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THE ECONOMICAL CONDITIONS FOR LONG FLIGHT. BY H. GLAUERT, OF THE ROYAL AIRCRAFT ESTABLISHMENT. PRESENTED BY THE DIRECTOR OF RESEARCH.

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# THE ECONOMICAL CONDITIONS FOR LONG FLIGHT.

By H. GLAUERT, of the R.A.E.

Presented by THE DIRECTOR OF RESEARCH.

*Reports and Memoranda, No.* 637. *June*, 1919.

SUMMARY.—(a) *Introductory.*—Report R. & M. 527 contains a mathematical treatment of the problem by Mr. A. Berry, based on rather simple assumptions, the conclusion reached being that the whole journey should be made at a certain best attitude.

(b) *Range of investigation.*—The present analysis is confined to the case of level flight, but is based on assumptions which approximate closely to the actual conditions. An attempt is also made to obtain the necessary modifications when the wind velocity is taken into account.

(c) *Conclusions.*—The general result of the analysis is to confirm Mr. Berry's conclusion that there is a best attitude which should be maintained throughout the flight. It is also advantageous to fly at as great a height as possible, to secure maximum range for a given load of fuel.



1. *Introduction.*—Early in 1918 Mr. A. Berry issued a report (R. & M. 527) dealing with the best conditions of flight to obtain a maximum range with a given supply of fuel. The problem was subjected to rigorous mathematical analysis, but was based on simple assumptions which in some respects did not approximate very closely to the actual conditions. The general conclusion attained was that it is best to fly at a certain attitude with the engine all out, thus climbing continually on account of the gradual loss of weight. If a limit be placed on the height, the aeroplane should climb "all out" to this height and then fly level at the same attitude by throttling the engine.

The present report is an attempt to deal with the problem on more rigorous assumptions. The discussion is confined mainly to the case of level flight, and conclusions are reached which are in general agreement with Mr. Berry's results. If either of two simple approximations to the engine horse-power curve be taken it is found that there is a best attitude at which the aeroplane should be flown throughout the journey. Also for a given total load it is advantageous to fly as high as possible, both to increase the range and to decrease the time of the journey.

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2. *Assumptions.*—The airscrew characteristics are assumed to be of the form suggested by Mr. L. Bairstow and Lieut. J. D. Coales (R. & M. 474). Thus we write :—

$$\left.\begin{aligned} k_T &= A_T\,(1-\lambda^2) \\ k_Q &= A_Q\,(a-\lambda^3) \\ \text{where} \qquad J &= V/nD \\ \lambda &= J/J_0 \\ a &= 1{\cdot}325 \end{aligned}\right\} \quad . \quad . \quad . \quad (1)$$

and $J_0$ is the value of J for zero thrust.

In accordance with various model tests on the interference between the airscrew and fuselage of an aeroplane, it is assumed that the increased drag of the structure due to the airscrew's slipstream is proportional to the thrust and may be absorbed into the thrust coefficient of the airscrew. The constant $A_T$ is therefore assumed to be chosen so as to include this slipstream correction.

For the engine unit, write

$P$ = engine power in ft.lbs. per sec. at revs. $n$,

$k$ = fuel consumption in lbs. per ft.lb.,

$\phi$ = law of variation of power with height,

and assume that, when throttled, the power falls off by the factor $\mu^2$ at the same revolutions $n$ and that the fuel consumption per ft.lb. increases to $k/\mu$. This assumption is made to allow for the less efficient working of the engine on throttle and represents the type of variation which actually occurs. In the case of any particular engine, for which the necessary data are available, the assumption should be suitably modified. The quantity $\mu$ is a variable representing the extent to which the engine is throttled, and its value must be between zero and unity.

3. *Conditions of level flight.*—In the case of a long flight, it is customary to choose some suitable height and to maintain this level throughout the journey. The initial climb and the final glide are then relatively unimportant, and it becomes important to examine the best method of conducting the level flight. The choice of the height and of the total weight is also of importance.

Consider an aeroplane with N engines, all working under the same conditions. Then, with the usual notation, the equations for level flight are

$$\left.\begin{aligned} \phi\mu^2 P &= 2\pi A_Q\,(a-\lambda^3)\,\rho n^3 D^5 \\ k_L \rho S V^2 &= W \\ k_D \rho S V^2 &= N A_T\,(1-\lambda^2)\,\rho n^2 D^4 \\ \frac{dW}{dt} &= -\,N k \phi \mu P \end{aligned}\right\} \quad . \quad . \quad . \quad (2)$$

These equations can be simplified considerably by the introduction of the following subsidiary variables and constants. Write

$$\left.\begin{aligned} w^2 &= W/\rho S D^2 J_o^2 \\ B_T &= S J_o^2 / N D^2 A_T \\ B_Q &= \phi / 2\pi\rho D^5 A_Q \\ K &= 2\pi N k D^2 A_Q / S J_o^3 \\ x &= k_L \\ y &= 1 + B_T k_D \\ F(n) &= P/n^3 \\ \xi &= \text{distance travelled} \end{aligned}\right\} \qquad (3)$$

It should be noted that $B_T$ and K are constants, $B_Q$ is proportional to $\phi/\sigma$ and normally decreases with the density, but is constant if the engine power varies directly as the density, $w$ is proportional to $\sqrt{W/\sigma}$, while $x$ and $y$ depend only on the attitude of the aeroplane.

