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DEPARTMENT OF THE NAVY
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DESCRIPTION AND USAGE OF GENERAL BENDING RESPONSE CODE 3 (GBRC3)

by

F. Henderson

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TECHNICAL NOTE

February 1968

AML-4-68

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ABSTRACT

The method of expanding General Bending Response Code 1 (GBRC1) to include the calculation of critical whirling frequencies and response of shafting systems is described. The extended computer program is designated General Bending Response Code 3 (GBRC3). Specifications for data, input forms and a practical application of the program are included.

ADMINISTRATIVE INFORMATION

The work reported here was carried out under Job Order number 1-823-343-01, Subproject SR 003 03 01, Task 10919.

I. INTRODUCTION

The method of analysis used in the development of General Bending Response Codes 1 and 2, references 2 and 3, Section II, is applied to the partial differential equations of motion for a whirling shaft as derived by Norman H. Jasper. Since the differential equations describe a coupling of flexural vibrations in two planes, the analysis results in a set of time-independent finite-difference equations in terms of deflections and moments occurring in these planes. General Bending Response Code 3 solves this system of difference equations for: (1) the response patterns of the shaft through a specified range of frequencies of the exciting forces and/or moments acting upon it, or (2) critical whirling frequencies and mode shapes of free vibration. The computed results consist of the components, absolute value, and phase angles of the vectors for deflection and moment in the two planes of shaft vibration.

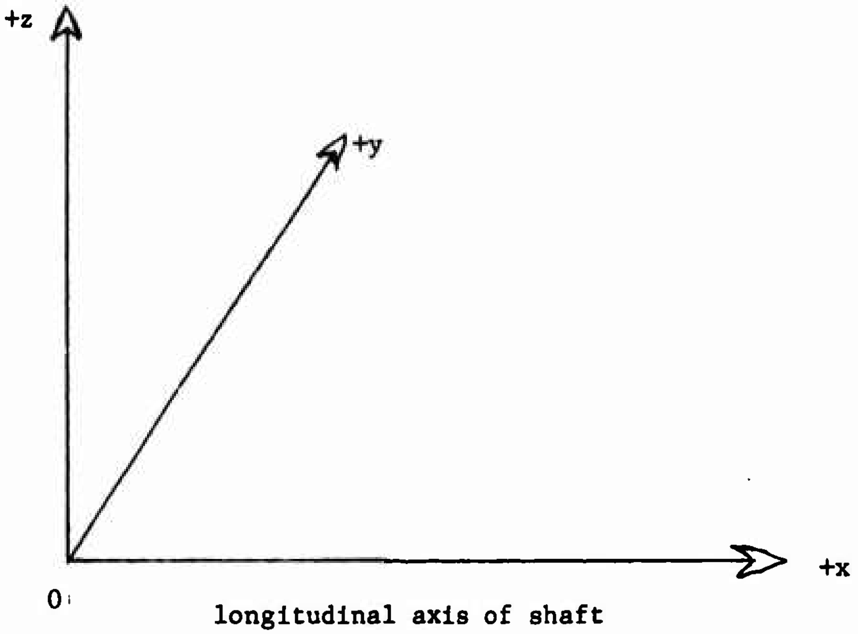
GBRC3 retains the provisions of previous codes in this series (references 2 and 3) for various types of damping and automatic calculation of frequency-dependent exciting forces and moments. Calculation of effective mass as a function of frequency is not included at this time. The code uses a FORTRAN IV compatible version of FORTRAN II and is currently being run with the IBSYS Operating System on the IBM 7090 computer at the Applied Mathematics Laboratory. An output tape for off-line plotting is written through use of the package of 7090 FORTRAN calling routines, AMPFOR, and the routine CXPLOT.

The facilities of GBRC3 along with GBRC1 and GBRC2 have been incorporated in a program package for ship vibration analysis, General Bending Response Program, which is in a testing stage.

II. ANALYSIS

The basic set of partial differential equations used for General Bending Response Code 3 were derived for a whirling shaft by Norman H. Jasper. The derivation is given in Section IV of reference 1. For the computer program, these equations were used in the form given in Section V of reference 1 and designated by [14].

As stated in reference 1, Section IV, the partial differential equations were derived for a shaft spinning with a uniform angular velocity, ω, about its longitudinal axis and having, simultaneously, a motion in two directions normal to the longitudinal shaft axis. The orientation of the shaft axis is shown below.



longitudinal axis of shaft

The set of partial differential equations for the whirling shaft is:

$$(2.1)\quad \frac{\partial V_y(x,t)}{\partial x} = -\mu(x)\frac{\partial^2 y(x,t)}{\partial t^2} - k_1(x)\,y(x,t) - c(x)\frac{\partial y(x,t)}{\partial t} + P_y(x,t)$$

$$(2.2)\quad \frac{\partial M_z(x,t)}{\partial x} = -V_y(x,t) - \tau(x)\omega\frac{\partial \alpha(x,t)}{\partial t} - \tau_d(x)\frac{\partial^2 \beta(x,t)}{\partial t^2} - - k_\beta(x)\,\beta(x,t) + Q_z(x,t)$$

$$(2.3)\quad \frac{\partial \beta(x,t)}{\partial x} = -\frac{M_z(x,t)}{EI(x)}$$

$$(2.4)\quad \frac{\partial y(x,t)}{\partial x} = \beta(x,t) - \frac{V_y(x,t)}{KAG(x)}$$

$$(2.5)\quad \frac{\partial V_z(x,t)}{\partial x} = -\frac{\mu(x)\partial^2 z(x,t)}{\partial t^2} - k_1(x)\,z(x,t) - c(x)\frac{\partial z(x,t)}{\partial t} + P_z(x,t)$$

$$(2.6)\quad \frac{\partial M_y(x,t)}{\partial x} = -V_z(x,t) + \tau(x)\omega\frac{\partial \beta(x,t)}{\partial t} - \tau_d(x)\frac{\partial^2 \alpha(x,t)}{\partial t^2} - k_\alpha(x)\,\alpha(x,t) + Q_y(x,t)$$

$$(2.7)\quad \frac{\partial \alpha(x,t)}{\partial x} = -\frac{M_y(x,t)}{EI(x)}$$

$$(2.8)\quad \frac{\partial z(x,t)}{\partial x} = \alpha(x,t) - \frac{V_z(x,t)}{KAG(x)}$$

Equations (2.1) through (2.4) are for the xy-plane and Equations (2.5) through (2.8) are for the xz-plane.

The notation for the equations is:

x	distance in the longitudinal direction measured from the origin of coordinates
t	time variable
y	displacement normal to x in the xy-plane of bending
z	displacement normal to x in the xz-plane of bending
α	slope of the projection of the normal to the shaft section in the xz-plane
β	slope of the projection of the normal to the shaft section in the xy-plane
V_y	shearing force in the direction of flexural vibration in the y-direction
V_z	shearing force in the direction of flexural vibration in the z-direction
M_z	bending moment relative to the z-axis
M_y	bending moment relative to the y-axis
μ	mass per unit length
τ_d	diametrical mass moment of inertia per unit length
KAG	shear rigidity
EI	bending rigidity
τ	polar mass moment of inertia per unit length
k_1	linear spring constant per unit length
k_β	rotatory spring constant per unit length acting in the xy-plane
k_α	rotatory spring constant per unit length acting in the xz-plane
c	damping coefficient
P_y	external force per unit length (excluding spring forces) in the y-direction
P_z	same as P_y except in the z-direction

Q_y	external moment per unit length relative to the y-axis
Q_z	external moment per unit length relative to the z-axis
ω	angular spin velocity about the longitudinal axis of the shaft

The departure from Jasper's notation for shear rigidity, mass per unit length and external moments was made to preserve consistency with the GBRC series notation.

The viscous damping term does not appear in the basic differential equations in reference 1.

The partial differential equations (2.1) through (2.8) are transformed to a set of differential equations in terms of the natural angular whirling frequency, Ω, by making the substitution $\omega = h\Omega$ and assuming that all time-dependent quantities are simple harmonic functions of time. Then

$$(2.9)\qquad \begin{array}{ll} M(x,t) = M(x)\, e^{i\Omega t} & \alpha(x,t) = \alpha(x) e^{i\Omega t} \\ y(x,t) = y(x)\, e^{i\Omega t} & \beta(x,t) = \beta(x) e^{i\Omega t} \\ V(x,t) = V(x)\, e^{i\Omega t} & z(x,t) = z(x) e^{i\Omega t} \\ P(x,t) = P(x)\, e^{i\Omega t} & Q(x,t) = Q(x)\, e^{i\Omega t}. \end{array}$$

Substituting (2.9) into Equations (2.1) through (2.8) performing differentiations, and dividing out $e^{i\Omega t}$ gives the set of differential equations,

$$\frac{dV_y(x)}{dx} = \mu(x)\Omega^2 y(x) - [k_1(x)]\,y(x) - i\Omega c(x)y(x) + P_y(x) \tag{2.10}$$

$$\frac{dM_z(x)}{dx} = -V_y(x) - i\tau(x)h\Omega^2\alpha(x) + \tau_d\Omega^2\beta(x) - [k_\beta(x)]\,\beta(x) + Q_z(x) \tag{2.11}$$

$$\frac{d\beta(x)}{dx} = -\frac{M_z(x)}{EI\ (x)} \tag{2.12}$$

$$\frac{dy(x)}{dx} = \beta(x) - \frac{V_y(x)}{KAG\ (x)} \tag{2.13}$$

$$\frac{dV_z(x)}{dx} = \mu(x)\Omega^2 z(x) - [k_1(x)]\,z(x) - i\Omega c(x)z(x) + P_z(x) \tag{2.14}$$

$$\frac{dM_y(x)}{dx} = -V_z(x) + i\tau(x)h\Omega^2\beta(x) + \tau_d(x)\Omega^2\alpha(x) - [k_\alpha(x)] + Q_y(x) \tag{2.15}$$

$$\frac{d\alpha(x)}{dx} = -\frac{M_y(x)}{EI(x)} \tag{2.16}$$

$$\frac{dz(x)}{dx} = \alpha(x) - \frac{V_z(x)}{KAG(x)} \quad . \tag{2.17}$$

Eliminating V_y between Equations (2.11) and (2.13) gives

$$\frac{dM_z(x)}{dx} - KAG(x)\,\frac{dy(x)}{dx} = -\,i\tau(x)h\Omega^2\alpha(x) + [\tau_d(x)\Omega^2 - KAG(x) - k_\beta(x)]\cdot \beta(x) + Q_z(x) \quad . \tag{2.18}$$

Eliminating V_z between Equations (2.15) and (2.17) gives

$$\frac{dM_y(x)}{dx} - KAG(x)\,\frac{dz(x)}{dx} = [\tau_d(x)\Omega^2 - KAG(x) - k_\alpha(x)]\,\alpha(x) + i\tau(x)h\Omega^2\beta(x) + Q_y(x) \quad . \tag{2.19}$$

Solving Equations (2.18) and (2.19) for $\alpha(x)$ and $\beta(x)$,

$$\alpha(x) = \left\{ - \frac{1}{\tau_d(x)\Omega^2 - KAG(x) - k_\beta(x)} \left[\frac{dM_z(x)}{dx} - KAG(x)\,\frac{dy(x)}{dx} - Q_z(x) \right] + \frac{1}{i\tau(x)h\Omega^2} \left[\frac{dM_y(x)}{dx} - KAG(x)\,\frac{dz(x)}{dx} - Q_y(x) \right] \right\} \left\{ \frac{i\tau(x)h\Omega^2}{\tau_d(x)\Omega^2 - KAG(x) - k_\beta(x)} + \frac{\tau_d(x)\Omega^2 - KAG(x) - k_\alpha(x)}{i\tau(x)h\Omega^2} \right\}^{-1}$$

or

$$(2.20) \qquad \alpha(x) = \frac{i\tau(x)h\Omega^2}{(\tau_d(x)\Omega^2 - KAG(x) - k_\beta(x))(\tau_d(x)\Omega^2 - KAG(x) - k_\alpha(x)) - (\tau(x)h\Omega^2)^2} \left[- \frac{dM_z(x)}{dx} + KAG(x)\,\frac{dy(x)}{dx} + Q_z(x) \right]$$

$$+ \frac{\tau_d(x)\Omega^2 - KAG(x) - k_\beta(x)}{(\tau_d(x)\Omega^2 - KAG(x) - k_\beta(x))(\tau_d(x)\Omega^2 - KAG(x) - k_\alpha(x)) - (\tau(x)h\Omega^2)^2} \left[\frac{dM_y(x)}{dx} - KAG(x)\,\frac{dz(x)}{dx} - Q_y(x) \right],$$

and

$$\beta(x) = \left\{ -\frac{1}{i\tau(x)h\Omega^2}\left[KAG(x)\,\frac{dy(x)}{dx} - \frac{dM_z(x)}{dx} + Q_z(x)\right]\right.$$

$$\left. + \frac{1}{(\tau_d(x)\Omega^2-KAG(x)-k_\alpha(x))}\left[\frac{dM_y(x)}{dx} - KAG(x)\,\frac{dz(x)}{dx} - Q_y(x)\right]\right\}$$

$$\left\{\frac{\tau_d(x)\Omega^2-KAG(x)-k_\beta(x)}{i\tau(x)h\Omega^2} + \frac{i\tau(x)h\Omega^2}{(\tau_d(x)\Omega^2-KAG(x)-k_\alpha(x))}\right\}^{-1}$$

or

$$(2.21)\quad \beta(x) = -\frac{(\tau_d(x)\Omega^2-KAG(x)-k_\alpha(x))}{(\tau_d(x)\Omega^2-KAG(x)-k_\beta(x))(\tau_d(x)\Omega^2-KAG(x)-k_\alpha(x))-(\tau(x)h\Omega^2)^2}$$

$$\left[KAG(x)\,\frac{dy(x)}{dx} - \frac{dM_z(x)}{dx} + Q_z(x)\right]$$

$$+\frac{i\tau(x)h\Omega^2}{(\tau_d(x)\Omega^2-KAG(x)-k_\beta(x))(\tau_d(x)\Omega^2-KAG(x)-k_\alpha(x))-(\tau(x)h\Omega^2)^2}$$

$$\left[\frac{dM_y(x)}{dx} - KAG(x)\,\frac{dz(x)}{dx} - Q_y(x)\right].$$

The expressions for $\alpha(x)$ and $\beta(x)$ can be easier written by defining;

$$E \equiv \frac{\tau_d(x)\Omega^2-KAG(x)-k_\alpha(x)}{(\tau_d(x)\Omega^2-KAG(x)-k_\beta(x))(\tau_d(x)\Omega^2-KAG(x)-k_\alpha(x))-(\tau(x)h\Omega^2)^2}\ ;$$

$$B \equiv \frac{\tau_d(x)\Omega^2-KAG(x)-k_\beta(x)}{(\tau_d(x)\Omega^2-KAG(x)-k_\beta(x))(\tau_d(x)\Omega^2-KAG(x)-k_\alpha(x))-(\tau(x)h\Omega^2)^2}\ ;$$

and

$$C = \frac{i\tau(x)h\Omega^2}{(\tau_d(x)\Omega^2-KAG(x)-k_\beta(x))(\tau_d(x)\Omega^2-KAG(x)-k_\alpha(x))-(\tau(x)h\Omega^2)^2} .$$

Using these definitions, Equations (2.20) and (2.21) can be rewritten as

$$\alpha(x) = C\left[-\frac{dM_z(x)}{dx} + KAG(x)\,\frac{dy(x)}{dx} + Q_z(x)\right] + B\left[\frac{dM_y(x)}{dx} - KAG(x)\,\frac{dz(x)}{dx} - Q_y(x)\right] \tag{2.22}$$

$$\beta(x) = -E\left[KAG(x)\,\frac{dy(x)}{dx} - \frac{dM_z(x)}{dx} + Q_z(x)\right] + C\left[\frac{dM_y(x)}{dx} - KAG(x)\,\frac{dz(x)}{dx} - Q_y(x)\right] . \tag{2.23}$$

From Equation (2.13),

$$V_y(x) = -\,KAG(x)\left[\frac{dy(x)}{dx} - \beta(x)\right] .$$

Substituting (2.23) for $\beta(x)$

$$V_y(x) = -\,KAG(x)\left\{\frac{dy(x)}{dx} + E\left[KAG(x)\,\frac{dy(x)}{dx} - \frac{dM_z(x)}{dx} + Q_z(x)\right] - C\left[\frac{dM_y(x)}{dx} - KAG(x)\,\frac{dz(x)}{dx} - Q_y(x)\right]\right\} . \tag{2.24}$$

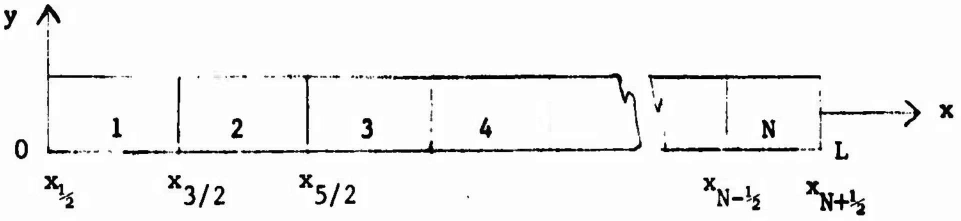
From Equation (2.17),

$$V_z(x) = KAG(x)\left[-\frac{dz(x)}{dx} + \alpha(x)\right] .$$

Substituting (2.22) for $\alpha(x)$,

$$(2.25) \quad V_z(x) = KAG(x)\left\{-\frac{dz(x)}{dx} + C\left[-\frac{dM_z(x)}{dx} + KAG(x)\frac{dy(x)}{dx} + Q_z(x)\right] + B\left[\frac{dM_y(x)}{dx} - KAG(x)\frac{dz(x)}{dx} - Q_y(x)\right]\right\}.$$

The shaft interval from $x = 0$ to $x = L$ is subdivided by abscissas $x_{\frac{1}{2}} = 0 < x_{3/2} < x_{5/2} < \dots < x_{N+\frac{1}{2}} = L$,



where L is the shaft length.

