UNCLASSIFIED

Copy # 2

660/18 Wal

AD-A954 192

DTIC FILE COPY

WATERTOWN ARSENAL

PRODUCTION DEPARTMENT

GUN DIVISION

REPORT NO. WGD-7

Plastic Strains in Thick Hollow Cylinders

Overstrained by Internal Pressure

BY

Donald H. Newhall
Capt., Ord. Dept.
Cold Work Section

DATE 24 January 1944

WATERTOWN ARSENAL
WATERTOWN, MASS.

WATERTOWN ARSENAL

Production Department

Gun Division

24 January 1944

Plastic Strains in Thick Hollow Cylinders Overstrained by Internal Pressure

by

Donald H. Newhall
Capt., Ord. Dept.
Cold Work Section

Approved:

F. S. Kohl, Major
Chief, Gun Division
Watertown Arsenal

REPORT NO. WGD-7

UNCLASSIFIED

NOMENCLATURE

e_i = Strain on the ID, inches per inch

e_o = Strain on the OD, inches per inch

FF = Flow Factor = $\frac{\Delta ID}{\Delta OD}$

e_t = Strain in the tangential direction, inches per inch

e_r = Strain in the radial direction, inches per inch

e_z = Strain in the longitudinal direction, inches per inch

S_t = Stress in the tangential direction, pounds per square inch

S_r = Stress in the radial direction, pounds per square inch

S_z = Stress in the longitudinal direction, pounds per square inch

a = Bore radius in a thick hollow cylinder

b = Outside radius in a thick hollow cylinder

r = General radius in a thick hollow cylinder

W = Wall ratio = $\frac{OD}{ID}$

μ = Poisson's ratio

L = Total length

h = General length in a solid

V = Volume of a solid

C = Constant

θ = Change in wall ratio with respect to height

PLASTIC STRAINS IN THICK HOLLOW CYLINDERS OVER-STRAINED BY INTERNAL PRESSURE

ABSTRACT

In this report, equations are derived relating wall ratio to plastic strains resulting from overstraining thick hollow cylinders. That the equations are applicable, is evidenced by comparison with statistical data from the inspection records of guns coldworked at Watertown Arsenal in the current production activities and by agreement with experimental test data. Equations are developed for

(1) Strain Ratio,

(2) Flow Factor,

(3) Longitudinal Strains,

(4) Longitudinal Shrinkage in Tapered Sections, and

(5) Strain Distribution Through the Cylinder Wall.

These equations have already been published but not derived in a Watertown Arsenal Report, "Selected Design Data Pertaining to Gun Tubes and High-Pressure Vessels" Report Number WGD-4.

INTRODUCTION

Attempts to develop accurate equations for plastic strains in thick hollow cylinders have met with little success in the past. Consequently empirical relations applicable to the deformations of gun tubes were established at Watertown Arsenal incidental to the early development of the coldworking process. A "strain ratio" was determined experimentally for cylinders of various wall ratios, which ratio was limited in the extent of its practical applications. Previous investigators, in attempting to extend the subject, have assumed that the value of Poisson's ratio changed abruptly in value from approximately (.285) in the elastic state to (.5) in the plastic state. Experimental data indicate that for strains of the magnitude encountered in the coldworking and auto-fretting processes a value of (.3) to (.333) seems to apply.

In this report new strain equations will be developed applicable to overstrained gun tubes and related high-pressure vessels. The applicability of these equations is evidenced by statistical data taken from inspection records of currently coldworked gun tubes and from experimental test data.

ORIGINAL FLOW DATA

The compilation of the early flow data at Watertown Arsenal was ably done under the direction of General Tracy Dickson and Dr. F. C. Langenberg by J. C. Solberg, now a Lt. Col. in the Ordnance Dept. The curve attributed to them in Fig. 1 was plotted from their records of tests of overstrained cylinders. The curve shows the relation between wall ratio and "strain ratio", the latter being the ratio of strain on the inside of a cylinder to that on the outside. Lt. Col. Solberg fitted the following equation to his test data:

$$\text{Strain Ratio} = \frac{e_i}{e_o} = \frac{w^2}{1.125} \qquad (1)$$

where e_i is the unit strain on the inside surface, e_o is the strain on the outside and w = wall ratio = (OD/ID). His experimental data are also plotted in Fig. 1.

The equation is a good approximation but is not theoretically correct for small wall ratios. This may be seen from a consideration of the value of strain ratio = 0.89 obtained from equation 1 when w=1. At that point the wall of the cylinder is infinitely thin and the OD and ID should have the same unit strain making the theoretical value of strain ratio equal to unity.

