Test and Analysis Branch

Research & Engineering Directorate

DIGITAL COMPUTER TECHNIQUE FOR SOLVING "WEIBULL" DISTRIBUTION FUNCTION PARAMETERS

RELIABILITY ENGINEERING REPORT

RTT-GEN- 65-1

BY

BEN TKATCH

FEBRUARY 1965

Army Tank Automotive Center
Warren, Michigan

TEST AND ANALYSIS BRANCH

RESEARCH AND ENGINEERING DIRECTORATE

U.S. ARMY TANK AUTOMOTIVE CENTER

WARREN, MICHIGAN

DIGITAL COMPUTER TECHNIQUE

For Solving

"WEIBULL" DISTRIBUTION FUNCTION

PARAMETERS

REPORT NO.

RTT-GEN-65-1

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ABSTRACT

It is recognized that data used in Reliability Engineering predictions, assessments, and verifications must be compiled and retained within an ADP data bank.

Equally true, the computer can store the mathematical equations necessary for Reliability Engineering. At the point in time when all equations are known and derived and can be programed against the data, rapid Reliability predictions, assessments and verifications will be possible.

In order to explore utilization of the computer several basic equations have been programed. These equations are presented in Annex A, this report.

PURPOSE

This report was prepared to inform USATAC management and engineers that parameters (unknowns), stated in earlier reports, and required for resolution of the "Weibull" distribution function, have been programed on the digital computer.

In view of the involved mathematics, the Systems Simulation Branch of the Research and Engineering Directorate are commended for having excelled in programing the equations. This assistance has been appreciated.

DIGITAL COMPUTER TECHNIQUE

For Solving

"WEIBULL" DISTRIBUTION FUNCTION PARAMETERS

INTRODUCTION

The Test & Analysis Branch has released several initial study reports on Reliability Engineering prediction, assessment and verification pertaining to Tank-Automotive Materiel.*

These reports have been based upon use of the Weibull distribution function. The formulae, adaptable for Tank-Automotive Reliability Engineering, contains three parameters which must be predetermined prior to use of the formulae. These parameters are (1) location (minimum life), (2) shape (slope) and (3) scale (characteristic life).

* See References at end of this report.

REPORT

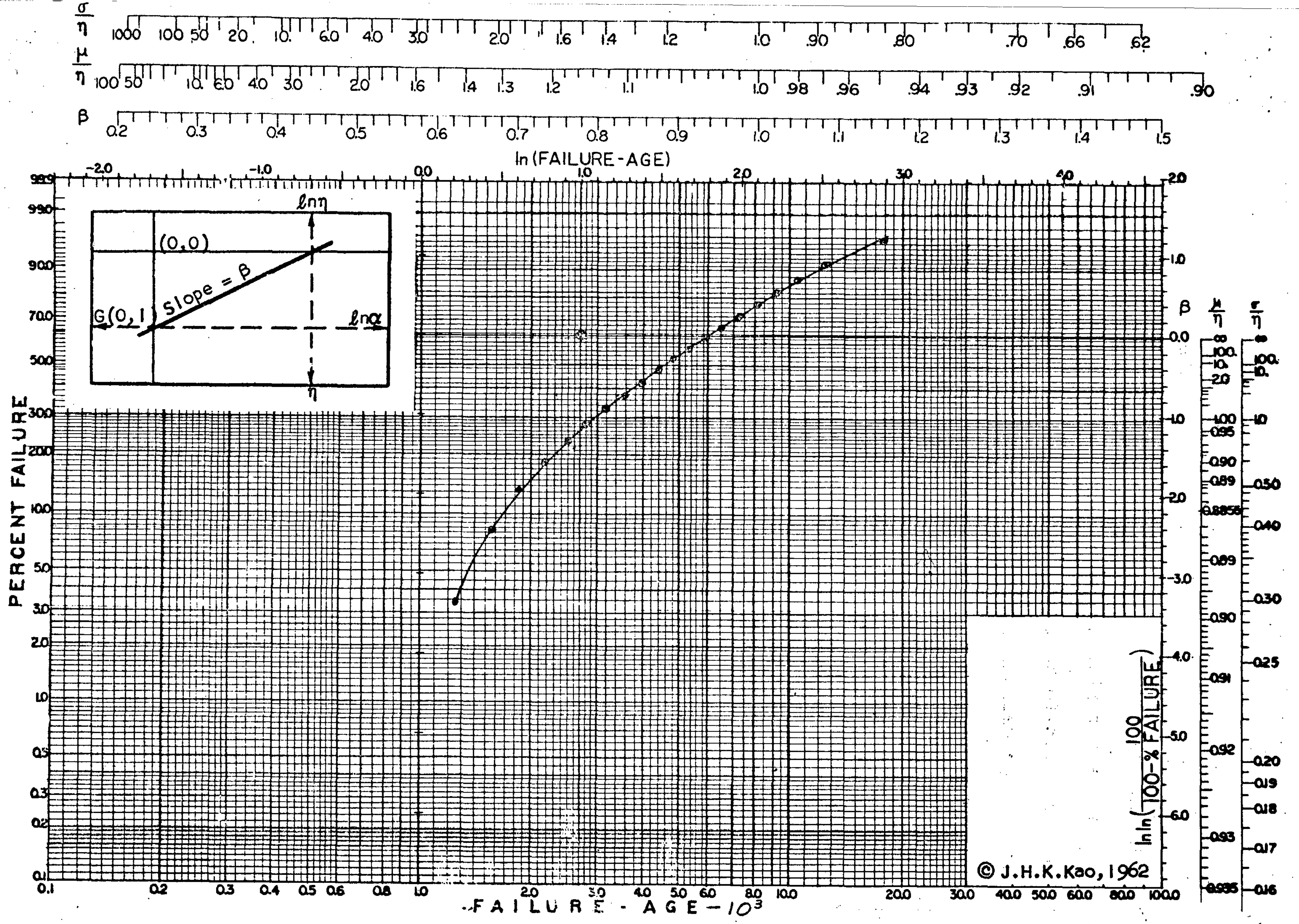
The usual method of determining parameters is by plotting failure data on special Weibull probability paper. For a large number of failure points, this method is tedious and time consuming and parameters are found by a "trial and error" plotting point method.

