HEAT TREATMENT

Quenching of Hollow Cylinders

John H. Hollomon
1st Lt., Ordnance Dept.

Clarence Zener
Physicist

February 5, 1943

WATERTOWN ARSENAL
WATERTOWN, MASS.
HEAT TREATMENT

Quenching of Hollow Cylinders

OBJECT

To develop a method of determining the conditions under which hollow steel cylinders will harden throughout.

SUMMARY OF RESULTS

1. The calculations of Grossmann of sizes of round bars and of flat plates having the same minimum hardness when quenched in different media have been modified by the introduction of a new basis for comparison.

2. The method of Grossmann for calculating the alloying elements necessary to harden a round bar throughout is extended to hollow cylinders by calculating for all quenching media the diameter of the round having the same cooling rate at the center as the minimum cooling rate in the wall of a hollow cylinder.

3. The amount of alloying elements necessary to harden the equivalent round throughout may be determined empirically by quenching round bars of different compositions and of varying sizes in the pre-elected medium or they may be estimated by the
method of Grossmann.

4. Experimentally determined sizes of solid cylinders having the same minimum hardness as hollow cylinders quenched only on the outside agree with the theory within 6 percent.

5. For the particular conditions of the experiments in which water was forced up the inside of small cylinders, the experimentally determined sizes of solid cylinders having the same minimum hardness as the hollow cylinders were 15 percent smaller than predicted by the theory in which the severity of quench on the inside and outside were assumed to be the same.

6. It may be inferred that the inside of the hollow cylinders of these experiments were quenched more effectively than the outside.

J. H. Hollomon
1st Lt., Ordnance Dept.

C. Zener
Physicist

APPROVED:

H. H. ZORNIG
Colonel, Ordnance Dept.
Director of Laboratory

UNANNOUNCED
INTRODUCTION

The method of Grossmann for calculating the effect of alloying elements on the hardenability of rounds has been described in detail in the literature. After calculating the multiplicative effects of the alloying elements, a series of curves can be determined for the effect of carbon content and grain size on the diameter which would just harden throughout in an ideal quenching medium. In an ideal quenching medium, the surface of the round is maintained at the temperature of the medium. This is approximated in practice by agitated or sprayed brine. In an actual quench, the diameter of the round which will just harden throughout is the smallest one having no "unhardened core" visible in either the fracture or the etch test. The center of such a hardened round is approximately 50 percent martensitic. After estimating the ideal diameter for some composition, the actual size of round which would just harden throughout may be determined from a series of curves for different quenching media which relate the actual size of round to this ideal diameter.

A series of curves prepared in a similar manner

1.-References to be found at end of report.
is presented herein in which the ideal round diameter equivalent to a hollow cylinder is plotted as a function of the severity of quench and wall ratio. From this equivalent ideal round diameter the alloying elements necessary to harden the hollow cylinder throughout may be calculated. The method outlined by Grossmann is admittedly only an approximate method of determining the composition having the desired hardenability, but it is the best method available for prediction. Knowing the size of round which will harden throughout (either by experiment or by estimation from Grossmann's data), the size of the equivalent hollow cylinder may be calculated readily. On the other hand if the size of the hollow cylinder is known the round size which must be hardened throughout in the given medium may also be calculated.

**DISCUSSION AND RESULTS**

A. Theoretical

In order to calculate the diameter of the solid steel cylinder which when quenched in a given medium will have the same hardness at its center as the minimum hardness in the wall of a hollow cylinder quenched in the same medium, it is first necessary to determine the conditions of time and temperature during cooling for which the hardness at the two positions of the two steel shapes will be the same.
Grossmann, in comparing the hardness distribution from the center to outside of solid cylinders found that the time required to reach half the difference between the quenching and final temperature during cooling determined the hardness; that is, if the outside of one solid cylinder of a given steel composition required the same time to cool half way from the quenching temperature to room temperature as the inside of some other cylinder of the same composition, the hardnesses at the specified positions would be very nearly the same. It was noted that the shapes of the cooling curves at different positions in solid rounds are quite dissimilar. If the cooling rate at the center of one bar is the same as that of the outside of another, the "half temperature times" will be widely different. Since the time spent below the critical temperature before transformation is of importance in determining the precise products of transformation, in comparisons where the initial parts of the cooling curves are of different shapes the time to reach half temperature is probably a better criterion than that of cooling rate. Grossmann and his collaborators presented some experimental evidence that cooling time is the more applicable criterion of hardenability when positions other than the centers of round sections are compared and assumed that this condition is also the more applicable
In comparing the cooling at the centers of different round bars.