Equations (2) now become

$$\left.\begin{aligned} \lambda^2 y &= 1 \\ n &= w\sqrt{y}/x \\ a - \lambda^3 &= B_Q \mu^2 F(n) \\ \frac{1}{W}\frac{dW}{d\xi} = \frac{2}{w}\frac{dw}{d\xi} &= -\frac{K}{x}\left\{B_Q\, y^{3/2}\,(a y^{3/2} - 1)\, F(n)\right\}^{\frac{1}{2}} \end{aligned}\right\} \qquad (4)$$

4. *Minimum fuel consumption.*—It is now possible to determine the conditions of level flight at any height to obtain the minimum fuel consumption per mile. The condition is that $\dfrac{dw}{d\xi}$ shall be a maximum, since the quantity is essentially negative, and we derive

$$\frac{1}{x} = \frac{3}{4}\frac{1}{y}\frac{dy}{dx}\left\{1 + \frac{a\,y^{3/2}}{a\,y^{3/2} - 1}\right\} + \frac{1}{2F}\frac{dF}{dn}\frac{n}{2}\left(\frac{1}{y}\frac{dy}{dx} - \frac{1}{x}\right)$$

or

$$\frac{x}{y}\frac{dy}{dx} = \frac{n\dfrac{dF}{dn} + 4F}{\dfrac{3F}{a\,y^{3/2} - 1} + \left(6F + n\dfrac{dF}{dn}\right)}$$

which can be written in the form

$$\frac{x}{y}\frac{dy}{dx} = \frac{\dfrac{d}{dn}(n\,P)}{\dfrac{3P}{a\,y^{3/2} - 1} + \dfrac{1}{n^2}\dfrac{d}{dn}(n^3 P)} \qquad (5)$$

Since $n^2$ is proportional to $W/\sigma$, the best attitude as determined by this equation will normally depend on the total weight and on the height. This general form is very complicated to work with in practice, but simple forms of P can be chosen which may be expected to lead to satisfactory results. In the first place it is to be noted that an excellent approximation to the power curve of any modern engines is obtained in the form

$$P = Bn - (A + Cn^2)$$

and this form leads to the condition

$$\frac{x\,dy}{y\,dx} = \frac{(a\,y^{3/2} - 1)\,(2Bn - A - 3\,C\,n^2)}{3\,a\,y^{3/2}\,(Bn - A - Cn^2) + (a\,y^{3/2} - 1)\,(Bn - 2\,C\,n^2)}$$

This result can be further simplified by taking the two approximations : (*a*) P = A, (*b*) P = B$n$. Neither of these simple cases are good approximations except over a small range of r.p.m. They have, however, the advantage that they are simple forms to use in analysis and that the actual power curve will at all points lie between these two limiting cases.

Equation (5) now simplifies to

$$P = A, \quad \frac{x\,dy}{y\,dx} = \frac{a\,y^{3/2} - 1}{3\,a\,y^{3/2}} \qquad (5a)$$

$$P = Bn, \quad \frac{x\,dy}{y\,dx} = \frac{2\,(a\,y^{3/2} - 1)}{4\,a\,y^{3/2} - 1} \qquad (5b)$$

and in these simple cases the best attitude of level flight is independent of the total weight and of the height. It is probable, therefore, that if the actual form of P for any engine be substituted in equation (5), the solution will also be approximately independent of these two variables.

To obtain the variation of the rate of fuel consumption with weight and height, we note that in the two simple cases

$$\frac{1}{W}\frac{dW}{d\xi} = -\sqrt{\frac{B_Q\,P}{n^3}}\,f(x)$$

and so derive the results

$$P = A, \quad \frac{1}{W}\frac{dW}{d\xi} \text{ varies as } \left(\frac{\phi^2\sigma}{W^3}\right)^{\frac{1}{4}}$$

$$P = Bn, \quad \frac{1}{W}\frac{dW}{d\xi} \text{ varies as } \left(\frac{\phi}{W}\right)^{\frac{1}{2}}$$

