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VOLUME II

FINITE ELEMENT ANALYSIS OF TRANSONIC FLOWS OVER THIN AIRFOILS
VOLUME II – PROGRAM USER'S MANUAL

LOCKHEED MISSILES & SPACE COMPANY, INC.
HUNTSVILLE RESEARCH & ENGINEERING CENTER
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AIR FORCE DYNAMICS LABORATORY
AIR FORCE WRIGHT AERONAUTICAL LABORATORIES
Air Force Systems Command
Wright-Patterson, Air Force Base, Ohio 45433

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A finite element program is described for computing steady and unsteady (oscillatory and transient) transonic flows over thin airfoils by solving directly the unsteady, nonlinear transonic potential equation based on small disturbance theory. The present numerical algorithm is developed using the concept of finite elements in conjunction with the least squares method of weighted residuals applied to both space and time. The basic element presently used is a product of an element in space and an element in time. The former has a cubic

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expansion inside each element, while the latter is a quadratic Lagrangian element. For each time step, the finite element discretization in both space and time results in a recurrence relationship in the form of a banded system of algebraic equations, which is solved by Gaussian elimination. The embedded shocks are smeared and a matching scheme for computing effectively flow over lifting airfoils is also incorporated in the program. The present computer program is composed of two parts: the first part (designated as UTRANL-I) generates, from a limited number of input cards, the necessary mesh information and, if desired, produces a CALCOMP mesh plot; the second part (UTRANL-II) carries out the analysis and displays the pressure coefficients along the chordline on printer plots. Two sample cases of flow over a NACA 64A 410 and a NACA 64A 006 airfoils are given to demonstrate the applicability and usage of the program. The solution procedures are found to be quite efficient and accurate, permitting the aerodynamic forces to be calculated to engineering accuracy in less than ten minutes CPU time on a CDC 6600 computer for the most time consuming case among all those studied.

FOREWORD

This report was prepared by personnel in the Engineering Sciences Section of the Lockheed Missiles & Space Company, Inc., Huntsville Research & Engineering Center, Huntsville, Alabama, for the Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio. The computer programs were developed under Project 1370, "Dynamic Problems in Flight Vehicles," Task 137004, "Design Analysis," Contract F33615-75-C-3125. Capt. Gerald Van Keuren, AFFDL/FBR, is the Air Force Project Engineer.

S.T.K. Chan was the principal investigator for the study, and H.C. Chen, as co-investigator, contributed to the development of the numerical solution method and developed most of the necessary computer programs. The study was conducted under the supervision of M.R. Brashears and B.H. Shirley.

The authors submitted this report in March 1976 as an AFFDL technical report to cover research performed from July 1975 to March 1976. The theory and numerical solution method for the present computer program is documented as AFFDL-TR-76-49, Vol.I.

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LIST OF SYMBOLS

Symbol	
a	local speed of sound
c	chord length of airfoil
C_p	pressure coefficient
$g(x)$	airfoil geometry at mean position
h	function describing the airfoil oscillation about mean steady position
H	radius of the computational domain
k	reduced frequency based on semi-chord
L_i	right-hand side vector of the resulting system of equations
$M = V/a$	local Mach number
M_∞	freestream Mach number
M_k	shape functions in time
N_i	shape functions in space
p	local static pressure
p_o	stagnation pressure
p_∞	freestream pressure
R	residual resulting from an approximate solution
S_{ij}	system coefficient matrix
u, v	perturbed velocity components in the x- and y-directions, respectively
V	flow speed
U_∞	freestream speed
$x = x'/c$ $y = y'/c$	nondimensional x and y coordinates
$t = t' \frac{U_\infty}{c}$	nondimensional time

LIST OF SYMBOLS (Continued)

Greek

α	mean angle of attack
β	$\sqrt{1 - M_\infty^2}$
Γ	total circulation strength
γ	ratio of specific heats (γ = 1.4 for air)
ϵ	small number used as convergence criterion
δ	amplitude of the oscillating flap
$\phi = \phi'/(U_\infty \cdot c)$	nondimensional perturbation velocity potential
ϕ_i^k	undetermined parameter at nodal point i and time level k
χ	integral expression for the squared errors over the entire flow domain
∇	gradient of a scalar function

Superscripts

+	upper surface of the airfoil or wake
-	lower surface of the airfoil or wake
k	time level k

Note: Subscripts after a comma indicate partial differentiations. In addition, a repeated index implies summation unless specified otherwise.

SECTION I
INTRODUCTION

In recent years, significant progress has been made toward developing a useful method for predicting steady transonic airloads over airfoils and to some degree for finite wings. Despite this progress, however, very few satisfactory methods have evolved for calculating both steady and unsteady transonic flows, as evidenced in a survey paper by Bland (Ref. 1). Only recently have numerical solutions via the finite difference technique been presented for two-dimensional airfoils executing harmonic motion. More recently, the finite element method has also been applied successfully to solve transonic flow problems (Refs. 2 and 3).

In the present study, the finite element technique is extended to compute both steady and unsteady transonic flows over lifting airfoils, based on small perturbation theory. Unlike most existing techniques for solving the small disturbance potential equation, the present approach solves directly the unsteady, nonlinear transonic flow equation, with both time derivative terms retained. Thus the present algorithm can be applied to compute a much wider class of transonic flow problems, including steady, oscillatory or transient solutions, either with or without angle of attack. For oscillatory flow, no assumption is made regarding the oscillating frequency, nor is the unsteady perturbation necessarily small compared to the mean steady solution.

The present numerical algorithm was developed using the concept of finite elements in conjunction with the method of weighted residuals. With the present approach, the embedded shocks are smeared. Also, a patching technique has been developed to match the finite element solution constructed in a moderately large domain with an asymptotic solution for the far field to avoid the necessity of using a very large domain otherwise needed in computations. The basic element presently used is the product of an element in space and an element in time. The former has a cubic expansion inside the element, with nodal unknowns representing the perturbed velocity potential and the two perturbed velocity components; while the latter is a quadratic Lagrangian element. The resulting system of algebraic equations, which relates the current time step solution to solutions at two previous time steps, is banded and can be solved conveniently by Gaussian elimination.

Several aspects of the present numerical algorithm are discussed herein. The small disturbance potential equation with associated boundary conditions and the related secondary unknowns are summarized in Section II. The numerical procedures employed in the present approach are discussed in Section III. More detailed discussion on the theory and the numerical solution method is presented in the first volume of the present report. In Section IV, the two parts of the computer code, namely, UTRANL-I and UTRANL-II are described separately, in the aspects of scope and flow chart, description of variables, subroutines used, and finally input and output. Two sample cases are given in Section V to demonstrate how to use these computer programs, which are listed in the Appendixes.

SECTION II
THEORY – ASSUMPTIONS AND BASIC EQUATIONS

The objective of the present study was to develop an efficient and accurate numerical algorithm for the analysis of steady and unsteady (oscillatory and transient) transonic flows over thin airfoils. With this objective in mind, the formulation was therefore based on the small disturbance but nonlinear transonic potential equation for inviscid, compressible fluid. The embedded shock is assumed to be weak and boundary layer effects are neglected. With these assumptions, the transonic flow problems under consideration can be stated in the following mathematical form.

Differential Equation

$$L(\phi) = \left[1 - M_\infty^2 - M_\infty^2 (\gamma + 1)\, \phi_{,x}\right] \phi_{,xx} + \phi_{,yy} - 2M_\infty^2\, \phi_{,xt} - M_\infty^2\, \phi_{,tt} = 0 \qquad (1)$$

Boundary Conditions

- Vanishing of disturbance at the far field, that is, at infinity

$$\nabla\phi = 0 \qquad (2)$$

- Flow tangency condition on the airfoil surface,

$$\phi_{,y} = \delta\,(h_{,x} + h_{,t}) + g_{,x}\,(1 + \phi_{,x}) \qquad (3)$$

- Unsteady Kutta condition in the wake to ensure pressure being continuous in the vortex sheet, namely,

$$\phi^+_{,t} + \phi^+_{,x} = \phi^-_{,t} + \phi^-_{,x} \tag{4a}$$

and

$$\phi^+_{,y} = \phi^-_{,y} \tag{4b}$$

In the above, ϕ = perturbed velocity potential function, M_∞ = freestream Mach number, γ = ratio of specific heats, taken to be 1.4 for air, g = geometry of the airfoil with angle of attack included, δ = amplitude of oscillation and h = function describing the airfoil oscillation. Equation (1) is in dimensionless form and the x-axis is aligned with the undisturbed flow direction. The dimensional (with primes) and nondimensional quantities are related by

$$x = \frac{x'}{c}, \quad y = \frac{y'}{c}, \quad t = t' \frac{U_\infty}{c} \text{ and } \phi = \frac{\phi'}{U_\infty c} \tag{5}$$

where c and U_∞ are the characteristic length and speed, which are currently taken as the chord length and the freestream speed, respectively. Consequently, the characteristic time is $\frac{c}{U_\infty}$, which is the time needed for flow at freestream speed to travel one chord length.

In actual computations, because only a finite domain can be considered in the analysis, an appropriate asymptotic solution instead of Eq. (2) must be imposed on the outer boundary of the finite element mesh. The asymptotic solution for a two-dimensional airfoil in steady flow has been derived by Klunker (Ref. 4) as

$$\phi = \frac{\Gamma}{2\pi} \left[\frac{\pi}{2} \operatorname{sgn}(y) + \tan^{-1} \left(\frac{x}{\beta y}\right)\right]$$

$$+\frac{1}{\pi\beta}\ \frac{x}{x^2+\beta^2 y^2}\int_c g(\xi)\,d\xi$$

$$+\frac{M_\infty^2(1+\gamma)}{4\pi\beta}\int_A u^2\ \frac{(x-\xi)}{(x-\xi)^2+\beta^2\,(y-\eta)^2}\,dA \qquad (6)$$

The first term on the right-hand side corresponds to a free vortex and represents the lifting effects; the second term corresponds to a source distribution whose strength is related to the airfoil thickness distribution $g(x)$; and the third term, which arises from the nonlinearity of the flow equations, has the form of a doublet with its strength given by the local value of u^2 and is to be integrated over the flow domain under consideration. Klunker also showed that the contributions from the thickness and the area integrals are of higher order effects. For these reasons, only the first term is used currently for the far field in computing flow over lifting airfoils.

Figure 1 shows the flow field, together with the unsteady transonic governing equation and corresponding boundary conditions, in a finite domain used in computations. Numerical experimentations indicate that a flow region with $H \geq 2.5c$ is generally required to yield solution with adequate accuracy. For flow with higher freestream Mach number, the flow region should be extended somewhat to account for effects from the enlarging supersonic flow pockets.

Once the flowfield solution in terms of the perturbed velocity potential has been obtained, all secondary unknowns can be calculated subsequently. These include

$$a = \left[\frac{\gamma-1}{2}\ (U_\infty^2 - V^2 - 2\phi_{,t}\,U_\infty) + \left(\frac{U_\infty}{M_\infty}\right)^2\right]^{1/2} \qquad (7)$$

$$M = \frac{V}{a} \qquad (8)$$

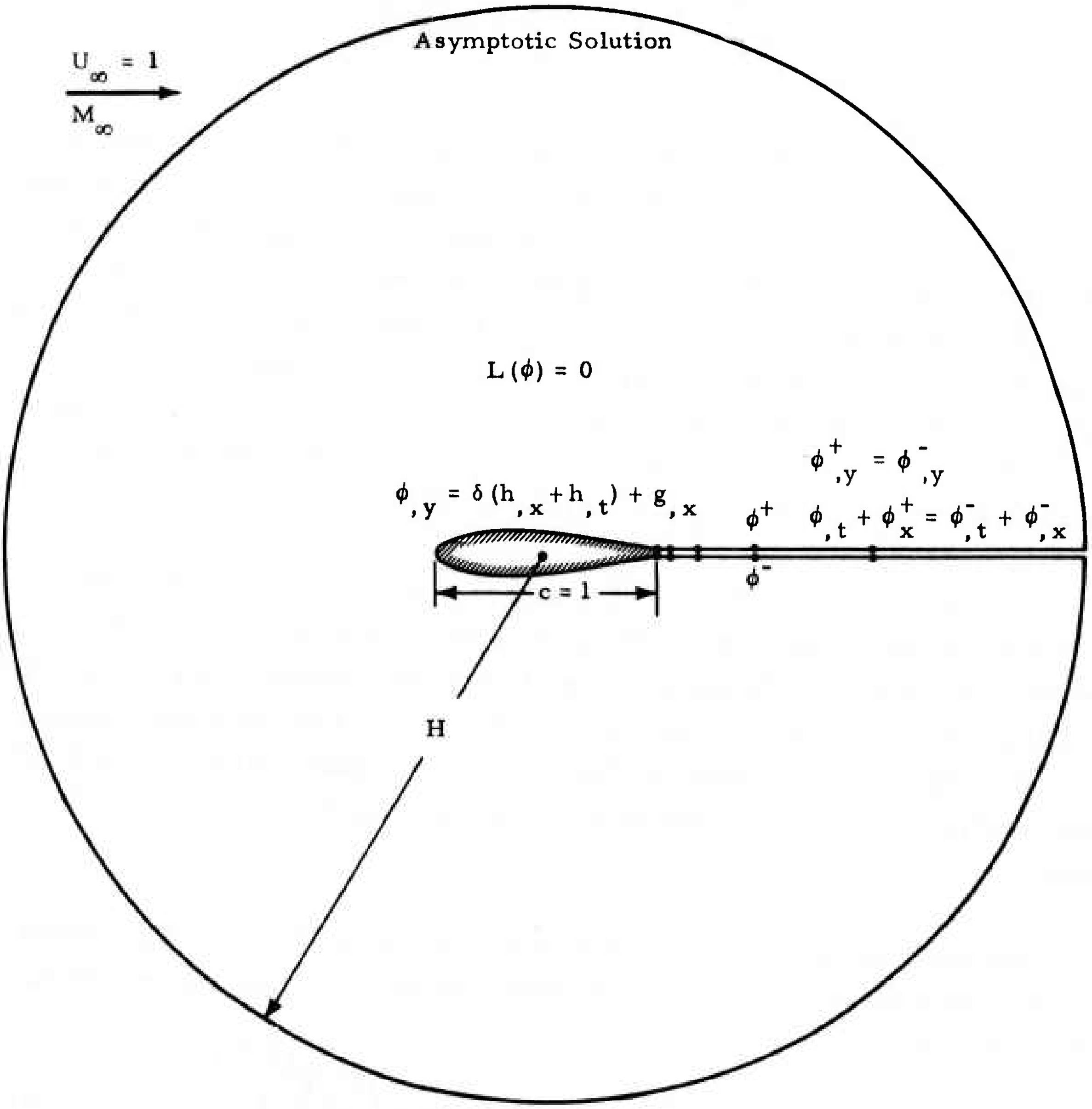


Figure 1 - Schematic of Flow Field

$$\frac{p}{p_o} = \frac{1}{\left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{\gamma}{\gamma - 1}}} \tag{9}$$

$$C_p = 2\phi_{,t} - 2\phi_{,x} \tag{10}$$

In the above, $U_\infty = 1$, the normalized freestream speed, a = local speed of sound, M = local Mach number, V = the total velocity, p = local static pressure, p_o = stagnation pressure and C_p = pressure coefficient.

SECTION III
NUMERICAL SOLUTION PROCEDURES

The concept of finite elements in conjunction with the least squares method of weighted residuals (MWR) is the basis of the present numerical procedures in solving the small disturbance unsteady transonic flow equation. The use of the least squares formulation ensures that the resulting matrix is positive definite and well conditioned. The numerical procedures including the finite element formulation, element description, the imposition of boundary conditions, and time marching procedures are described in the following subsections.

1. FINITE ELEMENT FORMULATION

The finite element method, in conjunction with the least squares method of weighted residuals, is used herein to solve numerically the unsteady small perturbation transonic equation. In this approach, a set of locally defined trial functions with undetermined parameters is assumed as the approximate solution, and the integral expression for the square of errors committed by the approximate solution is formulated. Then the integral of square errors in the time-space domain is minimized wtih respect to the undetermined parameters to yield a system of algebraic equations. In actual computations, the minimization process is performed at the element level and then an assembling process is invoked to obtain the system of algebraic equations.

Written in the usual manner with repeated indices implying summation, the approximate solution has the following form

$$\hat{\phi} = N_i M_k \phi_i^k \quad (i = 1 \text{ to } n,\ k = 1 \text{ to } 3) \tag{11}$$

where

$$N_i = N_i(x, y) \tag{12}$$

and

$$M_k = M_k(t) \tag{13}$$

Herein, N_i = shape functions in space, M_k = shape functions in time, ϕ_i^k = the undetermined parameter at node i and time level k, and n = total number of unknown parameters at one time level; three time levels are involved in the time marching process. As discussed in the first volume of this report, for stability reasons, the resulting residual is modified to take the following form

$$R = (\alpha N_{j,xx} + N_{j,yy})\, \phi_j^3 - B_j^k \phi_j^k \tag{14}$$

where

$$B_j^k = -M_\infty^2 (2 M_{k,t} N_{j,x} + M_{k,tt} N_j) \tag{15}$$

and α is approximated by

$$\alpha = (1 - M_\infty^2) - M_\infty^2 (1 + \gamma) N_{p,x} \phi_p^3 \tag{16}$$

From the expression of R, an integral expression for the square errors is obtained as

$$\chi = \iint R^2 \, dA\, dt \tag{17}$$

Upon minimization of χ with respect to the undetermined parameters at time level k = 3, one obtains

$$\iint \frac{\partial R}{\partial \phi_i^3} R \, dA\, dt = 0 \tag{18}$$

Letting

$$W_i = \frac{\partial R}{\partial \phi_i^3} = (\alpha N_{i,xx} + N_{i,yy}) - B_i^3 - M_\infty^2 (1+\gamma) N_{i,x} N_{\ell,xx} \phi_\ell^3 \qquad (19)$$

the resulting system of algebraic equations then becomes

$$\iint W_i \left[(\alpha N_{j,xx} + N_{j,yy}) - B_j^k \right] \phi_j^k \, dA\, dt = 0 \qquad (20)$$

Or in the form of a recurrence relationship, as

$$S_{ij} \phi_j^3 = L_i \qquad (21)$$

where

$$S_{ij} = \iint W_i [\alpha N_{j,xx} + N_{j,yy} - B_j^3] \, dA\, dt \qquad (22)$$

and

$$L_i = -\iint W_i [\alpha N_{j,xx} + N_{j,yy} - B_j^k] \phi_j^k \, dA\, dt, \; k = 2,3 \qquad (23)$$

Equation (21) will be used to solve for ϕ_j^3 in terms of ϕ_j^1 and ϕ_j^2.

As stated earlier, the system matrix is obtained by combining appropriate contributions from all the elements. The element matrices, in turn, are evaluated effectively by numerical integration to avoid the tedious and error prone algebraic manipulations. The Gaussian quadrature presently used has seven points in the spatial direction, and two points in the time direction. This quadrature formula can integrate exactly the product of a quintic polynomial in space and a cubic polynomial in time.

2. ELEMENT DESCRIPTION

The basic element presently used is the product of an element in space and an element in time. For the finite element approximation in the spatial directions, the nonconforming cubic triangular elements developed by Bazeley et al. (Ref. 5) are used. Also used in the program are quadrilateral elements constructed from these triangular elements. The element in time is a quadratic Lagrangian element.

The basic triangular element is shown in Fig. 2, which at each vertex has the function itself and its two first derivatives (velocity components) as undetermined parameters. This type of element was adapted mainly because boundary conditions of both Dirichlet and Neumann types can be imposed with

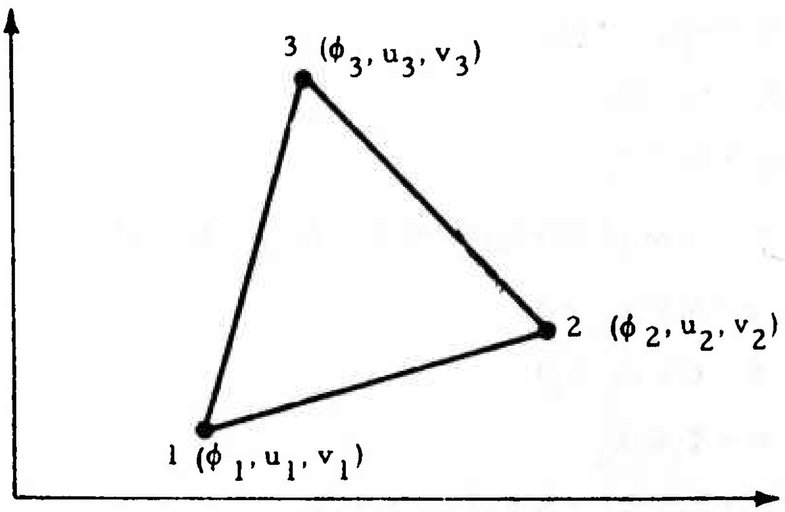


Figure 2 - Triangular Element with Undetermined Parameters

equal convenience. In addition, because velocities at nodes are treated as primary unknowns, secondary unknowns such as local Mach number, and pressure coefficients, etc., can be computed directly without resorting to numerical differentiation and thus assuring higher accuracy. Furthermore, the use of higher order elements can usually improve computational efficiency as evidenced in most finite element analyses.

In the element, the approximate solution is assumed as

$$\phi^e = N_i \, \phi_i \quad (i = 1 \text{ to } 9) \tag{24}$$

in which ϕ_i's are the nine undetermined parameters of ϕ and its first derivatives at the nodal points as shown in the figure and N_i's are the corresponding shape functions, which are expressed in terms of area coordinates.

Defined in the following are these shape functions and their first and second derivatives.

Letting

$$a_i = x_j y_k - x_k y_j$$

$$b_i = y_j - y_k$$

$$c_i = x_k - x_j$$

$$\Delta = \text{area of triangle 1-2-3} = (b_j c_k - b_k c_j)/2$$

$$\alpha = 0.5\,(c_k - c_j)$$

$$\beta = 0.5\,(b_j - b_k)$$

$$H = \zeta_i \zeta_j \zeta_k$$

$$H_x = b_i \zeta_j \zeta_k + b_j \zeta_k \zeta_i + b_k \zeta_i \zeta_j$$

$$H_y = c_i \zeta_j \zeta_k + c_j \zeta_k \zeta_i + c_k \zeta_i \zeta_j$$

$$H_{xx} = 2(\zeta_i b_j b_k + \zeta_j b_k b_i + \zeta_k b_i b_j)$$

$$H_{yy} = 2(\zeta_i c_j c_k + \zeta_j c_k c_i + \zeta_k c_i c_j)$$

with i=(1,2,3), j=(2,3,1), k=(3,1,2), then one has for ℓ=(1,4,7), i=(1,2,3),

$$N_{\ell} = \zeta_i^2 (3-2\zeta_i) + 2H$$

$$N_{\ell,x} = \frac{1}{2\Delta} \left[6b_i\zeta_i(1-\zeta_i) + 2H_x\right]$$

$$N_{\ell,y} = \frac{1}{2\Delta} \left[6c_i\zeta_i(1-\zeta_i) + 2H_y\right]$$

$$N_{\ell,xx} = \frac{1}{(2\Delta)^2} \left[6b_i^2(1-2\zeta_i) + 2H_{xx}\right]$$

$$N_{\ell,yy} = \frac{1}{(2\Delta)^2} \left[6c_i^2(1-2\zeta_i) + 2H_{yy}\right]$$

for ℓ=(2, 5, 8), i=(1, 2, 3)

$$N_{\ell} = \zeta_i^2(c_k\zeta_j - c_j\zeta_k) + \alpha H$$

$$N_{\ell,x} = \frac{1}{2\Delta} \left[2b_i\zeta_i(c_k\zeta_j - c_j\zeta_k) + 2\Delta\zeta_i^2 + \alpha H_x\right]$$

$$N_{\ell,y} = \frac{1}{2\Delta} \left[2c_i\zeta_i(c_k\zeta_j - c_j\zeta_k) + \alpha H_y\right]$$

$$N_{\ell,xx} = \frac{1}{(2\Delta)^2} \left[2b_i^2 (c_k\zeta_j - c_j\zeta_k) + 4b_i(2\Delta)\zeta_i + \alpha H_{xx}\right]$$

$$N_{\ell,yy} = \frac{1}{(2\Delta)^2} \left[2c_i^2(c_k\zeta_j - c_j\zeta_k) + \alpha H_{yy}\right]$$

for ℓ=(3, 6, 9), i=(1, 2, 3)

$$N_\ell = \zeta_i^2(b_j\zeta_k - b_k\zeta_j) + \beta H$$

$$N_{\ell,x} = \frac{1}{2\Delta}\left[2b_i\zeta_i(b_j\zeta_k - b_k\zeta_j) + \beta H_x\right]$$

$$N_{\ell,y} = \frac{1}{2\Delta}\left[2c_i\zeta_i(b_j\zeta_k - b_k\zeta_j) + 2\Delta\zeta_i^2 + \beta H_y\right]$$

$$N_{\ell,xx} = \frac{1}{(2\Delta)^2}\left[2b_i^2(b_j\zeta_k - b_k\zeta_j) + \beta H_{xx}\right]$$

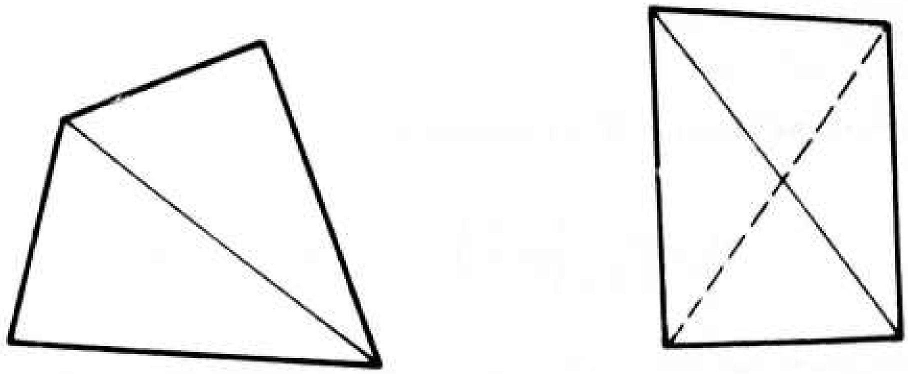
$$N_{\ell,yy} = \frac{1}{(2\Delta)^2}\left[2c_i^2(b_j\zeta_k - b_k\zeta_j) + 4c_i(2\Delta)\zeta_i + \beta H_{yy}\right]$$

The undetermined parameters are arranged in the order of ϕ_1, u_1, v_1, ϕ_2, u_2, v_2, ϕ_3, u_3 and v_3.

The elements constructed from the basic triangular elements are shown in Fig. 3 which are also used in the program. The quadrilaterals are used to save input data otherwise necessary, while the trapezoidals can be used to eliminate bias effects and to remove the improper upwind influence from the downwind station in the calculation of steady transonic flow. The element matrix for a quadrilateral is obtained by combining approximately the matrices for two triangles, while the matrix for a trapezoidal element is obtained by combining and averaging contributions from four (two left-running and two right-running) triangles. In any event, the present program selects automatically the type of element to use, according to the flow behavior in the element.

For the finite element approximation in time, the one presently used is a quadratic element using Lagrangian interpolation, as shown in Fig. 4. In the element, the approximate solution at node i is assumed as

$$\phi_i = M_k\,\phi_i^k \tag{25}$$



a. Quadrilateral Element   b. Trapezoidal Element

Figure 3 - Elements Constructed from Basic Triangles

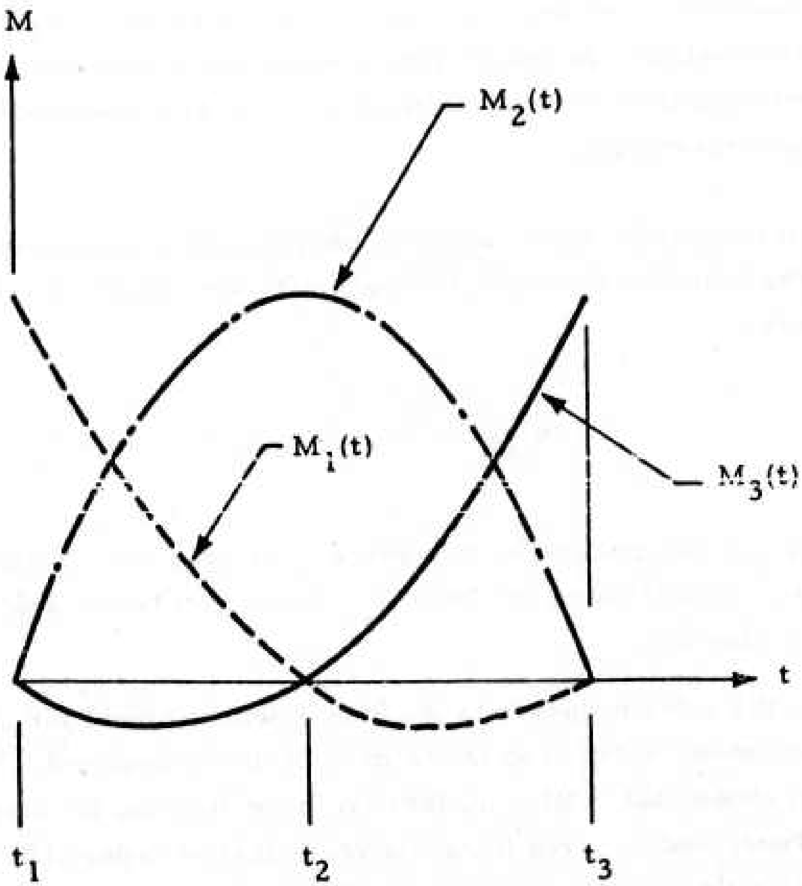


Figure 4 - Shape Functions in Time

where the shape function M_k is defined as

$$M_k = \prod_{k \neq j} \left(\frac{t - t_j}{t_k - t_j} \right) , \quad j = 1 \text{ to } 3, \quad k = 1 \text{ to } 3$$

The three shape functions in time are shown schematically in Fig.4.

3. THE IMPOSITION OF BOUNDARY CONDITIONS AND TIME MARCHING

As stated earlier, the imposition of boundary conditions for the present problem can be carried out conveniently because the elements presently used have function value and two first derivatives as primary unknowns. The associated boundary conditions thus can be treated as the essential type, i.e., having prescribed values. Standard finite element methodology is therefore followed by assembling first an unconstrained problem and then modifying the matrix equations accordingly.

On the airfoil, the boundary condition of flow tangency is imposed. This is accomplished by replacing the algebraic equation for $\phi_{,y}$ at node i originally generated by

$$(\phi_{,y})_i = \delta (h_{,x} + h_{,t})_i + g_{,x} (1 + \phi_{,x})_i , \tag{26}$$

where $(\phi_{,y})_i$ designates the unknown $\phi_{,y}$ at node "i". A linearized boundary condition is obtained by omitting $\phi_{,x}$ in the last term, which is to be imposed along the chordline.