Equation (2.10) is then integrated from $x_{n-\frac{1}{2}}$ to $x_{n+\frac{1}{2}}$ for $n = 1, 2, 3, \dots, N$, giving

$$V_{y_{n+\frac{1}{2}}} - V_{y_{n-\frac{1}{2}}} = \int_{x_{n-\frac{1}{2}}}^{x_{n+\frac{1}{2}}} \left[\mu(x)\Omega^2 - (k_1)_y - i\Omega c_y(x)\right] y(x)dx + \int_{x_{n-\frac{1}{2}}}^{x_{n+\frac{1}{2}}} P_y(x)dx ,$$

which can be approximated by

$$(2.26) \quad V_{y_{n+\frac{1}{2}}} - V_{y_{n-\frac{1}{2}}} = \left[(\mu\Delta x)_n\Omega^2 - (k_y)_n - i(c_y\Delta x)_n\Omega\right] y_n + P_{y_n}$$

where

$$(\mu\Delta x)_n = \int_{x_{n-\frac{1}{2}}}^{x_{n+\frac{1}{2}}} \mu(x)dx$$

$$(k_y)_n = (k_{1_y}\Delta x)_n = \int_{x_{n-\frac{1}{2}}}^{x_{n+\frac{1}{2}}} (k_1)_y \, dx$$

$$(c_y \Delta x) = \int_{x_{n-\frac{1}{2}}}^{x_{n+\frac{1}{2}}} c_y(x)dx$$

$$P_{y_n} = (P_y \Delta x)_n = \int_{x_{n-\frac{1}{2}}}^{x_{n+\frac{1}{2}}} P_y(x)dx$$

From Equation (2.24), $V_{y_{n+\frac{1}{2}}}$ is approximated for $n = 1, 2, 3, \ldots, N-1$ by

$$(2.27) \quad V_{y_{n+\frac{1}{2}}} = -\left(\frac{KAG}{\Delta x}\right)_{n,n+1} \Bigg\{ y_{n+1} - y_n + E_{n,n+1} \left[KAG_{n,n+1}(y_{n+1}-y_n) - (M_{z_{n+1}} - M_{z_n}) + Q_{z_{n,n+1}} \right] - C_{n,n+1} \left[M_{y_{n+1}} - M_{y_n} - KAG_{n,n+1}(z_{n+1} - z_n) - Q_{y_{n,n+1}} \right] \Bigg\},$$

where

$$\left(\frac{\Delta x}{KAG}\right)_{n,n+1} = \int_{x_n}^{x_{n+1}} \frac{dx}{KAG(x)}$$

$$Q_{z_{n,n+1}} = \int_{x_n}^{x_{n+1}} Q_z(x)dx$$

$$Q_{y_{n,n+1}} = \int_{x_n}^{x_{n+1}} Q_y(x)dx \quad ,$$

$$E_{n,n+1} = \frac{\tau_{d_{n,n+1}}\Omega^2 - KAG_{n,n+1} - k_{\alpha_{n,n+1}}}{(\tau_{d_{n,n+1}}\Omega^2 - KAG_{n,n+1} - k_{\beta_{n,n+1}})(\tau_{d_{n,n+1}}\Omega^2 - KAG_{n,n+1} - k_{\alpha_{n,n+1}}) - (\tau_{n,n+1}h\Omega^2)^2}$$

$$C_{n,n+1} = \frac{i(\tau_{n,n+1}h\Omega^2)}{(\tau_{d_{n,n+1}}\Omega^2 - KAG_{n,n+1} - k_{\beta_{n,n+1}})(\tau_{d_{n,n+1}}\Omega^2 - KAG_{n,n+1} - k_{\alpha_{n,n+1}}) - (\tau_{n,n+1}h\Omega^2)^2}$$

with

$$(\tau_d \Delta x)_{n,n+1} = \int_{x_n}^{x_{n+1}} \tau_d dx$$

$$(\tau \Delta x)_{n,n+1} = \int_{x_n}^{x_{n+1}} \tau dx$$

$$(k_\alpha \Delta x)_{n,n+1} = \int_{x_n}^{x_{n+1}} k_\alpha dx$$

$$(k_\beta \Delta x)_{n,n+1} = \int_{x_n}^{x_{n+1}} k_\beta dx$$

and

$$x_n = \tfrac{1}{2}\,(x_{n+\frac{1}{2}} + x_{n-\frac{1}{2}}) \ .$$

Substituting (2.27) into (2.26) gives

$$\begin{aligned}
(2.28)\qquad & -\left(\frac{KAG}{\Delta x}\right)_{n,n+1}\Bigg\{ y_{n+1} - y_n + E_{n,n+1}\Big[KAG_{n,n+1}\,(y_{n+1} - y_n) - \\
& \qquad (M_{z_{n+1}} - M_{z_n}) + Q_{z_{n,n+1}}\Big] \\
& \qquad -C_{n,n+1}\Big[M_{y_{n+1}} - M_{y_n} - KAG_{n,n+1}\,(z_{n+1} - z_n) - Q_{y_{n,n+1}}\Big]\Bigg\} \\
& +\left(\frac{KAG}{\Delta x}\right)_{n-1,n}\Bigg\{ y_n - y_{n-1} + E_{n-1,n}\Big[KAG_{n-1,n}\,(y_n - y_{n-1}) - \\
& \qquad (M_{z_n} - M_{z_{n-1}}) + Q_{z_{n-1,n}}\Big] \\
& \qquad -C_{n-1,n}\Big[M_{y_n} - M_{y_{n-1}} - KAG_{n-1,n}\,(z_n - z_{n-1}) - Q_{y_{n-1,n}}\Big]\Bigg\} \\
& \qquad = \Big[(\mu\Delta x)_n\Omega^2 - k_{y_n} - i(c_y\Delta x)_n\Omega\Big]\, y_n + P_{y_n}\,,
\end{aligned}$$

for n = 1, 2, ..., N-1.

Similarly, Equation (2.14) is integrated from $x_{n-\frac{1}{2}}$ to $x_{n+\frac{1}{2}}$ for $n = 1, 2, 3, \ldots, N-1$ giving

$$\begin{aligned}
V_{z_{n+\frac{1}{2}}} - V_{z_{n-\frac{1}{2}}} &= \int_{x_{n-\frac{1}{2}}}^{x_{n+\frac{1}{2}}} \Big[\mu(x)\Omega^2 - (k_1)_z - i\Omega c_z(x)\Big]\, z(x)\, dx \\
&\quad + \int_{x_{n-\frac{1}{2}}}^{x_{n+\frac{1}{2}}} P_z(x)dx\,,
\end{aligned}$$

which is then approximated by

$$(2.29) \qquad V_{z_{n+\frac{1}{2}}} - V_{z_{n-\frac{1}{2}}} = \left[(\mu\Delta x)_n\Omega^2 - (k_z)_n - i(c_z\Delta x)_n\Omega\right]z_n + P_{z_n} ,$$

where

$$(k_z)_n = (k_{1_z}\Delta x)_n = \int_{x_{n-\frac{1}{2}}}^{x_{n+\frac{1}{2}}} (k_1)_z \, dx$$

$$(c_z\Delta x)_n = \int_{x_{n-\frac{1}{2}}}^{x_{n+\frac{1}{2}}} c_z(x)dx$$

and

$$P_{z_n} = (P_z\Delta x)_n = \int_{x_{n-\frac{1}{2}}}^{x_{n+\frac{1}{2}}} P_z(x)dx.$$

From Equation (2.25), $V_{z_{n+\frac{1}{2}}}$ can be approximated for n = 1, 2, 3, ..., N-1 by

$$(2.30) \qquad V_{z_{n+\frac{1}{2}}} = \left(\frac{KAG}{\Delta x}\right)_{n,n+1} \Bigg\{ -(z_{n+1} - z_n) + C_{n,n+1}\Big[-(M_{z_{n+1}} - M_{z_n}) + KAG_{n,n+1}(y_{n+1} - y_n) + Q_{z_{n,n+1}}\Big] + B_{n,n+1}\Big[M_{y_{n+1}} - M_{y_n} - KAG_{n,n+1}(z_{n+1} - z_n) - Q_{y_{n,n+1}}\Big]\Bigg\},$$

where

$$B_{n,n+1} = \frac{\tau_{d_{n,n+1}}\Omega^2 - KAG_{n,n+1} - k_{\beta_{n,n+1}}}{(\tau_{d_{n,n+1}}\Omega^2 - KAG_{n,n+1} - k_{\beta_{n,n+1}})(\tau_{d_{n,n+1}}\Omega^2 - KAG_{n,n+1} - k_{\alpha_{n,n+1}}) - (\tau_{n,n+1}h\Omega^2)^2}$$

Substitution of (2.30) into (2.29) gives for n = 1, 2, ..., N-1

$$
\begin{aligned}
(2.31)\quad & \left(\frac{KAG}{\Delta x}\right)_{n,n+1} \Bigg\{ -(z_{n+1} - z_n) + C_{n,n+1}\Big[-(M_{z_{n+1}} - M_{z_n}) + KAG_{n,n+1} \\
& \qquad\qquad (y_{n+1} - y_n) + Q_{z_{n,n+1}}\Big] \\
& \quad + B_{n,n+1}\Big[M_{y_{n+1}} - M_{y_n} - KAG_{n,n+1}(z_{n+1} - z_n) - Q_{y_{n,n+1}}\Big]\Bigg\} \\
& -\left(\frac{KAG}{\Delta x}\right)_{n-1,n} \Bigg\{ -(z_n - z_{n-1}) + C_{n-1,n}\Big[-(M_{z_n} - M_{z_{n-1}}) + KAG_{n-1,n} \\
& \qquad\qquad (y_n - y_{n-1}) + Q_{z_{n-1,n}}\Big] \\
& \quad + B_{n-1,n}\Big[M_{y_n} - M_{y_{n-1}} - KAG_{n-1,n}\,(z_n - z_{n-1}) - Q_{y_{n-1,n}}\Big]\Bigg\} \\
& = \Big[(\mu\Delta x)_n\Omega^2 - k_{z_n} - i(c_z\Delta x)_n\Omega\Big] z_n + P_{z_n} \; .
\end{aligned}
$$

Integrating Equation (2.16) from $x_{n-\frac{1}{2}}$ to $x_{n+\frac{1}{2}}$ for n = 1,2,...,N,

$$
\alpha_{n+\frac{1}{2}} - \alpha_{n-\frac{1}{2}} = -\int_{x_{n-\frac{1}{2}}}^{x_{n+\frac{1}{2}}} \frac{M_y(x)}{EI(x)}\, dx \; ,
$$

and approximating the integral on the right side gives

$$(2.32) \qquad \alpha_{n+\frac{1}{2}} - \alpha_{n-\frac{1}{2}} = -\left(\frac{\Delta x}{EI}\right)_n M_{y_n}$$

where

$$\left(\frac{\Delta x}{EI}\right)_n = \int_{x_{n-\frac{1}{2}}}^{x_{n+\frac{1}{2}}} \frac{dx}{EI(x)} \quad .$$

Approximate $\alpha_{n+\frac{1}{2}}$ for $n = 1, 2, \ldots, N-1$ using (2.22),

$$(2.33) \quad \alpha_{n+\frac{1}{2}} = \left(\frac{1}{\Delta x}\right)_{n,n+1} \Bigg\{ C_{n,n+1}\Big[-(M_{z_{n+1}} - M_{z_n}) + KAG_{n,n+1} (y_{n+1} - y_n) + Q_{z_{\;n,n+1}}\Big] + B_{n,n+1}\Big[(M_{y_{n+1}} - M_{y_n}) - KAG_{n,n+1}(z_{n+1} - z_n) - Q_{y_{\;n,n+1}}\Big]\Bigg\} .$$

Substituting (2.33) into (2.32) gives for $n = 1,2,\ldots,N-1$

$$(2.34) \quad \left(\frac{1}{\Delta x}\right)_{n,n+1} \Bigg\{ C_{n,n+1}\Big[-(M_{z_{n+1}} - M_{z_n}) + KAG_{n,n+1} (y_{n+1}-y_n) + Q_{z_{\;n,n+1}}\Big] + B_{n,n+1}\Big[M_{y_{n+1}} - M_{y_n} - KAG_{n,n+1} (z_{n+1}-z_n) - Q_{y_{\;n,n+1}}\Big]\Bigg\}$$

$$-\left(\frac{1}{\Delta x}\right)_{n-1,n} \Bigg\{ C_{n-1,n}\Big[-(M_{z_n} - M_{z_{n-1}}) + KAG_{n-1,n} (y_n - y_{n-1}) + Q_{z_{\;n-1,n}}\Big] + B_{n-1,n}\Big[M_{y_n} - M_{y_{n-1}} - KAG_{n-1,n}(z_n - z_{n-1}) - Q_{y_{\;n-1,n}}\Big]\Bigg\}$$

$$= -\left(\frac{\Delta x}{EI}\right)_n M_{y_n} .$$

Integrating Equation (2.12) from $x_{n-\frac{1}{2}}$ to $x_{n+\frac{1}{2}}$ for $n = 1, 2, \ldots, N$,

$$\beta_{n+\frac{1}{2}} - \beta_{n-\frac{1}{2}} = -\int_{x_{n-\frac{1}{2}}}^{x_{n+\frac{1}{2}}} \frac{M_z(x)}{EI(x)}\, dx ,$$

and approximating the right side, one has

$$\beta_{n+\frac{1}{2}} - \beta_{n-\frac{1}{2}} = -\left(\frac{\Delta x}{EI}\right)_n M_{z_n} . \tag{2.35}$$

Approximating $\beta_{n+\frac{1}{2}}$ with (2.23) for $n = 1, 2, \ldots, N$,

$$\begin{aligned}\beta_{n+\frac{1}{2}} = -\left(\frac{1}{\Delta x}\right)_{n,n+1} \Bigg\{ & E_{n,n+1}\Big[KAG_{n,n+1}\,(y_{n+1} - y_n) - (M_{z_{n+1}} - M_z) \\ & \qquad + Q_{z\,n,n+1}\Big] \\ & - C_{n,n+1}\Big[M_{y_{n+1}} - M_{y_n} - KAG_{n,n+1}\,(z_{n+1} - z_n) - Q_{y\,n,n+1}\Big]\Bigg\},\end{aligned} \tag{2.36}$$

and substituting into (2.35) gives for $n = 1, 2, \ldots, N-1$,

$$\begin{aligned} & -\left(\frac{1}{\Delta x}\right)_{n,n+1} \Bigg\{ E_{n,n+1}\Big[KAG_{n,n+1}\,(y_{n+1} - y_n) - (M_{z_{n+1}} - M_z) + Q_{z\,n,n+1}\Big] \\ & \qquad - C_{n,n+1}\Big[(M_{y_{n+1}} - M_{y_n}) - KAG_{n,n+1}\,(z_{n+1} - z_n) - Q_{y\,n,n+1}\Big]\Bigg\} \\ & + \left(\frac{1}{\Delta x}\right)_{n-1,n} \Bigg\{ E_{n-1,n}\Big[KAG_{n-1,n}\,(y_n - y_{n-1}) - (M_{z_n} - M_{z_{n-1}}) + Q_{z\,n-1,n}\Big] \\ & \qquad - C_{n-1,n}\Big[(M_{y_n} - M_{y_{n-1}}) - KAG_{n-1,n}\,(z_n - z_{n-1}) - Q_{y\,n-1,n}\Big]\Bigg\} \\ & \qquad\qquad = -\left(\frac{\Delta x}{EI}\right)_n M_{z_n} .\end{aligned} \tag{2.37}$$

End Conditions. The conditions required at the ends of the shaft determine the values of

$$V_{y_{1\frac{1}{2}}},\ V_{z_{1\frac{1}{2}}},\ \alpha_{1\frac{1}{2}},\ \beta_{1\frac{1}{2}},\text{ and }V_{y_{N+\frac{1}{2}}},\ V_{z_{N+\frac{1}{2}}},\ \alpha_{N+\frac{1}{2}},\text{ and }\beta_{N+\frac{1}{2}}.$$

For free ends, the values imposed on V are

$$V_{y_{1\frac{1}{2}}} = V_{z_{1\frac{1}{2}}} = 0 \quad \text{for } V_y(0) = V_z(0) = 0$$

(2.38) and

$$V_{y_{N+\frac{1}{2}}} = V_{z_{N+\frac{1}{2}}} = 0 \quad \text{for } V_y(L) = V_z(L) = 0.$$

Since for a free end both shear and moment are 0, the following values are assigned to M,

$$M_{y_1} = M_{z_1} = 0$$

(2.39) and

$$M_{y_L} = M_{z_L} = 0\ .$$

If, on the other hand, the ends are fixed the conditions imposed on α and β are

$$\alpha_{1\frac{1}{2}} = \beta_{1\frac{1}{2}} = 0 \quad \text{for } \alpha(0) = \beta(0) = 0$$

(2.40) and

$$\alpha_{N+\frac{1}{2}} = \beta_{N+\frac{1}{2}} = 0 \quad \text{for } \alpha(L) = \beta(L) = 0.$$

In addition the values for y and z at the fixed end are

$$y_{1\frac{1}{2}} = z_{1\frac{1}{2}} = 0 \quad \text{for } y(0) = z(0) = 0$$

(2.41) and

$$y_N = z_N = 0 \quad \text{for } y(L) = z(L) = 0\ .$$

To arrange Equations (2.28), (2.31), (2.34) and (2.37) in a convenient form for solution the following is done:

a) Redefine $E_{n,n+1}$, $B_{n,n+1}$ and $C_{n,n+1}$,

$$E_{n,n+1} \equiv \left(\frac{1}{\Delta x}\right)_{n,n+1} E_{n,n+1}$$

$$B_{n,n+1} \equiv \left(\frac{1}{\Delta x}\right)_{n,n+1} B_{n,n+1}$$

$$C_{n,n+1} \equiv \left(\frac{1}{\Delta x}\right)_{n,n+1} C_{n,n+1} ,$$

and

$$E'_{n,n+1} = KAG_{n,n+1} \cdot E_{n,n+1}$$

$$B'_{n,n+1} = KAG_{n,n+1} \cdot B_{n,n+1}$$

$$C'_{n,n+1} = KAG_{n,n+1} \cdot C_{n,n+1}$$

$$E''_{n,n+1} = KAG^2_{n,n+1} \cdot E_{n,n+1}$$

$$B''_{n,n+1} = KAG^2_{n,n+1} \cdot B_{n,n+1}$$

$$C''_{n,n+1} = KAG^2_{n,n+1} \cdot C_{n,n+1}$$

$$L_{n,n+1} = \left(\frac{\Delta x}{KAG}\right)^{-1}_{n,n+1}$$

$$\zeta_n = \left(\frac{\Delta x}{EI}\right)_n$$

$$\delta'_{y_n} = \left[(\mu\Delta x)_n\Omega^2 - (k_y\Delta x)_n - i(c_y\Delta x)_n\Omega\right]$$

$$\delta'_{z_n} = \left[(\mu\Delta x)_n\Omega^2 - (k_z\Delta x)_n - i(c_z\Delta x)_n\Omega\right] .$$

b) Multiply Equations (2.28) and (2.34) by -1.

With a) and b) the final system of equations is, after collecting terms for the coefficients,

$$
\begin{aligned}
&(L_{n-1,n} + E''_{n-1,n})\, y_{n-1} - E'_{n-1,n}\, M_{z_{n-1}} + C''_{n-1,n}\, z_{n-1} - C'_{n-1,n}\, M_{y_{n-1}} \\
&\quad -(L_{n-1,n} + L_{n,n+1} + E''_{n-1,n} + E''_{n,n+1} - \delta'_{y_n})\, y_n + (E'_{n-1,n} + E'_{n,n+1})\, M_{z_n} \\
&\qquad - (C''_{n-1,n} + C''_{n,n+1})\, z_n \\
&\quad + (C'_{n-1,n} + C'_{n,n+1})\, M_{y_n} + (L_{n,n+1} + E''_{n,n+1})\, y_{n+1} - E'_{n,n+1}\, M_{z_{n+1}} \\
&\qquad + C''_{n,n+1}\, z_{n+1} - C'_{n,n+1}\, M_{y_{n+1}} \\
&\quad = E'_{n-1,n}\, Q_{z_{n-1,n}} - E'_{n,n+1}\, Q_{z_{n,n+1}} + C'_{n-1,n}\, Q_{y_{n-1,n}} - C'_{n,n+1}\, Q_{y_{n,n+1}} - P_{y_n}
\end{aligned}
\tag{2.42}
$$

$$
\begin{aligned}
&-E'_{n-1,n}\, y_{n-1} + E_{n-1,n}\, M_{z_{n-1}} - C'_{n-1,n}\, z_{n-1} + C_{n-1,n}\, M_{y_{n-1}} \\
&\qquad + (E'_{n-1,n} + E'_{n,n+1})\, y_n \\
&\quad - (E_{n-1,n} + E_{n,n+1} - \zeta_n)\, M_{z_n} + (C'_{n-1,n} + C'_{n,n+1})\, z_n - (C_{n-1,n} + C_{n,n+1})\, M_{y_n} \\
&\quad - E'_{n,n+1}\, y_{n+1} + E_{n,n+1}\, M_{z_{n+1}} - C'_{n,n+1}\, z_{n+1} + C_{n,n+1}\, M_{y_{n+1}} \\
&\quad = - E_{n-1,n}\, Q_{z_{n-1,n}} + E_{n,n+1}\, Q_{z_{n,n+1}} - C_{n-1,n}\, Q_{y_{n-1,n}} + C_{n,n+1}\, Q_{y_{n,n+1}}
\end{aligned}
\tag{2.43}
$$

$$\begin{aligned}(2.44)\quad & C''_{n-1,n}\, y_{n-1} - C'_{n-1,n}\, M_{z_{n-1}} - (L_{n-1,n} + B''_{n-1,n})\, z_{n-1} + B'_{n-1,n}\, M_{y_{n-1}} \\ & \qquad - (C''_{n-1,n} + C''_{n,n+1})\, y_n \\ & + (C'_{n-1,n} + C'_{n,n+1})\, M_{z_n} + (L_{n-1,n} + L_{n,n+1} + B''_{n-1,n} + B''_{n,n+1} - \delta'_{z_n})\, z_n \\ & - (B'_{n-1,n} + B'_{n,n+1})\, M_{y_n} + C''_{n,n+1}\, y_{n+1} - C'_{n,n+1}\, M_{z_{n+1}} - (L_{n,n+1} + B''_{n,n+1}) z_{n+1} \\ & + B'_{n,n+1}\, M_{y_{n+1}} \\ & = C'_{n-1,n}\, Q_{z_{n-1,n}} - C'_{n,n+1}\, Q_{z_{n,n+1}} - B'_{n-1,n}\, Q_{y_{n-1,n}} + B'_{n,n+1}\, Q_{y_{n,n+1}} + P_{z_n}\end{aligned}$$

$$\begin{aligned}(2.45)\quad & - C'_{n-1,n}\, y_{n-1} + C_{n-1,n}\, M_{z_{n-1}} + B'_{n-1,n}\, z_{n-1} - B_{n-1,n}\, M_{y_{n-1}} \\ & + (C'_{n-1,n} + C'_{n,n+1})\, y_n - (C_{n-1,n} + C_{n,n+1})\, M_{z_n} \\ & - (B'_{n-1,n} + B'_{n,n+1})\, z_n + (B_{n-1,n} + B_{n,n+1} - \zeta_n)\, M_{y_n} - C'_{n,n+1}\, y_{n+1} \\ & + C_{n,n+1}\, M_{z_{n+1}} + B'_{n,n+1}\, z_{n+1} - B_{n,n+1}\, M_{y_{n+1}} \\ & = - C_{n-1,n}\, Q_{z_{n-1,n}} + C_{n,n+1}\, Q_{z_{n,n+1}} + B_{n-1,n}\, Q_{y_{n-1,n}} - B_{n,n+1}\, Q_{y_{n,n+1}} .\end{aligned}$$

These equations come respectively from Equations (2.28), (2.37), (2.31) and (2.34). For $n = 1$ and $n = N$, the end conditions as specified by the data will be included in the system of equations.