DERIVATION OF STRAIN EQUATIONS

General

In the derivation of the following strain equations, it has been assumed that the condition of constant volume applies since the plastic strains involved are large compared to the elastic strains. The assumption of constant volume requires that the sum of the principal unit strains is zero. In the case of thick hollow cylinders the principal strains are tangential, radial and longitudinal and are denoted in the above order as e_t, e_r and e_z.

Nadai in his text on plasticity set up the following equations relating stress and strain in the plastic state paralleling Hooke's Law for the Elastic State:

$$\begin{aligned} e_t &= C\left[S_t - \mu\,(S_r + S_z)\right] \\ e_r &= C\left[S_r - \mu\,(S_t + S_z)\right] \qquad \text{----------- (2)} \\ e_z &= C\left[S_z - \mu\,(S_t + S_r)\right] \end{aligned}$$

His assumption that the value (C) is a constant, limits the value of μ to (.5) in order that the sum of the principal strains be equal to zero. His assumptions are, no doubt, good when extremely large deformations are made. Since the plastic strains produced in coldworking, proof testing and autofrettage are relatively small, his assumptions are modified in this presentation in the following way:

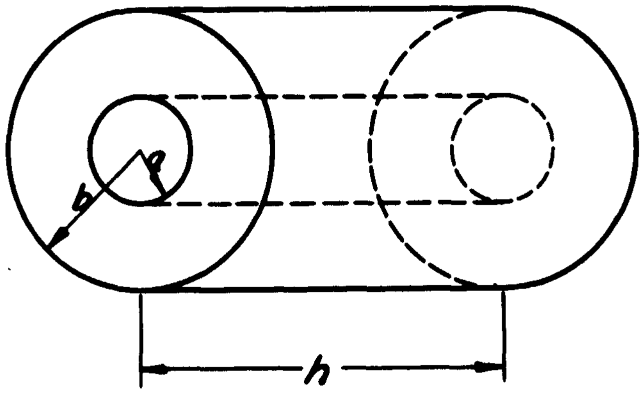
1) The assumption of constant volume holds since the plastic deformations, while relatively small, are still large in comparison to the elastic strains. As a result, the sum of the principal strains is zero.

2) Nadai's value (C) is not a constant until deformations larger than are encountered here are made.

3) Poisson's ratio, μ, in the plastic state gradually changes after the transition from the elastic state from a value of approximately (.285) to (.5). For the magnitude of plastic strains where this work applies, a smaller value of μ than (.5) exists.

Strain Ratio

The formula for strain ratio in tubes that are overstrained within the above assumptions may be derived in the following way:



WTN,639-6063 *Fig. 2*

The volume of metal in the cylinder (Fig. 2) is given by

$$V = \pi h(b^2 - a^2) \text{ - - - - - - - - - - - - - } (3)$$

The assumption of constant volume makes it possible to write

$$dV = 0 = 2\pi h(bdb - ada) + \pi(b^2 - a^2)\, dh$$

by letting

$e_i = \frac{da}{a} =$ Unit tangential strain at the bore surface

$e_0 = \frac{db}{b} =$ Unit tangential strain on the outside surface

$e_z = \frac{dh}{h} =$ Unit strain longitudinally

$W = \frac{b}{a} =$ Wall ratio

Noting that longitudinal strains are uniform over the cross section of the tube, the above equation for change in volume may be simplified to

$$e_i = W^2 e_0 + \frac{e_z}{2}(W^2 - 1) \text{ - - - - - - - - - } (4)$$

If the cylinder is free to move longitudinally during expansion, a uniaxial stress exists on the outside surface which makes it possible to write

$$e_z = -\mu e_0$$

By substituting this value for e_z in equation 4 that equation becomes, expressed as strain ratio

$$\text{Strain Ratio} = \frac{e_\perp}{e_0} = W^2\left(1 - \frac{\mu}{2}\right) + \frac{\mu}{2} \quad \text{- - - - (5)}$$

Flow Factor

It is frequently convenient in engineering problems to deal in total deformations instead of unit strains. The flow factor is a quantity involving total deformations which is only a slight modification of the strain ratio. Flow factor is defined as follows:

$$\text{F.F.} = \frac{\text{Permanent Enlargement of Pore}}{\text{Permanent Enlargement of the Outside Surface}}$$

$$\text{or F.F.} = \frac{e_\perp \times \text{ID}}{e_0 \times \text{OD}} = \frac{e_\perp}{We_0} \quad \text{- - - - - - - - - - - - - - - - (6)}$$

