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PAGES Text - 26
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REPORT
RAPPORT

REPORT / RAPPORT LTR-LT-92

DATE / DATE November 1978

FIG. / DIAG. 7

LABORATORY / LABORATOIRE

Low Temperature Laboratory

LAB. ORDER / COMM. LAB. 16329A

TABLES / TABLES 4

FILE / DOSSIER 3741-9

FOR / POUR ~~Internal~~ UNLIMITED par J. WALLACE.

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REFERENCE / RÉFÉRENCE

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AN APPRAISAL OF THE SINGLE ROTATING CYLINDER METHOD OF LIQUID WATER CONTENT MEASUREMENT

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SUMMARY

The theory of the single rotating cylinder method of liquid water content measurement is reviewed with particular emphasis on the various sources of possible error. The errors, although shown to be relatively small, may be reduced by taking into account the growth of the cylinder diameter resulting from the accreted ice, by employing a realistic value of ice density, and by using the actual median volume diameter of the droplet spectrum if this is known. Revised graphs for use with this method are presented.

Comparative tests in an icing wind tunnel between the rotating cylinder method and another (blade) method showed substantial agreement, thus giving confidence in the veracity of this method of liquid water content measurement.

1.0 INTRODUCTION

The recent development and introduction into service of new and sophisticated methods for the measurement of icing cloud parameters in an icing wind tunnel or engine test cell (see, for instance, Keller[1]) has led to some doubt being cast on the accuracy of the simple basic method of liquid water content measurement that has been employed for many years by a number of test and research agencies, namely the single rotating cylinder[2]. (JES)

This note will attempt to identify the possible sources of error in the rotating cylinder method and assess their relative importance. To complement the analysis, comparative icing wind tunnel tests using both the rotating cylinder method and the iced blade method (Section 4.1) have been made.

2.0 THEORY OF THE METHOD

From normal ice accretion considerations, in a small time increment, Δt, the mass of ice, Δw, accreting on a cylinder of radius r (assuming complete freezing of all impinging drops) is:

$$\Delta w = E(r)\, m V\, 2\, r\, \ell\, \Delta t \quad \ldots\ldots\ldots\ldots\ldots (1)$$

where $E(r)$ = collection efficiency, a function of r*
m = liquid water content
V = relative velocity between cylinder and cloud
ℓ = length of cylinder.

If the cylinder is rotated so that the accreted ice is distributed uniformly around the cylinder in a layer of thickness Δr, then another expression for the accreted ice mass may be written:

$$\Delta w = 2\pi \rho_1\, \ell\, r\, \Delta r \quad \ldots\ldots\ldots\ldots\ldots (2)$$

where ρ_1 = density of ice.

Eliminating Δw from eqns. 1 and 2, in the limit:

$$dr = \frac{E(r) m V}{\pi \rho_1}\, dt$$

If in time T the cylinder radius effectively increases as a result of the accretion of ice from an initial value r_1 to a final radius of r_f, then:

$$\int_{r_1}^{r_f} \frac{dr}{E(r)} = \int_0^T \frac{mV}{\pi \rho_1}\, dt \quad \ldots\ldots\ldots\ldots\ldots (3)$$

---

* The collection efficiency is a function of several variables including the velocity, droplet diameter, and the temperature and pressure of the airstream, but for the purposes of this derivation, only its functional relationship to the cylinder radius need be considered.

whence, if a constant value of collection efficiency, E, is assumed:

$$r_f - r_i = \frac{EmVT}{\pi \rho_i}$$

$$\text{or} \quad m = \frac{\pi \rho_i}{EVT}(r_f - r_i) \quad \text{..................(4)}$$

where the final radius, r_f, is as yet unknown, but may be found by taking eqn. 2 to the limit and integrating:

$$\int_0^w dw = 2\pi \rho_i \ell \int_{r_i}^{r_f} r\,dr$$

$$\text{whence} \quad w = \pi \rho_i \ell \,(r_f^2 - r_i^2)$$

$$\text{or} \quad r_f = \sqrt{\frac{w}{\pi \rho_i \ell} + r_i^2} \quad \text{..................(5)}$$

$$\text{thus} \quad m = \frac{\pi \rho_i}{EVT}\left[\sqrt{\frac{w}{\pi \rho_i \ell} + r_i^2} - r_i\right] \quad \text{.........(6)}$$

Eqn. 6 is the expression normally used for calculating the liquid water content from the weight of ice collected on the cylinder, and which was used to derive the figures 10 and 11 of Ref. 2 using the following values:

$$\rho_i = 800 \text{ kg/m}^3$$

$$r_i = 1.27 \times 10^{-3} \text{ m (0.05 inch)}$$

$$\ell = 5.08 \times 10^{-2} \text{ m (2 inch)}$$

and with E appropriate to a droplet diameter of 2×10^{-5} m (20 µm) and constant cylinder radius of r_i at velocity V, using the collection efficiency data of Langmuir and Blodgett[3].

These results have been used at NRC for many years, and likely at other establishments also, with little or no modification for differences in droplet size, density of ice, etc. The next section discusses the various discrepancies and sources of error that might arise in these results.

3.0 POSSIBLE DISCREPANCIES

3.1 Effect of Cylinder Growth on Collection Efficiency

As indicated above, the collection efficiency is a function of the cylinder radius (amongst other things), and so, rigorously, the functional relationship between E and r should be employed in eqn. 3; but because of the extreme complexity of this relationship such a course is not practical. However, the mean collection efficiency of the cylinder as its radius changes from r_i to r_f may be approximated with adequate accuracy by using the collection efficiency appropriate to the mean radius, $\bar{r}$.

The mean radius is simply derived from eqn. 5 as:

$$\bar{r} = \frac{r_i + r_f}{2} = \frac{\sqrt{\frac{w}{\pi \rho_i \ell} + r_i^2} + r_i}{2} \quad \text{............(7)}$$

When a 60-second collection time is used it is found that the use of the mean radius instead of the initial cylinder radius in the evaluation of E results, for LWCs of up to 1.0×10^{-3} kg/m^3 and velocities of up to 100 m/s, in a decrease in the value of E of up to 16%, with a corresponding increase in the measured LWC of the same proportion.

3.2 Effect of Density

The assumed value of ice density used to evaluate the mean radius (eqn. 7) and the liquid water content (eqn. 6) may represent a source of error. In Ref. 2 an ice density of 800 kg/m^3 is assumed; if, however, the true density in a particular case was 900 kg/m^3 (a 12.5% increase) the following errors result:

a) In mean radius, $\bar{r}$

for $w = 1 \times 10^{-4}$ kg error = 1.0%
$w = 8 \times 10^{-4}$ kg error = 3.1%

b) In LWC, using constant value of E

for $w = 1 \times 10^{-4}$ kg error = -1.2%
$w = 8 \times 10^{-4}$ kg error = -3.2%

c) In LWC, using $\bar{r}$ dependent value of E

for $w = 1 \times 10^{-4}$ kg error = -0.8%
$w = 8 \times 10^{-4}$ kg error = -2.8%

It is clear that an error in the assumed value of ice density results in a much lower percentage error in the computed liquid water content. This error is lowered slightly more when the collection efficiency is based on the mean radius rather than the initial cylinder radius.

The range of ice weights used here (i.e. 1 to 8×10^{-4} kg) covers most of the cases which are likely to occur in practice with this size of cylinder.

3.3 Effect of Droplet Size

The effect of droplet size on the collection efficiency is reflected directly as an error in the measured liquid water content should the actual droplet size differ from that used for the evaluation of E in the computation of eqn. 6.

Thus the use of curves such as presented in Ref. 2 based on collection efficiencies for cloud droplets of 20 micron diameter can lead to measurement errors should the effective droplet diameter differ from 20 microns. Such errors may be considerable, particularly at lower velocities, as Fig. 1 indicates.

These curves were calculated using equations 6 and 7 and Appendix A, and show the errors that would occur in the measured LWC value assuming that the weight of ice caught at any speed was that appropriate to a liquid water content of 0.5 g/m^3 with a droplet diameter of 20 microns. Thus if the droplet diameter differed from the 20 microns assumed, then a measurement error would result. For example, at 100 m/s the ice weight caught when the LWC is 0.5 g/m^3 and droplet diameter is 20 μm is calculated to be 0.507 g, but if this weight of ice were caught and the droplet size was in fact 15 μm and not 20 μm as assumed, the actual LWC would be 0.515 g/m^3 and the error in measurement would be -3%.

3.4 Effect of Droplet Size Spectrum

It is of interest to determine what magnitude of error results from using the median volume diameter of the droplet spectrum to determine the collection efficiency, rather than the effective efficiency determined by considering the actual spectral distribution.

By way of example, the Langmuir C and D distributions[3] and that presented by Cansdale[4] have been selected as typically representative distributions. The median volume diameter of each distribution is 20 microns, and the effective collection efficiency for each distribution on two cylinder diameters (2.54×10^{-3}m and 3.81×10^{-3}m), the latter to represent the mean diameter of a fairly heavily iced cylinder, and at 25 m/s (48.50 knots) velocity, have been computed and compared with that of a monodisperse cloud of 20 micron droplets. These results, which were calculated for an air temperature of -10°C and at an air pressure of 101.3 kPa, are tabulated in Table I and summarized as follows:

Distribution	Effective Collection Efficiency Cylinder Diameter - m 2.54×10^{-3}	 3.81×10^{-3}
Monodisperse	0.891	0.846
Langmuir C	0.872	0.824
Langmuir D	0.861	0.811
Cansdale	0.851	0.802

Use of the median volume diameter (or monodisperse distribution) is seen to overestimate the effective collection efficiency by some 2 to 5 percent, and hence to underestimate the liquid water content in the same proportion. These errors will become smaller with increased velocity.