On the outer boundary of the finite element mesh, for computing steady flow, asymptotic solution in the form of Eq. (6) is imposed. However, as it has been shown that, with a moderately large domain, the contribution from the thickness and the area integrals are of higher order effects, therefore only the first term representing the lifting effect is incorporated in the present program. The circulation strength, in turn, is updated systematically according to the potential jump calculated at the trailing edge. For unsteady flows, the circulation strength corresponding to mean steady flow is first computed and kept unchanged on the outer boundary. For symmetric airfoil without angle of attack, the circulation is set equal to zero.

The boundary conditions along the vortex wake, as defined by Eqs. (4a) and (4b), are imposed in the following way. To impose the first condition for a pair of nodes along the wake, the two finite element equations generated originally for $\phi^+_{,x}$ and $\phi^-_{,x}$ are properly combined to yield one equation, while the equation for $\phi^+_{,x}$ is replaced by the following equation:

$$\phi^{+(3)}_{,x} - \phi^{-(3)}_{,x} + M_{3,t}(\phi^{+(3)} - \phi^{-(3)})$$

$$= M_{2,t}(\phi^{-(2)} - \phi^{+(2)}) + M_{1,t}(\phi^{-(1)} - \phi^{+(1)}) \qquad (27)$$

where M is the shape function in time and the superscripts denote the time steps, while solutions for the first and the second time steps are known.

The second condition, Eq. (4b), can be imposed in a similar fashion. This condition, within the limit of small perturbation assumption, will ensure that flows just above and below the branch cut are tangent to each other. Therefore, flow at the branch cut behaves as an unsteady vortex sheet, and in the steady case this vortex sheet will vanish eventually.

In the present program, no special efforts have been devoted to treat the leading edge singularity. Any rigorous approach should consider also the invalidity of small perturbation theory in these regions, which is certainly beyond the scope of the present study. Nevertheless, these regions are relatively small and the flow is usually subsonic in nature, which implies any error committed is generally localized and becomes less important elsewhere. A possible remedy is to use very fine elements in that region and treat the leading edge as a singular point. In practical computations, however, an element with its area approaching zero might create a new numerical problem because all shape function derivatives involve element area as divisor. Therefore, caution must be taken in using extremely small elements.

With the equations properly assembled and boundary conditions imposed. the system of algebraic equations is finally solved by marching in time in the form

$$S_{ij} \ \phi_j^{(n)} = L_i \tag{27}$$

to solve for the solution $\phi^{(n)}$ at the nth time step. The term on the right-hand side is evaluated according to Eq. (23), with the solutions at the previous two time steps.

When a steady flow is to be computed, Eq. (27) is to be solved subject to certain prescribed convergence criteria. The one presently used is that the relative change of local Mach number between two consecutive time steps should be less than a prescribed small number for all the nodes in the flow field, that is

$$\left| \frac{M^{(n)} - M^{(n-1)}}{M^{(n)}} \right| < \epsilon \tag{28}$$

Numerical experimentations indicate that a solution with adequate accuracy is generally obtainable with ϵ in the range $0.001 \leq \epsilon \leq 0.005$.

SECTION IV
PROGRAM DESCRIPTIONS AND USAGE

As stated earlier, the present computer program is capable of analyzing both steady and unsteady (oscillatory and transient) transonic flows over thin airfoils by solving the small disturbance but nonlinear transonic potential equation. The numerical algorithm is based on utilizing the finite element technique in conjunction with the least squares method of weighted residuals. To solve a particular transonic flow problem, an appropriate finite element mesh must first be set up, followed by using the numerical procedures summarized in Section III. Thus the present programs are separated into two parts – the first part which generates the necessary mesh information from a limited number of input cards and the second part which carries out the analysis with element mesh generated in the first part. By doing so, the generated mesh can be fully inspected for its correctness prior to the analysis, and storage required in the second program can be accordingly determined to suit each particular problem. These two programs are designated UTRANL-I and UTRANL-II (Unsteady Transonic Flow by Lockheed, parts I and II, respectively) and are described in more detail in the following.

1. UTRANL-I

Scope and Flow Chart

This program reads in a limited number of input cards and generates mesh information such as element nodes, nodal coordinates, boundary nodes and airfoil slope, etc., to be used in the second program. The mesh is generated for the finite flow domain indicated in Fig. 1, with the outer boundary set to be a circle centered at the midchord of the airfoil. The present program generates initially a mesh based on the geometry of a 6% thick circular arc airfoil and, for airfoils of other geometric shape, the y-coordinate and surface slope of nodes on the airfoil are corrected accordingly, using a BLOCK

DATA which defines the geometric shape of the airfoil under consideration. Additionally, the present program has an option for plotting the generated mesh for quick inspection. A schematic flow chart of the program is shown in Fig. 5.

The program as presently dimensioned requires approximately 46_8 K words to run and can accommodate the following maxima:

- 400 elements
- 400 nodal points
- 100 nodes for each type of boundary conditions, and
- 10 nodes connected to any node in the mesh.

Description of Variables

TITLE	Array used to describe the problem under consideration.
IOPT	Array containing the option keys which are activated when read in as "1." Presently there are two keys in the program, IOPT (9) for using the BLOCK DATA to generate the required airfoil geometry and IOPT (12) for mesh plot option.
NDEL	Array containing element node points.
X	Array containing the x-coordinate of nodal points.
Y	Array containing the y-coordinate of nodal points.
NIDS	Array containing the actual number of boundary nodes for far field, on the line of symmetry, and along the airfoil surface.
NID	Array containing the nodal numbers for each type of boundary nodes mentioned above.
VAF	Array containing the airfoil slope for nodes on the airfoil surface.
NEM	Maximum number of elements allowed.
NPM	Maximum number of nodal points allowed.

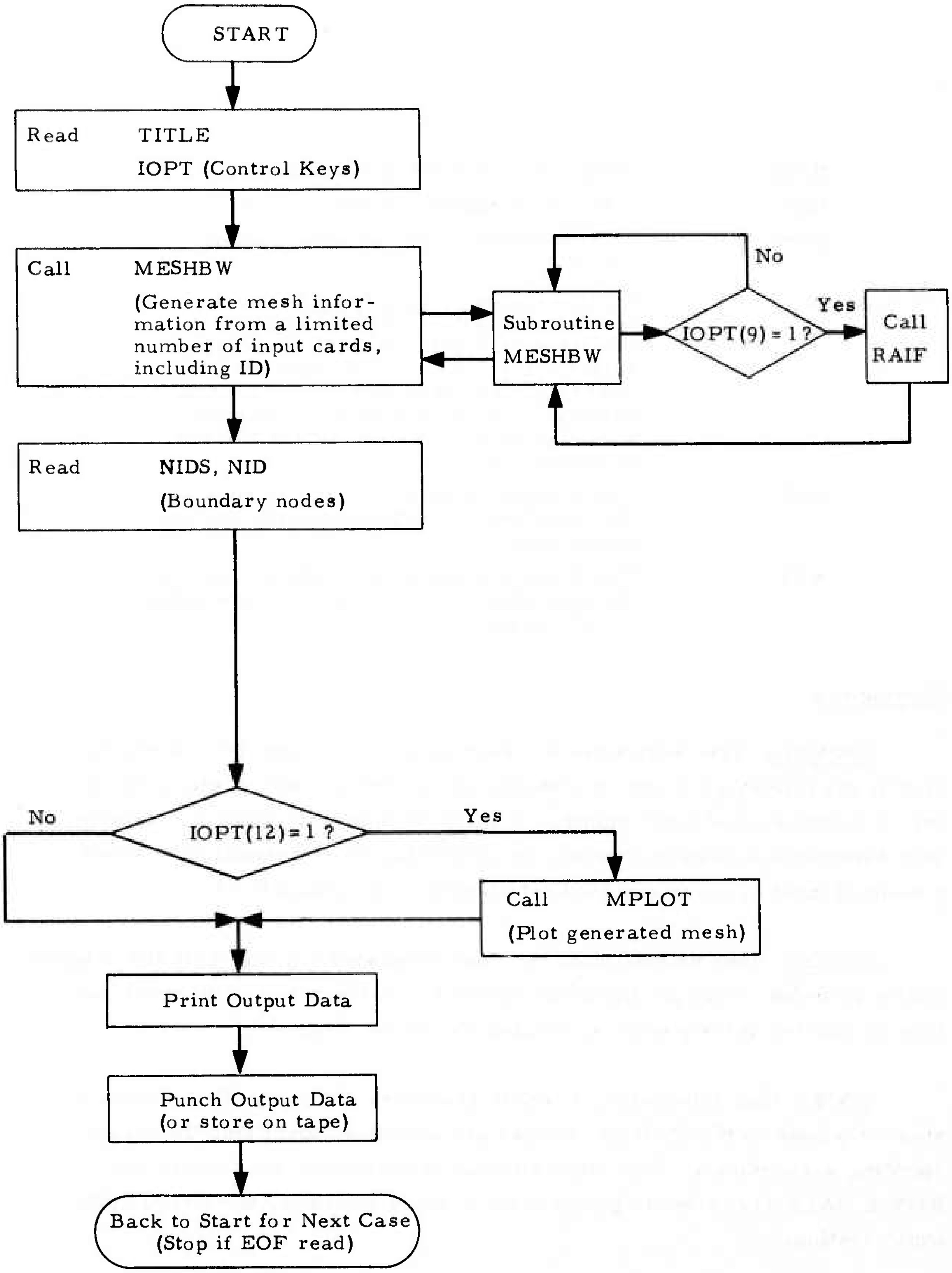


Figure 5 - Flow Chart of UTRANL-I

NELS	Total actual number of elements.
NPS	Total actual number of nodal points.
NBW	Full bandwidth of the resulting system matrix.
ID	Control key for calling the subroutine GOCR to compute a 6% thick circular arc airfoil geometry (y-coordinate and surface slope) and the corresponding far field boundary curve. Values on the upper surface or lower surface are computed according to ID = 1 or -1. GOCR will not be called if ID = 0.
XLE	The distance between the leading edge and the point farthest to the left in the computational domain.
XTE	The distance between the trailing edge and the point farthest to the right in the computational domain.

Subroutines

MESHBW: This subroutine is called to generate mesh information including the following: array of element nodes, total number of elements, x- and y-coordinates of nodal points, and the maximum difference in nodal numbers between two connected nodes. In generating this information, a small amount of input cards is required, which will be described later.

MPLOT: This routine plots the mesh generated together with the original node numbering, using the CALCOMP plotter. Correctness of the mesh can thus be checked quickly prior to running the second program.

GOCR: This subroutine, if called, generates the y-coordinate surface slope at a node on the 6% thick circular arc airfoil by specifying the corresponding x-coordinate. For other airfoils, the following subroutines and BLOCK DATA are called to generate the correct values for the airfoil under consideration.

RAIF: This subroutine is called when IOPT(9) = 1, to generate the y-coordinates and surface slope for nodes on the airfoil, based on available tabulated data for airfoil thickness distribution.

GMPG: This subroutine is called to compute the surface slope for preselected points on the airfoil, using the data given in BLOCK DATA.

BLOCK DATA: This data block defines the thickness distribution according to available tabulated data for the specific airfoil to be studied. This subroutine is to be used for any airfoil, provided the pertinent data statements are supplied.

Input and Output

All input and output are referred to the automatic numbered scheme, with input in the form of punch cards, and output in the form of printout and punch cards. If subroutine MPLOT is called, the mesh is also plotted for the convenience of inspection.

Input

Input cards to this program should be prepared and provided in the order described below.

A. TITLE CARD (12A6)

Col. 1-72 Description of the problem under study

B. OPTIONS CARD (40I2)

Col. 18 Punch "1" for airfoil other than the 6% thick circular arc.

Col. 24 Punch "1" if mesh plot is desired

C. ELEMENT CARDS (16I5)

These are cards supplied to generate element nodal numbers in groups. One card is needed for each group of elements whose nodes are related in a regular pattern. The number of elements in a group can vary from one to any positive integer number, depending on how the nodes are numbered originally. There can be as many cards as required to generate the element nodes, and a blank card is added at the end to terminate the process.

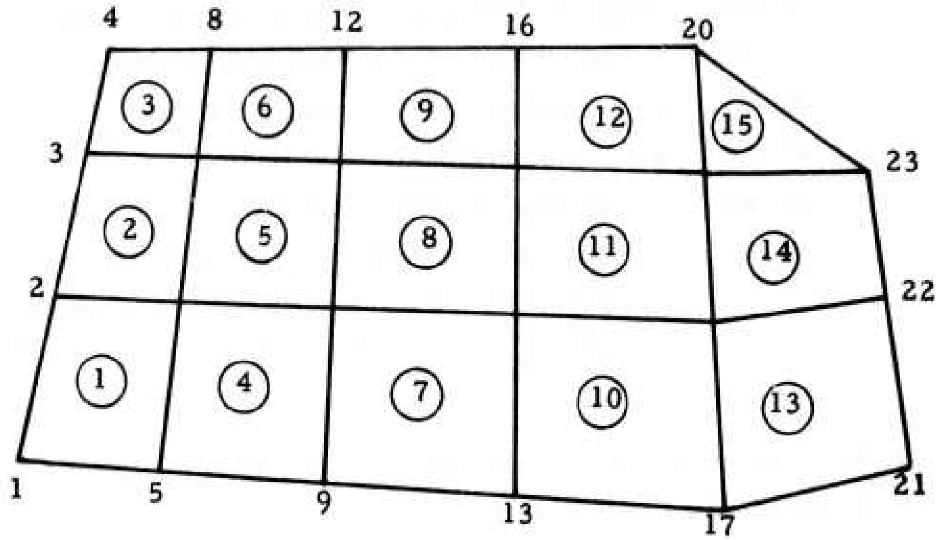
1-5	Number of nodes per element. Punch 3 for triangular elements, and punch 4 for quadrilateral and trapezoidal elements.
6-10	Number of elements in one direction (called first direction).
11-15	Number of elements in the other direction (called second direction).
16-20	Increment of nodal numbers in the first direction.
21-25	Increment of nodal numbers in the second direction.
26-30	First nodal number in the element
31-35	Second nodal number in the element
36-40	Third nodal number in the element
41-45	Fourth nodal number in the element ("0" for triangles)

As stated before, element nodes are presently ordered in the counterclockwise direction, and the upper left corner node should be taken as the first nodal point in an element located in the anticipated supersonic region. This is required to enact the assembling process correctly in the supersonic pocket for steady flow.

For example, to generate element nodes for the mesh shown on page 25, three input cards are required: one card for the first 12 elements, second card for the next two elements, and the third card for the triangular element. The three cards are:

4	4	3	4	1	2	1	5	6
4	1	2	0	1	18	17	21	22
3	1	1	0	0	20	19	23	0

(Ended by a blank card when all elements are covered)



D. CARD FOR TOTAL NO. OF NODES (I5)

Col. 1-5 Total number of nodes for the entire flow region.

E. CARD FOR LEADING AND TRAILING EDGE (2F10.0)

Col. 1-10 The distance between the leading edge and the point farthest to the left in the computational domain.

11-20 The distance between the trailing edge and the point farthest to the right in the computational domain.

F. NODE COORDINATE CARDS (3I5, 5F10.0, I5)

The x- and y-coordinates of nodes are also generated in connected groups. A connected group of nodes are those falling on a straight line with a constant ratio of nodal distances and constant increment in nodal numbers as well. The intermediate nodal points can thus be generated by linear interpolation. Again, the process is terminated by encountering a blank card.

Col.	Description
1-5	Node number of the beginning nodal point.
6-10	Total number of nodes on the line.
11-15	Increment of nodal numbers between two consecutive nodes.

16-25 Ratio of distances between three consecutive nodes.

26-35 x-coordinate of the beginning node.

36-45 y-coordinate of the beginning node.

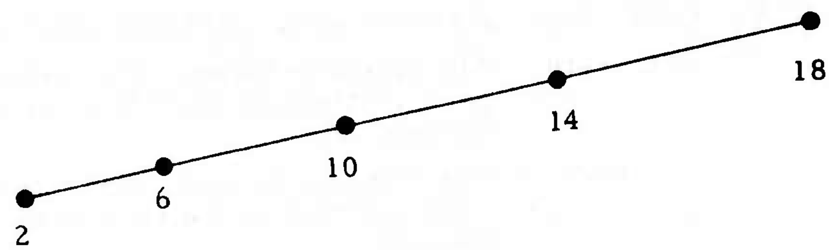
46-55 x-coordinate of the end node.

56-65 y-coordinate of the end node.

66-70 0 if a node does not fall on the airfoil surface.
1 for calling GOCR to compute y-coordinate and surface slope for the upper surface; -1 for those on the lower surface.

For example, to generate the coordinates for the nodes depicted below with a constant distance ratio of 1.2, the input card should read as

2 5 4 1.2 x_2 y_2 x_{18} y_{18}



G. CARD FOR NO. OF NODES ON BOUNDARIES (16I5)

Col.	Description
1-5	Number of nodes on the outer boundary (called NFARF)
6-10	Number of nodes on line of symmetry (called NWAKE)
11-15	Number of nodes on the airfoil surface (called NBODY)

H. CARDS FOR BOUNDARY NODES (16I5)

(a) Nodes on the outer boundary (NFARF entries)

(b) Nodes on line of symmetry (NWAKE entries)

Output

The output from this program is in the form of printout, punch cards, and also a plot if the mesh plot option is invoked.

The following items are printed in the order as described:

- Title of the problem under study
- Control keys specified in the options card
- Total number of elements, number of nodal points, and the full bandwidth
- Element numbers and element node points
- Nodal numbers and their corresponding x- and y-coordinates
- Nodes on each type of boundaries, and
- Slope for nodes on the airfoil surface.

The above items, except the first two, are then punched and saved as input to the second program (UTRANL-II).

2. UTRANL-II

Scope and Flow Chart

This program carries out the analysis following the numerical procedures described in Section III, with any mesh information generated and supplied through the first program. In summary this program solves the small perturbation transonic potential equation, by marching in time, via the least squares finite element technique. The embedded shock waves are smeared and the resulting system of algebraic equations, relating the solution of current time step to the solutions of previous two time steps, is solved by direct elimination. The present program has four options, one for inputting nonzero initial guess of the solutions, one for computing in the same run solutions for higher Mach number(s) using results computed for lower Mach number case(s), the third one for selecting the form of boundary conditions on the airfoil, and the last one for updating the circulation strength. A schematic flow chart of the program is shown in Fig. 6.

The program as presently dimensioned requires approximately 177_8K words for the program itself and can accommodate the following maxima:

- 173 elements
- 202 nodal points
- 50 nodes for each type of boundary conditions, and
- 84 in full bandwidth of the resulting matrix equations.

Description of Variables

TITLE	Array used to describe the problem under consideration
IOPT	Array containing the option keys which are activated when read in as "1." Presently there are four options: IOPT(1) for continuation for high Mach number cases while using existing results computed for earlier case, IOPT(2) for inputting non-zero initial guess, IOPT(3) for selecting the linearized boundary condition applied along the airfoil chordline, and IOPT(4) for updating the circulation strength.
NOD	Array containing element node points
S	The resulting system matrix with main diagonal terms stored in the column numbered NHBW. After solving, the column numbered NBW contains the solution for the current time step.
SL1	Array containing the solution for the first time step
SLP	Array containing the solution for the second time step.

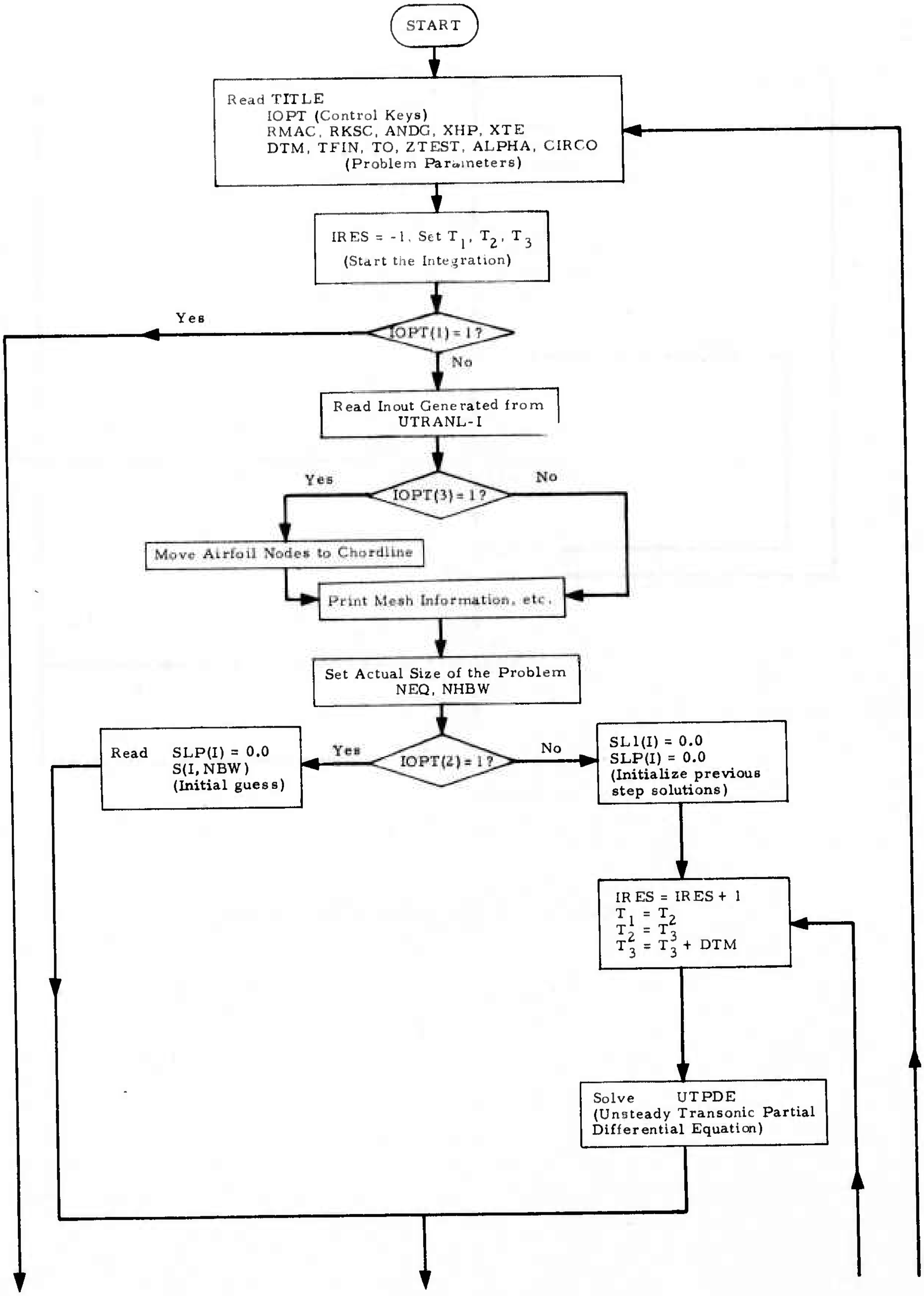


Figure 6 - Flow Chart of UTRANL-II (Continued)

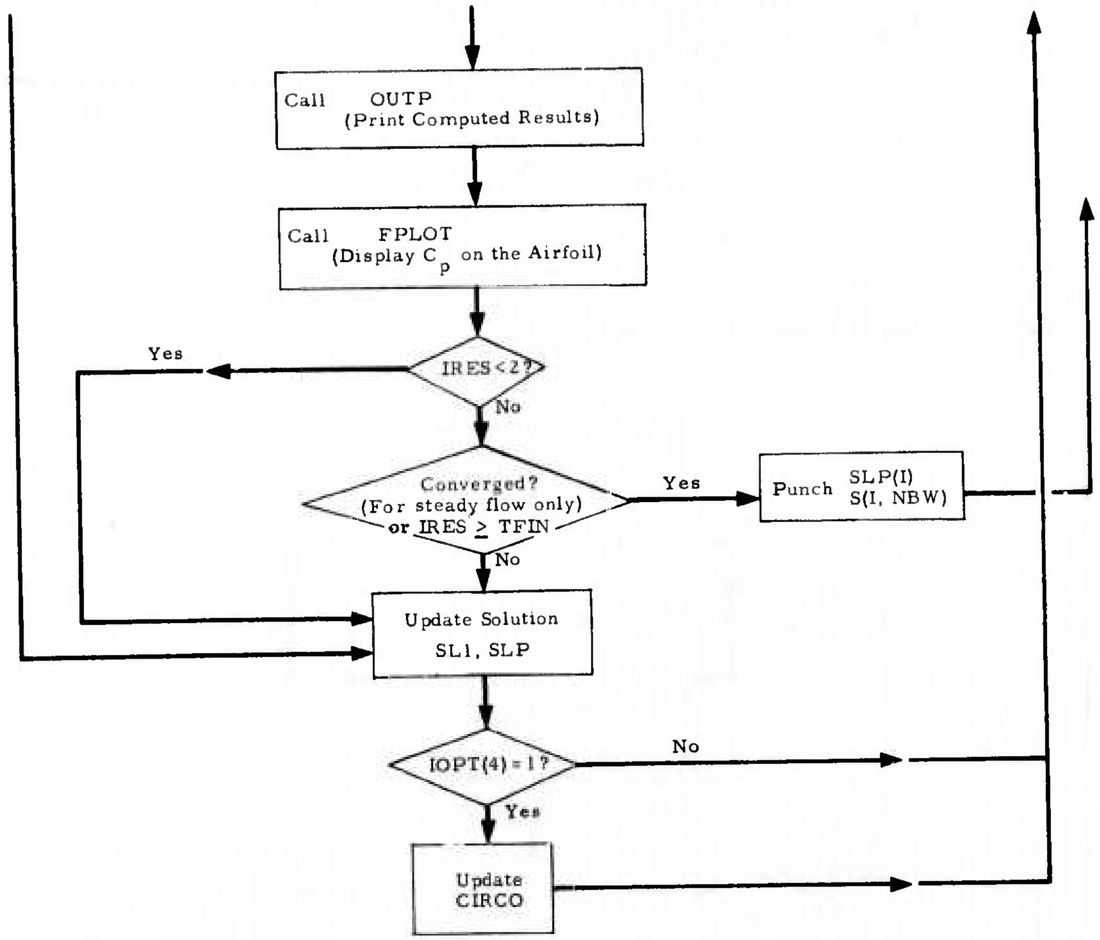


Figure 6 - Flow Chart of UTRANL-II (Completed)

X	Array containing the x-coordinate of nodal points
Y	Array containing the y-coordinate of nodal points.
RML	Array containing values of local Mach number at nodal points for the current time step.
COF	Array containing values of $M_\infty^2 + M_\infty^2 (1+\gamma)u$ evaluated at nodal points.
NIDS	Array containing the actual number of boundary nodes for far field, on the line of symmetry, and along the airfoil, respectively.
NID	Array containing the nodal numbers for each type of boundary nodes mentioned above.
VAF	Array containing the airfoil slope for nodes on the airfoil surface.
AR	Array to store values for use in subroutine FPLOT.
LR	Another array to be used in the subroutine FPLOT.
NEM	Maximum number of elements allowed.
NPM	Maximum number of nodal points allowed.
NCM	Maximum full bandwidth allowed in the resulting system matrix.
NRM	Maximum number of equations allowed.
NELS	Total actual number of elements.
NPS	Total actual number of nodal points.
NBW	Full bandwidth of the resulting system matrix.
NHBW	Half bandwidth of the resulting system matrix.
NEQ	Number of equations in the resulting system of equations.
ZTEST	Small number used to check convergence based on the relative change of local Mach number between two consecutive iterations in steady flow computations.
RMAC	Freestream Mach number.

RKSC	Reduced frequency based on semi-chord, and then converted into that based on chord length in computations. (RKSC = 0 for steady flow).
ANDG	Amplitude in degrees of the oscillating flap. (ANDG = 0 for steady flow).
XHP	x-coordinate of the hinge point.
XLE	x-coordinate of the trailing edge.
DTM	Desired number of time steps in each period of time, which is then converted into time increment Δt in computations. (For steady flow the input value of DTM is used as Δt).
TFIN	Total number of time steps to be computed.
TO	Initial time level, usually set equal to zero.
ALPHA	Angle of attack of the airfoil at mean steady position.
CIRCO	Circulation strength for airfoil at its mean position.
SQMAC	Square of the freestream Mach number.
IRES	Number of time-marching steps.
POT	Perturbed velocity potential.
UPT	Perturbed velocity component in the x-direction.
V	Perturbed velocity component in the y-direction.
U	Total velocity component in the x-direction.
PRA	The ratio of local static pressure to stagnation pressure.
CP	Pressure coefficient.
DELM	Change of local Mach number between two consecutive time steps.
CPU	Normalized unsteady pressure coefficient.

Subroutines

NEWK: This subroutine is called in the main program to generate system matrix for the entire flow field by assembling appropriately the element matrices.

The elements are, in general, a combination of triangles, quadrilaterals and trapezoids. For a 4-node element, this subroutine also determines either to treat it as a quadrilateral or as a trapezoid, depending on the local behavior of the governing equation being elliptic or otherwise. Subroutine EMQT is called to generate the element matrices.

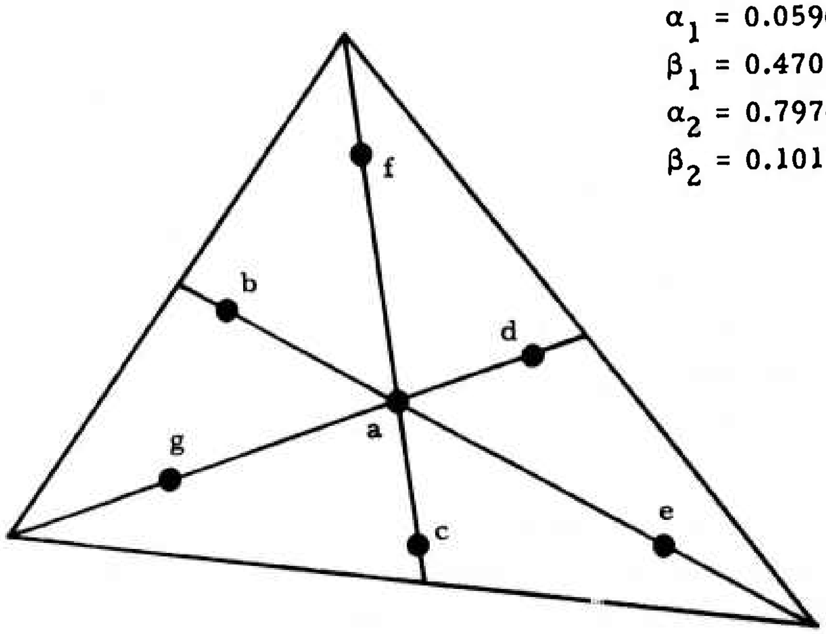
OUTP: This subroutine is called in the main program to compute the secondary parameters (Mach number, pressure coefficient, etc.), and to print all the computed results at all nodes. These include the perturbed velocity potential, total velocity components in the x- and y-directions, the value of M_∞^2 (1 + 2.4u), local Mach number, the ratio of local static pressure to stagnation pressure coefficient, and finally the change of local Mach number between two consecutive time steps. Also computed are the differences of pressure coefficients between the upper and lower airfoil surfaces.

