveness. Pages 50 to 55 are repeated here for completeness.

B. AN ALGORITHM FOR FIRST ORDER ESTIMATES

The algorithm presented herein enables first order estimates of laser reliability on outcomes of aerial combat between a laser armed aircraft and an attacking aircraft employing any feasible number of AAM's or gun firing attempts. Enemy SAM attacks on the laser armed aircraft are also accomodated, as well as any reasonable number of successive encounters. Friendly and enemy aircraft kill probabilities are directly computed, enabling direct determination of values of such MOE's as the exchange ratio (ER).

We start with the following definitions of terms:

defensive hassle = an encounter between a laser armed aircraft and any feasible number of successive attacking SAM's or AAM's, or gun firing passes. A defensive hassle terminates prior to the next offensive laser shot, e.g., a laser shot at the enemy aircraft.

offensive hassle = one or more laser shots at the enemy aircraft. The offensive hassle terminates prior to the next defensive hassle.

Figure E-1 presents the generic probability probability tree diagram for the kill calculations algorithm. Note that the order or number of offensive and defensive hassles is arbitrary, since we may set certain input parameters to values such that non-occurring hassles indicated on the Figure E-1 are accounted for, retaining algorithm validity.

Let

R_i = laser reliability* for the i^{th} laser firing,

P_i = probability that the laser kills the threat, on i^{th} laser firing

$\bar{P}_i = 1-P_{1-i}$

For the j^{th} offensive hassle (see figure E-1), we define:

Θ_j = probability that laser does not kill the enemy aircraft, and laser is operable at end of j^{th} offensive hassle; given surviving and operable laser and laser armed aircraft at start of j^{th} offensive hassle;

F_j = probability that laser does not kill the enemy aircraft, and laser is inoperable (failed) at end of j^{th} offensive hassle; given surviving and operable laser and laser armed aircraft at start of j^{th} offensive hassle;

E_j = probability that laser kills the enemy aircraft, given surviving and operable laser and laser armed aircraft at start of the j^{th} offensive hassle;

and

$$\Theta_j = R_j\bar{P}_j$$
$$F_j = \bar{R}_j$$
$$E_j = R_jP_j .$$

---

* The i^{th} firing is of time duration t_i, and R_i is determined from expressions for G(R) described in Chapter IV.

