U. S. NAVAL PROVING GROUND

DAHLGREN, VA.

REPORT NO. 1-43.

PENETRATION MECHANISMS

I. THE PENETRATION OF HOMOGENEOUS ARMOR BY UNCAPPED PROJECTILES AT 0° OBLIQUITY.

[redacted]

9 April, 1943.

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I. THE PENETRATION OF HOMOGENEOUS ARMOR BY UNCAPPED PROJECTILES AT 0° OBLIQUITY.

APPROVED:

David I. Hedrick
DAVID I. HEDRICK,
CAPTAIN, USN,
INSPECTOR OF ORDNANCE IN CHARGE.

PREFACE

AUTHORIZATION

This study is part of the program authorized in Bureau of Ordnance Letter NP9/A9(re3) of January 9, 1943, as Naval Proving Ground Research Project APL-2.

OBJECT

To provide a rational mechanical explanation of recent precisely determined laws of penetration.

SUMMARY

Certain experimental laws of penetration of homogeneous armor at 0° obliquity are presented, and theoretical interpretations are derived. In particular:

1. In the penetration of thick plates ($e/d > 0.3$) the quantity mV_L^2/d^3 is a linear function of e/d. This law is explained by Bethe's expanding-hole theory, modified to take account of the formation of petals on the back of the plate.

2. In the penetration of thick plates, if the residual energy E_R is plotted as a function of the striking energy E_S, a straight line results with a slope of about one. For a thick plate the slope of this line is less than unity; trials against a series of progressively thinner plates give slopes increasing as e/d decreases. These observations are explained when one considers the dynamic nature of projectile penetration; if the force with which the plate resists the projectile increases linearly with the projectile energy, the observed results follow. The slopes may be calculated quantitatively by an extension of Robertson's version of the Poncelet-Morin theory.

3. In the penetration of thin plates ($e/d < 0.3$), the predominant mechanism of failure is the bending back of plate material around the hole, comparable to the bending of the petals on the back of a thicker plate. This mechanism leads to a quadratic variation of mV_L^2/d^3 with e/d, which is in fair agreement with experiment. In this thin-plate theory, stretching and dishing are not included; they are relatively unimport...

at the upper end of the thin plate range, but contribute the bulk of the energy absorption in the thinnest plates, which lack the stiffness to absorb much energy by bending.

4. Additional qualitative results are given, including explanations of shatter at velocities well above the limit, shatter against thick plates, and the effect of projectile form on projectile breakage and on the phenomenon of punching.

CONTENTS

LIST OF FIGURES

I INTRODUCTION.

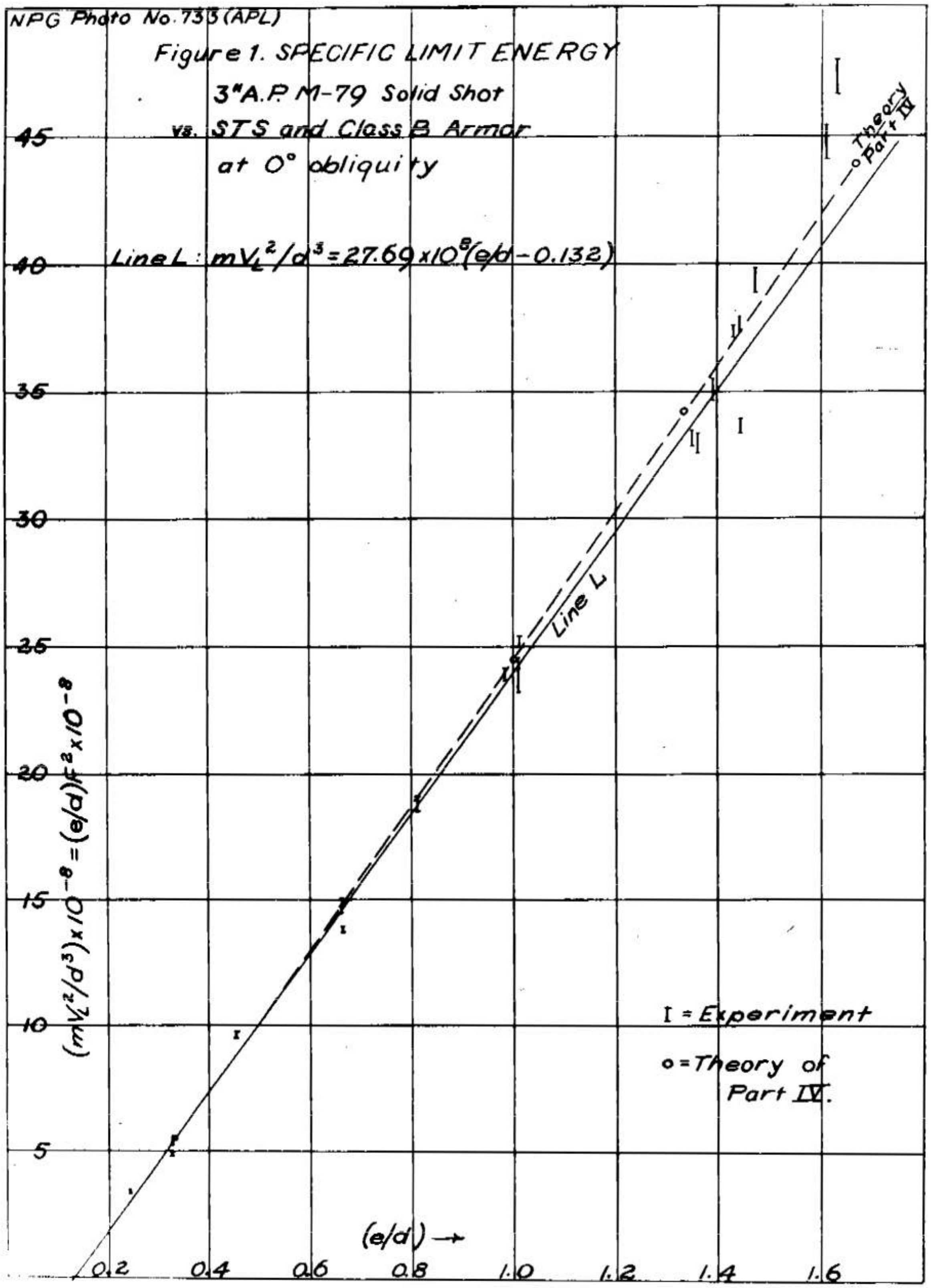
The experimental and theoretical study of projectile impact is authorized as Naval Proving Ground Research Project No. APL-2 in Bureau of Ordnance letter NP9/A9 (Re3) of January 9, 1943.

A complete general theory of penetration would enable us to choose the most desirable shapes of cap and projectile without laborious trial and error, to decide the optimum thickness and hardness of chill in Class A plate, and in general provide a guiding framework within which experimentation could be carried on with the greatest promise of success and with a minimum waste of effort.

It cannot be said that an adequate general theory of penetration is available, nor is there likely to be for some time; nevertheless, the theory presented in the present report represents a considerable advance over previous results in this field, being in better agreement with a wider range of more precise experimental results than are any known earlier theories. More refined methods of analysis are projected for the future, which should yield even more satisfactory results.

Experimental study of projectile impacts has yielded fairly complete information upon two points, namely (a) the way in which the limit energy for complete penetration (the Navy limit) varies with plate thickness, and (b) the way in which the residual energy after a complete penetration depends upon the striking energy. A discussion of the experimental results is presented in Appendix A. The particular experimental results applicable to this report are shown in Figures 1, 2 and 3; special points in connection with these figures will be discussed as they arise in the theory.