BCDS: This subroutine is called in the main program to impose all the relevent boundary conditions. These conditions include the far field condition, the airfoil flow tangency condition, and the condition of equal pressure along the wake.

EMQT: This subroutine generates element matrix for elements in the form of triangles, quadrilaterals and trapezoids. Element matrix for the latter two types is obtained by combining appropriately (and averaging for the last type) contributions from triangular elements which are in turn evaluated in subroutine EMTC.

EMTC: This subroutine evaluates the element matrix for a space-time element by numerical integration, presently with Gaussian quadrature. The one used in the program has 7 points in space and 2 points in time, which integrates exactly a quintic function in space and a cubic function in time.

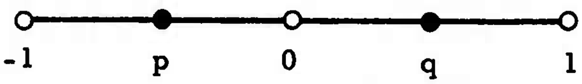
Shown below are the locations of the Gaussian points in space with their corresponding weights.



α_1 = 0.05961587
β_1 = 0.47014206
α_2 = 0.79742699
β_2 = 0.10128651

Points	Area Coordinates			Weights
a	1/3	1/3	1/3	0.225
b	α_1	β_1	β_1	
c	β_1	α_1	β_1	0.13239415
d	β_1	β_1	α_1	
e	α_2	β_2	β_2	
f	β_2	α_2	β_2	0.12593918
g	β_2	β_2	α_2	

Shown below are the Gaussian points in time with their corresponding weights.



Points	Normalized Coordinate	Weights
p	-0.55555556	0.5
q	+0.55555556	0.5

TDRU: This is a subroutine called by EMTC to evaluate the first and second derivatives of the shape functions in time at a Gaussian point.

DERV: This is a subroutine called by EMTC to evaluate the first and second derivatives of the shape functions in space at a Gaussian point, based on the expressions listed in Section III (2).

BNDEQ: This is an equation solver for banded nonsymmetric system of algebraic equations, utilizing the direct Gaussian elimination scheme. In this solver the system matrix is arranged in a twisted form such that main diagonal terms are stored in the column numbered NHBW and the right-hand side vector is in the column NBW. After solving, the column numbered NBW contains the solution vector.

FPLOT: FPLOT collects and stores data as it becomes available, and upon signal, produces a printer plot in practically any orientation and size. It should be regarded as a general purpose output routine for displaying output data in graphical form.

Description of Parameters

M1	Size of the main storage array, and it should never be larger than 200, which corresponds to 100 points to be plotted.
IPNT	Counter initialized — usually IPNT = 0 - in the calling program. It is incremented by 2 each time a new data point is entered in AR.
AR	Main storage array. It should be in a dimension statement in the calling program. For example, DIMENSION AR(800).
LR	An array of bytes used to hold the curve number. It should be dimensioned for M1/2. The type declaration LOGICAL*1 LR(400) should be in the calling program.
ISTOP	Flag used to signal that all data has been entered. ISTOP = 0 causes data to be stored. If ISTOP = -1 and NC = NCMAX the program immediately branches to the plotting section. If ISTOP = 1 and NC = NCMAX a data point is stored and then the plotting section is entered.
NC	Curve number for which data is being entered. It must be a positive integer less than 21.
NCMAX	The number of curves to appear on the graph. This is the largest value NC will have.
V1	The horizontal coordinate of the data point to be plotted.
V2	The vertical coordinate of the data point to be plotted.

Input and Output

Input

The bulk of input to this program is provided through UTRANL-I in the form of punch cards. To run several cases of various freestream Mach number and oscillating frequency but using the same mesh and boundary conditions, four additional cards should be furnished for each case to be computed. These cards should be provided in the following order

A. TITLE CARD (12A6)

Col. 1-72	Description of the problem under study

B. OPTIONS CARD (40I2)

Col. 2	Punch "1" if it is a continuation in computation from one case to another.
Col. 4	Punch "1" if cards for nonzero initial guess is furnished in the input data. If so, insert them following those generated from UTRANL-I.
Col. 6	Punch "1" if linearized boundary condition along the chordline is desired.
Col. 8	Punch "1" if circulation is to be updated.

NOTE: As seen in the flow chart, when IOPT(1) = 1, the next two options become inoperative.

C. PROBLEM PARAMETERS CARDS (8F10.0)

First Card

Col. 1-10	Freestream Mach number.
Col. 11-20	Reduced frequency based on semi-chord (0 for steady flow).
Col. 21-30	Amplitude in degrees for the oscillating flap. (0 for steady flow).
Col. 31-40	x-coordinate of the hinge point.
Col. 41-50	x-coordinate of the trailing edge.
Col. 51-60	Number of time steps desired in one period of time or time step size for steady flow.
Col. 61-70	Total number of time steps to be computed.
Col. 71-80	Initial time level counted in time steps. (Usually set equal to zero; with nonzero value when initial solutions are provided)

Second Card

Col. 1-10	Small number used to check convergence based on the relative change of local Mach number between two consecutive time steps, if flow approaches a steady flow, usually taken in the range from 0.005 to 0.001.
Col. 11-20	Mean angle of attack of the airfoil
Col. 21-30	Circulation for flow with the airfoil placed at its mean position.

For the first case of the run, punch cards from UTRANL-I are placed after the above cards. For subsequent cases using the same mesh, four additional cards as described above are required for each case.

<u>Output</u>

The output from this program include printout, punch cards and solutions for the last two time steps (or convergent solutions for steady flow).

The printouts are in the following order:

- Title of the problem under study
- Convergent limit (for steady flow)
- Control keys specified in the options card
- Total number of element, number of nodal points, and the full bandwidth of the resulting matrix equations
- Element numbers and element node points
- Nodal numbers and their corresponding x- and y-coordinates
- Boundary nodes for each type of boundaries
- Slope for nodes on the airfoil surface
- For each time step, the computed results at all nodal points are printed. These include the perturbed velocity potential, total velocity components in the x- and y-directions, the value of M_∞^2 $(1+2.4u)$,

local Mach number, the ratio of local static pressure to stagnation pressure coefficient, and finally the change of local Mach number between two consecutive time steps.

- For each time step, the unsteady pressures on the airfoil, for the upper and lower surfaces, as well as their difference, are printed. At the end of each period, the mean pressures on the airfoil, for the upper and lower surfaces, obtained by time average within a period, and their difference, are also printed.
- For steady case, the value of C_p along the airfoil is displayed graphically. For unsteady case, the difference of unsteady pressures across the airfoil is displaced graphically.

SECTION V
SAMPLE CASES

Two sample cases are presented herein to demonstrate the usage of the computer programs described in the previous section regarding input and typical output from the programs. The two problems are flow over a NACA 64 A006 airfoil with a quarter-chord oscillating flap and flow over a lifting NACA 64 A410 airfoil.

In computing the steady or unsteady transonic flow field for a given airfoil shape with the present programs, the following procedures are generally followed:

- A desired mesh is sketched with each node assigned a number. In order to save storage and computation time, nodal points should be numbered along the direction with less number of nodes, such as shown in Fig. 7.
- Based on the mesh sketched, an appropriate set of input cards is prepared and supplied to UTRANL-I to generate the required mesh information in the form of punch cards.
- The above punch cards, with four additional cards for each case and cards for non-zero initial guess if available, are used as input to UTRANL-II to compute the flow field.
- During the execution of UTRANL-II, results for each time step are printed, and solution for the last two time steps is saved in the form of punch cards for possible later use.

The following paragraphs describe in more detail the input and output for the two sample problems.

1. FLOW OVER A NACA 64 A006 AIRFOIL WITH AN OSCILLATING FLAP

This problem was analyzed using the mesh shown in Fig. 7, which consists of 173 elements together with 202 nodes. Summarized below are the

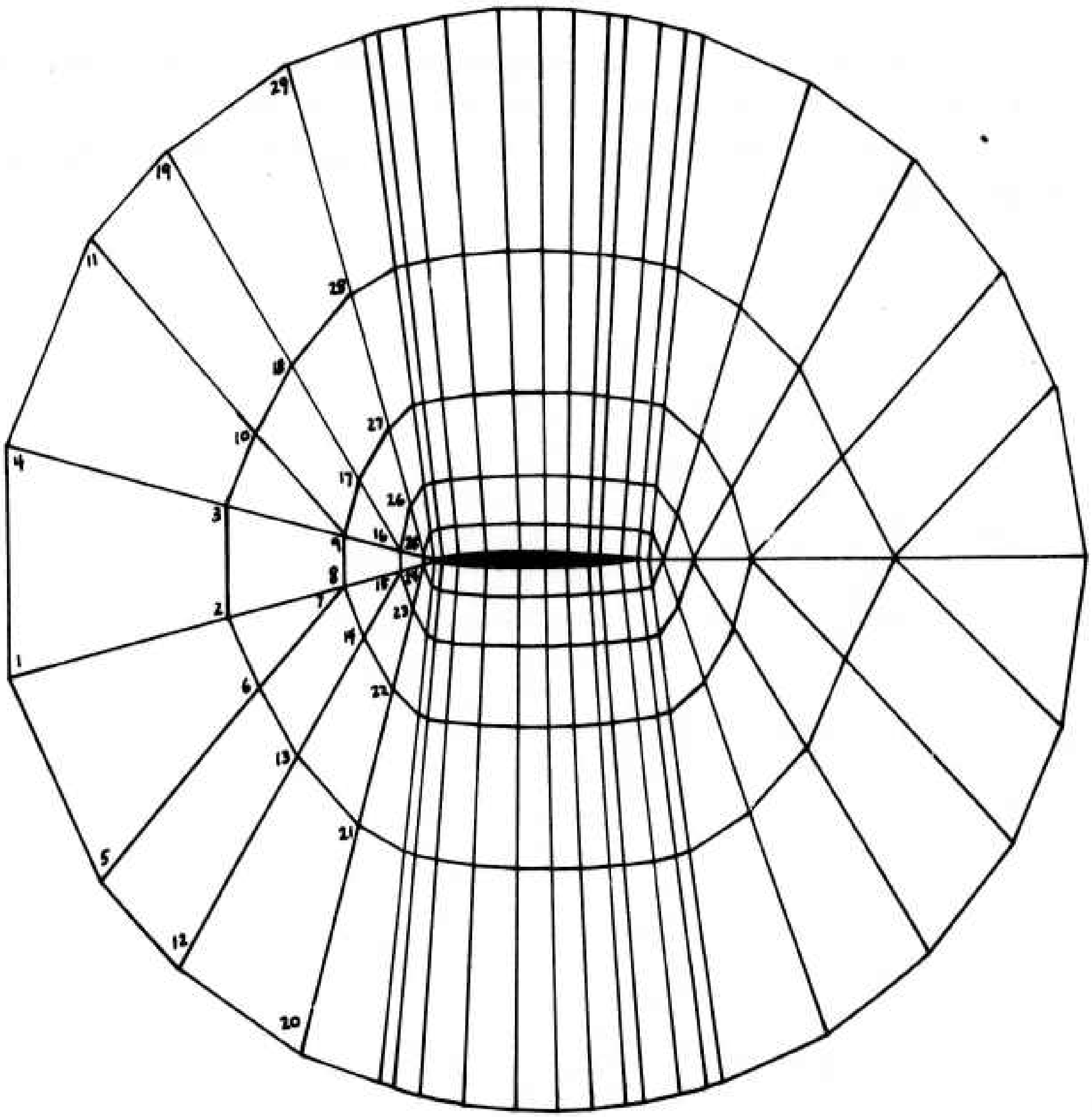


Figure 7 - Finite Element Mesh for NACA 64 A006 Airfoil
(173 elements, 202 nodes)

input and output to theUTRANL-I and UTRANL-II programs for the present problem.

UTRANL-I

Input

The necessary set of input cards is listed on the next two pages. Note that both options for plotting mesh and using BLOCK DATA are to be enacted, and the final mesh will have nodes on the actual airfoil surface rather than on the chordline.

Input Cards to UTRANL-I for NACA 64A006 Airfoil

UNSTEADY TRANSONIC FLOW--MESH L3 MODIFIED--173 ELEMENTS--202 NODES

			1	1				
4	1	1			4	1	2	3
4	1	1			3	2	7	8
4	1	1			8	7	14	15
4	1	1			15	14	23	24
4	1	1			24	23	34	35
4	1	1			2	1	5	6
3	1	1			7	2	6	
4	2	1	1		6	5	11	12
3	1	1			14	7	13	
4	3	1	1		12	11	19	20
3	1	1			23	14	22	
4	4	1	1		20	19	29	30
3	1	1			34	23	33	
3	1	1			24	35	36	
4	4	1	1		25	24	36	37
3	1	1			15	24	25	
4	3	1	1		16	15	25	26
3	1	1			8	15	16	
4	2	1	1		9	8	16	17
3	1	1			3	8	9	
4	1	1			4	3	9	10
4	5	11	1	12	30	29	41	42
4	5	11	1	12	36	35	47	48
4	4	1	1		162	161	173	174
3	1	1			166	165	177	
4	3	1	1		174	173	183	184
3	1	1			177	176	186	
4	2	1	1		184	183	191	192
3	1	1			186	185	193	
4	1	1			192	191	197	198
3	1	1			193	192	198	
3	1	1			198	197	201	
3	1	1			168	167	178	
4	4	1	1		169	168	178	179
3	1	1			179	178	187	
4	3	1	1		180	179	187	188
3	1	1			188	187	194	
4	2	1	1		189	188	194	195
3	1	1			195	194	199	
4	1	1			196	195	199	200
3	1	1			200	199	202	

(Blank Card)

```
  202
2.          3.
    2    2   -1   2.2      -1.42      -.255    -2.44     -.527
    7    3   -1   1.9       -.887     -.115    -2.035   -1.46
   14    4   -1   1.8       -.645     -.045    -1.68    -1.86
   23    5   -1   1.7       -.535     -.02     -1.13    -2.25
    3    2    1   2.2      -1.42       .255    -2.44      .527
    8    3    1   1.9       -.887      .115    -2.035    1.46
   15    4    1   1.8       -.645      .045    -1.68     1.86
   24    5    1   1.7       -.535      .02     -1.13     2.25
   34    6   -1   1.7       -.49                                      1
   35    6    1   1.7       -.49                                      1
   46    6   -1   1.7       -.45                                      1
   47    6    1   1.7       -.45                                      1
   58    6   -1   1.7       -.375                                     1
   59    6    1   1.7       -.375                                     1
   70    6   -1   1.7       -.25                                      1
   71    6    1   1.7       -.25                                      1
   82    6   -1   1.7       -.100                                    -1
   83    6    1   1.7       -.100                                     1
   94    6   -1   1.7       0.025                                    -1
   95    6    1   1.7       0.025                                     1
  106    6   -1   1.7        .125                                     1
  107    6    1   1.7        .125                                     1
  118    6   -1   1.7        .225                                     1
  119    6    1   1.7        .225                                     1
  130    6   -1   1.7        .275                                     1
  131    6    1   1.7        .275                                     1
  142    6   -1   1.7        .375                                     1
  143    6    1   1.7        .375                                     1
  154    6   -1   1.7        .450                                     1
  155    6    1   1.7        .450                                     1
  166    6   -1   1.7        .5                                       1
  167    6    1   1.7        .5                                       1
  177    5   -1   1.7        .565     -.00001   1.26    -2.16
  186    4   -1   1.7        .695     -.00001   1.73    -1.8
  193    3   -1   1.8        .955     -.00001   2.14    -1.3
  198    2   -1   2.        1.6       -.00001   2.38     -.78
  201    2    1   1.        2.5       -.00001   2.5       .00001
  178    5    1   1.7        .565      .00001   1.26     2.16
  187    4    1   1.7        .695      .00001   1.73     1.8
  194    3    1   1.8        .955      .00001   2.14     1.3
  199    2    1   2.        1.6        .00001   2.38      .78
(Blank Card)
   42   10   22
    1    4    5   10   11   18   19   28   29   40   41   52   53   64   65   76
   77   88   89  100  101  112  113  124  125  136  137  148  149  160  161  172
  173  182  183  190  191  196  197  200  201  202
  166  167  177  178  186  187  193  194  198  199
```

Output

As stated before, output of this program includes printouts, punch cards and a plot for the generated mesh.

Printouts and punch cards for the present mesh are listed on the next 11 pages.

Printouts from UTRANL-I for NACA 64 A006 Airfoil

UNSTEADY TRANSONIC FLOW--MESH L3 MODIFIED--173 ELEMENTS--202 NODES

0-0-0-0-0-0-0-0 1-0 -0-0

NO. OF ELEMENTS= 173 NO. OF NODES= 202 FULL BANDWIDTH= 84

ELE. NO. AND ELEMENT NODES

1	4	1	2	3
2	3	2	7	8
3	8	7	14	15
4	15	14	23	24
5	24	23	34	35
6	2	1	5	6
7	7	2	6	0
8	6	5	11	12
9	7	6	12	13
10	14	7	13	0
11	12	11	19	20
12	13	12	20	21
13	14	13	21	22
14	23	14	22	0
15	20	19	29	30
16	21	20	30	31
17	22	21	31	32
18	23	22	32	33
19	34	23	33	0
20	24	35	36	0
21	25	24	36	37
22	26	25	37	38
23	27	26	38	39
24	28	27	39	40
25	15	24	25	0
26	16	15	25	26
27	17	16	26	27
28	18	17	27	28
29	8	15	16	0
30	9	8	16	17
31	10	9	17	18
32	3	8	9	0
33	4	3	9	10
34	30	29	41	42
35	31	30	42	43

36	32	31	43	44
37	33	32	44	45
38	34	33	45	46
39	42	41	53	54
40	43	42	54	55
41	44	43	55	56
42	45	44	56	57
43	46	45	57	58
44	54	53	65	66
45	55	54	66	67
46	56	55	67	68
47	57	56	68	69
48	58	57	69	70
49	66	65	77	78
50	67	66	78	79
51	68	67	79	80
52	69	68	80	81
53	70	69	81	82
54	78	77	89	90
55	79	78	90	91
56	80	79	91	92
57	81	80	92	93
58	82	81	93	94
59	90	89	101	102
60	91	90	102	103
61	92	91	103	104
62	93	92	104	105
63	94	93	105	106
64	102	101	113	114
65	103	102	114	115
66	104	103	115	116
67	105	104	116	117
68	106	105	117	118
69	114	113	125	126
70	115	114	126	127
71	116	115	127	128
72	117	116	128	129
73	118	117	129	130
74	126	125	137	138
75	127	126	138	139
76	128	127	139	140
77	129	128	140	141
78	130	129	141	142
79	138	137	149	150
80	139	138	150	151
81	140	139	151	152
82	141	140	152	153
83	142	141	153	154
84	150	149	161	162
85	151	150	162	163
86	152	151	163	164
87	153	152	164	165

88	154	153	165	166
89	36	35	47	48
90	37	36	48	49
91	38	37	49	50
92	39	38	50	51
93	40	39	51	52
94	48	47	59	60
95	49	48	60	61
96	50	49	61	62
97	51	50	62	63
98	52	51	63	64
99	60	59	71	72
100	61	60	72	73
101	62	61	73	74
102	63	62	74	75
103	64	63	75	76
104	72	71	83	84
105	73	72	84	85
106	74	73	85	86
107	75	74	86	87
108	76	75	87	88
109	84	83	95	96
110	85	84	96	97
111	86	85	97	98
112	87	86	98	99
113	88	87	99	100
114	96	95	107	108
115	97	96	108	109
116	98	97	109	110
117	99	98	110	111
118	100	99	111	112
119	108	107	119	120
120	109	108	120	121
121	110	109	121	122
122	111	110	122	123
123	112	111	123	124
124	120	119	131	132
125	121	120	132	133
126	122	121	133	134
127	123	122	134	135
128	124	123	135	136
129	132	131	143	144
130	133	132	144	145
131	134	133	145	146
132	135	134	146	147
133	136	135	147	148
134	144	143	155	156
135	145	144	156	157
136	146	145	157	158
137	147	146	158	159
138	148	147	159	160
139	156	155	167	168

140	157	156	168	169
141	158	157	169	170
142	159	158	170	171
143	160	159	171	172
144	162	161	173	174
145	163	162	174	175
146	164	163	175	176
147	165	164	176	177
148	166	165	177	0
149	174	173	183	184
150	175	174	184	185
151	176	175	185	186
152	177	176	186	0
153	184	183	191	192
154	185	184	192	193
155	186	185	193	0
156	192	191	197	198
157	193	192	198	0
158	198	197	201	0
159	168	167	178	0
160	169	168	178	179
161	170	169	179	180
162	171	170	180	181
163	172	171	181	182
164	179	178	187	0
165	180	179	187	188
166	181	180	188	189
167	182	181	189	190
168	188	187	194	0
169	189	188	194	195
170	190	189	195	196
171	195	194	199	0
172	196	195	199	200
173	200	199	202	0

NODE	X(I)	Y(I)
1	-.24400E+01	-.52700E+00
2	-.14200E+01	-.25500E+00
3	-.14200E+01	.25500E+00
4	-.24400E+01	.52700E+00
5	-.20350E+01	-.14600E+01
6	-.12829E+01	-.57879E+00
7	-.88700E+00	-.11500E+00
8	-.88700E+00	.11500E+00
9	-.12829E+01	.57879E+00
10	-.20350E+01	.14600E+01
11	-.16800E+01	-.18600E+01
12	-.11248E+01	-.88639E+00
13	-.81636E+00	-.34550E+00
14	-.64500E+00	-.45000E-01
15	-.64500E+00	.45000E-01
16	-.81636E+00	.34550E+00
17	-.11248E+01	.88639E+00
18	-.16800E+01	.18600E+01
19	-.11300E+01	-.22500E+01
20	-.85168E+00	-.12069E+01
21	-.68796E+00	-.59326E+00
22	-.59165E+00	-.23232E+00
23	-.53500E+00	-.20000E-01
24	-.53500E+00	.20000E-01
25	-.59165E+00	.23232E+00
26	-.68796E+00	.59326E+00
27	-.85168E+00	.12069E+01
28	-.11300E+01	.22500E+01
29	-.77048E+00	-.23783E+01
30	-.64624E+00	-.13277E+01
31	-.57315E+00	-.70976E+00
32	-.53016E+00	-.34624E+00
33	-.50488E+00	-.13241E+00
34	-.49000E+00	-.66200E-02
35	-.49000E+00	.66200E-02
36	-.50488E+00	.13241E+00
37	-.53016E+00	.34624E+00
38	-.57315E+00	.70976E+00
39	-.64624E+00	.13277E+01
40	-.77048E+00	.23783E+01
41	-.70888E+00	-.23974E+01
42	-.59420E+00	-.13416E+01
43	-.52675E+00	-.72060E+00
44	-.48707E+00	-.35529E+00
45	-.46373E+00	-.14040E+00
46	-.45000E+00	-.13990E-01
47	-.45000E+00	.13990E-01
48	-.46373E+00	.14040E+00
49	-.48707E+00	.35529E+00
50	-.52675E+00	.72060E+00
51	-.59420E+00	.13416E+01
52	-.70888E+00	.23974E+01

53	-.59250E+00	-.24288E+01
54	-.49616E+00	-.13622E+01
55	-.43948E+00	-.73484E+00
56	-.40615E+00	-.36580E+00
57	-.38654E+00	-.14871E+00
58	-.37500E+00	-.21010E-01
59	-.37500E+00	.21010E-01
60	-.38654E+00	.14871E+00
61	-.40615E+00	.36580E+00
62	-.43948E+00	.73484E+00
63	-.49616E+00	.13622E+01
64	-.59250E+00	.24288E+01
65	-.39649E+00	-.24684E+01
66	-.33160E+00	-.13872E+01
67	-.29343E+00	-.75119E+00
68	-.27098E+00	-.37708E+00
69	-.25777E+00	-.15702E+00
70	-.25000E+00	-.27570E-01
71	-.25000E+00	.27570E-01
72	-.25777E+00	.15702E+00
73	-.27098E+00	.37708E+00
74	-.29343E+00	.75119E+00
75	-.33160E+00	.13872E+01
76	-.39649E+00	.24684E+01
77	-.15899E+00	-.24949E+01
78	-.13286E+00	-.14031E+01
79	-.11749E+00	-.76078E+00
80	-.10845E+00	-.38296E+00
81	-.10313E+00	-.16072E+00
82	-.10000E+00	-.29990E-01
83	-.10000E+00	.29990E-01
84	-.10313E+00	.16072E+00
85	-.10845E+00	.38296E+00
86	-.11749E+00	.76078E+00
87	-.13286E+00	.14031E+01
88	-.15899E+00	.24949E+01
89	.39766E-01	-.24997E+01
90	.33225E-01	-.14046E+01
91	.29378E-01	-.76035E+00
92	.27114E-01	-.38142E+00
93	.25783E-01	-.15851E+00
94	.25000E-01	-.27390E-01
95	.25000E-01	.27390E-01
96	.25783E-01	.15851E+00
97	.27114E-01	.38142E+00
98	.29378E-01	.76035E+00
99	.33225E-01	.14046E+01
100	.39766E-01	.24997E+01
101	.19869E+00	-.24921E+01
102	.16605E+00	-.13984E+01
103	.14685E+00	-.75511E+00
104	.13555E+00	-.37668E+00
105	.12891E+00	-.15407E+00

106	.12500E+00	-.23130E-01
107	.12500E+00	.23130E-01
108	.12891E+00	.15407E+00
109	.13555E+00	.37668E+00
110	.14685E+00	.75511E+00
111	.16605E+00	.13984E+01
112	.19869E+00	.24921E+01
113	.35704E+00	-.24744E+01
114	.29855E+00	-.13861E+01
115	.26415E+00	-.74592E+00
116	.24391E+00	-.36936E+00
117	.23200E+00	-.14785E+00
118	.22500E+00	-.17545E-01
119	.22500E+00	.17545E-01
120	.23200E+00	.14785E+00
121	.24391E+00	.36936E+00
122	.26415E+00	.74592E+00
123	.29855E+00	.13861E+01
124	.35704E+00	.24744E+01
125	.43586E+00	-.24617E+01
126	.36461E+00	-.13777E+01
127	.32269E+00	-.73998E+00
128	.29803E+00	-.36488E+00
129	.28353E+00	-.14423E+00
130	.27500E+00	-.14435E-01
131	.27500E+00	.14435E-01
132	.28353E+00	.14423E+00
133	.29803E+00	.36488E+00
134	.32269E+00	.73998E+00
135	.36461E+00	.13777E+01
136	.43586E+00	.24617E+01
137	.59250E+00	-.24288E+01
138	.49616E+00	-.13565E+01
139	.43948E+00	-.72575E+00
140	.40615E+00	-.35472E+00
141	.38654E+00	-.13646E+00
142	.37500E+00	-.80800E-02
143	.37500E+00	.80800E-02
144	.38654E+00	.13646E+00
145	.40615E+00	.35472E+00
146	.43948E+00	.72575E+00
147	.49616E+00	.13565E+01
148	.59250E+00	.24288E+01
149	.70888E+00	-.23974E+01
150	.59420E+00	-.13369E+01
151	.52675E+00	-.71309E+00
152	.48707E+00	-.34614E+00
153	.46373E+00	-.13028E+00
154	.45000E+00	-.33100E-02

155	.45000E+00	.33100E-02
156	.46373E+00	.13028E+00
157	.48707E+00	.34614E+00
158	.52675E+00	.71309E+00
159	.59420E+00	.13369E+01
160	.70888E+00	.23974E+01
161	.78582E+00	-.23733E+01
162	.65921E+00	-.13221E+01
163	.58474E+00	-.70370E+00
164	.54093E+00	-.33996E+00
165	.51516E+00	-.12599E+00
166	.50000E+00	-.13000E-03
167	.50000E+00	.13000E-03
168	.51516E+00	.12599E+00
169	.54093E+00	.33996E+00
170	.58474E+00	.70370E+00
171	.65921E+00	.13221E+01
172	.78582E+00	.23733E+01
173	.12600E+01	-.21600E+01
174	.93490E+00	-.11496E+01
175	.74366E+00	-.55528E+00
176	.63117E+00	-.20566E+00
177	.56500E+00	-.10000E-04
178	.56500E+00	.10000E-04
179	.63117E+00	.20566E+00
180	.74366E+00	.55528E+00
181	.93490E+00	.11496E+01
182	.12600E+01	.21600E+01
183	.17300E+01	-.18000E+01
184	.11949E+01	-.86941E+00
185	.88015E+00	-.32201E+00
186	.69500E+00	-.10000E-04
187	.69500E+00	.10000E-04
188	.88015E+00	.32201E+00
189	.11949E+01	.86941E+00
190	.17300E+01	.18000E+01
191	.21400E+01	-.13000E+01
192	.13782E+01	-.46429E+00
193	.95500E+00	-.10000E-04
194	.95500E+00	.10000E-04
195	.13782E+01	.46429E+00
196	.21400E+01	.13000E+01
197	.23800E+01	-.78000E+00
198	.16000E+01	-.10000E-04
199	.16000E+01	.10000E-04
200	.23800E+01	.78000E+00
201	.25000E+01	-.10000E-04
202	.25000E+01	.10000E-04

```
NODES AT FARFIELD
   1     4     5    10    11    18    19    28    29    40    41    52    53    64    65    76    77    88    89   100
 101   112   113   124   125   136   137   148   149   160   161   172   173   182   183   190   191   196   197   200
 201   202

NODES ON LINE OF SYMMETRY
 166   167   177   178   186   187   193   194   198   199

NODES ON THE AIRFOIL
  34    35    46    47    58    59    70    71    82    83    94    95   106   107   118   119   130   131   142   143
 154   155

SLOPE ALONG NODES ON AIRFOIL
   -.30800E+00     .30800E+00    -.13360E+00     .13360E+00    -.72800E-01     .72800E-01    -.33900E-01     .33900E-01
    .32000E-02    -.32000E-02     .34400E-01    -.34400E-01     .50000E-01    -.50000E-01     .61000E-01    -.61000E-01
    .63400E-01    -.63400E-01     .63600E-01    -.63600E-01     .63600E-01    -.63600E-01
```