Equation 1 in terms of flow factor would then become

$$\text{F.F.} = \frac{W}{1.125} \quad \text{- (7)}$$

and equation 5 would become

$$\text{F.F.} = W\left(1 - \frac{\mu}{2}\right) + \frac{\mu}{2W} \quad \text{- - - - - - - - - - - - - - - - (8)}$$

Longitudinal Strain

Longitudinal strain may be readily derived in terms of tangential strain at the bore and wall ratio from equation 4. Since $e_0 = -e_z/\mu$ equation 4 then may be expressed as

$$-\frac{e_\perp}{e_z} = \frac{W^2}{\mu}\left(1 - \frac{\mu}{2}\right) + \frac{1}{2} \quad \text{- - - - - - - - - - - - - - - - (9)}$$

By the same device Lt. Col. Solberg's equation 1 may be used to determine a value for longitudinal strain

$$\frac{e_l}{e_z} = -\frac{W^2}{1.125\mu} \qquad (10)$$

Shrinkage in Tapered Sections

In the design of coldworked guns, it is necessary to take into account longitudinal strains in a solid having a tapered outside surface and a concentric cylindrical hole. This may be done in the following manner:

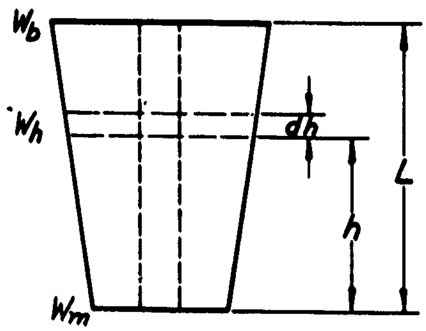


Fig. 3

Referring to Fig. 3

W_h = The wall ratio at any height h

W_b = The wall ratio at the breech

W_m = The wall ratio at the muzzle

L = The total length of the tapered cylinder

θ = The change of W with respect to h

h = Any height

The wall ratio at any height h is

$$W_h = W_m + \left(\frac{W_b - W_m}{L}\right)h$$

or

$$W_h = W_m + \theta h \qquad (11)$$

Differentiating equation 11 yields

$$dh = \frac{dW}{\theta} \text{ - } (12)$$

The total shrinkage of the tapered section is given by

$$\Delta L = \int_{0}^{L} e_z \, dh \text{ - - - - - - - - - - - - - - - - - } (13)$$

A value for e_z in terms of wall ratio and strain at the bore is given by equation 10; the value for dh in terms of wall ratio is given by equation 12. By making the substitutions in equation 13 and changing the limits to conform to the change in variable, the intergration yields

$$\Delta L = -\frac{1.125\mu e_i L}{W_b \; W_m} \text{ - - - - - - - - - - - - - - - } (14)$$

If equation 9 is used to express e_z a more complicated integral is derived. The final equations for ΔL derived in this manner for various values of μ are tabulated below:

$(\mu = 3/10)$

$$\Delta L = \frac{-.84 e_i L}{W_b - W_m}\left[\mathrm{TAN}^{-1}(2.38W_b) - \mathrm{TAN}^{-1}\, 2.38W_m\right] \text{ - - - - - } (15)$$

$(\mu = 1/3)$

$$\Delta L = \frac{-.895 e_i L}{W_b - W_m}\left[\mathrm{TAN}^{-1} 2.236W_b - \mathrm{TAN}^{-1} 2.236W_m\right] \text{ - - - - - } (16)$$

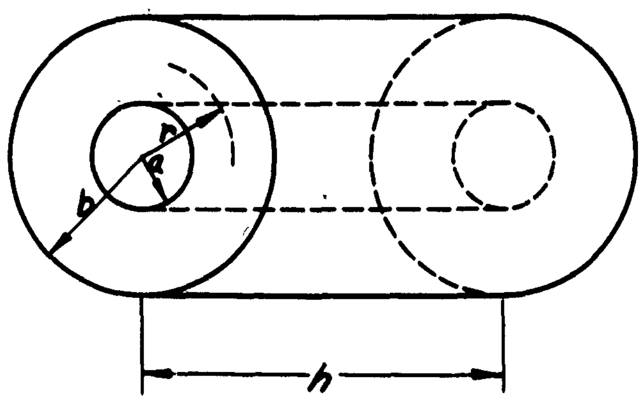
$(\mu = 1/2)$

$$\Delta L = \frac{-1.155 e_i L}{W_b - W_m}\left[\mathrm{TAN}^{-1} 1.732W_b - \mathrm{TAN}^{-1} 1.732W_m\right] \text{ - - - - } (17)$$

