DEPARTMENT OF THE SCIENTIFIC ADVISER TO THE ARMY COUNCIL

ARMY OPERATIONAL RESEARCH GROUP

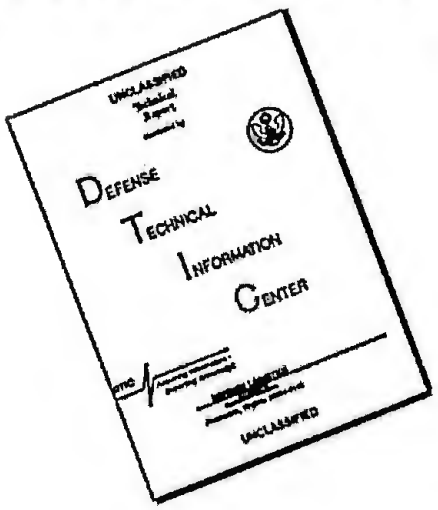
REPORT No. 6/52

TANK EFFECTIVENESS

A COMPARISON OF THE THEORETICAL MEASURE WITH OBSERVED BATTLE PERFORMANCE AND A FURTHER NOTE ON RATE OF FIRE.

1952

DISCLAIMER NOTICE



THIS DOCUMENT IS BEST QUALITY AVAILABLE. THE COPY FURNISHED TO DTIC CONTAINED A SIGNIFICANT NUMBER OF PAGES WHICH DO NOT REPRODUCE LEGIBLY.

ARMY OPERATIONAL RESEARCH GROUP

REPORT NO. 6/52

TANK EFFECTIVENESS

A COMPARISON OF THE THEORETICAL MEASURE WITH OBSERVED BATTLE PERFORMANCE AND A FURTHER NOTE ON RATE OF FIRE

CONTENTS

Study No. 262: Assessment of forms of A/tk defence

Requested by DWD

8G/Research/656(SA/AG)

UNCLASSIFIED
~~SECRET~~

ARMY OPERATIONAL RESEARCH GROUP

REPORT NO. 6/52

TANK EFFECTIVENESS

A COMPARISON OF THE THEORETICAL MEASURE WITH OBSERVED BATTLE PERFORMANCE AND A FURTHER NOTE ON RATE OF FIRE

Prepared by :- D. Benn
R.W. Shephard

ABSTRACT

In AORG Report No. 21/50 an attempt was made to derive on theoretical grounds a measure of Effectiveness that would be indicative of the value of a tank in the tank v. tank battle. In this paper, predictions made from this theoretical measure are compared with actual measures of performance obtained from battle data of World War II.

Although the data available have been too limited to allow a complete validation of the theory to be made, the comparisons that are presented are sufficient to show that the general trends and levels of performance predicted by the measure of Effectiveness are well indicative of what can be expected in battle; there is as yet no reason to suspect that any major modifications in the concept will be needed.

In view of certain comments that have been received, a modified expression has been derived which gives values for rates of fire in the formula for Effectiveness; although in many instances it corresponds to only a small order correction, it is suggested that this expression should be used in preference to the ratio of the cyclic rates of fire, as used hitherto. In the light of this, it has been thought desirable to recalculate the figures for the Effectiveness of British and Russian tanks given in AORG Report No. 11/51. It is found that the general effect of using the modified expression is to reduce slightly the Effectiveness of the British tanks; the Effectiveness of the Centurion 3 against the JS3, for example, is reduced from 1.3 to 1.2 (at 1000 yards); the general conclusions of Report No. 11/51, however, remain unaltered.

It is concluded that the measure of Effectiveness may now be used to give a good general indication of the relative merits of opposing tanks in battle. Confidence in future predictions of values for Effectiveness may perhaps now be considered to be limited primarily by the possible inaccuracies in the data for enemy equipments.

UNCLASSIFIED

ARMY OPERATIONAL RESEARCH GROUP

REPORT NO. 6/52

TANK EFFECTIVENESS

A COMPARISON OF THE THEORETICAL MEASURE WITH OBSERVED BATTLE PERFORMANCE AND A FURTHER NOTE ON RATE OF FIRE

Prepared by :- E. Benn
R.W. Shephard

INTRODUCTION

1. In an earlier paper* an attempt was made to derive on theoretical grounds a measure of performance that would be indicative of the value of a tank in the tank v. tank battle. This measure, termed Effectiveness, was defined as "the reciprocal of the number of tanks (of the given class) required per enemy tank to achieve parity in battle"; it was denoted by the symbol E. Thus in engagements between 'n' of the given class of tank and 'n x E' enemy tanks, the chances of success would, on an average, be the same for both sides.