Punch Cards from UTRANL-I for NACA 64 A006 Airfoil

```
 173  202   84   42   10   22
   4    1    2    3    3    2    7    8    8    7   14   15   15   14   23   24
  24   23   34   35    2    1    5    6    7    2    6    0    6    5   11   12
   7    6   12   13   14    7   13    0   12   11   19   20   13   12   20   21
  14   13   21   22   23   14   22    0   20   19   29   30   21   20   30   31
  22   21   31   32   23   22   32   33   34   23   33    0   24   35   36    0
  25   24   36   37   26   25   37   38   27   26   38   39   28   27   39   40
  15   24   25    0   16   15   25   26   17   16   26   27   16   17   27   28
   8   15   16    0    9    8   16   17   10    9   17   18    3    8    9    0
   4    3    9   10   30   29   41   42   31   30   42   43   32   31   43   44
  33   32   44   45   34   33   45   46   42   41   53   54   43   42   54   55
  44   43   55   56   45   44   56   57   46   45   57   58   54   53   65   66
  55   54   66   67   56   55   67   68   57   56   68   69   58   57   69   70
  66   65   77   78   67   66   78   79   68   67   79   80   69   68   80   81
  70   69   81   82   76   77   89   90   79   78   90   91   80   79   91   92
  81   80   92   93   82   81   93   94   90   89  101  102   91   90  102  103
  92   91  103  104   93   92  104  105   94   93  105  106  102  101  113  114
 103  102  114  115  104  103  115  116  105  104  116  117  106  105  117  118
 114  113  125  126  115  114  126  127  116  115  127  128  117  116  128  129
 118  117  129  130  126  125  137  138  127  126  138  139  128  127  139  140
 129  128  140  141  130  129  141  142  138  137  149  150  139  138  150  151
 140  139  151  152  141  140  152  153  142  141  153  154  150  149  161  162
 151  150  162  163  152  151  163  164  153  152  164  165  154  153  165  166
  36   35   47   48   37   36   48   49   38   37   49   50   39   38   50   51
  40   39   51   52   48   47   59   60   49   48   60   61   50   49   61   62
  51   50   62   63   52   51   63   64   60   59   71   72   61   60   72   73
  62   61   73   74   63   62   74   75   64   63   75   76   72   71   83   84
  73   72   84   85   74   73   85   86   75   74   86   87   76   75   87   88
  84   83   95   96   85   84   96   97   86   85   97   98   87   86   98   99
  88   87   99  100   96   95  107  108   97   96  108  109   98   97  109  110
  99   98  110  111  100   99  111  112  108  107  119  120  109  108  120  121
 110  109  121  122  111  110  122  123  112  111  123  124  120  119  131  132
 121  120  132  133  122  121  133  134  123  122  134  135  124  123  135  136
 132  131  143  144  133  132  144  145  134  133  145  146  135  134  146  147
 136  135  147  148  144  143  155  156  145  144  156  157  146  145  157  158
 147  146  158  159  148  147  159  160  156  155  167  168  157  156  168  169
 158  157  169  170  159  158  170  171  160  159  171  172  162  161  173  174
 163  162  174  175  164  163  175  176  165  164  176  177  166  165  177    0
 174  173  183  184  175  174  184  185  176  175  185  186  177  176  186    0
 184  183  191  192  185  184  192  193  186  185  193    0  192  191  197  198
 193  192  198    0  198  197  201    0  168  167  178    0  169  168  178  179
 170  169  179  180  171  170  180  181  172  171  181  182  179  178  187    0
 180  179  187  188  181  180  188  189  182  181  189  190  188  187  194    0
 189  188  194  195  190  189  195  196  195  194  199    0  196  195  199  200
 200  199  202    0
2.440E+00-5.270E-01-1.420E+00-2.550E-01-1.420E+00 2.550E-01-2.440E+00 5.270E-01
2.035E+00-1.460E+00-1.283E+00-5.788E-01-8.870E-01-1.150E-01-8.870E-01 1.150E-01
1.283E+00 5.788E-01-2.035E+00 1.460E+00-1.680E+00-1.860E+00-1.125E+00-8.864E-01
8.164E-01-3.455E-01-6.450E-01-4.500E-02-6.450E-01 4.500E-02-8.164E-01 3.455E-01
1.125E+00 8.864E-01-1.680E+00 1.860E+00-1.130E+00-2.250E+00-8.517E-01-1.207E+00
6.880E-01-5.933E-01-5.917E-01-2.323E-01-5.350E-01-2.000E-02-5.350E-01 2.000E-02
5.917E-01 2.323E-01-6.880E-01 5.933E-01-8.517E-01 1.207E+00-1.130E+00 2.250E+00
7.705E-01-2.378E+00-6.462E-01-1.328E+00-5.732E-01-7.098E-01-5.302E-01-3.462E-01
5.049E-01-1.324E-01-4.900E-01-6.620E-03-4.900E-01 6.620E-03-5.049E-01 1.324E-01
5.302E-01 3.462E-01-5.732E-01 7.098E-01-6.462E-01 1.328E+00-7.705E-01 2.378E+00
7.089E-01-2.397E+00-5.942E-01-1.342E+00-5.267E-01-7.206E-01-4.871E-01-3.553E-01
4.637E-01-1.404E-01-4.500E-01-1.399E-02-4.500E-01 1.399E-02-4.637E-01 1.404E-01
4.871E-01 3.553E-01-5.267E-01 7.206E-01-5.942E-01 1.342E+00-7.089E-01 2.397E+00
```

```
5.925E-01-2.429E+00-4.962E-01-1.362E+00-4.395E-01-7.348E-01-4.061E-01-3.658E-01
3.865E-01-1.487E-01-3.750E-01-2.101E-02-3.750E-01 2.101E-02-3.865E-01 1.487E-01
4.061E-01 3.658E-01-4.395E-01 7.348E-01-4.962E-01 1.362E+00-5.925E-01 2.429E+00
3.965E-01-2.468E+00-3.316E-01-1.387E+00-2.934E-01-7.512E-01-2.710E-01-3.771E-01
2.578E-01-1.570E-01-2.500E-01-2.757E-02-2.500E-01 2.757E-02-2.578E-01 1.570E-01
2.710E-01 3.771E-01-2.934E-01 7.512E-01-3.316E-01 1.387E+00-3.965E-01 2.468E+00
1.590E-01-2.495E+00-1.329E-01-1.403E+00-1.175E-01-7.608E-01-1.084E-01-3.830E-01
1.031E-01-1.607E-01-1.000E-01-2.999E-02-1.000E-01 2.999E-02-1.031E-01 1.607E-01
1.084E-01 3.830E-01-1.175E-01 7.608E-01-1.329E-01 1.403E+00-1.590E-01 2.495E+00
3.977E-02-2.500E+00 3.323E-02-1.405E+00 2.938E-02-7.604E-01 2.711E-02-3.814E-01
2.578E-02-1.585E-01 2.500E-02-2.739E-02 2.500E-02 2.739E-02 2.578E-02 1.585E-01
2.711E-02 3.814E-01 2.938E-02 7.604E-01 3.323E-02 1.405E+00 3.977E-02 2.500E+00
1.987E-01-2.492E+00 1.660E-01-1.398E+00 1.468E-01-7.551E-01 1.356E-01-3.767E-01
1.289E-01-1.541E-01 1.250E-01-2.313E-02 1.250E-01 2.313E-02 1.289E-01 1.541E-01
1.356E-01 3.767E-01 1.468E-01 7.551E-01 1.660E-01 1.398E+00 1.987E-01 2.492E+00
3.570E-01-2.474E+00 2.986E-01-1.386E+00 2.641E-01-7.459E-01 2.439E-01-3.694E-01
2.320E-01-1.478E-01 2.250E-01-1.755E-02 2.250E-01 1.755E-02 2.320E-01 1.478E-01
2.439E-01 3.694E-01 2.641E-01 7.459E-01 2.986E-01 1.386E+00 3.570E-01 2.474E+00
4.359E-01-2.462E+00 3.646E-01-1.378E+00 3.227E-01-7.400E-01 2.980E-01-3.649E-01
2.835E-01-1.442E-01 2.750E-01-1.444E-02 2.750E-01 1.444E-02 2.835E-01 1.442E-01
2.980E-01 3.649E-01 3.227E-01 7.400E-01 3.646E-01 1.378E+00 4.359E-01 2.462E+00
5.925E-01-2.429E+00 4.962E-01-1.356E+00 4.395E-01-7.257E-01 4.061E-01-3.547E-01
3.865E-01-1.365E-01 3.750E-01-8.080E-03 3.750E-01 8.080E-03 3.865E-01 1.365E-01
4.061E-01 3.547E-01 4.395E-01 7.257E-01 4.962E-01 1.356E+00 5.925E-01 2.429E+00
7.089E-01-2.397E+00 5.942E-01-1.337E+00 5.267E-01-7.131E-01 4.871E-01-3.461E-01
4.637E-01-1.303E-01 4.500E-01-3.310E-03 4.500E-01 3.310E-03 4.637E-01 1.303E-01
4.871E-01 3.461E-01 5.267E-01 7.131E-01 5.942E-01 1.337E+00 7.089E-01 2.397E+00
7.858E-01-2.373E+00 6.592E-01-1.322E+00 5.847E-01-7.037E-01 5.409E-01-3.400E-01
5.152E-01-1.260E-01 5.000E-01-1.300E-04 5.000E-01 1.300E-04 5.152E-01 1.260E-01
5.409E-01 3.400E-01 5.847E-01 7.037E-01 6.592E-01 1.322E+00 7.858E-01 2.373E+00
1.260E+00-2.160E+00 9.349E-01-1.150E+00 7.437E-01-5.553E-01 6.312E-01-2.057E-01
5.650E-01-1.000E-05 5.650E-01 1.000E-05 6.312E-01 2.057E-01 7.437E-01 5.553E-01
9.349E-01 1.150E+00 1.260E+00 2.160E+00 1.730E+00-1.800E+00 1.195E+00-8.694E-01
8.802E-01-3.220E-01 6.950E-01-1.000E-05 6.950E-01 1.000E-05 8.802E-01 3.220E-01
1.195E+00 8.694E-01 1.730E+00 1.800E+00 2.140E+00-1.300E+00 1.378E+00-4.643E-01
9.550E-01-1.000E-05 9.550E-01 1.000E-05 1.378E+00 4.643E-01 2.140E+00 1.300E+00
2.380E+00-7.800E-01 1.600E+00-1.000E-05 1.600E+00 1.000E-05 2.380E+00 7.800E-01
2.500E+00-1.000E-05 2.500E+00 1.000E-05
    1    4    5   10   11   18   19   28   29   40   41   52   53   64   65   76
   77   88   89  100  101  112  113  124  125  136  137  148  149  160  161  172
  173  182  183  190  191  196  197  200  201  202
  166  167  177  178  186  187  193  194  198  199
   34   35   46   47   58   59   70   71   82   83   94   95  106  107  118  119
  130  131  142  143  154  155
3.080E-01 3.080E-01-1.336E-01 1.336E-01-7.280E-02 7.280E-02-3.390E-02 3.390E-02
3.200E-03-3.200E-03 3.440E-02-3.440E-02 5.000E-02-5.000E-02 6.100E-02-6.100E-02
6.340E-02-6.340E-02 6.360E-02-6.360E-02 6.360E-02-6.360E-02
```

UTRANL-II Program

With the present mesh, two cases of flow are to be computed. These are

$$M_\infty = 0.804, \ k = 0.253 \quad \text{(Subcritical)}$$

$$M_\infty = 0.903, \ k = 0.228 \quad \text{(Supercritical)}$$

Input

Listed below are typical input cards to the UTRANL-II program to compute flow field for the above cases.

```
UNSTEADY  SOLUTION -- NACA 64A006 -- 202 NODES -- M=0.804 -- K=0.253

.804      0.253     1.1       0.25      0.5       16.       32.       0.
.0        0.        0.

(Insert cards generated in UTRANL-I)

UNSTEADY  SOLUTION -- NACA 64A006 -- 202 NODES -- M=0.903 -- K=0.228
1
.903      0.228     1.1       0.25      0.5       16.       32.       0.
.0        0.        0.
```

The above cards indicate there is no non-zero initial guess furnished to compute the case $M_\infty = 0.804$, while the case with higher Mach number will use results computed for lower Mach number as initial guess. In all cases, the nonlinear boundary condition on the actual airfoil surface is to be used, as suggested by IOPT(3) = 0 in the options card.

Output

The output from this program includes printouts and punch cards for the solution for the last two time steps.

The printouts are in the following order:

- Title of the problem under study
- Convergent limit (a positive number for steady flow)
- Control keys specified in the options card
- Reduced frequency, amplitude for the oscillating flap, x-coordinates for the hinge point and the trailing edge, freestream Mach number, initial time level when computation is started, mean angle of attack, circulation for the airfoil at its mean position
- Total number of elements, number of nodal points, and the full bandwidth of the resulting matrix equations
- Element numbers and element node points
- Nodal numbers and their corresponding x- and y-coordinates
- Boundary nodes for each type of boundary
- Slope for nodes on the airfoil surface
- For each time step, the computed results at all nodal points are printed, together with a printer plot displaying the pressure coefficient on the airfoil surface.
- For each time step, the unsteady pressures on the airfoil, for the upper and lower surfaces, and their differences, are printed. At the end of each period, the mean pressures on the airfoil, for the upper and lower surfaces, and their differences, are also printed.

The above items except problem parameters and computed results have also been printed in the UTRANL-I program, but are repeated here for completeness. For brevity, only results for a typical time step are listed on the next five pages for reference.

Printout for NACA 64 A006 at the 16th Time Step

MACH NUMBER= .864 TIME INCREMENT = .7751 TIME LEVEL = 12.42

TIME STEP = 16

NODE	PHIT	UCOM	VCOM	COF	LMAC	PRA	CPU	CP	DELM
1	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
2	-1.2835E-03	9.9781E-01	5.0412E-05	6.4301E-01	8.0199E-01	6.5473E-01	2.4983E-01	4.7964E-03	-1.5512E-04
3	-1.2452E-03	9.9787E-01	6.9288E-05	6.4311E-01	8.0204E-01	6.5469E-01	2.4486E-01	4.7010E-03	-1.9558E-04
4	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
5	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
6	-1.9215E-03	9.9504E-01	6.3855E-04	6.3872E-01	7.9948E-01	6.5636E-01	5.3950E-01	1.0358E-02	-1.9910E-04
7	-6.9287E-03	9.8064E-01	-4.7453E-04	6.1639E-01	7.8645E-01	6.6485E-01	2.0615E+00	3.9579E-02	-3.3111E-04
8	-6.8871E-03	9.8067E-01	8.3719E-04	6.1643E-01	7.8647E-01	6.6484E-01	2.0607E+00	3.9563E-02	-3.1937E-04
9	-1.8247E-03	9.9529E-01	-5.3737E-04	6.3910E-01	7.9970E-01	6.5622E-01	5.1748E-01	9.9348E-03	-2.3205E-04
10	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
11	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
12	-2.7723E-03	9.9391E-01	-1.1646E-03	6.3696E-01	7.9845E-01	6.5703E-01	6.5656E-01	1.2605E-02	-1.8750E-04
13	-8.2719E-03	9.7604E-01	-2.1052E-03	6.1017E-01	7.8284E-01	6.6720E-01	2.4769E+00	4.7553E-02	-2.1092E-04
14	-1.4775E-02	9.5157E-01	4.4832E-03	5.7128E-01	7.6039E-01	6.8181E-01	5.0954E+00	9.7825E-02	-8.8551E-05
15	-1.4756E-02	9.5150E-01	-4.4988E-03	5.7118E-01	7.6033E-01	6.8185E-01	5.1032E+00	9.7974E-02	-7.8253E-05
16	-8.1402E-03	9.7673E-01	2.4503E-03	6.1031E-01	7.8292E-01	6.6715E-01	2.4743E+00	4.7502E-02	-2.8128E-04
17	-2.6163E-03	9.9424E-01	1.1297E-03	6.3748E-01	7.9875E-01	6.5684E-01	6.2865E-01	1.2069E-02	-2.5172E-04
18	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
19	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
20	-3.5329E-03	9.9400E-01	-4.6645E-03	6.3711E-01	7.9855E-01	6.5697E-01	6.4273E-01	1.2339E-02	-8.8826E-05
21	-9.6975E-03	9.8308E-01	-6.9233E-03	6.2017E-01	7.8867E-01	6.6340E-01	1.7996E+00	3.4550E-02	-5.9339E-05
22	-1.7558E-02	9.5453E-01	-1.7955E-02	5.7588E-01	7.6318E-01	6.8000E-01	4.7827E+00	9.1821E-02	-1.4504E-06
23	-2.4782E-02	8.7553E-01	-2.3035E-02	4.4556E-01	6.8943E-01	7.2772E-01	1.3536E+01	2.5987E-01	3.8701E-05
24	-2.4778E-02	8.7546E-01	2.3240E-02	4.4544E-01	6.8936E-01	7.2776E-01	1.3544E+01	2.6003E-01	4.8438E-05
25	-1.7477E-02	9.5427E-01	1.8328E-02	5.7548E-01	7.6295E-01	6.8015E-01	4.8152E+00	9.2445E-02	-1.7188E-05
26	-9.4851E-03	9.8296E-01	7.1820E-03	6.1997E-01	7.8855E-01	6.6348E-01	1.8261E+00	3.5058E-02	-1.6401E-04
27	-3.4374E-03	9.9419E-01	4.4272E-03	6.3740E-01	7.9371E-01	6.5686E-01	6.3472E-01	1.2186E-02	-1.9060E-04
28	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
29	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
30	-3.7717E-03	9.9736E-01	-6.8978E-03	6.4231E-01	8.0161E-01	6.5497E-01	2.8894E-01	5.5473E-03	9.3190E-06
31	-1.0210E-02	9.9109E-01	-1.2035E-02	6.3259E-01	7.9596E-01	6.5865E-01	9.5951E-01	1.8421E-02	4.5301E-05
32	-1.7275E-02	9.7183E-01	-2.5163E-02	6.0271E-01	7.7881E-01	6.6982E-01	2.9752E+00	5.7138E-02	6.8372E-05
33	-2.4489E-02	9.4465E-01	-7.3400E-03	5.6054E-01	7.5425E-01	6.8581E-01	5.8122E+00	1.1159E-01	6.7558E-05
34	-3.5795E-02	7.3290E-01	-2.2573E-01	2.3204E-01	6.0075E-01	7.8355E-01	2.7872E+01	5.3511E-01	1.3430E-04
35	-3.5795E-02	7.3292E-01	2.2574E-01	2.3207E-01	6.0076E-01	7.8353E-01	2.7873E+01	5.3507E-01	1.1008E-04
36	-2.4464E-02	9.4431E-01	7.6212E-03	5.6002E-01	7.5395E-01	6.8601E-01	5.8500E+00	1.1231E-01	5.4252E-05
37	-1.7177E-02	9.7131E-01	2.5494E-02	6.0190E-01	7.7834E-01	6.7013E-01	3.0392E+00	5.8348E-02	7.0056E-05
38	-1.0007E-02	9.9057E-01	1.2212E-02	6.3178E-01	7.9547E-01	6.5897E-01	1.0304E+00	1.9782E-02	2.3546E-07
39	-3.5072E-03	9.9720E-01	6.5514E-03	6.4207E-01	8.0145E-01	6.5508E-01	3.1945E-01	6.1139E-03	-7.4902E-05
40	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
41	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
42	-3.7566E-03	9.9926E-01	-7.0111E-03	6.4527E-01	8.0334E-01	6.5385E-01	8.9332E-02	1.7151E-03	1.2839E-05
43	-1.0545E-02	9.8951E-01	-1.3510E-02	6.3015E-01	7.9455E-01	6.5957E-01	1.1220E+00	2.1540E-02	7.1079E-05
44	-1.7743E-02	9.9158E-01	-2.1988E-02	6.3335E-01	7.9655E-01	6.5827E-01	9.1576E-01	1.7601E-02	1.0871E-04
45	-2.7609E-02	9.1036E-01	-5.3850E-02	5.0734E-01	7.2559E-01	7.0441E-01	9.3829E+00	1.8014E-01	1.4455E-04
46	-4.1044E-02	9.0837E-01	-1.2136E-01	5.0426E-01	7.2927E-01	7.0203E-01	9.5911E+00	1.3414E-01	1.8340E-04
47	-4.1044E-02	9.0831E-01	1.2135E-01	5.0418E-01	7.2922E-01	7.0206E-01	9.5976E+00	1.3426E-01	6.6908E-05
48	-2.7656E-02	9.1044E-01	5.8997E-02	5.0685E-01	7.2531E-01	7.0459E-01	9.4186E+00	1.8082E-01	1.3813E-04
49	-1.7673E-02	9.9075E-01	2.2237E-02	6.3207E-01	7.9580E-01	6.5876E-01	1.0114E+00	1.9418E-02	1.4763E-04
50	-1.0372E-02	9.8871E-01	1.3523E-02	6.2891E-01	7.9381E-01	6.6005E-01	1.2217E+00	2.3455E-02	9.5322E-05

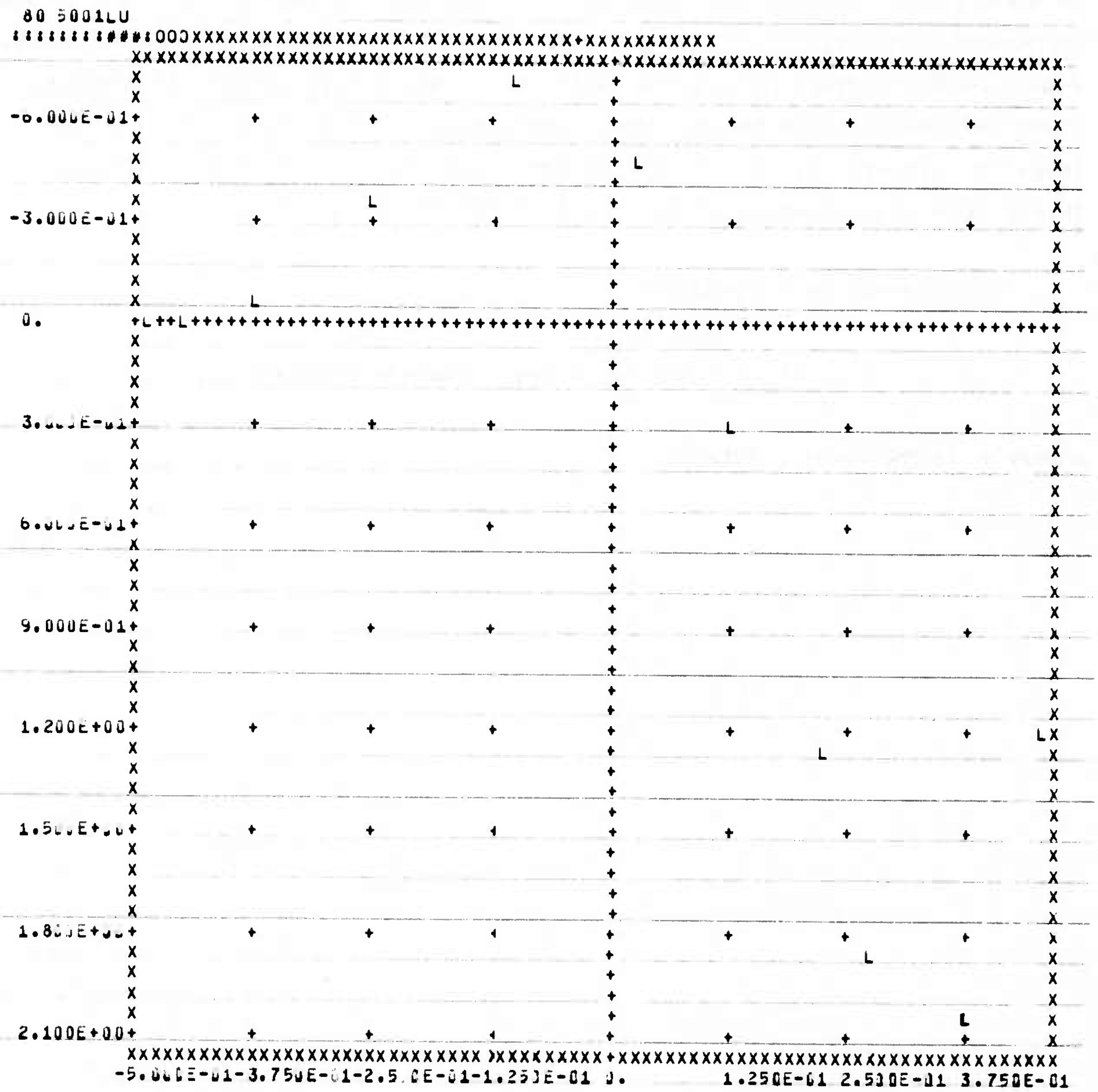
51	-3.3105E-03	9.9896E-01	6.6892E-03	6.4481E-01	8.1305E-01	6.5403E-01	1.3380E-01	2.5688E-03	-3.4545E-05
52	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
53	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
54	-3.7599E-03	9.9853E-01	-7.0442E-03	6.4413E-01	8.0268E-01	6.5428E-01	1.6388E-01	3.1463E-03	5.2172E-05
55	-1.0455E-02	1.0053E+00	-1.3447E-02	6.5461E-01	8.0886E-01	6.5025E-01	-5.2392E-01	-1.0059E-02	1.2994E-04
56	-1.8751E-02	9.8748E-01	-3.2235E-02	6.2699E-01	7.9310E-01	6.6052E-01	1.3399E+00	2.5724E-02	1.7803E-04
57	-3.0646E-02	9.9764E-01	-6.9389E-02	6.4133E-01	8.0346E-01	6.5376E-01	3.4737E-01	6.6806E-03	2.1630E-04
58	-3.9959E-02	1.0776E+00	-7.8452E-02	7.6637E-01	8.7823E-01	6.0526E-01	-8.0477E+00	-1.5450E-01	2.3857E-04
59	-3.9985E-02	1.0769E+00	7.8397E-02	7.6568E-01	8.7751E-01	6.0572E-01	-7.9662E+00	-1.5294E-01	1.7941E-04
60	-3.0665E-02	9.9626E-01	6.9338E-02	6.4062E-01	8.0275E-01	6.5423E-01	4.3154E-01	8.2850E-03	2.1695E-04
61	-1.8755E-02	9.8632E-01	3.2414E-02	6.2519E-01	7.9205E-01	6.6120E-01	1.4671E+00	2.8166E-02	3.3438E-04
62	-1.0367E-02	1.0040E+00	1.3605E-02	6.5266E-01	8.0770E-01	6.5100E-01	-3.7966E-01	-7.2889E-03	2.8260E-04
63	-3.6577E-03	9.9800E-01	6.7813E-03	6.4331E-01	8.[illegible]E-01	6.5460E-01	2.3145E-01	4.4436E-03	7.5526E-05
64	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
65	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
66	-3.2615E-03	1.0043E+00	-6.1827E-03	6.5313E-01	8.0794E-01	6.5085E-01	-4.4333E-01	-8.5114E-03	7.3700E-05
67	-6.9811E-03	1.0146E+00	-1.4172E-02	6.6906E-01	8.1736E-01	6.4472E-01	-1.4992E+00	-2.8783E-02	1.2689E-04
68	-1.6695E-02	1.0375E+00	-3.0733E-02	7.0453E-01	8.3865E-01	6.3088E-01	-3.8731E+00	-7.4358E-02	1.9582E-04
69	-2.3649E-02	1.1125E+00	-3.7152E-02	8.0551E-01	8.9962E-01	5.9150E-01	-1.0652E+01	-2.0450E-01	2.8387E-04
70	-2.7582E-02	1.1331E+00	-3.8412E-02	8.5290E-01	9.2879E-01	5.7288E-01	-1.3834E+01	-2.6559E-01	3.4875E-04
71	-2.7889E-02	1.1295E+00	3.8290E-02	8.4733E-01	9.2535E-01	5.7507E-01	-1.3470E+01	-2.5861E-01	1.0225E-03
72	-2.3949E-02	1.0987E+00	3.6969E-02	7.9955E-01	8.9599E-01	5.9383E-01	-1.0262E+01	-1.9701E-01	1.0372E-03
73	-1.6970E-02	1.0346E+00	3.1829E-02	7.0103E-01	8.3599E-01	6.3260E-01	-3.5805E+00	-6.8731E-02	9.3017E-04
74	-9.1442E-03	1.0123E+00	1.4504E-02	6.6543E-01	8.1523E-01	6.4610E-01	-1.2568E+00	-2.4129E-02	7.8769E-04
75	-3.2852E-03	1.0034E+00	6.1161E-03	6.5163E-01	8.0705E-01	6.5143E-01	-3.3531E-01	-6.4566E-03	3.0452E-04
76	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
77	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
78	-2.3465E-03	1.0072E+00	-4.6678E-03	6.5761E-01	8.1056E-01	6.4914E-01	-7.4492E-01	-1.4301E-02	-3.6966E-06
79	-5.0023E-03	1.0289E+00	-7.3729E-03	6.9124E-01	8.3038E-01	6.3625E-01	-2.9854E+00	-5.7315E-02	-2.4344E-04
80	-6.5938E-03	1.0796E+00	-4.4510E-03	7.6988E-01	8.7739E-01	6.0580E-01	-8.2550E+00	-1.5848E-01	-4.7030E-04
81	-6.5744E-03	1.1253E+00	-1.8961E-03	8.[illegible]E-01	9.2369E-01	5.7804E-01	-1.3015E+01	-2.4988E-01	-5.5668E-04
82	-6.1229E-03	1.1522E+00	3.6871E-03	8.8256E-01	9.4660E-01	5.6160E-01	-1.5819E+01	-3.0371E-01	-4.8634E-04
83	-7.2520E-03	1.1443E+00	-3.6617E-03	8.7027E-01	9.3906E-01	5.6637E-01	-1.5088E+01	-2.8967E-01	3.6903E-03
84	-7.7730E-03	1.1169E+00	2.0388E-03	8.2781E-01	9.1280E-01	5.8307E-01	-1.2235E+01	-2.3490E-01	3.5095E-03
85	-7.5494E-03	1.0731E+00	5.3221E-03	7.5976E-01	8.7138E-01	6.0968E-01	-7.6583E+00	-1.4702E-01	2.7988E-03
86	-5.5942E-03	1.0252E+00	8.2288E-03	6.8554E-01	8.2707E-01	6.3840E-01	-2.6547E+00	-5.0967E-02	1.6849E-03
87	-2.3045E-03	1.0058E+00	4.9365E-03	6.[illegible]E-01	8.[illegible]E-01	6.4996E-01	-6.1422E-01	-1.1792E-02	6.0663E-04
88	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
89	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
90	-7.6225E-04	1.0082E+00	-2.1428E-03	6.5913E-01	8.1145E-01	6.4857E-01	-8.4222E-01	-1.6170E-02	-1.6088E-04
91	-2.8981E-04	1.0325E+00	2.4204E-03	6.9684E-01	8.3365E-01	6.3412E-01	-3.3392E+00	-6.4103E-02	-9.2456E-04
92	3.1732E-03	1.0663E+00	1.3712E-02	7.4925E-01	8.6498E-01	6.1381E-01	-6.8250E+00	-1.3103E-01	-2.2319E-03
93	7.7455E-03	1.1017E+00	2.7652E-02	8.0423E-01	8.[illegible]E-01	5.9222E-01	-1.0502E+01	-2.0162E-01	-3.2063E-03
94	1.1994E-02	1.1302E+00	3.8879E-02	8.4841E-01	9.2595E-01	5.7469E-01	-1.3464E+01	-2.5849E-01	-3.5127E-03
95	9.7033E-03	1.1226E+00	-3.8619E-02	8.3667E-01	9.1902E-01	5.7910E-01	-1.2954E+01	-2.4871E-01	6.0148E-03
96	5.5530E-03	1.0942E+00	-2.6425E-02	7.9255E-01	8.9166E-01	5.9661E-01	-9.9835E+00	-1.9167E-01	5.5188E-03
97	1.3071E-03	1.0607E+00	-1.1887E-02	7.4065E-01	8.6004E-01	6.1701E-01	-6.4714E+00	-1.2424E-01	4.2186E-03
98	-1.5232E-03	1.0290E+00	-9.2941E-04	6.9142E-01	8.3058E-01	6.3612E-01	-3.1114E+00	-5.9734E-02	2.3567E-03
99	-1.2575E-03	1.0068E+00	2.8510E-03	6.[illegible]E-01	8.1318E-01	6.4939E-01	-7.3540E-01	-1.4119E-02	7.7772E-04
100	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
101	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
102	3.4583E-04	1.0082E+00	2.2636E-04	6.5910E-01	8.1141E-01	6.4859E-01	-8.3071E-01	-1.5948E-02	-3.3090E-04
103	3.1979E-03	1.0258E+00	8.4735E-03	6.8652E-01	8.2754E-01	6.3810E-01	-2.6127E+00	-5.3159E-02	-1.4305E-03
104	9.2567E-03	1.0450E+00	2.4429E-02	7.1615E-01	8.4528E-01	6.2657E-01	-4.5315E+00	-8.6999E-02	-3.6334E-03
105	1.6282E-02	1.0645E+00	4.1570E-02	7.4647E-01	8.6386E-01	6.1454E-01	-6.5244E+00	-1.2526E-01	-5.3802E-03
106	2.2255E-02	1.0764E+00	5.3821E-02	7.6496E-01	8.7548E-01	6.0703E-01	-7.7545E+00	-1.4888E-01	-6.1688E-03
107	1.9463E-02	1.0746E+00	-5.3729E-02	7.6212E-01	8.7434E-01	6.0777E-01	-8.0785E+00	-1.5519E-01	7.4042E-03
108	1.3585E-02	1.0614E+00	-4.0112E-02	7.4174E-01	8.6150E-01	6.1606E-01	-6.6951E+00	-1.2854E-01	6.4459E-03
109	6.9514E-03	1.0424E+00	-2.2357E-02	7.1222E-01	8.4331E-01	6.2785E-01	-4.6585E+00	-8.9437E-02	4.7813E-03
110	1.6051E-03	1.0234E+00	-6.6742E-03	6.8270E-01	8.2550E-01	6.3942E-01	-2.5837E+00	-4.9663E-02	2.5301E-03