The matrix formulation of Equations (2.42) through (2.45) is

(2.46) $$A\vec{z} = \vec{P}$$

as indicated in (2.20), (2.21) and (2.22) of reference 2 with the following definitions for the subvectors $\vec{z}_n$ of $\vec{z}$ and $\vec{P}_n$ of $\vec{P}$, and the submatrices $A_{i,j}$ of the matrix A:

(2.47) $$\vec{z}_n = \left| \begin{matrix} y_n \\ M_{z_n} \\ z_n \\ M_{y_n} \end{matrix} \right| , \quad \vec{P}_n = \left| \begin{matrix} \tilde{P}_n \\ \tilde{P}'_n \\ \tilde{P}''_n \\ \tilde{P}'''_n \end{matrix} \right|$$

with $$\tilde{P}_n = - P_{y_n} + E'_{n-1,n}\, Q_{z_{n-1,n}} - E'_{n,n+1}\, Q_{z_{n,n+1}} + C'_{n-1,n}\, Q_{y_{n-1,n}} - C'_{n,n+1}\, Q_{y_{n,n+1}}$$

$$\tilde{P}'_n = - E_{n-1,n}\, Q_{z_{n-1,n}} + E_{n,n+1}\, Q_{z_{n,n+1}} - C_{n-1,n}\, Q_{y_{n-1,n}} + C_{n,n+1}\, Q_{y_{n,n+1}}$$

$$\tilde{P}''_n = P_{z_n} + C'_{n-1,n}\, Q_{z_{n-1,n}} - C'_{n,n+1}\, Q_{z_{n,n+1}} - B'_{n-1,n}\, Q_{y_{n-1,n}} + B'_{n,n+1}\, Q_{y_{n,n+1}}$$

$$\tilde{P}'''_n = - C_{n-1,n}\, Q_{z_{n-1,n}} + C_{n,n+1}\, Q_{z_{n,n+1}} + B_{n-1,n}\, Q_{y_{n-1,n}} - B_{n,n+1}\, Q_{y_{n,n+1}} ,$$

and for n = 2, 3, ..., N-1 .

$$A_{n,n} = \begin{vmatrix} -(L_{n-1,n}+L_{n,n+1}+E''_{n-1,n}+E''_{n,n+1}-\delta'_{y_n}) & (E'_{n-1,n}+E'_{n,n+1}) & -(C''_{n-1,n}+C''_{n,n+1}) & (C'_{n-1,n}+C'_{n,n+1}) \\ (E'_{n-1,n}+E'_{n,n+1}) & -(E_{n-1,n}+E_{n,n+1}-\zeta_n) & (C'_{n-1,n}+C'_{n,n+1}) & -(C_{n-1,n}+C_{n,n+1}) \\ -(C''_{n-1,n}+C''_{n,n+1}) & (C'_{n-1,n}+C'_{n,n+1}) & (L_{n-1,n}+L_{n,n+1}+B''_{n-1,n}+B''_{n,n+1}-\delta'_{z_n}) & -(B'_{n-1,n}+B'_{n,n+1}) \\ (C'_{n-1,n}+C'_{n,n+1}) & -(C_{n-1,n}+C_{n,n+1}) & -(B'_{n-1,n}+B'_{n,n+1}) & (B_{n-1,n}+B_{n,n+1}-\zeta_n) \end{vmatrix}$$

$$A_{n,n+1} = \begin{vmatrix} (L_{n,n+1}+E''_{n,n+1}) & -E'_{n,n+1} & C''_{n,n+1} & -C'_{n,n+1} \\ -E'_{n,n+1} & E_{n,n+1} & -C'_{n,n+1} & C_{n,n+1} \\ C''_{n,n+1} & -C'_{n,n+1} & -(L_{n,n+1}+B''_{n,n+1}) & B'_{n,n+1} \\ -C'_{n,n+1} & C_{n,n+1} & B'_{n,n+1} & -B_{n,n+1} \end{vmatrix}$$

$$A_{n-1,n} = \begin{vmatrix} (L_{n-1,n}+E''_{n-1,n}) & -E'_{n-1,n} & C''_{n-1,n} & -C'_{n-1,n} \\ -E'_{n-1,n} & E_{n-1,n} & -C'_{n-1,n} & C_{n-1,n} \\ C''_{n-1,n} & -C'_{n-1,n} & -(L_{n-1,n}+B''_{n-1,n}) & B'_{n-1,n} \\ -C'_{n-1,n} & C_{n-1,n} & B'_{n-1,n} & -B_{n-1,n} \end{vmatrix} = A_{n,n-1}$$

and for n = 1 and n = N,

$$A_{1,1} = \begin{vmatrix} (L_{1,2} + E''_{1,2} - \delta'_{y_1}) & E'_{1,2} & -C''_{1,2} & C'_{1,2} \\ E'_{1,2} & -(E_{1,2}-\zeta_1) & C'_{1,2} & -C_{1,2} \\ -C''_{1,2} & C'_{1,2} & (L_{1,2}+B''_{1,2}-\delta'_{z_1}) & -B'_{1,2} \\ C'_{1,2} & -C_{1,2} & -B'_{1,2} & (B_{1,2}-\zeta_1) \end{vmatrix}$$

$$A_{N,N} = \begin{vmatrix} -(L_{N-1,N}+E''_{N-1,N}-\delta'_{y_N}) & E'_{N-1,N} & -C''_{N-1,N} & C'_{N-1,N} \\ E'_{N-1,N} & -(E_{N-1,N}-\zeta_N) & C'_{N-1,N} & -C_{N-1,N} \\ -C''_{N-1,N} & C'_{N-1,N} & (L_{N-1,N}+B_{N-1,N}-\delta'_{z_N}) & -B'_{N-1,N} \\ C'_{N-1,N} & -C_{N-1,N} & -B'_{N-1,N} & (B_{N-1,N}-\zeta_N) \end{vmatrix}$$

If the ends are free, ζ_1 and ζ_N are chosen sufficiently large that M_1 and M_N become zero for practical purposes. If the ends are fixed, the condition above on moment is released and a sufficiently large value is given to k_{y_1}, k_{z_1} and k_{y_N}, k_{z_N} in the expressions

$$\delta'_{y_1} = \left[(\mu\Delta x)_1\Omega^2 - k_{y_1} - ic_{y_1}\Omega\right],$$

$$\delta'_{z_1} = \left[(\mu\Delta x)_1 - k_{z_1} - ic_{z_1}\Omega\right] \qquad \text{and}$$

$$\delta'_{y_N} = \left[(\mu\Delta x)_N\Omega^2 - k_{y_N} - ic_{y_N}\Omega\right],$$

$$\delta'_{z_N} = \left[(\mu\Delta x)_N - k_{z_N} - ic_{z_N}\Omega\right] \quad \text{so that } y_1,\ z_1 \text{ and } y_N,\ z_N$$

are similarly forced close to 0.

The generalization of this matrix formulation to include more than one shaft coupled elastically proceeds in the way described on pages 10-12 of reference 2. The submatrices $A_{n,m}$ of the generalized A-matrix are set up according to the various types of elastic connections allowed between sections of the shaft(s). If a section numbered n is connected rigidly to a section numbered m the submatrix generated is

$$A_{n,m} = \begin{vmatrix} (L_{n,m}+E''_{n,m}) & -E'_{n,m} & C''_{n,m} & -C'_{n,m} \\ -E'_{n,m} & E_{n,m} & -C'_{n,m} & C_{n,m} \\ C''_{n,m} & -C'_{n,m} & -(L_{n,m}+B''_{n,m}) & B'_{n,m} \\ -C'_{n,m} & C_{n,m} & B'_{n,m} & -B_{n,m} \end{vmatrix} .$$

For a spring connection between sections n and m, one has,

$$A_{n,m}=\begin{vmatrix} (k_y)_{n,m}+i\Omega(c_y)_{n,m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -(k_z)_{n,m}-i\Omega(c_z)_{n,m} & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix},$$

where $(k_y)_{n,m}$ and $(k_z)_{n,m}$ are spring constants in the y and z directions respectively and $(c_y)_{n,m}$ and $(c_z)_{n,m}$ are the associated damping coefficients.

The diagonal submatrices are given by

$$A_{n,n}=-\sum_{\substack{m=1\\ m\neq n}}^{N} A_{n,m}+D_n ,$$

with

$$D_n=\begin{vmatrix} \delta'_{y_n} & 0 & 0 & 0 \\ 0 & \zeta_n & 0 & 0 \\ 0 & 0 & -\delta'_{z_n} & 0 \\ 0 & 0 & 0 & -\zeta_n \end{vmatrix}.$$

When a section n is connected through a spring to ground, one adds to $A_{n,n}$,

$$\begin{vmatrix} -(k_y)_n-i(c_y)_n\Omega & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & (k_z)_n+i(c_z)_n\Omega & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

where $(k_y)_n$ and $(k_z)_n$ are the spring constants for the connection and $(c_y)_n$ and $(c_z)_n$ are the associated damping coefficients.

III. Input Preparation and Definition of Parameters

The parameters which are specified for whirling calculations with GBRC3 are:

$(\mu\Delta x)_n$

$\left(\frac{\Delta x}{EI}\right)_n$

$(\Delta x)_{\frac{n,n+1}{n,m}}$

$\left(\frac{\Delta x}{KAG}\right)_{\frac{n,n+1}{n,m}}$

$(P_n)_y$

$(Q_{n,m})_y$

$(k_{n,m})_y$

$(c_{n,m})_y$

$(c_{n,m/\Omega})_y$

$(P_n)_z$

$(Q_{n,m})_z$

$(k_{n,m})_z$

$(c_{n,m})_z$

$(c_{n,m/\Omega})_z$

$$(\tau_d \Delta x)_{\frac{n,n+1}{n,m}}$$

$$(\tau \Delta x)_{\frac{n,n+1}{n,m}}$$

$$(k_\alpha \Delta x)_{\frac{n,n+1}{n,m}}$$

$$(k_\beta \Delta x)_{\frac{n,n+1}{n,m}}$$

h

Ω,

where μ, EI, KAG, $P_{\frac{y}{z}}$, $Q_{\frac{y}{z}}$, $k_{\frac{y}{z}}$, c, τ_d, τ, k_α and k_β are defined on pages and and h and Ω are defined on page . The subscripts n,n+1 or n,m show how a parameter is located in the system of divided shafts. For example, $\frac{n,n+1}{n,m}$ means an averaging of the parameter or its reciprocal over the shaft interval extending from the midpoint of section numbered n to the midpoint of the section numbered $\frac{n+1}{m}$. A single subscript n means that the parameter or its reciprocal is averaged over the interval of the shaft section numbered n. This location of parameters is consistent with that given in Appendix 2 of reference 1. Also included as data are the total number of shaft sections, section numbers, system numbers associated with sections and connections, end-condition designators, frequency range with an interval which can be used for response or natural frequency calculations, parameter scaling factors and various options provided by the program.

To accommodate the additional parameters required for the sections and special connections, the input forms used with GBRC1 (vibrations in one plane) are augmented to include formats for continuation cards which go with the basic data card types "41", "42," "43," "51," "52" and "53" described in Section IV of reference 2. The revised forms are shown in Section IV of this report.

The complete set of data card types to be used with GBRC3 is (cf. reference 2, Section IV):

1) Run Title Card

Columns	Contents
7 - 12	(62 H ΔΔ
13 - 72	Whatever title is to be associated with the run

2) Data Control Card

Columns		Contents
3 - 4	(Data card "type" is always designated by digits in these columns)	90
5 - 6 7 - 8		Columns 5, 6 contain number of data cards not counting continuation cards, of type given by columns 7, 8 which follow for this case.
9 - 10 11 - 12 etc.		Similar to previous four columns for next and each succeeding set of data cards of a particular type in the data deck.

3) Case Title Card

Columns	Contents
3 - 4	10
7 - 12	(62 H ΔΔ
13 - 72	Case title

4) Option Control Card

Columns	Contents
3 - 4	20
9 - 12	Op 1: Added mass option selector. 0 for whirling calculations.
13 - 16	Op 2: Selector for A-matrix and P-vector setup option. 0002 for whirling calculations.
17 - 20	Op 3: Selector for edit routine to be used at each frequency. 0 for present.
21 - 24	Op 4: Selector for plotting. 0001 Plots of the edited tables for each frequency are required. 0002 Plot of the summary edit is required. 0003 Both types of plots as given above are required.
25 - 28	Op 5: Selector for A-matrix routine. 0 if A-matrix is not printed. 0001 if A-matrix is to be printed.
29 - 32	Op 6: $\neq 0$, Plot displacements and/or moments versus sections for each frequency. 0 - No plot. 0001 - Plot displacements only. 0002 - Plot moments only. 0003 - Plot displacements and moments.
33 - 36	Op 7: $\neq 0$, Plot displacements and/or moments versus frequency for selected sections. 0 0001 0002 Same designation as above. 0003

Columns	Contents
41 - 44	Op 9: Selector to determine if forcing function is to be multiplied by Ω^2 (in radians). 0 - Force not multiplied. 0001 - Force is multiplied.
45 - 48	Op 10: Selector for natural frequency search. 0 - Forced response is obtained for all frequencies specified. 0001 - Natural frequencies and mode shapes are calculated.

5) Edit Control Card

Columns	Contents
3 - 4	21
9 - 12 13 - 16 17 - 20 21 - 24 25 - 28	Section numbers for those sections for which displacements and moments are to be tabulated versus frequency (summary edit).

6) General Data Card

Columns	Contents
3 - 4	30
7 - 8	Number of sections; maximum of 40 allowed with present version of GBRC3.
9 - 16	Starting frequency (Ω_1) in cps.
17 - 24	Upper limit for frequency (Ω_2) in cps.
25 - 32	Frequency interval ($\Delta\Omega$) in cps. Used as the increment for frequency either in a response calculation or a natural frequency search over the range specified by Ω_1 and Ω_2 above.
33 - 40	h, ratio of spinning frequency to whirling frequency

7) Section Parameter Cards

Columns	Contents
3 - 4	43
7 - 8	Section number
12	End condition - normally 0 Free end for section n, 1 for $V_y(n) = M_z(n) = 0$ and $V_z(n) = M_y(n) = 0$ Fixed end for section n, 2 for $\alpha(n) = \beta(n) = 0$ and $y(n) = z(n) = 0$
15 - 16	System number
17 - 24	Mass of the section
25 - 32	Water inertia
33 - 40	$(\Delta x/EI)_n$
41 - 48	$(\Delta x)_{n,n+1}$
49 - 56	$(\Delta x/KAG)_{n,n+1}$
57 - 64	$(\tau_d \Delta x)_{n,n+1}$
	Program notation, TAUD $\cdot$ Δx
65 - 72	$(P_n)_y$

8) Section Parameter Cards (Continuation)

Columns	Contents
1 - 16	Blank
17 - 24	$(\tau \Delta x)_{n,n+1}$
	Program notation, TAU $\cdot$ Δx

Columns	Contents
25 - 32	$(k_\alpha \Delta x)_{n,n+1}$ Program notation, KA · Δx
33 - 40	$(k_\beta \Delta x)_{n,n+1}$ Program notation, KB · Δx
41 - 48	$(P_n)_z$

9) Scaling Factor Cards for Section Parameters

Columns	Contents
3 - 4	41 if this card contains real parts of scaling factors. 42 if this card contains the imaginary parts of the scaling factors.
15 - 16	System number for which scaling factors on this card are to be applied.
17 - 72	Scaling factor for each parameter is given in those columns containing that parameter value on the section parameter cards.

10) Scaling Factor Cards for Section Parameters (Continuation)

Columns	Contents
1 - 16	Blank
17 - 48	Scaling factor for each parameter given in those columns containing that parameter value on the continuation cards associated with the section parameter cards.

11) Special Connection Parameter Cards

Columns	Contents
3 - 4	53

Columns		Contents
7 - 8		Section number for section <u>n</u>,
11 - 12		Section number for section <u>m</u>, where the sections n and m are connected either as adjacent (but $m \neq n+1$) sections of shaft or by a spring. If a section, n, is connected by spring to ground, then <u>m must be</u> 0. When numbering the shaft sections for whirling calculations one must insure that $\|n-m\| \leq 6$ <u>at present</u>.
15 - 16		System number.
17 - 24	(Specified for spring connections)	$(k_{n,m})_y$ Spring constant in y direction for the connection.
25 - 32		$(c_{n,m})_y$ Constant damping coefficient in y direction for the connection.
33 - 40		$(c_{n,m}/\Omega)_y$ Coefficient for frequency dependent damping in y direction for the connection.
41 - 48	(Specified for rigid connections only.)	$(\Delta x)_{n,m}$
49 - 56		$(\Delta x/KAG)_{n,m}$
57 - 64		$(\tau_d \Delta x)_{n,m}$
65 - 72		$(Q_{n,m})_y$

12) <u>Special Connection Parameter Cards</u> (Continuation)

Columns	Contents
1 - 16	Blank
17 - 24	$(\tau \Delta x)_{n,m}$
25 - 32	$(k_\alpha \Delta x)_{n,m}$
33 - 40	$(k_\beta \Delta x)_{n,m}$

Columns	Contents
41 - 48	$(k_{n,m})_z$
49 - 56	$(c_{n,m})_z$
57 - 64	$(c_{n,m}/\Omega)_z$
65 - 72	$(Q_{n,m})_z$

13) Scaling Factor Cards for Parameters in Special Connections

Columns	Contents
3 - 4	51 if this card contains real parts of scaling factors. 52 if this card contains the imaginary parts of scaling factors.
15 - 16	System number for which scaling factors on this card are to be applied.
17 - 72	Scaling factor for each parameter is given in those columns containing that parameter value on the special connection parameter cards.

14) Scaling Factor Cards for Parameters in Special Connections (Continuation)

Columns	Contents
1 - 16	Blank
17 - 72	Scaling factor for each parameter given in those columns containing that parameter value on the continuation cards associated with the special connection parameter cards.

15) Microfilm Recorder Plotting Characters

Columns	Contents
3 - 4	60
7 - 12	Blank
13 - 18	ΔΔΔΔΔ+

Columns	Contents
19 - 24	ΔΔΔΔΔ0
25 - 30	ΔΔΔΔΔx
31 - 36	ΔΔΔΔΔ-
37 - 42	ΔΔΔΔΔ.

16) Natural Frequency Card

Columns	Contents
3 - 4	70
9 - 10	Number of natural frequencies desired.

