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## ABSTRACT

This document describes a method for deriving K factors and includes instructions for applying them to reliability prediction. Supporting rationale and background material are also included. The method was developed by the Research and Engineering Division of the Boeing Aerospace Company as an independent research and development project. Field experience data at the Line Replaceable Unit (LRU) level were the basic data used in developing the method. Other applications of this K factor approach, such as Maintainability, will be documented and released separately.



Key Words

Failure Rate

K Factor

Reliability

Prediction



## Acknowledgement

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## 1.0 INTRODUCTION

In general, K factors (Logistic Performance Factors) are numbers which are used to adjust LRU (Line Replaceable Unit) field experience data from one environment to make predictions about the LRU performance in another environment. However this document deals only with failure rate K factors, shows that a definite need exists both for their use, and for research into their development, and details a method for calculating K factors. Sections 1 and 2 include all information necessary to understand the method and to begin to use it. Section 3 explains how the method was developed and validated, and will provide the reader with a better understanding of the usefulness and limitations of this K factor approach. Because this document deals with only one type of K factor, consider "K factor" and "failure rate K factor" synonymous throughout.

### 1.1 Objective

The initial step of this research effort was to develop and statistically validate a method whereby K factors could be calculated from field experience data. The second step was to use the method to produce a set of K factors and to further validate the method by checking these K factors against actual operational data. The third step was to see what applications in addition to failure rate prediction there would be for K factors calculated in this manner, especially in the areas of maintainability and system safety.

This report discusses the first two of these steps of the research effort. It includes step by step illustrations for applying the developed method to field experience data to produce results useful for Reliability prediction applications during the design phase of new systems. Results of the third step will be documented and released separately for each area, such as maintainability, that proves to be suitable for K factor application.

### 1.2 Background

Most aerospace programs are required by contract to perform complete reliability, maintainability, and system safety evaluations, and usually the contracts specify MIL-HDBK-217A dated 1965 and MIL-STD-756A dated 1963 to be used as reliability prediction guidelines. However MIL-HDBK-217A only lists failure rates and a few gross environmental K factors for some electronic piece parts, and MIL-STD-756A only lists gross environmental K factors for the group of electronic piece parts not covered in MIL-HDBK-217A (see Table 1.2-1, page 7). This means there are no K factors for use at the LRU level and only a few electronic piece part K factors. Further, considerable failure data has been generated from current state-of-the-art equipment that contradicts the listed K factors developed from data that was collected over 15 years ago on equipment that was designed and built well before that (Reference 3).

Table 1.2-1

MIL-STD-756A Environmental K Factors

| | | |
|---|---|---|
| Shipboard | | 1.0 |
| Manned Aircraft | | 6.5 |
| Missiles | | 80.0 |
| Satellite: | Launch and Boost Phase | 80.0 |
| | Orbit Phase | 1.0 |

Current factors are too gross for prediction purposes as system configuration and environmental applications become evident early in the design stages or even before that in the project planning stage of a program. The effects of this situation are reflected in AFLCP 800-3 dated April 1973. "While failure data collection has provided historical failure rates, insufficient effort has been made to date to calculate usable K factors. As a result forecasted failure rates may be highly inaccurate with unfavorable effects extending to LCC (Life Cycle Cost) and spares computations."

Another point that was important in defining the course of this research was that more and more emphasis is being placed on using equipment similar or equivalent to a single LRU or a group of LRU's in existing aerospace systems. Typically there is abundant experience data on equipment of this type, but not necessarily in the same application or environment as the new design.

Yet another point was that existing K factors did not provide a means of estimating their own validity. In other words, a possible range of values or "confidence limits" was not given. Part of the research was devoted, therefore, to attempt to establish some sort of confidence limits.

For these reasons, in 1973 a specific effort was initiated to determine what kinds and amounts of data were available at the LRU level, and to gather this data in a form that could be used to develop K factors. Then in 1974 with the knowledge of the kinds and amounts of data available and an approximate idea of the results obtainable, the research effort concentrated on developing a valid statistical method for calculating K factors which could be used with a quantifiable amount of confidence at the LRU level.

## 1.3 Scope

K factors have many applications, but in this document the primary emphasis and intended use for them is in reliability prediction studies for new aerospace design applications. Limited resources have restricted this phase of the research to developing and validating a method for use at the generic system level. However, as data improves and resources become available, it may be possible to look at subfactors such as complexity, mission type, duty cycle, etc. within generic systems to improve this method (refer to Section 3.3.4, Subfactors that Impact Reliability, page 35).

The method developed and validated in this effort is based on statistical techniques taken from texts included in the reference list. The statistical techniques are straightforward and easy to use with the aid of a computer or programmable calculator, and none are new or unproven.

## 2.0 METHOD PRESENTATION

A K factor (K) is the ratio of the same statistic (f()) taken from data sets from two different environments ($DS_a$, $DS_b$) and represents the fractional contribution to the statistic that is solely attributable to just the environmental differences between the two data sets:

$$K = \frac{f(DS_a)}{f(DS_b)}$$

For reliability this is better illustrated as

$$K = \frac{\hat{\lambda}_a}{\hat{\lambda}_b}$$

where $\hat{\lambda}_a$ and $\hat{\lambda}_b$ are the geometric mean failure rates for data sets a and b respectively.

In this study the data sets are either failure rates or MTBF's for LRU groups and the statistic is the geometric mean of the data set.

Reliability K factors are used when a failure rate prediction is needed for a particular item, but no failure history data is available on it in the desired application. Data on the item from another application ($\lambda_a$) can be adjusted by using the appropriate K factor. If the proper K factor has already been assigned, the calculation is simply:

$$\lambda_{predicted} = K \cdot \lambda_a$$

However, if the proper K factor is not available, a sampling of failure rate data from a few LRU groups within the general equipment classification from both the new and old environments must be gathered, first level K factors calculated, and a composite K factor calculated (see figure 2.3-1, page 15). Then the failure rate prediction would again be:

$$\lambda_{predicted} = \hat{K} \cdot \lambda_a$$

In a few special cases it may be both possible and advantageous to develop just one first level K factor from data on equipment belonging to the same LRU group in question. To be possible there must be sufficient data on the specific LRU group from two environments. To be advantageous just the one failure rate prediction in that general equipment classification should be



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required and there should be failure rate data on an identical LRU in the old environment. Otherwise, data on a similar LRU in the new application would be as good or better than factored data on a similar LRU in another application. In a rare case such as this the prediction equations would be,

$$K = \frac{\lambda_{New}}{\lambda_{Old}}$$

$$\lambda_{predicted} = K \cdot \lambda_a$$

## 2.1 General Technique

The geometric mean or nth root of the product of n values is the basic technique upon which this K factor method is based. Other measures of central tendancy were tried and are discussed in Section 3.1, (page 22) along with justification for choosing the geometric mean.

Calculating the geometric mean is most conveniently done by summing the logarithms of all the data points, dividing the sum by the number of data points and taking the antilog of the quotient to give the geometric mean. Further calculations, which are outlined in Section 2.3 (page 11), give confidence limits to the mean and subsequent K factors. Appendix I is a calculator program which can be used to do all of the above mentioned calculations. Outputs are geometric mean and mean confidence limits for any input data set. By using a computer or programmable calculator, time can be saved and chance for error in the many calculations is greatly reduced.

It is important to note that by using the geometric mean approach, failure rates or MTBF's work equally well as inputs, that is the resulting means and limits are exact reciprocals, a result that is not possible by any other averaging technique.

## 2.2 Assumptions

Field experience data is believed to be the best available failure data source. Nonetheless, it has some known drawbacks, for example, there are errors in reporting, individual times-to-failure are not known, and the distribution of failure rates is uncertain. Furthermore, field experience data does not reflect "true" or "absolute" reliability but, rather, reliability as it is affected by other factors. A modified Bayesian approach to the problem was therefore adopted in which, a priori, certain assumptions concerning the data were made with the reservation that subsequent research may require modification of, or may even invalidate, the assumptions. One assumption discussed in the previous section was that the geometric mean was the best measure of central tendency. In addition certain other basic assumptions have been made concerning the data used and its applicability to K factor determination.

(1) Experience data sets reflect an integration of all subfactors which affect reliability statistics.

(2) Reliability statistics vary primarily due to environmental effects, while other contributing effects tend to cancel when K factor ratios are taken.

(3) LRU's in a general equipment class are all affected similarly by changes in application, such that a single composite K factor will adequately represent the entire class.

(4) A direct relationship exists between failures and operating hours (constant failure rate).

(5) Failure rates are lognormally distributed.

It is important to remember that most analysis and trade studies which use these factors are made for comparative purposes early in the program, rather than for absolute values. Therefore certain errors in these K factors will not obscure the trade study results where the error in other considerations is often larger. However these assumptions do bring in some error, and for this reason they are discussed further in Section 3.3, Problem Areas.

## 2.3 Method Detailed

The following equations specify how the geometric mean (G. M.) and G. M. confidence intervals of a data set are calculated.

(1) Geometric mean, $\hat{a}$:

$$G.M. = \hat{a} = \sqrt[n]{\prod_{i=1}^{n} a_i} = 10^{\left(\frac{\sum_{i=1}^{n} \log_{10} a_i}{n}\right)} \quad \text{Note: } a_i = \lambda_i, MTBF_i, \text{or } K_i$$

(2) Log Variance, $s_L^2$ :

$$s_L^2 = \frac{\sum_{i=1}^{n}\left(\log_{10}a_i - \log_{10}\hat{a}\right)^2}{n-1} = \frac{n\sum_{i=1}^{n}\left(\log_{10}a_i\right)^2 - \left(\sum_{i=1}^{n}\log_{10}a_i\right)^2}{n(n-1)}$$

(3) Upper ($a_u$) and Lower ($a_L$) G.M. confidence limits at $1-\alpha$ level of confidence, such that

$$P\left[\hat{a}_L < a = 10^{(\log_{10}\hat{a})} < \hat{a}_u\right] = 1-\alpha$$

$$\hat{a}_u = 10^{(\log_{10}\hat{a}+\varepsilon)} \qquad \text{where, } \varepsilon = T\sqrt{s_L^2/n}$$

$$\hat{a}_L = 10^{(\log_{10}\hat{a}-\varepsilon)} \qquad \text{and, } T = t_{1-\alpha,\ n-1}*$$

$$\text{or, } T = t\alpha/2,\ n-1$$

(4) K factor, K; and Upper/Lower confidence limit K factors, $K_u/K_L$:

$$K = \frac{\hat{a}}{\hat{b}}, \qquad K_u = \frac{\hat{a}_u}{\hat{b}_L}, \qquad K_L = \frac{\hat{a}_L}{\hat{b}_u}$$

(5) Composite K factors, $\hat{K}$; and their Upper/Lower confidence limit K factors, $\hat{K}_u/\hat{K}_L$:

$\hat{K}$ = G.M. of $K_1, K_2 \ldots K_n$ (see equation #1)

$\hat{K}_u = \hat{a}_u$ and $\hat{K}_L = \hat{a}_L$ based on $K_1, K_2, K_3 \ldots K_n$

*Most t tables list $\alpha$ versus $\nu$, but Appendix II lists $1-\alpha$ versus $\nu=n-1$, where $\nu$ is degrees of freedom.

The foregoing equations show the relationships between raw data and their resulting K factors. Initially all failure rate data are sorted by LRU groups, each of which is defined by its unique construction and application. Generally the requirements are such that all data associated with a particular LRU group must come from LRU's which are at least similar if not identical in construction and application/environment. Then data from each LRU group is processed using equations 1, 2, and 3, yielding geometric mean and confidence limits for each group. (See Figure 2.4-2, page 21)

Next, first level K factors are developed, first by matching pairs of LRU groups that are nearly identical in construction but different in environment and secondly by applying equation #4 to the mean and limits previously developed for each matched pair. It is assumed that the LRU's in the matched LRU groups would have a common G.M. failure rate if used in the same environment, therefore the ratios (or K factors) developed using equation #4 measure the relative increase (decrease) in failure rate due to a more severe (less severe) environment. The resulting first level K factors can be identified by LRU group and two associated environments. (It is important to note which environment is used as the base when equation #4 is applied.) (See Table 2.3-1, page 14)

Often when only one failure rate prediction is needed, this is as far as the process needs to be followed (See Section 2.4 Example #2, page 19 for an example using first level K factors.) But for the majority of cases, a more general type K factor, described in this document as a composite K factor, would be more useful in mass application on a large program. These composite K factors are developed from the first level K factors by grouping them by identical environment combinations and then by further subdividing these groups into subgroups which are defined by the general equipment classification of the LRU groups. The order in which these first level K factor groups are sorted is not important as long as the members of each resulting subgroup have common classes of hardware and identical environment combinations. Equations 1, 2, and 3 are then applied to the first level K factor values to arrive at composite K factors and their confidence limits. (See Tables 2.3-1, page 14 and 2.4-1, page 18). This method of grouping permits equipment class composite K factors to be developed from first level K factors of LRU groups that do not have common G.M. failure rates. This is possible because each first level K factor is a ratio or index of severity which is independent of the gross magnitude of the failure rates. Therefore this process enables reliable K factors to be developed from a minimum sampling of failure rate data, as illustrated in Figure 2.3-1, page 15.