111	-3.3057E-04	1.0070E+00	9.3198E-04	6.5731E-01	8.1044E-01	6.4922E-01	-7.8439E-01	-1.5059E-02	8.2726E-04
112	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
113	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
114	1.3695E-03	1.0067E+00	2.5403E-03	6.5689E-01	8.1010E-01	6.4944E-01	-6.6877E-01	-1.2839E-02	-3.9347E-04
115	5.7762E-03	1.0165E+00	1.2185E-02	6.7202E-01	8.1897E-01	6.4367E-01	-1.5966E+00	-3.3653E-02	-1.5579E-03
116	1.3019E-02	1.0221E+00	2.8087E-02	6.8067E-01	8.2424E-01	6.4024E-01	-2.0512E+00	-3.9572E-02	-3.6931E-03
117	2.1087E-02	1.0264E+00	4.7019E-02	6.8736E-01	8.2871E-01	6.3731E-01	-2.4034E+00	-4.6142E-02	-7.8561E-03
118	2.8055E-02	1.0331E+00	6.3021E-02	6.9781E-01	8.3562E-01	6.3284E-01	-3.0499E+00	-5.8554E-02	-1.2491E-02
119	2.5251E-02	1.0365E+00	-6.3225E-02	7.0331E-01	8.3967E-01	6.3021E-01	-4.3090E+00	-8.2727E-02	1.2618E-02
120	1.8382E-02	1.0289E+00	-4.5832E-02	6.9129E-01	8.3182E-01	6.3531E-01	-3.4646E+00	-6.6515E-02	8.1213E-03
121	1.0614E-02	1.0225E+00	-2.6503E-02	6.8139E-01	8.2522E-01	6.3960E-01	-2.6875E+00	-5.1597E-02	4.3037E-03
122	3.9505E-03	1.0152E+00	-1.0372E-02	6.6995E-01	8.1807E-01	6.4425E-01	-1.7855E+00	-3.4279E-02	2.2677E-03
123	5.4209E-04	1.0059E+00	-9.9466E-04	6.5551E-01	8.0940E-01	6.4990E-01	-6.8517E-01	-1.3154E-02	7.5457E-04
124	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
125	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
126	1.7900E-03	1.0053E+00	3.3962E-03	6.5466E-01	8.0879E-01	6.5030E-01	-5.1175E-01	-9.8249E-03	-4.6477E-04
127	6.6728E-03	1.0115E+00	1.2995E-02	6.6432E-01	8.1443E-01	6.4662E-01	-1.0581E+00	-2.0315E-02	-1.5455E-03
128	1.4045E-02	1.0112E+00	2.8039E-02	6.6382E-01	8.1428E-01	6.4672E-01	-8.8804E-01	-1.7049E-02	-3.4144E-03
129	2.215+E-02	1.0091E+00	4.6935E-02	6.6051E-01	8.1284E-01	6.4766E-01	-5.3089E-01	-1.0192E-02	-5.7025E-03
130	2.9295E-02	1.0090E+00	6.3727E-02	6.6036E-01	8.1345E-01	6.4726E-01	-3.9176E-01	-7.5212E-03	-1.2868E-02
131	2.5743E-02	1.0157E+00	-6.4636E-02	6.7071E-01	8.2080E-01	6.4248E-01	-2.2811E+00	-4.3794E-02	1.2849E-02
132	1.9619E-02	1.0129E+00	-4.6727E-02	6.6640E-01	8.1724E-01	6.4480E-01	-1.8636E+00	-3.5778E-02	5.9874E-03
133	1.1669E-02	1.0119E+00	-2.7137E-02	6.6491E-01	8.1557E-01	6.4588E-01	-1.6262E+00	-3.1221E-02	3.9158E-03
134	4.7618E-03	1.0103E+00	-1.1329E-02	6.6240E-01	8.1367E-01	6.4712E-01	-1.3046E+00	-2.5046E-02	2.0714E-03
135	8.9303E-04	1.0045E+00	-1.7409E-03	6.5[illegible]E-01	8.0915E-01	6.5071E-01	-5.5066E-01	-1.0572E-02	7.2171E-04
136	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
137	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
138	2.4482E-03	1.0031E+00	4.4985E-03	6.5129E-01	8.0680E-01	6.5159E-01	-2.6846E-01	-5.1541E-03	-4.7000E-04
139	7.7618E-03	1.0043E+00	1.3144E-02	6.5304E-01	8.0777E-01	6.5096E-01	-2.5915E-01	-4.9754E-03	-1.3610E-03
140	1.4621E-02	9.9584E-01	2.4375E-02	6.3996E-01	8.0015E-01	6.5593E-01	7.7959E-01	1.4967E-02	-2.1702E-03
141	2.1681E-02	9.8126E-01	3.7858E-02	6.1734E-01	7.8721E-01	6.6436E-01	2.4624E+00	4.7275E-02	-3.2919E-03
142	2.8380E-02	9.6019E-01	6.0235E-02	5.9397E-01	7.7455E-01	6.7260E-01	4.1842E+00	8.0332E-02	-2.3771E-03
143	2.6471E-02	9.7177E-01	-6.3019E-02	6.0262E-01	7.8109E-01	6.6834E-01	2.1676E+00	4.1616E-02	2.9857E-03
144	1.9500E-02	9.8318E-01	-3.9299E-02	6.2032E-01	7.9009E-01	6.6248E-01	1.1324E+00	2.1740E-02	3.6856E-03
145	1.2284E-02	9.9545E-01	-2.3797E-02	6.3936E-01	8.0058E-01	6.5564E-01	2.0787E-02	3.9909E-04	2.4547E-03
146	5.7074E-03	1.0034E+00	-1.1435E-02	6.5162E-01	8.0738E-01	6.5121E-01	-6.2644E-01	-1.2027E-02	1.6441E-03
147	1.4272E-03	1.0027E+00	-2.6195E-03	6.5[illegible]E-01	8.06[illegible]E-01	6.51[illegible]E-01	-3.8876E-01	-7.4636E-03	5.7708E-04
148	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
149	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
150	2.7537E-03	1.0014E+00	5.2157E-03	6.4854E-01	8.0518E-01	6.5265E-01	-7.2035E-02	-1.3830E-03	-3.6405E-04
151	8.0806E-03	9.9979E-01	1.2172E-02	6.4610E-01	8.0367E-01	6.5363E-01	2.3445E-01	4.5012E-03	-1.0408E-03
152	1.4128E-02	9.8803E-01	2.0311E-02	6.2785E-01	7.9297E-01	6.6060E-01	1.6300E+00	3.1294E-02	-1.5087E-03
153	1.9551E-02	9.6112E-01	2.8811E-02	5.[illegible]E-01	7.68[illegible]E-01	6.7632E-01	4.5962E+00	8.8240E-02	-1.0831E-03
154	2.4624E-02	9.8367E-01	5.6803E-02	5.2800E-01	7.3661E-01	6.9727E-01	8.6362E+00	1.6580E-01	-2.2080E-04
155	2.2852E-02	9.2121E-01	-6.0532E-02	5.2418E-01	7.3595E-01	6.9768E-01	7.4112E+00	1.4228E-01	4.3519E-04
156	1.7355E-02	9.5858E-01	-2.9957E-02	5.8215E-01	7.6772E-01	6.7704E-01	3.6575E+00	7.3219E-02	1.3312E-03
157	1.1695E-02	9.8611E-01	-2.0296E-02	6.2487E-01	7.9209E-01	6.6117E-01	9.5539E-01	1.8361E-02	1.7129E-03
158	5.9057E-03	9.9835E-01	-1.0514E-02	6.4385E-01	8.0286E-01	6.5416E-01	-1.3245E-01	-2.5429E-03	1.1666E-03
159	1.5574E-03	1.0010E+00	-3.1062E-03	6.4792E-01	8.05[illegible]E-01	6.5277E-01	-2.1725E-01	-4.1708E-03	3.6626E-04
160	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
161	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
162	2.8885E-03	1.0004E+00	5.4245E-03	6.4705E-01	8.0431E-01	6.5322E-01	3.5004E-02	6.7203E-04	-3.2152E-04
163	8.1225E-03	9.9772E-01	1.1448E-02	6.4288E-01	8.0176E-01	6.5487E-01	4.6702E-01	8.9661E-03	-9.7427E-04
164	1.3520E-02	9.8508E-01	1.6841E-02	6.2326E-01	7.9023E-01	6.6239E-01	1.9561E+00	3.7554E-02	-1.0049E-03
165	1.7975E-02	9.7225E-01	2.0100E-02	6.0337E-01	7.7659E-01	6.6967E-01	3.4502E+00	6.6239E-02	-5.1583E-04
166	2.2457E-02	9.2797E-01	-1.6262E-03	5.3466E-01	7.3886E-01	6.9581E-01	8.1878E+00	1.5719E-01	7.7060E-04
167	1.8315E-02	9.1376E-01	-1.6262E-03	5.1263E-01	7.2766E-01	7.0307E-01	8.1878E+00	1.5719E-01	-5.1148E-04
168	1.5547E-02	9.6579E-01	-2.0512E-02	5.9334E-01	7.7398E-01	6.7297E-01	2.8929E+00	5.5540E-02	6.8403E-04
169	1.0968E-02	9.8304E-01	-1.6275E-02	6.2010E-01	7.8926E-01	6.6302E-01	1.2591E+00	2.4173E-02	1.1388E-03
170	5.8527E-03	9.9648E-01	-9.8771E-03	6.4096E-01	8.0117E-01	6.5526E-01	4.6416E-02	8.9113E-04	1.0348E-03

171	1.7265E-03	9.9972E-01	-3.1989E-03	6.4539E-01	8.0388E-01	6.5350E-01	-9.4736E-02	-1.3188E-03	2.8052E-04
172	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
173	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
174	3.4932E-03	9.9734E-01	5.4974E-03	6.4229E-01	8.0147E-01	6.5596E-01	4.7120E-01	7.7025E-03	-1.5266E-04
175	8.7228E-03	9.9066E-01	9.1758E-03	6.3183E-01	7.9521E-01	6.5914E-01	1.2898E+00	2.4763E-02	-2.1556E-04
176	1.3932E-02	9.8066E-01	1.2479E-02	6.1641E-01	7.8609E-01	6.6508E-01	2.5183E+00	4.8349E-02	-6.3943E-06
177	1.7513E-02	9.5907E-01	1.6638E-03	5.8291E-01	7.6649E-01	6.7785E-01	4.9407E+00	3.4855E-02	8.3796E-04
178	1.4423E-02	9.4531E-01	1.6638E-03	5.6111E-01	7.5532E-01	6.8511E-01	4.9407E+00	3.4855E-02	-7.4452E-04
179	1.0814E-02	9.7432E-01	-9.6912E-03	6.0658E-01	7.8144E-01	6.6812E-01	2.0636E+00	3.9613E-02	2.2573E-05
180	5.9777E-03	9.8979E-01	-6.9995E-03	6.3058E-01	7.9517E-01	6.5917E-01	6.6223E-01	1.2714E-02	9.6138E-05
181	1.7310E-03	9.9683E-01	-3.0495E-03	6.4150E-01	8.0130E-01	6.5518E-01	1.5955E-01	3.0632E-03	-7.6626E-05
182	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
183	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
184	3.6755E-03	9.9282E-01	4.2563E-03	6.3527E-01	7.9732E-01	6.5776E-01	9.2166E-01	1.7695E-02	7.2378E-04
185	9.3208E-03	9.9016E-01	7.9729E-03	6.3115E-01	7.9470E-01	6.5947E-01	1.4392E+00	2.7630E-02	7.2999E-04
186	1.4186E-02	9.8009E-01	3.5320E-03	6.1553E-01	7.8536E-01	6.6556E-01	2.7289E+00	5.2391E-02	9.2569E-04
187	9.4447E-03	9.6649E-01	3.5320E-03	5.9443E-01	7.7450E-01	6.7263E-01	2.7289E+00	5.2391E-02	-9.8489E-04
188	5.3768E-03	9.8533E-01	-2.3143E-03	6.2366E-01	7.9121E-01	6.6174E-01	1.0237E+00	1.9654E-02	-9.8316E-04
189	1.4671E-03	9.9724E-01	-1.7859E-03	6.4213E-01	8.0171E-01	6.5491E-01	7.0486E-02	1.3532E-03	-1.1652E-03
190	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
191	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
192	3.9564E-03	9.9111E-01	4.5085E-03	6.3262E-01	7.9574E-01	6.5880E-01	1.1366E+00	2.1822E-02	2.0394E-03
193	1.1070E-02	9.9269E-01	8.1965E-03	6.3508E-01	7.9683E-01	6.5809E-01	1.3472E+00	2.5865E-02	1.1031E-03
194	3.1889E-03	9.8058E-01	8.1965E-03	6.1628E-01	7.8712E-01	6.6441E-01	1.3472E+00	2.5865E-02	-1.3847E-03
195	1.3338E-03	9.9834E-01	4.5475E-04	6.4334E-01	8.0274E-01	6.5424E-01	-7.3381E-02	-1.4088E-03	-2.6398E-03
196	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
197	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
198	4.6830E-03	9.9736E-01	5.4924E-03	6.4232E-01	8.0142E-01	6.5510E-01	4.7450E-01	9.1097E-03	5.9301E-04
199	-7.5381E-04	9.9338E-01	5.4924E-03	6.3614E-01	7.9822E-01	6.5718E-01	4.7450E-01	9.1097E-03	-8.2926E-04
200	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
201	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.
202	0.	1.0000E+00	0.	6.4642E-01	8.0400E-01	6.5342E-01	0.	0.	0.

NODES =130 MAX(DELM) PCT = 1.237E+00

I	IU	IL	CP-UPPER	CP-LOWER	DCP	AVE-CP+	AVE-CP-	STEADY-DCP
						8.7094E-01	8.7101E-01	6.8710E-05
1	35	34	2.7870E+01	2.7872E+01	2.1987E-03			
						2.9992E-01	2.9972E-01	-2.0203E-04
3	47	46	9.5976E+00	9.5911E+00	-6.4648E-03			
						-2.4895E-01	-2.5149E-01	-2.5453E-03
5	59	58	-7.9662E+00	-8.0477E+00	-8.1450E-02			
						-4.2094E-01	-4.3231E-01	-1.1365E-02
7	71	70	-1.3470E+01	-1.3834E+01	-3.6367E-01			
						-4.7151E-01	-4.9435E-01	-2.2840E-02
9	83	82	-1.5088E+01	-1.5819E+01	-7.3089E-01			
						-4.0482E-01	-4.2075E-01	-1.5929E-02
11	95	94	-1.2954E+01	-1.3464E+01	-5.0974E-01			
						-2.5245E-01	-2.4233E-01	1.0123E-02
13	107	106	-8.0785E+00	-7.7545E+00	3.2394E-01			
						-1.3466E-01	-9.5310E-02	3.9346E-02
15	119	118	-4.3090E+00	-3.0499E+00	1.2591E+00			
						-7.1284E-02	-1.2242E-02	5.9041E-02
17	131	130	-2.2811E+00	-3.9176E-01	1.8893E+00			
						6.7739E-02	1.3075E-01	6.3019E-02
19	143	142	2.1676E+00	4.1842E+00	2.0166E+00			
						2.3160E-01	2.6988E-01	3.8282E-02
21	155	154	7.4112E+00	8.6362E+00	1.2250E+00			


-3.000E-01
0.
9.000E-01
1.200E+00
2.100E+00
1.250E-01
3.750E-01

2. FLOW OVER A NACA 64 A410 AIRFOIL

This problem was analyzed using the mesh shown in Fig. 8, which consists of 173 elements with 202 nodes. The procedures described for the NACA 64 A006 airfoil problem also apply here. However, the data coded on the DATA statement in the BLOCK DATA subroutine should correspond to NACA 64 A410. Also modified slopes, taken to be half of the slope at 0.5% chord, are imposed at the two leading edge nodes. Input cards (page 66) and some typical output (pages 67 through 71) for UTRANL-II are listed here for reference. The non-linear boundary condition on the actual airfoil surface is used.

The case to be computed is:

$$M_{\infty} = 0.700, \ \alpha = 2 \text{ deg} \qquad \text{(barely critical)}$$

where α is the angle of attack.

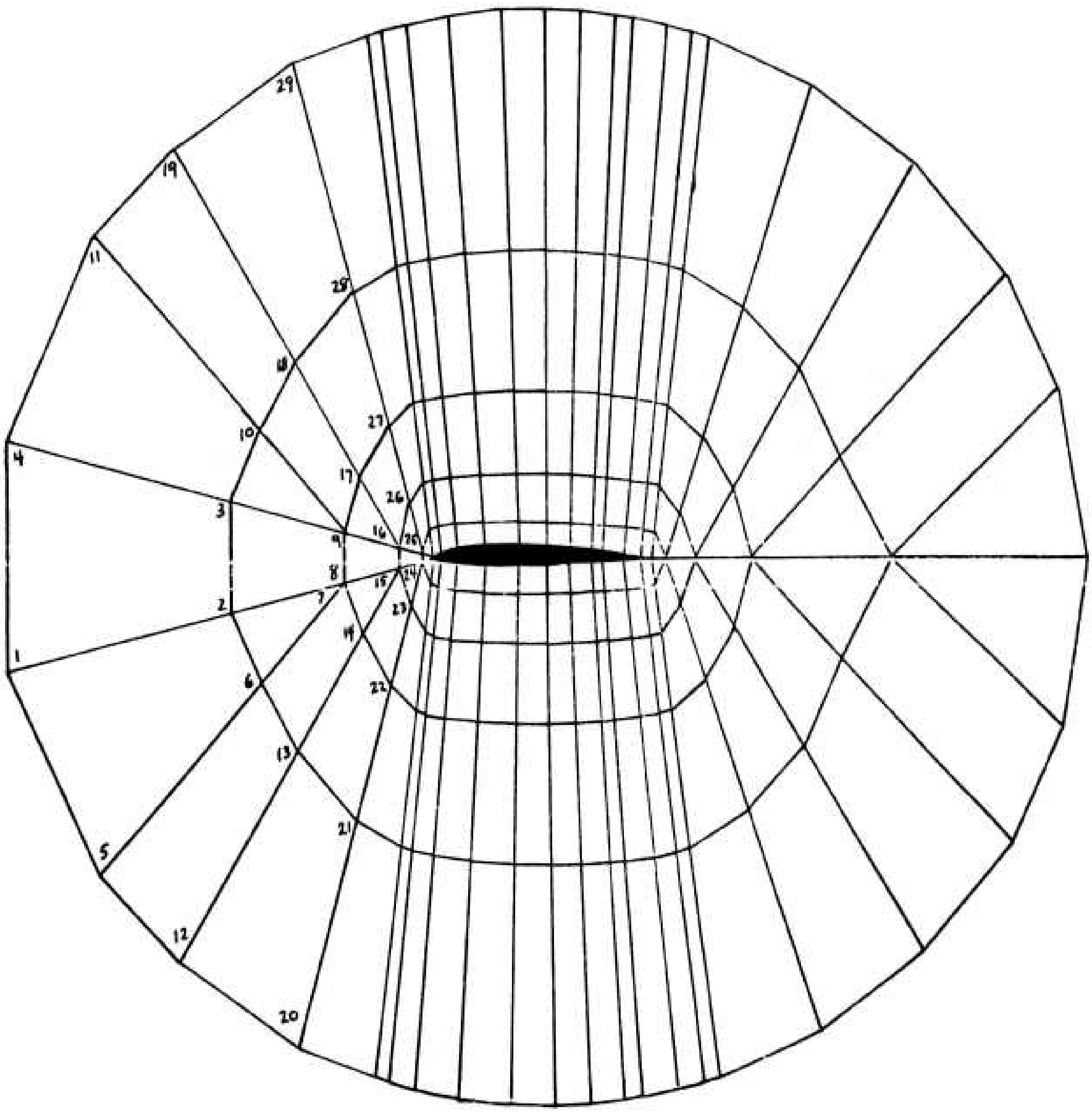


Figure 8 - Finite Element Mesh for NACA 64 A410 Airfoil
(173 elements, 202 nodes)

Input Cards to UTRANL-II for NACA 64 A410

```
LIFTING SOLUTION -- NACA 64A410 -- 202 NODES -- M=0.700
     1
.700      0.0       0.0       0.25      0.5       1.        20.       0.
.2        2.        0.
```

(Punched cards from UTRANL-I).

Results for the last time step are listed on the next five pages for reference.

Printout for NACA 64 A410 at Last Time Step

MACH NUMBER= .700 TIME INCREMENT =1.0000 TIME LEVEL = 16.00

TIME STEP = 16

NODE	PHIT	UCOM	VCOM	COF	LMAC	PRA	CPU	CP	DELM
1	1.7749E-01	9.9633E-01	1.6979E-02	4.8569E-01	6.9729E-01	7.2267E-01	7.5445E-03	7.5445E-03	1.8986E-05
2	1.7724E-01	9.9208E-01	2.2480E-02	4.8069E-01	6.9411E-01	7.2472E-01	1.6131E-02	1.6131E-02	-2.2333E-05
3	1.8891E-01	9.9454E-01	2.4202E-02	4.8358E-01	6.9602E-01	7.2348E-01	1.1221E-02	1.1221E-02	-2.0669E-05
4	1.9567E-01	1.0037E+00	1.6979E-02	4.9431E-01	7.0292E-01	7.1905E-01	-7.1023E-03	-7.1023E-03	1.4013E-05
5	1.5846E-01	9.8816E-01	1.6508E-02	4.7607E-01	6.9101E-01	7.2670E-01	2.3875E-02	2.3875E-02	2.4658E-05
6	1.6794E-01	9.8620E-01	2.6232E-02	4.7378E-01	6.8968E-01	7.2756E-01	2.7886E-02	2.7886E-02	-3.1071E-05
7	1.7024E-01	9.6759E-01	2.4449E-02	4.5189E-01	6.7543E-01	7.3668E-01	6.5224E-02	6.5224E-02	-8.0783E-05
8	1.7625E-01	9.7209E-01	2.7775E-02	4.5718E-01	6.7893E-01	7.3444E-01	5.6223E-02	5.6223E-02	-7.7399E-05
9	1.9643E-01	9.9529E-01	2.7516E-02	4.8446E-01	6.9666E-01	7.2307E-01	9.7413E-03	9.7413E-03	-2.7305E-05
10	2.1470E-01	1.0118E+00	1.6508E-02	5.0393E-01	7.0921E-01	7.1499E-01	-2.3433E-02	-2.3433E-02	8.3732E-06
11	1.4685E-01	9.8280E-01	1.5535E-02	4.6977E-01	6.8690E-01	7.2934E-01	3.4572E-02	3.4572E-02	2.8468E-05
12	1.5690E-01	9.7845E-01	2.6822E-02	4.6465E-01	6.8376E-01	7.3135E-01	4.3398E-02	4.3398E-02	-3.0750E-05
13	1.6186E-01	9.5627E-01	2.4893E-02	4.3857E-01	6.6683E-01	7.4216E-01	8.7875E-02	8.7875E-02	-8.3427E-05
14	1.6007E-01	9.3301E-01	3.6415E-02	4.1123E-01	6.4953E-01	7.5313E-01	1.3445E-01	1.3445E-01	-9.7306E-05
15	1.6279E-01	9.2522E-01	2.7649E-02	4.0206E-01	6.4343E-01	7.5697E-01	1.5004E-01	1.5004E-01	-9.2573E-05
16	1.8187E-01	9.8003E-01	4.0696E-02	4.6652E-01	6.8533E-01	7.3035E-01	4.0366E-02	4.0366E-02	-8.2867E-05
17	2.0576E-01	1.0027E+00	3.4661E-02	4.9323E-01	7.0256E-01	7.1928E-01	-5.1592E-03	-5.1592E-03	-2.9600E-05
18	2.2632E-01	1.0172E+00	1.5535E-02	5.1023E-01	7.1334E-01	7.1233E-01	-3.4130E-02	-3.4130E-02	4.6200E-06
19	1.2969E-01	9.7527E-01	1.2420E-02	4.6092E-01	6.8112E-01	7.3304E-01	4.9615E-02	4.9615E-02	3.3795E-05
20	1.4073E-01	9.6594E-01	2.4309E-02	4.4994E-01	6.7417E-01	7.3748E-01	6.8401E-02	6.8401E-02	-2.1860E-05
21	1.4965E-01	9.5060E-01	2.6866E-02	4.3120E-01	6.6212E-01	7.4515E-01	1.0042E-01	1.0042E-01	-7.5106E-05
22	1.4994E-01	9.1201E-01	1.2386E-02	3.8652E-01	6.3325E-01	7.6337E-01	1.7647E-01	1.7647E-01	-8.3627E-05
23	1.4726E-01	8.3896E-01	-2.7195E-02	3.0062E-01	5.7924E-01	7.9667E-01	3.2259E-01	3.2259E-01	-7.8049E-05
24	1.4762E-01	8.1463E-01	5.9969E-02	2.7201E-01	5.6267E-01	8.0663E-01	3.7124E-01	3.7124E-01	-7.6542E-05
25	1.6762E-01	9.7344E-01	7.6795E-02	4.5877E-01	6.8196E-01	7.3250E-01	5.3608E-02	5.3608E-02	-1.0790E-04
26	1.9358E-01	1.0167E+00	5.5177E-02	5.0960E-01	7.1398E-01	7.1191E-01	-3.2881E-02	-3.2881E-02	-9.8873E-05
27	2.2124E-01	1.0218E+00	4.0832E-02	5.1568E-01	7.1746E-01	7.0967E-01	-4.3321E-02	-4.3321E-02	-4.4749E-05
28	2.4347E-01	1.0247E+00	1.2420E-02	5.1908E-01	7.1913E-01	7.0859E-01	-4.9173E-02	-4.9173E-02	-5.4594E-07
29	1.1859E-01	9.7100E-01	9.3970E-03	4.5589E-01	6.7783E-01	7.3514E-01	5.8145E-02	5.8145E-02	3.6714E-05
30	1.3034E-01	9.6008E-01	2.1188E-02	4.4306E-01	6.6967E-01	7.4035E-01	8.0103E-02	8.0103E-02	-1.6229E-05
31	1.4021E-01	9.4080E-01	2.3226E-02	4.2038E-01	6.5509E-01	7.4961E-01	1.1881E-01	1.1881E-01	-5.8818E-05
32	1.4370E-01	9.2076E-01	4.9506E-03	3.9681E-01	6.3977E-01	7.5928E-01	1.5897E-01	1.5897E-01	-7.0728E-05
33	1.4330E-01	8.7999E-01	4.5164E-02	3.4886E-01	6.1014E-01	7.7776E-01	2.4054E-01	2.4054E-01	-7.4294E-05
34	1.3318E-01	6.4488E-01	-2.8472E-01	7.2382E-02	4.8171E-01	8.5319E-01	7.1075E-01	7.1075E-01	-2.5491E-05
35	1.3309E-01	6.4785E-01	2.6458E-01	7.5877E-02	4.7804E-01	8.5520E-01	7.0480E-01	7.0480E-01	-4.2808E-05
36	1.5490E-01	9.6049E-01	9.8686E-02	4.4353E-01	6.7363E-01	7.3783E-01	7.9546E-02	7.9546E-02	-1.1128E-04
37	1.7792E-01	1.0232E+00	9.2854E-02	5.1724E-01	7.2110E-01	7.0731E-01	-4.5810E-02	-4.5810E-02	-1.3057E-04
38	2.0448E-01	1.0494E+00	6.0851E-02	5.4806E-01	7.3958E-01	6.9534E-01	-9.8256E-02	-9.8256E-02	-1.1970E-04
39	2.3290E-01	1.0422E+00	4.0725E-02	5.3960E-01	7.3323E-01	6.9946E-01	-8.3966E-02	-8.3966E-02	-6.2177E-05
40	2.5458E-01	1.0290E+00	9.3970E-03	5.2411E-01	7.2241E-01	7.0647E-01	-5.7702E-02	-5.7702E-02	-3.3336E-06
41	1.1661E-01	9.7038E-01	8.7587E-03	4.5517E-01	6.7736E-01	7.3544E-01	5.9370E-02	5.9370E-02	3.7103E-05
42	1.2794E-01	9.5852E-01	2.0565E-02	4.4123E-01	6.6847E-01	7.4111E-01	8.3223E-02	8.3223E-02	-1.4411E-05
43	1.3694E-01	9.3039E-01	2.0824E-02	4.0814E-01	6.4719E-01	7.5460E-01	1.3963E-01	1.3963E-01	-4.9697E-05
44	1.4036E-01	9.2321E-01	1.5648E-02	3.9969E-01	6.4170E-01	7.5806E-01	1.5407E-01	1.5407E-01	-6.1015E-05
45	1.3676E-01	8.2985E-01	-1.6711E-02	2.8991E-01	5.7234E-01	8.0083E-01	3.4081E-01	3.4081E-01	-4.6999E-05
46	1.2411E-01	7.9313E-01	-1.3309E-01	2.4672E-01	5.5344E-01	8.1213E-01	4.1426E-01	4.1426E-01	-3.5613E-05
47	1.2789E-01	8.5750E-01	2.7259E-01	3.2242E-01	6.2404E-01	7.6912E-01	2.8552E-01	2.8552E-01	-1.0108E-04
48	1.5570E-01	9.9082E-01	1.4627E-01	4.7921E-01	7.0118E-01	7.2017E-01	1.8888E-02	1.8888E-02	-1.1958E-04
49	1.8130E-01	1.0549E+00	9.6932E-02	5.5451E-01	7.4597E-01	6.9119E-01	-1.0918E-01	-1.0918E-01	-1.5342E-04
50	2.0811E-01	1.0601E+00	6.2572E-02	5.6069E-01	7.4805E-01	6.8984E-01	-1.1972E-01	-1.1972E-01	-1.3840E-04