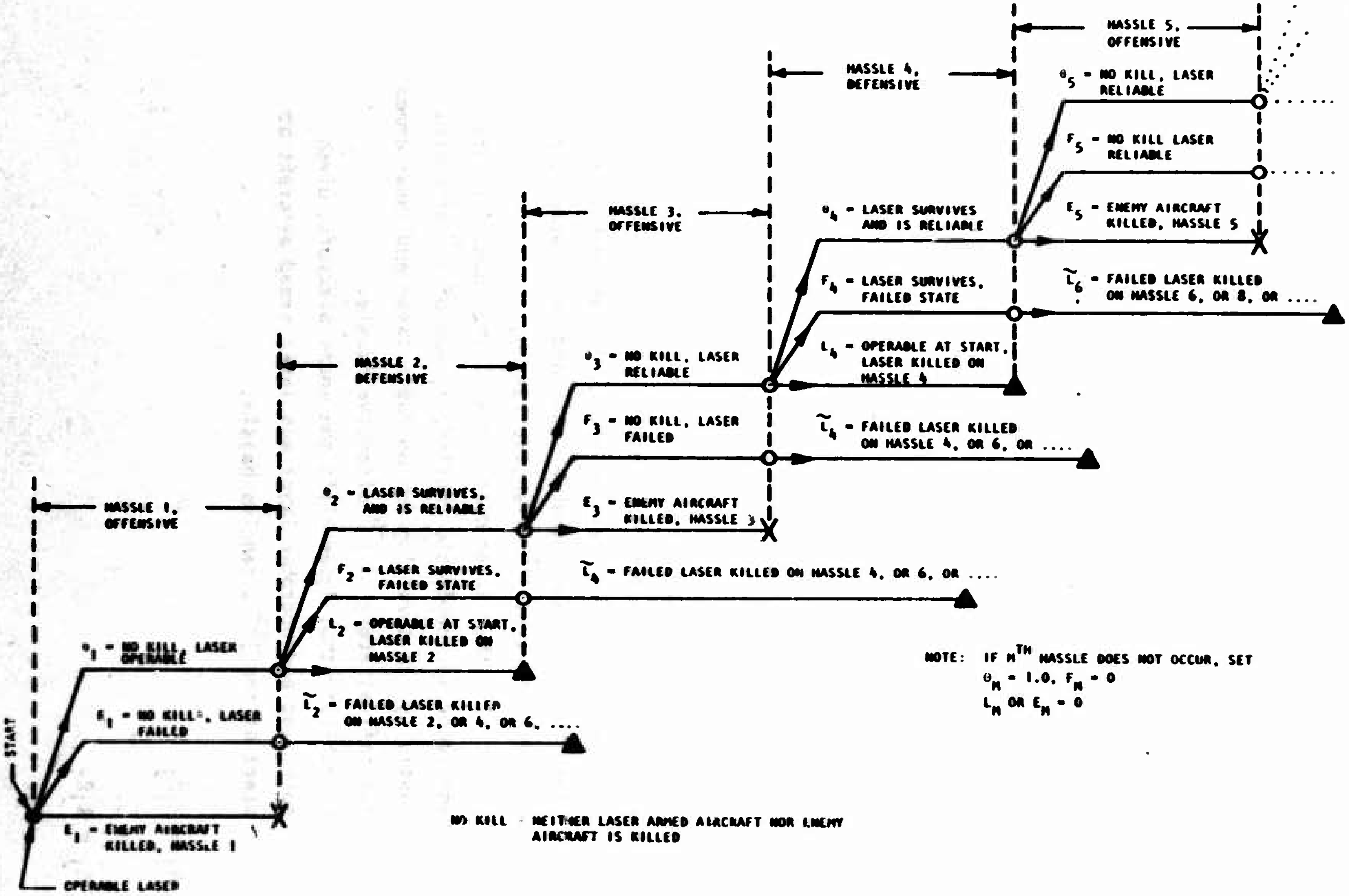

Figure E-1. Generic Probability Tree Diagram for Kill Calculations Algorithm and

$$\theta_j + F_j + E_j = 1.0, \text{ as required.}$$

We note that if the first hassle, for example, is a defensive hassle, we set $\theta_1 = 1.0$, $F_1 = E_1 = 0.0$.

In order to calculate the outcomes of defensive hassle 2, or indeed of any defensive hassle, we will now derive several outcome probability relations associated with the defensive hassle.

We define:

$W_i = 1 - $ (SSPK of the i^{th} threat in a defensive hassle*);

$\overline{W}_i = 1 - W_i$;

p_i = probability that the laser armed aircraft survives the i^{th} threat, and is reliable, given reliable laser and laser armed aircraft at start of i^{th} threat encounter in a defensive hassle;

then:

$$p_i = R_i P_i + R_i \overline{P}_i W_i$$

and we define:

$$z_i = \overline{R}_i W_i$$

---

* Note that the reliability of a threat SAM or AAM is usually included in the missile SSPK, since each such weapon is only employed once. However, the laser can be employed repeatedly, requiring explicit, detailed treatment of laser reliability as follows.

Figure E-2 presents the generic probability tree diagram for calculating S_{ij}, where:

S_{ij} = probability that the laser armed aircraft survives the 1st, and the 2nd, and the 3rd,........, and the i^{th} threat of the j^{th} defensive hassle; in operable or non-operable state; given that the laser is operable and laser armed aircraft is operable at start of the j^{th} defensive hassle.