A search of reliability text books and documents, for a computer or direct mathematical method to eliminate the "engineering guess work" required to solve parameters, proved fruitless. Thus it became necessary to derive equations.

Using the method of least squares, an implicit function was derived to determine the location (minimum life) parameter. This formula was programed on the Burrough's Datatron digital computer and quickly solved the location parameter to the desired accuracy. A direct solution of the shape and scale parameter was programed on the computer and determined after solving for the location parameter.

As an example, twenty (hypothetical) failure points (See Table I) were plotted on Figure 1 using median ranks for the ordinate. The curve concave downward indicates that a minimum life must be subtracted from each failure point before determining the Weibull distribution function parameters. The location (γ) parameter was determined by the computer to be 1035 (nearest whole number). Higher accuracy can be determined, if necessary.

Having the location (γ) parameter, the Weibull slope (β) and characteristic life (η) were directly determined by the computer to be 1.071 and 5005 respectively. On Figure 2, the Weibull distribution function is drawn as a straight line. This curve was drawn after solving for the location, shape and scale parameters on the computer. The population density function [f(t)], instantaneous failure rate, and reliability (R) curves were computed and plotted on Figures 3, 4 and 5 respectively.



ln (FAILURE-AGE)
PERCENT FAILURE
FAILURE - AGE - 10^3
β
μ/η
σ/η
ln ln (100 / (100-% FAILURE))
(0,0)
Slope = β
G(0,1)
ℓnη
ℓnα
η
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FIG. 1