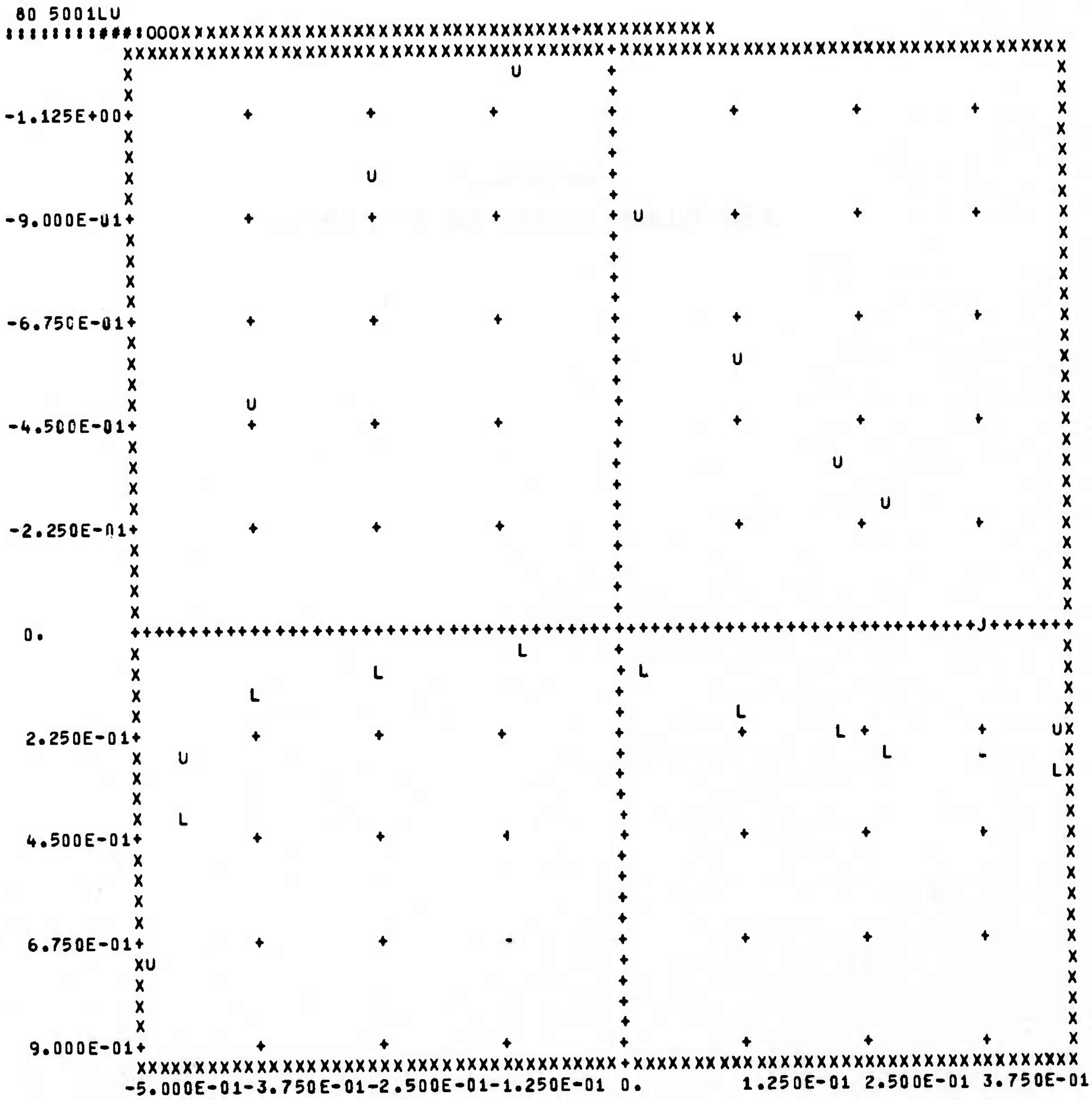
51	2.3591E-01	1.0458E+00	3.9228E-02	5.4384E-01	7.3599E-01	6.9767E-01	-9.1179E-02	-9.1179E-02	-6.9551E-05
52	2.5655E-01	1.0296E+00	8.7587E-03	5.2483E-01	7.2288E-01	7.0616E-01	-5.8928E-02	-5.8928E-02	-3.6966E-06
53	1.1284E-01	9.6934E-01	7.4790E-03	4.5394E-01	6.7656E-01	7.3596E-01	6.1455E-02	6.1455E-02	3.7731E-05
54	1.2307E-01	9.5142E-01	1.8934E-02	4.3287E-01	6.6305E-01	7.4456E-01	9.7439E-02	9.7439E-02	-8.3693E-06
55	1.3095E-01	9.3492E-01	1.8172E-02	4.1346E-01	6.5956E-01	7.5248E-01	1.3060E-01	1.3060E-01	-4.0412E-05
56	1.3279E-01	9.0077E-01	5.8106E-03	3.7331E-01	6.2479E-01	7.6866E-01	1.9896E-01	1.9896E-01	-4.6424E-05
57	1.2533E-01	8.7002E-01	-4.7872E-02	3.3714E-01	6.0285E-01	7.8225E-01	2.6048E-01	2.6048E-01	-4.1091E-05
58	1.1591E-01	9.4107E-01	-7.8407E-02	4.2070E-01	6.5753E-01	7.4806E-01	1.1839E-01	1.1839E-01	-5.7191E-05
59	1.3891E-01	1.2467E+00	1.9885E-01	7.8017E-01	9.1062E-01	5.8446E-01	-4.9294E-01	-4.9294E-01	-2.5482E-04
60	1.6153E-01	1.0933E+00	1.7392E-01	5.9966E-01	7.8359E-01	6.6671E-01	-1.8596E-01	-1.8596E-01	-2.0455E-04
61	1.8958E-01	1.1062E+00	1.0035E-01	6.1494E-01	7.8659E-01	6.6475E-01	-2.1192E-01	-2.1192E-01	-1.9721E-04
62	2.1542E-01	1.0896E+00	5.9476E-02	5.9067E-01	7.6792E-01	6.7691E-01	-1.7068E-01	-1.7068E-01	-1.5110E-04
63	2.4171E-01	1.0525E+00	3.6224E-02	5.5178E-01	7.4117E-01	6.9431E-01	-1.0467E-01	-1.0467E-01	-7.4100E-05
64	2.6032E-01	1.0307E+00	7.4790E-03	5.2606E-01	7.2368E-01	7.0564E-01	-6.1013E-02	-6.1013E-02	-4.2759E-06
65	1.0643E-01	9.6793E-01	5.1523E-03	4.5228E-01	6.7547E-01	7.3665E-01	6.4273E-02	6.4273E-02	3.8469E-05
66	1.1453E-01	9.4923E-01	1.5620E-02	4.3029E-01	6.6135E-01	7.4564E-01	1.0182E-01	1.0182E-01	5.3827E-06
67	1.2064E-01	9.2830E-01	1.1508E-02	4.0566E-01	6.4549E-01	7.5568E-01	1.4384E-01	1.4384E-01	-2.1273E-05
68	1.1938E-01	9.0542E-01	-9.1307E-03	3.7878E-01	6.2829E-01	7.6647E-01	1.8966E-01	1.8966E-01	-2.9972E-05
69	1.1390E-01	9.3352E-01	-3.1295E-02	4.1182E-01	6.4977E-01	7.5298E-01	1.3348E-01	1.3348E-01	-3.5262E-05
70	1.0829E-01	9.5499E-01	-4.6332E-02	4.3659E-01	6.6616E-01	7.4259E-01	9.1361E-02	9.1361E-02	-4.2664E-05
71	1.8662E-01	1.5041E+00	9.5890E-02	1.0828E+00	1.1275E+00	4.5252E-01	-1.0074E+00	-1.0074E+00	-7.6747E-04
72	1.9609E-01	1.3798E+00	1.0228E-01	9.3660E-01	1.0150E+00	5.1909E-01	-7.5882E-01	-7.5882E-01	-6.4125E-04
73	2.1321E-01	1.2137E+00	8.4912E-02	7.4136E-01	8.7245E-01	6.0899E-01	-4.2681E-01	-4.2681E-01	-3.2160E-04
74	2.3306E-01	1.1324E+00	4.7096E-02	6.4569E-01	8.0463E-01	6.5300E-01	-2.6420E-01	-2.6420E-01	-1.9591E-04
75	2.5246E-01	1.0653E+00	2.9177E-02	5.6680E-01	7.5099E-01	6.8793E-01	-1.3018E-01	-1.3018E-01	-8.6689E-05
76	2.6673E-01	1.0321E+00	5.1526E-03	5.2772E-01	7.2477E-01	7.0494E-01	-6.3831E-02	-6.3831E-02	-4.9351E-06
77	9.8577E-02	9.6693E-01	2.1074E-03	4.5111E-01	6.7470E-01	7.3714E-01	6.6254E-02	6.6254E-02	3.8703E-05
78	1.0400E-01	9.4695E-01	1.0695E-02	4.2761E-01	6.5957E-01	7.4677E-01	1.0637E-01	1.0637E-01	2.7065E-05
79	1.0741E-01	9.2463E-01	2.8998E-03	4.0137E-01	6.4268E-01	7.5744E-01	1.5117E-01	1.5117E-01	7.9426E-06
80	1.0591E-01	9.2448E-01	-7.3799E-03	4.0119E-01	6.4258E-01	7.5751E-01	1.5155E-01	1.5155E-01	-5.4085E-06
81	1.0369E-01	9.4429E-01	-1.1716E-02	4.2449E-01	6.5756E-01	7.4804E-01	1.1195E-01	1.1195E-01	-1.2019E-05
82	1.0246E-01	9.6858E-01	-9.5173E-03	4.5305E-01	6.7598E-01	7.3633E-01	6.3382E-02	6.3382E-02	-2.0147E-05
83	2.7077E-01	1.6146E+00	-4.0021E-02	1.2128E+00	1.2318E+00	3.9552E-01	-1.2284E+00	-1.2284E+00	-4.0596E-04
84	2.6500E-01	1.5127E+00	-2.3925E-02	1.0930E+00	1.1330E+00	4.4943E-01	-1.0246E+00	-1.0246E+00	-3.7376E-04
85	2.6023E-01	1.3462E+00	-1.4199E-03	8.9713E-01	9.8221E-01	5.3930E-01	-6.9164E-01	-6.9164E-01	-4.5311E-04
86	2.6035E-01	1.1709E+00	1.7313E-02	6.9098E-01	8.3503E-01	6.3323E-01	-3.4116E-01	-3.4116E-01	-2.6305E-04
87	2.6712E-01	1.0708E+00	1.7314E-02	5.8028E-01	7.5979E-01	6.8221E-01	-1.5308E-01	-1.5308E-01	-8.3930E-05
88	2.7459E-01	1.0331E+00	2.1074E-03	5.2889E-01	7.2553E-01	7.0445E-01	-6.5812E-02	-6.5812E-02	-5.0842E-06
89	9.1968E-02	9.6675E-01	-5.2892E-04	4.5090E-01	6.7456E-01	7.3723E-01	6.6607E-02	6.6607E-02	3.8300E-05
90	9.5001E-02	9.4552E-01	5.9829E-03	4.2593E-01	6.5846E-01	7.4748E-01	1.0922E-01	1.0922E-01	4.7368E-05
91	9.6630E-02	9.2822E-01	-1.4110E-03	4.0559E-01	6.4538E-01	7.5574E-01	1.4397E-01	1.4397E-01	3.9548E-05
92	9.5705E-02	9.2473E-01	-5.8669E-03	4.0149E-01	6.4276E-01	7.5739E-01	1.5103E-01	1.5103E-01	2.7274E-05
93	9.5821E-02	9.3318E-01	3.5217E-03	4.1143E-01	6.4912E-01	7.5338E-01	1.3416E-01	1.3416E-01	1.8695E-05
94	9.7524E-02	9.5177E-01	2.2008E-02	4.3328E-01	6.6336E-01	7.4436E-01	9.6989E-02	9.6989E-02	1.1413E-05
95	3.3815E-01	1.4431E+00	-1.3744E-01	1.0111E+00	1.0743E+00	4.8339E-01	-8.8535E-01	-8.8535E-01	-4.4515E-04
96	3.2209E-01	1.3812E+00	-1.1749E-01	9.3828E-01	1.0173E+00	5.1767E-01	-7.6156E-01	-7.6156E-01	-6.9746E-04
97	3.0260E-01	1.2850E+00	-6.7103E-02	8.2514E-01	9.3109E-01	5.7142E-01	-5.6919E-01	-5.6919E-01	-3.7824E-04
98	2.8587E-01	1.1761E+00	-1.9138E-02	6.9005E-01	8.3441E-01	6.3363E-01	-3.3956E-01	-3.3956E-01	-1.5357E-04
99	2.8012E-01	1.0790E+00	4.5078E-03	5.8291E-01	7.6149E-01	6.8113E-01	-1.5753E-01	-1.5753E-01	-6.5218E-05
100	2.8120E-01	1.0332E+00	-5.2892E-04	5.2910E-01	7.2567E-01	7.0436E-01	-6.6165E-02	-6.6165E-02	-4.6247E-06
101	8.6687E-02	9.6704E-01	-2.6282E-03	4.5124E-01	6.7479E-01	7.3709E-01	6.6025E-02	6.6025E-02	3.7582E-05
102	8.7825E-02	9.4612E-01	2.6375E-03	4.2664E-01	6.5891E-01	7.4719E-01	1.0799E-01	1.0799E-01	6.6137E-05
103	8.8314E-02	9.3029E-01	-4.5911E-03	4.0802E-01	6.4695E-01	7.5476E-01	1.3982E-01	1.3982E-01	6.9435E-05
104	8.7266E-02	9.2066E-01	-5.8485E-03	3.9670E-01	6.3970E-01	7.5932E-01	1.5916E-01	1.5916E-01	6.0751E-05
105	8.8075E-02	9.1671E-01	7.7869E-03	3.9205E-01	6.3675E-01	7.6118E-01	1.6709E-01	1.6709E-01	4.9413E-05
106	9.0828E-02	9.1520E-01	2.9152E-02	3.9027E-01	6.3593E-01	7.6169E-01	1.7012E-01	1.7012E-01	4.2888E-05
107	3.7556E-01	1.2876E+00	-1.7718E-01	8.2823E-01	9.4217E-01	5.6440E-01	-5.7457E-01	-5.7457E-01	8.5695E-04
108	3.5594E-01	1.2519E+00	-1.4916E-01	7.8620E-01	9.0913E-01	5.8542E-01	-5.0307E-01	-5.0307E-01	6.5467E-04
109	3.2966E-01	1.2069E+00	-9.9762E-02	7.3328E-01	8.6772E-01	6.1204E-01	-4.1305E-01	-4.1305E-01	2.4111E-04
110	3.0470E-01	1.1448E+00	-4.2127E-02	6.6023E-01	8.1440E-01	6.4664E-01	-2.8889E-01	-2.8889E-01	2.3978E-05

111	2.9049E-01	1.0762E+00	-5.6905E-03	5.7956E-01	7.5921E-01	6.8259E-01	-1.5184E-01	-1.5184E-01	-1.6810E-05
112	2.8646E-01	1.0330E+00	-2.6282E-03	5.2876E-01	7.2545E-01	7.0450E-01	-6.5583E-02	-6.5583E-02	-3.8745E-06
113	8.1450E-02	9.6770E-01	-4.6604E-03	4.5202E-01	6.7530E-01	7.3676E-01	6.4689E-02	6.4689E-02	3.6521E-05
114	8.0799E-02	9.4785E-01	-6.3387E-04	4.2867E-01	6.6021E-01	7.4636E-01	1.0452E-01	1.0452E-01	8.2456E-05
115	8.0182E-02	9.3231E-01	-7.8132E-03	4.1040E-01	6.4849E-01	7.5378E-01	1.3575E-01	1.3575E-01	1.0108E-04
116	7.8422E-02	9.1758E-01	-9.6479E-03	3.9307E-01	6.3741E-01	7.6076E-01	1.6530E-01	1.6530E-01	9.5249E-05
117	7.8504E-02	8.9871E-01	9.3289E-04	3.7089E-01	6.2324E-01	7.6962E-01	2.0306E-01	2.0306E-01	8.3034E-05
118	8.0659E-02	8.8116E-01	2.2014E-02	3.5024E-01	6.1035E-01	7.7762E-01	2.3817E-01	2.3817E-01	7.5103E-05
119	4.0154E-01	1.1868E+00	-2.0889E-01	7.0969E-01	8.6285E-01	6.1519E-01	-3.7313E-01	-3.7313E-01	5.4157E-04
120	3.7842E-01	1.1595E+00	-1.6808E-01	6.7752E-01	8.3548E-01	6.3294E-01	-3.1841E-01	-3.1841E-01	5.3227E-04
121	3.4952E-01	1.1388E+00	-1.1239E-01	6.5321E-01	8.1343E-01	6.4728E-01	-2.7702E-01	-2.7702E-01	3.8256E-04
122	3.2051E-01	1.1130E+00	-5.6816E-02	6.2284E-01	7.8948E-01	6.6288E-01	-2.2536E-01	-2.2536E-01	1.5954E-04
123	3.0042E-01	1.0702E+00	-1.5942E-02	5.7260E-01	7.5464E-01	6.8556E-01	-1.4002E-01	-1.4002E-01	2.6394E-05
124	2.9171E-01	1.0323E+00	-4.6604E-03	5.2798E-01	7.2494E-01	7.0483E-01	-6.4247E-02	-6.4247E-02	-2.7940E-06
125	7.8858E-02	9.6818E-01	-5.6343E-03	4.5258E-01	6.7566E-01	7.3653E-01	6.3739E-02	6.3739E-02	3.5864E-05
126	7.7379E-02	9.4890E-01	-2.3816E-03	4.2990E-01	6.6100E-01	7.4586E-01	1.0242E-01	1.0242E-01	9.0581E-05
127	7.6167E-02	9.3227E-01	-9.1890E-03	4.1034E-01	6.4846E-01	7.5380E-01	1.3583E-01	1.3583E-01	1.2122E-04
128	7.3853E-02	9.1507E-01	-1.2256E-02	3.9012E-01	6.3555E-01	7.6193E-01	1.7031E-01	1.7031E-01	1.2271E-04
129	7.3109E-02	8.9215E-01	-5.3277E-03	3.6316E-01	6.1835E-01	7.7267E-01	2.1618E-01	2.1618E-01	1.0774E-04
130	7.4368E-02	8.6739E-01	9.3447E-03	3.3405E-01	5.9996E-01	7.8403E-01	2.6571E-01	2.6571E-01	9.3241E-05
131	4.1111E-01	1.1298E+00	-2.2156E-01	6.4262E-01	8.1906E-01	6.4361E-01	-2.5913E-01	-2.5913E-01	4.7257E-04
132	3.8676E-01	1.1145E+00	-1.7135E-01	6.2470E-01	8.0004E-01	6.5599E-01	-2.2863E-01	-2.2863E-01	4.2525E-04
133	3.5711E-01	1.1069E+00	-1.1474E-01	6.1568E-01	7.8820E-01	6.6371E-01	-2.1325E-01	-2.1325E-01	3.4083E-04
134	3.2713E-01	1.0952E+00	-6.0733E-02	6.0190E-01	7.7552E-01	6.7197E-01	-1.8980E-01	-1.8980E-01	1.8096E-04
135	3.0500E-01	1.0632E+00	-1.9750E-02	5.6438E-01	7.4921E-01	6.8909E-01	-1.2605E-01	-1.2605E-01	4.3321E-05
136	2.9431E-01	1.0318E+00	-5.6343E-03	5.2742E-01	7.2457E-01	7.0507E-01	-6.3297E-02	-6.3297E-02	-2.1313E-06
137	7.3743E-02	9.6934E-01	-7.4790E-03	4.5394E-01	6.7656E-01	7.3596E-01	6.1409E-02	6.1409E-02	3.4396E-05
138	7.0663E-02	9.5026E-01	-4.9331E-03	4.3150E-01	6.6204E-01	7.4520E-01	9.9682E-02	9.9682E-02	1.0778E-04
139	6.8277E-02	9.3492E-01	-1.2236E-02	4.1347E-01	6.5049E-01	7.5252E-01	1.3049E-01	1.3049E-01	1.4936E-04
140	6.4663E-02	9.1776E-01	-1.9873E-02	3.9328E-01	6.3767E-01	7.6059E-01	1.6489E-01	1.6489E-01	1.6186E-04
141	6.1657E-02	8.9361E-01	-2.5956E-02	3.6136E-01	6.1747E-01	7.7321E-01	2.1922E-01	2.1922E-01	1.5085E-04
142	6.0343E-02	8.5406E-01	-2.0452E-02	3.1838E-01	5.9024E-01	7.8998E-01	2.9232E-01	2.9232E-01	1.2133E-04
143	4.2161E-01	1.0005E+00	-2.3252E-01	4.9057E-01	7.2094E-01	7.0742E-01	-6.1903E-04	-6.1903E-04	2.9600E-04
144	3.9679E-01	1.0323E+00	-1.5789E-01	5.2794E-01	7.3424E-01	6.9880E-01	-6.4165E-02	-6.4165E-02	2.7913E-04
145	3.6828E-01	1.0593E+00	-1.1054E-01	5.5971E-01	7.5046E-01	6.8827E-01	-1.1815E-01	-1.1815E-01	2.6316E-04
146	3.3817E-01	1.0763E+00	-6.4474E-02	5.7267E-01	7.5612E-01	6.8460E-01	-1.4015E-01	-1.4015E-01	2.0027E-04
147	3.1346E-01	1.0554E+00	-2.5314E-02	5.5516E-01	7.4316E-01	6.9301E-01	-1.1039E-01	-1.1039E-01	8.3981E-05
148	2.9942E-01	1.0307E+00	-7.4790E-03	5.2606E-01	7.2368E-01	7.0564E-01	-6.0967E-02	-6.0967E-02	-6.2253E-07
149	6.9973E-02	9.7038E-01	-8.7587E-03	4.5517E-01	6.7736E-01	7.3544E-01	5.9315E-02	5.9315E-02	3.3072E-05
150	6.5813E-02	9.5290E-01	-7.0621E-03	4.3461E-01	6.6406E-01	7.4392E-01	9.4377E-02	9.4377E-02	1.1636E-04
151	6.2601E-02	9.3828E-01	-1.5142E-02	4.1741E-01	6.5306E-01	7.5090E-01	1.2375E-01	1.2375E-01	1.7067E-04
152	5.7961E-02	9.1987E-01	-2.4933E-02	3.9576E-01	6.3935E-01	7.5954E-01	1.6064E-01	1.6064E-01	1.9011E-04
153	5.3029E-02	8.8763E-01	-3.5451E-02	3.5785E-01	6.1549E-01	7.7444E-01	2.2516E-01	2.2516E-01	1.8739E-04
154	4.9325E-02	8.4673E-01	-2.4949E-02	3.0976E-01	5.8492E-01	7.9322E-01	3.0696E-01	3.0696E-01	1.5751E-04
155	4.2022E-01	8.7663E-01	-2.0926E-01	3.4492E-01	6.2515E-01	7.6843E-01	2.4705E-01	2.4705E-01	1.8613E-04
156	3.9917E-01	9.7312E-01	-1.4138E-01	4.5839E-01	6.8721E-01	7.2914E-01	5.4095E-02	5.4095E-02	1.9000E-04
157	3.7363E-01	1.0281E+00	-1.0416E-01	5.2307E-01	7.2578E-01	7.0429E-01	-5.5885E-02	-5.5885E-02	2.0892E-04
158	3.4473E-01	1.0512E+00	-6.4585E-02	5.5025E-01	7.4122E-01	6.9428E-01	-1.0207E-01	-1.0207E-01	1.7780E-04
159	3.1911E-01	1.0469E+00	-2.9146E-02	5.4514E-01	7.3660E-01	6.9728E-01	-9.3387E-02	-9.3387E-02	9.0408E-05
160	3.0319E-01	1.0296E+00	-8.7587E-03	5.2483E-01	7.2288E-01	7.0616E-01	-5.8872E-02	-5.8872E-02	6.5559E-07
161	6.7504E-02	9.7116E-01	-9.5515E-03	4.5608E-01	6.7796E-01	7.3506E-01	5.7768E-02	5.7768E-02	3.2190E-05
162	6.2679E-02	9.5412E-01	-8.5165E-03	4.3604E-01	6.6499E-01	7.4333E-01	9.1932E-02	9.1932E-02	1.2246E-04
163	5.8926E-02	9.3874E-01	-1.6357E-02	4.1796E-01	6.5343E-01	7.5066E-01	1.2280E-01	1.2280E-01	1.8286E-04
164	5.3617E-02	9.2204E-01	-2.8241E-02	3.9832E-01	6.4106E-01	7.5847E-01	1.5626E-01	1.5626E-01	2.0310E-04
165	4.7807E-02	9.1121E-01	-4.5425E-02	3.8558E-01	6.3344E-01	7.6325E-01	1.7797E-01	1.7797E-01	2.3102E-04
166	4.1713E-02	8.7199E-01	-9.4095E-02	3.3946E-01	6.0710E-01	7.7963E-01	2.5642E-01	2.5642E-01	2.5635E-04
167	4.1476E-01	8.7194E-01	-9.4095E-02	3.3940E-01	6.0706E-01	7.7965E-01	2.5642E-01	2.5642E-01	7.1883E-05
168	3.9932E-01	9.7410E-01	-1.2098E-01	4.5954E-01	6.8587E-01	7.3000E-01	5.2114E-02	5.2114E-02	1.4358E-04
169	3.7596E-01	1.0138E+00	-9.5949E-02	5.0620E-01	7.1410E-01	7.1184E-01	-2.7222E-02	-2.7222E-02	1.9141E-04
170	3.4833E-01	1.0415E+00	-6.3245E-02	5.3875E-01	7.3354E-01	6.9926E-01	-8.2531E-02	-8.2531E-02	1.6866E-04

171	3.2248E-01	1.0403E+00	-3.0403E-02	5.3745E-01	7.3153E-01	7.0056E-01	-8.0312E-02	-8.0312E-02	8.2914E-05
172	3.0566E-01	1.0288E+00	-9.5515E-03	5.2392E-01	7.2229E-01	7.0655E-01	-5.7326E-02	-5.7326E-02	1.5698E-06
173	5.2613E-02	9.7691E-01	-1.3471E-02	4.6284E-01	6.8238E-01	7.3224E-01	4.6249E-02	4.6249E-02	2.5584E-05
174	4.8728E-02	9.5862E-01	-1.3358E-02	4.4134E-01	6.6845E-01	7.4112E-01	8.2870E-02	8.2870E-02	1.5220E-04
175	4.6568E-02	9.4196E-01	-2.2757E-02	4.2175E-01	6.5596E-01	7.4906E-01	1.1631E-01	1.1631E-01	2.0998E-04
176	4.2171E-02	9.2896E-01	-4.1938E-02	4.0645E-01	6.4665E-01	7.5494E-01	1.4241E-01	1.4241E-01	2.4833E-04
177	3.5728E-02	9.1167E-01	-7.3671E-02	3.8612E-01	6.3517E-01	7.6217E-01	1.7704E-01	1.7704E-01	2.7552E-04
178	4.0874E-01	9.1163E-01	-7.3671E-02	3.8607E-01	6.3514E-01	7.6218E-01	1.7704E-01	1.7704E-01	7.8917E-05
179	3.8817E-01	9.8017E-01	-8.7940E-02	4.6667E-01	6.8780E-01	7.2876E-01	3.9960E-02	3.9960E-02	1.3063E-04
180	3.6194E-01	1.0139E+00	-6.1089E-02	5.0634E-01	7.1211E-01	7.1312E-01	-2.7493E-02	-2.7493E-02	1.5800E-04
181	3.3714E-01	1.0260E+00	-3.4323E-02	5.2052E-01	7.2046E-01	7.0773E-01	-5.1564E-02	-5.1564E-02	8.4794E-05
182	3.2055E-01	1.0231E+00	-1.3471E-02	5.1716E-01	7.1787E-01	7.0941E-01	-4.5807E-02	-4.5807E-02	8.0489E-06
183	3.7953E-02	9.8357E-01	-1.5796E-02	4.7067E-01	6.8749E-01	7.2896E-01	3.2914E-02	3.2914E-02	1.8197E-05
184	3.4440E-02	9.6466E-01	-1.7986E-02	4.4844E-01	6.7310E-01	7.3816E-01	7.0762E-02	7.0762E-02	1.2785E-04
185	3.2780E-02	9.5371E-01	-3.2847E-02	4.3557E-01	6.6508E-01	7.4327E-01	9.2771E-02	9.2771E-02	2.4825E-04
186	2.7717E-02	9.4483E-01	-5.6930E-02	4.2512E-01	6.5922E-01	7.4700E-01	1.1065E-01	1.1065E-01	2.9781E-04
187	4.0057E-01	9.4481E-01	-5.6930E-02	4.2509E-01	6.5920E-01	7.4700E-01	1.1065E-01	1.1065E-01	7.7837E-05
188	3.7608E-01	9.9068E-01	-5.7162E-02	4.7834E-01	6.9365E-01	7.2501E-01	2.0078E-02	2.0078E-02	1.0441E-04
189	3.5147E-01	1.0115E+00	-3.3912E-02	5.0349E-01	7.0926E-01	7.1496E-01	-2.2646E-02	-2.2646E-02	9.4393E-05
190	3.3521E-01	1.0164E+00	-1.5796E-02	5.0933E-01	7.1274E-01	7.1271E-01	-3.2472E-02	-3.2472E-02	1.5298E-05
191	2.4310E-02	9.8987E-01	-1.6680E-02	4.7808E-01	6.9233E-01	7.2586E-01	2.0294E-02	2.0294E-02	1.1219E-05
192	2.0412E-02	9.7438E-01	-2.2543E-02	4.5987E-01	6.8058E-01	7.3339E-01	5.1304E-02	5.1304E-02	1.1866E-04
193	1.7846E-02	9.6885E-01	-4.0303E-02	4.5337E-01	6.7680E-01	7.3580E-01	6.2469E-02	6.2469E-02	3.1889E-04
194	3.9018E-01	9.6887E-01	-4.0303E-02	4.5340E-01	6.7682E-01	7.3579E-01	6.2469E-02	6.2469E-02	4.3387E-05
195	3.6609E-01	9.9900E-01	-3.2928E-02	4.8882E-01	6.9964E-01	7.2116E-01	2.2467E-03	2.2467E-03	4.0625E-05
196	3.4885E-01	1.0101E+00	-1.6680E-02	5.0192E-01	7.0789E-01	7.1584E-01	-1.9852E-02	-1.9852E-02	2.2125E-05
197	1.3654E-02	9.9446E-01	-1.6895E-02	4.8349E-01	6.9586E-01	7.2359E-01	1.1090E-02	1.1090E-02	6.0545E-06
198	5.1241E-03	9.8833E-01	-2.4126E-02	4.7628E-01	6.9127E-01	7.2654E-01	2.3305E-02	2.3305E-02	2.0316E-04
199	3.7796E-01	9.8851E-01	-2.4126E-02	4.7648E-01	6.9140E-01	7.2646E-01	2.3305E-02	2.3305E-02	-1.2177E-04
200	3.5951E-01	1.0055E+00	-1.6895E-02	4.9651E-01	7.0435E-01	7.1812E-01	-1.0648E-02	-1.0648E-02	2.7155E-05
201	1.6965E-07	1.0000E+00	-1.6965E-02	4.9000E-01	7.0011E-01	7.2086E-01	1.3592E-07	1.3592E-07	-2.6566E-07
202	3.7316E-01	1.0000E+00	-1.6965E-02	4.9000E-01	7.0010E-01	7.2087E-01	4.4207E-04	4.4207E-04	3.3263E-05