The first observation arising from an inspection of Figure 1 is that in the range $0.2 < e/d < 1.0$, the limit energy (or specific limit energy - see Appendix A) is a linear function of e/d. In reference (1) a theory of penetration is worked out essentially as follows: Two states of the plate are considered - first, the plate is intact, and second it has a hole in it. It is assumed as a first approximation that the energy expended by the projectile in making the hole is independent of the precise details of the process. It is therefore supposed that a small hole of radius $\underline{a}$ exists through the plate; this hole is expanded to a radius $\overline{b}$ by internal hydrostatic pressure, and the work required for the expansion is calculated. The limit of the work required is found as $\underline{a}$ approaches zero (no hole through the plate), and $\underline{b}$ approaches the radius r_o of the projectile. The work found in this way is assumed to be equal to the limit energy required



NPG Photo No. 733 (APL)
Figure 1. SPECIFIC LIMIT ENERGY
3" A.P. M-79 Solid Shot
vs. STS and Class B Armor
at 0° obliquity
Line L: $mV_L^2/d^3 = 27.69 \times 10^8 (e/d - 0.132)$
Theory Part IV
Line L
I = Experiment
o = Theory of Part IV.
$(mV_L^2/d^3) \times 10^{-8} = (e/d) F^2 \times 10^{-8}$
(e/d) →
45
40
35
30
25
20
15
10
5
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6

for a projectile of radius r_o just to pass through the plate. The result is

$$(1/2)mV_L^2 = 2\pi Y r_o^2 e, \quad \ldots\ldots\ldots \quad (1)$$

where m is the projectile mass, V_L its limit velocity, Y the yield-stress of the plate material, and e the plate thickness. To express this formula in terms of what may be called the specific limit energy*, equation (1) may be divided by $d^3/2$, which yields

$$mV_L^2/d^3 = \pi Y e/d, \quad \ldots\ldots\ldots \quad (2)$$

where d is the projectile diameter. On this theory, then, the specific limit energy is proportional to e/d; inspection of Fig. 1 shows, however, that the linear portion of the graph does not pass through the origin, but intersects the e/d axis at e/d = 0.132, the equation of this portion of the graph being

$$mV_L^2/d^3 = 27.69 \times 10^8(e/d - 0.132), \quad \ldots . \quad (3)$$

where the specific limit energy is in ft.-poundals/ft.^{3} The discrepancy between equations (2) and (3) can be cleared up by modifying the theory of reference (1) to take account of end effects - specifically, the opening out of petals on the back of the plate.

While there is certainly friction between projectile and plate, the friction is not very great, as is evidenced, for example, by the continued spin of the projectile during and after impact. As a first approximation it is therefore assumed that the force between the plate and projectile is everywhere normal to the surface of the projectile.

In the initial stages of penetration (Fig. 4) the forces exerted on the plate-material by the projectile have both forward and lateral components. Because of the "rigidity**" of the plate the forward motion of the plate material is inhibited, and the material is squeezed out laterally and thickened as shown in the figure, the displacements being approximately the same as in the thin plate theory of reference (1). This behavior will continue until the nose of the projectile has penetrated far enough so that the "rigidity" of the material still ahead of the projectile is insufficient to prevent bulging on the back of the plate. As the back

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* See Appendix A, p.24.

** The term rigidity is used to imply the resistance of the plate (or parts of it) to bending, as opposed to resistance to other types of deformation.

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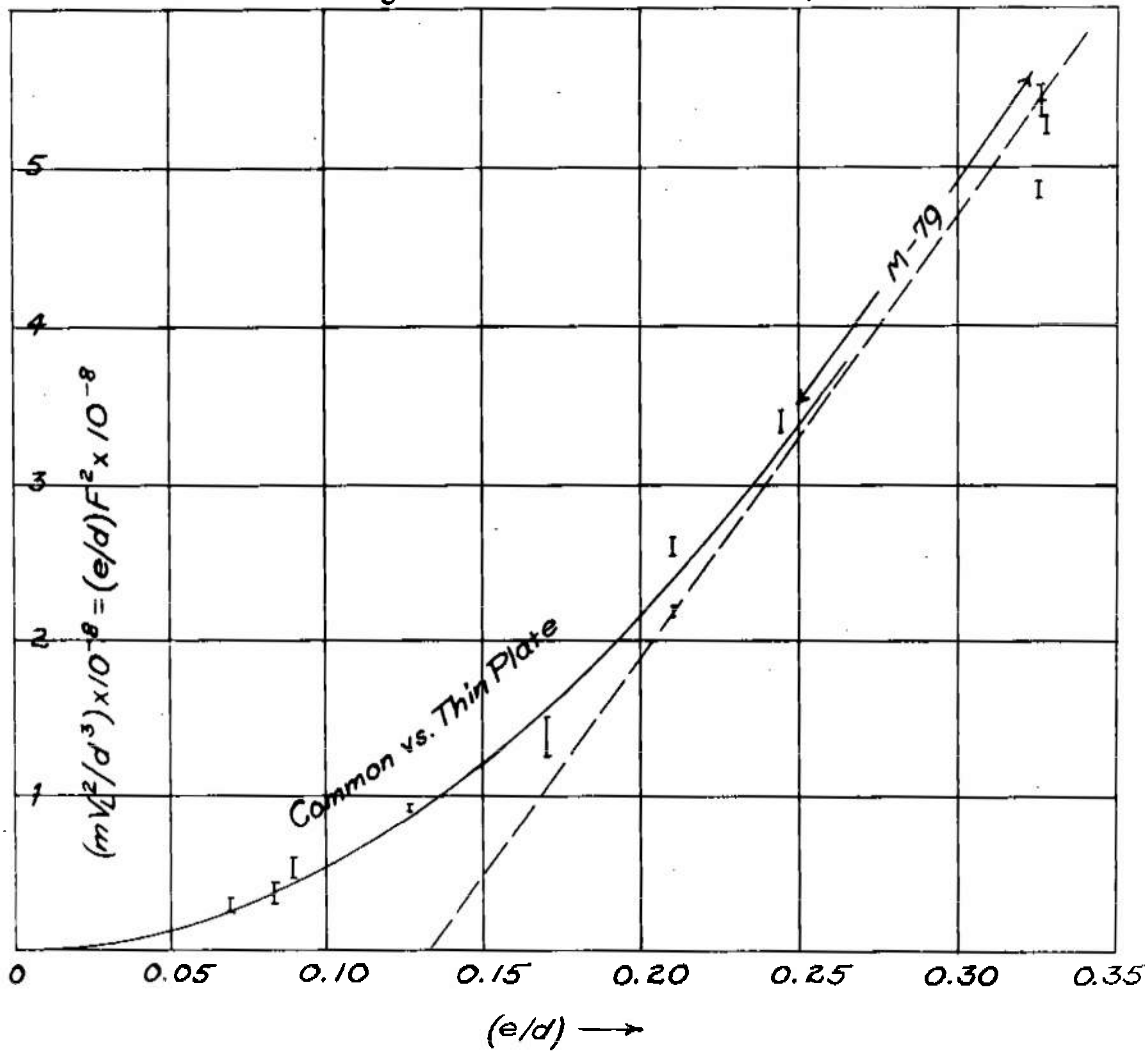
Figure 2. SPECIFIC LIMIT ENERGY

3" Common Mark 3 Projectiles
vs. Thin Homogeneous Armor at 0° Obliquity



bulges, material in the interior of the plate will be able to move forward as well as laterally, the displacements of this material being again approximately the same as in the thin plate theory of reference (a). In contrast, the material on the back of the plate will crack across the bulge and open out in petals (Fig. 5) a type of failure distinct in nature from the expanding-hole mechanism of Bethe.

The "rigidity" of the plate material ahead of the nose of the projectile will depend upon its thickness; when the nose reaches a certain distance from the back of the plate, bulging of the plate will set in and the transition from Bethe's mechanism to petal formation will begin. With a given caliber and shape of projectile, the distance between projectile nose and back of plate at which petalling sets in will presumably be independent of the plate thickness. Thus, a plate may be divided into two zones - a front zone in which the mechanism of failure is essentially that of Bethe, and a back zone in which the material fails by petalling. As one goes from thick plate to thinner plate, the thickness of the back zone remains constant, while the front zone gets thinner and thinner and disappears.

In Part II the thick plate case will be discussed quantitatively, and an equation of the same form as equation (3) will be derived for the specific limit energy. In Part IV the thin plate case (for plates so thin that Bethe's mechanism does not occur at all) will be taken up; it will be shown that equation (3) does not apply in this range of e/d, and the appropriate equation for thin plates will be derived and compared with the data of Figure 2.