From figure E-2, we see that:

$$S_{1j} = p_1 + z_1 \ , \ (j) \ ,$$

when the (j) symbol indicates that the parameters of the RHS of the equation are evaluated for the j^{th} defensive hassle, and

$$S_{2j} = p_1 p_2 + p_1 z_2 + z_1 W_2 \quad (j),$$

$$S_{3j} = p_1 p_2 p_3 + p_1 p_2 z_3 + p_1 z_2 W_3 + z_1 W_2 W_{e3} \quad (j),$$

$$S_{4j} = p_1 p_2 p_3 p_4 + p_1 p_2 p_3 z_4 + p_1 p_2 z_3 W_4$$
$$+ p_1 z_2 W_3 W_4 + z_1 W_2 W_3 W_4 \quad (j),$$

and, from the perceived pattern:

$$S_{nj} = \prod_{i=1}^{n} p_i + \left(\prod_{i=1}^{n-1} p_i\right) z_n + \left(\prod_{i=1}^{n-2} p_i\right) z_{n-1} W_n$$

$$+ \left(\prod_{i=1}^{n-3} p_i\right) z_{n-2} \left(\prod_{i=n-1}^{n} W_i\right)$$

$$+ \left(\prod_{i=1}^{n-4} pi\right) z_{n-3} \left(\prod_{i=n-2}^{n} W_i\right)$$