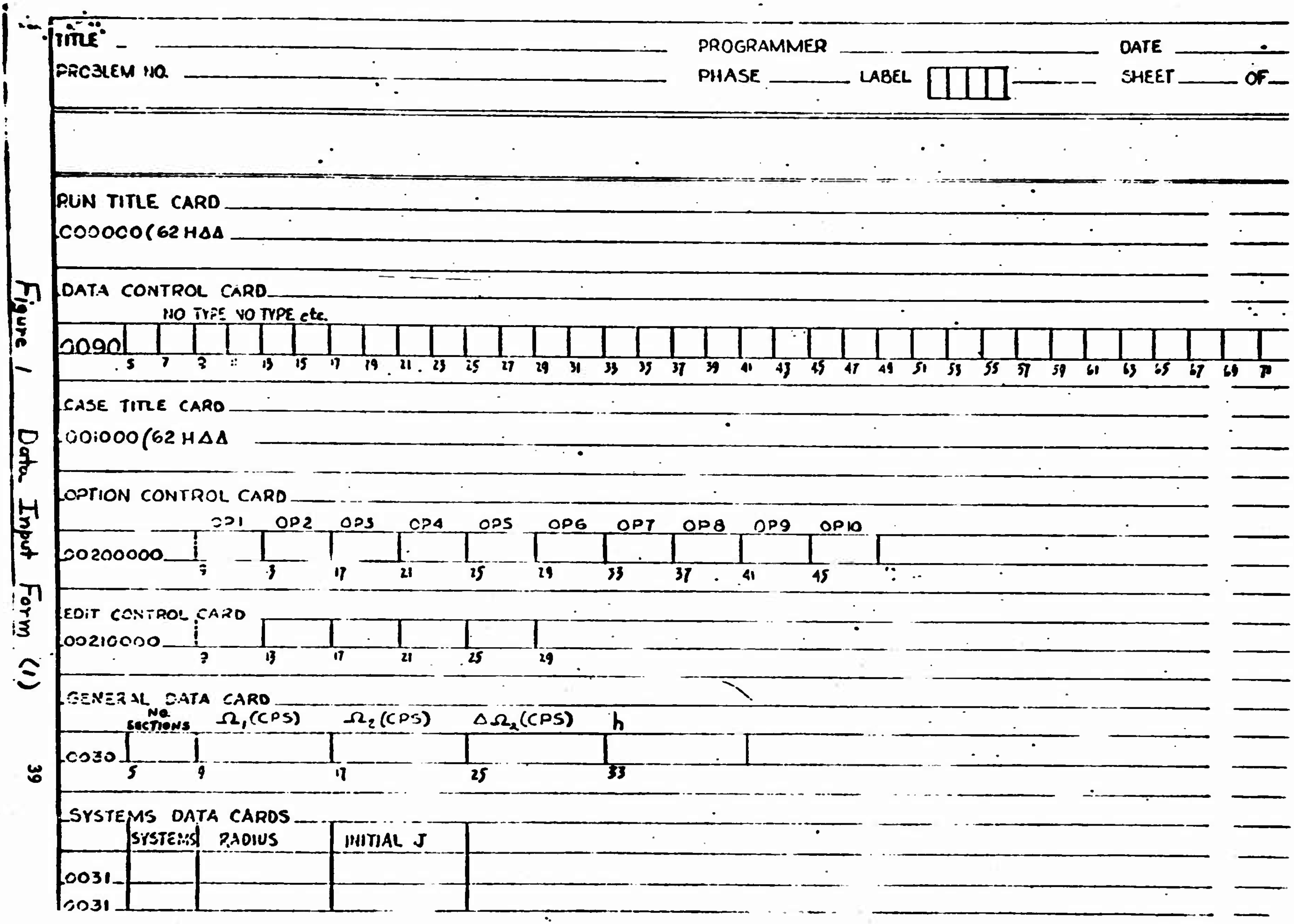
17) Begin a complete new set of data

Columns	Contents
3 - 4	98

18) End of data

Columns	Contents
3 - 4	99

IV. Data Input Forms



TITLE ______ PROGRAMMER ______ DATE ______
PROBLEM NO. ______ PHASE ______ LABEL ☐☐☐ ______ SHEET ______ OF ______

SECTION DATA CARDS

	5	9	13	17	25	33	41	49	57	65	73
REAL PART OF SCALING FACTORS											
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$\left(\Delta x / EI\right)_n$	$(\Delta x)_{n,n+1}$	$\left(\Delta x / KAG\right)_{n,n+1}$	$(TAUD \cdot \Delta x)_{n,n+1}$	$(P_n)_y$	
0041	0000	0000									
▨	▨	▨	▨	$(TAU \cdot \Delta x)_{n,n+1}$	$(KA \cdot \Delta x)_{n,n+1}$	$(KB \cdot \Delta x)_{n,n+1}$	$(P_n)_z$				
←BLANK→											
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$\left(\Delta x / EI\right)_n$	$(\Delta x)_{n,n+1}$	$\left(\Delta x / KAG\right)_{n,n+1}$	$(TAUD \cdot \Delta x)_{n,n+1}$	$(P_n)_y$	
0041	0000	0000									
▨	▨	▨	▨	$(TAU \cdot \Delta x)_{n,n+1}$	$(KA \cdot \Delta x)_{n,n+1}$	$(KB \cdot \Delta x)_{n,n+1}$	$(P_n)_z$				
←BLANK→											
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$\left(\Delta x / EI\right)_n$	$(\Delta x)_{n,n+1}$	$\left(\Delta x / KAG\right)_{n,n+1}$	$(TAUD \cdot \Delta x)_{n,n+1}$	$(P_n)_y$	
0041	0000	0000									
▨	▨	▨	▨	$(TAU \cdot \Delta x)_{n,n+1}$	$(KA \cdot \Delta x)_{n,n+1}$	$(KB \cdot \Delta x)_{n,n+1}$	$(P_n)_z$				
←BLANK→											
IMAGINARY PART OF SCALING FACTORS											
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$\left(\Delta x / EI\right)_n$	$(\Delta x)_{n,n+1}$	$\left(\Delta x / KAG\right)_{n,n+1}$	$(TAUD \cdot \Delta x)_{n,n+1}$	$(P_n)_y$	
0042	0000	0000									
▨	▨	▨	▨	$(TAU \cdot \Delta x)_{n,n+1}$	$(KA \cdot \Delta x)_{n,n+1}$	$(KB \cdot \Delta x)_{n,n+1}$	$(P_n)_z$				
←BLANK→											
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$\left(\Delta x / EI\right)_n$	$(\Delta x)_{n,n+1}$	$\left(\Delta x / KAG\right)_{n,n+1}$	$(TAUD \cdot \Delta x)_{n,n+1}$	$(P_n)_y$	
0042	0000	0000									
▨	▨	▨	▨	$(TAU \cdot \Delta x)_{n,n+1}$	$(KA \cdot \Delta x)_{n,n+1}$	$(KB \cdot \Delta x)_{n,n+1}$	$(P_n)_z$				
←BLANK→											

Figure 2 Data Input Form (2)

TITLE ______ PROGRAMMER ______ DATE ______

PROBLEM NO. ______ PHASE ______ LABEL ☐☐☐ ______ SHEET ______ OF ______

SECTION PARAMETER VALUES - UNSCALED

	5	9	13	17	25	33	41	49	57	65	73
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta X/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta X/KAG)_{n,n+1}$	$(TAUD \cdot \Delta X)_{n,n+1}$	$(P_n)_Y$	
0043											
▨	▨	▨	▨	$(TAU \cdot \Delta X)_{n,n+1}$	$(KA \cdot \Delta X)_{n,n+1}$	$(KB \cdot \Delta X)_{n,n+1}$	$(P_n)_Z$				
← BLANK →											
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta X/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta X/KAG)_{n,n+1}$	$(TAUD \cdot \Delta X)_{n,n+1}$	$(P_n)_Y$	
0043											
▨	▨	▨	▨	$(TAU \cdot \Delta X)_{n,n+1}$	$(KA \cdot \Delta X)_{n,n+1}$	$(KB \cdot \Delta X)_{n,n+1}$	$(P_n)_Z$				
← BLANK →											
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta X/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta X/KAG)_{n,n+1}$	$(TAUD \cdot \Delta X)_{n,n+1}$	$(P_n)_Y$	
0043											
▨	▨	▨	▨	$(TAU \cdot \Delta X)_{n,n+1}$	$(KA \cdot \Delta X)_{n,n+1}$	$(KB \cdot \Delta X)_{n,n+1}$	$(P_n)_Z$				
← BLANK →											
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta X/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta X/KAG)_{n,n+1}$	$(TAUD \cdot \Delta X)_{n,n+1}$	$(P_n)_Y$	
0043											
▨	▨	▨	▨	$(TAU \cdot \Delta X)_{n,n+1}$	$(KA \cdot \Delta X)_{n,n+1}$	$(KB \cdot \Delta X)_{n,n+1}$	$(P_n)_Z$				
← BLANK →											
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta X/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta X/KAG)_{n,n+1}$	$(TAUD \cdot \Delta X)_{n,n+1}$	$(P_n)_Y$	
0043											
▨	▨	▨	▨	$(THU \cdot \Delta X)_{n,n+1}$	$(KA \cdot \Delta X)_{n,n+1}$	$(KB \cdot \Delta X)_{n,n+1}$	$(P_n)_Z$				
← BLANK →											

Figure 3 Data Input Form (3)

TITLE ______________________ PROGRAMMER ______________ DATE __________

PROBLEM NO. ______________ PHASE ______ LABEL ☐☐☐ ______ SHEET ______

SPECIAL CONNECTION CARDS

5 9 13 17 25 33 41 49 57 65 73

REAL PART OF SCALING FACTORS

	n	m	SYSTEM	$(K_{n,m})_Y$	$(C_{n,m})_Y$	$(C_{n,m}/\Omega)_Y$	$(\Delta X)_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(\mathrm{TAUD}\cdot\Delta X)_{n,m}$	$(Q_{n,m})_Y$
0051										
← BLANK →				$(\mathrm{TAU}\cdot\Delta X)_{n,m}$	$(\mathrm{KA}\cdot\Delta X)_{n,m}$	$(\mathrm{KB}\cdot\Delta X)_{n,m}$	$(K_{n,m})_Z$	$(C_{n,m})_Z$	$(C_{n,m}/\Omega)_Z$	$(Q_{n,m})_Z$
	n	m	SYSTEM	$(K_{n,m})_Y$	$(C_{n,m})_Y$	$(C_{n,m}/\Omega)_Y$	$(\Delta X)_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(\mathrm{TAUD}\cdot\Delta X)_{n,m}$	$(Q_{n,m})_Y$
0051										
← BLANK →				$(\mathrm{TAU}\cdot\Delta X)_{n,m}$	$(\mathrm{KA}\cdot\Delta X)_{n,m}$	$(\mathrm{KB}\cdot\Delta X)_{n,m}$	$(K_{n,m})_Z$	$(C_{n,m})_Z$	$(C_{n,m}/\Omega)_Z$	$(Q_{n,m})_Z$
	n	m	SYSTEM	$(K_{n,m})_Y$	$(C_{n,m})_Y$	$(C_{n,m}/\Omega)_Y$	$(\Delta X)_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(\mathrm{TAUD}\cdot\Delta X)_{n,m}$	$(Q_{n,m})_Y$
0051										
← BLANK →				$(\mathrm{TAU}\cdot\Delta X)_{n,m}$	$(\mathrm{KA}\cdot\Delta X)_{n,m}$	$(\mathrm{KB}\cdot\Delta X)_{n,m}$	$(K_{n,m})_Z$	$(C_{n,m})_Z$	$(C_{n,m}/\Omega)_Z$	$(Q_{n,m})_Z$

IMAGINARY PART OF SCALING FACTORS

	n	m	SYSTEM	$(K_{n,m})_Y$	$(C_{n,m})_Y$	$(C_{n,m}/\Omega)_Y$	$(\Delta X)_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(\mathrm{TAUD}\cdot\Delta X)_{n,m}$	$(Q_{n,m})_Y$
0052										
← BLANK →				$(\mathrm{TAU}\cdot\Delta X)_{n,m}$	$(\mathrm{KA}\cdot\Delta X)_{n,m}$	$(\mathrm{KB}\cdot\Delta X)_{n,m}$	$(K_{n,m})_Z$	$(C_{n,m})_Z$	$(C_{n,m}/\Omega)_Z$	$(Q_{n,m})_Z$
	n	m	SYSTEM	$(K_{n,m})_Y$	$(C_{n,m})_Y$	$(C_{n,m}/\Omega)_Y$	$(\Delta X)_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(\mathrm{TAUD}\cdot\Delta X)_{n,m}$	$(Q_{n,m})_Y$
0052										
← BLANK →				$(\mathrm{TAU}\cdot\Delta X)_{n,m}$	$(\mathrm{KA}\cdot\Delta X)_{n,m}$	$(\mathrm{KB}\cdot\Delta X)_{n,m}$	$(K_{n,m})_Z$	$(C_{n,m})_Z$	$(C_{n,m}/\Omega)_Z$	$(Q_{n,m})_Z$

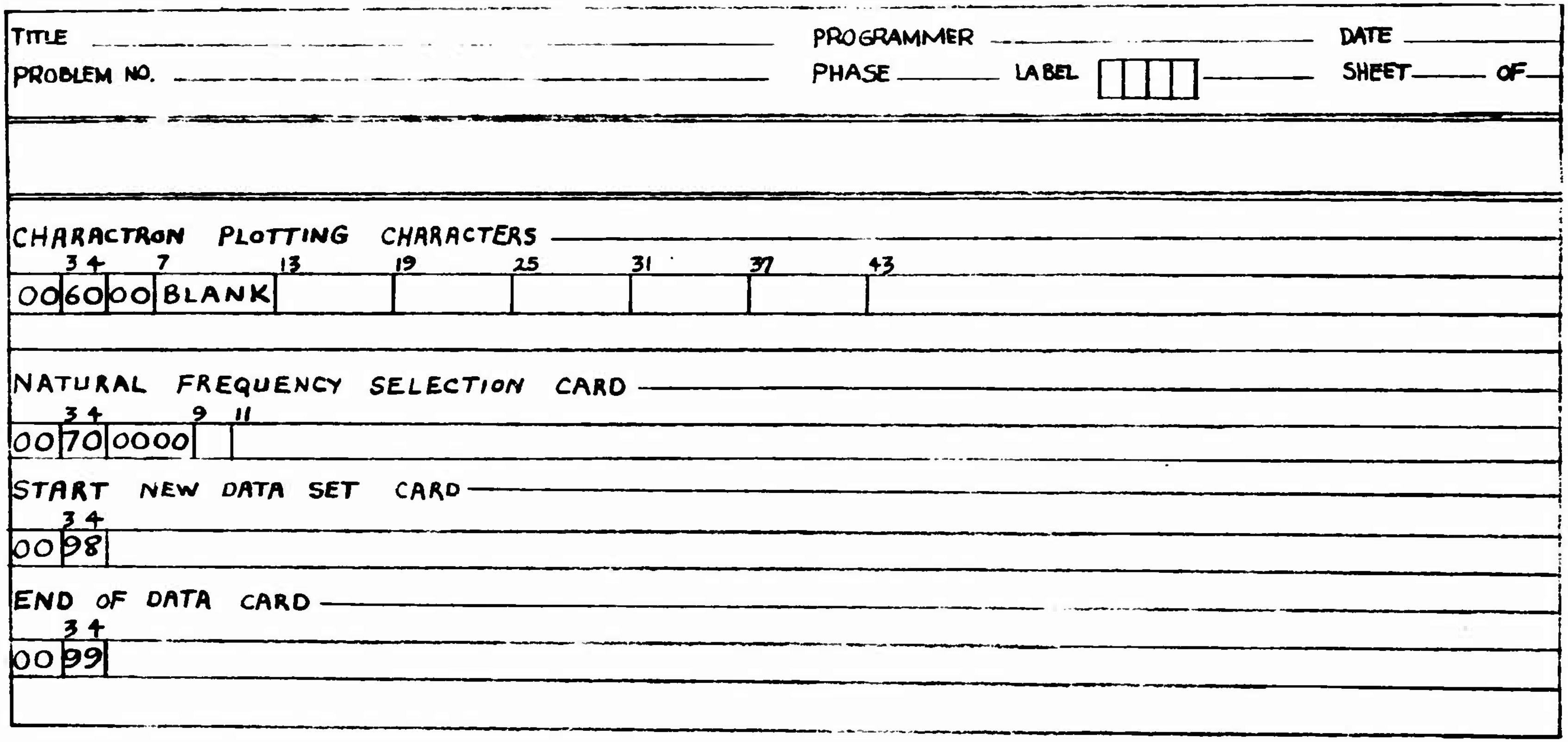
Figure 4 Data Input Form (4)

TITLE ______ PROGRAMMER ______ DATE ______

PROBLEM NO. ______ PHASE ______ LABEL ☐☐☐ ______ SHEET ____ OF ____

	SPECIAL CONNECTIONS			PARAMETER VALUES						
	5	9	13	17	25	33	41	49	57	65
	n	m	SYSTEM	$(K_{n,m})_Y$	$(C_{n,m})_Y$	$(C_{n,m}/\Omega)_Y$	$(\Delta X)_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(TAUD \cdot \Delta X)_{n,m}$	$(Q_{n,m})_Y$
0053										
				$(TAU \cdot \Delta X)_{n,m}$	$(KA \cdot \Delta X)_{n,m}$	$(KB \cdot \Delta X)_{n,m}$	$(K_{n,m})_Z$	$(C_{n,m})_Z$	$(C_{n,m}/\Omega)_Z$	$(Q_{n,m})_Z$
← BLANK →										
	n	m	SYSTEM	$(K_{n,m})_Z$	$(C_{n,m})_Y$	$(C_{n,m}/\Omega)_Y$	$(\Delta X)_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(TAUD \cdot \Delta X)_{n,m}$	$(Q_{n,m})_Y$
0053										
				$(TAU \cdot \Delta X)_{n,m}$	$(KA \cdot \Delta X)_{n,m}$	$(KB \cdot \Delta X)_{n,m}$	$(K_{n,m})_Z$	$(C_{n,m})_Z$	$(C_{n,m}/\Omega)_Z$	$(Q_{n,m})_Z$
← BLANK →										
	n	m	SYSTEM	$(K_{n,m})_Y$	$(C_{n,m})_Y$	$(C_{n,m}/\Omega)_Y$	$(\Delta X)_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(TAUD \cdot \Delta X)_{n,m}$	$(Q_{n,m})_Y$
0053										
				$(TAU \cdot \Delta X)_{n,m}$	$(KA \cdot \Delta X)_{n,m}$	$(KB \cdot \Delta X)_{n,m}$	$(K_{n,m})_Z$	$(C_{n,m})_Z$	$(C_{n,m}/\Omega)_Z$	$(Q_{n,m})_Z$
← BLANK →										
	n	m	SYSTEM	$(K_{n,m})_Y$	$(C_{n,m})_Y$	$(C_{n,m}/\Omega)_Y$	$(\Delta X)_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(TAUD \cdot \Delta X)_{n,m}$	$(Q_{n,m})_Y$
0053										
				$(TAU \cdot \Delta X)_{n,m}$	$(KA \cdot \Delta X)_{n,m}$	$(KB \cdot \Delta X)_{n,m}$	$(K_{n,m})_Z$	$(C_{n,m})_Z$	$(C_{n,m}/\Omega)_Z$	$(Q_{n,m})_Z$
← BLANK →										
	n	m	SYSTEM	$(K_{n,m})_Y$	$(C_{n,m})_Y$	$(C_{n,m}/\Omega)_Z$	$(\Delta X)_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(TAUD \cdot \Delta X)_{n,m}$	$(Q_{n,m})_Y$
0053										
				$(TAU \cdot \Delta X)_{n,m}$	$(KA \cdot \Delta X)_{n,m}$	$(KB \cdot \Delta X)_{n,m}$	$(K_{n,m})_Z$	$(C_{n,m})_Z$	$(C_{n,m}/\Omega)_Z$	$(Q_{n,m})_Z$
← BLANK →										

Figure 5 Data Input Form (5)



TITLE

PROBLEM NO.

PROGRAMMER

PHASE

LABEL

DATE

SHEET

OF

CHARACTRON PLOTTING CHARACTERS

1	3	5	7	13	19	25	31	37	43
00	60	00	BLANK						

NATURAL FREQUENCY SELECTION CARD

1	3	5	9	11
00	70	0000		

START NEW DATA SET CARD

1	3
00	98

END OF DATA CARD

1	3
00	99

Figure 6 Data Input Form (6)

V. Application with Comparison of Calculated Results

Two sample calculations have been made with GBRC3 in order to obtain results for comparing with an earlier AML whirling vibration program.

The data used in these calculations is for the propeller shaft of the USS Observation Island (EAG-154). The first three critical whirling frequencies are calculated for the shaft in air using for the ratio of spin to whirl a value of 1 in Problem number 1 and a value of ¼ in Problem 2.

The computed mode shapes of the shaft are presented in both tabular and graphic forms. The comparison of results is shown in Table 1 immediately following the output.