It will be noted that the theory of Part II (leading to an equation of the form of equation (3)) takes no account of dynamic effects in the penetration. In reference (2), H. P. Robertson presents a modified form of the Poncelet-Morin theory, and shows that it leads to a relation between the residual energy after penetration and the striking energy of the form

$$E_R = S(E_S - E_L), \qquad \ldots\ldots \qquad (4)$$

where E_R is the residual energy, E_S the striking energy, E_L the limit energy, and S is a constant, the value of S depending upon the value of e/d. This is a well known law of penetration, much used in limit determinations. Variations of equation (4) are often used in calculation - e.g., Fig. 3 is a plot of mV_R^2/e vs. mV_L^2/e, the actual plate thickness at the impact being used for the corresponding

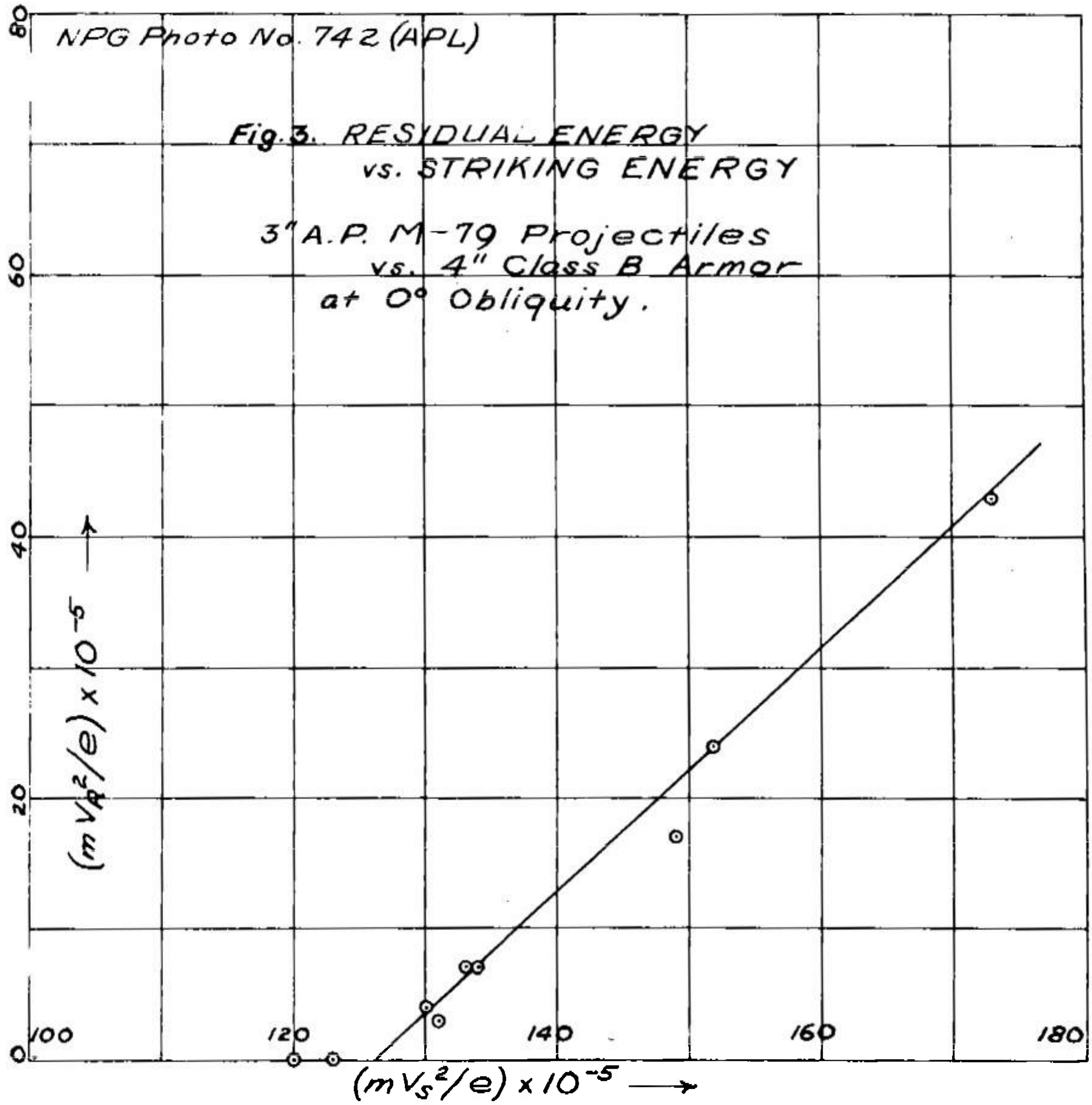

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Fig. 3. RESIDUAL ENERGY vs. STRIKING ENERGY
3" A.P. M-79 Projectiles vs. 4" Class B Armor at 0° Obliquity.
0
20
40
60
80
100
120
140
160
180
$(mV_R^2/e) \times 10^{-5}$ →
$(mV_S^2/e) \times 10^{-5}$ →

plotted point. This procedure tends to smooth out the variations in plate thickness encountered from round to round; this point is discussed further in Appendix A.

In Robertson's theory it is supposed that the projectile must not only overcome the cohesion of the plate material, but must also set the plate material in motion with a sufficient velocity to get it out of the path of the advancing projectile. On this theory the kinetic energy which must be imparted to the plate material is the determining factor in fixing the slope S in equation (4), and should have an effect upon the variation of the specific limit energy with e/d. Against thicker plates, higher striking velocities will be necessary, so the dynamic energy required should be greater, and departures from the line of equation (3) should be expected if the magnitude of the dynamic energy is significant. In qualitative agreement with this, it will be observed that the experimental points in Fig. 1 show a tendency to fall above the straight line for values of e/d greater than 1.

It has been objected that the work done in setting the plate material in motion will be recovered, as the kinetic energy produced in the plate will be utilized in expanding the projectile hole, so that no net effect will result from the fact that the penetration of a projectile is dynamic, and not static. However, in sections 6 and 15 of reference (1) a sufficient analysis of this situation is given to indicate that an increase in pressure on the ogive should result from the dynamic effects, with a consequent increase in the limit energy From this analysis it appears that the inertia in the elastic part of the plate is sufficient to prevent a reduction in pressure on the projectile until the bourrelet has passed the point in question in the plate.

As a matter of fact, the experiments described in reference (3) show that the plate will vibrate violently during impact, and attempts to find the force-curve for projectiles by spark photographs taken during impact (reference (4)) show that the projectile may experience violent longidutinal oscillations. If this is the case, the pressure between ogive and plate-material will oscillate rapidly, and any analysis of the mechanism of impact which omits consideration of these oscillations will necessarily be partial and incomplete. Part III of this report will be devoted to an analysis of the dynamic effects during penetration, along the lines laid down by Robertson.

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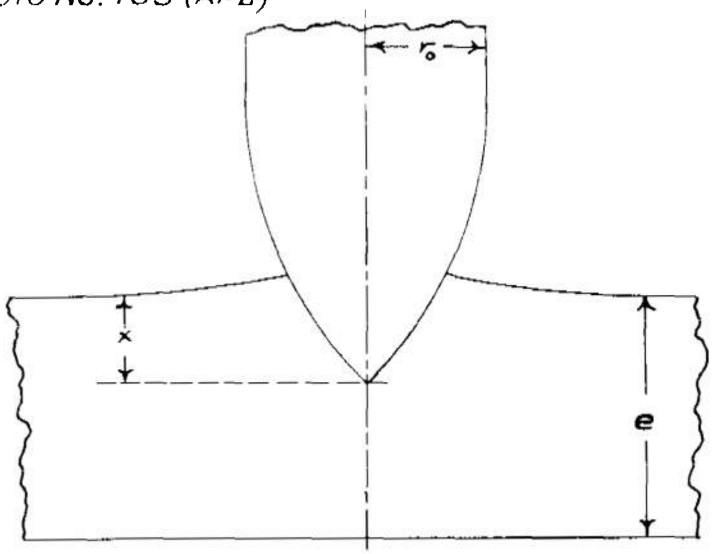


Figure 4
An Early Stage in the Penetration of a Thick Plate

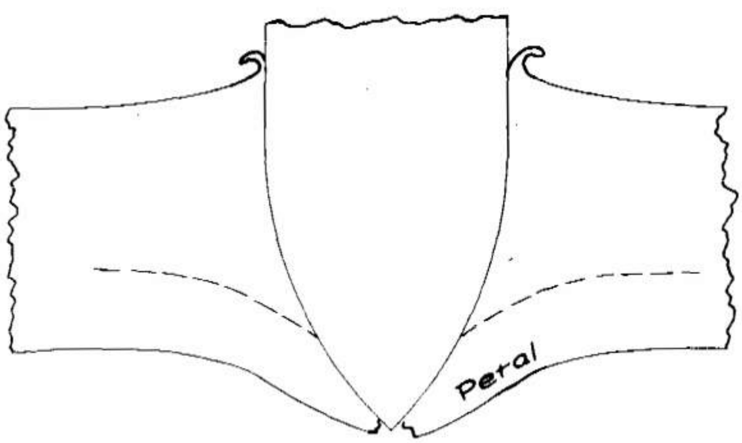
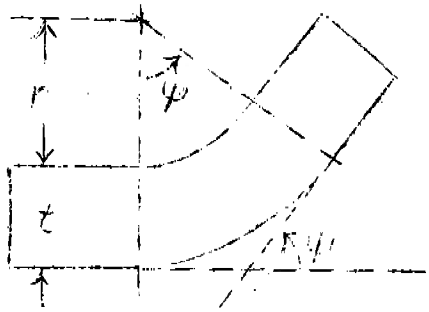


Figure 5

A Late Stage in the Penetration of a Thick Plate

II MODIFICATION OF BETHE'S THEORY TO INCLUDE PETALLING.