NODES =107 MAX(DELM) PCT = 8.569E-02

I	IU	IL	CP-UPPER	CP-LOWER	DCP	AVE-CP+	AVE-CP-	STEADY-DCP
						3.5240E-01	3.5537E-01	2.9720E-03
1	35	34	7.0480E-01	7.1075E-01	5.9440E-03			
						1.4276E-01	2.0713E-01	6.4372E-02
3	47	46	2.8552E-01	4.1426E-01	1.2874E-01			
						-2.4647E-01	5.9195E-02	3.0566E-01
5	59	58	-4.9294E-01	1.1839E-01	6.1133E-01			
						-5.0372E-01	4.5680E-02	5.4940E-01
7	71	70	-1.0074E+00	9.1361E-02	1.0988E+00			
						-6.1418E-01	3.1691E-02	6.4587E-01
9	83	82	-1.2284E+00	6.3382E-02	1.2917E+00			
						-4.4268E-01	4.8495E-02	4.9117E-01
11	95	94	-8.8535E-01	9.6989E-02	9.8234E-01			
						-2.8728E-01	8.5060E-02	3.7234E-01
13	107	106	-5.7457E-01	1.7012E-01	7.4469E-01			
						-1.8657E-01	1.1909E-01	3.0565E-01
15	119	118	-3.7313E-01	2.3817E-01	6.1131E-01			
						-1.2956E-01	1.3285E-01	2.6242E-01
17	131	130	-2.5913E-01	2.6571E-01	5.2483E-01			
						-3.0951E-04	1.4615E-01	1.4647E-01
19	143	142	-6.1903E-04	2.9232E-01	2.9294E-01			
						1.2352E-01	1.5348E-01	2.9954E-02
21	155	154	2.4705E-01	3.0696E-01	5.9909E-02			



COMPUTED CIRCULATION = .37316E+00

Appendix A
FORTRAN LISTING OF UTRANL-I

Appendix A

A FORTRAN listing of the source deck for the UTRANL-I program is presented in pages 74 through 82. The program, as presently configured, requires 46_8K words to execute.

```
      PROGRAM MAIN(INPUT=65,OUTPUT=131,PUNCH=65,TAPE5=INPUT,
     1     TAPE6=OUTPUT)
C     GENERATE 2-D MESH CONSISTING OF TRIANGLES AND QUADRILATERALS
C     TO INVOKE ANY OF THE FOLLOWING OPTIONS THE CORRESPONDING KEY
C     IOPT(I) IS READ IN AS 1
C     IOPT(9) --MESH GENERATED IS FOR AIRFOILS OTHER THAN THE 0.06
C     CIRCULAR-ARC AIRFOIL
C     IOPT(12)--PLOT MESH
C
      DIMENSION TITLE(12),IOPT(12)
      DIMENSION NDEL(400,4),X(400),Y(400)
      DIMENSION NIDS(3),NID(3,100),VAF(100)
      EQUIVALENCE (NIDS(1),NFARF),(NIDS(2),NWAKE),(NIDS(3),NBODY)
      NPM=400
      NEM=400
      NCN=10
C     READ AND GENERATE DATA
   10 READ (5,805) (TITLE(I),I=1,12)
      IF (EOF(5)) 2000,20
   20 CONTINUE
      READ 820, (IOPT(I),I=1,12)
      CALL MESHBW(NEM,NPM,NDEL,X,Y,NELS,NPS,MAXN,NID,VAF,IOPT)
      READ 825, NIDS
      DO 120 I=1,2
      NS=NIDS(I)
  120 READ 825, (NID(I,J),J=1,NS)
      NBW=6*(MAXN+1)
      IF (IOPT(12) .EQ. 1)  CALL MPLOT(NEM,NPM,NELS,NPS,NDEL,X,Y)
C     PRINT OUTPUT DATA
      PRINT 910, (TITLE(I),I=1,12)
      PRINT 820, (IOPT(I),I=1,12)
      PRINT 920, NELS,NPS,NBW
      PRINT 930
      DO 220 N=1,NELS
  220 PRINT 825, N,(NDEL(N,J),J=1,4)
      PRINT 935
      DO 230 I=1,NPS
  230 PRINT 940, I,          X(I),Y(I)
      PRINT 951, (NID(1,I),I=1,NFARF)
      PRINT 952, (NID(2,I),I=1,NWAKE)
      PRINT 953, (NID(3,I),I=1,NBODY)
      PRINT 955, (VAF(I),I=1,NBODY)
C     PUNCH OUTPUT DATA
      PUNCH 825, NELS,NPS,NBW,(NIDS(I),I=1,3)
      PUNCH 825, ((NDEL(I,J),J=1,4),I=1,NELS)
      PUNCH 840, (X(I),Y(I),I=1,NPS)
      DO 350 I=1,3
      NS=NIDS(I)
```

```
350 PUNCH 825, (NID(I,J),J=1,NS)
    PUNCH 840, (VAF(I),I=1,NBODY)
805 FORMAT (12A6)
820 FORMAT (40I2)
825 FORMAT (16I5)
840 FORMAT (1P8E10.3)
910 FORMAT (1H1,2X,12A6//)
920 FORMAT (1H0,-NO. OF ELEMENTS=',I4,'   NO. OF NODES=',I4,
   1      '   FULL BANDWIDTH=',I4//)
930 FORMAT ('OELE. NO. AND ELEMENT NODES'/)
935 FORMAT ('0    NODE',                6X,'X(I)',12X,'Y(I)'/)
940 FORMAT (I6,    2E15.5)
951 FORMAT ('ONODES AT FARFIELD'/(20I5))
952 FORMAT ('ONODES ON LINE OF SYMMETRY'/(20I5))
953 FORMAT ('ONODES ON THE AIRFOIL'/(20I5))
955 FORMAT ('OSLOPE ALONG NODES ON AIRFOIL'/(8E15.5))
    GO TO 10
2000 STOP
    END
```

```
      SUBROUTINE MESHBW(NEM,NPM,NDEL,X,Y,NELS,NPS,MAXN,
     1 NID,VAF,IOPT)
C     GENERATE MESH AND COMPUTE THE MAX. DIFFERENCE IN NODES
      DIMENSION NDEL(NEM,4),X(NPM),Y(NPM)
      DIMENSION IDUMP(4),DUMP(4)
      DIMENSION NID(3,100),VAF(100),IOPT(12)
 1020 FORMAT(16I5)
 1150 FORMAT (3I5,5F10.0, I5)
C     GENERATE ELEMENT NODE NUMBERS AND THEIR MAXIMUM DIFFERENCE
      DO 100 N=1,NEM
      DO 100 J=1,4
  100 NDEL(N,J)=0
      NELS = 0
      MAXN=0
      KOUNT=0
  120 READ 1020, NPE,KEI,KEJ,KDI,KDJ,IDUMP
      IF (NPE .EQ. 0)  GO TO 129
      J1=-KDJ
      DO 124 J=1,KEJ
      J1 = J1 + KDJ
      J2 = J1 - KDI
      DO 124 I=1,KEI
      NELS = NELS + 1
      J2 = J2 + KDI
      DO 124 K=1,NPE
  124 NDEL(NELS,K) = IDUMP(K) + J2
  125 CONTINUE
      DO 126 I=1,NPE
      DO 126 J=1,NPE
      NDIFF=NDEL(NELS,I)-NDEL(NELS,J)
  126 IF (IABS(NDIFF) .GT. MAXN)  MAXN=IABS(NDIFF)
      GO TO 120
  129 IF (NELS .GT. NEM)  GO TO 300
C     GENERATE NODAL COORDINATES
      READ 1020, NPS
      READ 1200, XLE,XTE
      XCENT=(XLE+XTE)*.5
      FACT=1./(XTE-XLE)
 1200 FORMAT (2F10.5)
      KOUNT=0
  130 READ 1150, ND,NNI,NDI,RATIO,DUMP,ID
      IF (ND .EQ. 0)  GO TO 140
      IF (ID.EQ.0) GO TO 132
      IF (KOUNT.NE.0) GO TO 131
      IF (IOPT(9).EQ.1) CALL RAIF (ID,DUMP(1),DUMP(2),VAFS)
  131 KOUNT=KOUNT+1
      NID(3,KOUNT)=ND
      CALL GOCR (ID,DUMP(1),DUMP(2),DUMP(3),DUMP(4),
     1 XCENT,FACT,VAFS)
      IF (IOPT(9).EQ.1) CALL RAF (ID,(DUMP(1)+.5)*100.,DUMP(2),VAFS)
      VAF(KOUNT)=VAFS
```

```
132 CONTINUE
    TEMP2 = 1.0
    IF (NNI .LT. 3)  GO TO 135
    J1 = NNI - 1
    TEMP1 = 1.0
    TEMP2 = 0.0
    DO 134 I=1,J1
    TEMP2 = TEMP2 + TEMP1
134 TEMP1 = RATIO*TEMP1
135 TEMP1 = (DUMP(3)-DUMP(1))/TEMP2
    TEMP2 = (DUMP(4)-DUMP(2))/TEMP2
    J1 = ND
    TEMP3 = 0.0
    TEMP4 = 0.0
    DO 137 I=1,NNI
    X(J1) = DUMP(1) + TEMP3
    Y(J1) = DUMP(2) + TEMP4
    J1 = J1 + NDI
    TEMP3 = TEMP3 + TEMP1
    TEMP4 = TEMP4 + TEMP2
    TEMP1 = RATIO*TEMP1
137 TEMP2 = RATIO*TEMP2
    GO TO 130
140 IF (NPS .GT. NPM)  GO TO 400
    RETURN
C   INSUFFICIENT ARRAY DIMENSION ASSIGNED
300 PRINT 310, NELS,NEM
310 FORMAT ('0TOO MANY ELEMENTS',5X,'NELS=',I4,5X,'NEM=',I4)
    CALL EXIT
400 PRINT 410, NPS,NPM
410 FORMAT ('0TOO MANY NODES', 5X,'NPS=',I4,5X,'NPM=',I4)
    CALL EXIT
    END
```

```
      SUBROUTINE MPLOT(XC,YC,NODE,NN,NE,NELS,NPS)
C
C     USE CALCOMP PLOTTER TO PLOT GENERATED MESH
C     CODED BY CAPT. GERRY VANKEUREN OF AFFDL. WRIGHT-PATTERSON AFB
C
      DIMENSION XC(NN),YC(NN),NODE(NE,4),XP(7),YP(7)
C     DEFINE PLOT SIZE AND PLACE PEN IN INITIAL POSITION
      DO 5 L=1,NPS
      XC(L)=XC(L)*8.
    5 YC(L)=YC(L)*8.
      CALL PLOT(0.0,-3.0,-3)
      CALL PLOT(1.0,3.0,-3)
C     DRAW EACH ELEMENT IN TURN
      DO 10 I=1,NELS
      NPE=5
      IF (NODE(I,4).EQ.0)  NPE=4
      NPP=NPE-1
      DO 9 J=1,NPP
      XP(J)=XC(NODE(I,J))
    9 YP(J)=YC(NODE(I,J))
      XP(NPE)=XP(1)
      YP(NPE)=YP(1)
      XP(NPE+1)=YP(NPE+1)=0.0
      XP(NPE+2)=YP(NPE+2)=1.0
   10 CALL LINE(XP,YP,NPE,1,0,0)
C     WRITE NODAL NUMBERS AND FINALLY END THE PLOT
      DO 20 L=1,NPS
      PL=FLOAT(L)
   20 CALL NUMBER(XC(L),YC(L),.14,PL,0,-1)
      CALL PLOTE
      RETURN
      END
```

```
      SUBROUTINE GOCR (I,    X,YO,XF,YF,XCR,FACT,DYDX)
C     GENERATING THE BODY GEOMETRY FOR A CIRCULAR-ARC AIRFOIL
      FY(X,R,H)=SQRT(R*R-X*X)-H
      DFY(X,Y,H)=-X/(Y+H)
      DATA R,H/4.1816667,4.1516667/
      XC=XCR*FACT
      R2=XC**2
      XO=X*FACT
      YO=FY(XO,R,H)*I
      DYDX=DFY(XO,YO*I,H)*I
      IF (ABS(DYDX).GE.1.E-10) GO TO 100
      IF (DYDX.GE.0.) DYDX=1.E-10
      IF (DYDX.LT.0.) DYDX=-1.E-10
  100 CONTINUE
      YP=-1./DYDX
      YYX=YO-YP*XO
      YYX2=YYX**2
      YP1=YP**2+1.
      DISC=YYX2*YP**2-YP1*(YYX2-R2)
      SD=SQRT(DISC)
      IF (YP*I .LT.0.) SD=-SD
      XF=(SD-YYX*YP)/YP1
      YF=SQRT(R2-XF*XF)*I
      YO=YO/FACT
      XF=XF/FACT
      YF=YF/FACT
      IF (ABS(XO+.5).GT.1.E-5) GO TO 120
      YO=0.
      GO TO 130
  120 IF (ABS(XO-.5).GT.1.E-5) GO TO 130
      YO=0.
  130 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE RAIF (ID,XI,YH,DYDX)
C     GENERATE Y-COORDINATE AND SURFACE SOLPE FOR
C     DATA GIVEN IN BLOCK DATA SUBROUTINE
      COMMON/AFG/XL(30),XU(30),YYL(30),YYU(30),NN
      DIMENSION          XX(30),YY(30),XP(30),YP(30)
     1,YPD(30,2),YPL(30),YPU(30),YYD(30,2)
     2,XXD(30,2),             XPD(30,2),XPL(30),XPU(30)
      EQUIVALENCE (YPD(1,1),YPL(1)),(YPD(1,2),YPU(1))
     1,(YYD(1,1),YYL(1))
     2,(XXD(1,1),XL(1))
     3,(XPD(1,1),XPL(1)),(XPD(1,2),XPU(1))
      N2=NN+1
      CALL GMPG (NN, XL,XPL,YYL,YPL)
      CALL GMPG (NN, XU,XPU,YYU,YPU)
      RETURN
      ENTRY RAF
      DO 760 I=1,N2
      IARG=(ID+3)/2
      XX(I)=XXD(I,IARG)
      XP(I)=XPD(I,IARG)
      YY(I)=YYD(I,IARG)
  760 YP(I)=YPD(I,IARG)
      DO 800 J=1,NN
      JP=J+1
      XJ=XP(J)
      XJP=XP(JP)
      IF ((XI-XJ)*(XI-XJP).LE.0.) DYDX =YP(J)+(XI-XJ)*(YP(JP)-YP(J))
     1 /(XJP-XJ)
      IF (J.GE.NN) GO TO 800
      XXJ=XX(J)
      XXJP=XX(JP)
      IF ((XI-XXJ)*(XI-XXJP).LE.0.) YH   =YY(J)+(XI-XXJ)
     1 *(YY(JP)-YY(J))/(XXJP-XXJ)
  800 CONTINUE
      YH=YH*.01
      IF (XI.GT.1.E-3) RETURN
      YH=0.
      DYDX=0.
      RETURN
      END
```

```
      SUBROUTINE GMPG (NN,XX,XP,YY,YP)
      DIMENSION           XX(30),YY(30),XP(30),YP(30)
      N1=NN-1
      N2=NN+1
      DO 700 I=1,N1
      IP=I+1
      YP(IP)=(YY(IP)-YY(I))/(XX(IP)-XX(I))
      XP(IP)=(XX(IP)+XX(I))*.5
  700 CONTINUE
      XP(1)=XX(1)
      YP(1)=YP(2)+(XP(1)-XP(2))*(YP(3)-YP(2))/(XP(3)-XP(2))
      XP(N2)=XX(NN)
      YP(N2)=YP(NN)+(XP(N2)-XP(NN))*(YP(NN)-YP(N1))/(XP(NN)-XP(N1))
      RETURN
      END


      BLOCK DATA
C     DATA FOR NACA 64A006 AIRFOIL
C     NN  NUMBER OF POINTS DEFINING THE THICKNESS DISTRIBUTION ON
C     EACH SURFACE OF THE AIRFOIL.
C     XL   ARRAY STORING X-COORDINATE, IN PERCENTAGE MEASURED FROM
C     LEADING EDGE, OF NODES ON THE LOWER SURFACE OF THE AIRFOIL.
C     XU   ARRAY STORING X-COORDINATE, IN PERCENTAGE MEASURED FROM
C     LEADING EDGE, OF NODES ON THE UPPER SURFACE OF THE AIRFOIL.
C     YYL ARRAY STORING Y-COORDINATE, IN PERCENTAGE, OF NODES ON
C     THE LOWER SURFACE OF THE AIRFOIL.
C     YYU ARRAY STORING Y-COORDINATE, IN PERCENTAGE, OF NODES ON
C     THE UPPER SURFACE OF THE AIRFOIL.
      COMMON/AFG/XL(30),XU(30),YYL(30),YYU(30),NN
      DATA NN/26/
      DATA (XL(I),I=1,26)/0.,.5,.75,1.25,2.5,5.,7.5,10.,15.,20.,25.,
     1 30.,35.,40.,45.,50.,55.,60.,65.,70.,75.,80.,85.,90.,95.,100./
      DATA (XU(I),I=1,26)/0.,.5,.75,1.25,2.5,5.,7.5,10.,15.,20.,25.,
     1 30.,35.,40.,45.,50.,55.,60.,65.,70.,75.,80.,85.,90.,95.,100./
      DATA (YYL(I),I=1,26)/0.,-.485,-.585,-.739,-1.016,-1.399,
     1 -1.684,-1.919,-2.283,-2.557,-2.757,-2.896,-2.977,-2.999,
     2 -2.945,-2.825,-2.653,-2.438,-2.188,-1.907,-1.602,-1.285,
     3 -.967,-.649,-.331,-.013/
      DATA (YYU(I),I=1,26)/0.,.485,.585,.739,1.016,1.399,1.684,1.919,
     1 2.283,2.557,2.757,2.896,2.977,2.999,2.945,2.825,2.653,2.438,
     2 2.188,1.907,1.602,1.285,.967,.649,.331,.013/
      END
```

```
      BLOCK DATA
C     DATA FOR NACA 64A410 AIRFOIL
C     NN  NUMBER OF POINTS DEFINING THE THICKNESS DISTRIBUTION ON
C     EACH SURFACE OF THE AIRFOIL.
C     XL  ARRAY STORING X-COORDINATE, IN PERCENTAGE MEASURED FROM
C     LEADING EDGE, OF NODES ON THE LOWER SURFACE OF THE AIRFOIL.
C     XU  ARRAY STORING X-COORDINATE, IN PERCENTAGE MEASURED FROM
C     LEADING EDGE, OF NODES ON THE UPPER SURFACE OF THE AIRFOIL.
C     YYL ARRAY STORING Y-COORDINATE, IN PERCENTAGE, OF NODES ON
C     THE LOWER SURFACE OF THE AIRFOIL.
C     YYU ARRAY STORING Y-COORDINATE, IN PERCENTAGE, OF NODES ON
C     THE UPPER SURFACE OF THE AIRFOIL.
      COMMON/AFG/XL(30),XU(30),YYL(30),YYU(30),NN
      DATA NN/26/
      DATA (XL(I),I=1,26)/0.,.65,0.918,1.441,2.724,5.251,7.77,
     1 10.263,15.252,20.23,25.2,30.166,35.129,40.09,45.05,50.011,
     2 54.975,59.943,64.915,69.892,74.574,79.849,84.852,89.896,
     3 94.947,100./
      DATA (XU(I),I=1,26)/0.,.35,.582,1.059,2.276,4.749,7.23,
     1 9.737,14.748,19.77,24.8,29.834,34.871,39.91,44.95,49.989,
     2 55.025,60.057,65.065,70.108,75.126,80.151,85.148,90.104,
     3 95.053,100./
      DATA (YYL(I),I=1,26)/0.,-.678,-.796,-.969,-1.251,-1.592,-1.919,
     1 -1.996,-2.244,-2.406,-2.499,-2.537,-2.518,-2.436,-2.266,
     2 -2.024,-1.736,-1.418,-1.056,-.76,-.46,-.229,-.132,-.076,
     3 -.048,-.021/
      DATA (YYU(I),I=1,26)/0.,.902,1.112,1.451,2.095,3.034,3.865,
     1 4.38,5.366,6.126,6.705,7.131,7.414,7.552,7.522,7.344,7.04,
     2 6.624,6.105,5.49,4.78,3.967,3.018,2.038,1.028,.021/
      END
```