The upper ($K_u$, $\hat{K}_u$) and lower ($K_L$, $\hat{K}_L$) confidence interval limits (K factors) are developed to give the user an idea of the dispersion of the failure rates used to calculate the K factors. A "worst case" condition was used for calculating confidence limits in which it was assumed that the two data sets would have actual values at the opposite extremes. If a 90% confidence level is chosen to calculate these K factors (typical for this type of calculation), this means that there is a .9 probability that the true K factor lies between the upper and lower confidence limits K factors. However, by the strict mathematical definition, it does not mean that there is a .9 probability that the actual failure rate of an LRU in a new application will be within these limits, although results of empirical testing do indicate that more than 90% of actual values will be within these limits when failure rate data on the same LRU is factored.

Table 2.3-1
Aircraft Electronics
Composite K Factor Work Table

| LRU | | Mil. Transport | | Civil Transport | | Fighter | | Bomber | |
|---|---|---|---|---|---|---|---|---|---|
| | | F.R. | K | F.R. | K | F.R. | K | F.R. | K |
| UHF/VHF | L.L. | 5.43 | | .58 | 0.05 | 12.50 * | 1.04 | 8.00 | 0.67 |
| | G.M. | 8.07 | 1 | 1.11 | 0.14 | 17.10 | 2.12 | 10.40 | 1.29 |
| | U.L. | 12.00 | | 2.26 | 0.42 | 23.30 | 4.29 | 13.60 | 2.50 |
| Radar Altimeter | L.L. | 3.01 | | 0.40 | 0.04 | 11.70 | 1.28 | 2.72 | 0.30 |
| | G.M. | 5.25 | 1 | 0.59 | 0.11 | 18.10 | 3.45 | 5.16 | 0.98 |
| | U.L. | 9.16 | | 0.86 | 0.29 | 28.00 | 9.30 | 9.76 | 3.24 |
| IFF | L.L. | 2.63 | | | | 4.46 | 0.73 | 1.40 | 0.23 |
| | G.M. | 4.02 | 1 | | | 6.41 | 1.59 | 3.94 | 0.98 |
| | U.L. | 6.14 | | | | 9.21 | 3.50 | 11.10 | 4.22 |
| TACAN | L.L. | 8.25 | | | | 11.80 | 0.91 | 7.09 | 0.55 |
| | G.M. | 10.30 | 1 | | | 16.30 | 1.58 | 12.80 | 1.24 |
| | U.L. | 12.90 | | | | 22.60 | 2.74 | 23.10 | 2.80 |
| Composite Electronics | L.L. | | | | 0.06 | | 1.34 | | 0.94 |
| | G.M. | | 1 | | 0.12 | ** | 2.07 | | 1.11 |
| | U.L. | | | | 0.27 | | 3.19 | | 1.33 |

* From Figure 2.4-2, page 21 and Table 3.1-1, page 24

** See Table 2.4-1, page 18

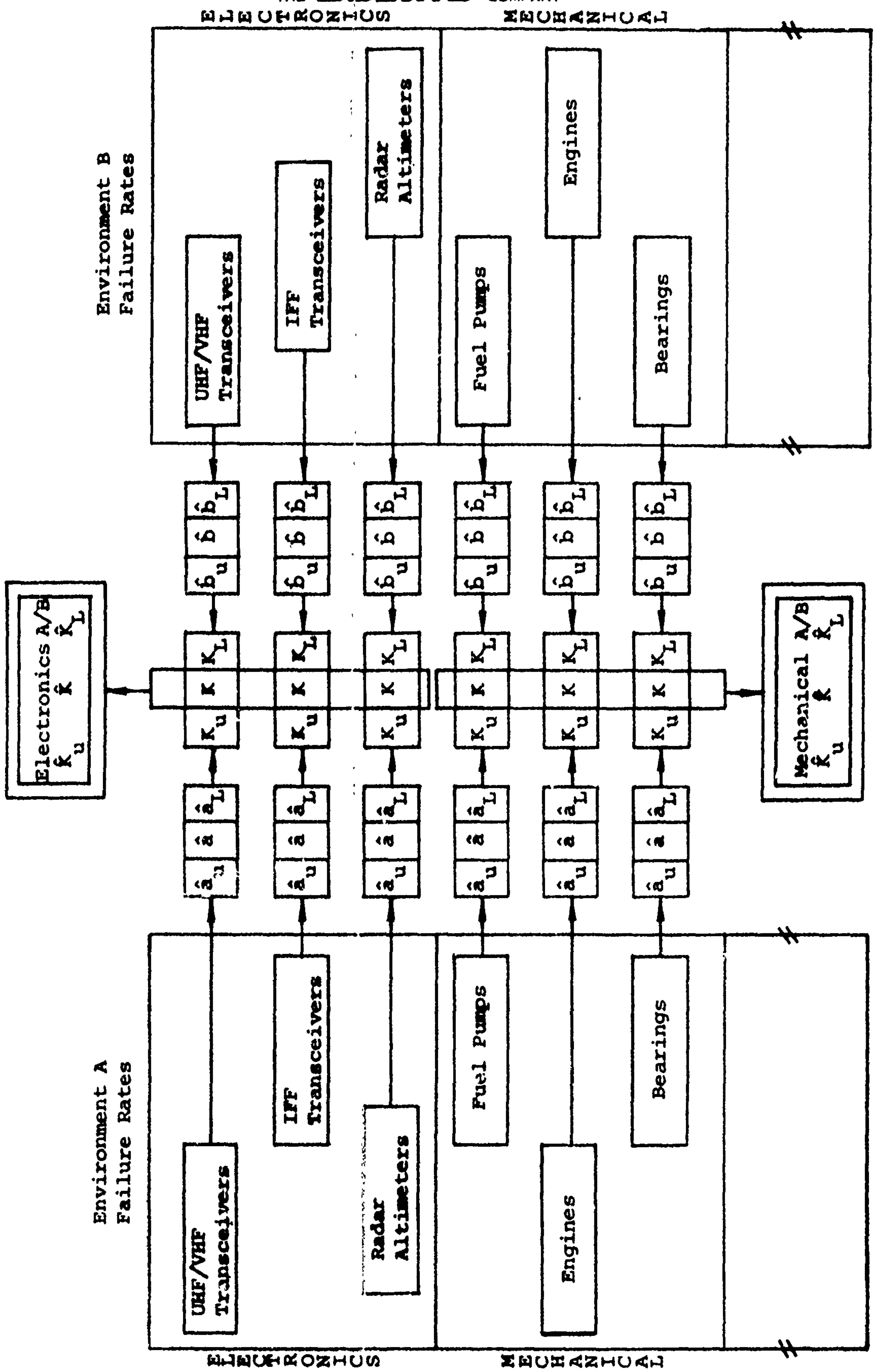


Figure: 2.3-1
Failure Rate Data Grouping for Composite K Factor Development

Throughout this report, common logarithms and corresponding powers of 10 have been used because of the ease in accessing tabular values, however natural logarithms and the exponential function work equally as well. Intermediate results, specifically the mean logarithms and standard deviations, are not the same, but the end results are identical, therefore it is important that one or the other approach be used exclusively. In fact there are many ways that the calculations, defined by the K factor equations at the beginning of this section, can be made, and the best way will depend on the user's individual situation. For convenience, a calculator program that performs geometric mean, confidence limit and frequency boundary limit calculations is included as Appendix I, and a simplified manual process is detailed in example #2 of Section 2.4 (page 20). If several K factors are to be calculated, computer aided processing will reduce the time required and will greatly reduce the chance for arithmetic errors.

## 2.4 Application to Reliability

The prime objective of the research effort was to develop and validate a useful K factor development method. This section is devoted to applying the developed method to reliability prediction.

A flow diagram, Figure 2.4-1 (page 17), illustrates how this method would be used in reliability prediction. Referring to the diagram, as soon as the need for a failure rate or MTBF prediction has been established, it must be determined what is the best kind of reliability data available. If the best data is failure history data on the same/similar item in another environment, a K factor adjustment by the method described herein would produce the desired results. In most cases a composite K factor table similar to Table 2.4-1 (page 17) would contain the appropriate K factor. However some programs have specific definitions of failure that are not compatible with the general form. In such a case a whole new set of composite K factors would need to be calculated by processing data according to the definition of failure set by the program. These program composite K factors may all be researched and calculated at one time to reduce the number of manhours needed to complete an entire set of K factors, and then logged in a reference file (the EAC maintains such a file) for use on the program and possibly for future programs. Sometimes only one K factor is needed and in these cases only one data set from each of the new and old applications need to be gathered to produce the K factor. At any rate, the end result of using the procedure illustrated in Figure 2.4-1 (page 17) is always a reliability prediction with the highest possible confidence.

**RELIABILITY PREDICTION FROM FIELD EXPERIENCE DATA**

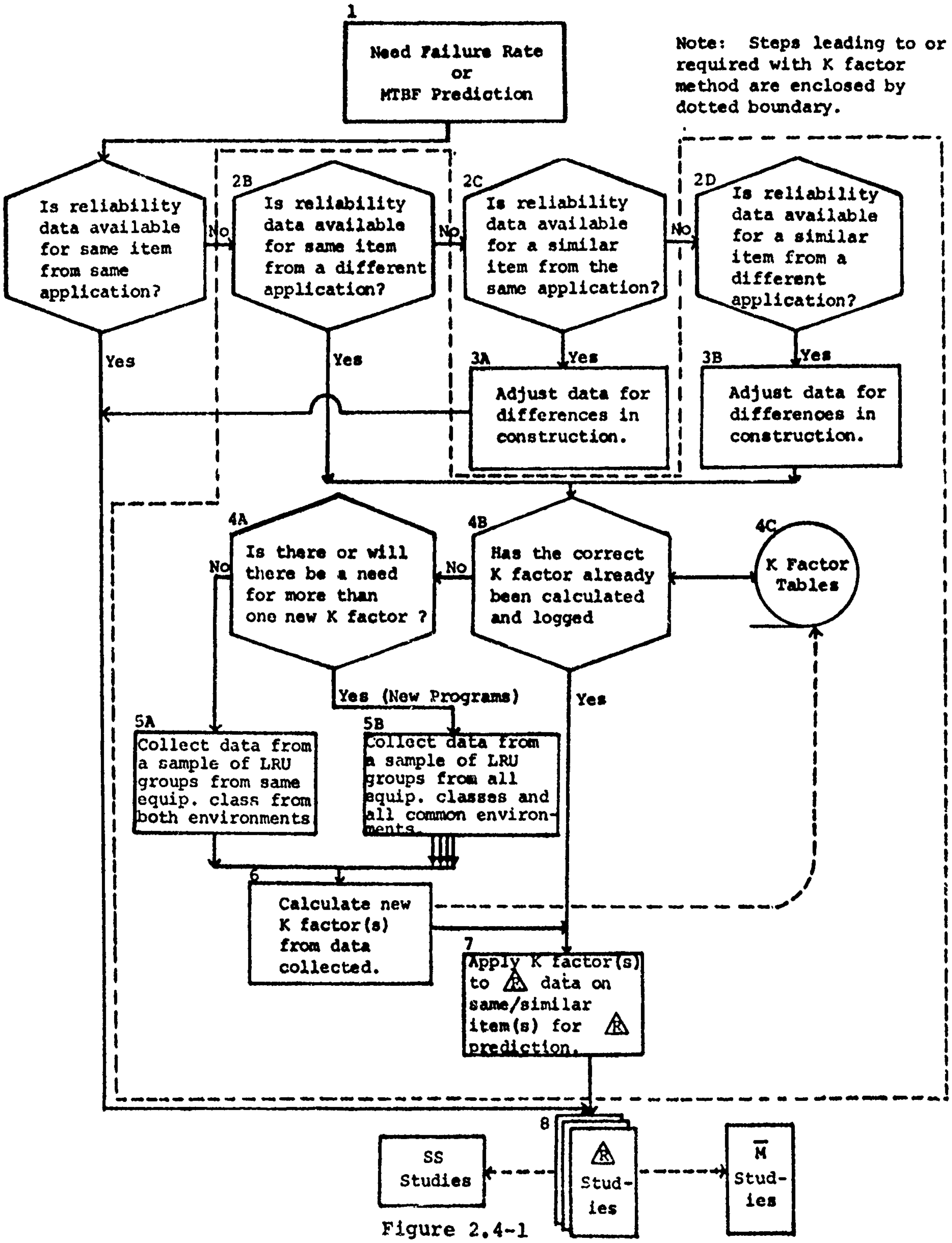


Figure 2.4-1

Example 1

Given: Need a failure rate for an AN/ARC-109 UHF transceiver for fighter aircraft application.

Assume that the transceiver has never flown in a fighter, but that 4.838 failures per 1000 flight hours were reported against the same transceiver in a C-5A military transport application.

Because a transceiver is electronic equipment, the electronics composite K factor for military transport to fighter from Table 2.4-1 (below) will be used and the calculations are as follows:

$$\lambda_{predicted} = \lambda_{C-5A} \times \hat{K}_{f/m.t.} = 4.838 \times 2.1 = 10.16$$ failures per 1000 flight hours

Table 2.4-1 is a preliminary composite K factor table formulated from actual data by the method developed in this study and used in this reliability prediction example. Note that K factors for military transport are unity and that all other application K factors are shown relative to the military transport. When such a table is developed for a reliability study on a new program, the new application could be used as the base, or a complete cross-reference table could be set up for each equipment class with no common base necessary.