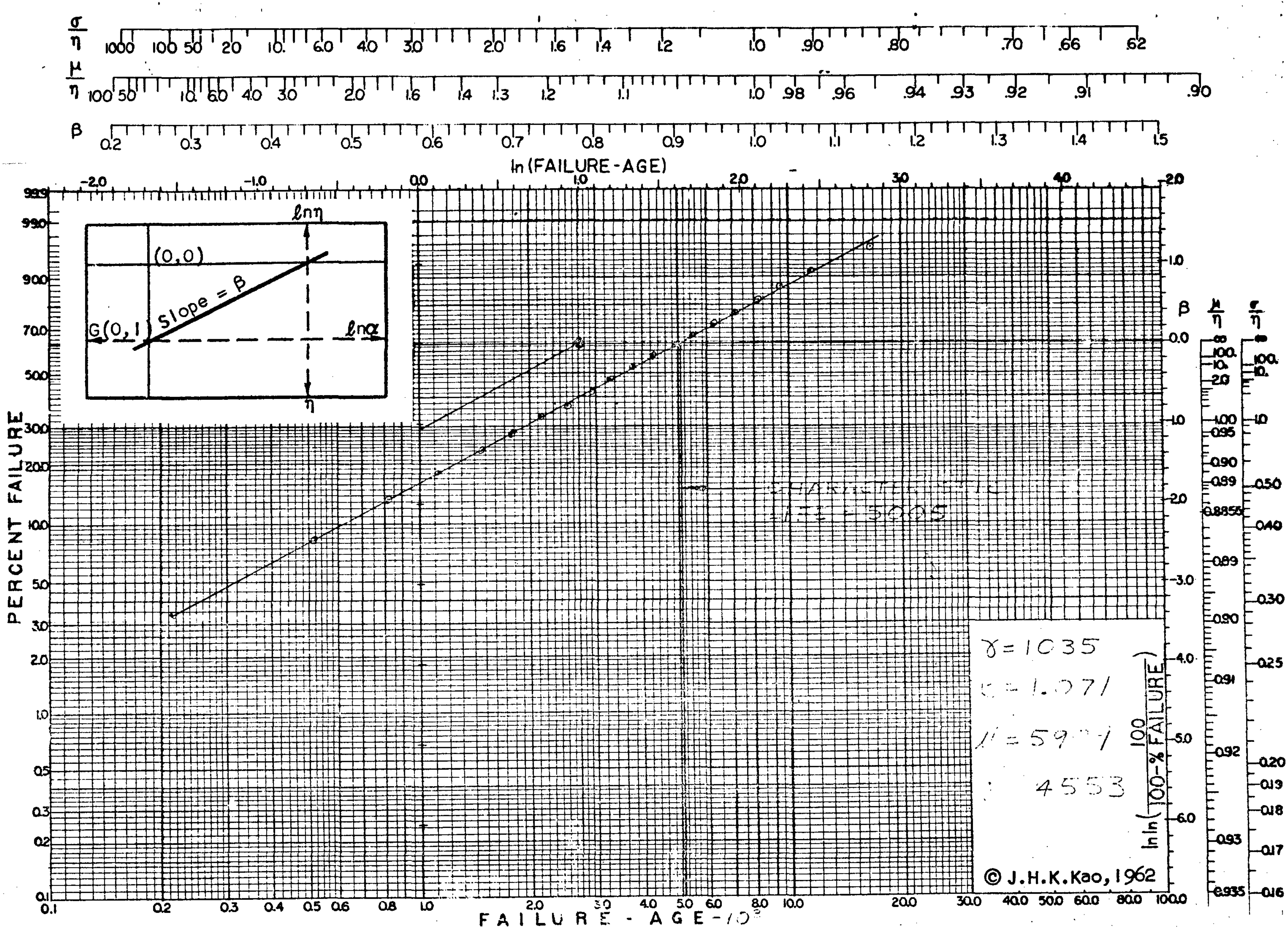

PERCENT FAILURE
FAILURE - AGE - 10³
ln (FAILURE-AGE)
β
Slope = β
(0,0)
G(0,1)
ℓnη
ℓnα
η
$\ln\ln\left(\frac{100}{100-\%\ \text{FAILURE}}\right)$
γ = 1035
= 1.071
= 5971
4553
© J.H.K.Kao, 1962