3.5 Reliability of Droplet Trajectory Data

Two sets of droplet trajectory data for cylinders are in common use, those of Langmuir and Blodgett[3] and those due to Brun et al[5]. The Brun data result in generally higher values of collection efficiencies than those of Langmuir, and the question arises as to which set of data to employ. However, more recent studies by Boeing and by Lockheed[6] both tend to support the Langmuir and Blodgett results (see Table II) with a result that this laboratory will continue to use these data in preference to those of Brun. Use of the Brun data could result in underestimation of the liquid water content by up to 10% at lower air velocities.

3.6 Effect of Surface Roughness

Under most conditions, the ice accretion on the rotating cylinder is macroscopically smooth, but when the Ludlam limit[7] is reached and a liquid water film exists on the surface of the ice, the surface tends to become bumpy. The effect of this roughness is twofold; it enhances both the collection efficiency and the convective heat transfer from the cylinder.

These effects only come into play under conditions where rotating cylinder measurements become marginal because of blow-off effects; however, evidence in the icing wind tunnel suggests that measurements are still reliable at temperatures a degree or two above the theoretical Ludlam limit. It seems probable that this is the result of surface roughness effects. Since this extension in the useful range of measurement cannot be readily quantified, it is still recommended that measurements outside the Ludlam limit be treated with the utmost caution.

3.7 The Possibility of Splash

An aspect of cloud water droplet impingement that does not appear to have received much attention is that of splashing on impact and the carrying away in the airstream of the splash droplets. Little experimental evidence currently exists to suggest that splashing is a significant mass loss mechanism, although it is mentioned in References 11 and 12, and discussed in more detail in References 13 and 14. No indication is given in Ref. 11 of the drop size or impact velocity, while Ref. 14 deals specifically with rain sized drops falling onto deep and shallow liquid layers. References 12 and 13 are more useful since they deal with cloud size droplets. Ref. 12 suggests that splashing occurs when the product of the droplet diameter d in microns and the velocity of impact V in metres per second exceeds 1000. Thus, based on this criterion, it would be expected that a 20-micron drop would not splash until the air velocity significantly exceeded 50 m/s. However, this splashing criterion was inferred from the charge imparted by the freezing of the impacted drop and not by direct evidence of splashing. The charge effects reported in Ref. 12, or evidence of splashing, were not observed in the experiments described in Ref. 13. This discrepancy is partly attributed to differences in the physical state of the accreting surfaces in the two experiments. As pointed out in Ref. 15, the conditions for splash must depend on the topography of the surface and the extent of wetting.

The resolution of this question of droplet splash is clearly of prime significance to the validity of measurements made by ice accretion instruments; however, until indisputable evidence to the contrary is forthcoming it will be assumed that splashing is either non-existent or of no significance.

4.0 ALTERNATIVE METHODS OF MEASUREMENT

4.1 Icing Blade Method

A similar basic method that has been in use in the U. K. for many years is the iced blade. In this method the narrow flat face of a metal blade of rectangular cross-section is exposed to the droplet flow for a short duration, usually one minute. The thickness of ice accreted is measured with a cold micrometer and the liquid water content determined from the expression:

$$m = \frac{\rho_i s}{E_b VT} \quad \text{.....................(8)}$$

where s = ice thickness
E_b = blade collection efficiency

It is usually assumed that $\rho_i \simeq E_b \simeq 0.9$ (using c.g.s. units), so that these terms cancel, and hence:

$$m = s/VT \quad \text{.....................(9)}$$

In practice the blade has a frontal width of about 1/8 inch (3.175×10^{-3}m), a depth of about 3/4 inch (1.9×10^{-2}m), and a length appropriate to the wind tunnel test section in which it is used.

It is assumed that in the short exposure period the ice accretion stays essentially flat and of the same width as the blade so that the collection efficiency remains constant and the mass per unit length of accreting ice may be expressed as:

$$m_1 = \rho_i st \quad \text{.....................(10)}$$

where t = blade width

Errors in this method of measurement result from:

1. The assumption of no change of shape as the ice accretes. Because a relatively small amount of ice is allowed to accrete, the change in shape tends to be minimized, and errors from this cause are probably small, both as far as change in collection efficiency and in the relation between ice thickness and ice mass (eqn. 10) are concerned. Quantitative values of possible errors cannot be readily deduced.

2. The assumption that ρ_i and E_b cancel in the simplified expression, eqn. 9. Generally, values of ρ_i might be expected to be 0.9 or less, probably less than 0.9 because measurements are usually made at low temperatures well

within the Ludlam limit. On the other hand, if the ribbon collection efficiency data of Langmuir and Blodgett[3] is used, it is found that the collection efficiency in this application ranges from a low of about 0.9 up to in excess of 0.99. It is therefore apparent that this assumption is only tenable under conditions approaching the Ludlam limit (so that $\rho_1 \simeq 0.9$) and with small droplet sizes at low velocities. For 20 µm droplets at a velocity of 100 m/s the collection efficiency is 0.98, and even if the density is as high as 0.9, this simplified expression would result in an overestimation of the liquid water content of 9%.

3. The assumption of an erroneous value of ice density, when eqn. 8 is used. It is clear that an error in the assumed ice density is reflected directly as an error of the same proportion in the measured liquid water content. This contrasts with the rotating cylinder method where errors in the density assumption result in a much reduced percentage error in the liquid water content value calculated.

4. The assumption of an erroneous droplet size on the value of collection efficiency in eqn. 8. Because of the higher collection efficiency of a ribbon relative to that of an equivalent sized cylinder, the effect of droplet size variation on collection efficiency is significantly less marked than in the cylinder case. This is illustrated by comparing Fig. 2 with Fig. 1 where a 3.175×10^{-3}m wide blade is compared with a 2.54×10^{-3}m diameter rotating cylinder. By the same token, the effect of using the collection efficiency corresponding to the median volume diameter, instead of the true effective collection efficiency for the droplet size spectrum, is correspondingly less.

4.2 Laser Particle Size Measuring System

Laser particle size measuring systems (Knollenberg probes) have been described elsewhere[8] and will not be described here. In general, two probes are necessary to cover the range of droplet sizes encountered in atmospheric cloud icing[1]. One probe, the FSSP (for "forward scattering spectrometer probe") as its name implies, uses the forward scattered light from a laser source to size and count droplets passing through its sample volume. This probe measures droplets in the 3 to 45 µm diameter range in 15 three-micron increments. The second probe, the OAP (for "optical array probe"), uses a linear array of photo-diodes and a shadowgraph technique to count and measure droplets in the 20 to 200 µm range in 15 twenty-micron increments.

To establish the number distribution of the complete droplet size spectrum, when drops larger than 45 µm are present, the distributions from the two probes are merged. It is almost always found that the distributions from the two probes do not merge smoothly as Fig. 4 of Ref. 1 demonstrates. This failure to merge smoothly must inevitably raise doubts regarding the accuracy of the overall distribution.

Calibration of these instruments is normally done with glass beads. In the case of the FSSP a correction has to be applied to allow for the difference in refractive indices of glass and water. Obviously the accuracy of the calibration procedures is of prime importance.

These laser probes are primarily designed for droplet size measurement and in this application can provide detail of cloud composition not possible with previous instruments. However, it is doubtful whether it is possible to derive improved accuracy over existing methods of liquid water content measurement by deriving it from the number distribution provided by these instruments, since any error in sizing results in three times the error in the computed volume. This is particularly significant for the larger droplets in the spectrum which contain the bulk of the water and which are sized by the OAP in rather wide 20 μm increments.

No attempt has been made here to analyse in detail the possible errors that might occur in this method of liquid water content measurement, but merely to point out that such measurements should be treated with some caution and not accepted as a definitive measure of liquid water content.

5.0 ICING WIND TUNNEL TESTS

Tests were made in the high speed icing wind tunnel of NRC's Low Temperature Laboratory using a rotating cylinder of 2.487×10^{-3}m diameter and 5.08×10^{-2}m long, and a blade of width 3.175×10^{-3}m, of depth 1.9×10^{-2}m, and of length 0.302 m.

Measurements were made at airspeeds of 25, 75 and 125 m/s with static air temperatures of -15°C and -20°C and at a variety of liquid water contents ranging from a nominal value of about 0.35 g/m^3 up to nearly 2 g/m^3. A number of runs at 100 m/s were made with constant water concentration but increasing temperature to assess the upper temperature at which measured values of LWC using the two methods became unreliable.

The procedure was to set up the tunnel conditions of airspeed and temperature and when stabilized to adjust the sprays for the nominal LWC based on the spray system calibration curves. The cylinder was then inserted into the centre of the tunnel test section and rotated at 1 Hz for a sampling period of either 60 seconds or 30 seconds, whichever was more appropriate to the icing rate in order to keep the weight of accreted ice within the range of 1 to 8×10^{-4} kg. The ice weight was determined by weighing the cylinder plus ice on a Mettler H54 balance that had previously been tared for the weight of the cylinder. The diameter of the ice deposit was also measured with a micrometer so that, together with its weight, the density of the ice could be determined. The tunnel conditions were maintained constant and the blade then inserted into the test section with its narrow dimension facing into the air flow for the same sampling period as used for the cylinder. The ice thickness on the face of the blade was measured with a cooled micrometer from the back of the blade, the blade depth then being subtracted from this measurement. On a number of occasions the width of the ice deposit on the blade was also measured. With the tunnel conditions still being maintained constant, an oiled slide sample was taken to determine the droplet size. Droplet samples were not necessarily taken if the conditions had not been significantly changed from the previous run (e.g. a change in temperature was not considered a significant change as far as any effect on droplet size was concerned).

Because of slight variation in the tunnel and spray conditions that could occur between taking the rotating cylinder and the blade measurements, and because of possible timing and measurement errors, it was decided after the first few runs to take more than one sampling of each condition with each device.