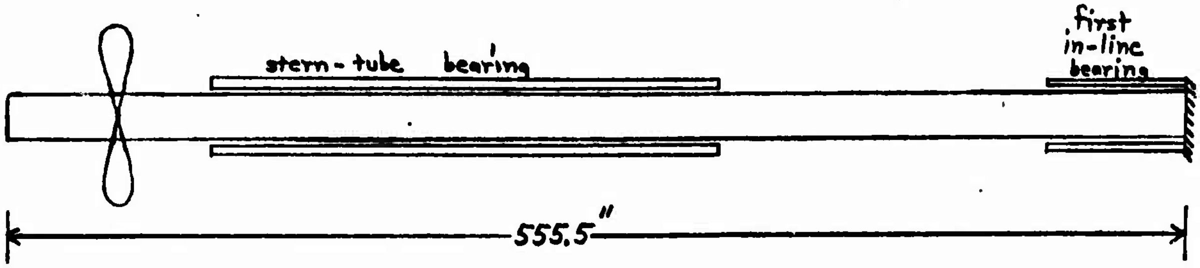


Figure 7

USS Observation Island (EAG-154) Propeller Shaft

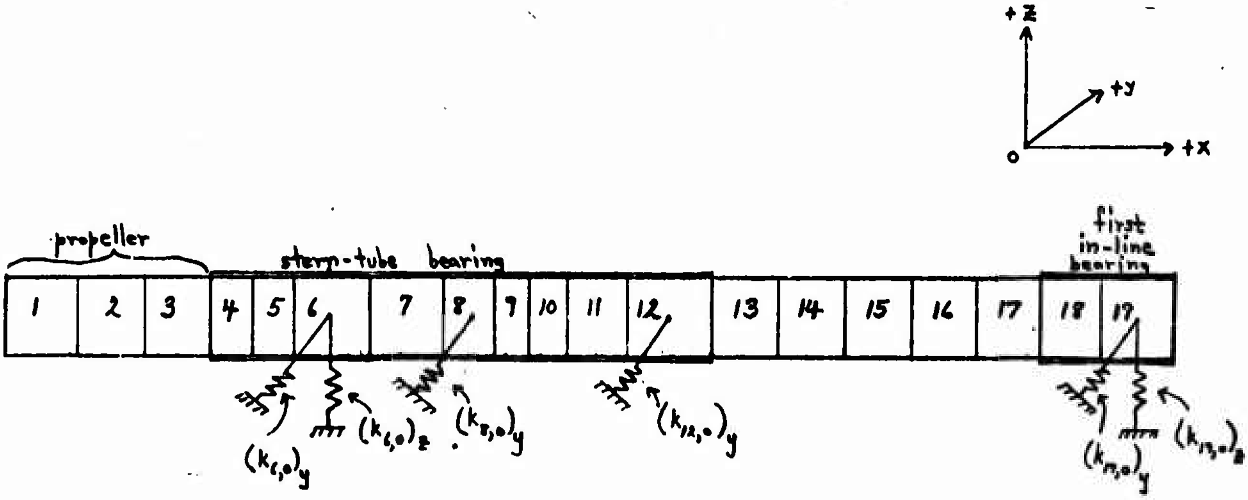


Figure 8

Mathematical Model of the Propeller Shaft Used for Whirling Calculations

Problem 1. The data is shown as it is prepared on the input forms from which the cards are keypunched.

For these calculations, the large spring constants required to bring about the constraint on deflection at the fixed end of the shaft are read in on a special connection card (type "53"). This card is seen as the last card filled in on the Data Input Form on page . In a later version of GBRC3 this additional data card will not be necessary. The value of such spring constants for fixed ends in the system will automatically be supplied by the program when the end condition designating integer 0002 is given with these sections on the Data Input Forms.

TITLE USS OBSERVATION, ISLAND, WHIRLING CALCULATION PROGRAMMER ______ DATE ______

PROBLEM NO. 1 ______ PHASE ______ LABEL ☐☐☐☐ ______ SHEET 1 OF 2

RUN TITLE CARD

C00000(62HΔΔ SAMPLE PROBLEM 1

DATA CONTROL CARD

NO TYPE NO TYPE etc.

0090	01	10	01	20	01	30	01	41	19	43	02	51	04	53	01	70	01	60
	5	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35	37	39

41 43 45 47 49 51 53 55 57 59 61 63 65 67 69 71

CASE TITLE CARD

001000(62HΔΔ USSΔOBSERVATIONΔISLAND,ΔWHIRLINGΔFREQUENCIES,-AIR,ΔH=1

OPTION CONTROL CARD

	OP1	OP2	OP3	OP4	OP5	OP6	OP7	OP8	OP9	OP10
00200000		0002		0001		0001				0001
	9	13	17	21	25	29	33	37	41	45

EDIT CONTROL CARD

00210000					
	9	13	17	21	25 29

GENERAL DATA CARD

	NO. SECTIONS	Ω_1(CPS)	Ω_2(CPS)	$\Delta\Omega_2$(CPS)	h
0030	0019	1.0	30.0	1.0	1.0
	5	9	17	25	33

SYSTEMS DATA CARDS

	SYSTEMS	RADIUS	INITIAL J
0031			
0031			

Data Input Form (1)

TITLE USS OBSERVATION ISLAND, WHIRLING CALCULATION PROGRAMMER ______ DATE ______
PROBLEM NO. 1 PHASE ______ LABEL ☐☐☐☐ ______ SHEET 2 OF 9

SECTION DATA CARDS

	5	9	13	17	25	33	41	49	57	65	73

REAL PART OF SCALING FACTORS

	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/KAG)_{n,n+1}$	$(TAUD\cdot\Delta x)_{n,n+1}$	$(P_n)_Y$
0041	0000	0000	0001	1.0		△1.0E-11	1.0	△△1.0E-9	△△△1.0E4	1.0
				$(TAU\cdot\Delta x)_{n,n+1}$	$(KA\cdot\Delta x)_{n,n+1}$	$(KB\cdot\Delta x)_{n,n+1}$	$(P_n)_Z$			
← BLANK →				△△△1.0E5		△△△1.0E9	1.0			
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/KAG)_{n,n+1}$	$(TAUD\cdot\Delta x)_{n,n+1}$	$(P_n)_Y$
0041	0000	0000								
				$(TAU\cdot\Delta x)_{n,n+1}$	$(KA\cdot\Delta x)_{n,n+1}$	$(KB\cdot\Delta x)_{n,n+1}$	$(P_n)_Z$			
← BLANK →										
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/KAG)_{n,n+1}$	$(TAUD\cdot\Delta x)_{n,n+1}$	$(P_n)_Y$
0041	0000	0000								
				$(TAU\cdot\Delta x)_{n,n+1}$	$(KA\cdot\Delta x)_{n,n+1}$	$(KB\cdot\Delta x)_{n,n+1}$	$(P_n)_Z$			
← BLANK →										

IMAGINARY PART OF SCALING FACTORS

	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/KAG)_{n,n+1}$	$(TAUD\cdot\Delta x)_{n,n+1}$	$(P_n)_Y$
0042	0000	0000								
				$(TAU\cdot\Delta x)_{n,n+1}$	$(KA\cdot\Delta x)_{n,n+1}$	$(KB\cdot\Delta x)_{n,n+1}$	$(P_n)_Z$			
← BLANK →										
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/KAG)_{n,n+1}$	$(TAUD\cdot\Delta x)_{n,n+1}$	$(P_n)_Y$
0042	0000	0000								
				$(TAU\cdot\Delta x)_{n,n+1}$	$(KA\cdot\Delta x)_{n,n+1}$	$(KB\cdot\Delta x)_{n,n+1}$	$(P_n)_Z$			
← BLANK →										

Data Input Form (2)

TITLE USS OBSERVATION ISLAND, WHIRLING CALCULATION PROGRAMMER ______ DATE ______
PROBLEM NO. 1 PHASE ______ LABEL ☐☐☐ ______ SHEET 3 OF 9

SECTION PARAMETER VALUES - UNSCALED

	5	9	13	17	25	33	41	49	57	65	73
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta X/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta X/KAG)_{n,n+1}$	$(TAUD\cdot\Delta X)_{n,n+1}$	$(P_n)_Y$	
0043	0001	0001	0001	51.9		15.31	32.0	Δ3.2 E-06	.005788		
▨	▨	▨	▨	$(TAU\cdot\Delta X)_{n,n+1}$	$(KA\cdot\Delta X)_{n,n+1}$	$(KB\cdot\Delta X)_{n,n+1}$	$(P_n)_Z$				
← BLANK →				.001158							
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta X/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta X/KAG)_{n,n+1}$	$(TAUD\cdot\Delta X)_{n,n+1}$	$(P_n)_Y$	
0043	0002		0001	51.9		Δ1.0 E-15	32.0	Δ3.2 E-06	7.235		
▨	▨	▨	▨	$(TAU\cdot\Delta X)_{n,n+1}$	$(KA\cdot\Delta X)_{n,n+1}$	$(KB\cdot\Delta X)_{n,n+1}$	$(P_n)_Z$				
← BLANK →				1.447							
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta X/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta X/KAG)_{n,n+1}$	$(TAUD\cdot\Delta X)_{n,n+1}$	$(P_n)_Y$	
0043	0003		0001	51.9		Δ1.0 E-15	32.0	Δ1.9 E-06	7.235		
▨	▨	▨	▨	$(TAU\cdot\Delta X)_{n,n+1}$	$(KA\cdot\Delta X)_{n,n+1}$	$(KB\cdot\Delta X)_{n,n+1}$	$(P_n)_Z$				
← BLANK →				1.447							
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta X/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta X/KAG)_{n,n+1}$	$(TAUD\cdot\Delta X)_{n,n+1}$	$(P_n)_Y$	
0043	0004		0001	1.996		1.2295	6.0	2.2108	.005788		
▨	▨	▨	▨	$(TAU\cdot\Delta X)_{n,n+1}$	$(KA\cdot\Delta X)_{n,n+1}$	$(KB\cdot\Delta X)_{n,n+1}$	$(P_n)_Z$				
← BLANK →				.001158							
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta X/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta X/KAG)_{n,n+1}$	$(TAUD\cdot\Delta X)_{n,n+1}$	$(P_n)_Y$	
0043	0005		0001	11.64		7.1721	35.0	12.896	.01491		
▨	▨	▨	▨	$(TAU\cdot\Delta X)_{n,n+1}$	$(KA\cdot\Delta X)_{n,n+1}$	$(KB\cdot\Delta X)_{n,n+1}$	$(P_n)_Z$				
← BLANK →				.002982							

Data Inbut Form (3)

TITLE USS OBSERVATION ISLAND, WHIRLING CALCULATION PROGRAMMER ________ DATE ________

PROBLEM NO. 1 ________ PHASE ________ LABEL ☐☐☐ ________ SHEET 4 OF 9

SECTION PARAMETER VALUES - UNSCALED

	5	9	13	17	25	33	41	49	57	65	73
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/KAG)_{n,n+1}$	$(TAUD \cdot \Delta x)_{n,n+1}$	$(P_n)_Y$	
0043	0006		0001	11.64		7.1721	35.0	12.896	.01491		
				$(TAU \cdot \Delta x)_{n,n+1}$	$(KA \cdot \Delta x)_{n,n+1}$	$(KB \cdot \Delta x)_{n,n+1}$	$(P_n)_Z$				
← BLANK →				.002982							
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/KAG)_{n,n+1}$	$(TAUD \cdot \Delta x)_{n,n+1}$	$(P_n)_Y$	
0043	0007		0001	11.64		7.1721	35.0	12.896	.01491		
				$(TAU \cdot \Delta x)_{n,n+1}$	$(KA \cdot \Delta x)_{n,n+1}$	$(KB \cdot \Delta x)_{n,n+1}$	$(P_n)_Z$				
← BLANK →				.002982							
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/KAG)_{n,n+1}$	$(TAUD \cdot \Delta x)_{n,n+1}$	$(P_n)_Y$	
0043	0008		0001	4.99		3.0738	15.0	5.5269	.02233		
				$(TAU \cdot \Delta x)_{n,n+1}$	$(KA \cdot \Delta x)_{n,n+1}$	$(KB \cdot \Delta x)_{n,n+1}$	$(P_n)_Z$				
← BLANK →				.004466							
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/KAG)_{n,n+1}$	$(TAUD \cdot \Delta x)_{n,n+1}$	$(P_n)_Y$	
0043	0009		0001	4.99		3.0738	15.0	5.5269	.02233		
				$(TAU \cdot \Delta x)_{n,n+1}$	$(KA \cdot \Delta x)_{n,n+1}$	$(KB \cdot \Delta x)_{n,n+1}$	$(P_n)_Z$				
← BLANK →				.004466							
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/KAG)_{n,n+1}$	$(TAUD \cdot \Delta x)_{n,n+1}$	$(P_n)_Y$	
0043	0010		0001	4.99		3.0738	15.0	5.5269	.02233		
				$(TAU \cdot \Delta x)_{n,n+1}$	$(KA \cdot \Delta x)_{n,n+1}$	$(KB \cdot \Delta x)_{n,n+1}$	$(P_n)_Z$				
← BLANK →				.004466							

Data Inbut Form (3)

TITLE USS OBSERVATION ISLAND, WHIRLING CALCULATION PROGRAMMER ________ DATE ________

PROBLEM NO. 1 PHASE ________ LABEL ☐☐☐ ________ SHEET 5 OF 9

SECTION PARAMETER VALUES - UNSCALED

	5	9	13	17	25	33	41	49	57	65	73
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/KAG)_{n,n+1}$	$(TAUD \cdot \Delta x)_{n,n+1}$	$(P_n)_Y$	
0043	0011		0001	15.0		9.2213	45.0	16.580	.2921		
				$(TAU \cdot \Delta x)_{n,n+1}$	$(KA \cdot \Delta x)_{n,n+1}$	$(KB \cdot \Delta x)_{n,n+1}$	$(P_n)_Z$				
← BLANK →				.05842							
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/KAG)_{n,n+1}$	$(TAUD \cdot \Delta x)_{n,n+1}$	$(P_n)_Y$	
0043	0012		0001	10.3		6.3525	31.0	11.422	.1093		
				$(TAU \cdot \Delta x)_{n,n+1}$	$(KA \cdot \Delta x)_{n,n+1}$	$(KB \cdot \Delta x)_{n,n+1}$	$(P_n)_Z$				
← BLANK →				.02186							
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/KAG)_{n,n+1}$	$(TAUD \cdot \Delta x)_{n,n+1}$	$(P_n)_Y$	
0043	0013		0001	10.3		6.3526	31.0	11.422	.1093		
				$(TAU \cdot \Delta x)_{n,n+1}$	$(KA \cdot \Delta x)_{n,n+1}$	$(KB \cdot \Delta x)_{n,n+1}$	$(P_n)_Z$				
← BLANK →				.02186							
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/KAG)_{n,n+1}$	$(TAUD \cdot \Delta x)_{n,n+1}$	$(P_n)_Y$	
0043	0014		0001	10.3		6.3525	31.0	11.422	.1093		
				$(TAU \cdot \Delta x)_{n,n+1}$	$(KA \cdot \Delta x)_{n,n+1}$	$(KB \cdot \Delta x)_{n,n+1}$	$(P_n)_Z$				
← BLANK →				.02186							
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta x/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta x/KAG)_{n,n+1}$	$(TAUD \cdot \Delta x)_{n,n+1}$	$(P_n)_Y$	
0043	0015		0001	10.3		6.3525	31.0	11.422	.1093		
				$(TAU \cdot \Delta x)_{n,n+1}$	$(KA \cdot \Delta x)_{n,n+1}$	$(KB \cdot \Delta x)_{n,n+1}$	$(P_n)_Z$				
← BLANK →				.02186							

Data Inbut Form (3)

TITLE USS OBSERVATION ISLAND, WHIRLING CALCULATION PROGRAMMER ________ DATE ________

PROBLEM NO. 1 ________ PHASE ________ LABEL ☐☐☐☐ ________ SHEET 6 OF 9

SECTION PARAMETER VALUES - UNSCALED

	5	9	13	17	25	33	41	49	57	65	73
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta X/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta X/KAG)_{n,n+1}$	$(TAUD \cdot \Delta X)_{n,n+1}$	$(P_n)_Y$	
0043	0016		0001	10.3		6.3525	31.0	11.422	.1093		
				$(TAU \cdot \Delta X)_{n,n+1}$	$(KA \cdot \Delta X)_{n,n+1}$	$(KB \cdot \Delta X)_{n,n+1}$	$(P_n)_Z$				
← BLANK →				.02186							
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta X/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta X/KAG)_{n,n+1}$	$(TAUD \cdot \Delta X)_{n,n+1}$	$(P_n)_Y$	
0043	0017		0001	10.3		6.3525	31.0	11.422	.1093		
				$(TAU \cdot \Delta X)_{n,n+1}$	$(KA \cdot \Delta X)_{n,n+1}$	$(KB \cdot \Delta X)_{n,n+1}$	$(P_n)_Z$				
← BLANK →				.02186							
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta X/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta X/KAG)_{n,n+1}$	$(TAUD \cdot \Delta X)_{n,n+1}$	$(P_n)_Y$	
0043	0018		0001	10.3		6.3525	31.0	11.422	.1093		
				$(TAU \cdot \Delta X)_{n,n+1}$	$(KA \cdot \Delta X)_{n,n+1}$	$(KB \cdot \Delta X)_{n,n+1}$	$(P_n)_Z$				
← BLANK →				.02186							
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta X/EI)_n$	$(\Delta X)_{n,n+1}$	$(\Delta X/KAG)_{n,n+1}$	$(TAUD \cdot \Delta X)_{n,n+1}$	$(P_n)_Y$	
0043	0019	0002	0001	15.0		9.2213					
				$(TAU \cdot \Delta X)_{n,n+1}$	$(KA \cdot \Delta X)_{n,n+1}$	$(KB \cdot \Delta X)_{n,n+1}$	$(P_n)_Z$				
← BLANK →											
	SECTION NO.	END CONDN.	SYSTEM	MASS	WATER INERTIA	$(\Delta X/EI)_n$	$(\Delta x)_{n,n+1}$	$(\Delta X/KAG)_{n,n+1}$	$(TAUD \cdot \Delta X)_{n,n+1}$	$(P_n)_Y$	
0043											
				$(TAU \cdot \Delta X)_{n,n+1}$	$(KA \cdot \Delta X)_{n,n+1}$	$(KB \cdot \Delta X)_{n,n+1}$	$(P_n)_Z$				
← BLANK →											

Data Input Form (3)

TITLE USS OBSERVATION ISLAND, WHIRLING CALCULATION PROGRAMMER ______ DATE ______

PROBLEM NO. 1 PHASE ______ LABEL ☐☐☐ ______ SHEET 7 OF 9

SPECIAL CONNECTION CARDS

5 9 13 17 25 33 41 49 57 65 73

REAL PART OF SCALING FACTORS

	n	m	SYSTEM	$(K_{n,m})_Y$	$(C_{n,m})_Y$	$(C_{n,m}/\Omega)_Y$	$(\Delta X)_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(TAUD \cdot \Delta X)_{n,m}$	$(Q_{n,m})_Y$
0051			0001	ΔΔΔ1.0E6						
				$(TAU \cdot \Delta X)_{n,m}$	$(KA \cdot \Delta X)_{n,m}$	$(KB \cdot \Delta X)_{n,m}$	$(K_{n,m})_Z$	$(C_{n,m})_Z$	$(C_{n,m}/\Omega)_Z$	$(Q_{n,m})_Z$
← BLANK →							ΔΔΔ1.0E9			
	n	m	SYSTEM	$(K_{n,m})_Y$	$(C_{n,m})_Y$	$(C_{n,m}/\Omega)_Y$	$(\Delta X)_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(TAUD \cdot \Delta X)_{n,m}$	$(Q_{n,m})_Y$
0051			0002	ΔΔ1.0E16						
				$(TAU \cdot \Delta X)_{n,m}$	$(KA \cdot \Delta X)_{n,m}$	$(KB \cdot \Delta X)_{n,m}$	$(K_{n,m})_Z$	$(C_{n,m})_Z$	$(C_{n,m}/\Omega)_Z$	$(Q_{n,m})_Z$
← BLANK →							ΔΔ1.0E16			
	n	m	SYSTEM	$(K_{n,m})_Y$	$(C_{n,m})_Y$	$(C_{n,m}/\Omega)_Y$	$(\Delta X)_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(TAUD \cdot \Delta X)_{n,m}$	$(Q_{n,m})_Y$
0051										
				$(TAU \cdot \Delta X)_{n,m}$	$(KA \cdot \Delta X)_{n,m}$	$(KB \cdot \Delta X)_{n,m}$	$(K_{n,m})_Z$	$(C_{n,m})_Z$	$(C_{n,m}/\Omega)_Z$	$(Q_{n,m})_Z$
← BLANK →										