The folding back of the petals may be considered as the bending back of a collection of bars by the advancing projectile.



Consider a bar of thickness t, bent without buckling through an angle ψ, the radius of the inside of the bend being r. If the inside of the bar does not buckle, no extension of the inside occurs in bending. The extension of the outer surface is $(r + t)\psi - r\psi = t\psi$. The mean extension is $(1/2) t\psi$, as the average over the bar is linear. The work of bending is obtained by multiplying the extension by the cross-section A and by the yield stress Y:

$$W = (1/2)\, t\psi A\, Y \qquad \ldots\ldots\ldots \qquad (5)$$

The same result is obtained regardless of whether or not buckling is assumed.

Observation of the petals on the back of a plate shows that they are bent back through an angle of some 60°, along a circumference roughly twice that of the projectile hole. To calculate the work done in folding back the petals on their bases, $4\pi r_o t$ is substituted for A in equation (5), and $\pi/3$ replaces ψ:

$$W_p = (t/2 \cdot \pi/3)\,(4\pi r_o t)Y$$

or

$$W_p = 2\pi Y(\pi r_o t^2/3), \qquad \ldots\ldots\ldots \qquad (6)$$

where r_o is the radius of the projectile at the bourrelet. The yield stress in equation (6) may differ somewhat from that used in Bethe's formula (equation (1)), although hardness surveys around impact holes indicate that the difference should be small, the work hardening being about the same at the roots of the petals as through the depth of the plate.

As suggested in the introduction the penetration is considered to follow Bethe's mechanism up to the point at which petalling sets in, the work done in this part of the penetration being

$$W_B = 2\pi Y r_o^2(e - t) \quad \ldots\ldots\ldots \qquad (7)$$

This work is combined with that calculated by equation (6) to get the total amount of work done in penetrating the plate, which should equal the limit energy:

$$E_L = 2\pi Y r_o^2(e - t) + 2\pi Y(\pi r_o t^2/3) \quad \ldots \qquad (8)$$

To make further progress, t must be evaluated. It is supposed that for a given size of projectile t is constant over the range of e/d for which equation (3) holds. Furthermore, the basic assumption underlying the analysis of penetration data by means of the F-coefficient is that if equation (3) (or any similar equation) holds for a given 3" projectile, it will also hold for a similar projectile of any other caliber. On this basis one may reason that t must be proportional to r_o - i.e., must be the same fraction of r_o for 3" projectiles as for 14" projectiles. It is therefore supposed that

$$t = (3/\pi)\, k r_o, \quad \ldots\ldots\ldots \qquad (9)$$

where k is arbitrary, and the factor $3/\pi$ is introduced merely to simplify the algebraic manipulations. Substitution from equation (9) in equation (8) yields

$$E_L = 2\pi Y\left[r_o^2 e - (3k/\pi) r_o^3 + (\pi/3)(9k^2/\pi^2) r_o^3\right]$$

or

$$E_L = 2\pi Y\left[r_o^2 e - (3/\pi) r_o^3 k(1 - k)\right] \quad \ldots\ldots \qquad (10)$$

The quantity k is of course positive, and to agree with equation (3) the quantity k (1 - k) must also be positive, so that $0 < k < 1$. If the plate fails at the back in the easiest way possible -- i.e., so as to absorb minimum energy from the projectile -- k(1 - k) must be a maximum so that k = 1/2. In this case

$$t = 3\, r_o/2\pi \quad \ldots\ldots\ldots \qquad (11)$$

For $2\, r_o = d = 3''$, t - 0"7; actual petals appear to be nearer 1" thick at the base. If the value of t from equation (11) is substituted in equation (10), the limit energy is found to be

$$E_L = 2\pi Y\ (r_o^2 e - 3r_o^3/4\pi)$$

or

$$E_L = (\pi Y/2)\ d^2\ (e - d\ 3/8\pi) \quad .... \qquad (12)$$

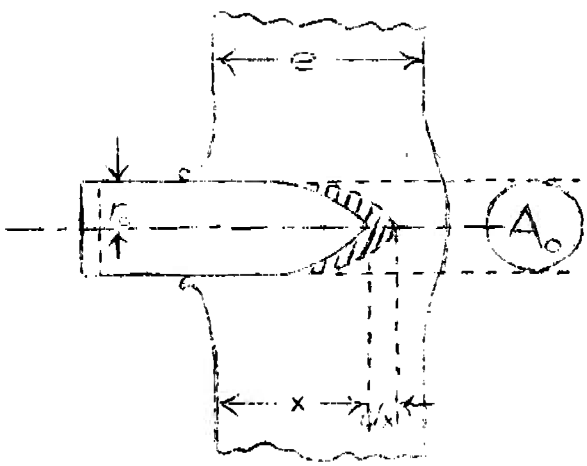
From this the specific limit energy is found to be

$$mV_L^2/d^3 = \pi Y(e/d - 3/8\pi) \quad ...... \qquad (13)$$

This line intercepts the e/d axis at $3/8\pi$, or 0.119; the line of equation (3) intercepts the e/d axis at 0.132, so equation (13) gives an intercept about 10% too low. The agreement is satisfactory, considering the rough methods used in calculating the petalling energy. A more refined treatment of petalling is projected for the future. The question of the yield stress will be discussed later in this paper.

III DYNAMIC EFFECTS IN ARMOR PENETRATION.

In reference (2) H. P. Robertson has shown that a law of resistance of the Poncelet-Morin form leads to an equation for the residual energy of the form of equation (4). In this section Robertson's theory will be developed further in a form suitable for numerical calculation.



Suppose a projectile penetrating a target - consider first a case where the bourrelet has entered the target Let the depth of penetration be x, and the cross-section of the projectile be $A_o = \pi r_o^2$. In advancing a distance dx, the projectile will displace directly a volume of material dV = $A_o$dx. Work will be done on this material in disrupting it, and in setting it in motion with a speed sufficient to displace it from the path of the projectile. This directly displaced material will in turn set other plate material in motion, - a point to which we will return later. If ρ' is the density of the target material and v_o' the mean velocity of the directly-displaced material, the work done on the projectile in advancing a distance dx will be of the form

$$dE = -\left[a\ dV + (1/2)(\rho' dV)v_o'^2\right], \quad \ldots. \qquad (14)$$

where a-assumed constant and called by Robertson the "shatter coefficient"-- is of the nature of a yield stress. The resistance function is then of the form

$$dE/dx = -A_o(a + (1/2)\rho' v_o'^2) \quad \ldots. \qquad (15)$$

In the earlier and later stages of the penetration the cross-section (the projection of the area of ogive in contact with the plate) will not be the full cross-section of the projectile; in the early stages it will depend only upon x, while when the nose emerges from the back the cross-section depends not only upon x, but also upon the plate-thickness e. Thus a more general form of the resistance function would be

$$dE/dx = -A(x,e)(a + (1/2)\rho' v_o'^2), \quad \ldots. \qquad (16)$$

where A(x,e) is the variable cross-section.