In each case the rate of fuel consumption expressed as a fraction of the total weight decreases as the height or the weight increases. These two conditions, however, are in opposition. For a given total load it is advantageous to fly as high as possible, but the maximum height attainable decreases as the weight increases. If we assume as a rough approximation that the engine power varies as the density, *i.e.*, that $\phi = \sigma$, the value of $\sigma$ at the ceiling will be proportional to the total weight W, since equations (4) indicate that $w$ will be constant. This assumes that the flight is made at the best attitude and that the only modification to the aeroplane is in the load carried. In practice it might be found advantageous to make corresponding changes in the wing area and in the propeller characteristics, but the present discussion is confined to determining the best load to be carried in an aeroplane whose other characteristics are already fixed. It follows for this type of variation that the percentage rate of fuel consumption is independent of the load carried. The lighter aeroplane will fly at a slower indicated speed and at a greater height, and these two factors will compensate, so that the true speed is unaltered. These considerations show that, from the point of view of economical flying, the load carried is unimportant, but it is essential that the aeroplane shall fly at its best attitude as determined by equation (5) and at the greatest possible height. It follows also that the weight per horse power of the aeroplane should be chosen to give as low a ceiling as is compatible with the other conditions, such as safety.

5. *Level flight at constant attitude.*—For a long flight it is not necessarily advantageous to fly at each instant in such a manner that the rate of fuel consumption is a minimum. The previous analysis has dealt only with this latter point and it is now necessary to deal with the more general problem. It has been shown above that, in the case of two simple forms of the power curve P, the best attitude is independent of the weight. This suggests the investigation of the case where the whole flight is made at a constant attitude and at a constant height. It will then be possible to determine the attitude which will give the maximum range for a certain amount of fuel.

Returning to equations (4) we note that $x$, $y$, and $\lambda$ are constant, while $n$, $w$, and $\mu$ vary. It follows that the total range for a reduction in weight from $W_1$ to $W_2$ can be determined by the equation

$$\frac{K\sqrt{B_0}}{2}\,\xi = \frac{x}{\sqrt{\{y^{3/2}(a\,y^{3/2}-1)\}}}\int_{n_2}^{n_1}\frac{dn}{n\sqrt{F(n)}} \quad . \qquad (6)$$

where $n_1$ and $n_2$ are the initial and final values of $n$, and depend on the values of $W_1$ and $W_2$ respectively.

Now introduce a subsidiary function $\psi$, defined by the differential equation

$$\frac{d\psi}{dn} = \frac{1}{n\sqrt{F(n)}} = \frac{n}{\sqrt{P}} \quad . \quad . \quad . \qquad (7)$$

and we obtain

$$\frac{K\sqrt{B_0}}{2}\,\xi = \frac{x(\psi_1-\psi_2)}{\sqrt{\{y^{3/2}(a\,y^{3/2}-1)\}}} \quad . \quad . \qquad (8)$$

The condition for maximum range with a given load of fuel, or for minimum fuel consumption for a given range and a given initial or final load, can now be expressed as

$$\frac{1}{x}+\frac{\left(n\frac{d\psi}{dn}\right)_1-\left(n\frac{d\psi}{dn}\right)_2}{\psi_1-\psi_2}\,\frac{\left(\frac{x}{y}\frac{dy}{dx}-1\right)}{2x}=\frac{3}{4}\frac{1}{y}\frac{dy}{dx}\left[1+\frac{a\,y^{3/2}}{a\,y^{3/2}-1}\right]$$

or

$$\frac{x}{y}\frac{dy}{dx}=\frac{2(\psi_1-\psi_2)-\left\{\left(n\frac{d\psi}{dn}\right)_1-\left(n\frac{d\psi}{dn}\right)_2\right\}}{\frac{3(\psi_1-\psi_2)}{2(a\,y^{3/2}-1)}+3-\left\{\left(n\frac{d\psi}{dn}\right)_1-\left(n\frac{d\psi}{dn}\right)_2\right\}}$$

which can be reduced to the form

$$\frac{x}{y}\frac{dy}{dx}=\frac{\left(n^3\frac{d}{dn}\frac{\psi}{n^2}\right)_1-\left(n^3\frac{d}{dn}\cdot\frac{\psi}{n^2}\right)_2}{\left\{\left(n^4\frac{d}{dn}\frac{\psi}{n^3}\right)_1-\left(n^4\frac{d}{dn}\frac{\psi}{n^3}\right)_2\right\}-\frac{3}{2}\frac{\psi_1-\psi_2}{a\,y^{3/2}-1}} \qquad (9)$$

This formula corresponds to equation (5) derived above, and both are subject to the limiting condition that level flight at the attitude so determined is possible at the height chosen. In other words, for a given weight the maximum height, at which level flight of this nature is possible, must be determined from equations (4) by the condition that $\mu$ never exceeds unity (engine all out).