IMAGINARY PART OF SCALING FACTORS

	n	m	SYSTEM	$(K_{n,m})_Y$	$(C_{n,m})_Y$	$(C_{n,m}/\Omega)_Y$	$(\Delta X)_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(TAUD \cdot \Delta X)_{n,m}$	$(Q_{n,m})_Y$
0052										
				$(TAU \cdot \Delta X)_{n,m}$	$(KA \cdot \Delta X)_{n,m}$	$(KB \cdot \Delta X)_{n,m}$	$(K_{n,m})_Z$	$(C_{n,m})_Z$	$(C_{n,m}/\Omega)_Z$	$(Q_{n,m})_Z$
← BLANK →										
	n	m	SYSTEM	$(K_{n,m})_Y$	$(C_{n,m})_Y$	$(C_{n,m}/\Omega)_Y$	$(\Delta X)_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(TAUD \cdot \Delta X)_{n,m}$	$(Q_{n,m})_Y$
0052										
				$(TAU \cdot \Delta X)_{n,m}$	$(KA \cdot \Delta X)_{n,m}$	$(KB \cdot \Delta X)_{n,m}$	$(K_{n,m})_Z$	$(C_{n,m})_Z$	$(C_{n,m}/\Omega)_Z$	$(Q_{n,m})_Z$
← BLANK →										

TITLE USS OBSERVATION ISLAND, WHIRLING CALCULATION PROGRAMMER ______ DATE ______

PROBLEM NO. 1 ______ PHASE ______ LABEL ☐☐☐ ______ SHEET 8 OF 9

SPECIAL CONNECTIONS PARAMETER VALUES

	5	9	13	17	25	33	41	49	57	65	73
	n	m	SYSTEM	$(K_{n,m})_y$	$(C_{n,m})_y$	$(C_{n,m}/\Omega)_y$	$(\Delta X)_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(TAUD\cdot\Delta X)_{n,m}$	$(Q_{n,m})_y$	
0053	0006	0000	0001	9.703							
▨	▨	▨	▨	$(TAU\cdot\Delta X)_{n,m}$	$(KA\cdot\Delta X)_{n,m}$	$(KB\cdot\Delta X)_{n,m}$	$(K_{n,m})_z$	$(C_{n,m})_z$	$(C_{n,m}/\Omega)_z$	$(Q_{n,m})_z$	
← BLANK →							2.568				
	n	m	SYSTEM	$(K_{n,m})_y$	$(C_{n,m})_y$	$(C_{n,m}/\Omega)_y$	$(\Delta X)_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(TAUD\cdot\Delta X)_{n,m}$	$(Q_{n,m})_y$	
0053	0008	0000	0001	4.366							
▨	▨	▨	▨	$(TAU\cdot\Delta X)_{n,m}$	$(KA\cdot\Delta X)_{n,m}$	$(KB\cdot\Delta X)_{n,m}$	$(K_{n,m})_z$	$(C_{n,m})_z$	$(C_{n,m}/\Omega)_z$	$(Q_{n,m})_z$	
← BLANK →											
	n	m	SYSTEM	$(K_{n,m})_y$	$(C_{n,m})_y$	$(C_{n,m}/\Omega)_y$	$(\Delta X)_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(TAUD\cdot\Delta X)_{n,m}$	$(Q_{n,m})_y$	
0053	0012	0000	0001	1.31							
▨	▨	▨	▨	$(TAU\cdot\Delta X)_{n,m}$	$(KA\cdot\Delta X)_{n,m}$	$(KB\cdot\Delta X)_{n,m}$	$(K_{n,m})_z$	$(C_{n,m})_z$	$(C_{n,m}/\Omega)_z$	$(Q_{n,m})_z$	
← BLANK →											
	n	m	SYSTEM	$(K_{n,m})_y$	$(C_{n,m})_y$	$(C_{n,m}/\Omega)_y$	$(\Delta X)_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(TAUD\cdot\Delta X)_{n,m}$	$(Q_{n,m})_y$	
0053	0019	0000	0002	1.0							
▨	▨	▨	▨	$(TAU\cdot\Delta X)_{n,m}$	$(KA\cdot\Delta X)_{n,m}$	$(KB\cdot\Delta X)_{n,m}$	$(K_{n,m})_z$	$(C_{n,m})_z$	$(C_{n,m}/\Omega)_z$	$(Q_{n,m})_z$	
← BLANK →							1.0				
	n	m	SYSTEM	$(K_{n,m})_y$	$(C_{n,m})_y$	$(C_{n,m}/\Omega)_y$	$(\Delta X)_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(TAUD\cdot\Delta X)_{n,m}$	$(Q_{n,m})_y$	
0053											
▨	▨	▨	▨	$(TAU\cdot\Delta X)_{n,m}$	$(KA\cdot\Delta X)_{n,m}$	$(KB\cdot\Delta X)_{n,m}$	$(K_{n,m})_z$	$(C_{n,m})_z$	$(C_{n,m}/\Omega)_z$	$(Q_{n,m})_z$	
← BLANK →											
	n	m	SYSTEM	$(K_{n,m})_y$	$(C_{n,m})_y$	$(C_{n,m}/\Omega)_y$	$(\Delta X)_{n,m}$	$(\Delta X/KAG)_{n,m}$	$(TAUD\cdot\Delta X)_{n,m}$	$(Q_{n,m})_y$	
0053											
▨	▨	▨	▨	$(TAU\cdot\Delta X)_{n,m}$	$(KA\cdot\Delta X)_{n,m}$	$(KB\cdot\Delta X)_{n,m}$	$(K_{n,m})_z$	$(C_{n,m})_z$	$(C_{n,m}/\Omega)_z$	$(Q_{n,m})_z$	
← BLANK →											

Data Input Form (5)