Let du be the mean displacement of the plate material displaced when the projectile advances the infinitesimal distance dx. The volume of displaced material is A(x,e)dx, and the material is displaced outward through an area equal to the lateral (surface) area of the embedded portion of the ogive, which area may be designated by L(x,e). Therefore, since L(x,e)du = A(x,e)dx,

$$du/dx = A(x,e)/L(x,e),$$

so that

$$v_o'/v = A(x,e)/L(x,e) \quad \ldots. \qquad (17)$$

and

$$v_o'^2 = \left[A(x,e)/L(x,e)\right]^2 v^2 \quad \ldots. \qquad (18)$$

or

$$v_o'^2 = \gamma(x,e)\ v^2 \quad \ldots. \qquad (19)$$

where

$$\gamma(x,e) = \left[A(x,e)/L(x,e)\right]^2.$$

In the early stages of the penetration, when only the end of the nose is embedded, γ will be fairly large, approximating 1 for blunt projectiles, and smaller for more pointed projectiles. As the nose of the projectile emerges from the back of the plate γ will take smaller and smaller values, reaching zero as the bourrelet clears the plate. An approximation to the mean value of γ through the plate will be the square of the ratio of the lateral area of the entire ogive to the cross-section of the projectile; the area of the ogive is calculated as shown in Appendix B. The average γ is then

$$\gamma_o = (A_o/L_c)^2 \quad \ldots\ldots\ldots \qquad (20)$$

If plate and projectile are of the same material, as is usually the case for major caliber projectiles, $\rho' = \rho$. For the projectile, the mass may be represented by

$$m = \pi r_o^2 \lambda \rho, \quad \ldots\ldots\ldots \qquad (21)$$

which defines λ, the "effective length" of the projectile; λ will usually be different from the actual length. Equation (21) is solved for ρ, and the result substituted in equation (16), together with the substitution for $v_o'^2$ from equations (18) and (20), yielding

$$dE/dx = -A(x,e)\left[a + (1/2)(m/\pi r_o^2 \lambda)\ \gamma_o v^2\right] \ .. \ (22)$$

or

$$dE/dx = -A(x,e)(a + b_o E) \quad \ldots\ldots\ldots \qquad (23)$$

where $E = mV^2/2$ and $b_o = \gamma_o/\pi r_o^2 \lambda$

Equation (23) is easily solved by separation of variables:

$$dE/(a + b_o E) = -A(x,e)dx,$$

$$(1/b_o)\ \ln\ (a + b_o E) = -\int_o^x A(x,e)dx + \text{const.}$$

For $x = 0$, $E = E_S$, the striking energy, so the constant is $(1/b_o)\ \ln\ (a + b_o E_S)$. Substitution and transposition of the constant yields

$$\left(\frac{1}{b_o}\right)\ \ln\left(\frac{a + b_o E}{a + b_o E_S}\right) = -\int_o^x A(x,e)dx$$

Let the projectile pass completely through the plate, so that

$$\int A(x,e)dx = V_o,$$

the volume of the hole through the plate, which is $\pi r_o^2 e$, and $E = E_R$, the residual energy.
Thus,

$$\ln\left(\frac{a + b_o E_R}{a + b_o E_S}\right) = -b_o V_o$$

Taking anti-logarithms,

$$a + b_o E_R = (a + b_o E_S)\ \exp(-b_o V_o)\ \ldots. \qquad (24)$$

$$b_o V_o = (\gamma_o/\pi r_o^2 \lambda)(\pi r_o^2 e) = \gamma_o e/\lambda\ .$$

Thus,

$$E_R = -a/b_o + (1/b_o)(a + b_o E_S)\ \exp\ (-\gamma_o e/\lambda)\ \ldots \qquad (25)$$

When the projectile strikes with just the limit energy E_L, the residual energy is zero, so that equation (24) becomes

$$a = (a + b_o E_L)\ \exp(-b_o V_o)\ \ldots. \qquad (24a)$$

Substitution of this value of a in equation (25) yields

$$E_R = \left[\exp(-\gamma_o e/\lambda)\right](E_S - E_L)\ \ldots. \qquad (26)$$

This equation is essentially that obtained by Robertson in reference (2), and is of the same form as equation (4), as desired. However, before introducing any numberical results certain corrections need to be made.

As previously noted, the projectile must displace material not only of the volume V_o of the hole through the plate, but also surrounding material out to some radius r_1, which is taken provisionally as the radius of the plastic region in the plate; at this point the displacements will be small, as they must correspond to strains within the elastic limit. In Bethe's thin-plate theory, in the static case, the radius of the plastic region is about $3.3r_o$. Bethe points out that the plastic region will be somewhat smaller in dynamic penetrations, but does not say how much. It will be assumed herein that $r_1 = 3r_o$. The volume V_o previously used is therefore to be replaced by $(3)^2 V_o = 9V_o$, and e/λ in equation (26)(and related equations) is to be multiplied by 9.

These factors represent one element of the necessary corrections, modifying the equations insofar as they involve the volume of displaced plate material. The mean velocity of this larger volume of displaced material need not bear the same relation to the projectile velocity as previously found. In default of more exact knowledge, it is supposed that the displacement of any portion of the plate material falls off linearly with distance of its initial position from the axis of the projectile-hole. Thus, if u_o is the displacement of a point initially on the axis ($r = o$), that of a point at $r_1 = 3r_o$ is zero, and for intermediate positions the displacement is

$$u = u_o \, (r_1 - r)/r_1 \quad \ldots\ldots \qquad (27)$$

Averaging over the entire volume, which is symmetrical about the axis of the projectile hole, it is found that

$$\overline{u^2} = \int_o^{r_1} u^2 \cdot 2\pi r dr \Big/ \int_o^{r_1} 2\pi r dr, \quad \ldots\ldots \qquad (28)$$

with the result

$$\overline{u^2} = u_o^{\,2}/6 \quad \ldots\ldots \qquad (29)$$

The velocity being proportional to the displacement,

$\overline{v'^2} = v_o'^{\,2}/6$, so that one should use not γ_o', but $\gamma_o'/6$. When this substitution is made, together with $9e/\lambda$ for e/λ, equation (26) becomes

$$E_R = \exp(-3\gamma_o' e/2\lambda)(E_S - E_L) \quad \ldots\ldots \qquad (30)$$

There are two obvious criticisms of this calculation. The precise law of fall-off of displacement may not be very much like that represented by equation (27). Furthermore, it is to be expected that the law would vary with depth through the plate, as should also the radius of the plastically worked region, due to the change in mechanism from piercing to petalling. The calculation is not applicable to thin plates, for which dishing is important.

The most accurately determined slope available for a thick plate at 0° obliquity is for the 3-inch A.P. M-79 projectile vs. 4-inch Class B plate, six points giving a slope of 0.93. For the projectile in question, the actual diameter is 2"99, and the ogival radius is 5 "; insertion of these values in equation (1) of Appendix B yields a lateral area of ogive of 24.80 sq.in. The cross-section of the projectile is 7.02 sq.in. so that

$$\gamma_o = (A_o/L_o)^2 = (7.02/24.80)^2 = 0.0801.$$

For a 15-lb. projectile, λ = 7.5 in. Thus, against a 4-inch plate

$$3\gamma_o e/2\lambda = 3(0.0801)(4)/2(7.5) = 0.0687.$$

The corresponding slope is therefore 0.934, which agrees with the experimental value within the errors of observation.

Other values of the slope have not been determined with equal precision. However, it has been well established by firing at the Armor and Projectile Laboratory that the slope increases with decreasing e/d, which is in accord with equation (30). As a matter of fact, equation (30) predicts a slope approaching unity as e approaches zero, whereas for thin plate (e/d<0.3, say) the slope is greater than one. This higher value of the slope for thin plate is associated with the marked dishing which occurs around the impact hole, with which the present theory is not designed to deal.