2. This concept is, by definition, concerned only with performance in battle; the basic theory was therefore developed primarily in terms of fire-power and protection; no account was taken of mobility (the term being used in its widest sense), since this was considered unlikely to be an important factor in the battle itself.

3. An expression for Effectiveness was derived which was characteristic of the guns and armour of the tanks concerned and did not depend on the numbers engaged. In order to derive this expression, certain simplifying assumptions were made; for example, factors such as tactical skill, surprise, concealment, etc., were ignored, since their influence is quite distinct from that of the inherent differences between the tanks. Such factors will, however, give an advantage to one side or the other in any particular battle. It will be clear therefore that the measure derived is concerned only with average values and cannot be used directly to predict the outcome of a particular battle. It can perhaps better be used to predict the overall average outcome of a number of battles.

4. Any appraisement of the value of a measure of this form, and of the adequacy of the assumptions on which it is based, must rest ultimately on an assessment of the accuracy of the predictions that can be made. Such an assessment will be described in this paper. Certain predictions made from the basic theory of Effectiveness will be compared with actual measures of performance obtained from battle data for Allied and German tanks in World War II.

---

* AORG Report No. 21/50. "Tanks in the A/Tk Role : A Measure of Effectiveness".

DATA ON BRITISH AND GERMAN TANKS

Battle data from World War II

5. A considerable amount of information on tank v. tank actions of the last war has been derived from an analysis of Allied war diaries. Full details of this information are given in ÆORU Report No. 33: "Tank Battle Analysis" and in an ÆORG Memorandum : "A Survey of the Tank Warfare in Europe" (to be published shortly). For about 100 of the actions recorded, the details are sufficient to indicate the types of tank involved, the initial allied and enemy strengths, and the casualties suffered by each side. Some 20 of these actions are one v. one battles, and the data for these must, by the very nature of the source of information, constitute a biased sample; this group of actions has not therefore been considered here. Details of the remaining 79 actions, which form the basis of the present analysis, are given in Table 1 below.

TABLE 1

Tank Casualties in German v. British Tank Actions

(X_0 and A_0 are numbers of German and British tanks committed; X and A are numbers remaining.)

Serial No.	Tanks Engaged	Average Range (yards)	Numbers of tanks				Sample of actions
			X_0	X	A_0	A	
1	Mixed v. Sherman (17 pr) (a)	600 - 1000	35	18	49	42	9
2	Pz Kw IV v. Sherman Cromwell (75 mm) (b)	1000 - 1500	11	5	14	9	6
3	Pz Kw V v. Sherman (75 mm + 17 pr) (c)	600 - 1000	53	22	172	160	17
4	Mixed v. Sherman (75 mm + 17 pr)	600 +	105	65	186	137	15
5	Pz Kw VI v. Sherman (75 mm + 17 pr) (d)	600	67	40	148	126	20
6	Pz Kw V & VI v. Sherman Cromwell (75 mm) (e)	300	19	7	32	21	12

Notes :-
(a) Average proportions of German tanks, Pz Kw IV:V:VI :: 4:5:2
(b) Average proportion Sherman : Cromwell was 4:1
(c) Average proportion Sherman 75 mm : Sherman 17 pr was 3.: 1
(d) Pz Kw VI includes models VI(E) and VI(B) in the ratio 3 : 1
(e) Average proportion Pz Kw V : VI was 5 : 2

6. The basic steps in the calculation of numerical values for Effectiveness have been fully described in AC&G Report No. 11/51: "Assessment of Forms of Anti-tank Defence: Effectiveness of British and Russian Tanks"; the figures quoted here have been obtained in a similar way.* Published data for German tanks and guns have been used whenever possible; when such data were not available equivalent British figures have been used. The rates of fire for the German equipments have been based on estimates given by British tank users familiar with the German vehicles and their armament.

7. Full details of the calculated values of Effectiveness for the main German and British tanks are given in Appendix A. The figures quoted there have been used to give estimated values of E for the particular combinations of tanks of Table 1; the results obtained are given in Table 2 below.

TABLE 2.