Plastic Strain Distribution

Feferring to Fig. 4 the volume before expansion bcunded by the bore radius (a) and any radius (r) is given by

$$V = \pi h(r^2 - a^2)$$



WTN.639-6065 Fig. 4

Since no change in volume is bought about

$$dv = 0 = 2\pi h(rdr - ada) + (r^2 - a^2)\pi dh \text{ - - - } (18)$$

The change in radius after expansion is given by $dr = e_t r$ where e_t = unit tangential strain at radius (r) and the change in bore radius is given by $da = ae_1$ where e_1 again equals the unit strain at the bore. By substituting these values in equation 18, noting that $dh/h = e_z$, simplifying, and solving for e_t the following equation is derived

$$e_t = \frac{a^2}{r^2} e_1 - \left(1 - \frac{a^2}{r^2}\right) \frac{e_z}{2} \text{ - - - - - - - - - - - - - - - } (19)$$

Since the sum of the principle strains is zero:

$$e_r = -(e_t + e_z)$$

therefore

$$e_r = -\frac{a^2}{r^2} e_1 - \frac{e_z}{2}\left(1 + \frac{a^2}{r^2}\right) \text{ - - - - - - - - - - - - - } (20)$$

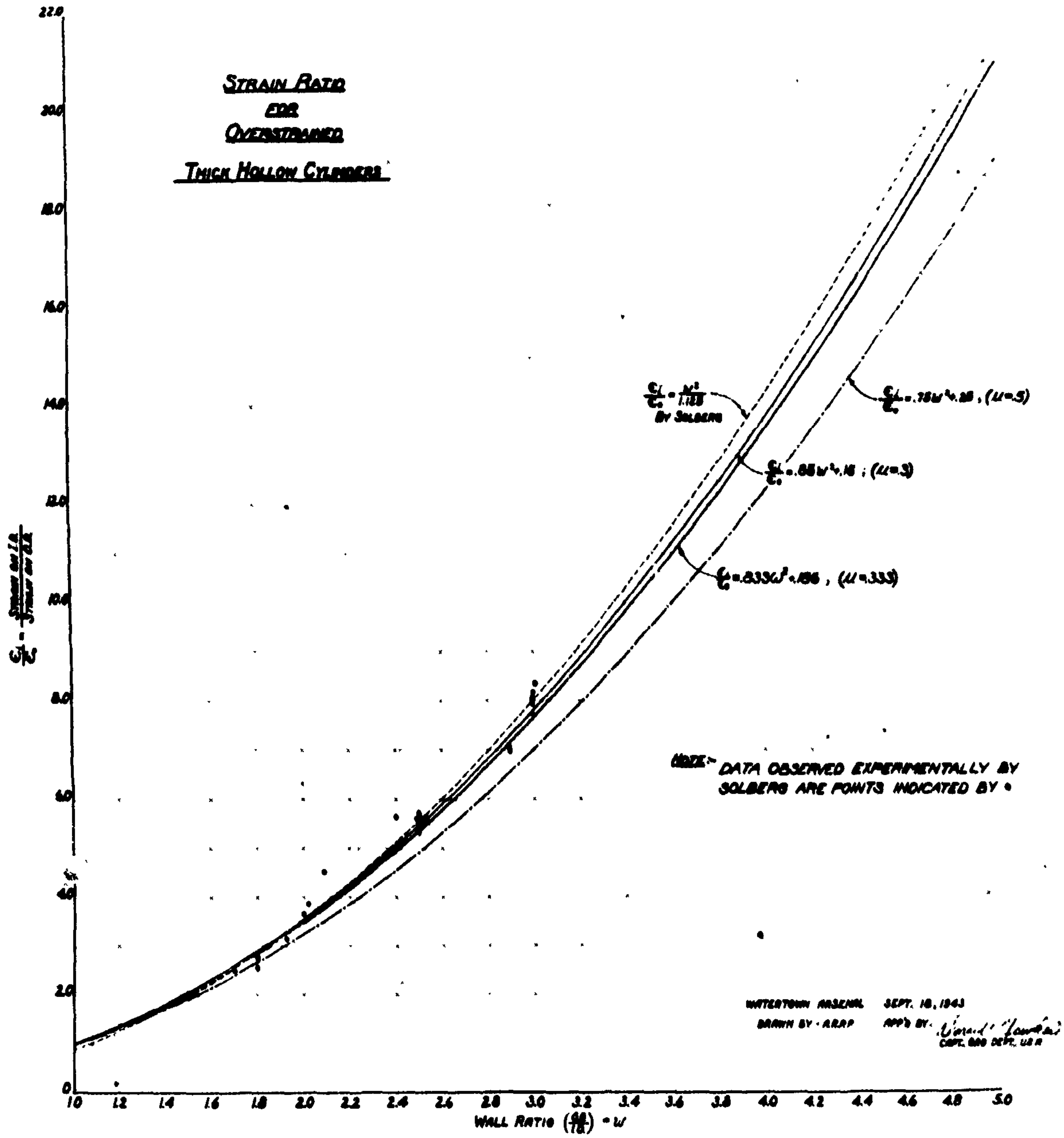
DISCUSSION

Figure 1 shows a comparison of the strain ratio equations 1 and 5 with Col. Solberg's original experimental strain ratio data which was obtained from tests on cylinders having wall ratios from $W = 1$ to $W = 3$. The experimental data agree essentially with all of the curves except the one plotted from equation 5 with Poisson's ratio assumed equal to one-half. The differences in values obtained from these equations are more clearly indicated in the frequency distribution curves (Figs. 5, 6, and 7) showing shrinkage in gun tubes as a result of coldworking. In these figures total shrinkage calculated by use of equation 14 is compared with that actually measured in a large number of gun tubes. These data indicate that using a value of Poisson's ratio of 3/10 to 1/3 in equation 5 will lead to computed values of shrinkage that are in close agreement with experimental test results.