In certain important cases, the rate of cooling in the appropriate temperature range may be specified by a single parameter, $c$, in the manner indicated below:

$$\frac{dT}{dt} = -cT,$$

where $T$ is the temperature of the element referred to that of the quenching medium. Throughout the temperature range in which Eq. (1) is obeyed a plot of $T$ vs. $t$ on semi-log paper will yield a straight line. Such a plot is presented as Fig. 1 for the centers of solid cylinders from the data of Russell. As the quenching proceeds the curves approach straight lines. The less severe the quench or rather the smaller the product of $H$ (severity of quench) times $D$ (diameter of the round) the more rapidly is the asymptote approached. For each different product, $HD$, in this graph the diameter ($D$) of the bar has been chosen which renders the various asymptotes parallel. The centers of the bars represented in Fig. 1 quenched in different media have the same "$c$" while their half temperature times are quite different, having the ratio 2.4:3.0:4.0 for $HD$ equal to 0,1, and respectively. Since most of the differences in half temperature times occurs before the
critical temperature is reached, it is not significant. For this example the quenching temperature was assumed to be 1700°F, the critical temperature, 1600°F, and the temperature of the nose of the "S" curve 1050°F. The half temperature time starting from the critical rather than the initial temperature are in the ratio of 2.1:2.1:2.4 which is not far from the ratio 1:1:1. Since the difference between the initial and critical temperatures is variable and very difficult to fix in actual quenching practice it may be concluded that when center hardenability are to be compared the slopes of the linear portion of the curves of Fig. 1 are a better measure of the hardenability than the half temperature times. Using this new criterion the ratio of the round diameter quenched in a medium of quench severity \( H D_{\text{SH}} \) to the round diameter quenched in an ideal medium \( D_{S00} \) is plotted as a function of \( H D_{\text{SH}} \) as curve a in Fig. 2. This curve is to be compared with Fig. 3 taken from Grossmann's calculations for which the criterion of equal half temperature times was used.

The use of curve a in Fig. 2 may be illustrated by an example. Let us assume that the severity of quench, \( H \), has been measured or estimated and is equal to 2. Assuming that it has also been determined experimentally by an etch test or hardness survey that
a 2.5" round bar of a given steel will harden throughout when quenched in this medium, we wish to know what size round bar would harden in a medium of \( H = 1 \), and \( H = \infty \) (ideal medium). It is first necessary to multiply \( H \) by \( \frac{D_{SH}}{D_S} \), which = 5. From Fig. 2a is read the value of the ratio \( \frac{D_{SH}}{D_S} (0.81) \). We obtain \( D_{S\infty} \) by dividing \( D_{SH} (2.5") \) by this ratio. The ideal diameter is therefore equal to 3.1". A 3.1" diameter bar of the given steel would harden throughout in a medium of infinite severity of quench. To obtain the diameter which would harden through in a medium of severity \( H = 1 \), the value of \( \frac{D_{SH}}{D_{S\infty}} \) is first read from Fig. 2a. For a value of \( HD_{SH} = 2.5 \) this ratio is equal to .71. The ratio of \( \frac{D_{SH} = 1/D_{SH} = 2}{} \) is found by dividing .71 by .61. This fraction when multiplied by 2.5" \( (D_{SH} = 2) \) is equal to the diameter of the bar which would just harden through in a medium of \( H = 1 \).