Effectiveness of German v. British Tanks

Serial No.	Tanks	Range (yards)	Theoretical Effectiveness	
			Rangefinder	Visual
1	Mixed v. Sherman (17 pr)	600 - 1000	1.0(5)	1.0(5)
2	Pz Kw IV v. Sherman Cromwell (75 mm)	1000 - 1500	1.0	1.4
3	Pz Kw V v. Sherman (75 mm + 17 pr)	600 - 1000	1.4(5)	1.8
4	Mixed v. Sherman (75 mm + 17 pr)	600 +	1.4	1.8
5	Pz Kw VI v. Sherman (75mm + 17 pr)	600	1.6	1.7
6	Pz Kw V & VI v. Sherman Cromwell (75 mm)	300	1.7	1.7

COMPARISON OF THEORETICAL PREDICTIONS WITH BATTLE PERFORMANCE

Introductory

8. In the basic theory outlined in AC&G Report No. 21/50, the following equation was developed :-

$$A_0^2 - A^2 = E^2 (X_0^2 - X^2) \quad \ldots\ldots\ldots (1)$$

where A_0, X_0 are the initial allied and enemy strengths,
A, X are the numbers remaining,
E is the effectiveness of the enemy tank with respect to the allied tank.

E is calculated directly from the gun and armour characteristics of the opposing tanks.

* The figures have actually been calculated using the 2nd. method of Appx. B, loc. cit.

SECRET

DISCLASSIFIED

9. From the nature of the form and development of the theory, and from its use of average values, the battle equation (1) should be directly applicable to the outcome of the tank battle as a whole - provided always that the random factors of surprise, time, etc., tend to average out, and that the individual actions, which together constitute the tank battle, do not progress so far that one or other of the sides is annihilated. This would seem to be so for the sample of actions considered in this paper; the evidence on 'firing first', for instance, indicates that the number of actions in which the British fired first was about equal to the number in which the Germans fired first; neither, in the main, did the battles go to annihilation. The calculated values of E given in Table 2 can therefore be compared directly with the battle data of Table 1, through the battle equation.

10. There are two ways in which this comparison of theory with practice can be presented :-

(a) by a direct comparison of the theoretical values for E with the values of $\left\{\frac{A_0^2 - A^2}{X_0^2 - X^2}\right\}^{\frac{1}{2}}$ calculated from the battle data;

(b) by a comparison of the observed casualties with numbers calculated from theoretical values for E in conjunction with the given initial strengths.

These two comparisons will now be considered in turn.

Comparison in terms of E

11. Table 3 compares, for each group of actions, the theoretical values of E with values of $\left\{\frac{A_0^2 - A^2}{X_0^2 - X^2}\right\}^{\frac{1}{2}}$ calculated from the battle data.

TABLE 3

Serial No	Tanks	Observed value of $\left\{\frac{A_0^2 - A^2}{X_0^2 - X^2}\right\}^{\frac{1}{2}}$	Theoretical Effectiveness	
			Rangefinder	Visual
1	Mixed v. Sherman (17 pr)	0.8(5)	1.0(5)	1.0(5)
2	Pz Kw IV v. Sherman Cromwell (75 mm)	1.1	1.0	1.4
3	Pz Kw V v. Sherman (75 mm + 17 pr)	1.3	1.4(5)	1.8
4	Mixed v. Sherman (75 mm + 17 pr)	1.5	1.4	1.8
5	Pz Kw VI v. Sherman (75 mm + 17 pr)	1.4(5)	1.6	1.7
6	Pz Kw V & VI v. Sherman Cromwell (75 mm)	1.4	1.7	1.7

12. It will be seen that in general the observed values agree more closely with the theoretical figures for E based on rangefinder accuracy than with those based on visual range estimation. Thus it would appear that rangefinder figures are the more truly representative of battle effectiveness; this is not unreasonable since it is to be expected that large range errors, if they exist, will be quickly eliminated as the battle develops.

Comparison in terms of casualties

13. Making use of the battle equation and the theoretical values of E, expected values of X and A have been calculated for the observed initial strengths X_o and A_o; the predicted values are compared with those observed, for each of the groups of actions considered, in Table 4 below. Full details of the method of calculation are given in Appendix B.

TABLE 4

Serial No.	Tanks engaged	Source	Numbers of tanks			
			X_o	X	A_o	A
1	Mixed v. Sherman (17 pr)	Battle data Prediction	35	18 20.8	49	42 39.2
2	Pz Kw IV v. Sherman Cromwell (75 mm)	Battle data Prediction	11	5 4.3	14	9 9.7
3	Pz Kw V v. Sherman (75 mm + 17 pr)	Battle data Prediction	53	22 24.2	172	160 157.8
4	Mixed v. Sherman (75 mm + 17 pr)	Battle data Prediction	105	65 60.0	186	137 142.0
5	Pz Kw VI v. Sherman (75 mm + 17 pr)	Battle data Prediction	67	40 42.9	148	126 123.1
6	Pz Kw V & VI v. Sherman Cromwell (75 mm)	Battle data Prediction	19	7 10.6	32	21 17.4
	TOTALS	Battle data Prediction	290	157 162.8	601	495 489.2

Note :- Predictions are based on the rangefinder figures for Effectiveness.