It is pointed out that equation 14 (derived from equation 1) yields values of shrinkage in close agreement with those obtained from either equation 15, 16, or 17 (which were derived from equation 5) when the same value of Poisson's ratio is employed in both cases. For practical purposes equation 14 involves simpler computations and appears to be equally accurate for the usual range of wall ratios employed in gun tube design.

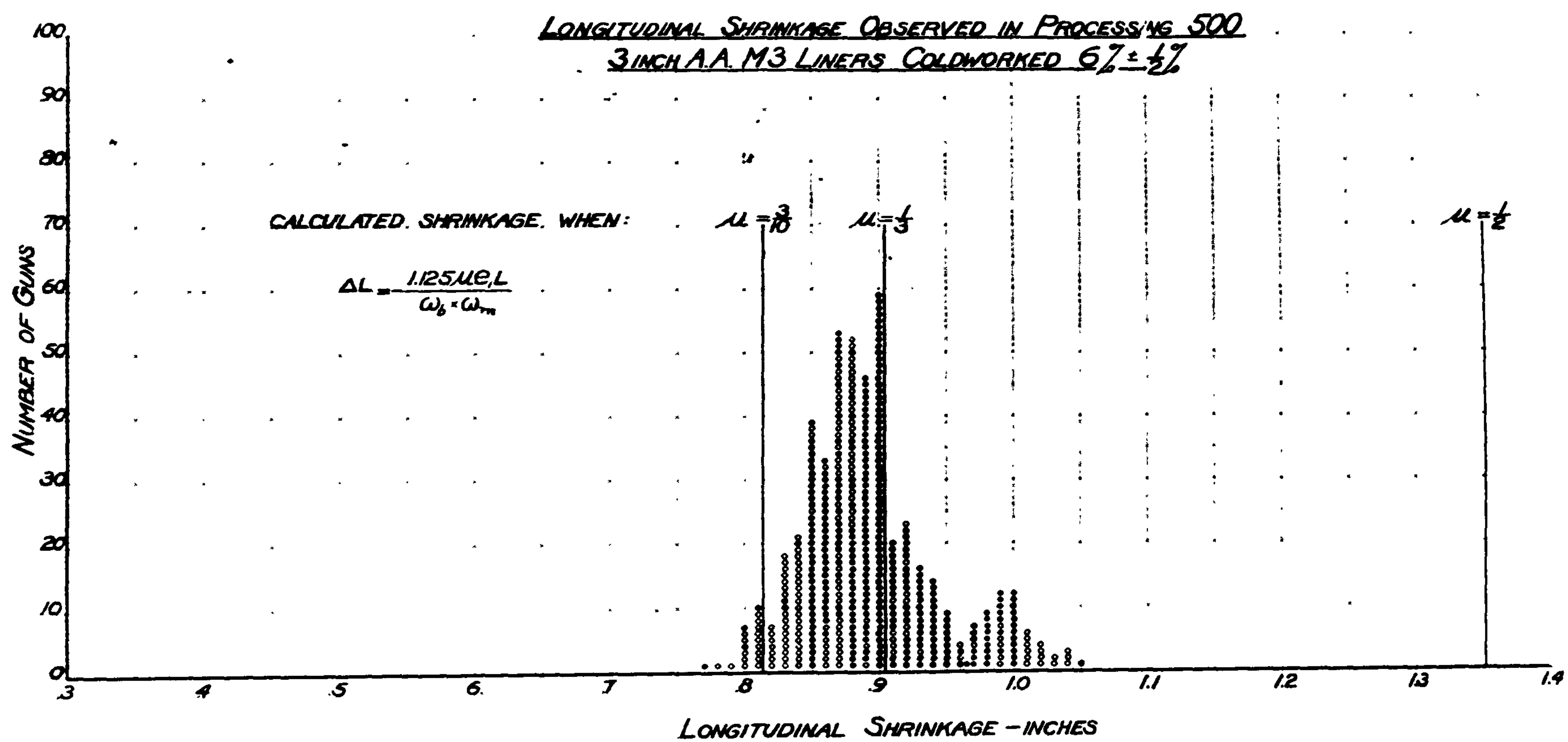
Figure 8 shows a typical "True Stress Strain" curve recorded under the direction of Lt. Herbert Hollomon in the Watertown Arsenal Laboratory. The method of determining this curve was reported in Watertown Arsenal Laboratory Report #111.2/16 entitled "A Description of an Automatic Recorder for Tensile Machines" by Dr. Zener and Mr. Van Winkle. The slope of the upper portion of the curve does not "flatten out" until approximately 15% extension is reached. The average equivalent strain through the wall of a coldworked tube is considerably less than 15%. For strains greater than 15%, the effective value of μ probably approaches one-half. Based upon the hypothesis that for less than 15% extension μ is a function of the amount of strain, Poisson's ratio would not be constant throughout the wall of an autofretted or coldworked gun since there is more strain at the bore surface than at any other point. The net effect, however, appears to be approximately the same as though μ were assumed constant throughout the tube wall with an average value of approximately one-third.