The tunnel conditions and the raw measurements for each run are given in Table III, together with the ice density as determined by the weight and diameter of ice on the rotating cylinder using the relation:

$$\rho_i = \frac{w}{\pi \ell (r_f^2 - r_i^2)}$$

$$\text{or} \quad \rho_i = \frac{2.506 \times 10^4 w}{(d_f^2 - 6.185)} \text{ kg/m}^3$$

where w = weight of ice in grams
d_f = final ice diameter in mm

6.0 DISCUSSION OF TUNNEL TESTS

6.1 Measurement Procedures

It is necessary that both the rotating cylinder and the blade be cooled to a temperature below 0°C before being exposed to the droplet stream in the wind tunnel test section. On Run No. 1 the cylinder had not been adequately cooled, with the result that during the first few seconds exposure, the impinging water drops failed to freeze immediately and coalesced into a number of large drops on the cylinder surface. These subsequently froze, but resulted in a knobbly ice at the end of the run. None of the coalesced drops were observed to blow off in the moderately low air velocity (25 m/s) so that the LWC derived from this test is not thought to be in substantial error (c.f. blade result in Table IV); however, a serious error in the measured diameter of the ice accretion resulted because the micrometer measured only across the peaks of the protuberances. No ice density determination was therefore made.

At the end of each sampling, both the cylinder and the blade were immediately returned to the freezer cabinet for the necessary micrometer measurements. The iced cylinder was then transferred to the balance tray for weighing at room temperature. The ice melts during this process but this is of no consequence since the tray retains the water. Evaporation losses are negligible. In normal use, when measuring the LWC with the rotating cylinder, no measurement is made of the final ice diameter, and the iced cylinder is transferred directly from the tunnel working section to the balance tray in a matter of seconds.

The iced blade on the other hand has to be returned immediately to the freezer chest to prevent the ice from melting, and the micrometer measurements have to be made within the chest. This was found to be a disadvantage of this method, since, besides being rather awkward, it was difficult to be sure that a consistent touch was applied to the cold micrometer with cold fingers. Because of the droplet collection characteristics of a flat plate, the ice on the front of the blade had a concave frontal profile between two lobes, in the manner shown in the sketch:

These lobes were rather fragile, and if too heavy a touch was used, the micrometer anvil would tend to crush them. Langmuir and Blodgett provide data on the total collection efficiency, E_M, and on the stagnation point collection efficiency, β_0, for a ribbon. Use of the total collection efficiency would result in some overestimation of the LWC, whereas use of the stagnation point collection efficiency would underestimate the LWC unless the lobes were deliberately crushed in order to measure to the frontal hollow. This latter procedure, which was not tried, would probably give more accurate results if one could be sure of applying just the right pressure to the micrometer.

Because of the high value of the total collection efficiency, E_M, in this application, little error should result by using this efficiency value in conjunction with the ice thickness as measured to the points of the lobes, as was done for the results presented in this report.

6.2 Test Results

The ice density results presented in Table III range in value from 730 kg/m^3 (Run 21) up to 960 kg/m^3 (Run 8). Obviously a measurement error was made in the latter case since the density of pure ice at 0°C is 917 kg/m^3, a value exceeded by none of the other measurements. The value of 730 kg/m^3 in Run 21 is also most likely the result of a gross measurement error since it does not conform to the values obtained for Runs 22 and 23 under the same conditions. These two extreme measurements have therefore been discarded.

On all runs made at temperatures of -10°C or above (i.e. Runs 32 to 34 and 37 to 41), the ice surface on the cylinder became progressively more pebbly with increasing temperature. The result on the ice density measurements was an apparent decrease in density with increasing temperature, owing to the ice diameter measurements being taken across the highlights of the rough surface. Consequently, these measurements are also discarded as erroneous.

The remaining 40 density measurements have a range from 820 to 910 kg/m^3 with no significant trend with air velocity or LWC. The mean value of these measurements is 880 kg/m^3 with a standard deviation of 20 kg/m^3 or 2.3%.

Hence, an ice density of 880 kg/m^3 will be adopted as the ice density value for use in both the rotating cylinder and blade calculations.

The results of these calculations are given in Table IV.

For the rotating cylinder, equation 6 was used with the following values:

$$\rho_i = 880 \text{ kg/m}^3$$

$$r_i = 1.244 \times 10^{-3} \text{ m}$$

$$\ell = 5.08 \times 10^{-2} \text{ m}$$

The collection efficiency, E, was calculated according to the procedure of Appendix A using the mean cylinder radius (eqn. 7) and the median volume diameter of the measured droplet distribution as shown in Table III.

The results of the blade calculations using equation 8 are also given in Table IV. Again, the value of ice density, ρ_i, of 880 kg/m^3 has been used, and the collection efficiency has been calculated using the procedure of Appendix B for a blade width of 3.175×10^{-3} m.

Table IV also lists the values of LWC as read from the graphs of Reference 2 for the rotating cylinder weight measurements, and the values of LWC calculated using the simplified blade equation (eqn. 9).

A perusal of Table IV indicates generally good agreement between the rotating cylinder results (using eqn. 6) and the blade results (using eqn. 8). Two runs, No. 3 and No. 23, both at 25 m/s, showed abnormally high differences between the two methods, i.e. differences of 35% and 21% of the cylinder reading respectively, for which no ready explanation is evident. The ice density measurements for these two runs are fairly reasonable suggesting that the rotating cylinder ice weight measurements were not in great error; while at the same time the LWC value given by the blade method in Run 23 is consistent with Runs 21 and 22. The only rational explanation would seem to be that unexplained perturbations occurred in the actual tunnel water concentrations for a period during each of these runs.

For the remainder of the results, with the exception of those runs at -10°C or higher, differences of no greater than 8% occurred. The differences were greatest at the two extremes of speed (i.e. at 25 m/s and 125 m/s). At 25 m/s the blade gave readings that were on the average 5.4% lower than those of the cylinder, with a standard deviation of 1.7% (runs 3 and 23 were not included). At 75 m/s, the blade readings averaged 0.5% below the cylinder readings with a standard deviation of 3.8%. At 100 m/s, those runs at -15°C and -20°C only, showed the average excess of the cylinder readings over the blade readings to be 0.8% with standard deviation 2.3%; and finally at 125 m/s the blade measurements exceeded the cylinder measurements by an average 2.4% with standard deviation 5.6%.

Thus excellent agreement was achieved at the intermediate speeds, and acceptable agreement at the two extreme speeds.

Excellent agreement is also seen to exist between the results obtained for the rotating cylinder using the graphs of Reference 2 and those given by eqn. 6, in spite of the possible errors introduced by the assumptions used to derive the graphs of Ref. 2. Nowhere is the discrepancy greater than 5%.

Much larger discrepancies are evident when the results of the simplified blade calculation (eqn. 9) are compared with those obtained using equation 8. The simplified calculation gave values of LWC which ranged from 10 to 13% larger than those given by the more exact expression (eqn. 8). The use of the simplified expression is therefore to be deprecated.

Runs 30 to 45 demonstrate the effect of temperature on the measured values of LWC derived using both the cylinder and the blade. The tunnel sprays were set to provide a nominal LWC of 0.6 g/m^3 and a droplet size of 20 μm at 100 m/s true airspeed. These conditions were held constant while the static air temperature was raised in steps from -15°C to -6°C, after which it was dropped to -20°C for a final check. The manner in which the measured LWC value using each device varies with temperature is shown in Fig. 3.

The results indicate that the rotating cylinder is somewhat more temperature tolerant than the blade. Under the conditions of the test the rotating cylinder does not under-read the LWC until the static air temperature is higher than -9°C, regardless of sampling time. The plotted results suggest, however, that the blade under-reads at temperatures higher than about -11°C if the sampling time is restricted to 30 seconds, but with a 60-second sampling time the reading appears to begin to drop off at about -15°C. There is obviously a trade-off between the lower accuracy achievable in measuring the smaller ice accretion thickness of the shorter sampling time and the inaccuracy introduced as a result of the time dependent change in ice shape when longer sampling times are used. Clearly, if close to the Ludlam limit, the trade-off is in favour of the shorter sampling time. An indication of the time dependent effect is given by plotting the ice width measurements for these runs (30-45) as in Fig. 4. Even at -20°C some broadening of the accretion with time is evident. As the temperature rises the width increases at the longer sampling time at the expense of the forward growth, creating at -10°C broad "wings". The width does not continue to increase at temperatures above -10°C, but levels out, possibly as a result of water or particles of ice being blown off the ice "wings".

The effect of changing the median volume diameter of the droplet spectrum was investigated in Runs 46 to 51. For these runs, the LWC values calculated using equations 6 and 8 with the measured droplet median volume diameter are compared with those values given by equations 6 and 8 when a droplet m.v.d. of 20 μm is used throughout. The errors that are introduced by assuming a droplet size of 20 μm are not great and are in agreement with Figs. 1 and 2.

7.0 LWC GRAPHS

Figures 5 and 6 are graphs of liquid water content versus weight of ice caught in 60 and 30 seconds respectively at various airspeeds. The following parameters are employed:

Cylinder diameter: 2.489×10^{-3} m (0.098 in.)

Cylinder length: 5.08×10^{-2} m (2.0 in.)

Droplet diameter: 20 μm

Ice density: 880 kg/m^3

The curves were computed using equation 6, with the collection efficiency based on the mean radius (eqn. 7) and derived according to Appendix A for an air temperature of -10 °C and air pressure of 101.3 kPa.

8.0 LUDLAM LIMIT

The Ludlam limit is that critical liquid water content above which accretion type icing instruments become inaccurate and underestimate the liquid water content. This occurs when the rate of supercooled droplet impingement on the instrument exceeds that which can be completely frozen by the net rate of heat loss at the icing surface, resulting in the possible run-off or blow-off of some of the impinging water.

As originally formulated by Ludlam (Ref. 9) for a rotating cylinder, the rate of water impingement was averaged over the whole cylinder surface to provide a mean water impingement rate per unit area of cylinder surface. Similarly, a mean convective heat transfer coefficient for the whole cylinder surface was used. These assumptions, which might apply to a cylinder that is rotated very rapidly, are too optimistic for a non-rotating cylinder or one that is rotated slowly (i.e. about 1 Hz) as in the case considered here.