The relation between the thickness of a plate quenched in a medium, \( H \), and the ideal plate thickness is presented as curve b in Fig. 2. This curve is also based on the equivalence of the "c" rather than on the equivalence of half temperature times. The ideal round size equivalent to the ideal plate thickness may be determined from the following relation. (Appendix)

\[
D_{S\infty} = 1.55L_{\infty}
\]

where \( L_{\infty} \) is the ideal plate thickness.
The calculation of the ideal round diameter equivalent to a hollow cylinder is more complicated, for the outside and inside diameters can be varied independently. The equivalent round diameter is determined for a given wall ratio and for a severity of quench measured by the product of \( H \) times the difference between the inside and outside diameters.

The results of calculations (Appendix) based on this new criterion are presented as Fig. 4 in which the ratio of equivalent round diameter \( D_{SH} \) to outside diameter \( D_o \) is plotted as a function of \( D_1 / D_o \) for different values of \( H \) times \( (D_o - D_1) \) where \( D_1 \) is the inside diameter. \( D_{SH} \) is the diameter of the round which has the same cooling rate at its center as the minimum cooling rate in the wall of the hollow cylinder when quenched in a medium of severity \( H \). The use of curve a of Fig. 3 and Fig. 4 can be best understood through an example; let us assume that the equivalent ideal diameter of a round is to be determined for the following situation:

- Severity of quench \( H = 1 \)
- Outside diameter \( D_o = 4" \)
- Inside diameter \( D_1 = 2" \)

\[
\frac{D_1}{D_o} = 0.5 \\
D_o - D_1 = 2" \\
H(D_o - D_1) = 2
\]

The value of \( D_{SH}/D_o \) corresponding to a wall ratio of 0.5 and a value of \( H(D_o - D_1) \) of 2 is read from Fig. 4. \( D_{SH}/D_o \) in this case is 0.43 and when multiplied by
$D_0(4")$, an equivalent round diameter of 1.72" is obtained. By employing the graph of Fig. 2a and the product $D_{SH}(1.72)$ and $H(1)$, $D_{SH}/D_{SOO}(0.63)$ is found. $D_{SH}$ divided by this ratio fixes the ideal round diameter equivalent to the hollow cylinder (2.7").

B. Experimental

In order to determine how closely the theoretical results outlined in the previous section approximate the actual relationships between the sizes of hollow cylinders and rounds having the same minimum hardness, experiments described in this section have been performed.

The ideal critical diameter of a steel of the following composition was calculated by the method of Grossmann.

<table>
<thead>
<tr>
<th>C</th>
<th>Mo</th>
<th>Si</th>
<th>S</th>
<th>P</th>
<th>Cr</th>
<th>Cu</th>
<th>Mo</th>
<th>Al</th>
</tr>
</thead>
<tbody>
<tr>
<td>.40</td>
<td>.79</td>
<td>.43</td>
<td>.070</td>
<td>.016</td>
<td>.14</td>
<td>.105</td>
<td>.16</td>
<td>.004</td>
</tr>
</tbody>
</table>

The critical diameter in mildly agitated water was measured by quenching from 1575° F (neutral atmosphere) rounds of various diameters from 1\(\frac{1}{2}\)" to 2\(\frac{3}{8}\)" all machined from a 2\(\frac{3}{8}\)" round bar. In all cases the length of the solid cylinder was 4 times the diameter. The water in the quenching tank was agitated by a stream of water entering from the bottom through a 1\(\frac{1}{4}\)" diameter aperture at about 25 gal. per minute. The diameters of the unhardened cores were determined after sectioning.
by etching with a hot sulfuric-hydrochloric acid mixture. The critical diameter of the steel and the K value of the medium were then determined by the method of Grossmann and were found to be 1.5" and 1.4 respectively. The ideal diameter calculated from the composition was 2.2" while the measured diameter was equal to 2.1". The ratio $D_1/D_0$ of the hollow cylinder with outside diameter of 2.5" and for both inside and outside quenched which would be equivalent to this solid round (1.5") was read from Fig. 4 and $D_1/D_0$ is equal to .3. The wall ratio $(D_1/D_0)$ for the equivalent hollow cylinder for the case in which only the outside is quenched was found from Fig. 4 to be .3. Five hollow cylinders with wall ratios $(D_0 = 2.5")$ less than each of these two critical values were machined from the 2.5" round stock. One set of five hollow cylinders was quenched from 1575°F into the water and supported directly over the aperture. Thin plates were welded over the ends of the second set of hollow cylinders and these were quenched as nearly like the solid cylinders as possible.