In view of the corrections to equation (26), corresponding changes will be required in equation (23) and the other equations derived from it. Because of the greater volume of material actually displaced, b_o should be replaced by a larger b,

$$b = 9\ b_o,$$

or in general, $b = (r_1^2/r_o^2)b_o$. At the same time, a smaller γ should be used in calculating b, $\gamma = \gamma_o/6$, i.e.,

$$b = 9(\gamma/\pi r_o^2\lambda) = 3\gamma_o/2\pi r_o^2\lambda \quad \ldots\ldots \quad (31)$$

When this expression for b is introduced into equation (24a), one obtains

$$a = (a + b\ E_L)\ \exp(-3\gamma_o e/2\lambda),$$

whence

$$E_L = (a/b)\left[\exp(3\gamma_o e/2\lambda) - 1\right] \quad \ldots\ldots \quad (32)$$

Superficially this equation bears no resemblance to the desired form, equation (3), but it can be improved, bringing it into a form very similar to equation (1). Substitute for b in equation (32), obtaining

$$E_L = (2\pi r_o{}^2 a\lambda/3\gamma_o)\left[\exp(3\gamma_o e/2\lambda) - 1\right]$$

$$= \pi r_o{}^2 a \left[\frac{\exp(3\gamma_o e/2\lambda) -1}{3\gamma_o/2\lambda}\right],$$

or

$$E_L = \pi r_o{}^2 ae \left[\frac{\exp(3\gamma_o e/2\lambda) -1}{3\gamma_o e/2\lambda}\right] \qquad ..... \qquad (33)$$

If the factor πa is identified with $2\pi Y$ of equation (1), this expression is similar to equation (1) except for the factor in brackets. This factor is of the form

$$f(z) = \left[\exp(\alpha z)-1\right]/\alpha z, \qquad ..... \qquad (34)$$

where $z = e/d$, and $\alpha = 3\gamma_o d/2\lambda = 0.0516$ for the 3" A.P. M-79 projectile. f(z) may be tabulated for various plate-thicknesses:

e, in	z	αz	$\exp(\alpha z)-1$	f(z)
1	0.333	0.0172	0.0173	1.008
2	0.667	0.0344	0.0350	1.023
3	1.000	0.0516	0.0530	1.027
4	1.333	0.0688	0.0712	1.035
5	1.667	0.0865	0.0903	1.044

It is seen from this table that the factor in brackets in equation (33) will not produce any marked deviation from linearity. The departure of the curve of equation (33) from linearity has been suggested in Figure 1 by fitting the point for e/d = 0.33 to the line L; the ordinates for the other values of e/d shown in the table were obtained by multiplying the corresponding ordinates for the line L by f(e/d)/f(0.33 These points are shown in Fig. 1 as open circles, and the broken curve is drawn through them. The agreement in the range of e/d from 1.6 to 1.7 is actually better than it appears, as the experimental points were obtained on a plate known from larger caliber firing to be of exceptional quality.

Referring to the question of the intercept of the curve of equation (33), as compared to the experimental curve of the type exemplified by equation (3), in equation (23) for the resistance,

$$dE/dx = -A(x,e)(a + b_0 E)$$

it should be noted that a is treated as a constant, the static part of the resistance varying only with the variable area of impression A(x,e). The analysis of petalling in Part II shows that the resistance should depend not only upon A(x,e), but also upon the type of failure occurring at the particular depth of penetration considered; a is at best an expression of the average static resistance. Equations of the type (12), (13) show that a plate resists penetration as though its effective thickness is not e, but $(e - d \cdot 3/8\pi)$, and one should identify πa not with $2\pi Y$, but with

$$2\pi Y(e-3d/8\pi)/e$$

Thus, the final expression for the limit energy becomes

$$E_L = 2\pi Y r_0^2 (e-3d/8\pi)\left[\frac{\exp(\alpha e/d)-1}{\alpha e/d}\right]$$

or

$$mV_L^2/d^3 = \pi Y(e/d - 3/8\pi) \quad \exp(+\alpha e/d)-1 \; /(\alpha e/d) \quad ..(35)$$

When this is compared with equation (3), it is seen that

$$\pi Y = 27.69 \times 10^8 \text{ ft.-poundals/ft.}^3,$$

whence

$$Y = 8.81 \times 10^8 \text{ poundals/sq.ft.}$$

Translated into the usual terms, the yield-stress is thus

$$Y = 1.90 \times 10^5 \text{ lb./sq.in.}$$

Strictly speaking the coefficient of Y in equation (35) will depend upon the particular theory of plasticity used. Bethe has shown that if the theory of von Mises is employed, instead of that of Mohr (which he used in deriving equation

(1)), the energy expression involves not πY, but $1.15 \pi Y$. With this modification the value of Y obtained by comparison with experiment would be

$$Y = 1.65 \times 10^5 \text{ lb./sq.in.}$$

This value of the yield stress is about twice the value obtained for armor steels in static tension tests.

In static tension tests, the yield stress is found at the onset of plastic yielding. Normally the change in yield stress resulting from work hardening is not determined, but it is considerable. In the penetration of armor work hardening will occur at an early stage in the expansion of the hole, and the mean yield stress should certainly be well above the usual test. Furthermore, the high rates of strain during projectile impact will result in higher values of the yield stress than those found in the usual engineering tests. Some data on this speed effect are given in reference (1); experiments now underway under the auspices of Division 2 of the National Defense Research Committee should provide further information on this point.

Work hardening and the effect of rate of strain on the yield stress will modify Bethe's theory of thin plates. In his theory the plate thickens around the impact hole, in such a way that at any radius r within a certain value the thickness of plate h at that r is given by a formula of the type $2\pi rh$ = const. This formula is designed to apply to a static penetration with a constant yield stress, and results from the fact that every elementary ring in the armor must have equal and opposite forces acting upon its inner and outer surfaces; as the pressure in the plastic region is fixed by the yield stress, the outer and inner surface of every elementary ring must be equal. However, hardness patterns taken around impact holes show that the hardness is greatest near the hole, falling off into the body of the plate; presumably the yield stress varies in a similar manner. This variation in yield stress will make it unnecessary for the thickness to increase so rapidly as one proceeds from the body of the plate towards the impact; the condition $2\pi rh$ = const. is replaced by the condition $2\pi rhY$ = const., where Y is now a variable. This formula will apply only in static penetrations; in dynamic penetrations where the plate material is being accelerated, the force pressing outward on the inside of any elementary ring must exceed the inward force on the outside of the ring. In consequence one does not expect a plate to thicken as much around an impact hole as in Bethe's thin-plate theory, a point in agreement with the observation of actual impacts.

IV THE PENETRATION OF THIN PLATES.

The theory of Part II applies only to plates thicker than a certain critical value. Below the critical thickness the plate fails essentially by petalling, the expanding-hole mechanism of Bethe not being present at all. While in such thin plates dishing normally extends for some distance out from the impact, the major deformation occurs near the hole. Provisionally it may be supposed that equation (6) applies, where the thickness t is now the entire plate-thickness e, and the work of petalling is the limit energy:

$$E_L = 2\pi Y(\pi r_o e^2/3) \quad \ldots\ldots \quad (36)$$

This expression is quadratic in e, and gives zero energy at zero e/d, as should be the case. This curve should join that of the linear expression (12), or its equivalent

$$E_L = 2\pi Y\,(r_o^2 e - 3r_o^3/4\pi)$$

at some value of e, the two being tangent at the point of intersection. The value of e is found by solving simultaneously:

$$2\pi Y(\pi r_o e^2/3) = 2\pi Y(r_o^2 e - 3r_o^3/4\pi),$$

which reduces to

$$(2\pi e)^2 - 2(2\pi e)(3r_o) + (3r_o)^2 = 0.$$

This equation has a double root,

$$e = 3r_o/2\pi,$$

agreeing with equation (11),

or $$e/d = 3/4\pi = 0.239 \quad \ldots\ldots \quad (37)$$

Figure 2 is a graph of the specific limit energy for 3" Common Mark 3 projectiles versus thin homogeneous plate at 0° obliquity; these projectiles have the same ogival radius as the 3" A.P. M-79 projectile. The linear graph for the M-79 projectile against thicker plate is shown as the dashed line. The graph drawn in for the 3" Commons is the least-squares parabola for the plotted points. The closest approach of the two graphs is at e/d = 0.259. The discrepancy between the experimental graph and the calculation of equation (36) may be interpreted as due to the neglect in the latter case of the energy of dishing.

The slopes of the graphs of E_L vs. e are obtained by differentiation:

$$dE_L/de = 4\pi^2 Yr_o e/3$$

(from equation (36)), and

$$dE_L/de = 2\pi Yr_o^2$$

from the linear graph. If in the equation for the slope of the parabola the value of e from equation (37), is substituted, it is seen that the slopes are the same - i.e., the theoretical curves are tangent at the point of intersection.