At the other extreme, to consider only the case where incomplete freezing of the impinging water at the stagnation point just occurs, will result in too conservative a value.

A more reasonable assumption might be to consider only the front half of the cylinder. Accordingly, this approach is taken here. In the heat balance equation, the water impingement rate is taken as the mean over the front half cylinder,

$$\text{i.e.} \quad W = 2EmV/\pi \quad kg/m^2s$$

The local heat transfer coefficient over the front of an isothermal cylinder is given (Ref. 10) by:

$$Nu = 1.14\, Pr^{0.4}\, Re^{0.5} \left(1 - \left(\frac{\theta}{90}\right)^3\right)$$

$$= Re_D^{0.5} \left(1 - \left(\frac{\theta}{90}\right)^3\right) \quad \text{for air.}$$

where Nu = Nusselt No. (hD/K)
Pr = Prandtl No. of air
Re_D = Reynolds No. of cylinder
θ = Angle measured from stagnation point (degrees)

By integrating this expression between 0° and 90°, an average coefficient for the front half cylinder is approximated:

$$Nu = 0.75\, Re_D^{0.5}$$

$$\text{whence} \quad h = 6.90\, Re_D^{0.5}\ W/m^2 k$$

for a cylinder of diameter 2.489×10^{-3} m at -15°C.

With these assumptions, the results of the Ludlam limit calculations are as presented in Fig. 7.

It is seen that at 100 m/s and -10°C the Ludlam limit is 0.59 g/m^3 which is in satisfactory agreement with the experimental data of Fig. 3.

9.0 CONCLUSIONS

The theory of the single rotating cylinder method of liquid water content measurement has been reviewed with particular emphasis being placed on the various sources of possible error. The individual errors are seen to be relatively small, and some cancellation between various sources of error may occur so that the overall error may also be relatively small.

The errors inherent in the formulation of the graphs of Ref. 2 can be reduced by using an effective (mean) iced cylinder diameter, an improved estimate of ice density, and the actual median volume diameter of the droplet spectrum if this is known. New graphs incorporating the first two of these three refinements are presented.

The close agreement with the blade method of measurement obtained in icing wind tunnel tests tends to give confidence in both methods of measurement as long as assumptions regarding collection efficiency and ice density are not unreasonable, as is the case with the simplified blade equation (eqn. 9), the use of which is deprecated. The rotating cylinder is useful over a wider range of temperatures than the blade.

Loss of water by splashing of the impacting droplets is an unknown factor which may be significant at higher velocities and with large droplets.

These tests provided an improved estimate of ice density (880 kg/m^3) appropriate to the rotating cylinder method.

Appendices provide formulations for the derivation of the collection efficiency of cylinders and ribbons without resort to charts, and hence suitable for computer applications.

The overall conclusion drawn is that the single rotating cylinder method provides a simple and accurate method of liquid water content measurement in applications where ready access to the droplet laden airstream is possible.

10.0 REFERENCES

1. R. G. Keller — Measurement and Control of Simulated Environmental Icing Conditions in an Outdoor, Free Jet, Engine Ground Test Facility. NATO, AGARD Conference Proceedings No. 236, Icing Testing for Aircraft Engines, London, 1978; pp. 7-1 to 7-13.

2. C. K. Rush, R. L. Wardlaw — Icing Measurements with a Single Rotating Cylinder. NRC, NAE Lab. Report LR-206, Sept. 1957.

3. I. Langmuir, K. B. Blodgett — A Mathematical Investigation of Water Droplet Trajectories. US AAF Tech. Report 5418, February 1946.

4. J. T. Cansdale, I. I. McNaughtan — Calculation of Surface Temperature and Ice Accretion Rate in a Mixed Water Droplet/Ice Crystal Cloud. RAE Technical Report 77090, June 1977.

5. R. J. Brun, W. Lewis, P. J. Perkins, J. S. Serafini — Impingement of Cloud Droplets on a Cylinder and Procedure for Measuring Liquid-Water Content and Droplet Sizes in Supercooled Clouds by Rotating Multi-cylinder Method. NACA Report 1215, 1955.

6. J. B. Werner — Private communication.

7. D. Fraser, C. K. Rush, D. Baxter — Thermodynamic Limitations of Ice Accretion Instruments. NRC, NAE Laboratory Report LR-32, Aug. 1952.

8. R. G. Knollenberg — Comparative Liquid Water Content Measurements of Conventional Instruments with an Optical Array Spectrometer. Jour. Appl. Meteorology, Vol. 11, April 1972, pp. 501-508.

9. F. H. Ludlam — The Heat Economy of a Rimed Cylinder. Quart. J. Roy. Met. Soc., Vol. 77, 1951, pp. 663-666.

10. H. H. Sogin — A Design Manual for Thermal Anti-Icing Systems. USAF, WADC Technical Report 54-313, 1954.

11. M. G. Scherberg — Discussion of Paper by M. E. Goldstein et al, Momentum and Heat Transfer in Laminar Flow of Gas with Liquid-Droplet Suspension over a Circular Cylinder. Trans. ASME, Jour. of Heat Transfer, May 1967, pp. 185-194.

12. J. Latham, B. J. Mason — Generation of Electric Charge Associated with the Formation of Soft Hail in Thunderstorms. Proc. Roy. Soc., A, Vol. 260, 1961, pp. 537-549.

13. J. L. Brownscombe, J. Hallett — Experimental and Field Studies of Precipitation Particles formed by the Freezing of Supercooled Water. Q. J. Roy. Met. Soc., Vol. 93, 1967, pp. 455-473.

14. W. C. Macklin, G. J. Metaxas — Splashing of Drops on Liquid Layers. Jour. of Applied Physics, Vol. 47, 1976, pp. 3963-3970.

15. I. Langmuir — Supercooled Water Droplets in Rising Currents of Cold Saturated Air. G. E. Report RL-223, 1944 (re-issued 1949). Also, Collected Works of Irving Langmuir, Vol. 10, Pergamon Press, 1960, pp. 199-334.

TABLE I. Effective Collection Efficiencies for Various Droplet Size Distributions having Median Volume Diameter of 20 Microns on Cylinders of 2.54×10^{-3} and 3.81×10^{-3} Diameter at a Speed of 25 m/s.

1. Langmuir C Distribution

Mid-band Droplet Diameter d_d µm	Fraction of Total Volume F	Cylinder dia.= 2.54×10^{-3} m: Collection Efficiency E	Cylinder dia.= 2.54×10^{-3} m: F x E	Cylinder dia.= 3.81×10^{-3} m: E	Cylinder dia.= 3.81×10^{-3} m: F x E
8.4	.05	0.649	0.0325	0.554	0.0277
12.2	.10	0.779	0.0779	0.703	0.0703
15.4	.20	0.840	0.1680	0.779	0.1558
20.0	.30	0.891	0.2673	0.846	0.2538
25.2	.20	0.923	0.1846	0.890	0.1780
30.2	.10	0.942	0.0942	0.916	0.0916
36.2	.05	0.956	0.0478	0.936	0.0468
		$E_{eff} = \sum = 0.872$		$E_{eff} = \sum = 0.824$	

2. Langmuir D Distribution

d_d µm	F	Cylinder dia.= 2.54×10^{-3} m: E	Cylinder dia.= 2.54×10^{-3} m: F x E	Cylinder dia.= 3.81×10^{-3} m: E	Cylinder dia.= 3.81×10^{-3} m: F x E
6.2	.05	0.521	0.0261	0.423	0.0212
10.4	.10	0.729	0.0729	0.643	0.0643
14.2	.20	0.821	0.1642	0.754	0.1508
20.0	.30	0.891	0.2673	0.846	0.2538
27.4	.20	0.933	0.1866	0.902	0.1804
34.8	.10	0.954	0.0954	0.932	0.0932
44.4	.05	0.968	0.0484	0.953	0.0477
		$E_{eff} = \sum = 0.861$		$E_{eff} = \sum = 0.811$	

3. Canadale Distribution

d_d µm	F	Cylinder dia.= 2.54×10^{-3} m: E	Cylinder dia.= 2.54×10^{-3} m: F x E	Cylinder dia.= 3.81×10^{-3} m: E	Cylinder dia.= 3.81×10^{-3} m: F x E
5	.06	0.427	0.0256	0.316	0.0190
10	.10	0.715	0.0715	0.637	0.0637
15	.19	0.834	0.1585	0.771	0.1465
20	.29	0.891	0.2584	0.846	0.2453
25	.18	0.922	0.1660	0.888	0.1598
30	.08	0.941	0.0753	0.915	0.0732
35	.05	0.954	0.0477	0.933	0.0467
40	.035	0.963	0.0337	0.945	0.0331
45	.015	0.969	0.0145	0.954	0.0143
		$E_{eff} = \sum = 0.851$		$E_{eff} = \sum = 0.802$	

4. Monodisperse 20 µm Distribution

	Cylinder dia.= 2.54×10^{-3} m	Cylinder dia.= 3.81×10^{-3} m
	E = 0.891	E = 0.846

TABLE II. Comparison of Cylinder Water Catch Data

K_o	Comparison of Theoretical Cylinder Collection Efficiency			
	Langmuir and Blodgett	Brun NACA TR-1215	Lockheed	Boeing
0.9	0.38	0.41	0.39	
1.5	0.54	0.57	0.54	
1.7	0.55	0.62	0.55	0.55
2.2	0.63	0.67	0.64	
2.8	0.68	0.74	0.70	
4.7	0.78	0.82	0.79	
13.9	0.92	0.92	0.92	

(Based on Lockheed California Co. information, Ref. 6)