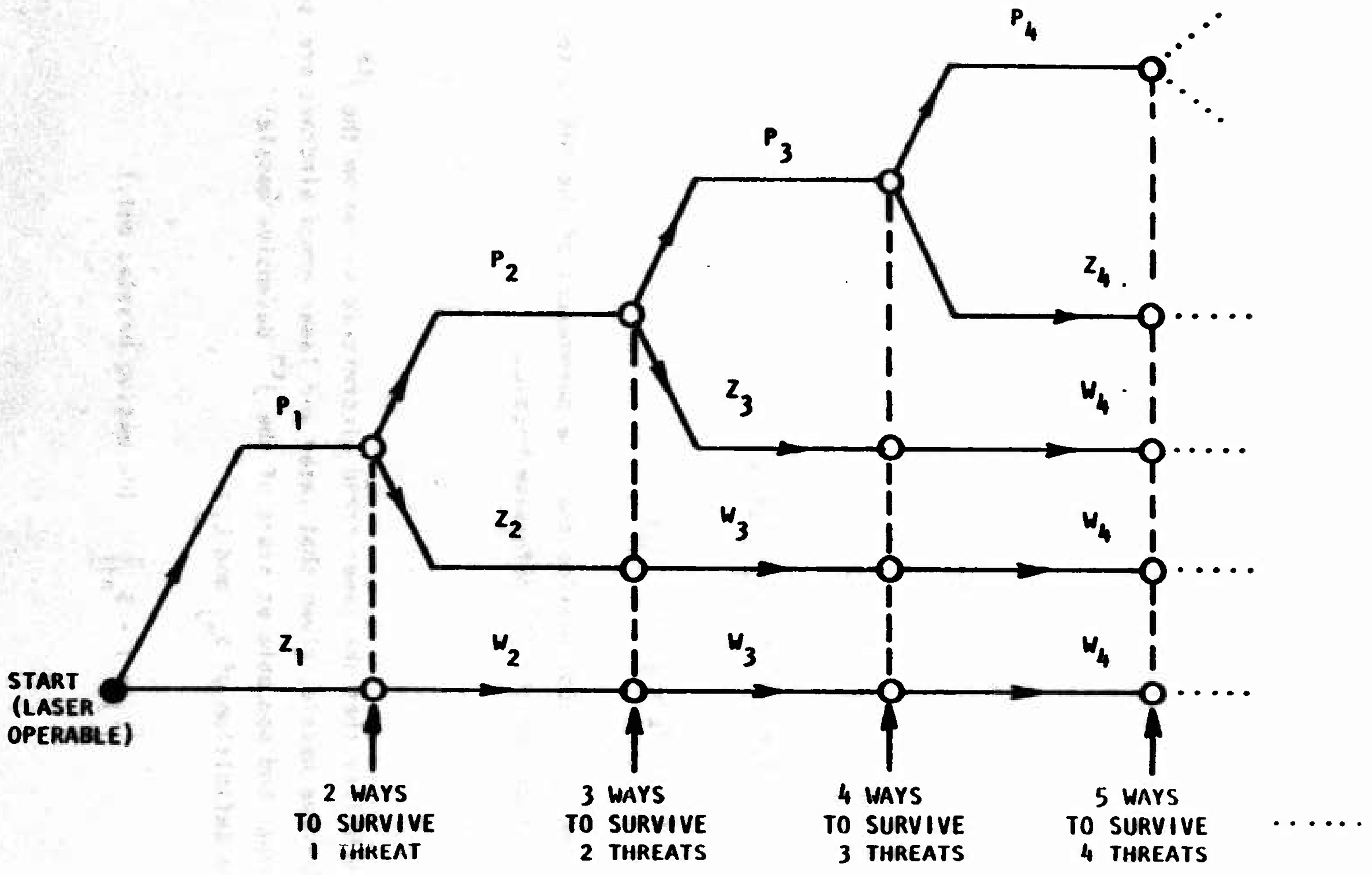

Figure E-2. Generic Probability Tree Diagram for Laser Armed Aircraft Survival in a Defensive Hassle

$$+ \quad \cdots \qquad\qquad + z_1 \prod_{i=2}^{n} W_n \quad ; \quad (j),$$

with $n \geq 2$, integer . (For $n=1$, $S_{1j} = p_1 + z_1$),

For example, with $n=6$:

$$\begin{aligned} S_{6j} = {} & p_1p_2p_3p_4p_5p_6 + p_1p_2p_3p_4p_5z_6 \\ & + p_1p_2p_3p_4z_5W_6 + p_1p_2p_3z_4W_5W_6 \\ & + p_1p_2z_3W_4W_5W_6 + p_1z_2W_3W_4W_5W_6 \\ & + z_1W_2W_3W_4W_5W_6 \quad ; \quad (j) \end{aligned}$$

where, again, the (j) symbol implies that the parameters of the RHS of the equation have values for the j^{th} defensive hassle.

Let

L_j = probability that the laser armed aircraft is killed on the j^{th} defensive hassle, given that laser and laser armed aircraft are surviving and operable at start of the j^{th} defensive hassle:

then, from the definition of S_{nj} and L_j:

$$\left[L_j = 1 - S_{nj}\right] \qquad \text{(defensive hassles only)}$$

Let

F_j = probability that the laser survives the j^{th} defensive hassle and in a failed state, given that the laser and laser armed aircraft are surviving and operable at the start of the j^{th} defensive hassle;

then, from the definition of F_j and p_i and S_{nj}:

$$\left[F_j = S_{nj} - \prod_{i=1}^{n} p_i \right] \quad ;$$

Let

θ_j = probability that the laser survives the j^{th} defensive hassle and in an operable state; given that the laser and laser armed aircraft are surviving and operable at the start of the j^{th} defensive hassle,

then, from the definition of p_i;

$$\left[\theta_j = \prod_{i=1}^{n} p_i \right] \quad (j)$$

and: $L_j + F_j + \theta_j = 1.0$, as required.

Let

$\tilde{L}_2$ = probability that the laser, surviving hassle 1 but in a failed condition, is killed on defensive hassle: 2, or 4, or 6,...., then for M_2 threats in hassle 2, M_4 threats in hassle 4, ..., we have:

$$\tilde{L}_2 = 1 - \underbrace{\left(\prod_{i=1}^{M_2} w_i\right)}_{\substack{\text{not}\\ \text{killed}\\ \text{on}\\ \text{Hassle}\\ 2}} \underbrace{\left(\prod_{i=1}^{M_4} w_i\right)}_{\substack{\text{not}\\ \text{killed}\\ \text{on}\\ \text{Hassle}\\ 4}} \underbrace{\left(\prod_{i=1}^{M_6} w_i\right)}_{\substack{\text{not}\\ \text{killed}\\ \text{on}\\ \text{Hassle}\\ 6}} \cdots$$

Finally, with all values of θ_j, F_j, E_j, L_j and $\tilde{L}$ calculated from the above relations, we let:

K_R = probability that the enemy aircraft is killed

K_B = probability that the friendly, laser armed aircraft is killed,

and referencing Figure 1, we see that:

$$K_R = E_1 + \theta_1\theta_2E_3 + \theta_1\theta_2\theta_3\theta_4E_5 + \ldots$$

$$K_B = F_1\tilde{L}_2 + \theta_1L_2 + \theta_1L_2 + \theta_1F_2\tilde{L}_4 + \theta_1\theta_2F_3L_4 + \theta_1\theta_2\theta_3\tilde{L}_4 + \ldots,$$

where the K_R and K_B are derived from listing the mutually exclusive, exhaustive ways in which the kills can occur.