Table 2.4-1

Aircraft Composite K Factors

| Application | Mechanical/ Hydraulic | | | Electro-Mechanical | | | Electronic | | |
|---|---|---|---|---|---|---|---|---|---|
| | $\hat{K}_u$ | $\hat{K}$ | $\hat{K}_L$ | $\hat{K}_u$ | $\hat{K}$ | $\hat{K}_L$ | $\hat{K}_u$ | $\hat{K}$ | $\hat{K}_L$ |
| Military Transport | | 1 | | | 1 | | | 1 | |
| Bomber | 8.1 | 3.2 | 1.1 | 2.0 | 1.0 | 0.5 | 2.7 | 1.5 | 1.1 |
| Fighter | 4.6 | 2.0 | 0.8 | 3.0 | 1.5 | 0.7 | 3.2 | 2.1* | 1.3 |
| Helicopter | 1.8 | 0.8 | 0.3 | 5.9 | 2.9 | 1.4 | 1.4 | 0.9 | 0.5 |
| Commercial Transport | 1.8 | 0.6 | 0.2 | 1.6 | 0.4 | 0.1 | 0.2 | 0.1 | 0.1 |

* From Table 2.3-1

In reality, the AN/ARC-109 had been used in the F-111A fighter aircraft with a field demonstrated failure rate of 12.68 failures per 1000 flight hours, well within the expected range of the predicted value of 10.16 failures per 1000 flight hours.

The Experience Analysis Center (EAC) of the Boeing Aerospace Company, maintains a log and file of all K factors developed using this approach. It may already have single K factors or K factor tables for different programs. The EAC should be contacted before indiscriminately using any K factor, since the definition of failure may vary from program to program and for different data sets.

Example 2

Given: Need a failure rate for an AN/ARC-109 UHF transceiver for fighter aircraft application.

The basic requirements are the same here as in Example 1, but with one added restriction - assume that the military transport to fighter composite K factors have not been calculated, therefore Table 2.4-1 (page 17) cannot be used. Now a UHF transceiver is electronic equipment and according to assumption #3 Section 2.2 (page 11) a composite electronics K factor would be applicable, as illustrated in Example 1, but assume there is sufficient data on both transport and fighter UHF/VHF transceivers to produce an accurate first level transceiver K factor, as is really the case here. Then first because the item in question belongs to the same LRU group as other transceivers, and secondly because it takes much more data to calculate a composite K factor, a first level transceiver LRU group K factor would be best, in this case. (On a large program where a full set of composite K factors would be available, the electronics K factor would be used, as in Example 1, to eliminate retrieving additional data.)

The next step is to collect and process transceiver failure rate data from both applications. Figure 2.4-2 (page 21) lists such data and shows the necessary calculations for processing the fighter data. The same steps were used to calculate mean and mean confidence intervals for the military transport transceivers, but only the results are shown. The prediction of 10.22 is again well within range of the actual rate for this transceiver on the F-111A fighter aircraft of 12.68.

The following steps were used in Figure 2.4-2 (page 21) to calculate the data set means and mean confidence limits. The steps are marked with numbered circles in the figure for ease in following the procedure.

1. List failure rates.
2. List the logarithms of the failure rates.
3. Sum the logarithms.
4. Divide the sum by the number of entries.
5. Take the antilog of the quotient to get the geometric mean.
6. List the differences between the logs of the individual failure rates and the log of the geometric mean.
7. Square and list the difference for #6.
8. Sum the squares.
9. Divide the sum by one less than the number of entries to obtain the variance.
10. From the t-table (Appendix II) find the value of t corresponding to the desired confidence level, 90%, and the appropriate degrees of freedom, n-1.
11. Compute $\varepsilon$, the deviation from the sample mean, from the formula $\varepsilon = t\sqrt{s_L^2/N}$.
12. Compute the upper 90% mean confidence limit.
13. Compute the lower 90% mean confidence limit. (Repeat steps 1 through 13 for second data set.)
14. Compute K factors, K, $K_u$, $K_L$.
15. Compute predicted failure rate.
16. Log K factors.

| i | A/C | $\lambda_i \times 10^3$ ① | $\log \lambda_i$ ② | $\log \lambda_i - \log \hat{\lambda}$ ⑥ | $(\log \lambda_i - \log \hat{\lambda})^2$ ⑦ |
|---|---|---|---|---|---|
| 1 | F-4B | 25.78 | 1.411283 | .179519 | .032227 |
| 2 | F-4E | 15.66 | 1.194792 | -.036971 | .001367 |
| 3 | F-111A | 12.68 | 1.103119 | -.128644 | .016549 |
| 4 | F-111F | 6.27 | 0.797268 | -.434495 | .188786 |
| 5 | A-7A | 22.24 | 1.347135 | .115371 | .013310 |
| 6 | A-7B | 23.77 | 1.376029 | .144265 | .020812 |
| 7 | A-7D | 18.67 | 1.271144 | .039380 | .001551 |
| 8 | A-7E | 22.56 | 1.353339 | .121575 | .014780 |
| Sums | | ③ | 9.854109 | ⑧ | .289382 |

④ $\log \hat{\lambda} = \frac{\Sigma \log \lambda_i}{n} = \frac{9.854109}{8} = 1.231764$

⑤ $\hat{\lambda} = 10^{(\log \hat{\lambda})} = 10^{(1.231764)} = 17.05$

⑨ $s_L^2 = \frac{\Sigma(\log \lambda_i - \log \hat{\lambda})^2}{n-1} = \frac{.289382}{7} = 0.041340$

⑩ At 90% confidence level, $t_{90\%,7} = 1.895$ (see Appendix II)

⑪ $\varepsilon = T\sqrt{s_L^2/n} = 1.895\sqrt{.041340/8} = 0.136223$

⑫ $\hat{\lambda}_u = 10^{(\log \hat{\lambda} + \varepsilon)} = 10^{(1.231764 + .136223)} = 23.33$

⑬ $\hat{\lambda}_L = 10^{(\log \hat{\lambda} - \varepsilon)} = 10^{(1.231764 - .136223)} = 12.46$

---

UHF/VHF Transceivers Military Transport $\hat{\lambda} = 8.07,\ \hat{\lambda}_u = 12.0,\ \hat{\lambda}_L = 5.43$

UHF/VHF Transceivers Fighter $\hat{\lambda} = 17.05,\ \hat{\lambda}_u = 23.33,\ \hat{\lambda}_L = 12.46$

AN/ARC-109 C-5A Failure rate 4.838

⑭ $K = \frac{\hat{\lambda}_f}{\hat{\lambda}_{m.t.}} = \frac{17.05}{8.07} = \underline{2.11},\ K_u = \frac{\hat{\lambda}_{uf}}{\hat{\lambda}_{Lm.t.}} = \frac{23.33}{5.43} = 4.30,$

$K_L = \frac{\hat{\lambda}_{Lf}}{\hat{\lambda}_{um.t.}} = \frac{12.46}{12.0} = 1.04$

⑮ $\lambda_{predicted} = \lambda_{C-5A} \times K = 4.838 \times 2.11 = \underline{10.22}$

Figure 2.4-2
Fighter Aircraft Transceivers

## 3.0 VALIDATION

This section includes all of the mathematical approaches investigated for possible use in K factor development, and the acceptance tests used to test the different approaches, plus discussion of problems encountered in this K factor research effort.

### 3.1 Methods Investigated

The following techniques for processing field experience data to calculate K factors were tested and evaluated, and will be discussed individually in this section.

- Arithmetic Mean
- Linear Correlation
- Non-Linear Correlation
- Forced Correlation
- Geometric Mean

#### 3.1.1 Arithmetic Mean

In the 1973 phase of this research effort, the arithmetic mean of each of the data sets accumulated was caculated to get a quick estimate of the K factors that could be produced. The arithmetic mean is symbolized as follows:

$$\bar{a} = \frac{\sum_{i=1}^{n} a_i}{n}$$

This measure of central tendancy is the simplest, but is weighted to a great extent toward the high end and produces results derived from failure rates that are not equivalent to results derived from MTBF's (see Figure 3.2-2, page 30, and Table 3.1-1, page 24).

Because of the bias and the resulting non-equivalence of results, this method was rejected.

#### 3.1.2 Linear Correlation

The linear correlation technique gives the slope of a straight line approximation to the data points, plus the arithmetic mean is an intermediate result, but the technique is more complicated than simply an arithmetic mean. Also a correlation factor can be calculated to give a more quantitative judgement as to how the points fit the straight line approximation. However, because the slope of the line is not useful in any practical application and the mean has the same bias mentioned in Section 3.1.1, (above ), this method was also rejected.

### 3.1.3 Non-Linear Correlation

Two curve types, exponential $Y_c = ab^x$ and power curve $Y_c = ax^b$, were tested by the least squares method to try to approximate the data points. The power curve appeared to fit the data better than the exponential and also better than the linear correlation. Also the geometric mean was an intermediate result in the calculations and appeared to be more centrally located than the arithmetic mean. However, no explanation (except for random chance) could be made for the powers of x that were calculated, and because this method is quite complex it was also rejected.

### 3.1.4 Forced Correlation

The same power curve approximation described in Section 3.1.3 (above) was again tried, but this time the parameter b was set at 1 in all cases, such that the result is forced to a constant rate of the form $Y_c = ax$, where $a = \lambda$ when the statistics are failure rates. This method has the same complexity as the non-linear correlation, and again there is no significant practical advantage.

It should be noted that the linear, non-linear, and forced correlations were all curvilinear attempts at representing sets of data pairs. Because of the nature of the data and the assumptions that were made, namely assuming a constant failure rate, the data is really only one dimensional. Therefore these techniques yielded some results that were either invalid or had no application, and they were paid for by added complexity.

### 3.1.5 Geometric Mean

The geometric mean approach selected in this effort is actually the nth root of n products approach, calculated using logarithms as described in Sections 2.3 (page 11) and 2.4 (page 16). The geometric mean has the advantage of being less biased toward the high end than the arithmetic mean. This is true because the geometric mean is the mean, median, and mode of the logarithms of a perfect lognormal distribution (see Figure 3.2-2, page 30). The reliability data investigated appears to be distributed lognormally. An example of a chi-square test indicating that the data is distributed lognormally is included in Section 3.2.3 (page 27). It is felt that failure rates and MTBF's are distributed lognormally because they are actually ratios of failures versus time, bounded by zero on the low end and unbounded above which forces them to be skewed to the right. Also, all of the numbers from which K factors will be developed will be ratios such as failure rates (failures per $10^n$ hours), MTBF's (hours per failure), maintenance actions per failure, maintenance manhours per maintenance action, etc., and according to the statistical texts listed as references 6, 7, and 8, the geometric mean is especially useful when applied to pure ratios such as these. This is true because it makes no difference which way the ratio is taken, the results are equivalent. For example, take the illustration of the geometric mean calculation for the eight fighter UHF/VHF transceivers from Example 2 in Section 2.4 (page 21). Table 3.1-1 summarizes those results, plus results for the corresponding MTBF based calculations for the geometric mean and arithmetic mean.

Table 3.1-1
Reciprocals of Geometric Mean vs Arithmetic Mean

DATA POINTS

| | λ | MTBF=1/λ x 1000 |
|---|---|---|
| 1 | 25.78 | 38.79 |
| 2 | 15.66 | 63.86 |
| 3 | 12.68 | 78.86 |
| 4 | 6.27 | 159.49 |
| 5 | 22.24 | 44.96 |
| 6 | 23.77 | 42.07 |
| 7 | 18.67 | 53.56 |
| 8 | 22.56 | 44.32 |

Note: All λ's are in failures per 1000 hours and all MTBF's are in hours per failure.

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) |
|---|---|---|---|---|---|---|---|
| | Geometric Mean Approach | | | Arithmetic Mean Approach | | | |
| | λ | MTBF | 1/MTBF=λ' | λ | MTBF | 1/MTBF=λ' | 1/λ=MTBF' |
| Mean | 17.055 | 58.632 | 17.055 | 18.456 | 65.714 | 15.218 | 54.182 |
| Upper 90% Limit | 23.334 | 42.855 | 23.334 | 22.861 | 38.842 | 25.746 | 43.743 |
| Lower 90% Limit | 12.466 | 80.217 | 12.466 | 14.051 | 92.588 | 10.800 | 71.169 |



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By using the G.M. approach the failure rate and MTBF results are all equivalent (Column 1 equals Column 3). However, under the Arithmetic Mean Approach, Column 4 does not equal Column 6 nor does Column 5 equal Column 7 and this will always be the case. Here they differ by as much as 40%, but many examples have been found where the difference is more than 100%. Therefore, if the arithmetic mean approach were used, two different sets of reliability K factors would have to be developed, one for use with failure rates and one for MTBF's. Likewise, similar situations would result for other RM&SS K factors, all of which are based on ratios that could just as easily be interchanged.

The geometric mean eliminates this problem by producing results that are compatible no matter how the ratio is taken. The geometric mean is the best measure of central tendency for a log-normal distribution. Further, the geometric mean is straightforward and lends itself easily to calcuation of mean confidence limits and expected frequency distributions of the data points, both of which are necessary to make objective judgements concerning the data collected and the K factors produced. For these reasons the geometric mean has been selected as the averaging technique for K factor development.

### 3.2 Acceptance Tests

The initial effort in 1974 was directed toward trying to validate a K factor technique, and the initial hypothesis tested was as follows:

> $H_O$ = This group of observed failure rates is a sample from a population having a failure rate which is approximately the mean of the observed failure rates.

Acceptance w'uld constitute validation of the technique, but it must be recognized that the classical dilemma existed, mainly the double risk of accepting a false hypothesis ($\beta$) or rejecting a true one ($\alpha$).