TABLE III — BASIC TEST RESULTS

Run No.	Air Velocity m/s	Static Air Temp. °C	Nominal LWC g/m³	Sample Time s	Droplet MVD µm	Rotating Cylinder: Ice Weight g	Rotating Cylinder: Ice Diameter mm	Rotating Cylinder: Ice Density kg/m³	Blade: Ice Thickness mm	Blade: Ice Width mm	Remarks
1	25	-15	0.8	60	28	.1657	4.36	--	1.245	3.60	Cylinder ice knobbly - discard
2	25	-15	0.8	60		.1564	3.26	880	1.160	3.49	
3	25	-15	0.5	60	17	.0945	3.00	840	0.545		
4	75	-15	0.35	60		.2459	3.65	860	1.895	3.925	
5	75	-16.8	0.35	60	23	.2640	3.77	820	1.885		
6	75	-17.8	0.35	60		.2570	3.65	900	1.965		
7	75	-20	0.7	60		.5612	4.66	900	3.685	4.25	
8	75	-20	0.7	60	20	.5510	4.53	960	3.635		
9	75	-20	0.7	60		.5500	4.64	900	3.575	4.20	
10	75	-20	1.0	30		.3716	4.08	890	2.515		
11	75	-20	1.0	30	18	.3630	4.06	880	2.585		
12	75	-20	1.0	30		.3678	4.06	890	2.575		
13	125	-20	0.35	60		.4205	4.29	860	--		No blade measurement.
14	125	-20	0.35	30	24	.1859	3.41	860	1.585		
15	125	-20	0.35	60		.4433	4.34	880	--		No blade measurement.
16	125	-20	0.7	30	17	.3937	4.19	870	2.855		
17	125	-20	0.7	30		.4036	4.20	880	2.965		
18	125	-20	1.0	30		.5775	4.77	870	4.005	3.55	
19	125	-20	1.0	30	19	.6015	4.82	890	3.555		
20	125	-20	1.0	30		.5735	4.82	840	3.635	4.79	
21	25	-20	0.4	60		.0715	2.94	730	0.595	3.40	
22	25	-20	0.4	60	19	.0745	2.91	820	0.655	3.625	
23	25	-20	0.4	60		.0870	2.94	890	0.600		
24	25	-17.3	1.0	60		.2204	3.54	870	1.605		
25	25	-17.3	1.0	60	20	.2227	3.54	880	1.610		
26	25	-17.3	1.0	60		.2227	3.54	880	1.625		

TABLE III -- BASIC TEST RESULTS (cont'd.)

Run No.	Air Velocity m/s	Static Air Temp. °C	Nominal LWC g/m³	Sample Time s	Droplet MVD µm	Rotating Cylinder: Ice Weight g	Rotating Cylinder: Ice Diameter mm	Rotating Cylinder: Ice Density kg/m³	Blade: Ice Thickness mm	Blade: Ice Width mm	Remarks
27	25	-18	2.0	60		.4290	4.31	870	2.845		
28	25	-17.3	2.0	60	24	.4180	4.27	870	2.755	4.07	
29	25	-17	2.0	60		.4110	4.22	880	2.800		
TESTS TO INVESTIGATE EFFECT OF TEMPERATURE											
30	100	-15	0.6	60		.6565	4.97	890	3.945	5.00	
31	100	-15	0.6	60		.6425	4.92	890	3.905		
35	100	-15	0.6	60		.6695	5.05	870	4.005	4.76	
36	100	-15	0.6	30		.2740	3.725	890	2.035	3.84	
32	100	-10	0.6	60		.6630	5.12	830	3.265	7.87	Ice slightly lumpy.
33	100	-10	0.6	60		.6435	5.10	810	3.275	7.80	Small chip of ice lost off cylinder.
34	100	-10	0.6	30		.2835	3.76	890	1.855	5.45	
38	100	-8.5	0.6	30	20	.2660	3.72	870	1.635	5.41	Ice slightly pebbly.
37	100	-7.5	0.6	30		.2170	3.54	850	1.585	5.03	Ice lumpy.
41	100	-7.5	0.6	30		.2480	3.66	860	1.515	5.01	Ice lumpy.
39	100	-6	0.6	30		.2245	3.70	750	1.255	5.20	Clear lumpy ice.
40	100	-6	0.6	30		.2155	3.66	750	1.265	4.96	Clear lumpy ice.
42	100	-20	0.6	30		.2785	3.73	900	2.085	3.62	
43	100	-20	0.6	30		.2710	3.73	880	2.005	3.83	
44	100	-20	0.6	30		.2730	3.73	890	2.085	3.80	
45	100	-20	0.6	60		.6430	4.90	910	3.975	4.14	
TESTS TO INVESTIGATE EFFECT OF DROPLET SIZE											
46	75	-15	0.5	60	21.5	.3265	3.91	900	2.365	3.78	Nominal drop size = 20µm
47	75	-15	0.5	60		.3265	3.92	890	2.435	3.97	
48	75	-15	0.5	60	13.6	.2970	3.79	910	2.115	3.69	Nominal drop size = 15µm
49	75	-15	0.5	60		.2880	3.77	900	2.035		
50	75	-15	0.5	60	33.6	.3600	4.08	860	2.565	4.36	Nominal drop size - 30µm
51	75	-15	0.5	60		.3570	4.05	880	2.495	4.23	

TABLE IV. DERIVED RESULTS

Run No.	Air Velocity m/s	Static Air Temp. °C	Droplet MVD μm	Sampling Time s	Calculated L.W.C. - g/m^3			
					Rot. Cylinder		Blade	
					Eqn. 6	Ref. 2	Eqn. 8	Eqn. 9
1	25	-15	28	60	.80	.82	.75	.83
2	25	-15		60	.76	.77	.70	.77
3	25	-15	17	60	.52	.50	.34	.36
21	25	-20		60	.39	.38	.37	.40
22	25	-20	19	60	.41	.39	.40	.44
23	25	-20		60	.47	.45	.37	.40
24	25	-17.3		60	1.08	1.06	.98	1.07
25	25	-17.3	20	60	1.09	1.08	.99	1.07
26	25	-17.3		60	1.09	1.08	1.00	1.08
27	25	-18		60	1.83	1.85	1.73	1.90
28	25	-17.3	24	60	1.79	1.82	1.67	1.84
29	25	-17		60	1.77	1.79	1.70	1.87
4	75	-15		60	.37	.37	.38	.42
5	75	-16.8	23	60	.39	.39	.38	.42
6	75	-17.8		60	.38	.38	.39	.44
7	75	-20		60	.73	.71	.74	.82
8	75	-20	20	60	.72	.70	.73	.81
9	75	-20		60	.72	.70	.72	.79
10	75	-20		30	1.06	1.01	1.01	1.12
11	75	-20	18	30	1.04	1.00	1.04	1.15
12	75	-20		30	1.05	1.00	1.03	1.14
13	125	-20		60	.34	.33	—	—
14	125	-20	24	30	.34	.34	.38	.42
15	125	-20		60	.35	.34	—	—
16	125	-20		30	.66	.63	.69	.78
17	125	-20	17	30	.67	.64	.71	.79
18	125	-20		30	.88	.85	.96	1.07
19	125	-20	19	30	.91	.88	.85	.95
20	125	-20		30	.88	.84	.87	.97

TABLE IV. DERIVED RESULTS (cont'd.)

Run No.	Air Velocity m/s	Static Air Temp. °C	Droplet MVD μm	Sampling Time s	Calculated L.W.C. - g/m³ Rot. Cylinder Eqn.6	Rot. Cylinder Ref. 2	Blade Eqn. 8	Blade Eqn. 9
				EFFECT OF TEMPERATURE				
30	100	-15		60	.61	.59	.59	.66
31	100	-15		60	.60	.58	.58	.65
35	100	-15		60	.62	.60	.60	.67
36	100	-15		30	.61	.59	.61	.68
32	100	-10		60	.62	.60	.49	.54
33	100	-10		60	.60	.58	.49	.55
34	100	-10		30	.62	.60	.56	.62
38	100	-8.5		30	.59	.57	.49	.55
37	100	-7.5	20	30	.50	.48	.47	.53
41	100	-7.5		30	.56	.56	.45	.51
39	100	-6		30	.51	.49	.38	.42
40	100	-6		30	.49	.47	.38	.42
42	100	-20		30	.61	.60	.62	.70
43	100	-20		30	.60	.58	.60	.67
44	100	-20		30	.60	.58	.62	.70
45	100	-20		60	.60	.58	.59	.66
				EFFECT OF DROPLET SIZE				
					Eqn.6	Eqn.6 using d = 20 μm	Eqn.8	Eqn.8 using d = 20 μm
46	75	-15	21.5	60	.47	.47	.47	.47
47	75	-15		60	.47	.47	.49	.49
48	75	-15	13.6	60	.46	.44	.43	.42
49	75	-15		60	.45	.43	.42	.41
50	75	-15	33.6	60	.50	.51	.51	.51
51	75	-15		60	.49	.51	.49	.50