14. This form of presentation is perhaps more interesting than that of Table 3, since it provides a convenient means of testing statistically the differences between the two lots of figures. The complete form of analysis is given in Appendix D. It is there shown that for the data as a whole, and for the individual groups of actions, the differences between the theoretical figures and the observed figures are no greater than could reasonably be expected to occur by chance.

15. There is one other point that may be mentioned here; it is generally accepted that the side which fires first thereby gains an advantage. A limited analysis of some of the present data (Appendix B) confirms that the side which fired first did gain an appreciable advantage, equivalent to an increase in Effectiveness of from 30 to 50%.

DISCUSSION

16. The sample of actions on which the present analysis has been based is neither as large nor as detailed as could have been wished for; for this reason it has not been possible to assess with certainty the significance of the individual factors or assumptions involved in the derivation of the measure of Effectiveness. It can however be said that the general agreement between theoretical predictions and observed battle performance is good, and that there is reasonable evidence to confirm the assumption that numerical strengths and casualties follow a 'square law' (rather than, for example, a 'linear law').

17. It is apparent that the use of rangefinder figures (for accuracy) provides the more truly representative value for E. As has been previously noted (para 12), this is perhaps not unexpected. It should however be remembered that the present analysis has been concerned specifically with British v. German actions, and it is known that, in general, the average standards and abilities of the combatants were comparable. If on some future occasion two sides of markedly different ability were to be compared, the rangefinder figures should probably no longer be used; a value of E enhanced in favour of the better-trained side would probably be more realistic.

18. There is one further tentative deduction that can be drawn. It was previously suggested that mobility was not of major importance in the battle itself, and no allowance has been made for mobility as such in the calculation of E. There is no evidence from the present analysis that would indicate the need for modifying this assumption. This does not, of course, suggest that any assessment of the overall value of a tank (as distinct from its battle effectiveness) should neglect mobility.

SUMMARY

19. Although the data at present available have been too limited to allow a complete validation of the theory to be made, the comparisons that have been presented are sufficient to show that the general trends and levels of performance predicted by the measure of Effectiveness are well indicative of what can be expected in battle. In this connection it is again worth stressing that E, and predictions made from it, are average measures; they should not be applied indiscriminately to particular actions, where factors of the moment may have an important influence.

PART II: A FURTHER NOTE ON RATE OF FIRE

REVISED VALUES FOR RATES OF FIRE IN THE FORMULA FOR EFFECTIVENESS

Theoretical background

20. The calculations of values of Effectiveness are based on the following formula :-

$$E = \left(\frac{r_A}{r_X}\right)^{\frac{1}{2}} \cdot \left(\frac{P_{AX}}{P_{XA}}\right)^{\frac{1}{2}}$$

where E is the Effectiveness of A with respect to X,

P_{AX} is the average chance that a round from A will hit and kill X (Similarly P_{XA}),

r_A and r_X are the average numbers of rounds fired in unit time by A and X respectively.

21. So far, in the calculation of numerical values for E, r_A/r_X has been given a value equal to the ratio of the normal rates of aimed fire of the equipments (R_A/R_X). In certain recent comments, however, it has been suggested that this might not always provide an accurate picture of battle performance, and that, unless due allowances are made for time spent in switching from target to target, for example, the results may well be biased unduly in favour of the tank with the higher rate of fire.

22. These implications have been examined in Appendix C, and it is there suggested that the value of r_A/r_X would be more correctly given by

$$\frac{R_A}{R_X}\left\{\frac{R_X S_X P_{XA} + 60\,(1 - P_{XA})}{R_A S_A P_{AX} + 60\,(1 - P_{AX})}\right\}$$

where R is the normal rate of aimed fire (rounds/min);
S is the average time required to switch from one target to another in the tank battle (measured in seconds, from the last round fired at one target to the first round fired at the next).

A further consideration of the battle data

23. The theoretical figures for Effectiveness given in Part I, Table 2, have been recalculated in accordance with the formula given above; full details are given in Appendix D. The modified figures, together with the original figures of Table 3, are presented in Table 5 below.