V. DISCUSSION.

Most firing at armor plate is done for proof, and limits are not determined, except accidentally. If a limit velocity is determined for a plate, it is usually at an obliquity and value of e/d approximating those expected in service - if the value of e/d is small, θ is large, and vice versa. It is only recently that accurate limits have been found, with a single type of projectile at a fixed obliquity, covering the entire practicable range of e/d. This series of determinations provides a much more satisfactory basis for theoretical discussion than has hitherto been available.

Various empirical formulas have been proposed in the past for the calculation of plate limits; dimensional considerations have suggested the type form:

$$1/2mV_L^2 = \beta Ye^n d^{3-n},$$

where β is dimensionless, and n is chosen to give the best fit to the available experimental data. In terms of specific energy, this formula becomes

$$mV_L^2/d^3 = \alpha(e/d)^n,$$

where α is now a constant having the dimensions of stress. It is clear that dimensional considerations do not restrict such empirical formulas to a single term, and that in general one could write

$$mV_L^2/d^3 = \sum_n \alpha_n (e/d)^n.$$

Equations of the type of equation (3) represent the simplest case of an empirical formula with more than one term. A study conducted at the Naval Proving Ground based upon date obtained at the Naval Proving Ground, the Naval Research Laboratory, and the Princeton Range has established the type formula

$$mV_L^2/d^3 = -\alpha + \beta(e/d) \quad \ldots\ldots \quad (38)$$

for considerable ranges of e/d, and obliquities of 30° or less. This type form is unquestionably superior to the older formulas of the deMarre type, as far as the attack of homogeneous armor ir concerned.

The theory presented in this report provides a rational explanation of equation (38), and in addition predicts the breakdown of this equation at low values of e/d, and the smaller deviations from equation (38) at large values of e/d. It should be noted that no ad hoc hypotheses were introduced into the theory to attain numerical agreement with equation (3) or the slope of equation (4). The assumed petal-thickness, which determines the intercept on the e/d axis in equation (3), was found by a minimizing principle; the mean value of γ and the size of the worked region, which determine the slope to be used in equation (4), were found geometrically and from Bethe's theory.

The thin-plate form of Bethe's expanding-hole theory has been used in preference to his thick-plate theory, even for plates of $e/d > 1$; this choice is based upon studies of etchings and hardness patterns of sections through impacts, which studies will be presented in a subsequent report. These studies show that the type of displacement within the plate and the extent of the work hardened region are consistent only with the thin-plate mechanism, for plates corresponding to e/d values up to 1.4.

Certain additional consequences of the theory may be noted:

1. As the resistance function (equation (23)) increases with the striking energy, a projectile, barely able to penetrate a given plate at a given striking velocity without marked deformation, may be expected to experience greater stresses at higher impact velocities, so may be broken when striking the plate at velocities well above the limit. A qualitative explanation of the phenomenon of shatter is thus obtained. A general quantitative discussion is not possible, as shatter must be determined as much by the quality of the projectile considered as by the mechanical laws of penetration.

2. In extension of item 1 above, one can understand why

the probability of shatter at the limit will be greater in the attack upon thicker plates; the striking velocity for penetration being greater for thick plates than for thinner plates, the initial resistance experienced by the projectile will be greater in attacking thick plates, and may set up sufficient stresses to shatter the projectile.

3. The resistance-function increases linearly with γ; the shorter the ogive, the larger the value of γ, and the greater the resistance encountered on impact. One would expect a blunt projectile to be more likely to shatter on a heavy plate than a long, pointed projectile - an observation in accordance with the facts, as long as the obliquity is near zero.

4. The projectile sets up both forward and lateral components of force as it penetrates the plate. With projectiles of conventional form against thicker plates, the forward components will set up shearing stresses, but the "rigidity" of the plate will be sufficient to prevent the shearing out of a punching, and the penetration will proceed essentially according to the mechanism envisaged in this report. However, if a blunter projectile is used or if the conventional projectile is used against a thinner plate, the shearing stresses may be great enough for the plate to fail by punching.

To be more concrete, the 3" A.P. M-79 projectile, with a simple 5" radius ogive, will pierce (as opposed to punching) plates of any thickness yet fired at (e/d values up to about 1.6). A 3" projectile with a hemispherical nose (1.5 radius ogive) will knock a punching, slightly smaller in diameter than the projectile, out of any plate it can penetratetrate without shattering. Projectiles of intermediate form - having compound ogives of the type used on most 3" A.P. projectiles - penetrated a 1.95 plate by a piercing action but when fired against a 0.73 plate produced small punchings in front of the bluntest portion of the projectile nose, the hole being enlarged to the full size of the projectile by petalling.

Punching in Class B plates has been observed only with blunt projectiles - i.e., projectiles having a relatively large value of γ, so that they experience a high initial resistance. Punching invariably occurs with Class A plates, where the hard face causes a very high initial resistance, as shown by the ability of Class A plate to break projectiles. One may therefore assume for purposes of investigation that punching will always occur in any plate, however thick, if it offers high enough resistance to the projectile; this high resistance may be a consequence of projectile form or of plate hardness, or of a combination of these two factors.

5. Experiment has shown (reference (5)) that, as long as punching does not occur, the limit energy of a projectile

varies in an inverse way with its length of ogive. This is explained qualitatively by the decrease in the value of γ as the ogive is lengthened, which results in a lower average resistance to the projectile and a consequently lower limit energy. On this basis, one would expect little difference between projectiles of different form against thin plates, where the dynamic terms are least, an expectation which is fulfilled - see reference (5) - as long as punching does not occur.

6. If petals break off at the base without bending through a considerable angle, or if a button is thrown from the back of the plate, the energy absorbed in the back zone of the plate will be less than given by equation (6), and the plate limit will be lower than for ductile plate of the same thickness. However, this explanation of the lower limit of brittle plate does not give any information as to why the plate is brittle in the first place.

VI. RECOMMENDATIONS.

Further research on this problem should include a more precise method of calculating the energy of petalling, and a better method of evaluating γ. The question of dishing in thin plates must be dealt with, together with the associated question of the slopes of the graphs of E_R vs. E_S. The shearing action immediately adjacent to the impact should be considered; it might help to clear up the apparent discrepancy between the intercept of Figure 1 and that calculated in Part II, and also the apparent large value of the yield stress. Hardness surveys and etchings of sections through impacts are expected to provide additional data which should guide further researches.

VII REFERENCES.

(1) Frankford Arsenal Report, Attempt of a Theory of Armor Penetration, by H. A. Bethe. (May 23, 1941)

(2) National Defense Research Committee Report No. A-16, The Mechanics of Armor Penetration: Residual Velocity, by H. P. Robertson.

(3) Taylor Model Basin Report 494. Acceleration Measurements on Armor Plate under Projectile Impact, Assembly F, by J. S. Parkinson.

(4) Frankford Arsenal Report 52-B, Forces during Armor Penetration by Caliber .30 A.P. Cores, by Colin M. Hudson and William J. Kroeger.

(5) Naval Proving Ground Report No. 2-43, The Effect of Nose Shape on the Ballistic Performance of 15-lb. 3" AP Solid Shot against Homogeneous Armor Plate.

VIII APPENDIX A.

EXPERIMENTAL DATA.

The determination of accurate and significant ballistic data is a difficult problem. The testing of armor and projectiles is very expensive, and as the determination of an accurate limit velocity by a close "straddle" requires a considerable number of rounds, such limit determinations have not usually been made on all plates tested. When funds have permitted a substantial number of limit determinations, practical considerations have dictated the conditions, which are chosen to approximate the values of e/d and θ likely to be encountered in service. In general, plates corresponding to large values of e/d are tested at low obliquities, and vice versa.

In view of these conditions, it is clear that the first analyses of data had to be statistical in character, with individual limit energies departing considerably from the best-fitting curve. In particular, when limit velocities and limit energies are being determined, variations in plate-thickness, obliquity of impact and projectile mass will occur from round to round. Variations in obliquity will be particularly hard to control when firing at thin plates at high obliquities, where the dished area around one impact hole will extend practically to the next point of impact. It has therefore been necessary to devise a means of comparing successive rounds, and of comparing limits on different plates of nearly the same thickness at obliquities differing by a few degrees.