The Rockwell "C" hardness distribution for each of the hollow and solid cylinders was measured and the minimum hardnesses are plotted in Fig. 3. A typical hardness survey for both inside and outside quenching is presented in Fig. 7. The size of the round which
would have the same minimum hardness as each of the hollow cylinders was determined from Fig. 2. The experimental determinations and the corresponding theoretical values read from Fig. 4 are tabulated in Table I. The difference between theory and experiment for the hollow cylinders with only the outside quenched is less than 6 percent. For the case of both inside and outside quenching the difference is about 15 percent, the theory predicting a larger equivalent round size. For these small size cylinders, the inside quenching is more efficacious than the outside for the particular conditions of our experiment. In these experiments, it appears that the severity of quench is larger on inside than the outside and the equivalent solid cylinder is necessarily smaller than that predicted assuming that the inside and outside of the hollow cylinder were quenched identically.
TABLE 1

Comparison of Theoretical and Experimental Equivalent Solid Cylinders (For Some Minimum Hardness)

<table>
<thead>
<tr>
<th>Hollow Cylinder Only Outside Quenched</th>
<th>Diameter of Equivalent Solid Cylinder (Inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D_1 (Inches)</td>
<td>D_0 (Inches)</td>
</tr>
<tr>
<td>----------------</td>
<td>--------------</td>
</tr>
<tr>
<td>1.00</td>
<td>2.48</td>
</tr>
<tr>
<td>1.15</td>
<td>2.48</td>
</tr>
<tr>
<td>1.25</td>
<td>2.48</td>
</tr>
<tr>
<td>1.35</td>
<td>3.45</td>
</tr>
<tr>
<td>1.50</td>
<td>2.48</td>
</tr>
</tbody>
</table>

Both Inside & Outside Quenched

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>Exp.</th>
<th>Theo.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.38</td>
<td>2.48</td>
<td>1.55</td>
<td>1.75</td>
</tr>
<tr>
<td>1.50</td>
<td>2.48</td>
<td>1.44</td>
<td>1.62</td>
</tr>
<tr>
<td>1.63</td>
<td>2.48</td>
<td>1.31</td>
<td>1.54</td>
</tr>
<tr>
<td>1.75</td>
<td>2.48</td>
<td>1.20</td>
<td>1.45</td>
</tr>
<tr>
<td>1.88</td>
<td>2.48</td>
<td>1.10</td>
<td>1.35</td>
</tr>
</tbody>
</table>
APPENDIX A
CALCULATIONS

1. General Theory

The purpose of this section is to find the relation between the equivalent ideal critical diameter \(D_{s*}\) of a solid cylinder and of a hollow cylinder of inside diameter \(D_1\) and outside diameter \(D_o\), when quenched in some medium. This relation will contain as parameters the severity of the quench in the actual medium, \(K\), and the wall ratio \((a)\) of the inside diameter over the outside diameter. Thus

\[D_s = F(a, K, D_o).\]

The mathematical problem to be solved is the determination of the constant \(c\) (Eq. 1) for hollow and for solid cylinders, for various severities of quench.