We can then compute ER = exchange ratio, where

$$ER = \frac{K_R}{K_B},$$

pertaining to repeated occurrences of a scenario having the specified hassles, hassle sequence and hassle parameter sets.

Appendix F provides two worked examples for a defensive, offensive. defensive hassle sequence; with 2 AAM's as threats in each defensive hassle; with ohly HEL reliability varied between examples. (NOTE: Any offensive/defensive hassle sequence is accomodated by the algorithm).

APPENDIX F

COMBAT EFFECTIVENESS EXAMPLES

HASSLE SEQUENCE: DEFENSIVE, OFFENSIVE, DEFENSIVE.

EXAMPLE:

PROBLEM 1. **GIVEN:**

Hassle 1, offensive: Does not occur, thus:

$\theta_1 = 1.0$

$F_1 = 0.0$

$E_1 = 0.0$

Hassle 2, defensive: 2 AAM threats:

$W_1 = 0.8$ $P_1 = 0.95$ $R_1 = 0.90$

$W_2 = 0.7$ $P_2 = 0.97$ $R_2 = 0.93$

Hassle 3, offensive:

$R_3 = 0.90$

$P_3 = 0.98$

Hassle 4, defensive: 2 AAM threats:

$W_1 = 0.9$ $P_1 = 0.98$ $R_1 = 0.92$

$W_2 = 0.85$ $P_2 = 0.99$ $R_2 = 0.92$

Solution

Step 1: We just compute the values of:
S_{12}, S_{22}, S_{14}, S_{24}, as follows:

For S_{12}:

$$S_{12} = p_1 + z_1 \qquad (2)$$

$$z_1 = \bar{R}_1 W_1$$

$$p_1 = R_1 P_1 + R_1 \bar{P}_1 W_1 \quad ,$$

thus:

$$\boxed{z_1 = (.10)(.8) = .08} \quad ,$$

$$p_1 = R_1 P_1 + R_1 \bar{P}_1 W_1$$

$$= (.90)(.95) + (.90)(.05)(.8)$$

$$= .8550 + .360$$

$$\boxed{p_1 = .8910}$$

$$S_{12} = .8910 + .800$$

$$\boxed{S_{12} = .9710} \quad ,$$

For S_{22}:

$$S_{22} = p_1 p_2 + p_1 z_2 + z_1 W_2 \qquad (2)$$

p_1 = .8910 from S_{12} calculations

z_1 = .08 from S_{12} calculations

$$p_2 = R_2 p_2 + R_2 \bar{P}_2 W_2$$

$$= (.93)(.97) + (.93)(.03)(.7)$$

$$= .9021 + .0195$$

$$\boxed{p_2 = .9207}$$

$$z_2 = \bar{R}_2 W_2$$

$$z_2 = (.07)(.7)$$

$$\left[z_2 = .049 \right]$$

$$\therefore \quad S_{22} = (.8910)(.9207) + (.8910)(.049) + (.08)(.70)$$

$$= .8203 + .0437 + .0560$$

$$\left[S_{22} = .9200 \right]$$

<u>For S_{14}:</u>

$$S_{14} = p_1 + z_1 \qquad (4)$$

$$z_1 = \bar{R}_1 W_1 \quad ,$$

$$p_1 = R_1 P_1 + R_1 \bar{P}_1 W_1 \quad ,$$

thus:

$$\left[z_1 = (.08)(.9) = .072 \right]$$

$$p_1 = (.92)(.98) + (.92)(.02)(.9)$$

$$= .9016 = .0165$$

$$\left[p_1 = .9181 \right] \quad ,$$

and:

$$S_{14} = .9181 + .0720$$

$$\left[S_{14} = .9901 \right]$$

For S_{24}:

$$S_{24} = p_1 p_2 + P_1 z_2 + z_1 W_2 \qquad (4)$$

$p_1 = .9181$, from S_{14} calculations

$z_1 = .072$, from S_{14} calculations

$$z_2 = \bar{R}_2 W_2$$

$$= (.08)(.85)$$

$$\left[z_2 = .068 \right]$$

$$P_2 = R_2 P_2 + R_2 \bar{P}_2 W_2 = (.92)(.99) + (.92)(.01)(.85)$$

$$= .9108 + .0078$$

$$\left[P_2 = .9186 \right]$$

and $$S_{24} = (.9181)(.9186) + (.9181)(.068) + (.072)(.85)$$

$$S_{24} = .8434 + .0624 + 0612$$

$$\left[S_{24} = .9670 \right]$$

Step 2: Compute and/or list θ_j, F_j, E_j, L_j, $\tilde{L}_j$, for all j:

$\theta_1 = 1.0$, given;

$F_1 = 0.0$, given;

$E_1 = 0.0$, given;

$$L_2 = 1-S_{22}$$

$$L_2 = 1-.9200$$

$$\left[L_2 = .0800\right] \quad ;$$

$$F_2 = S_{22} - \prod_{i=1}^{2} p_i$$

$$F_2 = .9200 - (.8910)(.9207) = .9200 - .8203$$

$$\left[F_2 = .0997\right] ;$$

$$\theta_2 = \prod_{i=1}^{2} p_i$$

$$\left[\theta_2 = .8203\right];$$

$$\theta_3 = R_3\bar{P}_3 = (.90)(.02)$$

$$\left[\theta_3 = .018\right] ;$$

$$\left[F_3 = \bar{R}_3 = .10\right] \quad ;$$

$$E_3 = R_3P_3 = (.90)(.98)$$

$$\left[E_3 = .882\right] \quad ;$$

$$\tilde{L}_2 = 1 - \left(\prod_{i=1}^{M_2} W_i\right)\left(\prod_{i=1}^{M_4} W_i\right)$$

$$= 1 - \left[(.8)(.7)\right]\left[(.9)\ (.85)\right]$$

$$= 1 - (.56)(.765) = 1 - .4284$$

$$\left[\tilde{L}_2 = .5716\right]$$

$$\tilde{L}_4 = 1 - S_{24} = 1 - .9670$$

$$\left[\tilde{L}_4 = .033\right]$$

Step 3. Compute K_R and K_B:

$$K_R = E_1 + \theta_1 \theta_2 E_3$$

$$= 0 + (1.0)(.8203)(.882)$$

$$\left[K_R = .7235\right]$$

$$K_B = F_1 \tilde{L}_2 + \theta_1 L_2 + \theta_1 F_2 \tilde{L}_4 + \theta_1 \theta_2 F_3 \tilde{L}_4 + \theta_1 \theta_2 \theta_3 L_4$$

$$= (0)(.5716) + (1.0)(.08) + (1.0)(.0997)(.033)$$

$$+ (1.0)(.8203)(.10)(.235) + (1.0)(.8203)(.018)(.033)$$

$$= 0 + .08 + .0032 + .0193 + .0005$$

$$\left[K_B = .1030\right]$$

Step 4. Compute ER

$$ER = \frac{.7235}{.1030}$$

$$\left[ER = 7.024 \right]$$

PROBLEM 2. GIVEN:

Same as Problem 1, except $R_1 = R_2 = 1.0$ for Hassle 2; and $R_1 = R_2 = 1.0$ for Hassle 4; and $R_3 = 1.0$ for Hassle 3.

Solution:

Step 1: Calculate S_{12}, S_{22}, S_{14}, and S_{24}.