FIG. 2

WEIBULL POPULATION
DENSITY FUNCTION

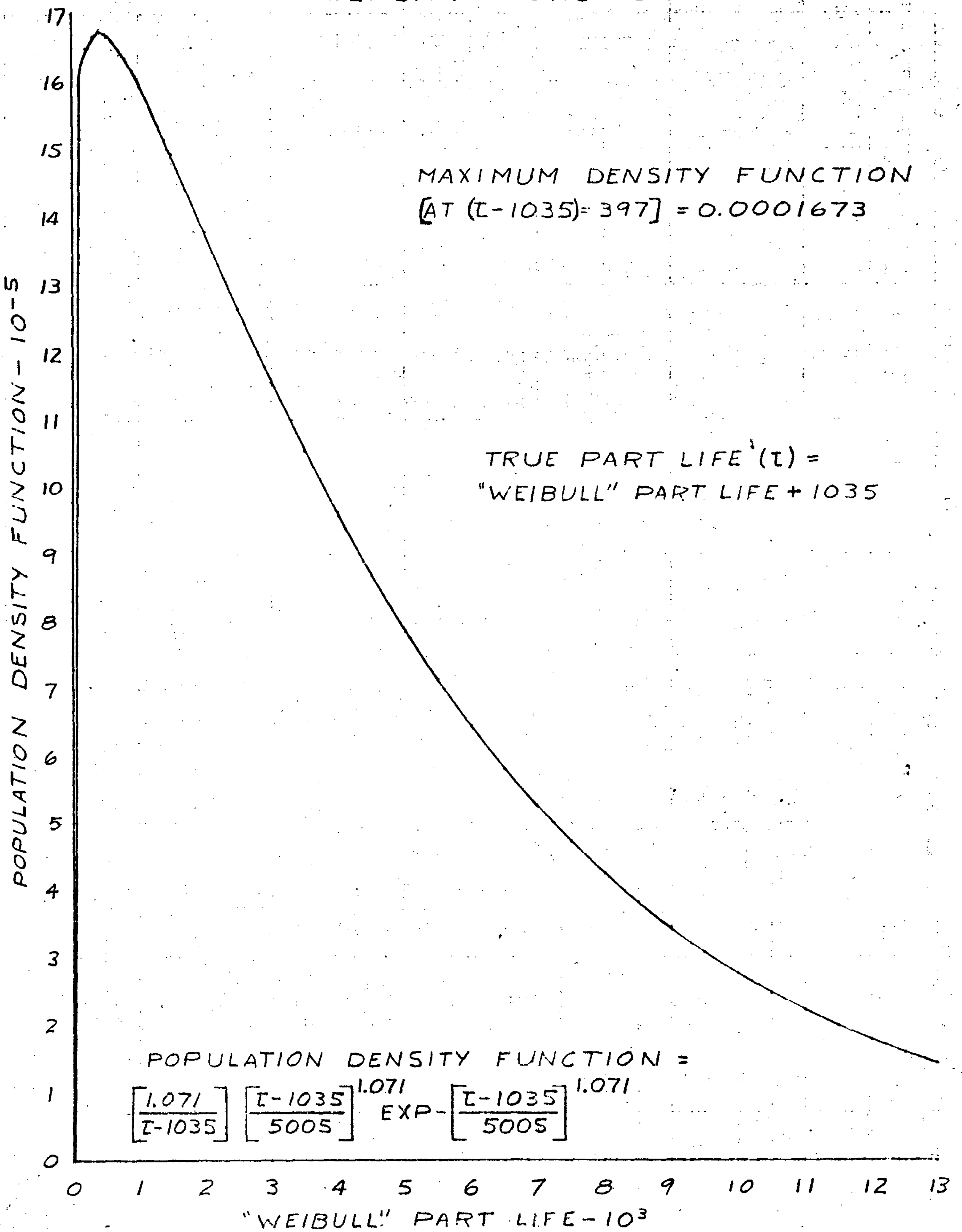


FIG. 3

INSTANTANEOUS RATE
OF FAILURE

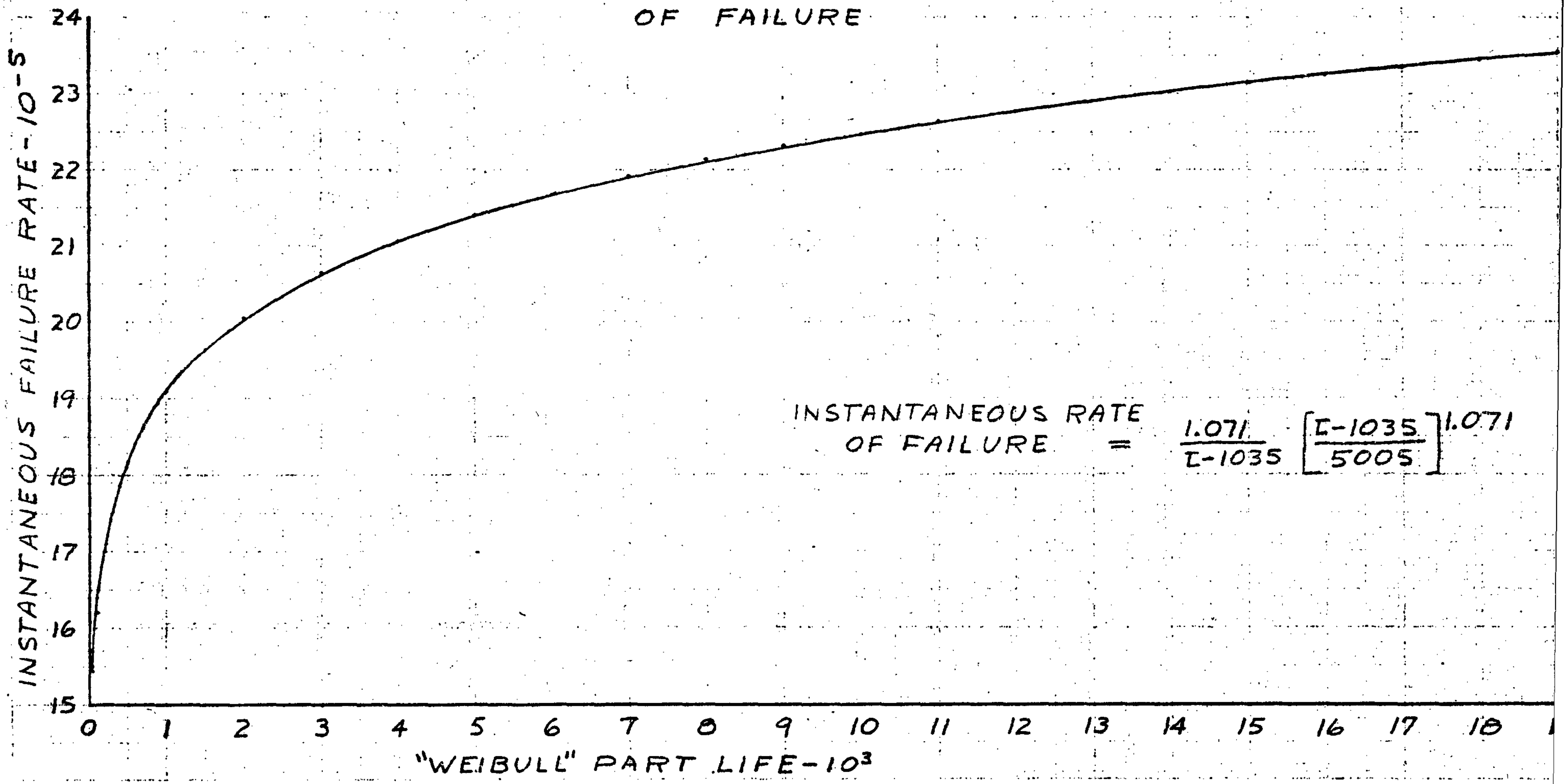
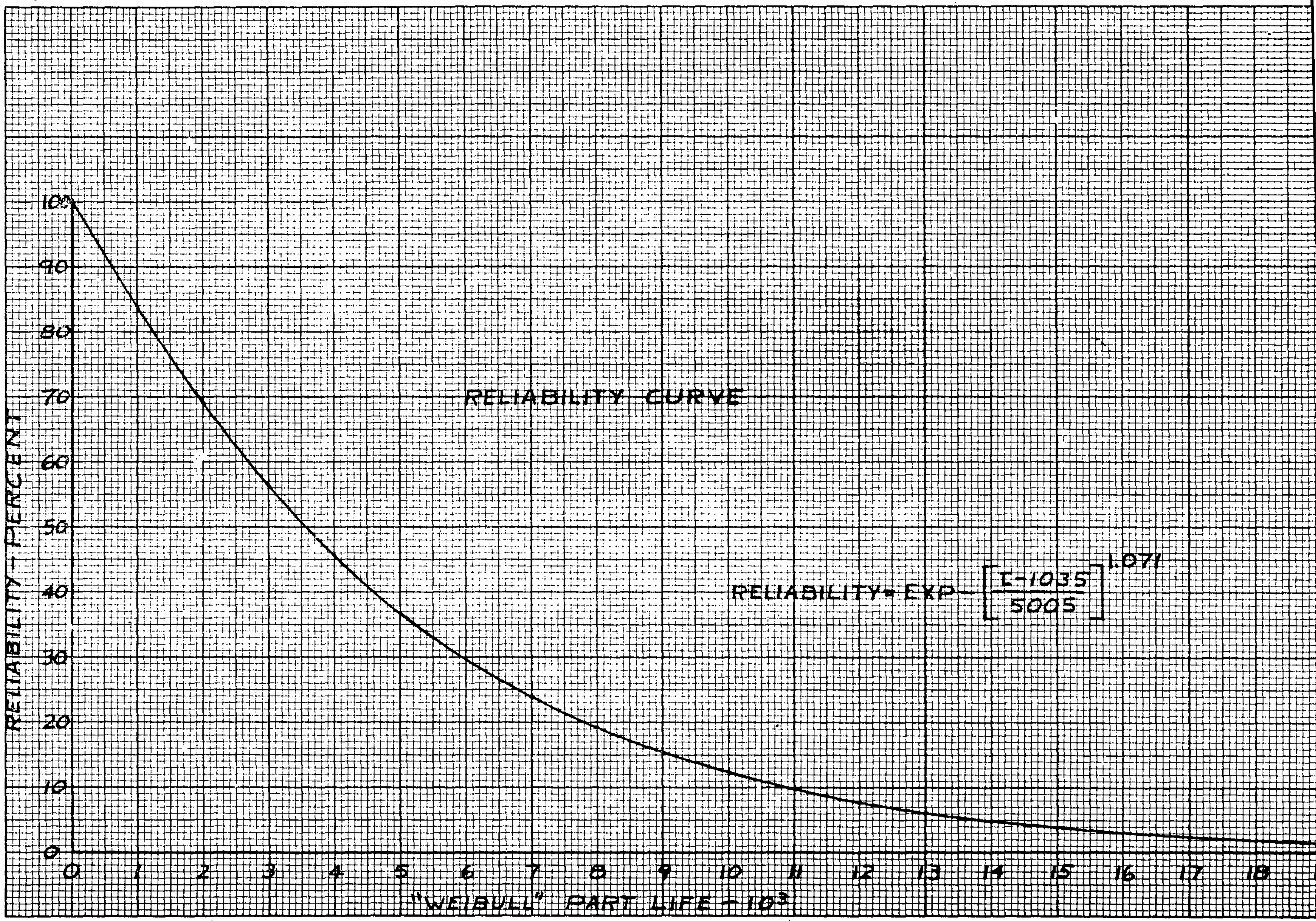


FIG. 4



RELIABILITY CURVE
RELIABILITY = EXP — [(I-1035) / 5005]^1.071
RELIABILITY - PERCENT
0
10
20
30
40
50
60
70
80
90
100
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
"WEIBULL" PART LIFE - 10³

FIG. 5

TABLE I

(Used for Plotting Figures 1 and 2)

n	Median Rank (in Percent) F	Test Miles t	t-γ*
1	3.4	1250	215
2	8.3	1550	515
3	13.2	1850	815
4	18.1	2150	1115
5	23.0	2500	1465
6	27.9	2800	1765
7	32.8	3150	2115
8	37.7	3550	2515
9	42.6	3950	2915
10	47.5	4350	3315
11	52.5	4800	3765
12	57.4	5300	4265
13	62.3	5900	4865
14	67.2	6500	5465
15	72.1	7250	6215
16	77.0	8100	7065
17	81.9	9150	8115
18	86.8	10500	9465
19	91.7	12450	11415
20	96.6	17900	16865