The equations presented in this report are not theoretically perfect but serve in estimating plastic strains with sufficient accuracy for practical engineering purposes. It is suggested that variations of Poisson's ratio and related plastic deformation are phenomena worthy of a more careful study from which still more precise and more academic strain equations may be derived, leading to a better understanding of the plastic behavior of metals.



WTN.639-6062

FIGURE 1



WATERTOWN ARSENAL

DRAWN BY J. ROMAN

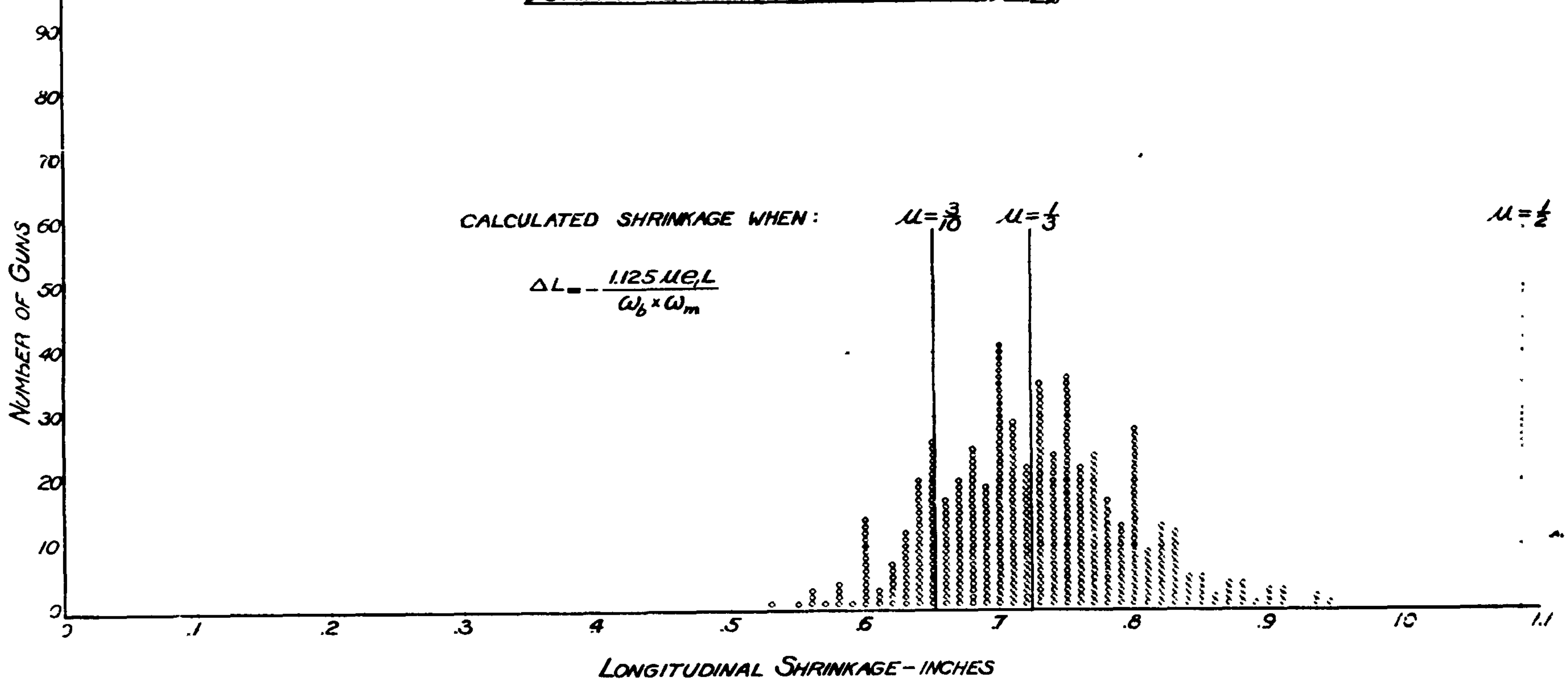
OCTOBER 12, 1943

WTN.639-6066

FIGURE 5

WTN.639-6667

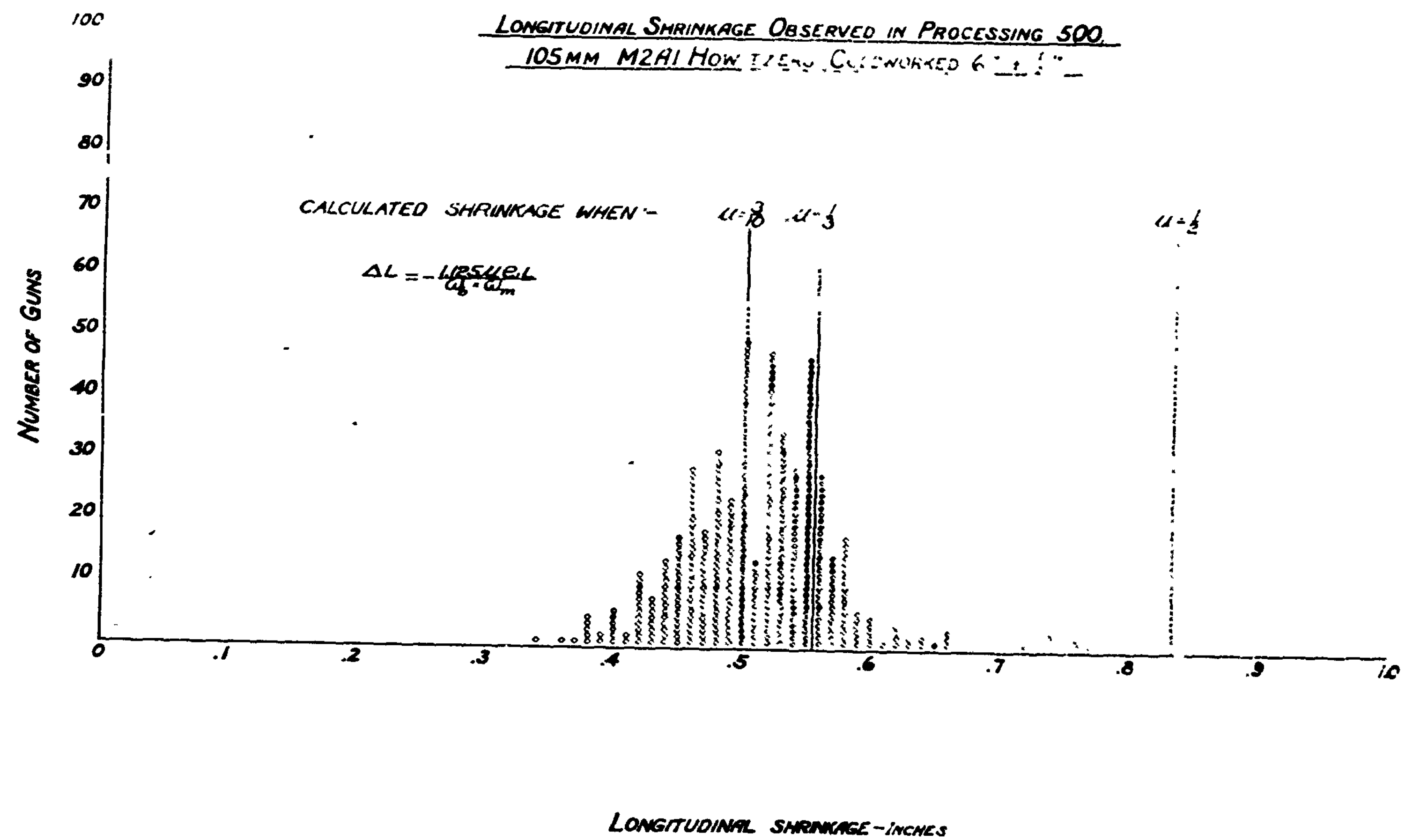
LONGITUDINAL SHRINKAGE OBSERVED IN PROCESSING 500
90MM M1 GUN TUBES COLDWORKED 6% ± ½%