TABLE 5

Serial No.	Observed value of $\left\{\frac{A_0^2 - A^2}{X_0^2 - X^2}\right\}^{\frac{1}{2}}$	Theoretical Effectiveness	
		Modified value	Original value (Table 3)
1	0.8(5)	1.0	1.0(5)
2	1.1	1.0	1.0
3	1.3	1.3	1.4(5)
4	1.5	1.3	1.4
5	1.4(5)	1.5	1.6
6	1.4	1.5	1.7

(Values of E are for ranges known with R/F accuracy)

24. In general the agreement between theoretical prediction and observed performance is improved when the modified factor for rate of fire is used; it will be seen, however, that, in the present instances, the differences between the original and modified values for Effectiveness are small.

DISCUSSION

25. In the light of the comparison presented above, it may be concluded that, although the expression of para 22 corresponds in many cases to only a small order correction, it does in fact provide a more correct basis for the evaluation of the factor for rates of fire in the formula for Effectiveness. It is therefore suggested that, in the determination of values for Effectiveness, this expression should be used in preference to the ratio of the cyclic rates of fire, as used hitherto.

26. For the sake of completeness it has been thought desirable to recalculate the figures of AORG Report No. 11/51 (Effectiveness of British and Russian Tanks), making use of the modified expression for rates of fire. The revised figures are presented in Appendix E. It is found that the general effect of using the modified expression is to reduce slightly the Effectiveness of the British tanks; the Effectiveness of the Centurion 3 against the JS3, for example, is reduced from 1.3 to 1.2 (at 1000 yards); the general conclusions of Report No. 11/51, however, remain unaltered.

GENERAL CONCLUSIONS

27. It is concluded that predictions made from the measure of Effectiveness derived in AORG Report No. 21/50 are in good agreement with observed battle performance, and there is as yet no reason to suspect that any major modifications in the concept will be needed.

28. A modified expression has been derived which gives values for rates of fire in the formula for Effectiveness; although in many instances it corresponds to only a small order correction, it is suggested that this expression should be used in preference to the ratio of the cyclic rates of fire, as used hitherto.

29. It is concluded that the measure of Effectiveness may now be used to give a good general indication of the relative merits of opposing tanks in battle. Confidence in future predictions of values for Effectiveness may perhaps now be considered to be limited primarily by the possible inaccuracies in the data for enemy equipments.

ACKNOWLEDGEMENT

30. The assistance of Major R.W. Eccles, who prepared the figures of Appendix A, is gratefully acknowledged.

A.W.S. Ross

Superintendent A.O.R.G.

March 1952

ML

~~SECRET~~

UNCLASSIFIED

Appendix A:

EFFECTIVENESS OF GERMAN V. BRITISH TANKS.

1. Table 6 below gives values for the Effectiveness of the German PzKw IV, V, VI(E), and VI(B), with respect to the Cromwell 75mm and Sherman 75mm and 17pr. Ranges of engagement have been selected to correspond to those given in the battle data of Table 1. In each instance two values of E are quoted: one for ranges known with range-finder accuracy (m.d. of $15R^2$); the other for ranges estimated visually (m.d. of 250R).

Table 6

Effectiveness of German v. British Tanks.

German Tank.	Range (Yards)	VERSUS Sherman (17pr. APCBC; 7rds/min)		Sherman (75mm. APC; 12rds/min)		Cromwell (75mm. APC; 10rds/min)	
		R/F	Visual	R/F	Visual	R/F	Visual
PzKw IV(H) (75mm KwK40; 9rds/min)	1000	0.90	0.85	1.10	1.35	1.35	1.65
	1500			0.90	1.25	1.50	2.10
PzKw V(G) (75mm KwK42; 7rds/min)	300			1.55	1.55	1.85	1.85
	600	1.20	1.20	1.55	2.00	1.85	2.30
	1000	1.20	1.30	1.70	2.50		
PzKw VI(E) (88mm KwK36; 5rds/min)	300			1.65	1.65	1.90	1.90
	600	0.90	0.80	1.60	1.75	1.90	2.00
	1000	0.90	0.75				
PzKw VI(B) (88mm KwK43; 5rds/min)	300			2.70	2.70	3.20	3.20
	600	1.55	1.65	2.70	3.35	3.20	3.90
	1000	1.75	1.90				

All figures have been rounded off to the nearest 0.05

Appendix B

PREDICTION OF CASUALTIES

Method of calculation

1. It is required to compare the observed values of X and A given in the battle data with those that would be predicted from the generalised battle equation:-

$$A_o^2 - A^2 = E^2(X_o^2 - X^2) \quad \text{----------(1)}$$

For any selected group of actions, E will be known (by calculation); X_o and A_o will be given. Thus an expected numerical relationship between X and A can be postulated. If the value of (X + A) is chosen to agree with the observed total number remaining, unique values of X and A can be predicted which will provide unbiased theoretical estimates of the casualties in the group of actions considered.