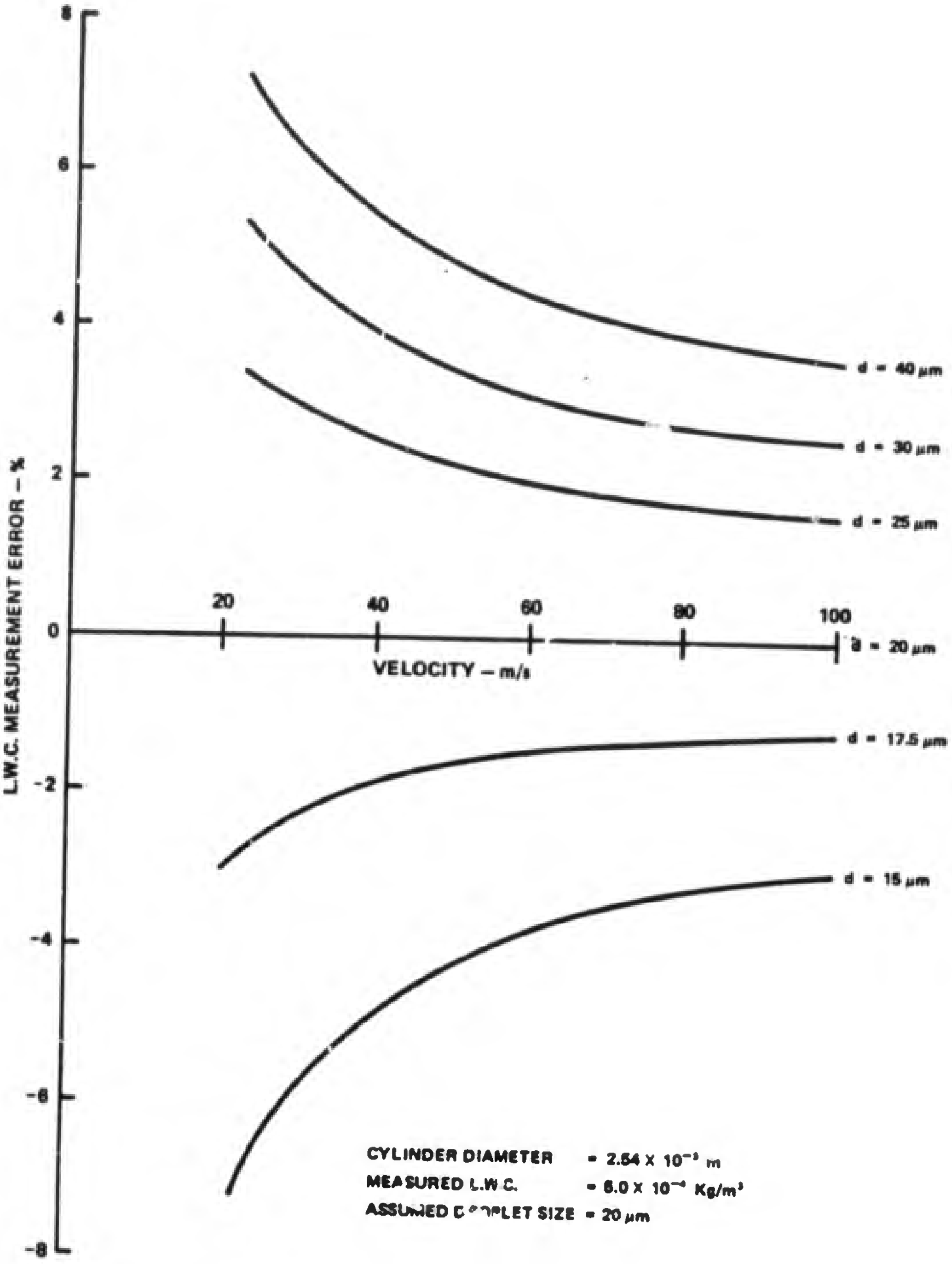

FIG. 1: MEASUREMENT ERRORS INTRODUCED WHEN DROPLET SIZE DIFFERS FROM THAT ASSUMED IN COMPUTING LIQUID WATER CONTENT USING THE ROTATING CYLINDER METHOD

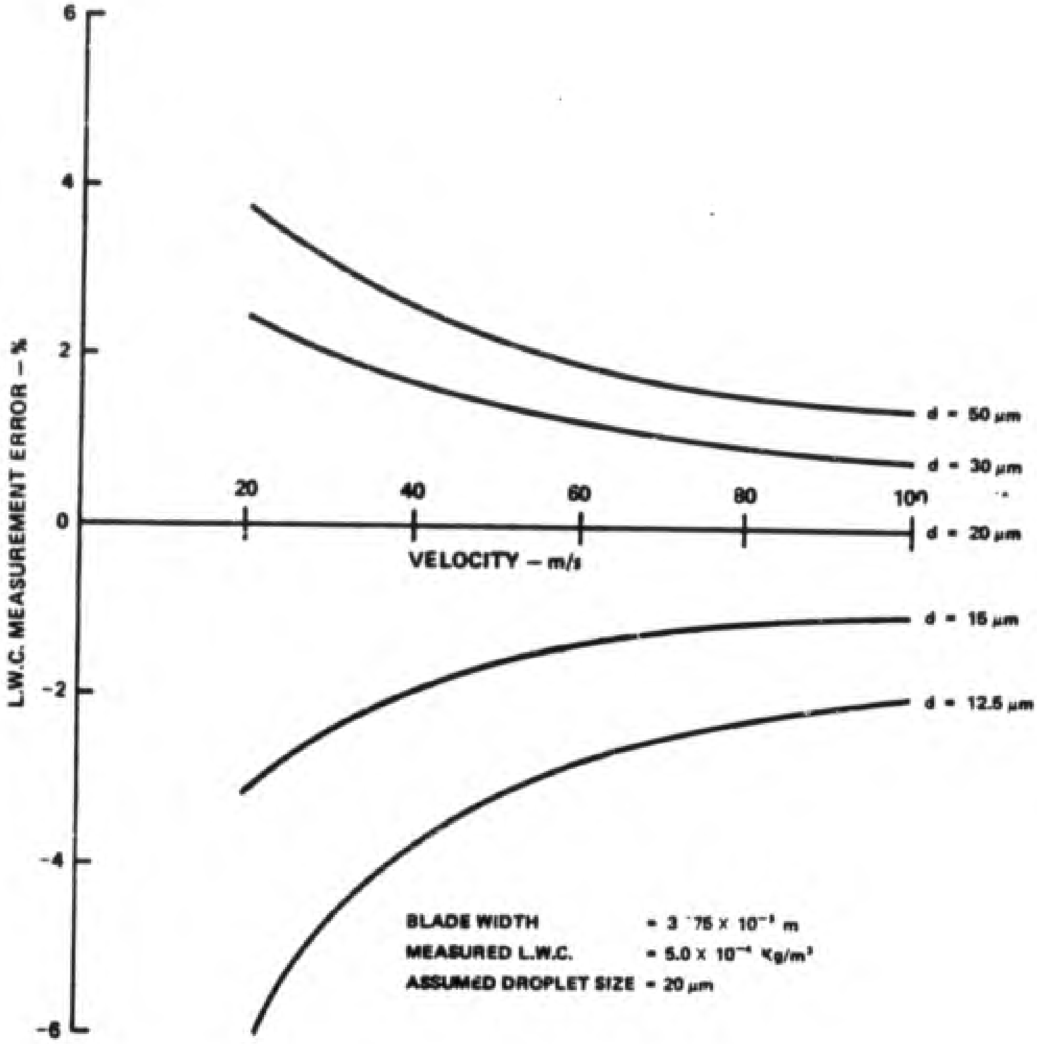

FIG. 2: MEASUREMENT ERRORS INTRODUCED WHEN DROPLET SIZE DIFFERS FROM THAT ASSUMED IN COMPUTING LIQUID WATER CONTENT USING THE ICED BLADE METHOD