The differential equation for heat flow is

\[
\frac{\partial T}{\partial t} = k\nabla^2 T.
\]  

In this equation, \(k\) is the thermal diffusion coefficient, \(\nabla^2\) is the Laplacian operator. Since we are considering the special case in which Eq. (1) is satisfied, we shall set

\[T = T_o \left\{ \exp \left( -ct \right) \right\} U(r).\]

Here, as in Eq. (1), \(T\) is the temperature of element referred to that of the quenching medium, \(U\) is an unknown
function of coordinates, and c is a constant to be determined.

Substitution of Eq. (3) into Eq. (2) gives Eq. (5) below as the differential equation for \( U(r) \); with

\[ m^2 = c/k. \]

(4)

Only those solutions of this differential equation are desired which satisfy the appropriate boundary condition. The boundary condition which has been found to be fairly accurate is given by Newton's law of cooling. This law states that the flow of heat across a boundary is proportional to the temperature drop across that boundary. Since the flow of heat is proportional to the temperature gradient, the appropriate boundary condition is that the temperature gradient at the boundary is proportional to the difference in temperature between the solid and the quenching medium. The constant of proportionality is taken as \(-h\), the gradient being taken in the direction from the solid to the quenching medium. The constant \( h \) is equal to twice the \( H \) used by Grossmann. This boundary condition may be written as in Eq. (6). Eqs. (5) and (6) below form the complete mathematical formulation of the problem.

**Differential equation for \( U \):**

\[ \nabla^2 U = -m^2 U. \]

(5)

**Boundary condition for \( U \):**

gradient, \( U = -hU \) at surface. \( \) (6)
2. Plates and Solid Cylinders

The solution of these equations have previously been given for the cases of a plate and of a solid cylinder. In the case of the plate, of thickness 2L,
\[ U = \cos mx, \]
where \( m \) is the lowest root of the equation
\[ \tan mL = h/m. \]  \hspace{1cm} (7)

In the case of a solid cylinder of radius \( r_s \)
\[ U = J_0(mr), \]
where \( m \) is the lowest root of the equation
\[ J_1(mr_s)/J_0(mr_s) = h/m. \]  \hspace{1cm} (3)

Eqs. (7) and (8) have no solution for arbitrary values of \( m \), but only for a discrete set of values. In the present case one is interested only in the lowest value of \( m \). The higher values are important only when the transient effects are being studied.

a. Limit \( h \to 0 \).

In the case of a very poor quenching medium, \( h \to 0 \) and also \( m \to 0 \). The left members of Eqs. (7) and (8) may therefore be replaced by the first term in a Taylor expansion. We obtain
\[ mL = h/m, \]  \hspace{1cm} (7a)
\[ \frac{1}{2} mr_s = h/m. \]  \hspace{1cm} (8a)

The ratio of the diameter of the solid cylinder to
the thickness of equivalent plate is therefore given by the equation

\[ \frac{r_g}{L} = 2. \]  

(9)

b. Limit \( h = \infty \).

The case of an ideal quench is obtained by setting \( h = \infty \). Eqs. (7) and (8) now become, with \( r_{s\infty} \) written for \( r_g \),

\[ mL_{\infty} = \pi/2, \]  

(7b)

\[ mr_{s\infty} = 2.404. \]  

(8b)

In this case we obtain the ratio

\[ \frac{r_{s\infty}}{L_{\infty}} = 1.53. \]  

(10)

The observed value of the ratio \( r_{s}/L \) lies between the above two extreme values of 1.53 and 2.0, while the ratio as calculated by Grossmann's method of half temperature time is 1.57.3

By means of Eqs. (8) and (8b), one can obtain the ratio \( r_{s}/r_{s\infty} \) of the radius of a solid cylinder in an actual quench to the radius of a solid cylinder in an ideal quench. From Eq. (8) \( h/m \) is obtained as a function of \( mr_{s} \). Upon forming the quotient \( mr_{s}/2.404 \) we obtain, by Eq. (8b), \( r_{s}/r_{s\infty} \). Upon forming the product \( mr_{s} \times (h/m) \) we obtain \( hr_{s} \). The resulting relation between \( r_{s}/r_{s\infty} \) and \( hr_{s} \) is given as Fig. 2. In this figure the first quantity is written as \( D_{s}/D_{s\infty} \), the latter quantity as \( HD_{s\infty} \). In an identical manner, one
can obtain from Eqs. (7) and (7b) the ratio \( \frac{L_u}{L_\infty} \) of the thickness of a plate in an actual quench to the equivalent thickness in an ideal quench. The result is given as Fig. 5a.

c. Intermediate quenches.