*γ = 1035 Miles

TABLE II
(Used for Plotting Figures 3, 4 and 5)

Weibull Part Life t-γ	Reliability R=1-F	Instantaneous Rate of Failure Z	Population Density Function f(t)
0	1.0000		
50	.9928	.0001543	.0001532
100	.9850	.0001621	.0001596
150	.9769	.0001668	.0001629
200	.9687	.0001702	.0001649
250	.9604	.0001730	.0001661
300	.9521	.0001752	.0001668
350	.9438	.0001771	.0001672
400	.9354	.0001788	.0001673
450	.9270	.0001803	.0001672
500	.9187	.0001817	.0001669
600	.9020	.0001841	.0001660
700	.8855	.0001861	.0001648
800	.8691	.0001879	.0001633
900	.8528	.0001894	.0001616
1000	.8368	.0001909	.0001597
1100	.8209	.0001922	.0001577
1200	.8052	.0001934	.0001557
1300	.7898	.0001945	.0001536
1400	.7745	.0001955	.0001514
1500	.7595	.0001964	.0001492

TABLE III
(Used for Plotting Figures 3, 4, and 5)

t*	t-γ	R= 1-F	Z	f(t)
2535	1500	.759	.0001964	.0001492
3035	2000	.688	.0002005	0001379
3535	2500	.622	.0002037	.0001266
4035	3000	.561	.0002064	.0001158
4535	3500	.506	.0002086	.0001055
5035	4000	.455	.0002106	.0000959
5535	4500	.410	.0002124	.0000870
6035	5000	.368	.0002140	.0000788
6535	5500	.331	.0002154	.0000713
7035	6000	.297	.0002168	.0000644
7535	6500	.266	.0002180	.0000581
8035	7000	.239	.0002192	.0000523
8535	7500	.214	.0002202	.0000471
9035	8000	.192	.0002213	.0000424
9535	8500	.171	.0002222	.0000381
10035	9000	.153	.0002231	.0000342
10535	9500	.137	.0002240	.0000307
11035	10000	.123	.0002248	.0000276
11535	10500	.110	.0002256	.0000247
12035	11000	.0978	.0002263	.0000221
12535	11500	.0874	.0002270	.0000198
13035	12000	.0780	.0002277	.0000178
13535	12500	.0696	.0002284	.0000159
14035	13000	.0621	.0002290	.0000142
14535	13500	.0553	.0002296	.0000127
15035	14000	.0493	.0002302	.0000114
15535	14500	.0440	.0002308	.0000101
16035	15000	.0392	.0002314	.0000091
16535	15500	.0349	.0002319	.0000081
17035	16000	.0311	.0002324	.0000072
17535	16500	.0276	.0002329	.0000064
18035	17000	.0246	.0002334	.0000057
18535	17500	.0219	.0002339	.0000051
19035	18000	.0195	.0002344	.0000046
19535	18500	.0173	.0002348	.0000041
20035	19000	.0154	.0002353	.0000036
20535	19500	.0137	.0002357	.0000032
21035	20000	.0122	.0002361	.0000029

* True Part Life = "Weibull" Part Life +1035 Miles.

REFERENCES

1. Production Vehicle Reliability Estimation from Pilot Vehicle Failure Analysis
   REV-GEN-64-1 — P. Myron, P.E.

2. Reliability Estimation From Pilot Vehicle Failure Analysis
   RTT-GEN-64-2 — Ben Tkatch

3. Reliability: Its Definitions and Terms
   RTT-GEN-64-3 — W. Simkovitz, P.E.

4. Vehicle Power Train Reliability
   RTT-GEN-64-4 — P. Myron, P.E.

ANNEX A

Derivation of location, shape and scale parameters of Weibull Distribution Function using method of least squares.

1) $\ln \ln \frac{1}{1-F} = \beta \ln(\tau-\gamma) - \beta \ln \eta$

γ = LOCATION PARAMETER

β = SHAPE PARAMETER

η = CHARACTERISTIC LIFE

$\Sigma = \sum_{i=1}^{N}$

2) $G = \Sigma \left[\ln \ln \frac{1}{1-F_i} - \ln \ln \frac{1}{1-F}\right]^2$

$= \Sigma \left[\ln \ln \frac{1}{1-F_i} - \beta \ln(\tau-\gamma) + \beta \ln \eta\right]^2$

TAKING PARTIAL DERIVATIVES OF EQ 2 TO EACH PARAMETER

3) $\frac{\partial G}{\partial \beta} = \Sigma 2 \left[\ln \ln \frac{1}{1-F_i} - \beta \ln(\tau-\gamma) + \beta \ln \eta\right]\left[\ln \eta - \ln(\tau-\gamma)\right] = 0$

3a) $\Sigma \left[\ln \ln \frac{1}{1-F_i}\right]\left[\ln \frac{\eta}{(\tau-\gamma)}\right] + \Sigma \beta \left[\ln \frac{\eta}{(\tau-\gamma)}\right]^2 = 0$