In general, equation (9) will have a solution differing from that of equation (5). It can be shewn, however, that the two formulæ are identical if P is of the form $n^r$ where $r$ has any value other than 3. Thus for the two simple cases previously considered, the attitudes determined by equation (9) are those already given by equations (5*a*) and (5*b*). Since the actual power curve lies within the limits of these two simple cases, the error due to adopting a similar conclusion in the general case will probably be small.

6. *Effect of wind.*—So far the aeroplane has been considered to be flying in still air, and important modifications to the analysis are necessary when the wind velocity is taken into account. Divide the wind velocity into two components—a head wind $V_h$ and a cross wind $V_c$. Also write

$$V_c = v\mathrm{DJ}_0$$
$$V_h = u\mathrm{DJ}_0$$

Then the velocity of the aeroplane along its path is

$$\sqrt{V^2-V_c^2}-V_h$$

or

$$\frac{V}{n}\left\{\sqrt{n^2-y\,v^2}-\sqrt{y\,u^2}\right\}$$

The expression for $\frac{dW}{d\xi}$ given in equations (4) must be divided by the ratio of this velocity to V in order to obtain the modified form. Proceeding as before we derive the condition for the minimum rate of fuel consumption at any instant, and it reduces to the equation

$$\frac{x}{y}\frac{dy}{dx}\left\{\frac{3P}{a\,y^{3/2}-1}+\frac{1}{n^2}\frac{d}{dn}(n^3\,P)\right\}-\frac{d}{dn}(n\,P)$$

$$= 2P\,\frac{\sqrt{y\,u^2\,(n^2-y\,v^2)+y\,v^2}}{\sqrt{y\,u^2\,(n^2-y\,v^2)+y\,v^2}-n^2} \quad . \quad . \quad (10)$$

This condition replaces (5) for deciding the best attitude of level flight. If the head and cross winds are considered separately and if the simple forms are assumed for P, the following results are obtained :—

(1) head wind only

$$P = A, \quad \frac{x}{y}\frac{dy}{dx} = \frac{a\,y^{3/2}-1}{3\,a\,y}\;\frac{V-3\,V_h}{V-V_h}$$

$$P = Bn, \quad \frac{x}{y}\frac{dy}{dx} = \frac{2\,(a\,y^{3/2}-1)}{4\,a\,y^{3/2}-1}\;\frac{V-2\,V_h}{V-V_h}$$

(2) cross wind only

$$P = A, \quad \frac{x}{y}\frac{dy}{dx} = \frac{a\,y^{3/2}-1}{3\,a\,y^{3/2}}\;\frac{V^2-3\,V^2}{V^2-V_c^2}$$

$$P = Bn, \quad \frac{x}{y}\frac{dy}{dx} = \frac{2\,(a\,y^{3/2}-1)}{4\,a\,y^{3/2}-1}\;\frac{V^2-2\,V_c^2}{V^2-V_c^2}$$

Both for a head wind and for a cross wind the effect is to make a smaller value of $x$ desirable, and this corresponds to a higher speed. In each case the attitude now depends on the weight, since the speed at any definite attitude falls off with the weight during a long flight. In consequence it is no longer true that the attitude should be maintained constant, and the nature of the variation may be derived from the conditions given above. It should be noted in this connection that a cross wind has a relatively small effect compared with a head or following wind of the same velocity.

7. *General conclusions.*—The problem discussed in the present paper is that of determining the best method of flying an aeroplane whose aerodynamic constants and airscrew characteristics are fixed, the variables being the speed, total load and altitude at which the journey is made. Modifications to the aeroplane structure or airscrew form are not considered.

The analysis has been confined to the case of level flight, but this is the most important aspect of the problem since on a long flight the greatest part of the journey will probably be made at a height chosen from the following considerations :—

(1) a certain minimum height, say 5,000 ft., is desirable to allow a reasonably large area for landing in case of engine failure ;

(2) an upper limit will be placed on the height by considerations of the comfort of the pilot and passengers ;

(3) on particular routes the height may be decided by meteorological conditions, such as the direction and velocity of the prevailing winds.

To obtain maximum range or minimum fuel consumption in still air, the aeroplane should be flown at a definite attitude which is almost independent of the load carried and of the height. The most advantageous loading of the aeroplane will be such that the chosen height for the journey is the greatest height at which the aeroplane can fly level at this attitude. Owing to the gradual decrease in total weight owing to fuel consumption this maximum height will increase during the course of the journey, and it would appear to be better to allow the aeroplane to climb slowly on this account, rather than to maintain the initial height by throttling the engine.

These conclusions are in good agreement with those obtained by Mr. Berry, that it is best to fly at a certain attitude with the engine all out, but the above analysis indicates a certain increased range if the journey is made at a great height.

It should be noted also that the design of the aeroplane can be modified to suit the conditions indicated by the analysis. In particular the advance per revolution is constant for level flight at a definite attitude and the airscrew can therefore be designed to give its maximum efficiency throughout the flight instead of at top speed as usually occurs.

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