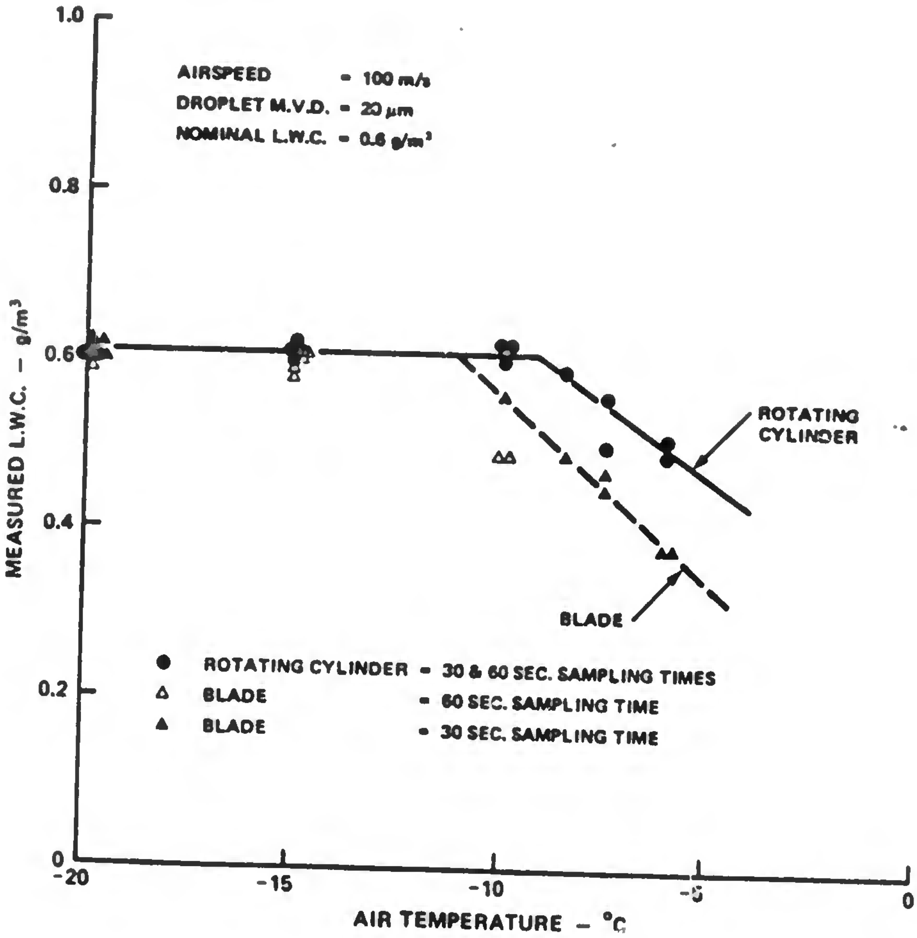

FIG. 3: EFFECT OF TEMPERATURE ON MEASURED VALUE OF LIQUID WATER CONTENT USING ROTATING CYLINDER AND BLADE METHODS

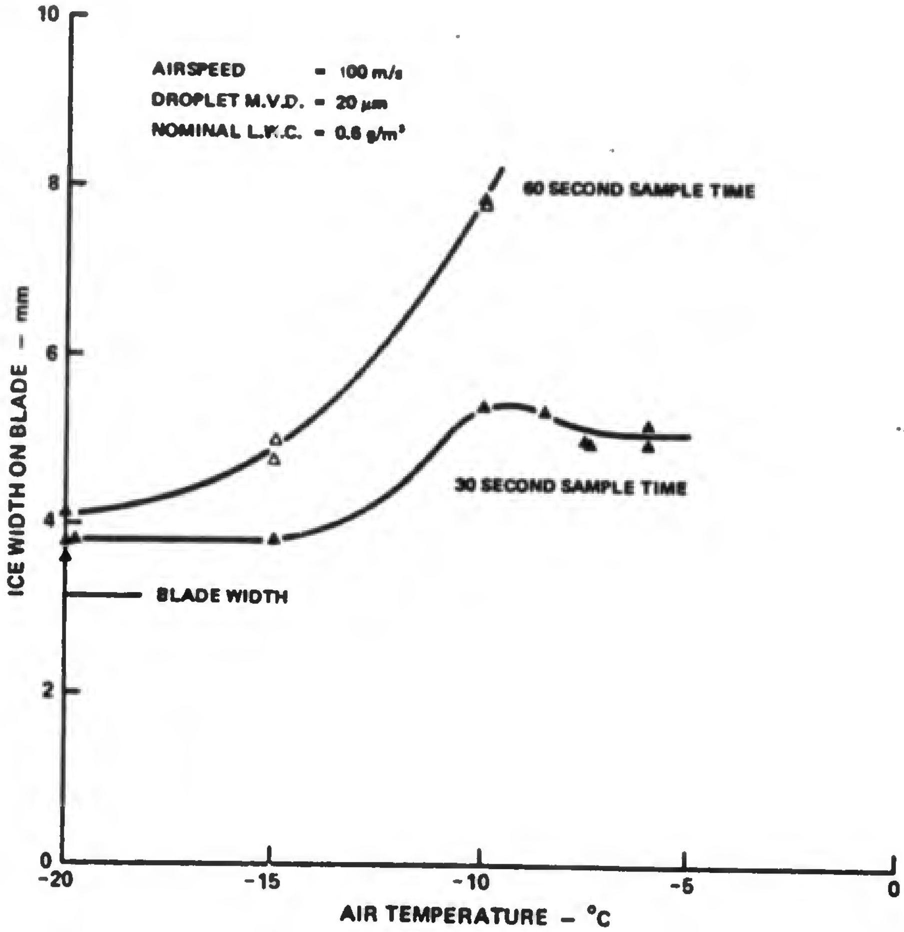

FIG. 4: EFFECT OF TEMPERATURE AND SAMPLING TIME ON WIDTH OF ICE ACCRETION ON BLADE

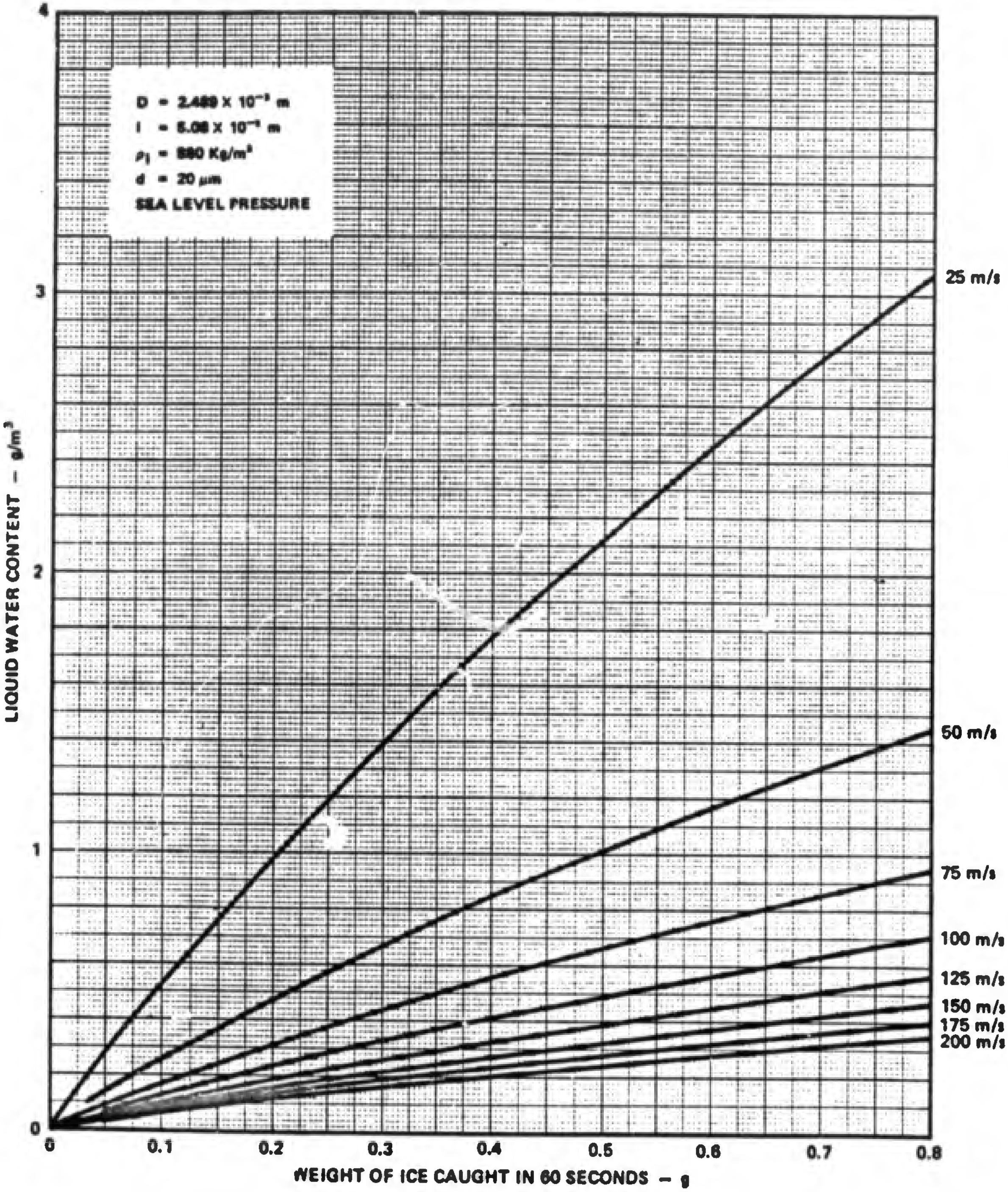

FIG. 5: LIQUID WATER CONTENT VERSUS WEIGHT OF ICE CAUGHT IN 60 SECONDS AT VARIOUS AIRSPEEDS BY ROTATING CYLINDER METHOD

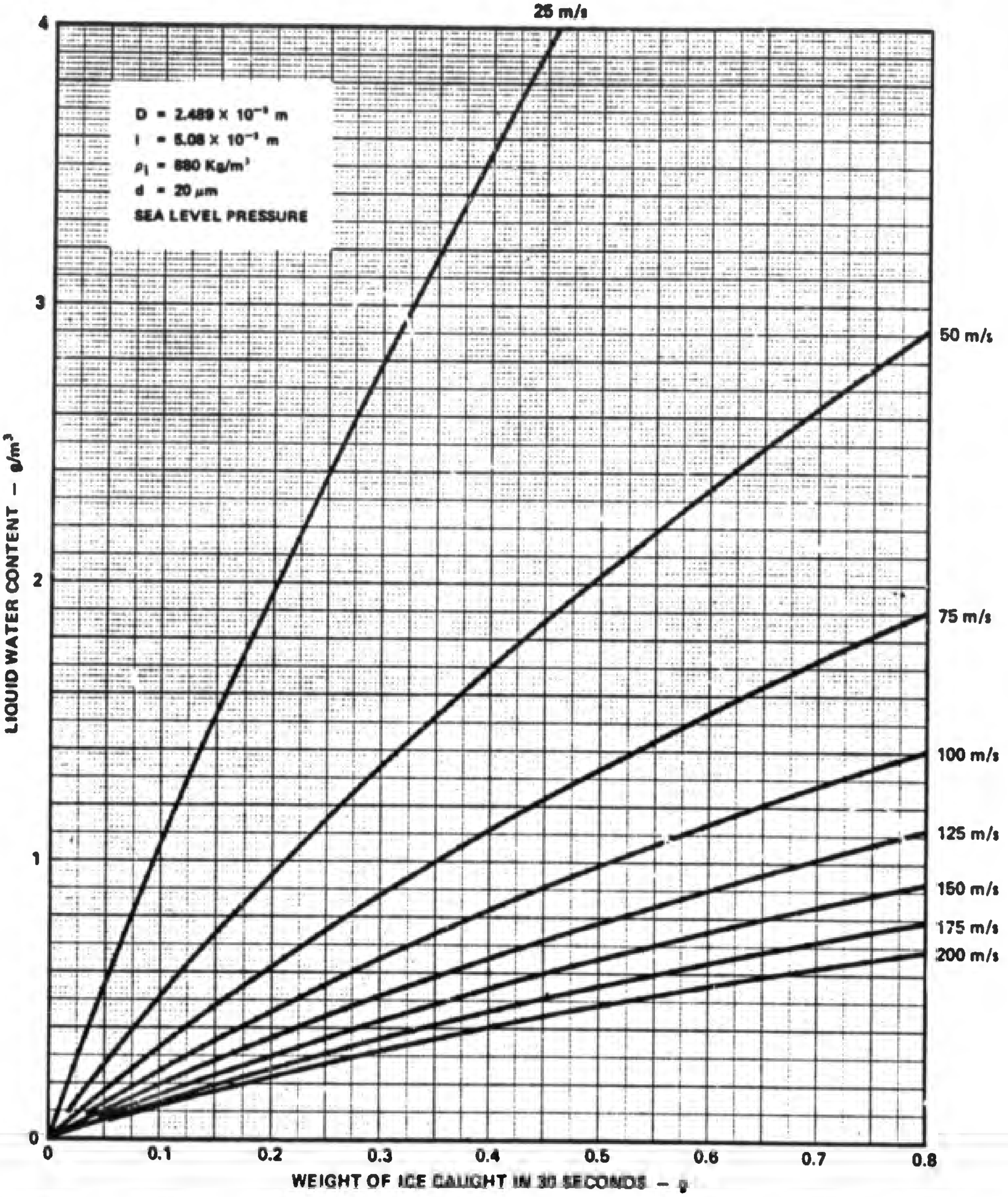

FIG. 6: LIQUID WATER CONTENT VERSUS WEIGHT OF ICE CAUGHT IN 30 SECONDS AT VARIOUS AIRSPEEDS BY ROTATING CYLINDER METHOD

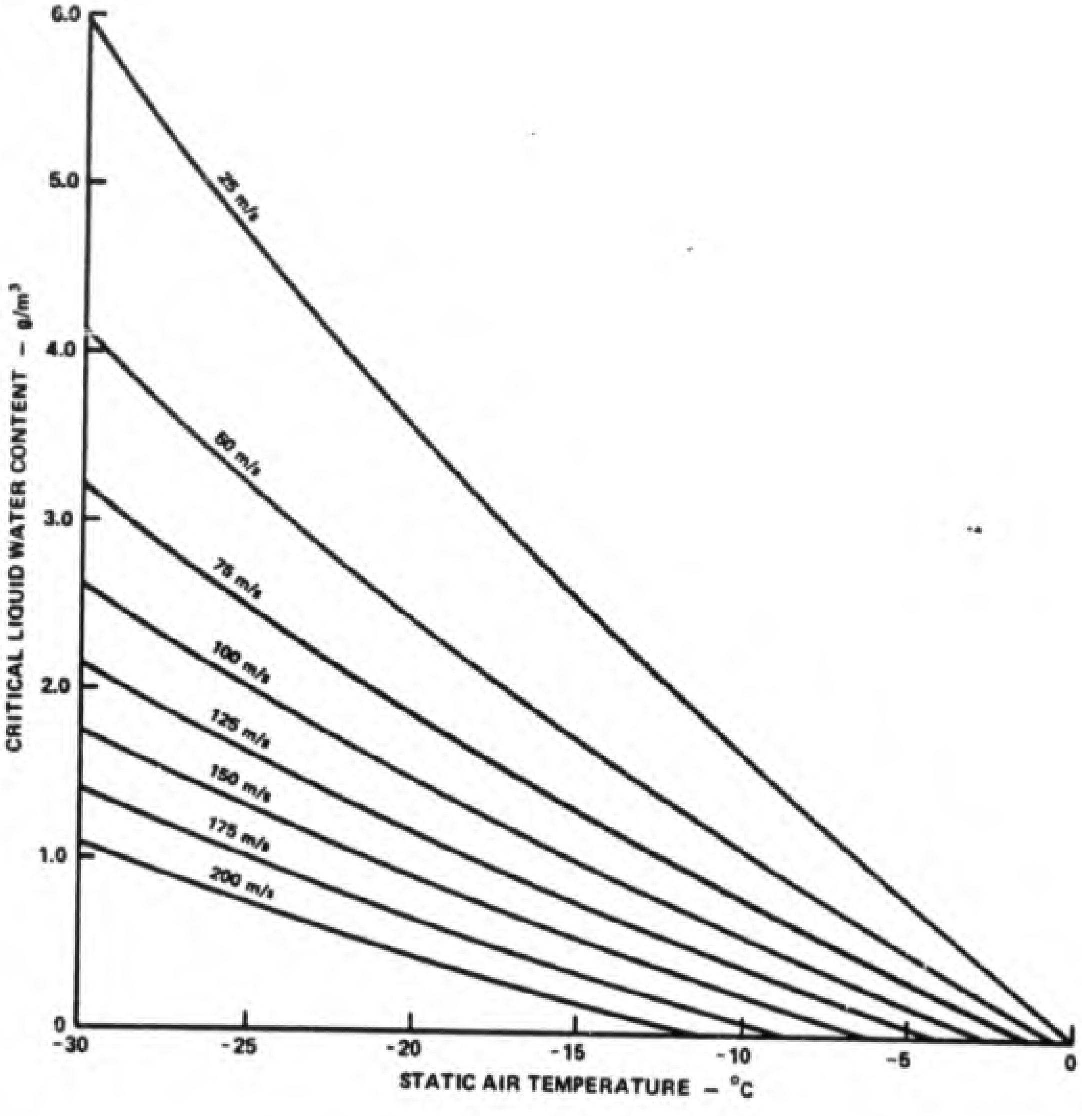

FIG. 7: CRITICAL LIQUID WATER CONTENT (LUDLAM LIMIT) FOR A 2.489 X 10^{-3} m CYLINDER

APPENDIX A

ANALYTICAL DETERMINATION OF CYLINDER COLLECTION EFFICIENCY

The analytical determination of cylinder collection efficiency for application to the single rotating cylinder calculations is based on that of Langmuir and Blodgett[3]. This makes use of the inertia parameter K, defined as

$$K = 1/9 \ \frac{\rho_w d^2 V}{\mu D}$$

where ρ_w = density of water, kg/m^3
d = droplet diameter, m
V = air velocity, m/s
μ = dynamic viscosity of air, kg/ms (Ns/m^2)
D = cylinder diameter, m

If evaluated at -10°C, the value of K obtained is within 3% of its true value from 0°C to -20°C.

$$\text{i.e.} \quad K = d^2V/(1.5 \times 10^{-7} D)$$

This method of collection efficiency determination provides for two ranges of the inertia parameter K.

<u>For low values of K</u> (i.e. $K \leqslant 3.0$) use is made of the K_o parameter which is defined by:

$$K_o = 0.125 + (K - 0.125) \ \lambda/\lambda_s \ \ldots\ldots\ldots\ldots\ldots\ldots (\text{L \& B eqn. 40})$$

where λ_s is the range which the droplet would have as a projectile released in still air at a velocity equal to the free stream velocity used to evaluate K, assuming that Stokes' law holds, and λ is the true range. The ratio λ/λ_s can be calculated from the droplet Reynolds number, Re_d, by the equation:

$$\lambda/\lambda_s = (1/Re_d) \int_0^{Re_d} (1/(C_D Re/24)) \, dRe \ \ldots\ldots\ldots (\text{L \& B eqn. 41})$$

A table of values of λ/λ_s as a function of Re_d is given in Table I of Langmuir and Blodgett. An empirical expression that is within 3/4% of the tabulated values of λ/λ_s over the droplet Reynolds number range of interest in this study, i.e. $20 \leqslant Re_d \leqslant 500$, is:

$$\lambda/\lambda_s = 1/(1 + 0.1206 \ Re_d^{0.59})$$