The ratio \( \frac{r_s}{L} \) for intermediate quenches has been obtained as follows. For a series of values of \( \frac{h}{m} \), the corresponding values of \( mL \) and \( mr_s \) were found from Eqs. (7) and (8), respectively. By forming the ratio 
\[
\frac{(mr_s/ml)}{r_s/L},
\]
and the product \( 2(mL) \cdot (h/m) = 2Lh \), \( r_s/L \) was obtained as a function of \( 2Lh \). The transition of \( r_s/L \), which will be denoted by \( x(h) \), from 2 to 1.53 as \( h \) goes from 0 to \( \infty \), is best presented by a graph giving \( (x-x_\infty)/(x_0-x_\infty) \) as a function of \( 2Lh \). This is given as Fig. 5.

3. Hollow Cylinder, Inside and Outside Quenched

In a hollow cylinder the general solution of Eq. (5) must be used, namely
\[
U(r) = C_0 J_0(mr) + C_1 N_0(mr), \tag{11}
\]
where \( J_0 \) and \( N_0 \) are Bessel functions of the first and second type, respectively. The two constants must be so chosen as to satisfy the boundary condition (6) at both the inner and the outer surfaces.

The boundary condition (6) applied to both the inner and outer surfaces, gives two simultaneous linear homogeneous equations in the two unknowns \( C_0 \) and \( C_1 \).
These equations are soluble only if the determinant of the coefficients is zero. This condition gives the following equation, which corresponds to Eq. (8) for a solid cylinder.

\[
\frac{m J_0'(mr_0) + h J_0(mr_0)}{m N_0'(mr_0) + h N_0(mr_0)} = \frac{m J_0'(amr_0) - h J_0(amr_0)}{m N_0'(amr_0) - h N_0(amr_0)} .
\] (12)

a. Limit \( h = 0 \).

One obtains the extreme case of a poor quench by letting \( h \), and hence also \( m \), approach zero. In this case each Bessel function may be replaced by the first term in the Taylor expansion. Using the expansion given in reference (11), one reduces Eq. (12) to

\[
\frac{\frac{1}{2} mr_0(1-a)}{\frac{1}{2} amr_0} = \frac{h}{m} .
\] (12a)

Upon dividing Eq. (8a) by Eq. (12a) one obtains

\[
\frac{r_0}{r_{eq}} = 1-a
\] (13)

as the ratio of the radius of the round to that of an equivalent hollow cylinder.

b. Limit \( h = \infty \).

The extreme case of an ideal quench is obtained by letting \( h \) approach \( \infty \) in Eq. (12). In this case Eq. (12) reduces to

\[
\frac{J_0'(mr_0)}{N_0'(mr_0)} = \frac{J_0'(amr_0)}{N_0'(amr_0)} .
\] (12b)
zero, one obtains the equation

\[ \frac{m J'_o(mr_0) + h J'_o(mr_0)}{m N'_o(mr_0) + h N'_o(mr_0)} = \frac{J'_o(amr_0)}{N'_o(amr_0)} \]  

(14)

\[ a. \text{ Limit } h = 0. \]

Proceeding as in Section 3a, we retain only the first terms in the Taylor expansion of the functions in Eq. (14). In this case Eq. (14) reduces to

\[ \frac{1}{m} mr_0 \left( 1 - a^2 \right) = h/m. \]  

(14a)

Upon comparing this equation with Eq. (8a), we obtain

\[ \frac{r_s}{r_o} = \left( 1 - a^2 \right). \]