A search of various statistical texts was made to identify methods of testing hypothesis. In addition, the problem was discussed with various people knowledgeable in the fields of statistical methods and reliability. Several possible testing methods were examined of which three at first appeared promising. These were explored in more detail, and are outlined in the following sections.

#### 3.2.1 Cumulative Frequency Distribution, "d" Test.

A cumulative-percentage histogram is drawn for the observations in a sample of failure rates. Then two parallel polygons are drawn above and below the histogram at a distance which depends upon the level of confidence desired to support the statement that "the cumulative frequency distribution of the population is in this band." Figure 3.2-1 ( **below** ) shows a basic cumulative-percentage histogram for failure rates of UHF/VHF transceivers with 95% confidence limits.

This test has the advantage of establishing confidence limits, but otherwise it is not particularly attractive. For instance, it does not directly test the stated hypothesis; and it presents the data in a form, cumulative frequency polygons, which is not generally used in this sort of application and would be unfamiliar to users.

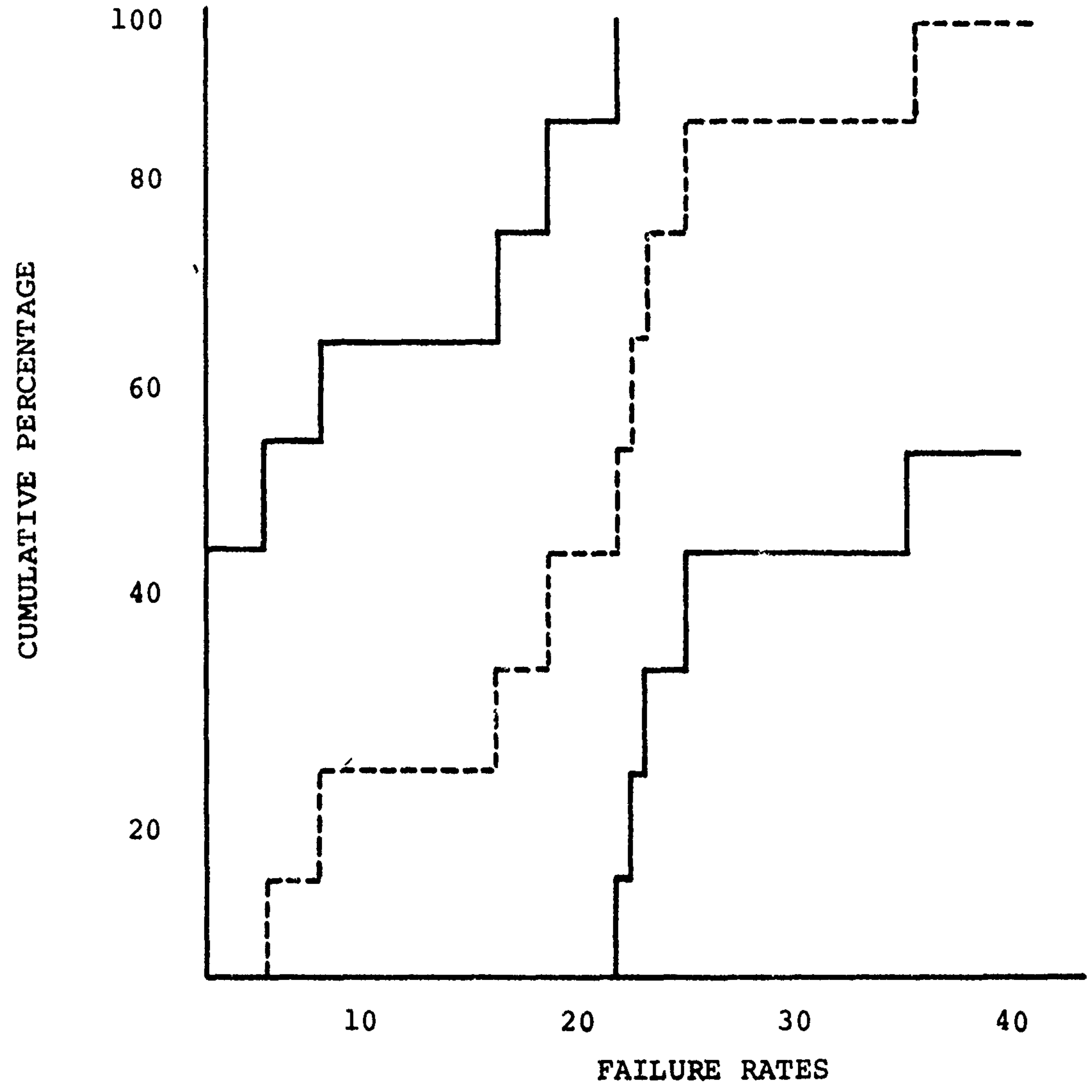


Figure 3.2-1
UHF/VHF Transceiver
Cumulative Frequency Histogram

### 3.2.2 Significant Ratio, "t" Ratio

This test provides an estimate of the probability, at a predetermined level of confidence, that a sample could have come from a certain population. The t ratio is:

$$t = \frac{\bar{x} - \mu}{\hat{\sigma}_{\bar{x}}}$$

where $\bar{x}$ is the sample mean, $\mu$ is the population mean, and $\hat{\sigma}_{\bar{x}}$ is an estimate of the standard error based on an estimate of the population standard deviation.

For the data at hand neither the population mean, $\mu$, nor the individual values which make up $\bar{x}$ are known. Although estimates of these values can be drived, the net result is estimates of estimates, leading to such a degree of uncertainty that the usefulness of this test was doubtful. Also the test is most appropriate when applied to a normal distribution, whereas the distribution of the data at hand is not a normal distribution. No illustration of this test is given.

### 3.2.3 Chi-square ($\chi^2$) test,

The chi-square test is useful in a variety of cases. It can be used to compare an observed parameter of a sample with the corresponding known or estimated parameter of the population from which the sample was taken.

$$\chi^2 = \frac{\sum_{i=1}^{n} (\bar{x} - \mu)^2}{\mu}$$

The value of $\chi^2$ thus obtained is compared with the expected value of $\chi^2$ determined by the sample size and desired confidence level taken from a chi-square table of values. This will give a probability that the sample came from the population it was assumed to be from. For the example shown in Table 3.2-1 (page 28), $\bar{X}$ would be the failure rate of an LRU in one type/model aircraft, and $\mu$ would be the sample mean failure rate for all aircraft in the sample.

A typical calculation for the chi-square test is shown in Table 3.2-1 (page 28). Referring to the table, the probability of the sample coming from the assumed population is .001, hence the hypothesis would be rejected. However an examination of a plot of the failures as a function of operating hours indicates that rejection may be the wrong conclusion.

| I.D. No. | Application A | Operating Hours | Failures | MTBF | Failures/$10^3$ O.H. $\overline{X}$ | $\overline{X} - \mu$ | $(\overline{X} - \mu)^2$ | $\frac{(\overline{X} - \mu)^2}{\mu}$ |
|---|---|---|---|---|---|---|---|---|
| F-5 | F-4B | 106,276 | 2740 | 39 | 25.78 | 4.31 | 18.57 | .86 |
| F-6 | F-4E | 100,319 | 1571 | 64 | 15.66 | -5.81 | 33.76 | 1.57 |
| F-8 | F-111A | 13,950 | 177 | 79 | 12.68 | -8.79 | 77.26 | 3.60 |
| F-9 | F-111F | 6,055 | 38 | 159 | 6.27 | -15.20 | 231.04 | 10.76 |
| F-1 | A-7A | 63,905 | 1421 | 45 | 22.24 | .77 | .59 | .03 |
| F-2 | A-7B | 99,646 | 2369 | 42 | 23.77 | 2.30 | 5.29 | .25 |
| F-3 | A-7D | 38,508 | 719 | 54 | 18.67 | -2.80 | 7.84 | .37 |
| F-4 | A-7E | 156,763 | 3537 | 44 | 22.56 | 1.09 | 1.19 | .06 |
| | | | | | | | | |
| | TOTAL | 585,422 | 12572 | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| CLASS MTBF = 47<br>CLASS FAILURE RATE = 21.47 = $\mu$ | | | | | | | $X^2$ =<br>P = | 17.50<br>.001 |

TABLE 3.2-1
UFH/VHF - FIGHTERS
CHI-SQUARE TEST

While only one example has been shown, it is typical of the results obtained by the chi-square test. The test was applied 19 times with the result that the hypothesis was rejected ($P < 0.01$) ten times, accepted marginally ($0.01 < P < 0.05$) four times and accepted ($P > .05$) only five times.

Hypothesis testing previously discussed, tacitly assumed the data were approximately normally distributed, but frequency plots of the data show that normality may be a poor assumption, (see Figure 3.2-2, page 30). Upon further examination of the data, the lognormal distribution appeared to be the best candidate for further testing. Another application of the chi-square test, testing the expected distribution of data points instead of the expected values, produces acceptable results. The hypothesis for this test is: $H_o$ = The data are lognormally distributed, and the statistic tested is:

$$\chi^2 = \Sigma \frac{(f-f_e)^2}{f_e}$$

where f is the actual partition frequency, and $f_e$ is the expected partition frequency.

By using the common logarithms of the data points (already listed for calculating the geometric mean) and a standard normal probability table, Appendix III, it is easy to calculate the partition boundary limits, the actual partition frequencies, and the expected partition frequencies.

The partition boundary limits are defined by the following equation:

$$\text{partition boundary limit} = 10^{(\log 10(\hat{x}) \pm z s_L^2)}$$

where z is a function of $\alpha$ taken from a standard normal table, $\hat{x}$ is the geometric mean, and $s_L^2$ is the log variance defined in Section 2.3 (page 11). The partition boundary limits are also outputs of the computer program for calculating the geometric mean, Appendix I.

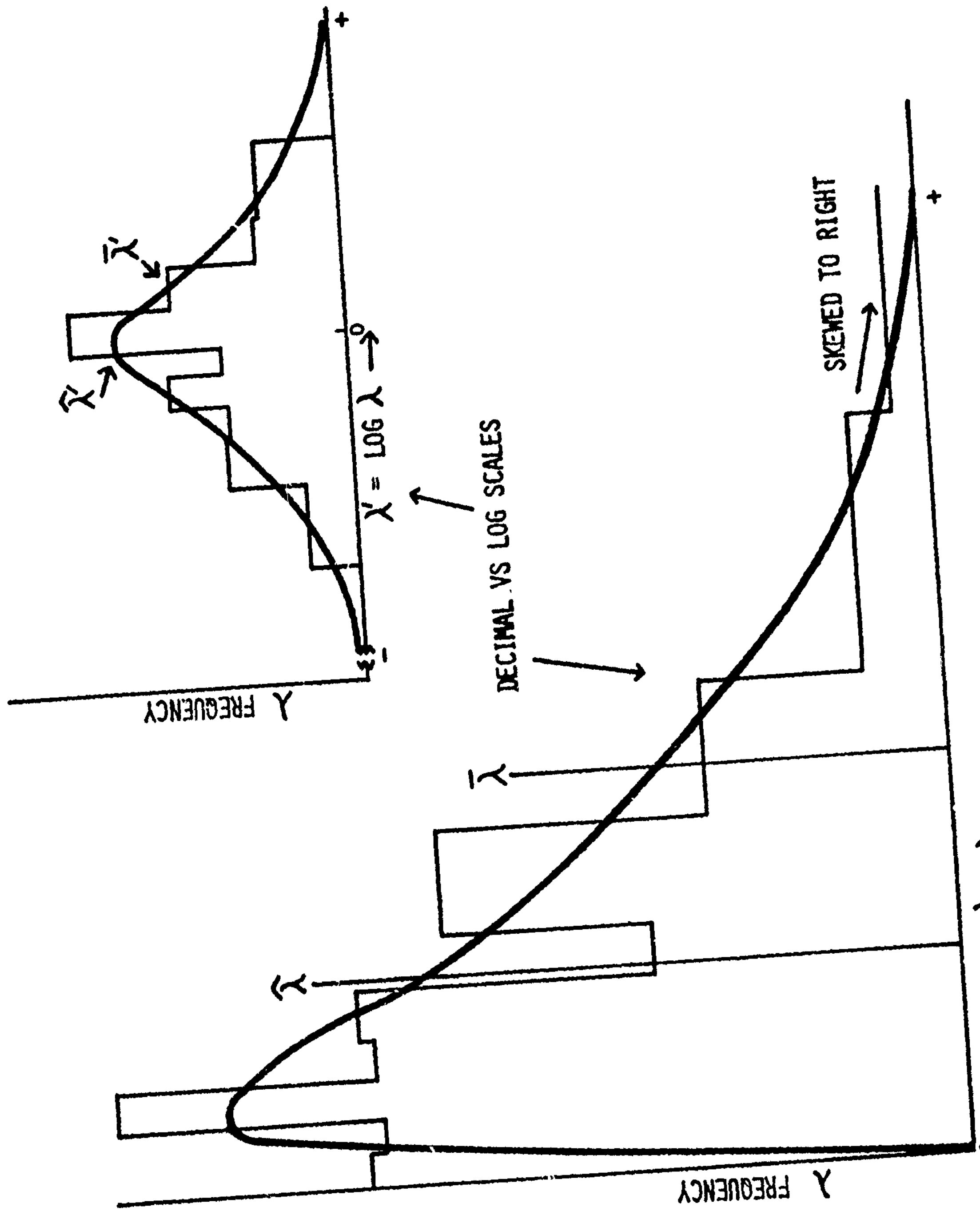


Figure 3.2-2
Frequency Distribution for 87 Gyroscope Failure Rates

Table 3.2-2

Chi-square test on frequency distribution of 87 gyroscope failure rates.

| n | z | Partition Lower Limit | $f_e$ | f | $\frac{(f-f_e)^2}{f_e}$ |
|---|---|---|---|---|---|
| 1 | 1.645 | 5.718 | 4.35 | 4 | 0.028 |
| 2 | 1.0365 | 2.832 | 8.7 | 10 | 0.194 |
| 3 | .6745 | 1.864 | 8.7 | 6 | 0.838 |
| 4 | .3854 | 1.335 | 8.7 | 10 | 0.194 |
| 5 | .1256 | .989 | 8.7 | 14 | 3.229 |
| 6 | -.1256 | .740 | 8.7 | 6 | 0.838 |
| 7 | -.3854 | .548 | 8.7 | 9 | 0.010 |
| 8 | -.6745 | .393 | 8.7 | 7 | 0.332 |
| 9 | -1.0365 | .258 | 8.7 | 9 | 0.010 |
| 10 | -1.645 | .128 | 8.7 | 6 | 0.838 |
| 11 | $-\infty$ | 0 | 4.35 | 6 | 0.626 |
| | | | | 87 | $\chi^2$ = 7.137 P > .50 |

$\hat{x} = .8555$ $\log_{10} \hat{x} = .06727$ $s_L^2 = .5015$ $v = n-3 = 8$

The results of the chi-square example in Table 3.2-2 would lead to acceptance of the hypothesis that the data are lognormally distributed. Further testing of the hypothesis with 10 other data sets produced acceptance at the .25 probability level 9 out of 10 times and marginal acceptance at the .01 probability level the tenth time. Several tests showed probabilities greater than .75.