For S_{12}:

$$S_{12} = p_1 + z_1 \qquad (2)$$

$$z_1 = \bar{R}_1 W_1$$

$$p_1 = R_1 P_1 + R_1 \bar{P}_1 W_1$$

thus:

$$\left[z_1 = (0)(W_1) = 0 \right]$$

$$p_1 = (1)(.95) + (1)(.05)(.8)$$

$$= .95 + .04$$

$$\left[p_1 = .99 \right]$$

$$S_{12} = .99 + 0$$

$$\left[S_{12} = .99 \right]$$

For S_{22}:

$$S_{22} = p_1 p_2 + p_1 z_1 + z_1 W_2 \qquad (2)$$

$$p_1 = .99, \text{ from } S_{12} \text{ calculations}$$

$$z_1 = 0, \text{ from } S_{12} \text{ calculations}$$

$$p_2 = R_2 p_2 + R_2 \bar{P}_2 W_2$$

$$= (1)(.97) + (1)(.03)(.7)$$

$$= 97 + .021$$

$$\left[p_2 = .991\right]$$

$$z_2 = \bar{R}_2 W_2 = (0)(W_2) = 0$$

$$\therefore \quad S_{22} = (.99)(.991) + (.99)(0) + (0)(W_2)$$

$$\left[S_{22} = .9811\right]$$

For S_{14}:

$$S_{14} = p_1 + z_1 \qquad (4)$$

$$\left[z_1 = \bar{R}_1 W_1 = (0)(W_1) = 0\right]$$

$$p_1 = R_1 \bar{F}_1 + R_1 \bar{F}_1 W_1$$

$$= (1)(.98) + (1)(.02)(.9) = .98 + .018$$

$$\left[p_1 = .998\right] ,$$

and

$$\left[S_{14} = 998 + 0 = .998\right] ,$$

For S_{24}:

$$S_{24} = p_1 p_2 + p_1 Z_2 + Z_1 W_2$$

$p_1 = .998$, from S_{14} calculation,

$Z_1 = 0$, from S_{14} calculation;

$$p_2 = R_2 P_2 + R_2 \overline{P}_2 W_2$$

$$= (1)(.99) + (1)(.01)(.85) = .99 + .0085$$

$$\left[p_2 = .9985\right]$$

and,

$$S_{24} = (.998)(.9985) + (.998)(0) + (0)(W_2)$$

$$\left[S_{24} = .9965\right]$$

Step 2. Compute List Θ_j, F_j, E_j, L_j, $\tilde{L}_j$, for all j.

$\Theta_1 = 1.0$, given

$F_1 = 0.0$, given

$E_1 = 0.0$, given;

$$L_2 = 1 - S_{22} = 1 - .991$$

$$\left[L_2 = .009\right]$$

$$F_2 = S_{22} - \prod_{i=1}^{2} p_i = .991 - (.99)(.991)$$

$$\left[F_2 = .0099\right]$$

$$\left[\ \theta_2 = \prod_{i=1}^{2} P_i = .9811\ \right]$$

$$\left[\theta_3 = R_3\bar{P}_3 = (1)(.02) = .02\right]$$

$$\left[F_3 = \bar{R}_3 = 0\right]$$

$$\left[E_3 = R_3P_3 = (1)(.98) = .98\right]$$

$\tilde{L}_2 = .5716$, (same as for previous example problem);

$\tilde{L}_4 = .235$, (same as for previous example problem),

$$L_4 = 1 - S_{24} = 1 - .9965$$

$$\left[L_4 = .0035\right]$$

Step 3. Compute K_R and K_B:

$$K_R = E_1 + \theta_1\theta_2E_3$$

$$= 0 + (1.0)(.9811)(.98)$$

$$\left[K_R = .9615\right]$$

$$K_B = F_1\tilde{L}_2 + \theta_1L_2 + \theta_1F_2\tilde{L}_4 + \theta_1\theta_2\theta_3\tilde{L}_4$$

$$= (0)(.5716) + (1.0)(.009) + (1.0)(.0099)(.235)$$

$$+ (1.0)(.9811)(0)(.235) + (1.0)(.9811)(.02)(.0035)$$

$$= 0 + .0090 + .0023 + 0 + .0001$$

$$\left[K_B = .0114\right]$$

Step 4. Compute ER = exchange ratio:

$$ER = \frac{.9615}{.0114}$$

$$\left[ER = 84.34\right]$$

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