This relation is given in Fig. 4.

\[ b. \text{ Limit } h = \infty. \]

In the case of an ideal quench, \( h = \infty \), Eq. (14) reduces to

\[ \frac{J'_o(mr_0)}{N'_o(mr_0)} = \frac{J'_o(amr_0)}{N'_o(amr_0)}. \]  

(14b)

This equation has been solved graphically in the same manner as was Eq. (12b). The result is given in Fig. 4.

\[ c. \text{ Intermediate Quenching Rates} \]

When \( a = 1 \), the distribution of temperature in the hollow cylinder becomes identical to the temperature
Since \( m \) may be replaced by \( 2.404/r_{s\infty} \) from Eq. (8b), the ratio \( r_s/r_0 \) may be determined as a function of \( a \). This function has been determined by a graphical method, and is given in Fig. 4.

2. Intermediate Quenching

The variation of the ratio \( x = r_s/r_0 \) with severity of quench may best be studied through the ratio \( \frac{x-x_\infty}{x_0-x_\infty} \). At least in the case of a thin hollow cylinder, \( a = 1 \), the dependence of this ratio upon \( h (r_0-r_1) \) will be given by the graph in Fig. 5 which was derived originally from a comparison of rounds with plates in Section 2 c. A numerical calculation for \( r_s/r_0 \) was carried out for the cases \( a = 0.1 \) and \( a = 0.5 \). The results showed that Fig. 5 was applicable to values of \( a \) at least as low as 0.1. The intermediate lines in Fig. 4 have been based upon Fig. 5.

4. Hollow Cylinder, Only Outside Quenched

In the case of a hollow cylinder quenched only on the outside, we must again use the general solution given in Eq. (11). The constants \( C_0 \) and \( C_1 \) are to be so chosen that the boundary condition (6) is satisfied with \( h \) set equal to zero at the inner boundary. The two conditions corresponding to the two boundaries lead to two linear homogeneous equations in \( C_0 \) and \( C_1 \). These equations are soluble only when the determinant of the coefficients is zero. Upon equating this determinant to
distribution throughout one-half of a plate quenched on both sides, whose thickness is double the real thickness of the hollow cylinder. The transition from $h \rightarrow 0$ to $h \rightarrow \infty$ is therefore obtained from Fig. 5 provided the abscissa is now $2H \cdot (D_0 - D_1)$ in place of $H(D_0 - D_1)$. 
BIBLIOGRAPHY


4. W. E. Jominy: Metal Progress, December, 1941.


11. E. Jahnske and F. Emde: Funktionentafeln. (Berlin 1926)
FIG. 2

EQUIVALENCE OF ROUND IN AN ACTUAL QUENCH TO A ROUND IN AN IDEAL QUENCH
(FOR EQUAL COOLING RATES)

EQUIVALENCE OF PLATE IN AN ACTUAL QUENCH TO PLATE IN AN IDEAL QUENCH
(FOR EQUAL COOLING RATES)
Fig. 4

**Equivalence of Solid and Hollow Cylinders**

\[ \frac{D_{SH}}{D_0} \]

- \( D_{SH} = \text{Diameter of equivalent solid cylinder.} \)
- \( D_0 = \text{Outside diameter of hollow cylinder.} \)
- \( D_i = \text{Inside diameter of hollow cylinder.} \)
FIG. 6
MINIMUM HARDNESS OF
SOLID & HOLLOW CYLINDERS
AS A FUNCTION OF DIMENSION
(EXPERIMENTAL)

HOLLOW CYLINDERS
QUENCHED ONLY OUTSIDE

HOLLOW CYLINDERS
QUENCHED OUTSIDE & INSIDE

SOLID CYLINDERS

ROCKWELL "C" HARDNESS

DIAMETER OF HOELE OF HOLLOW CYLINDER (D = 2.48") OR DIAMETER
OF SOLID CYLINDER
Fig. 7

Typical hardness survey for hollow cylinder quenched inside & outside.