3b) $\beta = \dfrac{\left[\Sigma \ln \ln \frac{1}{1-F_i}\right]\left[\ln \frac{(\tau-\gamma)}{\eta}\right]}{\Sigma \left[\ln \frac{\eta}{(\tau-\gamma)}\right]^2}$

4) $\frac{\partial G}{\partial \eta} = \Sigma 2 \left[\ln \ln \frac{1}{1-F_i} - \beta \ln(\tau-\gamma) + \beta \ln \eta\right]\left[\frac{\beta}{\eta}\right] = 0$

4a) $\Sigma \frac{\beta}{\eta} \ln \ln \frac{1}{1-F_i} + \Sigma \frac{\beta^2}{\eta} \ln \frac{\eta}{(\tau-\gamma)} = 0$

4b) $\beta = \dfrac{\Sigma \ln \ln \frac{1}{1-F_i}}{\Sigma \ln \frac{(\tau-\gamma)}{\eta}}$

5) $\frac{\partial G}{\partial \gamma} = \Sigma 2 \left[\ln \ln \frac{1}{1-F_i} - \beta \ln(\tau-\gamma) + \beta \ln \eta\right]\left[\frac{\beta}{\tau-\gamma}\right] = 0$

5a) $\Sigma \frac{\beta}{(\tau-\gamma)} \ln \ln \frac{1}{1-F_i} + \Sigma \frac{\beta^2}{(\tau-\gamma)} \ln \frac{\eta}{(\tau-\gamma)} = 0$

5b) $\beta = \dfrac{\Sigma \dfrac{\ln \ln \frac{1}{1-F_i}}{(\tau-\gamma)}}{\Sigma \dfrac{\ln \frac{(\tau-\gamma)}{\eta}}{(\tau-\gamma)}}$

SETTING EQ 3b EQUAL TO EQ 4b

$$6)\quad \beta = \frac{\Sigma\left[\mathrm{LN\,LN}\frac{1}{1-F'}\right]\left[\mathrm{LN}(\tau-\gamma)-\mathrm{LN}\,\eta\right]}{\Sigma\left[\mathrm{LN}\,\eta-\mathrm{LN}\,(\tau-\gamma)\right]^2} = \frac{\Sigma\,\mathrm{LN\,LN}\frac{1}{1-F'}}{\Sigma\left[\mathrm{LN}(\tau-\gamma)-\mathrm{LN}\,\eta\right]}$$

$$7)\quad \frac{\Sigma\left[\mathrm{LN\,LN}\frac{1}{1-F'}\right]\left[\mathrm{LN}(\tau-\gamma)\right]-\mathrm{LN}\,\eta\,\Sigma\,\mathrm{LN\,LN}\frac{1}{1-F'}}{\Sigma\,\mathrm{LN}^2\eta-2\,\mathrm{LN}\,\eta\,\Sigma\,\mathrm{LN}(\tau-\gamma)+\Sigma\left[\mathrm{LN}(\tau-\gamma)\right]^2} = \frac{\Sigma\,\mathrm{LN\,LN}\frac{1}{1-F'}}{\Sigma\,\mathrm{LN}\,(\tau-\gamma)-\Sigma\,\mathrm{LN}\,\eta}$$

NOTE $\Sigma\,\mathrm{LN}\,\eta = N\,\mathrm{LN}\,\eta$

$\Sigma\,\mathrm{LN}^2\eta = N\,\mathrm{LN}^2\eta$

MULTIPLING OUT EQ. 7 AND SOLVING FOR $\Sigma\,\mathrm{LN\,LN}\frac{1}{1-F'}$ AND $\mathrm{LN}\,\eta$

$$8)\quad \Sigma\,\mathrm{LN\,LN}\frac{1}{1-F'} = \frac{\left[\Sigma\,\mathrm{LN}\,(\tau-\gamma)-N\,\mathrm{LN}\,\eta\right]\left[\Sigma\,\mathrm{LN\,LN}\frac{1}{1-F'}\;\mathrm{LN}\,(\tau-\gamma)\right]}{\Sigma\left[\mathrm{LN}\,(\tau-\gamma)\right]^2-\mathrm{LN}\,\eta\,\Sigma\,\mathrm{LN}\,(\tau-\gamma)}$$

$$9)\quad \mathrm{LN}\,\eta = \frac{\Sigma\left[\mathrm{LN\,LN}\frac{1}{1-F'}\right]\left[\mathrm{LN}\,(\tau-\gamma)\right]\Sigma\,\mathrm{LN}\,(\tau-\gamma)-\Sigma\left[\mathrm{LN}\,(\tau-\gamma)\right]^2\Sigma\,\mathrm{LN\,LN}\frac{1}{1-F'}}{N\,\Sigma\left[\mathrm{LN\,LN}\frac{1}{1-F'}\right]\left[\mathrm{LN}\,(\tau-\gamma)\right]-\left[\Sigma\,\mathrm{LN}\,(\tau-\gamma)\right]\left[\Sigma\,\mathrm{LN\,LN}\frac{1}{1-F'}\right]}$$