Both the cumulative frequency distribution and significance ratio (d-test and t-ratio) were not considered appropriate for the problem at hand. The first chi-square test, on the other hand, appeared promising but turned out to be inconclusive. However the second chi-square test indicates that the data are lognormally distributed, which is strong evidence leading toward validation of all the formulas in Section 2 (page 9).

### 3.2.4 Empirical

Examples 1 and 2 in Section 2.4 (page 18) are typical of the good results that were obtained by empirical testing, and similar results will be obtained by carefully using this method. That is, not only do the K factors have to be properly calculated, but care must also be exercised in adjusting the failure rates of similar equipment for differences in construction, if failure data on the same equipment is not available.

### 3.2.5 Data Acceptance/Rejection

Failure rates based on field experience data have been observed to vary by a factor of 10 or greater within a set of samples presumed to have come from the same population. This led to the question of whether or not to consider extreme data points as being outside the main body of data and therefore to reject them from the calculations.

This problem of inclusion or deletion of extreme data points was approached in the following manner. Techniques for processing data with extreme values when sample sizes are small were reviewed. The technique selected was an r-test (described in Chapter 16 of reference 7) which is based on a ratio comparison of the distance from the end data points to their neighbors to the total range of all the data points. This ratio establishes a probability, at a desired confidence level, that the end observation is from the same population as the others. This test also requires a normal distribution, therefore the common logarithms of the data should be used with the table and ratio formulas in Appendix IV in applying this test.

This test should be used primarily to identify data points that should be rechecked to determine, if possible, the reason for the large deviation. The decision to accept or reject an extreme data point would then be made on the basis of the recheck.

## 3.3 Problem Areas

Certain problems or potential problems were discovered during the course of this effort and others were pointed out by specialists in the fields of RM&SS who reviewed the method before release. Such problems are listed and discussed in this section:

3.3.1 Data Limitations
3.3.2 Application Requirements
3.3.3 Assumptions (Section 2.2)
3.3.4 Application Limitations

### 3.3.1 Data Limitations

Field experience data is the foundation on which this K factor development effort is built, but even though the data is the best available, it does have shortcomings.

First, the data has reporting errors. These errors can be introduced by the person reporting the failure, maintenance action, or accident/incident, or by key punch operators, or anyone else along the line of data collection. Many obvious errors or this kind have been found and corrected to increase the validity of the data, but some remain undetected. Because most of the data is homogeneous, reporting errors will tend to cancel out in the K factor ratio process.

Sometimes there is the problem of finding field experience data for the desired application. Small data sets of 3 to 10 individual points are typical both for individual LRU's in an LRU group, used to make first level K factors, and for LRU groups in an equipment class, used to make composite K factors. However the small sample sizes are reflected in the confidence intervals associated with each K factor so that it is clear how much confidence can be placed in them.

### 3.3.2 Application Requirements

The greatest concern presented by RM&SS specialists who reviewed the initial draft of this K facto· approach was whether or not the K factors would fit the requi:ements of their particular program.

The reliability people were particularly concerned with the definition of failure that would be used to determine the reliability K factors. Apparently definitions vary from program to program and even within a program. However this has no effect on the validity of this K factor development method, because the raw input data can be processed in any manner to meet the definition of failure determined by a program, and a whole new set of reliability K factors can be calculated from this data by exactly the same method. Variations in reliability K factors due to changes in failure definition have not been investigated, therefore it is possible that the definition of failure has little effect on K factors. At any rate, the method is applicable to any program.

### 3.3.3 Assumptions (from Section 2.2, page 10)

Several basic assumptions were outlined in Section 2.2 concerning the data used and its applicability to K factor determination and these will be further discussed in this section.

1. Experience data sets reflect an integration of all subfactors which affect reliability statistics.

2. Reliability statistics vary primarily due to environmental effects while other contributing effects tend to cancel out when K factor ratios are taken.

Because experience data reflects all factors, there will be normal random variations in failure rates due to other than the basic application effects. Since the data sets are homogeneous, these other variations will tend to cancel in the K factor ratio taking process. The added uncertainties due to these normal variations will be reflected by slightly larger confidence intervals.

3. LRU's in a general equipment class are all affected similarly by changes in application such that one composite K factor will adequately represent the entire class.

An LRU class will be affected similarly by changes in application, because equipment with similar construction will have the same modes of failure and approximately the same number of failures depending on complexity and part count. However, the equipment construction can vary considerably even within a class and a composite K factor is only an average of the entire class.

4. That a direct relationship exists between failures and operating hours (constant failure rate).

Reliability "Bath Tub" Curve

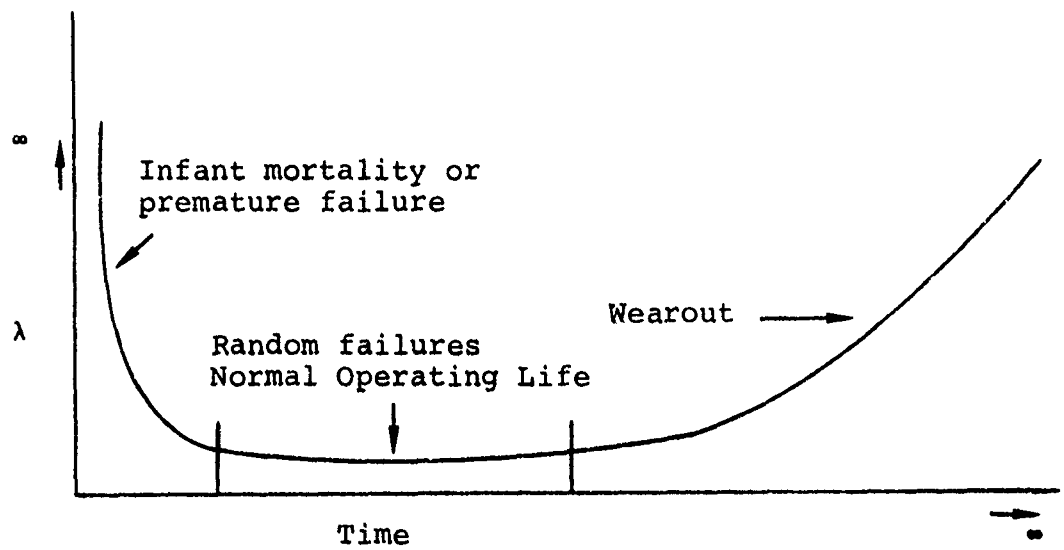


Figure 3.3-1

It is presumed that the equipment under consideration is operated in the central region of the reliability "Bath Tub" curve depicted by random failures occuring at a constant rate. This is accomplished by adequate screening and burn-in to catch the premature failures and time-scheduled removals to eliminate wearout problems. A problem comes from trying to identify the proper parameter to use as the time base. Operating hours has been used where possible but flight hours has been used for airborne environments and they both exclude storage, dormant, standby, warm up and checkout times. Further, some LRU's exhibit failures more as a function of cycles of operation than operating hours. Therefore cycles or some other measure could be better for some LRU's. However, in the past, system operating hours has proven to be a convenient base to work with and has produced satisfactory results, even though some error is introduced.

5. Failure rates are lognormally distributed.

In order to do any accurate hypothesis testing or statistical processing, it is necessary to make an assumption as to how the data is distributed. In Section 3.2.3 (page 27) an effort was made to show that failure rates appear to be lognormally distributed, which would make their common logarithms normally distributed. This is convenient because most statistical tests require that a sample be normally distributed. Because lognormality of all data sets is not proven, there may be another distribution that better describes some data sets. With little deviation from the lognormal in the samples investigated, the search for another distribution that might be better was not continued.

### 3.3.4 Subfactors That Impact Reliability

Application/environment K factors as developed in this study are really the integration of many parameters or subfactors which are all reflected in the field experience data, as stated in Assumption #1, Section 2.2 (page 11). It is important to recognize that these factors exist and that they do impact field failure rates, but it is not yet known how to evaluate and quantify their relative impact. Below is a list of subfactors that affect reliability. The list is not complete, but it does include many of the known subfactors.

| | | |
|---|---|---|
| complexity | state-of-the-art | personnel skill level |
| temperature | weapon system | on-off cycles |
| vibration | operating command | design stress level |
| utilization | repairability | grade of parts |
| duty cycle | mission type | burn-in |
| | phase of mission | |

### 3.3.5 Application Limitations

K factors developed by this technique are intended for only one purpose - to predict the failure rate of a device in an environment or application for which no failure data on that device currently exists. Any use of K factors other than for the one intended would result in a trade off of accuracy for other factors, some of which could conceivably include ease in data handling or time savings.

As a general rule to follow, when a failure rate is required, available data and K factors should be used as necessary to make a prediction. Failure to use factual data in prediction has resulted in many availability, reliability, maintainability and safety problems in current systems.



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## 4.0 CONCLUSIONS AND RECOMMENDATIONS

4.1 The following conclusions were drawn concerning the development and use of K factors:

1. A method for deriving environmental adjustment failure rate K factors has been developed.

2. The method has been validated by empirical testing against actual failure rates from field experience data.

3. The method is valid for any program or major equipment since new K factors can be calculated to fit the definitions and requirements set for each program.

### 4.2 Recommendations

1. In order to satisfy the requirement of processing selected data sets according to the specific definitions and requirements of each program, field experience data should be put in a mechanized file accessible by remote terminal for low cost, repetitive, rapid retrieval with convenient variable processing options.

2. Subfactors which have major impact on reliability should be investigated to determine their relative impact on the total K factor.

3. The basic technique developed in this research may be applicable to development of other types of K factors and application to other areas should be considered as the need arises and the resources become available.

## NOMENCLATURE

| | |
|---|---|
| $\alpha$ | - Type I error. The probability of rejecting a true hypothesis. |
| Application | - Intended use of an equipment (see Environment). |
| $\bar{\alpha}, \bar{\lambda}$ | - The arithmetic mean of a sample, i.e. the sum of all observations divided by the number of observations. |
| $\beta$ | - Type II error. The probability of accepting a false hypothesis. |
| Class | - A group of items alike in some way (see Equipment Classification, General). |
| Composite K Factor | - K factor developed from first level K factors taken from a small sampling of LRU groups within a general equipment classification and applicable to entire equipment classification. |
| Confidence Interval | - A range of values estimated from a random sample on the premise that the range will encompass a sought for true parameter of the sampled population a given percentage of times if the sampling process were to be repeated many times. |
| Confidence Level | - The percentage figure that expresses the probability or proportion of times a statement should be correct or that an estimated parameter lies within the given confidence interval. |
| Confidence Limits | - The upper and lower extremes of a confidence interval. |
| Environment | - The aggregate of all the conditions and influences which affect the operation of equipment, e.g. physical location, operating characteristics, shock, vibration, etc. Syn. application. |
| Equipment Classification, General | - Broadest grouping of equipment similarity, based solely on construction by predominant piece part classification. Examples: Electronic, Hydraulic, Mechanical, Electro-Mechanical, etc. |

| | |
|---|---|
| Failure Rate, $\lambda$ | - A figure of merit expressing the frequency of failure occurrences which can be observed over any specified time interval or number of operating cycles; e.g. average failures per 1000 flight hours. (see MTBF) |
| Field [Experience] Data | - Data accumulated as a result of normal operations; as opposed to data collected from laboratory controlled tests, accelerated life tests, etc. |
| First Level K Factor | - A K factor developed from failure rates taken from an LRU group and applicable only to equipment within the group. |
| Frequency Distribution Function | - (see Probability Distribution Function). |
| General Equipment Classification | - (see Equipment Classification, General). |
| [illegible], $\hat{\lambda}$, G. M. | - The geometric mean of a sample, i.e. the nth root of the product of n observations, (no observation can be zero). |
| K Factor | - 1. Any Logistics Performance factor. 2. Failure rate K factors are used to predict failure rates by utilizing failure rate data from the same/similar equipment from different applications and adjusting it for environmental differences. |
| Line Replaceable Unit, LRU | - 1. An equipment or assembly that is removed as a single unit and taken to a shop or similar facility for repair or maintenance. 2. A specific equipment, unique in construction and function. |
| LRU Group or Family | - LRU's with similar construction, similar functions and approximately equal failure rates. Failure rates from an LRU family are used to develop a single first level K factor. Examples: Hydraulic actuators, gyroscopes, check valves, etc. |

| | |
|---|---|
| MTBF | - The total number of operating hours of a population of equipments divided by the total number of failures within the population during the measured period of time. In most cases of interest, MTBF is the reciprocal of failure rate, MTBF = $1/\lambda$. |
| Parts, Piece Parts | - An article which is an element of an LRU or a subassembly of an LRU, and is of such construction that it is not practical or economically amenable to further disassemble for maintenance purposes. Examples: resistor, transformer, bearing. |
| pdf | - (see Probability Density Function) |
| Probability density function, pdf | - A curve or equation specifying the probability that a random variable will have a specific value. |
| Reliability Prediction | - To estimate beforehand the expected reliability value (failure rate) of an LRU. |
| Subfactors | - Identifiable effects that contribute to the overall K factor, but which have not been evaluated in this research effort. A K factor is an integration of all subfactors some of which include - utilization, duty cycle, vibration, temperature, etc. |