$$\text{whence } K_o = 0.125 + (K - 0.125)/(1 + 0.1206 \ Re_d^{0.59})$$

and, modifying Langmuir's eqn. 33 to remove the need for the corrections given in Table V of L & B, the collection efficiency is given by:

$$E = 0.457 \left(\log_{10} 8K_o\right)^{1.634}$$

For higher values of K (i.e. $K > 3.0$) use is made of Langmuir's parameter H_E, defined by:

$$H_E = 1 + 0.5708 \left(C_D Re/24\right) - 0.73 \times 10^{-4} \, Re_d^{\,1.38} \quad \text{... (L \& B eqn. 43)}$$

Values of $C_D Re/24$ as a function of Re are given in Table I of Langmuir and Blodgett, together with the empirical equation:

$$C_D Re/24 = 1 + 0.197 \, Re^{0.63} + 2.6 \times 10^{-4} \, Re^{1.38} \quad \text{... (L \& B eqn. 22)}$$

Greater accuracy in the range of Reynolds number $20 \leq Re_d \leq 500$ is afforded by a slight modification to this equation, as follows:

$$C_D Re/24 = 1 + 0.212 \, Re^{0.6} + 2.6 \times 10^{-4} \, Re^{1.38}$$

whence $$H_E = \pi/2 + 0.121 \, Re_d^{\,0.6} + 0.754 \times 10^{-4} \, Re_d^{\,1.38}$$

Finally, the collection efficiency is given by:

$$E = K/(K + H_E) \quad \text{................(L \& B eqn. 42)}$$

APPENDIX B

ANALYTICAL DETERMINATION OF BLADE COLLECTION EFFICIENCY

The analytical determination of the collection efficiency for application to the blade method of liquid water content measurement is based on the Langmuir and Blodgett[3] calculations for ribbons. This makes use of the inertia parameter K, defined as:

$$K = 1/9 \; \frac{\rho_w d^2 V}{\mu t}$$

where ρ_w = density of water, kg/m^3
d = droplet diameter, m
V = air velocity, m/s
μ = dynamic viscosity of air, kg/ms (Ns/m^2)
t = blade width, m

If evaluated at -10°C, the value of K obtained is within ±3% of its true value from 0°C to -20°C.

$$\text{i.e.} \quad K = d^2 V/(1.5 \times 10^{-7}\, t)$$

In this determination of collection efficiency two ranges of K are addressed.

<u>Low values of K</u> (i.e. $K \leqslant 1.1$). Use is made of the K_o parameter as in Appendix A.

$$\text{i.e.} \quad K_o = 0.125 + (K - 0.125)/(1 + 0.1206\, Re_d^{\,0.59})$$

The collection efficiency is then determined by:

E_b = [illegible].35 $(\log_{10} 8K_o)^2$ ....(L & B eqn. 54)

<u>Higher values of K</u> (i.e. $K > 1.1$). Use is made of a parameter H_E, but differing from that for cylinder collection efficiency, defined by:

$$H_E = 0.48 + 0.046\, Re_d^{\,0.63} \quad \text{................(L \& B eqn. 57)}$$

Whence the collection efficiency is determined by:

$$E_b = K/(K + H_E) \quad \text{..................(L \& B eqn. 56)}$$