2. For example: consider the group of actions under Serial 3 of Table 1.

$$X_o = 53 \qquad A_o = 172$$

$$(X = 22) \qquad (A = 160)$$

$$E = 1.45$$

The predicted relationship between X and A is therefore:-

$$(172^2 - A^2) = 1.45^2(53^2 - X^2)$$

When X + A = 182,

$$172^2 - (182 - X)^2 = 1.45^2(53^2 - X^2)$$

Whence,

$$X = 24.20$$

and

$$A = 157.80$$

These values may be compared with the observed values of 22 and 160.

Test of Significance

3. The extent of agreement between the values calculated by the above method and those observed in practice can be investigated statistically.

Thus: for the example quoted above:-

Table 7

	German	British	Totals
Casualties	31 (28.80)	12 (14.20)	43
Numbers Remaining	22 (24.20)	160 (157.80)	(X+A) 182
Totals	(X_o) 53	(A_o) 172	-

Predicted figures in brackets.

SECRET

UNCLASSIFIED

$$\chi^2 = 2.2^2 \left[\frac{1}{28.8} + \frac{1}{14.2} + \frac{1}{24.2} + \frac{1}{157.8} \right]$$

$$= 0.75$$

For one degree of freedom, $P \sim 0.4$; that is, differences as large as those observed could well arise 4 times out of 10, solely from chance variations in sampling.

Results

4. Results for Serials 1 - 6 are given below.

Table 8

Serial No.	Source	Numbers of tanks				χ^2	Significance of difference
		X_o	X	A_o	A		
1	Battle data Prediction	35	18 20.85	49	42 39.15	1.99	P = 0.16 NOT Significant
2	Battle data Prediction	11	5 4.32	14	9 9.68	0.33	P = 0.57 NOT Significant
3	Battle data Prediction	53	22 24.20	172	160 157.80	0.75	P = 0.39 NOT Significant
4	Battle data Prediction	105	65 60.00	186	157 142.00	1.71	P = 0.19 NOT Significant
5	Battle data Prediction	67	40 42.90	148	126 123.10	0.95	P = 0.33 NOT Significant
6	Battle data Prediction	19	7 10.57	32	21 17.43	4.32	P = 0.04 Possibly Significant
	SUMMED (χ^2 = (6 degrees of freedom)					10.05	P = 0.12 NOT Significant

Effects of Firing first

5. The numbers of actions for which the side opening fire first is known are few, and only for Serials 3 and 5 has an analysis been possible. The data are given in Table 9 below.

Table 9

Serial No.	Side firing first	Source	Numbers of tanks				Sample of actions	$\left\{\frac{A_o^2 - A^2}{X_o^2 - X^2}\right\}^{\frac{1}{2}}$	E (Table 3)
			X_o	X	A_o	A			
3a	A	Battle data Prediction	27	12 17.00	51	46 41.00	7	0.9	1.4(5)
3b	X	Battle data Prediction	10	5 3.82	50	47 48.18	5	2.0	1.4(5)
5a	A	Battle data Prediction	21	12 14.05	55	51 48.95	7	1.2	1.6
5b	X	Battle data Prediction	21	17 12.95	44	31 35.05	5	2.5	1.6

The predicted values are calculated as in para. 1, assuming neither side fired first.

6. It will be seen that the side which fired first consistently gained an advantage. The figures in the last two columns give an indication of the extent of this advantage; thus, for the actions considered, the gain from firing first was equivalent to an increase in Effectiveness of approximately 30 - 50%.

Appendix C

RATE OF FIRE IN THE FORMULA FOR EFFECTIVENESS

1. Consider a battle between forces A and X. It is required to determine the average number of rounds (r_A) that will be fired by a tank of side A in unit time.

2. In the development of the theory of Effectiveness it was assumed that a tank would immediately engage a new target as soon as its previous target had been destroyed. Let S_A be the average time required for a tank of side A to switch from one target to another: more precisely, it is the average time (in seconds) between the last round fired at one target and the first round fired at the next.