## APPENDIX I

A computer program for use with an HP 9100A programmable calculator. Outputs are: geometric mean ($\hat{a}$), mean confidence limits ($\hat{a}_u$, $\hat{a}_L$) and/or partition boundary limits ($P_u$, $P_L$).

| STEP | CODE | KEY | EXPLANATION | STEP | CODE | KEY | EXPLANATION |
|---|---|---|---|---|---|---|---|
| 00 | 20 | clear | clear | 24 | 15 | f | CONTINUE ← (A) |
| 01 | 23 | x→( ) | d,e,f,x,y,z | 25 | 27 | ↑ | $-(\Sigma a_i)^2$ |
| 02 | 17 | d | registers. | 26 | 32 | chg sign | |
| 03 | 01 | 1 | Set registers for start. | 27 | 36 | * | |
| 04 | 27 | ↑ | | 28 | 12 | e | |
| 05 | 02 | 2 | z = 1<br>y = 2<br>x = 3 | 29 | 27 | ↑ | $n\Sigma a_i^2$ |
| 06 | 27 | ↑ | | 2a | 17 | d | |
| 07 | 03 | 3 | | 2b | 36 | * | |
| 08 | 41 | STOP | ENTER $x = a_i$<br>CONTINUE | 2c | 25 | ↓ | $s_L^2 \cdot n(n-1)$ |
| 09 | 45 | PRINT | Print $a_i$ ← | 2d | 33 | + | |
| 0a | 27 | ↑ | Save $a_i$ | 30 | 17 | d | |
| 0b | 17 | d | | 31 | 27 | ↑ | |
| 0c | 27 | ↑ | Add 1 to | 32 | 01 | 1 | |
| 0d | 01 | 1 | counter, | 33 | 34 | - | $n(n-1)$ |
| 10 | 33 | + | i. | 34 | 17 | d | |
| 11 | 40 | y→( ) | | 35 | 36 | * | |
| 12 | 17 | d | | 36 | 25 | ↓ | |
| 13 | 22 | Roll↑ | Return and | 37 | 35 | ÷ | $s_L^2$ |
| 14 | 27 | ↑ | repeat $a_i$. | 38 | 25 | ↓ | |
| 15 | 75 | log x | Accumulate | 39 | 76 | √x | |
| 16 | 27 | ↑ | log $a_i$ in f | 3a | 23 | x→() | |
| 17 | 36 | * | and | 3b | 14 | b | STORE $s_L$ in register b. |
| 18 | 60 | acc + | $(\log a_i)^2$ in e | 3c | 15 | f | |
| 19 | 30 | x⟳y | Ready display. | 3d | 27 | ↑ | log $\hat{a}$ |
| 1a | 17 | d | $z = a_i$<br>$y = \log a_i$<br>$x = 1$ | 40 | 17 | d | |
| 1b | 41 | STOP | If entries completed | 41 | 35 | ÷ | |
| 1c | 43 | IF FLAG | SET FLAG | 42 | 40 | y→() | STORE log $\hat{a}$ in |
| 1d | 02 | 2 | and go to | 43 | 15 | f | register f. |
| 20 | 04 | 4 | next step. | | | | |
| 21 | 44 | GO TO()() | Otherwise, | | | | |
| 22 | 00 | 0 | enter next entry. | | | | |
| 23 | 11 | 9 | $x = a_i + 1$<br>Continue → (A) | | | | |

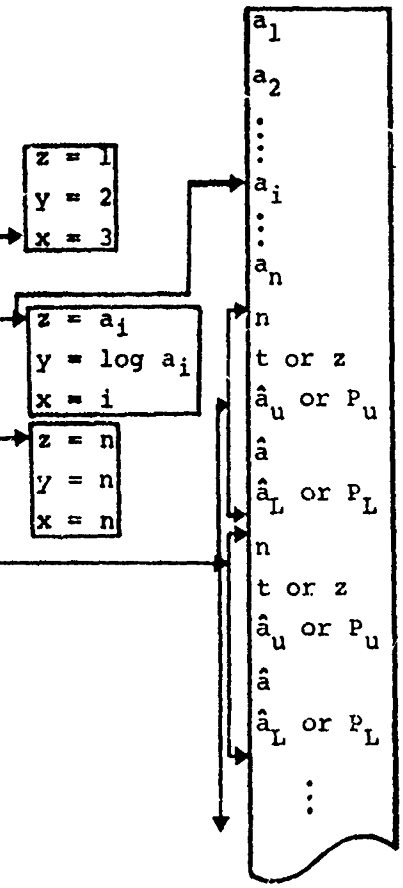
(Continued)

| STEP | CODE | KEY | EXPLANATION |
|---|---|---|---|
| 44 | 01 | 1 | |
| 45 | 00 | 0 | |
| 46 | 65 | ln x | |
| 47 | 36 | * | â |
| 48 | 25 | ↓ | |
| 49 | 74 | $e^x$ | |
| 4a | 23 | x→() | STORE â in register e. |
| 4b | 12 | e | |
| 4c | 17 | d | Ready display z = n, y = n, x = n ← (B) |
| 4d | 27 | ↑ | |
| 50 | 27 | ↑ | |
| 51 | 41 | STOP | ENTER x=t or SET FLAG x=z CONTINUE |
| 52 | 23 | x→() | STORE t or z in register c. |
| 53 | 16 | c | |
| 54 | 27 | ↑ | Save t or z. |
| 55 | 17 | d | Print n. |
| 56 | 45 | Print | |
| 57 | 16 | c | Print t or z. |
| 58 | 45 | Print | |
| 59 | 14 | b | |
| 5a | 36 | * | s·t or s·z |
| 5b | 43 | IF FLAG | If flag is set, z test to be done. |
| 5c | 06 | 6 | |
| 5d | 03 | 3 | |
| 60 | 17 | d | |
| 61 | 76 | $\sqrt{x}$ | $\sqrt{n}$ |
| 62 | 35 | + | $\epsilon$ |
| 63 | 27 | ↑ | Save $\epsilon$. |
| 64 | 25 | ↓ | |

| STEP | CODE | KEY | EXPLANATION |
|---|---|---|---|
| 65 | 15 | f | |
| 66 | 33 | ÷ | |
| 67 | 01 | 1 | |
| 68 | 00 | 0 | |
| 69 | 65 | ln x | Print $\hat{a}_u$. |
| 6a | 36 | * | |
| 6b | 25 | ↓ | |
| 6c | 74 | $e^x$ | |
| 6d | 45 | Print | |
| 70 | 12 | e | Print â. |
| 71 | 45 | Print | |
| 72 | 15 | f | |
| 73 | 30 | x⇄y | |
| 74 | 34 | − | |
| 75 | 01 | 1 | |
| 76 | 00 | 0 | |
| 77 | 65 | ln x | Print $\hat{a}_L$. |
| 78 | 36 | * | |
| 79 | 25 | ↓ | |
| 7a | 74 | $e^x$ | |
| 7b | 45 | Print | |
| 7c | 44 | GO TO ()() | Return for new t or z. → (B) |
| 7d | 04 | 4 | |
| 80 | 16 | c | |
| 81 | 46 | END | End of Program. |

PROGRAM EXECUTION STEPS

1. SWITCH ON.
2. PRINTER x ON.
3. SWITCH RUN.
4. GO TO 0,0.
5. SWITCH PROGRAM.
6. ENTER PROGRAM B.
7. SWITCH RUN.
8. END
9. CONTINUE
*10. ENTER [x = $a_i$]
11. CONTINUE
12. REPEAT STEPS 10→11, or
13. SET FLAG
14. CONTINUE
*15. ENTER [x = z] SET FLAG, or
ENTER [x = t]
16. CONTINUE
17. RETURN TO STEP 15, or
18. END
19. RETURN TO STEP 9, or
20. SWITCH PRINTER OFF
21. SWITCH OFF




*Note: Use CLEAR X key only,
CLEAR key destroys program.

# APPENDIX II

## Condensed t-table

| n-1 | Two Sided Confidence Limit, % 1 - α | | | | | n-1 |
|---|---|---|---|---|---|---|
| | 80 | 90 | 95 | 99 | 99.5 | |
| 1 | 3.078 | 6.314 | 12.706 | 63.657 | 127.32 | 1 |
| 2 | 1.886 | 2.920 | 4.303 | 9.925 | 14.089 | 2 |
| 3 | 1.638 | 2.353 | 3.182 | 5.841 | 7.453 | 3 |
| 4 | 1.533 | 2.132 | 2.776 | 4.604 | 5.598 | 4 |
| 5 | 1.476 | 2.015 | 2.571 | 4.032 | 4.773 | 5 |
| 6 | 1.440 | 1.943 | 2.447 | 3.707 | 4.317 | 6 |
| 7 | 1.415 | 1.895 | 2.365 | 3.499 | 4.029 | 7 |
| 8 | 1.397 | 1.860 | 2.306 | 3.355 | 3.832 | 8 |
| 9 | 1.383 | 1.833 | 2.262 | 3.250 | 3.690 | 9 |
| 10 | 1.372 | 1.812 | 2.228 | 3.169 | 3.581 | 10 |
| 11 | 1.363 | 1.796 | 2.201 | 3.106 | 3.497 | 11 |
| 12 | 1.356 | 1.782 | 2.179 | 3.055 | 3.428 | 12 |
| 13 | 1.350 | 1.771 | 2.160 | 3.012 | 3.372 | 13 |
| 14 | 1.345 | 1.761 | 2.145 | 2.977 | 3.326 | 14 |
| 15 | 1.341 | 1.753 | 2.131 | 2.947 | 3.286 | 15 |
| 16 | 1.337 | 1.746 | 2.120 | 2.921 | 3.252 | 16 |
| 17 | 1.333 | 1.740 | 2.110 | 2.898 | 3.222 | 17 |
| 18 | 1.330 | 1.734 | 2.101 | 2.878 | 3.197 | 18 |
| 19 | 1.328 | 1.729 | 2.093 | 2.861 | 3.174 | 19 |
| 20 | 1.325 | 1.725 | 2.086 | 2.845 | 3.153 | 20 |
| 21 | 1.323 | 1.721 | 2.080 | 2.831 | 3.135 | 21 |
| 22 | 1.321 | 1.717 | 2.074 | 2.819 | 3.119 | 22 |
| 23 | 1.319 | 1.714 | 2.069 | 2.807 | 3.104 | 23 |
| 24 | 1.318 | 1.711 | 2.064 | 2.797 | 3.090 | 24 |
| 25 | 1.316 | 1.708 | 2.060 | 2.787 | 3.078 | 25 |
| 26 | 1.315 | 1.706 | 2.056 | 2.779 | 3.067 | 26 |
| 27 | 1.314 | 1.703 | 2.052 | 2.771 | 3.056 | 27 |
| 28 | 1.313 | 1.701 | 2.048 | 2.763 | 3.047 | 28 |
| 29 | 1.311 | 1.699 | 2.045 | 2.756 | 3.038 | 29 |
| 30 | 1.310 | 1.697 | 2.042 | 2.750 | 3.030 | 30 |
| 40 | 1.303 | 1.684 | 2.021 | 2.704 | 2.971 | 40 |
| 60 | 1.296 | 1.671 | 2.000 | 2.660 | 2.915 | 60 |
| 120 | 1.289 | 1.658 | 1.980 | 2.617 | 2.860 | 120 |
| ∞ | 1.282 | 1.645 | 1.960 | 2.576 | 2.807 | ∞ |

# APPENDIX III

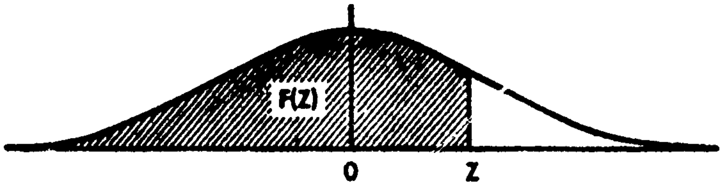



**TABLE 2-II CUMULATIVE NORMAL DISTRIBUTION**

| z | .000 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| .0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| .1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5336 | .5675 | .5714 | .5753 |
| .2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| .3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| .4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| .5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| .6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| .7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| .8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| .9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9206 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9699 | .9608 | .9616 | .9625 | .9533 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9876 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |

$$F(Z) = \int_{-\infty}^{Z} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \, dz = 1 - \alpha$$