SETTING EQ 4b EQUAL TO EQ 5b

$$10)\quad \Sigma\,\mathrm{LN\,LN}\frac{1}{1-F'} = \frac{\left[\Sigma\,\frac{\mathrm{LN\,LN}\frac{1}{1-F'}}{(\tau-\gamma)}\right]\left[\Sigma\,\mathrm{LN}\,(\tau-\gamma)-N\,\mathrm{LN}\,\eta\right]}{\Sigma\,\frac{\mathrm{LN}\,(\tau-\gamma)}{(\tau-\gamma)}-\mathrm{LN}\,\eta\,\Sigma\,\frac{1}{\tau-\gamma}}$$

SUBSTITUTING $\mathrm{LN}\,\eta$ OF EQ9 INTO EQ 10 AND REDUCING

$$11)\quad \Sigma\,\mathrm{LN\,LN}\frac{1}{1-F'} = \frac{\left[\Sigma\,\frac{\mathrm{LN\,LN}\frac{1}{1-F'}}{(\tau-\gamma)}\right]\left\{\left[\Sigma\,\mathrm{LN\,LN}\frac{1}{1-F'}\right]\right\}\left\{N\,\Sigma\left[\mathrm{LN}\,(\tau-\gamma)\right]^2-\left[\Sigma\,\mathrm{LN}\,(\tau-\gamma)\right]^2\right\}}{\left[\Sigma\,\frac{\mathrm{LN}\,(\tau-\gamma)}{(\tau-\gamma)}\right]\left[N\Sigma\,\mathrm{LN\,LN}\frac{1}{1-F'}\,\mathrm{LN}\,(\tau-\gamma)-\Sigma\,\mathrm{LN}\,(\tau-\gamma)\,\Sigma\,\mathrm{LN\,LN}\frac{1}{1-F'}\right]-\left\{\Sigma\,\frac{1}{\tau-\gamma}\right\}\left\{\Sigma\,\mathrm{LN\,LN}\frac{1}{1-F'}\,\mathrm{LN}(\tau-\gamma)\,\Sigma\,\mathrm{LN}\,(\tau-\gamma)-\Sigma\left[\mathrm{LN}(\tau-\gamma)\right]^2\Sigma\,\mathrm{LN\,LN}\frac{1}{1-F'}\right\}}$$

SIMPLIFYING EQ. 11

$$12)\ \Sigma\, \mathrm{LN}\, \mathrm{LN} \frac{1}{1-F_i} = \frac{\left\{\Sigma \frac{\mathrm{LN}\,\mathrm{LN}\frac{1}{1-F_i}}{(\tau-\gamma)}\right\}\left\{N\Sigma\left[\mathrm{LN}(\tau-\gamma)\right]^2-\left[\Sigma\,\mathrm{LN}(\tau-\gamma)\right]^2\right\}+\left\{\Sigma\left[\mathrm{LN}\,\mathrm{LN}\frac{1}{1-F_i}\right]\left[\mathrm{LN}(\tau-\gamma)\right]\right\}\left\{\Sigma\frac{1}{(\tau-\gamma)}\left[\Sigma\,\mathrm{LN}(\tau-\gamma)\right]-N\Sigma\frac{\mathrm{LN}(\tau-\gamma)}{(\tau-\gamma)}\right\}}{\left\{\Sigma\frac{1}{(\tau-\gamma)}\Sigma\left[\mathrm{LN}(\tau-\gamma)\right]^2\right\}-\left\{\Sigma\frac{\mathrm{LN}(\tau-\gamma)}{(\tau-\gamma)}\right\}\left\{\Sigma\,\mathrm{LN}(\tau-\gamma)\right\}}$$

SUBSTITUTING LN η OF EQ 9 INTO EQ 4b AND REDUCING

$$13)\quad \beta = \frac{N\Sigma\left[\mathrm{LN}\,\mathrm{LN}\frac{1}{1-F_i}\right]\left[\mathrm{LN}(\tau-\gamma)\right]-\left[\Sigma\,\mathrm{LN}(\tau-\gamma)\right]\left[\Sigma\,\mathrm{LN}\,\mathrm{LN}\frac{1}{1-F_i}\right]}{N\Sigma\left[\mathrm{LN}(\tau-\gamma)\right]^2-\left[\Sigma\,\mathrm{LN}(\tau-\gamma)\right]^2}$$

EQ. 12 IS AN IMPLICIT FUNCTION SOLVING FOR THE VALUE OF THE LOCATION PARAMETER (γ).

EQ. 9 AND EQ. 13 ARE EXPLICIT FUNCTIONS SOLVING FOR CHARACTERISTIC LIFE (η) AND SHAPE PARAMETER (β) RESPECTIVELY AFTER SOLVING EQ. 12 FOR THE LOCATION PARAMETER

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