TITLE ______ PROGRAMMER ______ DATE ______

PROBLEM NO. ______ PHASE ______ LABEL ☐☐☐☐ ______ SHEET 9 OF 9

CHARACTRON PLOTTING CHARACTERS

	3 4	7	13	19	25	31	37	43
00	60	00	BLANK	ΔΔΔΔΔ+	ΔΔΔΔΔ0	ΔΔΔΔΔX	ΔΔΔΔΔ−	ΔΔΔΔΔ•

NATURAL FREQUENCY SELECTION CARD

	3 4		9 11
00	70	0000	03

START NEW DATA SET CARD

	3 4
00	98

END OF DATA CARD

	3 4
00	99
	✓

Data Input Form (6)

```
GBRC3 OCT 7, 1966
SAMPLE PROBLEM 1

DATA CONTROL CARD

NC  TYPE
 1   10
 1   20
 1   30
 1   41
19   43
 2   51
 4   53
 1   70
 1   60

CASE TITLE-USS OBSERVATION ISLAND, WHIRLING FREQUENCIES.-AIR, H=1

OPTION DATA
 20      -0   2  -0   1  -0   1  -0  -0  -0   1

GENERAL DATA - NUMBER OF SECTIONS     19
               WHIRLING FREQUENCY RANGE FROM        1.000 CPS  TO    30.000 CPS
               FREQUENCY INTERVAL    1.000 CPS
               RATIO OF SPIN TO WHIRL =    1.000

REAL PARTS OF SCALING FACTORS

SECTION-END CONDN-SYSTEM    MASS      WATER INERTIA    DX/EI(N)      DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)    P(N)/Y
                      1   1.0000E 00   -0.            1.0000E-11    1.0000E 00   1.0000E-09    1.0000E 04     1.0000E 00

                        TAU*DX(N,N+1) KA*DX(N,N+1)  KB*DX(N,N+1)     P(N)/Z
                         1.0000E 05   -0.           1.0000E 09    1.0000E 00

PARAMETER VALUES FOR EACH SECTION - UNSCALED

SECTION-END CONDN-SYSTEM    MASS      WATER INERTIA    DX/EI(N)      DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)    P(N)/Y
     1         1      1   5.1900E 01   -0.            1.5310E 01    3.2000E 01   3.2000E-06    5.7880E-03   -0.

                        TAU*DX(N,N+1) KA*DX(N,N+1)  KB*DX(N,N+1)     P(N)/Z
                         1.1580E-03   -0.           -0.          -0.

                            MASS      WATER INERTIA    DX/EI(N)      DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)    P(N)/Y
     2         0      1   5.1900E 01   -0.            1.0000E-15    3.2000E 01   3.2000E-06    7.2350E 00   -0.

                        TAU*DX(N,N+1) KA*DX(N,N+1)  KB*DX(N,N+1)     P(N)/Z
                         1.4470E 00   -0.           -0.          -0.

                            MASS      WATER INERTIA    DX/EI(N)      DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)    P(N)/Y
     3         0      1   5.1900E 01   -0.            1.0000E-15    3.2000E 01   1.9000E-06    7.2350E 00   -0.

                        TAU*DX(N,N+1) KA*DX(N,N+1)  KB*DX(N,N+1)     P(N)/Z
                         1.4470E 00   -0.           -0.          -0.
```

```
                  MASS        WATER INERTIA    DX/EI(N)       DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)       P(N)/Y
 4     0     1    1.9960E 00   -0.              1.2295E 00     6.0000E 00    2.2108E 00    5.7880E-03    -0.
                 TAU*DX(N,N+1) KA*DX(N,N+1)   KB*DX(N,N+1)       P(N)/Z
                  1.1580E-03   -0.             -0.             -0.

                  MASS        WATER INERTIA    DX/EI(N)       DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)       P(N)/Y
 5     0     1    1.1640E 01   -0.              7.1721E 00     3.5000E 01    1.2896E 01    1.4910E-02    -0.
                 TAU*DX(N,N+1) KA*DX(N,N+1)   KB*DX(N,N+1)       P(N)/Z
                  2.9820E-03   -0.             -0.             -0.

                  MASS        WATER INERTIA    DX/EI(N)       DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)       P(N)/Y
 6     0     1    1.1640E 01   -0.              7.1721E 00     3.5000E 01    1.2896E 01    1.4910E-02    -0.
                 TAU*DX(N,N+1) KA*DX(N,N+1)   KB*DX(N,N+1)       P(N)/Z
                  2.9820E-03   -0.             -0.             -0.

                  MASS        WATER INERTIA    DX/EI(N)       DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)       P(N)/Y
 7     0     1    1.1640E 01   -0.              7.1721E 00     3.5000E 01    1.2896E 01    1.4910E-02    -0.
                 TAU*DX(N,N+1) KA*DX(N,N+1)   KB*DX(N,N+1)       P(N)/Z
                  2.9820E-03   -0.             -0.             -0.

                  MASS        WATER INERTIA    DX/EI(N)       DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)       P(N)/Y
 8     0     1    4.9900E 00   -0.              3.0738E 00     1.5000E 01    5.5269E 00    2.2330E-02    -0.
                 TAU*DX(N,N+1) KA*DX(N,N+1)   KB*DX(N,N+1)       P(N)/Z
                  4.4660E-03   -0.             -0.             -0.

                  MASS        WATER INERTIA    DX/EI(N)       DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)       P(N)/Y
 9     0     1    4.9900E 00   -0.              3.0738E 00     1.5000E 01    5.5269E 00    2.2330E-02    -0.
                 TAU*DX(N,N+1) KA*DX(N,N+1)   KB*DX(N,N+1)       P(N)/Z
                  4.4660E-03   -0.             -0.             -0.

                  MASS        WATER INERTIA    DX/EI(N)       DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)       P(N)/Y
10     0     1    4.9900E 00   -0.              3.0738E 00     1.5000E 01    5.5269E 00    2.2330E-02    -0.
                 TAU*DX(N,N+1) KA*DX(N,N+1)   KB*DX(N,N+1)       P(N)/Z
                  4.4660E-03   -0.             -0.             -0.

                  MASS        WATER INERTIA    DX/EI(N)       DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)       P(N)/Y
11     0     1    1.5000E 01   -0.              9.2213E 00     4.5000E 01    1.6580E 01    2.9210E-01    -0.
                 TAU*DX(N,N+1) KA*DX(N,N+1)   KB*DX(N,N+1)       P(N)/Z
                  5.8420E-02   -0.             -0.             -0.

                  MASS        WATER INERTIA    DX/EI(N)       DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)       P(N)/Y
12     0     1    1.0300E 01   -0.              6.3525E 00     3.1000E 01    1.1422E 01    1.0930E-01    -0.
```

```
                         TAU*DX(N,N+1) KA*DX(N,N+1)  KB*DX(N,N+1)    P(N)/Z
                          2.1860E-02    -0.           -0.           -0.

                          MASS        WATER INERTIA    DX/EI(N)       DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)      P(N)/Y
  13      0         1     1.0300E 01    -0.            6.3525E 00     3.1000E 01    1.1422E 01    1.0930E-01    -0.

                         TAU*DX(N,N+1) KA*DX(N,N+1)  KB*DX(N,N+1)    P(N)/Z
                          2.1860E-02    -0.           -0.           -0.

                          MASS        WATER INERTIA    DX/EI(N)       DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)      P(N)/Y
  14      0         1     1.0300E 01    -0.            6.3525E 00     3.1000E 01    1.1422E 01    1.0930E-01    -0.

                         TAU*DX(N,N+1) KA*DX(N,N+1)  KB*DX(N,N+1)    P(N)/Z
                          2.1860E-02    -0.           -0.           -0.

                          MASS        WATER INERTIA    DX/EI(N)       DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)      P(N)/Y
  15      0         1     1.0300E 01    -0.            6.3525E 00     3.1000E 01    1.1422E 01    1.0930E-01    -0.

                         TAU*DX(N,N+1) KA*DX(N,N+1)  KB*DX(N,N+1)    P(N)/Z
                          2.1860E-02    -0.           -0.           -0.

                          MASS        WATER INERTIA    DX/EI(N)       DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)      P(N)/Y
  16      0         1     1.0300E 01    -0.            6.3525E 00     3.1000E 01    1.1422E 01    1.0930E-01    -0.

                         TAU*DX(N,N+1) KA*DX(N,N+1)  KB*DX(N,N+1)    P(N)/Z
                          2.1860E-02    -0.           -0.           -0.

                          MASS        WATER INERTIA    DX/EI(N)       DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)      P(N)/Y
  17      0         1     1.0300E 01    -0.            6.3525E 00     3.1000E 01    1.1422E 01    1.0930E-01    -0.

                         TAU*DX(N,N+1) KA*DX(N,N+1)  KB*DX(N,N+1)    P(N)/Z
                          2.1860E-02    -0.           -0.           -0.

                          MASS        WATER INERTIA    DX/EI(N)       DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)      P(N)/Y
  18      0         1     1.0300E 01    -0.            6.3525E 00     3.1000E 01    1.1422E 01    1.0930E-01    -0.

                         TAU*DX(N,N+1) KA*DX(N,N+1)  KB*DX(N,N+1)    P(N)/Z
                          2.1860E-02    -0.           -0.           -0.

                          MASS        WATER INERTIA    DX/EI(N)       DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)      P(N)/Y
  19      2         1     1.5000E 01    -0.            9.2213E 00    -0.            -0.            -0.           -0.

                         TAU*DX(N,N+1) KA*DX(N,N+1)  KB*DX(N,N+1)    P(N)/Z
                         -0.            -0.           -0.           -0.

REAL PARTS OF SCALING FACTORS

   N       M       SYSTEM   K(N,M)/Y      C(N,M)/Y      C(N,M)/W/Y    DX(N,M)        DX/KAG(N,M)   TAUD*DX(N,M)   Q(N,M)/Y
                      1     1.0000E 06    -0.           -0.           -0.            -0.           -0.            -0.

                         TAU*DX(N,M)   KA*DX(N,M)     KB*DX(N,M)      K(N,M)/Z      C(N,M)/Z      C(N,M)/W/Z     Q(N,M)/Z
```

```
               -0.          -0.          -0.          1.0000E 09   -0.          -0.          -0.

                K(N,M)/Y     C(N,M)/Y     C(N,M)/W/Y   DX(N,M)      DX/KAG(N,M)  TAUD*DX(N,M) Q(N,M)/Y
            2   1.0000E 16  -0.          -0.          -0.          -0.          -0.          -0.

               TAU*DX(N,M)  KA*DX(N,M)   KB*DX(N,M)    K(N,M)/Z     C(N,M)/Z    C(N,M)/W/Z   Q(N,M)/Z
               -0.          -0.          -0.          1.0000E 16   -0.          -0.          -0.

PARAMETER VALUES FOR SPECIAL CONNECTIONS

     N       M      SYSTEM  K(N,M)/Y     C(N,M)/Y     C(N,M)/W/Y   DX(N,M)      DX/KAG(N,M)  TAUD*DX(N,M) Q(N,M)/Y
     6       0        1     9.7030E 00  -0.          -0.          -0.          -0.          -0.          -0.

               TAU*DX(N,M)  KA*DX(N,M)   KB*DX(N,M)    K(N,M)/Z     C(N,M)/Z    C(N,M)/W/Z   Q(N,M)/Z
               -0.          -0.          -0.          2.5680E 00   -0.          -0.          -0.

                K(N,M)/Y     C(N,M)/Y     C(N,M)/W/Y   DX(N,M)      DX/KAG(N,M)  TAUD*DX(N,M) Q(N,M)/Y
     8       0        1     4.3660E 00  -0.          -0.          -0.          -0.          -0.          -0.

               TAU*DX(N,M)  KA*DX(N,M)   KB*DX(N,M)    K(N,M)/Z     C(N,M)/Z    C(N,M)/W/Z   Q(N,M)/Z
               -0.          -0.          -0.          -0.          -0.          -0.          -0.

                K(N,M)/Y     C(N,M)/Y     C(N,M)/W/Y   DX(N,M)      DX/KAG(N,M)  TAUD*DX(N,M) Q(N,M)/Y
    12       0        1     1.3100E 00  -0.          -0.          -0.          -0.          -0.          -0.

               TAU*DX(N,M)  KA*DX(N,M)   KB*DX(N,M)    K(N,M)/Z     C(N,M)/Z    C(N,M)/W/Z   Q(N,M)/Z
               -0.          -0.          -0.          -0.          -0.          -0.          -0.

                K(N,M)/Y     C(N,M)/Y     C(N,M)/W/Y   DX(N,M)      DX/KAG(N,M)  TAUD*DX(N,M) Q(N,M)/Y
    13       0        2     1.0000E 00  -0.          -0.          -0.          -0.          -0.          -0.

               TAU*DX(N,M)  KA*DX(N,M)   KB*DX(N,M)    K(N,M)/Z     C(N,M)/Z    C(N,M)/W/Z   Q(N,M)/Z
               -0.          -0.          -0.          1.0000E 00   -0.          -0.          -0.
NFREQ= 3
 CHARACTRON PLOTTING CHARACTERS
CHAR(1)=
CHAR(2)=      +
CHAR(3)=      O
CHAR(4)=      X
CHAR(5)=      -
CHAR(6)=      .
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SBRC3
          SAMPLE PROBLEM 1

          USS OBSERVATION ISLAND, WHIRLING FREQUENCIES,-AIR, H=1

FREQUENCY       6.37 CPS

                                                  XY-PLANE

SECTION                    DEFLECTIONS                                                      MOMENTS
          REAL PART   IMAG PART    ABS VALUE    PHASE ANGLE  REAL PART   IMAG PART    ABS VALUE    PHASE ANGLE
    1     1.0000E 00   0.          1.0000E 00   0.           7.4734E-06 -0.           7.4734E-06 -0.
    2     7.6085E-01 -0.           7.6085E-01 -0.           -2.6623E 06 -0.           2.6623E 06  0.
    3     5.2170E-01 -0.           5.2170E-01 -0.           -1.4111E 07 -0.           1.4111E 07  0.
    4     2.8256E-01  0.           2.8256E-01  0.           -2.6948E 07 -0.           2.6948E 07  0.
    5     2.3928E-01  0.           2.3928E-01  0.           -2.8097E 07 -0.           2.8097E 07  0.
    6     5.7307E-02  0.           5.7307E-02  0.           -3.4937E 07  0.           3.4937E 07  0.
    7    -2.9807E-02 -0.           2.9807E-02  0.           -2.2349E 07  0.           2.2349E 07  0.
    8    -6.0814E-02 -0.           6.0814E-02  0.           -9.7383E 06  0.           9.7383E 06  0.
    9    -7.1077E-02 -0.           7.1077E-02  0.           -8.3130E 06  0.           8.3130E 06  0.
   10    -7.7505E-02 -0.           7.7505E-02  0.           -6.8778E 06  0.           6.8778E 06  0.
   11    -8.0757E-02 -0.           8.0757E-02  0.           -5.4320E 06  0.           5.4320E 06  0.
   12    -6.7943E-02 -0.           6.7943E-02  0.           -9.9223E 05  0.           9.9223E 05  0.
   13    -5.8165E-02 -0.           5.8165E-02  0.           -6.5124E 05  0.           6.5124E 05  0.
   14    -4.7093E-02 -0.           4.7093E-02  0.           -2.7409E 05  0.           2.7409E 05  0.
   15    -3.5473E-02 -0.           3.5473E-02  0.            1.3124E 05  0.           1.3124E 05  0.
   16    -2.4105E-02 -0.           2.4105E-02  0.            5.5637E 05  0.           5.5637E 05  0.
   17    -1.3827E-02 -0.           1.3827E-02  0.            9.9298E 05  0.           9.9298E 05  0.
   18    -5.5029E-03 -0.           5.5029E-03  0.            1.4332E 06  0.           1.4332E 06  0.
   19    -1.3696E-12 -0.           1.3696E-12  0.            1.8703E 06  0.           1.8703E 06  0.

                                                  XZ-PLANE

SECTION                    DEFLECTIONS                                                      MOMENTS
          REAL PART   IMAG PART    ABS VALUE    PHASE ANGLE  REAL PART   IMAG PART    ABS VALUE    PHASE ANGLE
    1    -0.           3.3229E 00  3.3229E 00   9.0000E 01 -0.            2.5434E-05  2.5434E-05  9.0000E 01
    2    -0.           2.5090E 00  2.5090E 00   9.0000E 01  0.           -8.8498E 06  8.8498E 05 -9.0000E 01
    3    -0.           1.6951E 00  1.6951E 00   9.0000E 01 -0.           -2.9058E 07  2.9058E 07 -9.0000E 01
    4    -0.           8.8122E-01  8.8122E-01   9.0000E 01 -0.           -5.3779E 07  5.3779E 07 -9.0000E 01
    5    -0.           7.3119E-01  7.3119E-01   9.0000E 01 -0.           -5.7557E 07  5.7557E 07 -9.0000E 01
    6     0.           3.4221E-04  3.4221E-04   9.0000E 01 -0.           -8.0057E 07  8.0057E 07 -9.0000E 01
    7     0.          -5.1821E-01  5.1821E-01  -9.0000E 01 -0.           -7.1797E 07  7.1797E 07 -9.0000E 01
    8     0.          -8.5640E-01  8.5640E-01  -9.0000E 01 -0.           -6.3197E 07  6.3197E 07 -9.0000E 01
    9     0.          -9.7217E-01  9.7217E-01  -9.0000E 01 -0.           -5.9411E 07  5.9411E 07 -9.0000E 01
   10     0.          -1.0605E 00  1.0605E 00  -9.0000E 01 -0.           -5.5507E 07  5.5507E 07 -9.0000E 01
   11     0.          -1.1232E 00  1.1232E 00  -9.0000E 01 -0.           -5.1475E 07  5.1475E 07 -9.0000E 01
   12     0.          -1.0972E 00  1.0972E 00  -9.0000E 01 -0.           -3.8154E 07  3.8154E 07 -9.0000E 01
   13     0.          -1.0040E 00  1.0040E 00  -9.0000E 01 -0.           -2.8413E 07  2.8413E 07 -9.0000E 01
   14     0.          -8.5460E-01  8.5460E-01  -9.0000E 01 -0.           -1.8154E 07  1.8154E 07 -9.0000E 01
   15     0.          -6.6931E-01  6.6931E-01  -9.0000E 01 -0.           -7.4566E 06  7.4566E 06 -9.0000E 01
   16     0.          -4.6921E-01  4.6921E-01  -9.0000E 01 -0.            3.5846E 06  3.5846E 06  9.0000E 01
   17     0.          -2.7607E-01  2.7607E-01  -9.0000E 01 -0.            1.4865E 07  1.4865E 07  9.0000E 01
   18     0.          -1.1217E-01  1.1217E-01  -9.0000E 01 -0.            2.6286E 07  2.6286E 07  9.0000E 01
   19     0.          -3.6793E-11  3.6793E-11  -9.0000E 01 -0.            3.7760E 07  3.7760E 07  9.0000E 01
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GBRC3
         SAMPLE PROBLEM 1

         USS OBSERVATION ISLAND, WHIRLING FREQUENCIES,-AIR, H=1

FREQUENCY      8.08 CPS

                                                 XY-PLANE

SECTION                      DEFLECTIONS                                                  MOMENTS
          REAL PART   IMAG PART    ABS VALUE   PHASE ANGLE  REAL PART   IMAG PART    ABS VALUE   PHASE ANGLE
    1     1.0000E 00  0.           1.0000E 00  0.           7.2791E-06 -0.           7.2791E-06 -0.
    2     7.6707E-01 -0.           7.6707E-01 -0.          -4.2835E 06 -0.           4.2835E 06  0.
    3     5.3414E-01 -0.           5.3414E-01 -0.          -1.2052E 07 -0.           1.2052E 07  0.
    4     3.0120E-01  0.           3.0120E-01  0.          -2.2108E 07 -0.           2.2108E 07  0.
    5     2.5847E-01  0.           2.5847E-01  0.          -2.3966E 07 -0.           2.3966E 07  0.
    6     6.9281E-02  0.           6.9281E-02  0.          -3.5073E 07  0.           3.5073E 07  0.
    7    -2.3227E-02 -0.           2.3227E-02  0.          -2.2724E 07  0.           2.2724E 07  0.
    8    -5.8683E-02 -0.           5.8683E-02  0.          -1.0351E 07  0.           1.0351E 07  0.
    9    -7.0517E-02 -0.           7.0517E-02  0.          -8.8801E 06  0.           8.8801E 06  0.
   10    -7.8252E-02 -0.           7.8252E-02  0.          -7.3952E 06  0.           7.3952E 06  0.
   11    -8.2572E-02 -0.           8.2572E-02  0.          -5.8953E 06  0.           5.8953E 06  0.
   12    -7.1015E-02 -0.           7.1015E-02  0.          -1.2523E 06  0.           1.2523E 06  0.
   13    -6.1628E-02 -0.           6.1628E-02  0.          -8.8123E 05  0.           8.8123E 05  0.
   14    -5.0487E-02 -0.           5.0487E-02  0.          -4.6065E 05  0.           4.6065E 05  0.
   15    -3.8424E-02 -0.           3.8424E-02  0.           6.6603E 02  0.           6.6603E 02  0.
   16    -2.6351E-02 -0.           2.6351E-02  0.           4.9321E 05  0.           4.9321E 05  0.
   17    -1.5240E-02 -0.           1.5240E-02  0.           1.0075E 06  0.           1.0075E 06  0.
   18    -6.1095E-03 -0.           6.1095E-03  0.           1.5349E 06  0.           1.5349E 06  0.
   19    -1.7277E-12 -0.           1.7277E-12  0.           2.0683E 06  0.           2.0683E 06  0.

                                                 XZ-PLANE

SECTION                      DEFLECTIONS                                                  MOMENTS
          REAL PART   IMAG PART    ABS VALUE   PHASE ANGLE  REAL PART   IMAG PART    ABS VALUE   PHASE ANGLE
    1     0.         -4.1186E-01   4.1186E-01 -9.0000E 01  0.          -3.1069E-06   3.1069E-06 -9.0000E 01
    2     0.         -3.1244E-01   3.1244E-01 -9.0000E 01  0.           1.7626E 06   1.7626E 06  9.0000E 01
    3     0.         -2.1302E-01   2.1302E-01 -9.0000E 01 -0.           2.7283E 06   2.7283E 06  9.0000E 01
    4     0.         -1.1360E-01   1.1360E-01 -9.0000E 01 -0.           4.6066E 06   4.6066E 06  9.0000E 01
    5     0.         -9.5023E-02   9.5023E-02 -9.0000E 01 -0.           5.3614E 06   5.3614E 06  9.0000E 01
    6     0.         -5.9375E-05   5.9375E-05 -9.0000E 01 -0.           9.8704E 06   9.8704E 06  9.0000E 01
    7    -0.          6.8161E-02   6.8161E-02  9.0000E 01 -0.           9.0444E 06   9.0444E 06  9.0000E 01
    8    -0.          1.1365E-01   1.1365E-01  9.0000E 01 -0.           8.1479E 06   8.1479E 06  9.0000E 01
    9    -0.          1.2938E-01   1.2938E-01  9.0000E 01 -0.           7.7415E 06   7.7415E 06  9.0000E 01
   10    -0.          1.4154E-01   1.4154E-01  9.0000E 01 -0.           7.3104E 06   7.3104E 06  9.0000E 01
   11    -0.          1.5031E-01   1.5031E-01  9.0000E 01 -0.           6.8520E 06   6.8520E 06  9.0000E 01
   12    -0.          1.4809E-01   1.4809E-01  9.0000E 01 -0.           5.2184E 06   5.2184E 06  9.0000E 01
   13     0.          1.3625E-01   1.3625E-01  9.0000E 01 -0.           3.9696E 06   3.9696E 06  9.0000E 01
   14     0.          1.1654E-01   1.1654E-01  9.0000E 01 -0.           2.6082E 06   2.6082E 06  9.0000E 01
   15     0.          9.1665E-02   9.1665E-02  9.0000E 01 -0.           1.1506E 06   1.1506E 06  9.0000E 01
   16     0.          6.4495E-02   6.4495E-02  9.0000E 01  0.          -3.8282E 05   3.8282E 05 -9.0000E 01
   17     0.          3.8059E-02   3.8059E-02  9.0000E 01  0.          -1.9694E 06   1.9694E 06 -9.0000E 01
   18     0.          1.5490E-02   1.5490E-02  9.0000E 01  0.          -3.5874E 06   3.5874E 06 -9.0000E 01
   19     0.          5.2592E-12   5.2592E-12  9.0000E 01  0.          -5.2180E 06   5.2180E 06 -9.0000E 01
```

```
GBRC3
          SAMPLE PROBLEM 1

          USS OBSERVATION ISLAND, WHIRLING FREQUENCIES.-AIR, H=1

FREQUENCY      23.54 CPS

                                                   XY-PLANE

SECTION                    DEFLECTIONS                                                     MOMENTS
           REAL PART  IMAG PART     ABS VALUE   PHASE ANGLE  REAL PART  IMAG PART     ABS VALUE   PHASE ANGLE
   1     1.0000E 00  0.            1.0000E 00  0.           2.0567E-06 -0.           2.0567E-06 -0.
   2     9.3419E-01 -0.            9.3419E-01 -0.          -3.6702E 07 -0.           3.6702E 07  0.
   3     8.6837E-01 -0.            8.6837E-01 -0.           5.0479E 07  0.           5.0479E 07  0.
   4     8.0256E-01  0.            8.0256E-01  0.           1.0610E 08  0.           1.0610E 08  0.
   5     7.7524E-01  0.            7.7524E-01  0.           8.6797E 07  0.           8.6797E 07  0.
   6     3.9547E-01  0.            3.9547E-01  0.          -3.3477E 07  0.           3.3477E 07  0.
   7     1.4793E-01  0.            1.4793E-01  0.          -2.3544E 07  0.           2.3544E 07  0.
   8    -4.1007E-02 -0.            4.1007E-02  0.          -1.5215E 07  0.           1.5215E 07  0.
   9    -1.1593E-01 -0.            1.1593E-01  0.          -1.5401E 07  0.           1.5401E 07  0.
  10    -1.8368E-01 -0.            1.8368E-01  0.          -1.5335E 07  0.           1.5335E 07  0.
  11    -2.4425E-01 -0.            2.4425E-01  0.          -1.4841E 07  0.           1.4841E 07  0.
  12    -3.6304E-01 -0.            3.6304E-01  0.          -1.6658E 07  0.           1.6658E 07  0.
  13    -4.1657E-01 -0.            4.1657E-01  0.          -2.3786E 07  0.           2.3786E 07  0.
  14    -4.2218E-01 -0.            4.2218E-01  0.          -2.4947E 07  0.           2.4947E 07  0.
  15    -3.7758E-01  0.            3.7758E-01  0.          -2.0358E 07 -0.           2.0358E 07  0.
  16    -2.9191E-01  0.            2.9191E-01  0.          -1.1043E 07 -0.           1.1043E 07  0.
  17    -1.8375E-01  0.            1.8375E-01  0.           1.3004E 06  0.           1.3004E 06  0.
  18    -7.7670E-02  0.            7.7670E-02  0.           1.4526E 07  0.           1.4526E 07  0.
  19    -2.1163E-11 -0.            2.1163E-11  0.           2.6325E 07  0.           2.6325E 07  0.

                                                   XZ-PLANE

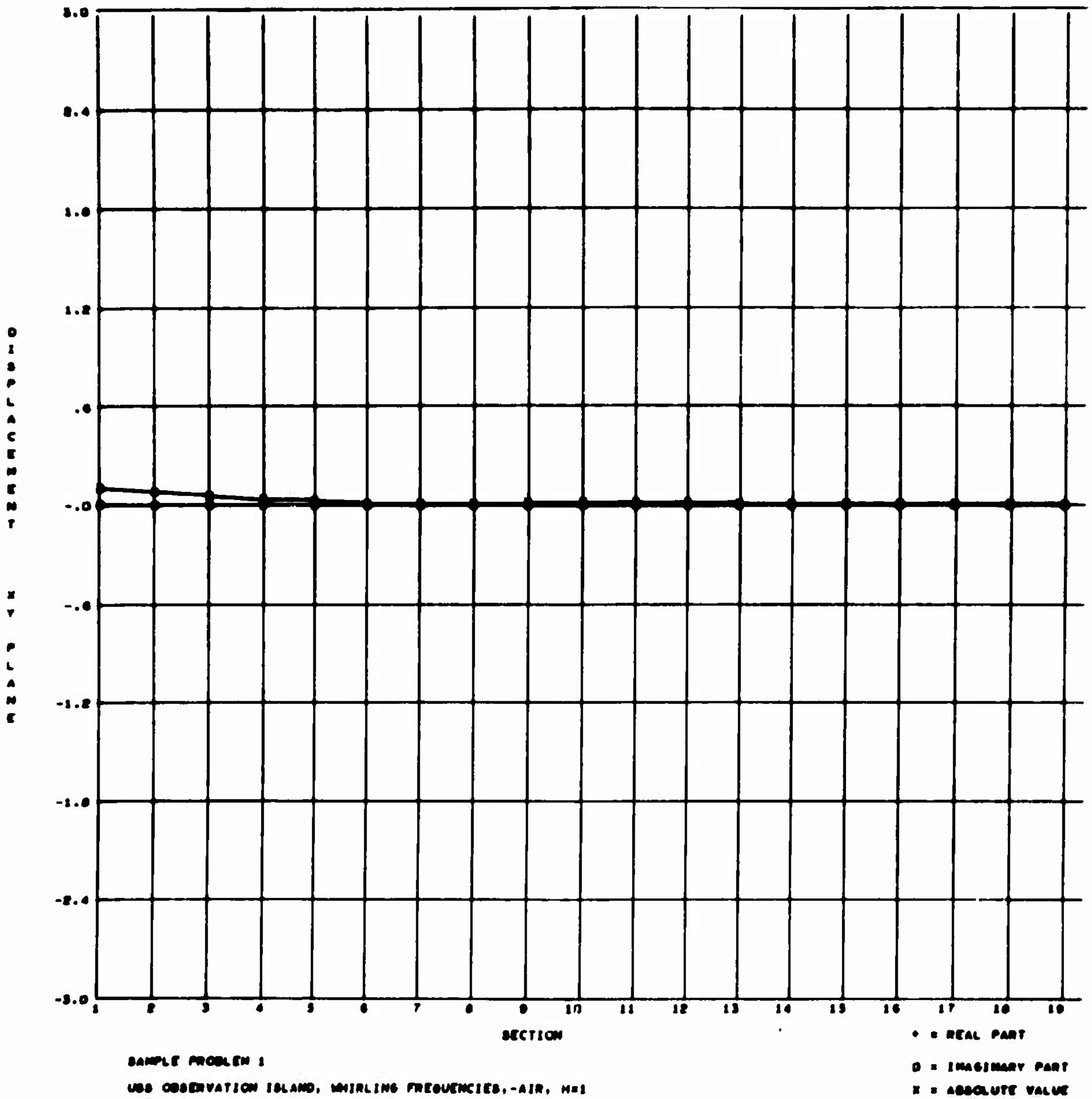
SECTION                    DEFLECTIONS                                                     MOMENTS
           REAL PART  IMAG PART     ABS VALUE   PHASE ANGLE  REAL PART  IMAG PART     ABS VALUE   PHASE ANGLE
   1      0.         -4.3748E 00  4.3748E 00 -9.0000E 01  0.        -5.0913E-05  5.0913E-05 -9.0000E 01
   2      0.         -2.7456E 00  2.7456E 00 -9.0000E 01  0.         1.5906E 08  1.5906E 08  9.0000E 01
   3      0.         -1.1164E 00  1.1164E 00 -9.0000E 01 -0.         4.9194E 08  4.9194E 08  9.0000E 01
   4     -0.          5.1285E-01  5.1285E-01  9.0000E 01 -0.         8.6539E 08  8.6539E 08  9.0000E 01
   5     -0.          7.7513E-01  7.7513E-01  9.0000E 01 -0.         9.2142E 08  9.2142E 08  9.0000E 01
   6      0.         -1.0479E-02  1.0479E-02 -9.0000E 01 -0.         1.2410E 09  1.2410E 09  9.0000E 01
   7      0.         -4.2583E 00  4.2583E 00 -9.0000E 01 -0.         6.1854E 08  6.1854E 08  9.0000E 01
   8      0.         -1.0045E 01  1.0045E 01 -9.0000E 01 -0.         3.3902E 07  3.3902E 07  9.0000E 01
   9      0.         -1.2534E 01  1.2534E 01 -9.0000E 01  0.        -2.0080E 08  2.0080E 08 -9.0000E 01
  10      0.         -1.4924E 01  1.4924E 01 -9.0000E 01  0.        -4.1493E 08  4.1493E 08 -9.0000E 01
  11      0.         -1.7113E 01  1.7113E 01 -9.0000E 01  0.        -6.0455E 08  6.0455E 08 -9.0000E 01
  12      0.         -2.1078E 01  2.1078E 01 -9.0000E 01 -0.        -9.2426E 08  9.2426E 08 -9.0000E 01
  13      0.         -2.1935E 01  2.1935E 01 -9.0000E 01 -0.        -9.9394E 08  9.9394E 08 -9.0000E 01
  14      0.         -2.0779E 01  2.0779E 01 -9.0000E 01 -0.        -9.0877E 08  9.0877E 08 -9.0000E 01
  15      0.         -1.7779E 01  1.7779E 01 -9.0000E 01 -0.        -6.7695E 08  6.7695E 08 -9.0000E 01
  16      0.         -1.3400E 01  1.3400E 01 -9.0000E 01 -0.        -3.1981E 08  3.1981E 08 -9.0000E 01
  17      0.         -8.3577E 00  8.3577E 00 -9.0000E 01 -0.         1.3149E 08  1.3149E 08  9.0000E 01
  18      0.         -3.5523E 00  3.5523E 00 -9.0000E 01 -0.         6.4098E 08  6.4098E 08  9.0000E 01
  19      0.         -1.7116E-09  1.7116E-09 -9.0000E 01 -0.         1.1743E 09  1.1743E 09  9.0000E 01
```

```
 GBRC3 OCT 7, 1966
 SAMPLE PROBLEM 1

DATA CONTROL CARC

NO  TYPE
```