Appendix B
FORTRAN LISTING OF UTRANL-II

Appendix B

A FORTRAN listing of the source deck for the UTRANL-II program is presented in pages 85 through 102. The program, as presently configured, requires 177_8K words to execute.

```
      PROGRAM MAIN (INPUT=65,OUTPUT=131,PUNCH=65,TAPE5=INPUT,
     1   TAPE6=OUTPUT)
C     UNSTEADY TRANSONIC FLOW ANALYSIS BY FINITE ELEMENT METHOD
C     USING LEAST SQUARES WITH TRIANGULAR AND QUADRILATERAL ELEMENTS
C     IOPT(1)=1, USE RESULTS OF PREVIOUS CASE AS STARTING SOLUTION
C     WHILE THE OTHER OPTION IS IGNORED
C     IOPT(2)=1, READ IN NON-ZERO INITIAL GUESS
C     IOPT(3)=1, LINEARIZED BOUNDARY COND. IMPOSED ON CHORDLINE
C     IOPT(4)=1, UPDATING THE CIRCULATION
C
      COMMON S
      COMMON /COM1/AM(3),AMT(3),AMTT(3),T1,T2,T3,DT
      DIMENSION NOD(173,4),S(606,84),SLP(606),SL1(606),X(202),
     1 Y(202),RML(202),COF(202),NID(3,50),VAF(50)
      DIMENSION TITLE(12),IOPT(12),NIDS(3),AR(80)
      LOGICAL LR(40)
      EQUIVALENCE (NIDS(1),NFARF),(NIDS(2),NWAKE),(NIDS(3),NBODY)
      DATA GAMMA/1.40/
      NPM=202
      NEM=173
      NRM=NPM*3
      NCM=84
      NBCT=50
      EXP=-GAMMA/(GAMMA-1.0)
      CONST=0.5*(GAMMA-1.0)
C
C     READ TITLE, CONTROL KEYS, AND PROBLEM PARAMETERS
C
  100 READ (5,845) (TITLE(I),I=1,12)
      IF (EOF(5)) 2000,105
  105 CONTINUE
      READ(5,820) (IOPT(I),I=1,12)
      READ 835, RMAC,RKSC,ANDG,XHP,XTE,DTM,TFIN,TO,ZTEST,ALPHA,CIRCO
C
C     PRINT TITLE, CONTROL KEYS, AND PROBLEM PARAMETERS
C
      PRINT 910,  (TITLE(I),I=1,12)
      PRINT 820,  (IOPT(I),I=1,12)
      PRINT 822, RKSC,ANDG,XHP,XTE,RMAC,TFIN,TO,ALPHA,CIRCO,ZTEST
      IRES=-1
      IPRD=DTM
      SQMAC=RMAC**2
      RKSC=RKSC*2.
      ANDG=ANDG*3.1415926/180.
      ALPHA=ALPHA*3.1415926/180.
      IF (RKSC.GT.0.) DTM=6.2831853/(DTM*RKSC)
      NIFT=TFIN
      T1=(TO-3.)*DTM
      T2=T1+DTM
      T3=T2+DTM
      IF (IOPT(1) .EQ. 1)  GO TO 382
C
C     READ AND GENERATE MESH INFORMATION
C
      READ 825, NELS,NPS,NBW,(NIDS(I),I=1,3)
      READ 825, ((NOD(I,J),J=1,4),I=1,NELS)
      READ 840, (X(I),Y(I),I=1,NPS)
      DO 110 I=1,3
```

```
      NS=NIDS(I)
  110 READ 825, (NID(I,J),J=1,NS)
      READ 840, (VAF(I),I=1,NBODY)
C
C     IF IOPT(3)=1, MOVE NODES ON AIRFOIL SURFACE TO CHORDLINE POSITION
C     AND APPLY LINEARIZED BOUNDARY CONDITION, OTHERWISE IMPOSE
C     NONLINEAR BOUNDARY CONDITION ON THE AIRFOIL SURFACE
C
      IF (IOPT(3) .NE. 1)  GO TO 116
      NEVEN=NBODY/2*2
      DO 115 J=1,NEVEN,2
      I=NID(3,J)
      Y(I)=-0.015
      I=NID(3,J+1)
  115 Y(I)= 0.015
  116 CONTINUE
C
C     PRINT GENERATED DATA
C
      NHBW=NBW/2
      NEQ=3*NPS
  200 PRINT 920, NELS,NPS,NBW
      PRINT 930
      DO 220 N=1,NELS
  220 PRINT 825, N,(NOD(N,J),J=1,4)
      PRINT 935
      DO 230 I=1,NPS
  230 PRINT 940, I,          X(I),Y(I)
      PRINT 951, (NID(1,I),I=1,NFARF)
      PRINT 952, (NID(2,I),I=1,NWAKE)
      PRINT 953, (NID(3,I),I=1,NBODY)
      PRINT 955, (VAF(I),I=1,NBODY)
      DO 244 II=1,NPS
      RML(II)=RMAC
  244 CONTINUE
C
C     READ NONZERO INITIAL GUESS OR PROCEED WITH ZERO SOLUTION
C
      IF (IOPT(2) .NE. 1)  GO TO 250
      READ 840, (SLP(I),I=1,NEQ)
      READ 840, (S(I,NBW),I=1,NEQ)
      GO TO 295
  250 DO 260 I=1,NEQ
      SL1(I)=0.
  260 SLP(I)=0.0
C
C     INTEGRATION  START HERE AND CHECK IF NIFT IS EXCEEDED.  IF SO,
C     PRINT FAIL TO CONVERGE AND PROCEED TO NEXT CASE. OTHERWISE,
C     CONTINUE TO INTEGRATE
C
  265 T1=T2
      T2=T3
      T3=T3+DTM
      IRES=IRES+1
C     FORMULATE SYSTEM OF ALGEBRAIC EQUATIONS
      DO 266 I=1,NEQ
      DO 266 J=1,NBW
  266 S(I,J)=0.0
```

```
      CALL TDRV (0.,SGMAC)
      CALL NEWK(SGMAC,NRM,NCM,NEG,NBW,NEM,NELS,NOD,SLP,S,COF,NPM,X,Y,
     1 SLI)
C     IMPOSE B.C. FOR FARFIELD, LINE OF SYMMETRY, AND ON AIRFOIL
      CALL BDYCON (NIDS,NID,NBCT,VAF,X,Y,NPM,S,SLP,NRM,NCM,
     1 NEG,NBW,ALPHA,RMAC,CIRCO,KEY,XHP,ANDG,RKSC,SLI,IOPT)
C     CALL BANDED SOLVER TO SOLVE THE SYSTEM OF EQUATIONS
      CALL BNDEQ(S,NRM,NCM,NEG,NHBW)
C
C     PRINT COMPUTED RESULTS
C
  295 PRINT 970, RMAC,DTM,T3
      CALL TDRV(1.,SGMAC)
      CALL         OUTP  (S,SLP,SLI,RML,COF,SGMAC,CONST,EXP,
     1 ANDG,IRES,NPS,NBW,NRM,PCTE,NID,NBODY,NBCT,IPRD)
      PRINT 985
      IPNT=0
      ISTOP=0
      NEVEN=NBODY/2*2
      DO 320 J=1,NEVEN,2
      IF (J.GE.NEVEN-1) ISTOP=1
      I=NID(3,J)
      IF (IOPT(4).EQ.1) GO TO 315
      CP=S(J,3)
      CALL FPLOT (80,IPNT,AR,LR,ISTOP,1,I,X(I),CP)
      GO TO 320
  315 CONTINUE
      CP =S(I,1)
      CALL FPLOT (80,IPNT,AR,LR,ISTOP,1,2,X(I),CP)
      I =NID(3,J+1)
      CP =S(I,1)
      CALL FPLOT (80,IPNT,AR,LR,ISTOP,2,2,X(I),CP)
  320 CONTINUE
C
C     CHECK CONVERGENCE, IF SO, PUNCH THE FINAL SOLUTION AND
C     PROCEED TO NEXT CASE, OTHERWISE, UPDATE SOLUTION AND CONTINUE
C     TO INTEGRATE
C
      IF (IRES.LT.1.OR.PCTE.GE.ZTEST) GO TO 382
      IF (IRES.GE.NIFT) GO TO 600
      GO TO 700
  600 PRINT 980
  700 PUNCH 840, (SLP(I),I=1,NEQ)
      PUNCH 840, (S(I,NBW),I=1,NEQ)
      IF (IOPT(4).EQ.1) PRINT 325, CIRCO
      GO TO 100
  382 CONTINUE
      DO 390 I=1,NEQ
      SLI(I)=SLP(I)
  390 SLP(I)=S(I,NBW)
C
C     IF IOPT(4)=1, UPDATE THE CIRCULATION
C
      IF (IOPT(4).EQ.0) GO TO 265
      IL=NID(2,1)*3-2
      IU=NID(2,2)*3-2
      CIRCO=SLP(IU)-SLP(IL)
      PRINT 325, CIRCO
```

```
      GO TO 265
  325 FORMAT (1H0,-COMPUTED CIRCULATION =',E14.5/)
  820 FORMAT (40I2)
  822 FORMAT (1H0,- RKSC =',F9.5,4X,'ANDG =',F9.5,5X,'XHP =',F9.5,
     1 3X,'XTE =',F9.5,3X,'RMAC =',F9.5,3X,'TFIN =',F9.5,3X,
     2 'TO =',F9.5/1H ,'ALPHA =',F9.5,3X,'CIRCO =',F9.5,3X,
     3 'ZTEST =',F9.5/)
  825 FORMAT (16I5)
  835 FORMAT (8F10.0)
  840 FORMAT (1P8E10.3)
  845 FORMAT (1H ,13A6)
  910 FORMAT (1H1,12A6/)
  920 FORMAT (1H0,-NO. OF ELEMENTS=',I4,'  NO. OF NODES=',I4,
     1          '    FULL BANDWIDTH=',I4//)
  930 FORMAT ('0ELE. NO. AND ELEMENT NODES'/)
  935 FORMAT ('0OLD NODE',                 10X,'X(I)',12X,'Y(I)'/)
  940 FORMAT (    I10,2E15.5)
  951 FORMAT ('0NODES AT FARFIELD'/(20I5))
  952 FORMAT ('0NODES ON LINE OF SYMMETRY'/(20I5))
  953 FORMAT ('0NODES ON THE AIRFOIL'/(20I5))
  955 FORMAT ('0SLOPE ALONG NODES ON AIRFOIL'/(8E15.5))
  970 FORMAT (1H1, -MACH NUMBER=',F6.3,
     1     5X,'TIME INCREMENT =',F6.4,5X,'TIME LEVEL =',F6.2/)
  980 FORMAT ('0FAIL TO CONVERGE IN SPECIFIED NO. OF ITERATIONS')
  985 FORMAT (1H1)
 2000 STOP
      END
```

```
      SUBROUTINE OUTP (S,SLP,SL1,RML,COF,SQMAC,CONST,EXP,
     1 ANDG,IRES,NPS,NBW,NRM,PCTE,NID,NBODY,NBCT,IPRD)
C
C     PRINT OUT RESULTS FOR THE CURRENT TIME STEP
C
      COMMON /COM1/AM(3),AMT(3),AMTT(3),T1,T2,T3,DT
      DIMENSION S(NRM,NBW),SLP(NRM),SL1(NRM),RML(NPS),COF(NPS)
     1,           NID(3,NBCT)
      PRINT 975, IRES
      IND=0
      PCTE=0.
      DO 305 J=1,NPS
      I1=3*J
      I2=I1-2
      POT=S(I2,NBW)
      UPT=S(I1-1,NBW)
      V=S(I1,NBW)
      U=UPT+1.0
      QSQ=U*U+V*V
      PHT=POT*AMT(3)+SLP(I2)*AMT(2)+SL1(I2)*AMT(1)
      ASQ=CONST*(1.0-QSQ-2.*PHT)+1./SQMAC
      RM=SQRT(QSQ/ASQ)
      PRA=(1.+CONST*RM**2)**EXP
      CPS=-2.*UPT
      CPU=-2.*PHT
      CP=CPS+CPU
      CPU=CP
      IF (ABS(ANDG).GT.1.E-5) CPU    =CP/ANDG
      S(J,1)=CPU
      COF(J)=SQMAC*(1.0+2.4*UPT)
      DELM=RM-RML(J)
      PADM=ABS(DELM)*100.
      RML(J)=RM
      IF (PADM.LE.PCTE) GO TO 305
      IND=J
      PCTE=PADM
```

```
305 PRINT 978, J,POT,U,V,COF(J),RML(J),PRA,CPU,    CP,DELM
    PRINT 810,IND,PCTE
    INPRD= IRES    /IPRD*IPRD-IRES
    PRINT 985
    NEVEN=NBODY/2*2
    DO 500 I=1,NEVEN,2
    I1=I+1
    IL=NID(3,I)
    IU=NID(3,I1)
    DCP=S(IL,1)-S(IU,1)
    S(I,3)=DCP
    IF (INPRD.NE. 0) GO TO 400
    S(I,2) =(S(I,2) +S(IL,1)*.5)/IPRD
    S(I1,2)=(S(I1,2)+S(IU,1)*.5)/IPRD
    DSUM=S(I,2)-S(I1,2)
    PRINT 988, S(I1,2),S(I,2),DSUM
    S(I,2) =S(IL,1)*.5
    S(I1,2)=S(IU,1)*.5
    GO TO 500
400 S(I,2) =S(I,2) +S(IL,1)
    S(I1,2)=S(I1,2)+S(IU,1)
500 PRINT 990, I,IU,IL,S(IU,1),S(IL,1),DCP

810 FORMAT (1H0,-NODES =',I3,5X,'MAX(DELM) PCT =',1PE10.3/)
975 FORMAT (1H0,4X,-TIME STEP =',I4/
   1 1H0,3X,'NODE-,7X,'PHIT',10X,'UCOM',10X,'VCOM',11X,'COF',10X,
   2 'LMAC',11X,-PRA-,11X,'CPU',12X,'CP',10X,'DELM'/)
978 FORMAT (I6,1P9E14.4)
985 FORMAT (1H0,8X,-I',6X,'IU',8X,'IL',7X,'CP-UPPER',7X,
   1 'CP-LOWER',7X,-DCP',8X,'AVE-CP+',8X,'AVE-CP-',5X,'STEADY-DCP'/)
988 FORMAT  (75X,1P3E15.4)
990 FORMAT (3I10,1P3E15.4)
    RETURN
    END
```

```
      SUBROUTINE BDYCON(NIDS,NID,NBCT,VAF,X,Y,NPM,S,SLP,NRM,NCM,
     1    NEQ,NBW,ALPHA,RMAC,CIRCO,KEY,XHP,ANDG,RKSC,SL1,IOPT)
C
C     IMPOSITIONS OF THE BOUNDARY CONDITIONS
C
      COMMON /COM1/AM(3),AMT(3),AMTT(3),T1,T2,T3,DT
      DIMENSION NIDS(3),NID(3,NBCT),VAF(NBCT),X(NPM),Y(NPM),
     1 S(NRM,NCM),SLP(NRM),SL1(NRM),IOPT(12)
      DATA PI/3.1415926/
      NBW1=NBW-1
      NHBW=NBW/2
      NFARF=NIDS(1)
      NPAIR=NIDS(2)/2
      NBODY=NIDS(3)
      CNST=0.5*CIRCO/PI
      ROOT=SQRT(1.0-RMAC*RMAC)
      DO 120 J=1,NBODY
      IE3=NID(3,J)
      IE=3*IE3
      QUAN=VAF(J)-ALPHA
      XREDU=X(IE3)-XHP
      DQUAN=0.
      IF (XREDU.LT.0.) GO TO 90
      ARGU=RKSC*T3+1.5707963
      DQUAN =   ANDG*(COS(ARGU)-XREDU*          RKSC*SIN(ARGU))
      IF (XREDU.LE.1.E-5) DQUAN=DQUAN*.5
   90 CONTINUE
      IF (IOPT(3).EQ.1) GO TO 96
      DO 95 K=2,NBW1
   95 S(IE-1,K)=S(IE-1,K)+QUAN*S(IE,K-1)
      S(IE-1,NBW)=S(IE-1,NBW)+QUAN*S(IE,NBW)
   96 CONTINUE
      DO 100 K=1,NBW
  100 S(IE,K)=0.0
      S(IE,NHBW)=1.0
      S(IE,NBW)=QUAN+DQUAN
      IF (IOPT(3).EQ.1) GO TO 120
      S(IE,NHBW-1)=-QUAN
  120 CONTINUE
      CALL TDRV (1.)
      DO 220 N=1,NPAIR
      IL=3*NID(2,2*N-1)-3
      IU=3*NID(2,2*N)-3
      NC=IL-IU+NHBW
      DO 210 M=2,3
      IE=IU+M
      IEL=IL+M
      ID=IU-IL
      IDP=ID+1
      DO 190 K=IDP,NBW1
  190 S(IEL,K)=S(IEL,K)+S(IE,K-ID)
      S(IEL,NBW)=S(IEL,NBW)+S(IE,NBW)
      DO 200 K=1,NBW
  200 S(IE,K)=0.0
      S(IE,NHBW)=1.0
      S(IE,NC)=-1.0
      IF (M.NE.2) GO TO 210
      S(IE,NHBW-1)=AMT(3)
```

```
      S(IE,NC-1)= -AMT(3)
      S(IE,NBW)=AMT(2)*(SLP(IEL-1)-SLP(IE-1))+AMT(1)*(SLI(IEL-1)
     1 -SLI(IE-1))
  210 CONTINUE
  220 CONTINUE
      DO 340 N=1,NFARF
      I=NID(1,N)
      J=3*I-3
      YMOD=ROOT*Y(I)
      RSQ=YMOD**2+X(I)**2
      YY=-YMOD
      THETA=ATAN2(YY,X(I))
      IF (THETA .LT. 0.0)  THETA=THETA+2.*PI
      DO 310 K=1,3
      IE=J+K
      DO 300 L=1,NBW
  300 S(IE,L)=0.0
  310 S(IE,NHBW)=1.0
      S(J+1,NBW)= CNST*THETA
      S(J+2,NBW)=CNST*YMOD/RSQ
      S(J+3,NBW)=-CNST*ROOT*X(I)/RSQ
  340 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE FPLOT(M1,IPNT,AR,LR,ISTOP,NC,NCMAX,V1,V2)
C
C     RESULTS PLOTTING
C
      LOGICAL   LM(2),LN(120),LP(4),LX(4),LR(1),LC(20)
      INTEGER RI(2),ST(2),SG(2)
      DIMENSION AR(M1),N(120),RHO(30),Z(15),IOFF(2),ISP(2),SF1(2),SF2(2)
     1,ID(2),ISG(2),II(4),IP(4),IM(4),IX(4),IR(202),IA(238)
      EQUIVALENCE (IP(1),LP(1)),(IM(1),LM(1)),(LN(1),N(1),RHO(1)),
     1(IX(1),LX(1)),(N120,RI(1)),(N58,RI(2))
      DATA (Z(I),I=1,15)/1.,1.25,1.5,1.75,2.,2.5,3.,3.5,4.,4.5,5.,6.,
     17.,8.,9./
      DATA (ISP(I),I=1,2),(IX(I),I=1,4),(IP(I),I=1,4),(IM(I),I=1,4)/
     110,5,4*1HX,4*1H+,4*0/
      DATA ID(1),ID(2),ISG(1),ISG(2)/120,58,1,-1/
      DATA LN(1),II,LC(1),LC(2)/1,80,50,0,1,1HL,1HO/
      IF(ISTOP)173,172,172
  172 J=IPNT+2
      IF(J.GT.M1)GO TO 173
      IPNT=J
      AR(J-1)=V1
      AR(J)=V2
      DO 10 K=1,4
   10 IM(K) = NC
      J=J/2
      LR(J)=LM(2)
  173 IF(NC.LT.NCMAX)RETURN
      IF(ISTOP.EQ.0)RETURN
C 103 READ(5,1)LN(1),(II(I),I=1,4),(LC(I),I=1,20),(LN(I),I=2,52)
    1 FORMAT(A1,2I3,2I1,71A1)
      WRITE(6,1)LN(1),(II(I),I=1,4),(LC(I),I=1,20),(LN(I),I=2,52)
      DO 171 I=1,2
      RI(I)=ID(I)
      SG(I)=ISG(I)
      IF(II(I).NE.0)RI(I)=II(I)
      IF(II(I+2).EQ.1)SG(I)=-SG(I)
  171 ST(I)=(RI(I)+1-SG(I)*(RI(I)-1))/2
      IF(RI(1).GT.120)RI(1)=120
      IF(RI(2).GT.238)RI(2)=238
      N20 = N120/6
      N120 = 6* N20
      DO 3 I=1,2
      AMIN=AR(I)
      AMAX=AR(I)
      DO 7 J=I,IPNT,2
      AA1=AR(J)
      IF(AMIN.GT.AA1)AMIN=AA1
    7 IF(AMAX.LT.AA1)AMAX=AA1
      IF (ABS(AMIN) .LT. .000001)  AMIN = 0.0
      IF (ABS(AMAX) .LT. .000001)  AMAX = 0.0
      R=AMAX-AMIN
      IF (ABS(R).LT.1.E-9.AND.ABS(AMAX).LT.1.E-9) R=1.0
      IF (ABS(R).LT.1.E-9.AND.AMAX.LT.0.)         R=-AMAX
      IF (ABS(R).LT.1.E-9.AND.AMAX.GT.0.)         R= AMAX
      DO 22 J=1,15
      B=ALOG10(R/(RI(I)-2)/Z(J))
      M=B
      IF(B.LT.0)M=M-1
```

```
      C=Z(J)*10.**(M+1)
      B=AMIN/Z(J)/10.**(M+1)
      I1=B
      IF(B.LT.0)I1=I1-1
      IF((RI(I)-2+I1)*C-AMIN)18,19,19
   18 C=10.*C
   19 IF(J.EQ.1)SMIN=C
   22 IF(C.LT.SMIN)SMIN=C
      SF1(I)=(1./SMIN)*SG(I)
      B=AMIN/SMIN
      M=B
      IF(B.LT.0)M=M-1
      SF2(I)=ST(I)-M*SG(I)
      RHO(I)=SF2(I)+0.5
      M=SF2(I)
      DO 25 J=1,10
      IF(M-J-((M-J)/ISP(I))*ISP(I))25,3,25
   25 CONTINUE
    3 IOFF(I)=J
      DO 101 I=1,N58
  101 IA(I)=0
       DO 102 J=1,IPNT,2
      IR(J)=SF1(1)*AR(J)+RHO(1)
      IT=SF1(2)*AR(J+1)+RHO(2)
      IF(J.NE.1)GO TO 109
      IR(IPNT+2)=2
      IR(J+1)=0
  108 I3=IT
      GO TO 102
  109 IF(IT-I3)104,105,105
  104 IR(J+1)=IR(IPNT+2)
      IR(IPNT+2)=J+1
      GO TO 108
  105 I=IT+1
  106 I=I-1
      I1=IA(I)
      IF(I1)106,106,107
  107 IR(J+1)=IR(I1)
      IR(I1)=J+1
  102 IA(IT)=J+1
      LAST=IPNT+2
      JJ=IOFF(2)
      LZH=SF2(1)
      LZV=SF2(2)
      DO 100 I=1,N58
      DO 40 J=1,N120
   40 N(J)= 1H
      IF((I-1)*(I-N58))140,151,140
  151 DO 141 J=1,N120
  141 N(J) = IX(1)
  140 LN(1)=LX(1)
      LN(N120)=LX(1)
      IF(LZH.LE.0.OR.LZH.GT.N120)GO TO 131
      LN(LZH)=LP(4)
  131 NB=1
      IF(I.NE.JJ)GO TO 35
      NB=2
      JJ=JJ+5
```

```
      I3=IOFF(1)
      DO 32 J=I3,N120,10
   32 LN(J)=LP(4)
      IF(I.NE.LZV)GO TO 35
      DO 135 J=1,N120
  135 N(J) = IP(1)
   35 I3=IA(I)
      IF(I3.EQ.0)GO TO 121
  120 LAST=IR(LAST)
      I2=IR(LAST-1)
      I1=LAST/2
      LM(2)=LR(I1)
      IMM = IM(2)
      LN(I2)=LC(IMM)
      IF(LAST.NE.I3)GO TO 120
  121 GO TO (38,41),NB
   38 WRITE(6,39)(N(J),J=1,N120)
   39 FORMAT(11H           ,120A1)
      GO TO 100
   41 AA1=I
      VALUE=(AA1-SF2(2))/SF1(2)
      WRITE(6,42)VALUE,(N(J),J=1,N120)
   42 FORMAT(1H ,1PE10.3,120A1)
  100 CONTINUE
      I3=IOFF(1)
      J=0
      DO 49 I=I3,N120,10
      J=J+1
      AA=I
   49 RHO(J)=(AA-SF2(1))/SF1(1)
      IF(IOFF(1)-5)62,62,63
   62 WRITE(6,50)(RHO(I),I=1,J)
   50 FORMAT(10X,12(1PE10.3))
      RETURN
   63 IF(J.GE.12)J=11
      WRITE(6,64)(RHO(I),I=1,J)
   64 FORMAT(16X,11(1PE10.3))
      RETURN
      END
```

```
      SUBROUTINE NEWK(SQMAC,NRM,NCM,NEQ,NBW,NEM,NELS,NOD,SLP,S,
     1     COEF,NPM,X,Y,SL1)
C     GENERATE SYSTEM MATRIX
      DIMENSION COEF(NPM),X(NPM),Y(NPM),XQ(4),YQ(4)
      COMMON/COM5/AQ(12,12),BQ(12,12),AS(9,9),BS(9,9)
      COMMON/COM9/CSQA(12),CSQB(12),CSLA(9),CSLB(9)
      DIMENSION NOD(NEM,4),S(NRM,NCM),SLP(NRM),SL1(NRM),PMD(12,2)
      NHBW=NBW/2
      DO 480 N=1,NELS
      I1=1
      IF (NOD(N,4)) 402,402,404
  402 NPEL=3
      NTRS=1
      GO TO 410
  404 NPEL=4
      NTRS=4
      I2=0
      DO 408 I=1,4
      NI=NOD(N,I)
  408 IF (COEF(NI) .GT. 1.00)    I2=I2+1
      IF (I2 .EQ. 0)  NTRS=2
      IF (N.LE.5) NTRS=4
  410 DO 425 I=1,NPEL
      NI=NOD(N,I)
      XQ(I)=X(NI)
      YQ(I)=Y(NI)
      DO 425 J=1,3
      IS=3*(NI-1)+J
      IE=3*(I-1)+J
      PMD(IE,1)=SL1(IS)
  425 PMD(IE,2)=SLP(IS)
      CALL         EMQT(XQ,YQ,PMD, SQMAC,NTRS)
      DO 450 I=I1,NPEL
      NR=3*(NOD(N,I)-1)
      IE=3*(I-1)
      DO 450 II=1,3
      NR=NR+1
      IE=IE+1
      S(NR,NBW)=S(NR,NBW)+CSQA(IE)+CSQB(IE)
      DO 450 J=1,NPEL
      NC=3*(NOD(N,J)-1)-NR+NHBW
      JE=3*(J-1)
      DO 450 JJ=1,3
      NC=NC+1
      JE=JE+1
  450 S(NR,NC)=S(NR,NC)+AQ(IE,JE)+BQ(IE,JE)
  480 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE EMQT(XQ,YQ,PMQD,SQMAC,NTRS)
C     GENERATE MATRIX FOR A QUADRILATERAL OR TRIANGLE
      COMMON/COM5/AQ(12,12),BQ(12,12),AS(9,9),BS(9,9)
      COMMON/COM9/CSQA(12),CSQB(12),CSLA(9),CSLB(9)
      DIMENSION XQ(4),YQ(4),XT(3),YT(3),MP(3,4)
      DIMENSION PMQD(12,2),PMTD(9,2)
      DATA MP/1,2,3,3,4,1,2,3,4,4,1,2/
      FTOR=1.0
      IF (NTRS .EQ. 4)  FTOR=0.5
      DO 100 I=1,12
      CSQA(I)=0.
      CSQB(I)=0.
      DO 100 J=1,12
      AQ(I,J)=0.
      BQ(I,J)=0.
  100 CONTINUE
      NTRA=1
      IF (NTRS.EQ.1.OR.NTRS.EQ.4) GO TO 210
      NTRA=3
      NTRS=4
      DO 200 I=1,4
      IF (YQ(I).LT. 1.E-5) GO TO 200
      NTRA=1
      NTRS=2
      GO TO 210
  200 CONTINUE
  210 CONTINUE
      DO 150 II=NTRA,NTRS
      DO 105 I=1,3
      NI=MP(I,II)
      IT=3*(I-1)
      IQ=3*(NI-1)
      DO 102 J=1,3
      IT=IT+1
      IQ=IQ+1
      DO 102 K=1,2
  102 PMTD(IT,K)=PMQD(IQ,K)
      XT(I)=XQ(NI)
  105 YT(I)=YQ(NI)
      CALL EMTC   (XT,YT,PMTD,SQMAC)
      DO 130 K=1,3
      NR=3*(MP(K,II)-1)
      IE=3*(K-1)
      DO 130 KK=1,3
      NR=NR+1
      IE=IE+1
      CSQA(NR)=CSQA(NR)+CSLA(IE)*FTOR
      CSQB(NR)=CSQB(NR)+CSLB(IE)*FTOR
      DO 130 L=1,3
      NC=3*(MP(L,II)-1)
      JE=3*(L-1)
      DO 130 LL=1,3
      NC=NC+1
      JE=JE+1
      AQ(NR,NC)=AQ(NR,NC)+AS(IE,JE)*FTOR
      BQ(NR,NC)=BQ(NR,NC)+BS(IE,JE)*FTOR
  130 CONTINUE
  150 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE EMTC   (XL,YL,PELD,SQMAC)
C     EVALUATE ELEMENT MATRIX FOR A TRIANGLE BY GAUSSIAN QUADRATURE
      COMMON /COM1/AM(3),AMT(3),AMTT(3),T1,T2,T3,DT
      COMMON/COM5/AQ(12,12),BQ(12,12),AS(9,9),BS(9,9)
      COMMON/COM9/CSQA(12),CSQB(12),CSLA(9),CSLB(9)
      DIMENSION DNX(9),P(9),Q(9),NP(5),B(3),C(3),XL(3),YL(3),S(3),
     1          GPA(9),DNXX(9),DNYY(9),SN(9),PELD(9,2)
      DIMENSION EINT(3,7),WT(7)
      DIMENSION S2(2),W2(2),PD(2),UD(2)
      DATA MMAX/2/,S2/-.57735027,.57735027/,W2/2*.5/
      DATA LMAX/7/,WT/0.225,3*0.13239415,3*0.12593918/
      DATA EINT/3*0.33333333,0.05961587,3*0.47014206,0.05961587,
     1    3*0.47014206,0.05961587,0.79742699,3*0.10128651,
     2    0.79742699,3*0.10128651,0.79742699/
      DATA NP/1,2,3,1,2/,GAMMA/1.40/
      DO 2 I=1,9
      CSLA(I)=0.
      CSLB(I)=0.
      DO 2 J=1,9
      AS(I,J)=0.
      BS(I,J)=0.
    2 CONTINUE
      DO 4 I=1,3
      J=NP(I+1)
      K=NP(I+2)
      B(I)=YL(J)-YL(K)
    4 C(I)=XL(K)-XL(J)
      AREA=0.5*(B(2)*C(3)-B(3)*C(2))
      VOLUME=AREA*DT
      CST1=1.0-SQMAC
      CST2=SQMAC*(1.0+GAMMA)
      DO 100 L=1,LMAX
      DO 10 I=1,3
   10 S(I)=EINT(I,L)
      CALL DERV(AREA,B,C,S,DNX,DNXX,DNYY,SN)
      U=0.
      UX=0.
      DO 30 I=1,9
      PELDI2=PELD(I,2)
      U=U+DNX(I)*PELDI2
   30 UX=UX+DNXX(I)*PELDI2
      ALPHA=CST1-CST2*U
      DO 40 I=1,9
      P(I)=ALPHA*DNXX(I)+DNYY(I)
   40 Q(I)=P(I)-CST2*UX*DNX(I)
      DO 45 I=1,2
      PD(I)=0.
      UD(I)=0.
      DO 45 J=1,9
      PELDJI=PELD(J,I)
      PD(I)=PD(I)+PELDJI*SN(J)
      UD(I)=UD(I)+PELDJI*DNX(J)
```

```
45 CONTINUE
   DO 100 M=1,MMAX
   CALL TDRV (S2(M),SQMAC)
   WEIG = WT(L)*W2(M)*VOLUME
   DO 48 J=1,9
48 GPA(J)=P(J)-(AMT(3)*DNX(J)+AMTT(3)*SN(J))

   SUM=0.
   DO 50 J=1,2
50 SUM=SUM +                    (AMT(J)*UD(J)+AMTT(J)*PD(J))
   DO 60 I=1,9
   CSA=-WEIG*                    (AMT(3)*DNX(I)+AMTT(3)*SN(I))
   CSB=WEIG * Q(I)
   CSLA(I)=CSLA(I)+CSA*SUM
   CSLB(I)=CSLB(I)+CSB*SUM
   DO 60 J=1,9
   AS(I,J)=AS(I,J)+CSA*GPA(J)
   BS(I,J)=BS(I,J)+CSB*GPA(J)
60 CONTINUE
100 CONTINUE
   RETURN
   END


   SUBROUTINE TDRU (QPN,SQMAC)
   COMMON /COM1/AM(3),AMT(3),AMTT(3),T1,T2,T3,DT
   RMA=SQMAC*2.
   RMB=RMA
   DT1=T2-T1
   DT2=T3-T2
   DT=DT1+DT2
   DN1=1./(DT*DT1)
   DN2=-1./(DT1*DT2)
   DN3=1./(DT*DT2)
   RETURN
   ENTRY TDRV
   T=(T1+T3+DT*QPN)*.5
   AM(1)=(T-T2)*(T-T3)*DN1
   AM(2)=(T-T1)*(T-T3)*DN2
   AM(3)=(T-T1)*(T-T2)*DN3
   AMT(1)=(T*2.-T2-T3)*DN1*RMA
   AMT(2)=(T*2.-T3-T1)*DN2*RMA
   AMT(3)=(T*2.-T1-T2)*DN3*RMA
   AMTT(1)=DN1*RMB
   AMTT(2)=DN2*RMB
   AMTT(3)=DN3*RMB
   RETURN
   END
```

```
      SUBROUTINE DERV(AREA,B,C,S,DNX,DNXX,DNYY,SN)
C     EVALUATE THE DERIVATIVES OF SHAPE FUNCTIONS AT A GAUSSIAN POINT
      DIMENSION B(3),C(3),S(3),DNX(9),DNXX(9),DNYY(9),NP(5),SN(9)
      DATA NP/1,2,3,1,2/
      TWOA=2.*AREA
      TWOASQ=TWOA**2
      H=S(1)*S(2)*S(3)
      DO 200 I=1,3
      J=NP(I+1)
      K=NP(I+2)
      SI=S(I)
      SJ=S(J)
      SK=S(K)
      BI=B(I)
      BJ=B(J)
      BK=B(K)
      CI=C(I)
      CJ=C(J)
      CK=C(K)
      SISQ=SI*SI
      BISQ=BI*BI
      CISQ=CI*CI
      ALFA=0.5*(CK-CJ)
      BETA=0.5*(BJ-BK)
      HX=BI*SJ*SK+BJ*SK*SI+BK*SI*SJ
      HXX=2.*(SI*BJ*BK+SJ*BK*BI+SK*BI*BJ)
      HYY=2.*(SI*CJ*CK+SJ*CK*CI+SK*CI*CJ)
      CSS=6.*SI*(1.-SI)
      CS=6.*(1.-2.*SI)
      L=3*I-2
      SN(L)=SISQ*(3.-2.*SI)+2.*H
      DNX(L)=BI*CSS+2.*HX
      DNXX(L)=BISQ*CS+2.*HXX
      DNYY(L)=CISQ*CS+2.*HYY
      CS=CK*SJ-CJ*SK
      L=L+1
      SN(L)=SISQ*CS+ALFA*H
      DNX(L)=2.*BI*SI*CS+TWOA*SISQ+ALFA*HX
      DNXX(L)=2.*BISQ*CS+4.*BI*TWOA*SI+ALFA*HXX
      DNYY(L)=2.*CISQ*CS+ALFA*HYY
      BS=BJ*SK-BK*SJ
      L=L+1
      SN(L)=SISQ*BS+BETA*H
      DNX(L)=2.*BI*SI*BS+BETA*HX
      DNXX(L)=2.*BISQ*BS+BETA*HXX
  200 DNYY(L)=2.*CISQ*BS+4.*CI*TWOA*SI+BETA*HYY
      DO 300 I=1,9
      DNX(I)=DNX(I)/TWOA
      DNXX(I)=DNXX(I)/TWOASQ
  300 DNYY(I)=DNYY(I)/TWOASQ
      RETURN
      END
```

```
      SUBROUTINE BNDEQ(A,NRMAX,NCMAX,N,ITERM)
C     EQUATION SOLVER FOR BANDED NON-SYMMETRIC SYSTEM OF EQUATIONS
C     USING GAUSSIAN ELIMINATION
C     SOLUTION STORED IN THE COLUMN  A(N,2*ITERM)
      DIMENSION A(NRMAX,NCMAX)
      CERO=1.E-06
      PARE = CERO**2
      NBND=2*ITERM
      NBM = NBND - 1
C     BEGINS ELIMINATION OF THE LOWER LEFT
      DO 1000 I=1, N
      IF ( ABS(A(I,ITERM))  .LT.  CERO)  GO TO 410
      GO TO 430
  410 IF ( ABS(A(I,ITERM))  .LT.  PARE)   GO TO 1600
      PRINT 420, A(I,ITERM), I
  420 FORMAT (34H WARNING--ILL-CONDITIONED A-MATRIX,E16.6,5X,I4)
  430 IPIT=I+ITERM-1
      JLAST=MINO(IPIT,N)
      L = ITERM + 1
      DO 500 J=I, JLAST
      L = L - 1
      IF ( ABS(A(J,L))  .LT.  PARE)  GO TO 500
      B = A(J,L)
      DO 450 K=L, NBND
  450 A(J,K) = A(J,K) / B
      IF (I  .EQ.  N)  GO TO 1200
  500 CONTINUE
      L=0
      JFIRST = I + 1
      IF (JLAST  .LE.  I)  GO TO 1000
      DO 900 J= JFIRST, JLAST
      L=L+1
      ITL=ITERM-L
      IF ( ABS(A(J,ITL)) .LT. PARE)  GO TO 900
      JML=J-L
      DO 600 K=ITERM, NBM
      KML=K-L
  600 A(J,KML)=A(JML,K)-A(J,KML)
      A(J,NBND)=A(JML,NBND)-A(J,NBND)
      NIT=N-ITERM+1
      IF (I .GE. NIT)  GO TO 900
      DO 800 K=1, L
      NBNDK=NBND-K
  800 A(J,NBNDK)=-A(J,NBNDK)
```

```
  900 CONTINUE
 1000 CONTINUE
C      BACK-SUBSTITUTION
 1200 L = ITERM - 1
      DO 1500 I=2, N
      DO 1500 J=1, L
      ITJ=ITERM+J
      NPI=N+1-I
      NPIJ=NPI+J
      IF (NPIJ .GT. N)  GO TO 1500
      A(NPI,NBND)=A(NPI,NBND)-A(NPIJ,NBND)*A(NPI,ITJ)
 1500 CONTINUE
      RETURN
C     PRINT THE ENTIRE MATRIX IF ZERO ON MAIN DIAGONAL

 1600 PRINT 1601
 1601 FORMAT (22H ZERO ON MAIN DIAGONAL)
      DO 1602 I=1, N
 1602 PRINT 1603, (A(I,J), J=1, NBND)
 1603 FORMAT (10E12.4)
      STOP
      END
```

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