## APPENDIX IV*

| Statistic | Number of Observations k | Critical Values $\alpha$ = .30 | $\alpha$ = .20 | $\alpha$ = .10 | $\alpha$ = .05 | $\alpha$ = .02 | $\alpha$ = .01 | $\alpha$ = .005 |
|---|---|---|---|---|---|---|---|---|
| $r_{10} = \frac{X_2 - X_1}{X_k - X_1}$ | 3 | .684 | .781 | .886 | .941 | .976 | .988 | .994 |
| | 4 | .471 | .560 | .679 | .765 | .846 | .889 | .926 |
| | 5 | .373 | .451 | .557 | .642 | .729 | .780 | .821 |
| | 6 | .318 | .386 | .482 | .560 | .644 | .698 | .740 |
| | 7 | .281 | .344 | .434 | .507 | .586 | .637 | .680 |
| $r_{11} = \frac{X_2 - X_1}{X_{k-1} - X_1}$ | 8 | .318 | .385 | .479 | .554 | .631 | .683 | .725 |
| | 9 | .288 | .352 | .441 | .512 | .587 | .635 | .677 |
| | 10 | .265 | .325 | .409 | .477 | .551 | .597 | .639 |
| $r_{21} = \frac{X_3 - X_1}{X_{k-1} - X_1}$ | 11 | .391 | .442 | .517 | .576 | .638 | .679 | .713 |
| | 12 | .370 | .419 | .490 | .546 | .605 | .642 | .675 |
| | 13 | .351 | .399 | .467 | .521 | .578 | .615 | .649 |
| $r_{22} = \frac{X_3 - X_1}{X_{k-2} - X_1}$ | 14 | .370 | .421 | .492 | .546 | .602 | .641 | .674 |
| | 15 | .353 | .402 | .472 | .525 | .579 | .616 | .647 |
| | 16 | .338 | .386 | .454 | .507 | .559 | .595 | .624 |
| | 17 | .325 | .373 | .438 | .490 | .542 | .577 | .605 |
| | 18 | .314 | .361 | .424 | .475 | .527 | .561 | .589 |
| | 19 | .304 | .350 | .412 | .462 | .514 | .547 | .575 |
| | 20 | .295 | .340 | .401 | .450 | .502 | .535 | .562 |
| | 21 | .287 | .331 | .391 | .440 | .491 | .524 | .551 |
| | 22 | .280 | .323 | .382 | .430 | .481 | .514 | .541 |
| | 23 | .274 | .316 | .374 | .421 | .472 | .505 | .532 |
| | 24 | .268 | .310 | .367 | .413 | .464 | .497 | .524 |
| | 25 | .262 | .304 | .360 | .406 | .457 | .489 | .516 |

* From W. J. Dixon, "Processing Data for Outliers," Biometrics, Vol. 9 (1953), p. 74.

# APPENDIX V

## Cumulative Chi-Square Distribution

$$F(u) = \int_0^u \frac{x^{(n-2)/2} e^{-x/2}\, dx}{2^{n/2}(n-2)/2\,!}$$

| n \ F | .005 | .010 | .025 | .050 | .100 | .250 | .500 | .750 | .900 | .950 | .975 | .990 | .995 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | .0000393 | .000157 | .000982 | .00393 | .0158 | .102 | .455 | 1.32 | 2.71 | 3.84 | 5.02 | 6.63 | 7.88 |
| 2 | .0100 | .0201 | .0506 | .103 | .211 | .575 | 1.39 | 2.77 | 4.61 | 5.99 | 7.38 | 9.21 | 10.6 |
| 3 | .0717 | .115 | .216 | .352 | .584 | 1.21 | 2.37 | 4.11 | 6.25 | 7.81 | 9.35 | 11.3 | 12.8 |
| 4 | .207 | .297 | .484 | .711 | 1.06 | 1.92 | 3.36 | 5.39 | 7.78 | 9.49 | 11.1 | 13.3 | 14.9 |
| 5 | .412 | .554 | .831 | 1.15 | 1.61 | 2.67 | 4.35 | 6.63 | 9.24 | 11.1 | 12.8 | 15.1 | 16.7 |
| 6 | .676 | .872 | 1.24 | 1.64 | 2.20 | 3.45 | 5.35 | 7.84 | 10.6 | 12.6 | 14.4 | 16.8 | 18.5 |
| 7 | .989 | 1.24 | 1.69 | 2.17 | 2.83 | 4.25 | 6.35 | 9.04 | 12.0 | 14.1 | 16.0 | 18.5 | 20.3 |
| 8 | 1.34 | 1.65 | 2.18 | 2.73 | 3.49 | 5.07 | 7.34 | 10.2 | 13.4 | 15.5 | 17.5 | 20.1. | 22.0 |
| 9 | 1.73 | 2.09 | 2.70 | 3.33 | 4.17 | 5.90 | 8.34 | 11.4 | 14.7 | 16.9 | 19.0 | 21.7 | 23.6 |
| 10 | 2.16 | 2.56 | 3.25 | 3.94 | 4.87 | 6.74 | 9.34 | 12.5 | 16.0 | 18.3 | 20.5 | 23.2 | 25.2 |
| 11 | 2.60 | 3.05 | 3.82 | 4.57 | 5.58 | 7.58 | 10.3 | 13.7 | 17.3 | 19.7 | 21.9 | 24.7 | 26.8 |
| 12 | 3.07 | 3.57 | 4.40 | 5.23 | 6.30 | 8.44 | 11.3 | 14.8 | 18.5 | 21.0 | 23.3 | 26.2 | 28.3 |
| 13 | 3.57 | 4.11 | 5.01 | 5.89 | 7.04 | 9.30 | 12.3 | 16.0 | 19.8 | 22.4 | 24.7 | 27.7 | 29.8 |
| 14 | 4.07 | 4.66 | 5.63 | 6.57 | 7.79 | 10.2 | 13.3 | 17.1 | 21.1 | 23.7 | 26.1 | 29.1 | 31.3 |
| 15 | 4.60 | 5.23 | 6.26 | 7.26 | 8.55 | 11.0 | 14.3 | 18.2 | 22.3 | 25.0 | 27.5 | 30.6 | 32.8 |
| 16 | 5.14 | 5.81 | 6.91 | 7.96 | 9.31 | 11.9 | 15.3 | 19.4 | 23.5 | 26.3 | 28.8 | 32.0 | 34.3 |
| 17 | 5.70 | 6.41 | 7.56 | 8.67 | 10.1 | 12.8 | 16.3 | 20.5 | 24.8 | 27.6 | 30.2 | 33.4 | 35.7 |
| 18 | 6.26 | 7.01 | 8.23 | 9.39 | 10.9 | 13.7 | 17.3 | 21.6 | 26.0 | 28.9 | 31.5 | 34.8 | 37.2 |
| 19 | 6.84 | 7.63 | 8.91 | 10.1 | 11.7 | 14.6 | 18.3 | 22.7 | 27.2 | 30.1 | 32.9 | 36.2 | 38.6 |
| 20 | 7.43 | 8.26 | 9.59 | 10.9 | 12.4 | 15.5 | 19.3 | 23.8 | 28.4 | 31.4 | 34.2 | 37.6 | 40.0 |
| 21 | 8.03 | 8.90 | 10.3 | 11.6 | 13.2 | 16.3 | 20.3 | 24.9 | 29.6 | 32.7 | 35.5 | 38.9 | 41.4 |
| 22 | 8.64 | 9.54 | 11.0 | 12.3 | 14.0 | 17.2 | 21.3 | 26.0 | 30.8 | 33.9 | 36.8 | 40.3 | 42.8 |
| 23 | 9.26 | 10.2 | 11.7 | 13.1 | 14.8 | 18.1 | 22.3 | 27.1 | 32.0 | 35.2 | 38.1 | 41.6 | 44.2 |
| 24 | 9.89 | 10.9 | 12.4 | 13.8 | 15.7 | 19.0 | 23.3 | 28.2 | 33.2 | 36.4 | 39.4 | 43.0 | 45.6 |
| 25 | 10.5 | 11.5 | 13.1 | 14.6 | 16.5 | 19.9 | 24.3 | 29.3 | 34.4 | 37.7 | 40.6 | 44.3 | 46.9 |
| 26 | 11.2 | 12.2 | 13.8 | 15.4 | 17.3 | 20.8 | 25.3 | 30.4 | 35.6 | 38.9 | 41.9 | 45.6 | 48.3 |
| 27 | 11.8 | 12.9 | 14.6 | 16.2 | 18.1 | 21.7 | 26.3 | 31.5 | 36.7 | 40.1 | 43.2 | 47.0 | 49.6 |
| 28 | 12.5 | 13.6 | 15.3 | 16.9 | 18.9 | 22.7 | 27.3 | 32.6 | 37.9 | 41.3 | 44.5 | 48.3 | 51.0 |
| 29 | 13.1 | 14.3 | 16.0 | 17.7 | 19.8 | 23.6 | 28.3 | 33.7 | 39.1 | 42.6 | 45.7 | 49.6 | 52.3 |
| 30 | 13.8 | 15.0 | 16.8 | 18.5 | 20.6 | 24.5 | 29.3 | 34.8 | 40.3 | 43.8 | 47.0 | 50.9 | 53.7 |