3. It can be assumed that, in the course of the battle, the tank is likely to fire 1, 2, 3,.....rounds against a target with relative frequencies given approximately by $P_{AX}, P_{AX}(1 - P_{AX}), P_{AX}(1 - P_{AX})^2, \ldots\ldots$, where P_{AX} is the average chance that a round will hit and kill an enemy tank. To a first approximation, therefore, it will on an average fire

$$1.P_{AX} + 2.P_{AX}(1 - P_{AX}) + 3.P_{AX}(1 - P_{AX})^2 + \ldots\ldots\ldots\ldots\text{rounds}$$

in

$$S_A.P_{AX} + (S_A + {}^{60}/_{R_A})P_{AX}(1 - P_{AX}) + (S_A + {}^{120}/_{R_A})P_{AX}(1 - P_{AX})^2 + \ldots\text{secs}$$

where R_A is the normal rate of aimed fire (rds/min).

4. Thus the number of rounds fired in unit time is given approximately by:-

$$r_A = \frac{P_{AX} + 2.P_{AX}(1 - P_{AX}) + 3P_{AX}(1 - P_{AX})^2 + \ldots\ldots\ldots\ldots\ldots}{S_A P_{AX} + (S_A + {}^{60}/_{R_A})P_{AX}(1 - P_{AX}) + (S_A + {}^{120}/_{R_A})P_{AX}(1 - P_{AX})^2 \ldots}$$

$$= \frac{{}^{1}/_{P_{AX}}}{S_A + {}^{60}/_{R_A}({}^{1}/_{P_{AX}} - 1)}$$

$$= \frac{R_A}{R_A S_A P_{AX} + 60(1 - P_{AX})}$$

5. In the determination of Effectiveness, therefore, the value of r_A/r_X will be given by:-

$$\frac{R_A}{R_X}\left\{\frac{R_X S_X P_{XA} + 60(1 - P_{XA})}{R_A S_A P_{AX} + 60(1 - P_{AX})}\right\}$$

Appendix D

EFFECTIVENESS OF GERMAN V. BRITISH TANKS USING REVISED VALUES FOR RATES OF FIRE

1. The figures of Table 6 have been revised in accordance with the formula of Appendix C, para. 5. The following data have been used:-

Table 10

Tank (A)	R_A (rds/min)	Assumed S_A (secs)	Range (yards)	Value of P_{AX}			
				PzKw IV	PzKw V	PzKw VI (E)	PzKw VI (B)
Sherman (17 pr)	7	15	600		0.525	0.603	0.209
			1000	0.798	0.503	0.599	0.180
Sherman (75 mm)	12	12	300		0.186	0.118	0.044
			600		0.186	0.114	0.044
Cromwell (75 mm)	10	12	1000	0.330	0.153		
			1500	0.215			

Table 11

Tank (X)	R_X (rds/min)	Assumed S_X (secs)	Range (yards)	Value of P_{Xi}	
				Sherman	Cromwell
PzKw IV	9	14	1000	0.521	0.691
			1500	0.229	0.533
PzKw V	7	15	300	0.765	0.928
			600	0.750	0.892
			1000	0.748	
PzKw VI (E)	5	18	300	0.750	0.839
			600	0.713	0.839
			1000	0.708	
PzKw VI (B)	5	18	300	0.772	0.893
			600	0.772	0.893
			1000	0.768	

Note: Values of P are for rangefinder accuracy

2. The modified figures for Effectiveness are given in Table 12 below.

Table 12

Effectiveness of German v. British tanks using revised values for rates of fire

(R/F accuracy only)

German Tank	Range (yards)	VERSUS		
		Sherman (17 pr)	Sherman (75 mm)	Cromwell (75 mm)
PzKw IV	1000	0.90	1.05	1.20
	1500		0.90	1.30
PzKw V	300		1.40	1.55
	600	1.15	1.40	1.55
	1000	1.15	1.50	
PzKw VI (E)	300		1.50	1.70
	600	0.95	1.50	1.70
	1000	0.95		
PzKw VI (B)	300		2.35	2.70
	600	1.50	2.35	2.70
	1000	1.60		

All figures have been rounded off to the nearest 0.05

UNCLASSIFIED

Appendix E

REVISED VALUES OF EFFECTIVENESS FOR BRITISH V. RUSSIAN TANKS.

1. The figures in Tables 1-5 of ACRG Report No. 11/51 have been recalculated using the modified expression for rates of fire derived in Appendix C. The following are the relevant data:-

Tank	E (rds/min)	Assumed S (secs)
Centurion 3 (20pr)	12	12
Centurion 2 (17pr)	12	12
Comet (77mm)	12	12
JS3 (122mm)	3	23
T34 (85mm)	7½	12
JS3 (88mm)	3	23
JS3 (85mm)	7½	15

It is thought that the values for S will, if anything, have favoured the Russian tanks.