WATERTOWN ARSENAL

OCTOBER 1, 1943

FIGURE 6

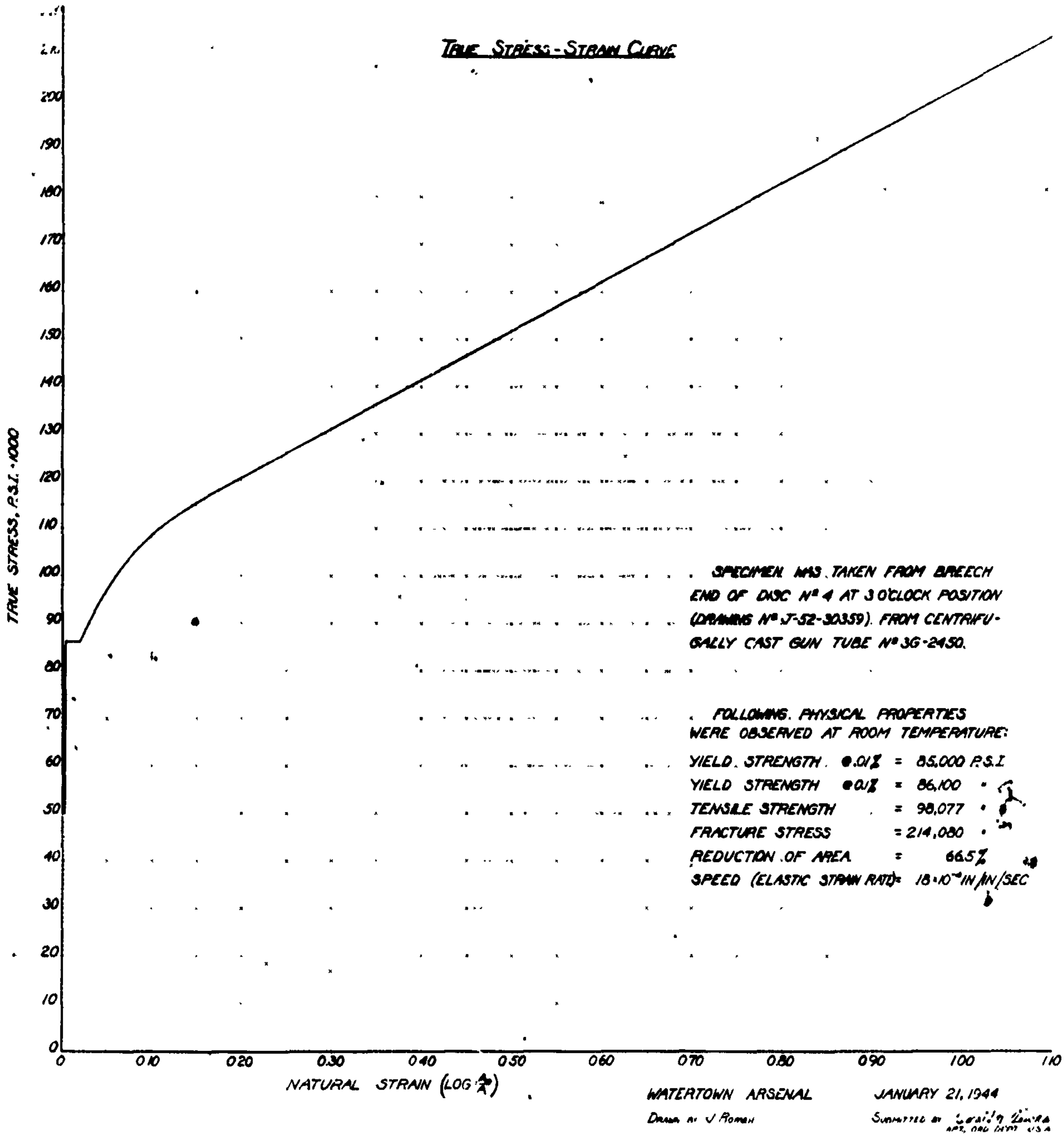


WATERTOWN ARSENAL
DRAWN BY J ROWAN
A R P PAUL

OCTOBER 1C, 1943

WTN.639-6068

FIGURE 7



WTN.639-6069

FIGURE 8