## APPENDIX VI
### Common Logarithms

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 0000 | 0043 | 0086 | 0128 | 0170 | 0212 | 0253 | 0294 | 0334 | 0374 |
| 11 | 0414 | 0453 | 0492 | 0531 | 0569 | 0607 | 0645 | 0682 | 0719 | 0755 |
| 12 | 0792 | 0828 | 0864 | 0899 | 0934 | 0969 | 1004 | 1038 | 1072 | 1106 |
| 13 | 1139 | 1173 | 1206 | 1239 | 1271 | 1303 | 1335 | 1367 | 1399 | 1430 |
| 14 | 1461 | 1492 | 1523 | 1553 | 1584 | 1614 | 1644 | 1673 | 1703 | 1732 |
| 15 | 1761 | 1790 | 1818 | 1847 | 1875 | 1903 | 1931 | 1959 | 1987 | 2014 |
| 16 | 2041 | 2068 | 2095 | 2122 | 2148 | 2175 | 2201 | 2227 | 2253 | 2279 |
| 17 | 2304 | 2330 | 2355 | 2380 | 2405 | 2430 | 2455 | 2480 | 2504 | 2529 |
| 18 | 2553 | 2577 | 2601 | 2625 | 2648 | 2672 | 2695 | 2718 | 2742 | 2765 |
| 19 | 2788 | 2810 | 2833 | 2856 | 2878 | 2900 | 2923 | 2945 | 2967 | 2989 |
| 20 | 3010 | 3032 | 3054 | 3075 | 3096 | 3118 | 3139 | 3160 | 3181 | 3201 |
| 21 | 3222 | 3243 | 3263 | 3284 | 3304 | 3324 | 3345 | 3365 | 3385 | 3404 |
| 22 | 3424 | 3444 | 3464 | 3483 | 3502 | 3522 | 3541 | 3560 | 3579 | 3598 |
| 23 | 3617 | 3636 | 3655 | 3674 | 3692 | 3711 | 3729 | 3747 | 3766 | 3784 |
| 24 | 3802 | 3820 | 3838 | 3856 | 3874 | 3892 | 3909 | 3927 | 3945 | 3962 |
| 25 | 3979 | 3997 | 4014 | 4031 | 4048 | 4065 | 4082 | 4099 | 4116 | 4133 |
| 26 | 4150 | 4166 | 4183 | 4200 | 4216 | 4232 | 4249 | 4265 | 4281 | 4298 |
| 27 | 4314 | 4330 | 4346 | 4362 | 4378 | 4393 | 4409 | 4425 | 4440 | 4456 |
| 28 | 4472 | 4487 | 4502 | 4518 | 4533 | 4548 | 4564 | 4579 | 4594 | 4609 |
| 29 | 4624 | 4639 | 4654 | 4669 | 4683 | 4698 | 4713 | 4728 | 4742 | 4757 |
| 30 | 4771 | 4786 | 4800 | 4814 | 4829 | 4843 | 4857 | 4871 | 4886 | 4900 |
| 31 | 4914 | 4928 | 4942 | 4955 | 4969 | 4983 | 4997 | 5011 | 5024 | 5038 |
| 32 | 5051 | 5065 | 5079 | 5092 | 5105 | 5119 | 5132 | 5145 | 5159 | 5172 |
| 33 | 5185 | 5198 | 5211 | 5224 | 5237 | 5250 | 5263 | 5276 | 5289 | 5302 |
| 34 | 5315 | 5328 | 5340 | 5353 | 5366 | 5378 | 5391 | 5403 | 5416 | 5428 |
| 35 | 5441 | 5453 | 5465 | 5478 | 5490 | 5502 | 5514 | 5527 | 5539 | 5551 |
| 36 | 5563 | 5575 | 5587 | 5599 | 5611 | 5623 | 5635 | 5647 | 5658 | 5670 |
| 37 | 5682 | 5694 | 5705 | 5717 | 5729 | 5740 | 5752 | 5763 | 5775 | 5786 |
| 38 | 5798 | 5809 | 5821 | 5832 | 5843 | 5855 | 5866 | 5877 | 5888 | 5899 |
| 39 | 5911 | 5922 | 5933 | 5944 | 5955 | 5966 | 5977 | 5988 | 5999 | 6010 |
| 40 | 6021 | 6031 | 6042 | 6053 | 6064 | 6075 | 6085 | 6096 | 6107 | 6117 |
| 41 | 6128 | 6138 | 6149 | 6160 | 6170 | 6180 | 6191 | 6201 | 6212 | 6222 |
| 42 | 6232 | 6243 | 6253 | 6263 | 6274 | 6284 | 6294 | 6304 | 6314 | 6325 |
| 43 | 6335 | 6345 | 6355 | 6365 | 6375 | 6385 | 6395 | 6405 | 6415 | 6425 |
| 44 | 6435 | 6444 | 6454 | 6464 | 6474 | 6484 | 6493 | 6503 | 6513 | 6522 |
| 45 | 6532 | 6542 | 6551 | 6561 | 6571 | 6580 | 6590 | 6599 | 6609 | 6618 |
| 46 | 6628 | 6637 | 6646 | 6656 | 6665 | 6675 | 6684 | 6693 | 6702 | 6712 |
| 47 | 6721 | 6730 | 6739 | 6749 | 6758 | 6767 | 6776 | 6785 | 6794 | 6803 |
| 48 | 6812 | 6821 | 6830 | 6839 | 6848 | 6857 | 6866 | 6875 | 6884 | 6893 |
| 49 | 6902 | 6911 | 6920 | 6928 | 6937 | 6946 | 6955 | 6964 | 6972 | 6981 |
| 50 | 6990 | 6998 | 7007 | 7016 | 7024 | 7033 | 7042 | 7050 | 7059 | 7067 |
| 51 | 7076 | 7084 | 7093 | 7101 | 7110 | 7118 | 7126 | 7135 | 7143 | 7152 |
| 52 | 7160 | 7168 | 7177 | 7185 | 7193 | 7202 | 7210 | 7218 | 7226 | 7235 |
| 53 | 7243 | 7251 | 7259 | 7267 | 7275 | 7284 | 7292 | 7300 | 7308 | 7316 |
| 54 | 7324 | 7332 | 7340 | 7348 | 7356 | 7364 | 7372 | 7380 | 7388 | 7396 |
| 55 | 7404 | 7412 | 7419 | 7427 | 7435 | 7443 | 7451 | 7459 | 7466 | 7474 |
| 56 | 7482 | 7490 | 7497 | 7505 | 7513 | 7520 | 7528 | 7536 | 7543 | 7551 |
| 57 | 7559 | 7566 | 7574 | 7582 | 7589 | 7597 | 7604 | 7612 | 7619 | 7627 |
| 58 | 7634 | 7642 | 7649 | 7657 | 7664 | 7672 | 7679 | 7686 | 7694 | 7701 |
| 59 | 7709 | 7716 | 7723 | 7731 | 7738 | 7745 | 7752 | 7760 | 7767 | 7774 |
| 60 | 7782 | 7789 | 7796 | 7803 | 7810 | 7818 | 7825 | 7832 | 7839 | 7846 |
| 61 | 7853 | 7860 | 7868 | 7875 | 7882 | 7889 | 7896 | 7903 | 7910 | 7917 |
| 62 | 7924 | 7931 | 7938 | 7945 | 7952 | 7959 | 7966 | 7973 | 7980 | 7987 |
| 63 | 7993 | 8000 | 8007 | 8014 | 8021 | 8028 | 8035 | 8041 | 8048 | 8055 |
| 64 | 8062 | 8069 | 8075 | 8082 | 8089 | 8096 | 8102 | 8109 | 8116 | 8122 |
| 65 | 8129 | 8136 | 8142 | 8149 | 8156 | 8162 | 8169 | 8176 | 8182 | 8189 |
| 66 | 8195 | 8202 | 8209 | 8215 | 8222 | 8228 | 8235 | 8241 | 8248 | 8254 |
| 67 | 8261 | 8267 | 8274 | 8280 | 8287 | 8293 | 8299 | 8306 | 8312 | 8319 |
| 68 | 8325 | 8331 | 8338 | 8344 | 8351 | 8357 | 8363 | 8370 | 8376 | 8382 |
| 69 | 8388 | 8395 | 8401 | 8407 | 8414 | 8420 | 8426 | 8432 | 8439 | 8445 |
| 70 | 8451 | 8457 | 8463 | 8470 | 8476 | 8482 | 8488 | 8494 | 8500 | 8506 |
| 71 | 8513 | 8519 | 8525 | 8531 | 8537 | 8543 | 8549 | 8555 | 8561 | 8567 |
| 72 | 8573 | 8579 | 8585 | 8591 | 8597 | 8603 | 8609 | 8615 | 8621 | 8627 |
| 73 | 8633 | 8639 | 8645 | 8651 | 8657 | 8663 | 8669 | 8675 | 8681 | 8686 |
| 74 | 8692 | 8698 | 8704 | 8710 | 8716 | 8722 | 8727 | 8733 | 8739 | 8745 |
| 75 | 8751 | 8756 | 8762 | 8768 | 8774 | 8779 | 8785 | 8791 | 8797 | 8802 |
| 76 | 8808 | 8814 | 8820 | 8825 | 8831 | 8837 | 8842 | 8848 | 8854 | 8859 |
| 77 | 8865 | 8871 | 8876 | 8882 | 8887 | 8893 | 8899 | 8904 | 8910 | 8915 |
| 78 | 8921 | 8927 | 8932 | 8938 | 8943 | 8949 | 8954 | 8960 | 8965 | 8971 |
| 79 | 8976 | 8982 | 8987 | 8993 | 8998 | 9004 | 9009 | 9015 | 9020 | 9025 |
| 80 | 9031 | 9036 | 9042 | 9047 | 9053 | 9058 | 9063 | 9069 | 9074 | 9079 |
| 81 | 9085 | 9090 | 9096 | 9101 | 9106 | 9112 | 9117 | 9122 | 9128 | 9133 |
| 82 | 9138 | 9143 | 9149 | 9154 | 9159 | 9165 | 9170 | 9175 | 9180 | 9186 |
| 83 | 9191 | 9196 | 9201 | 9206 | 9212 | 9217 | 9222 | 9227 | 9232 | 9238 |
| 84 | 9243 | 9248 | 9253 | 9258 | 9263 | 9269 | 9274 | 9279 | 9284 | 9289 |
| 85 | 9294 | 9299 | 9304 | 9309 | 9315 | 9320 | 9325 | 9330 | 9335 | 9340 |
| 86 | 9345 | 9350 | 9355 | 9360 | 9365 | 9370 | 9375 | 9380 | 9385 | 9390 |
| 87 | 9395 | 9400 | 9405 | 9410 | 9415 | 9420 | 9425 | 9430 | 9435 | 9440 |
| 88 | 9445 | 9450 | 9455 | 9460 | 9465 | 9469 | 9474 | 9479 | 9484 | 9489 |
| 89 | 9494 | 9499 | 9504 | 9509 | 9513 | 9518 | 9523 | 9528 | 9533 | 9538 |
| 90 | 9542 | 9547 | 9552 | 9557 | 9562 | 9566 | 9571 | 9576 | 9581 | 9586 |
| 91 | 9590 | 9595 | 9600 | 9605 | 9609 | 9614 | 9619 | 9624 | 9628 | 9633 |
| 92 | 9638 | 9643 | 9647 | 9652 | 9657 | 9661 | 9666 | 9671 | 9675 | 9680 |
| 93 | 9685 | 9689 | 9694 | 9699 | 9703 | 9708 | 9713 | 9717 | 9722 | 9727 |
| 94 | 9731 | 9736 | 9741 | 9745 | 9750 | 9754 | 9759 | 9763 | 9768 | 9773 |
| 95 | 9777 | 9782 | 9786 | 9791 | 9795 | 9800 | 9805 | 9809 | 9814 | 9818 |
| 96 | 9823 | 9827 | 9832 | 9836 | 9841 | 9845 | 9850 | 9854 | 9859 | 9863 |
| 97 | 9868 | 9872 | 9877 | 9881 | 9886 | 9890 | 9894 | 9899 | 9903 | 9908 |
| 98 | 9912 | 9917 | 9921 | 9926 | 9930 | 9934 | 9939 | 9943 | 9948 | 9952 |
| 99 | 9956 | 9961 | 9965 | 9969 | 9974 | 9978 | 9983 | 9987 | 9991 | 9996 |

## APPENDIX VII

Failure Rate Data for 87 Aircraft Gyroscopes

| # | Flight Hours | Failures | Failures/ 1000 Hrs. | # | Flight Hours | Failures | Failures/ 1000 Hrs. |
|---|---|---|---|---|---|---|---|
| 1 | 63,905 | 55 | .861 | 45 | 97,318 | 298 | 3.062 |
| 2 | 63,905 | 78 | 1.221 | 46 | 17,787 | 123 | 6.915 |
| 3 | 63,905 | 56 | .876 | 47 | 17,787 | 53 | 2.980 |
| 4 | 99,646 | 103 | 1.034 | 48 | 37,013 | 280 | 7.565 |
| 5 | 99,646 | 91 | .913 | 49 | 37,013 | 129 | 3.485 |
| 6 | 99,646 | 117 | 1.174 | 50 | 52,947 | 352 | 6.648 |
| 7 | 156,763 | 74 | .472 | 51 | 52,947 | 116 | 2.191 |
| 8 | 156,763 | 65 | .415 | 52 | 142,190 | 59 | .415 |
| 9 | 156,763 | 70 | .447 | 53 | 142,190 | 18 | .127 |
| 10 | 106,276 | 82 | .772 | 54 | 142,190 | 28 | .197 |
| 11 | 106,276 | 159 | 1.496 | 55 | 142,190 | 156 | 1.097 |
| 12 | 106,276 | 122 | 1.148 | 56 | 142,190 | 206 | 1.449 |
| 13 | 192,241 | 102 | .531 | 57 | 142,190 | 138 | .971 |
| 14 | 192,241 | 193 | 1.004 | 58 | 142,190 | 200 | 1.407 |
| 15 | 192,241 | 96 | .499 | 59 | 121,609 | 10 | .082 |
| 16 | 16,977 | 221 | 13.018 | 60 | 121,609 | 41 | .337 |
| 17 | 33,954 | 15 | .442 | 61 | 121,609 | 338 | 2.779 |
| 18 | 50,931 | 2 | .039 | 62 | 121,609 | 159 | 1.307 |
| 19 | 100,319 | 33 | .329 | 63 | 121,609 | 186 | 1.529 |
| 20 | 100,319 | 101 | 1.007 | 64 | 21,527 | 26 | 1.208 |
| 21 | 100,319 | 60 | .598 | 65 | 43,054 | 68 | 1.579 |
| 22 | 20,807 | 80 | 3.845 | 66 | 43,054 | 11 | .255 |
| 23 | 58,481 | 25 | .427 | 67 | 64,581 | 21 | .325 |
| 24 | 58,481 | 85 | 1.453 | 68 | 27,575 | 44 | 1.596 |
| 25 | 58,481 | 70 | 1.197 | 69 | 55,150 | 21 | .381 |
| 26 | 20,330 | 88 | 4.329 | 70 | 55,150 | 63 | 1.142 |
| 27 | 79,899 | 60 | .751 | 71 | 82,725 | 25 | .302 |
| 28 | 79,899 | 161 | 2.015 | 72 | 284,382 | 893 | 3.140 |
| 29 | 79,899 | 107 | 1.339 | 73 | 284,382 | 70 | .246 |
| 30 | 6,055 | 3 | .495 | 74 | 568,764 | 29 | .051 |
| 31 | 38,508 | 5 | .130 | 75 | 568,764 | 259 | .455 |
| 32 | 38,508 | 4 | .104 | 76 | 853,146 | 1060 | 1.242 |
| 33 | 40,939 | 24 | .586 | 77 | 70,279 | 294 | 4.183 |
| 34 | 40,939 | 49 | 1.197 | 78 | 70,279 | 11 | .157 |
| 35 | 40,939 | 30 | .733 | 79 | 140,558 | 292 | 2.077 |
| 36 | 13,950 | 15 | 1.075 | 80 | 140,558 | 48 | .341 |
| 37 | 98,584 | 224 | 2.272 | 81 | 210,837 | 93 | .441 |
| 38 | 98,584 | 37 | .375 | 82 | 317,109 | 570 | 1.797 |
| 39 | 338,854 | 227 | .670 | 83 | 317,109 | 844 | 2.662 |
| 40 | 338,854 | 18 | .053 | 84 | 634,218 | 933 | 1.471 |
| 41 | 38,116 | 132 | 3.463 | 85 | 951,327 | 687 | .722 |
| 42 | 38,116 | 24 | .630 | 86 | 92,806 | 345 | 3.717 |
| 43 | 73,650 | 272 | 3.693 | 87 | 139,209 | 34 | .244 |
| 44 | 73,650 | 50 | .679 | 87 | 11,986,565 | 13,316 | 134.084 |

$\hat{x} = .855 \qquad s_L^2 = .2516$

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## ACTIVE SHEET RECORD

| SHEET NO | REV LTR | ADDED SHEETS SHEET NO. | REV LTR | SHEET NO. | REV LTR | SHEET NO. | REV LTR | ADDED SHEETS SHEET NO. | REV LTR | SHEET NO. | REV LTR |
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