If projectiles are of the same form and strike the plate with zero yaw and do not deform, the variables affecting the limit velocity V_L will be the projectile mass m, the obliquity θ, the plate thickness e, the yield stress Y of the plate, and the projectile diameter d. These variables must be functionally related in some way, thus

$$\phi\,(m, V_L, Y, e, d, \theta) = 0.$$

If this function is to be expressed explicitly, the requirements of dimensional homogeneity necessitate the grouping of the variables in dimensionless combinations. e/d and θ are dimensionless. mV_L^2 has the dimensions of energy; Yed^2, which is proportional to the energy per unit volume of the material displaced from the projectile hole, also has the dimensions of energy, which suggests the use of mV_L^2/Yed^2 as another dimensionless combination. Thus one may try to find a relation of the form

$$\psi\,(mV_L^2/Yed^2, \theta, e/d) = 0,$$

or what is equivalent,

$$\psi_1(m^{1/2}V_L/Y^{1/2}\ e^{1/2}d, \theta, e/d) = 0$$

This implicit relation may be written more explicitly

$$\frac{m^{1/2}V_L}{e^{1/2}d} = F_o(e/d, \theta),$$

where the constant $Y^{1/2}$ has been absorbed into F_o. It is found by experiment that this function varies only slowly with e/d and m, and to that extent is suitable for comparison of different rounds on different plates. However, the variation with θ is rapid, and to iron this out it has been found best to introduce a factor of $\cos\theta$, which is of course dimensionless:

$$F(e/d, \theta) = m^{1/2}V_L\cos\theta/e^{1/2}d$$

This function is the well-known F-coefficient developed at the Naval Proving Ground in 1932 and since used constantly in the analysis of data obtained at the Naval Proving Ground and elsewhere. The F-coefficient is not a constant but varies slowly with e/d and θ. It is useful for the direct comparison of plates under closely similar conditions, and the average F-coefficient, evaluated as a function of these variables, serves as a valuable reference for comparisons of plate qualities over large ranges of plate thickness and obliquity, and as a basis for writing armor specifications.

Earlier analyses of penetration data in terms of the F-coefficient were based almost entirely on values of e/d and θ of principal service interest, and with a wide variety of projectile designs. Hence, they are not entirely adequate in range for any single projectile from the standpoint of fundamental investigation. The development of the residual velocity technique for the determination of limit velocities, described in reference (2), has enabled the determination of limits with only one or two rounds--except at high obliquities where more may be required--and together with systematic firing at small scale (.30 and .50 caliber and 3") has made feasible the accumulation of sufficient data for the determination of accurate curves of F as a function of e/d and θ, over wide ranges for particular projectiles.

From a mechanical point of view, the limit energy is more significant than the limit velocity. The specific limit energy is readily obtained from the F-coefficient, as

$$(e/d)F^2 = mV_L{}^2\cos^2\theta/d^3.$$

At obliquities near zero, cosθ is practically 1; to get the actual energy at higher obliquities, one would use

$$(e/d)F^2 \sec^2\theta = mV_L{}^2/d^3.$$

Figures 1 and 2 give the experimental data for one type of uncapped projectile at 0°.

In the plotting of residual energy graphs (of the type of equation (4)) it is found that less dispersion is obtained if an F^2 plot is made, using the quantities.

$$F_S{}^2 = mV_S{}^2\cos^2\theta/ed^2,$$

$$F_R{}^2 = mV_R{}^2\cos^2\theta/ed^2,$$

and

$$F^2 = mV_L{}^2\cos^2\theta/ed^2.$$

The equation equivalent to (4) is then

$$F_R{}^2 = S(F_S{}^2 - F^2).$$

If one or more quantities stay constant from round to round, they may be cancelled from this equation; for example, in the firing basic to the plot of Figure 3, d and cosθ were practically constant, so the plot shows

$$\frac{mV_R{}^2}{e} = S\left(\frac{mV_S{}^2}{e} - \frac{mV_L{}^2}{e}\right)$$

The term specific limit energy has been introduced because the weight of a projectile in pounds is usually not very far from half the cube of its diameter in inches:

$$w = d^3/2.$$

Therefore,

$$(mV_L{}^2/2)/(d^3/2)$$

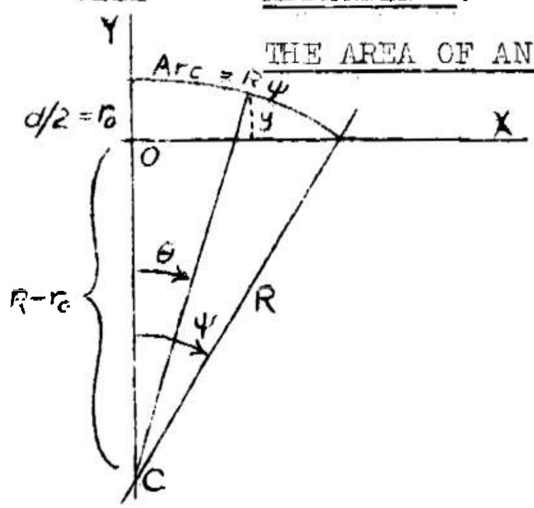
is approximately numerically equal to the limit-energy per pound of projectile weight. For projectiles of a given form,

$$mV_L{}^2/d^3$$

is strictly proportional to the energy per pound of projectile weight, regardless of the units used for d. In calculations using the F-coefficient, d is in feet, rather than inches, and m is in pounds, rather than slugs, so that while $(e/d)F^2$ is an expression of the specific energy, it is not numerically equal to the energy in ft.-lb. per lb. of projectile weight. The utility of the specific energy arises from the fact that, like the F-coefficient, except for small scale-effects, it can be used to compare results for projectiles of the same form and different caliber. Thus, if a 3" plate is attacked by the 3" model of the 8" A.P. Mark 11, the specific limit energy should be about the same as required for an 8" A.P. Mark 11 against an 8" plate at the same obliquity, or if not, the differences would be attributable to different plate qualities.

VIII APPENDIX B.

THE AREA OF AN OGIVE.



A simple ogive is generated by revolving an arc of a circle about its chord; in the figure, an arc of radius R with center at C is revolved about the line OX. The nose of the projectile will be represented by that half of the ogive lying to the right of OY. By a Theorem of Pappus, the area of a surface of revolution is the product of the length of the revolving arc by the circumference described by the center of gravity of the arc. Thus, if $\bar{y}$ is the mean y along the arc, and L_o is the desired lateral area of the ogive,

$$L_o = 2\pi \bar{y} \cdot R\psi$$

Here
$$\bar{y} = \int y ds / \int ds,$$

where the element of arc ds is Rdθ, and $y = R\cos\theta - (R - r_o) = R\left[\cos\theta - (1 - d/2R)\right]$.

Therefore

$$\bar{y} = \frac{\int_0^{\psi} R\left[\cos\theta - (1-d/2R)\right] Rd\theta}{\int_0^{\psi} Rd\theta}$$

$$= R\left[\sin\psi/\psi - (1-d/2R)\right]$$

In this formula $\psi = \cos^{-1}(1-d/2R)$, so can be found from the specified form of the projectile. Then

$$L_o = 2\pi R^2 \psi \left[\sin\psi/\psi - (1-d/2R) \right]$$

or

$$L_o = 2\pi R^2 \left[\sin\psi - \psi \cos\psi \right] \quad \ldots\ldots \quad (1)$$

This is probably the most convenient equation to use for calculation, the value of ψ being found and substituted in the formula. If ψ is eliminated from equation (1), the formula becomes

$$L_o = 2\pi R^2 \left[(d/2R)\sqrt{(4R/d)-1} - (1-d/2R)\cos^{-1}(1-d/2R) \right] \quad ..(2)$$

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U.S. NAVAL PROVING GROUND, DAHLGREN, VA.

PENETRATION MECHANISM - I - THE PENETRATION OF HOMOGENEOUS ARMOR BY UNCAPPED PROJECTILES AT 0° OBLIQUITY - AND APPENDIXES A AND B (1-43)

DAVID I. HEDRICK 9 APRIL '43 30 PP. DIAGR, GRAPHS

ORDNANCE AND ARMAMENT (22)
ARMOR (5)

ARMOR PLATE - PENETRATION
ARMOR PLATE - PHYSICAL PROPERTIES
PROJECTILES - PENETRATION

2 2
2

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