2. The revised figures for Effectiveness are presented in the following Tables; figures in brackets are the original figures of Report No. 11/51.

Table 13 (Table 1 of Report No. 11/51)
Effectiveness:- British v. Russian Tanks.

Allied Tank	Range (yards)	versus JS3 (122mm) Aiming Point: Centre of Hull	versus JS3 (122mm) Aiming Point: Vul. Area	versus T34 (85mm) Aiming point: Centre of Hull	versus T34 (85mm) Aiming point: Vul. Area
Centurion 3 (20pr)	600	1.1 (1.3)	1.4 (1.9)	1.6 (2.3)	1.6 (2.3)
	1000	1.2 (1.5)	1.4 (1.8)	1.8 (2.5)	1.8 (2.5)
	1500	1.4 (1.6)	1.7 (2.0)	2.1 (2.7)	2.0 (2.8)
Comet (77mm)	600	0.8 (0.9)	0.8 (0.9)	1.1 (1.3)	1.1 (1.3)
	1000	0.8 (0.9)	0.8 (0.8)	1.0 (1.1)	1.0 (1.1)
	1500	0.8 (0.8)	0.8 (0.8)	0.9 (0.9)	0.9 (0.9)

Table 14 (Table 2 of Report No. 11/51)
Effectiveness:- Centurion 2 v. JS3.

Allied Tank	Range (yards)	versus JS3 (122 mm)
Centurion 2 (17 pr.)	600	1.1 (1.2)
	1000	0.9 (1.0)
	1500	1.0 (1.0)

Table 15 (Table 3 of Report No. 11/51)
Effectiveness:- Centurion 3 v. JS3 with 88mm and 85mm.

Allied Tank	Range (yards)	Versus JS3 (88mm)	(85mm)
Centurion 3 (20pr.)	600	1.1 (1.3)	1.2 (1.3)
	1000	1.2 (1.3)	1.3 (1.4)
	1500	1.3 (1.5)	1.5 (1.7)

Table 16 (Table 4 of Report No. 11/51)
Relative Effectiveness:- Centurion 3/Comet on the basis of JS3 & T34/85

Allied Tanks	Range (yards)	On the basis of:- JS3	T34/85
Centurion 3 v. Comet	600	1.4 (1.5)	1.5 (1.7)
	1000	1.4 (1.5)	1.8 (2.2)
	1500	1.8 (2.1)	2.5 (3.0)

Table 17 (Table 5 of Report No. 11/51)
Relative Effectiveness:- JS3/T34 on the basis of Centurion 3 and Comet

Enemy Tanks	Range (yards)	On the basis of:- Centurion 3	Comet
JS3 v. T34/85	600	1.4 (1.8)	1.3 (1.5)
	1000	1.5 (1.9)	1.2 (1.3)
	1500	1.4 (1.7)	1.1 (1.3)

3. It will be seen that the general effect of using the modified expression for rates of fire has been to reduce slightly the Effectiveness of the British tanks; in the important case of Centurion 3 v. JS3, for example, the Effectiveness at 1000 yards has been reduced from 1.3 to 1.2 (the modified ratio for rates of fire being about 3.1/1, compared with the original ratio of 4/1). However, the general conclusions of Report No. 11/51 still stand:-

(a) the Centurion 3 is slightly superior to the JS3 and is superior to the T34/85;

(b) the Comet is inferior to the JS3 and is about equivalent to the T34/85.

Distribution

SA/AC	12	
DMO	1	
DMT	1	
DGMT	2	
Military College of Science	1	
RAC Centre	1	
STT	1	
DWD	2	
DED	1	
DRAC	2	
DRA	1	
E-in-C	1	
D. Inf.	1	
Ministry of Defence		
DRP Staff	1	
Ministry of Supply		
P.O.B.	1	
DGFV	1	
DVR(D)	1	
DFV(A)	1	
FVDE	1	
FVDE/RW	1	
FVPE	1	
ADE	2	(one for CT2)
TPA3/TIB	3	
Defence Research Liaison Canada	6	
Australian Army Staff	4	
NZ Army Liaison Staff	4	
British Joint Services Mission, Washington	5	
ORO Representative, Officer Group One, US Army	8	
ORS (BAOR)	2	
ORS (FARELF)	2	