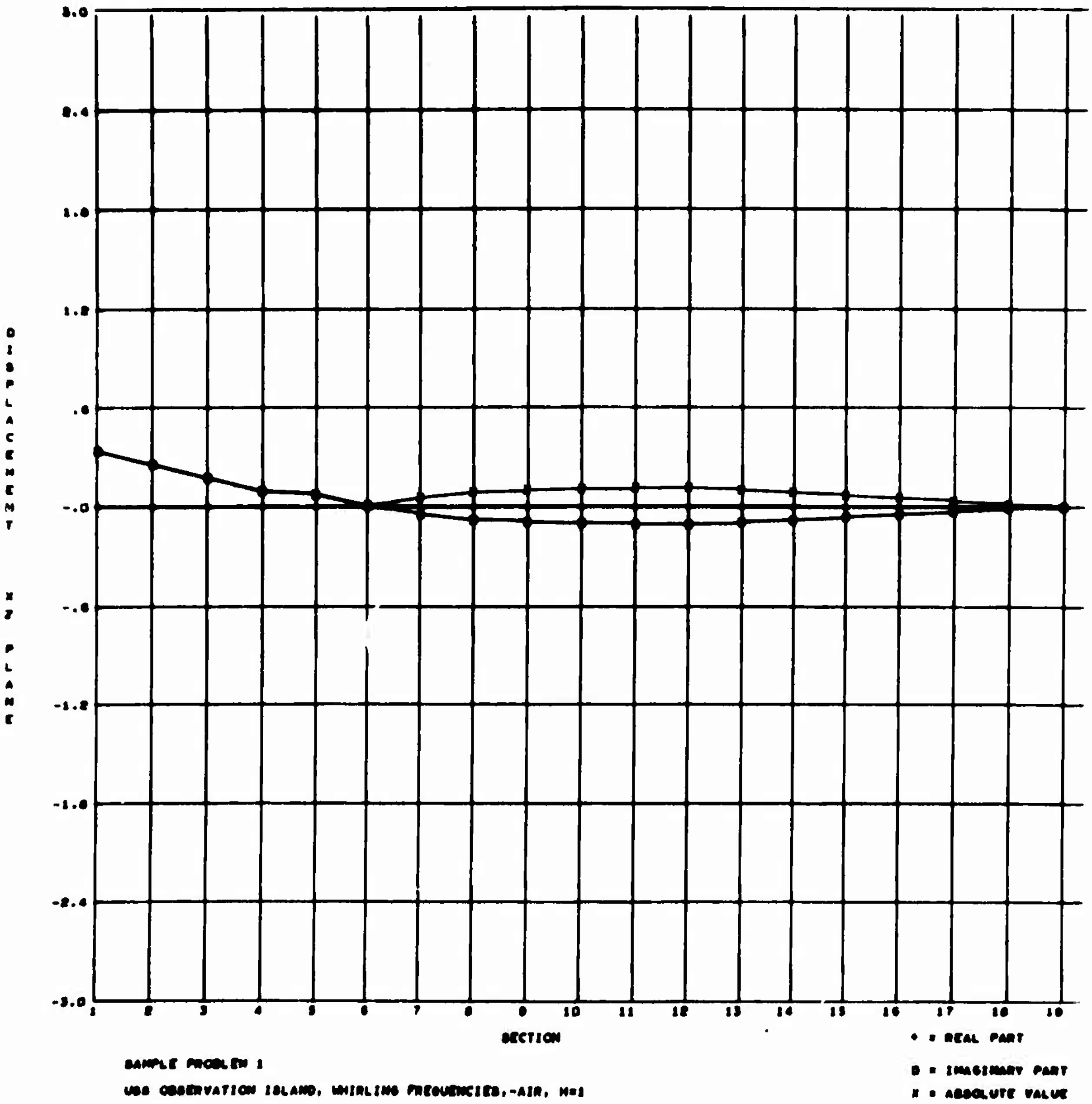
END GBRC3 RUN



SAMPLE PROBLEM 1

USS OBSERVATION ISLAND, WHIRLING FREQUENCIES,-AIR, M=1

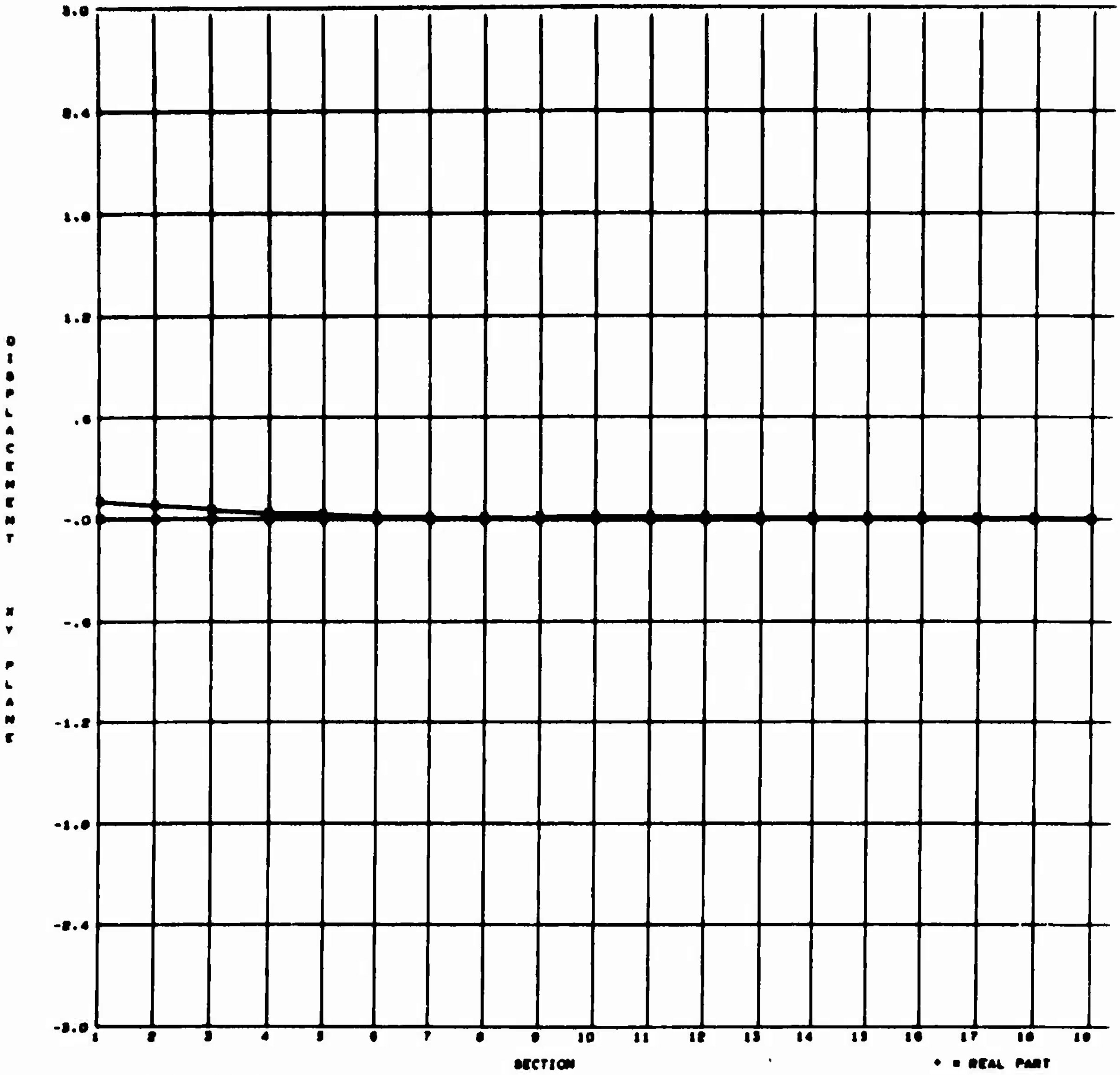
FREQUENCY = 6.37 CPS SCALING FOR DISPLACEMENT = 10. E 1
SYSTEM = 1



SAMPLE PROBLEM 1

USS OBSERVATION ISLAND, WHIRLING FREQUENCIES,-AIR, N=1

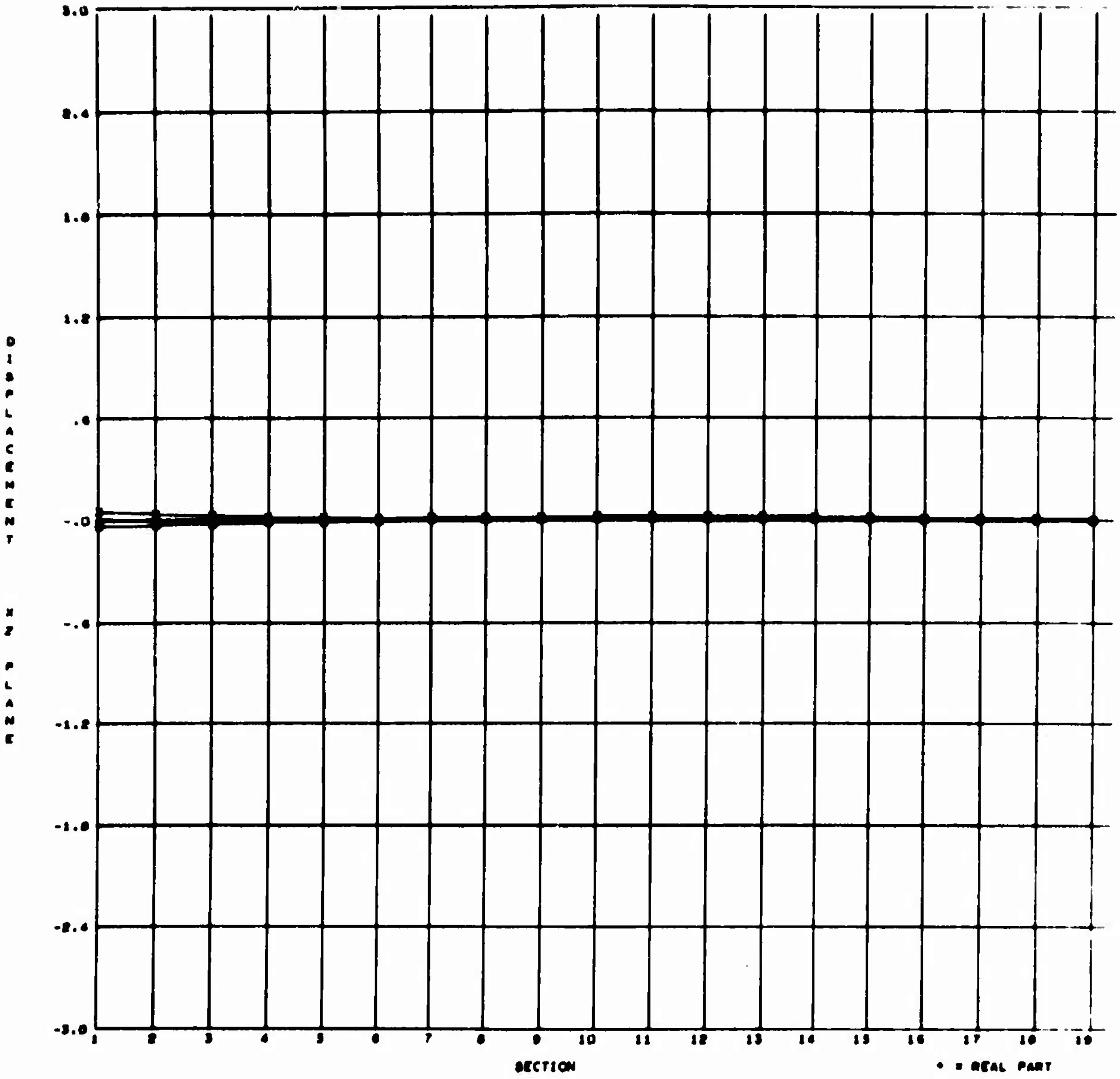
FREQUENCY = 6.37 CPS SCALING FOR DISPLACEMENT = 10. E 1
SYSTEM = 1



SAMPLE PROBLEM 1

USS OBSERVATION ISLAND, WHIRLING FREQUENCIES,-AIR, M=1

FREQUENCY = 0.00 CPS SCALING FOR DISPLACEMENT = 10. E1
SYSTEM = 1



+ = REAL PART

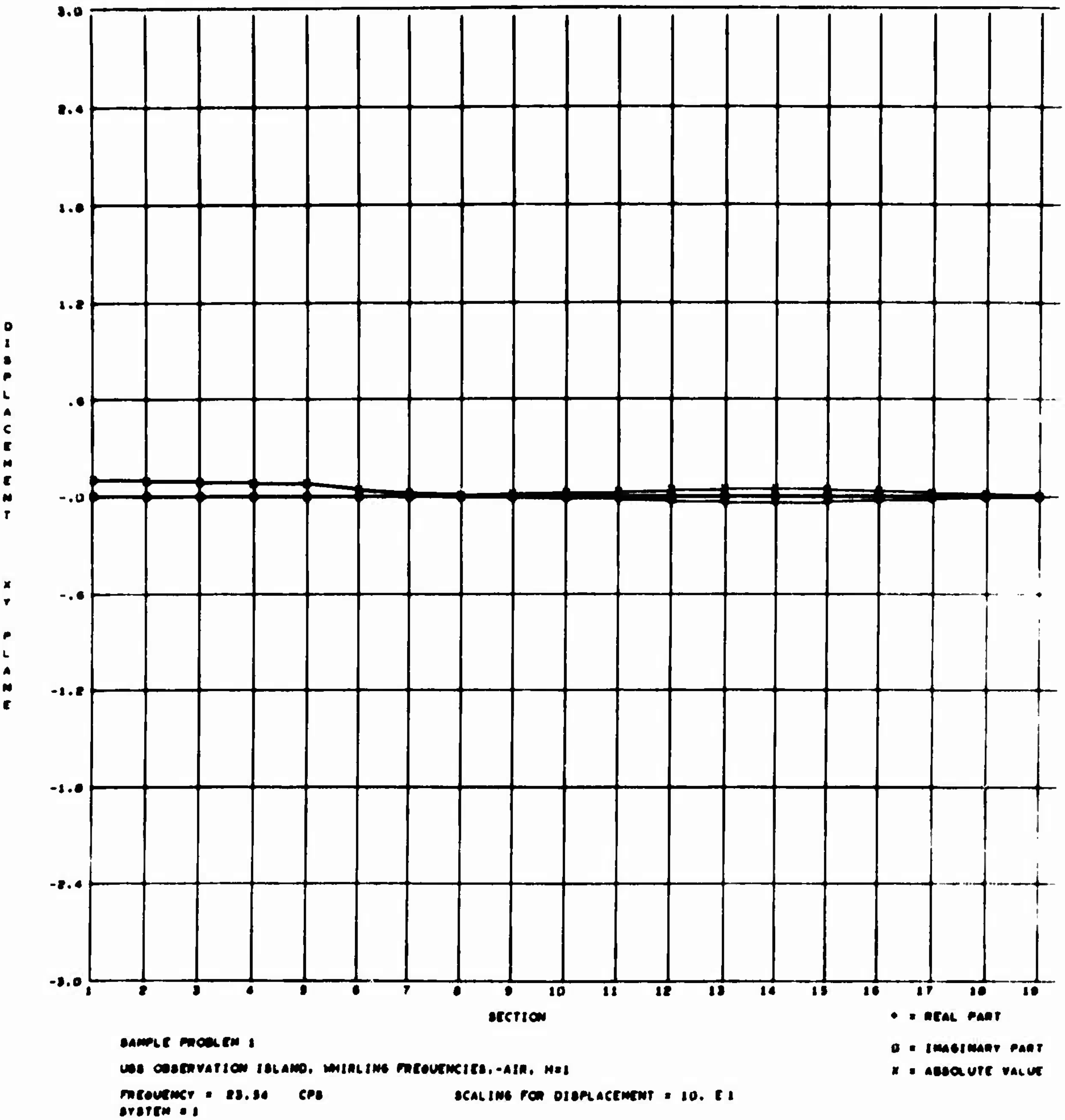
O = IMAGINARY PART

X = ABSOLUTE VALUE

SAMPLE PROBLEM 1

USS OBSERVATION ISLAND, WHIRLING FREQUENCIES,-AIR, M=1

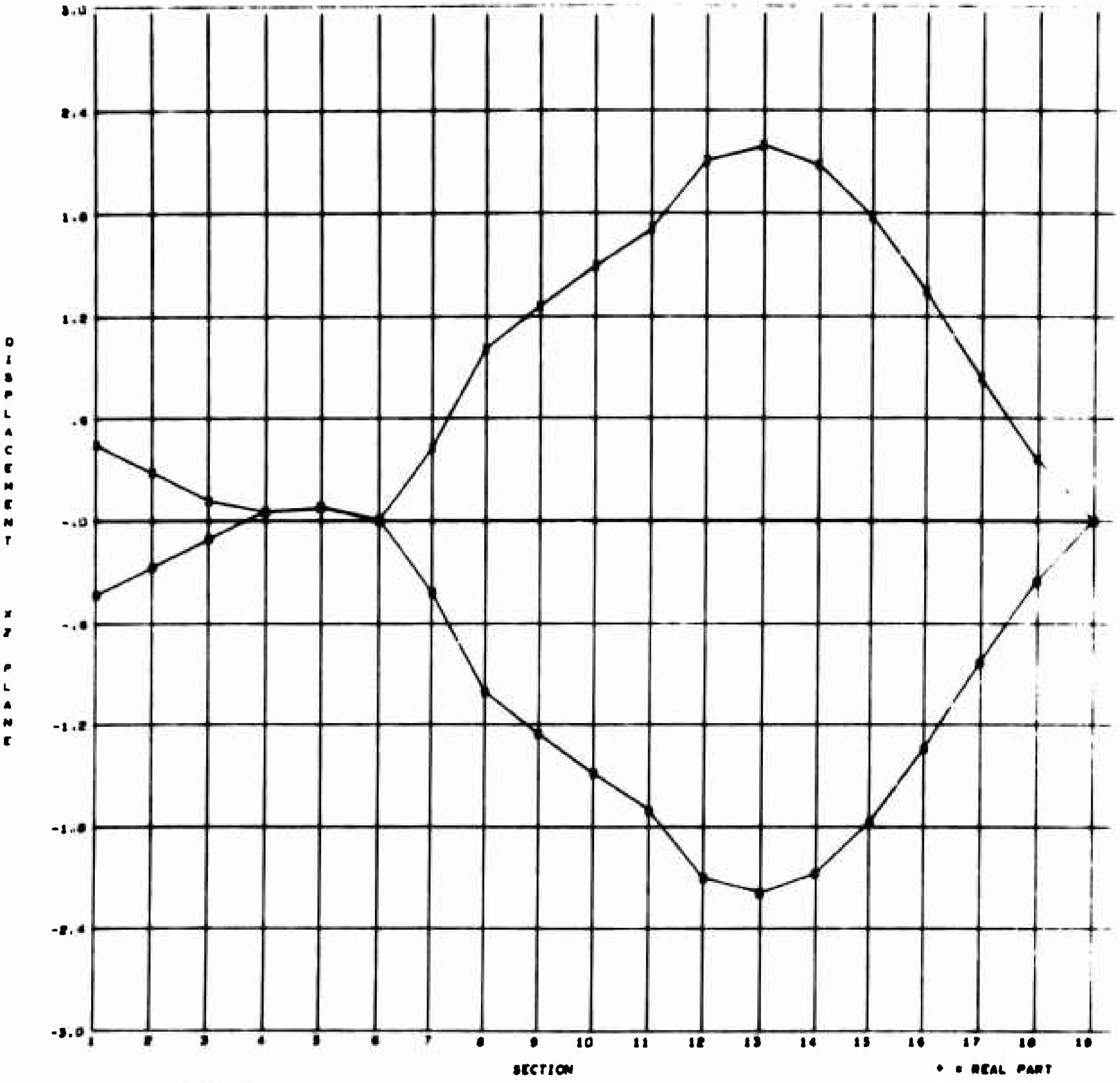
FREQUENCY = 0.00 CPS SCALING FOR DISPLACEMENT = 10. E 1
SYSTEM = 1



SAMPLE PROBLEM 1

USS OBSERVATION ISLAND, WHIRLING FREQUENCIES,-AIR, N=1

FREQUENCY = 23.54 CPS SCALING FOR DISPLACEMENT = 10. E 1
SYSTEM = 1



SAMPLE PROBLEM 1

USS OBSERVATION ISLAND, WHIRLING FREQUENCIES,-AIR, N=1

FREQUENCY = 23.34 CPS SCALING FOR DISPLACEMENT = 10. E 1
SYSTEM = 1

Problem 2. The data for this calculation is exactly that of Problem 1. except that h is specified as 0.25.

It should be noted that both of these calculations can be done in one running on the computer by using the following sequence of cards after the last card of shaft data for Problem 1.:

1)

```
00 90 0110 0130
```

2)

```
001000(62H△△TITLE FOR THIS CASE
```

3)

```
003000191.0  30.0   1.0     0.25
```

4)

```
0099
```

This affects a saving in computer time since only the case title and values on the type "30" card are restored in memory, all other data remaining as it was in the previous problem.

Since the output presented here is from calculations performed at different stages of the program's testing, it doesn't reflect this facility.

```
 GBRC3 CCT 7. 1966
 SAMPLE PROBLEM 2

DATA CONTROL CARD

NO  TYPE
 1   10
 1   20
 1   30
 1   41
19   43
 2   51
 4   53
 1   70
 1   60

CASE TITLE-USS OBSERVATIUN ISLAND. WHIRLING FREQUENCIES.-AIR. H=1/4

CPTION DATA
 20      -0   2  -0   1  -0   1  -0  -0  -0   1

GENERAL DATA - NUMBER OF SECTIONS     19
                 WHIRLING FREQUENCY RANGE FROM        1.000 CPS  TO    30.000 CPS
                 FREQUENCY INTERVAL    1.000 CPS
                 RATIC OF SPIN TO WHIRL =    0.250


REAL PARTS OF SCALING FACTORS

SECTION-END CONDN-SYSTEM     MASS      WATER INERTIA     DX/EI(N)       DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)     P(N)/Y
                       1   1.0000E 00   -0.              1.0000E-11    1.0000E 00    1.0000E-09    1.0000E 04     1.0000E 00

                         TAU*DX(N,N+1) KA*DX(N,N+1)  KB*DX(N,N+1)       P(N)/Z
                           1.0000E 05   -0.              1.0000E 09    1.0000E 00


PARAMETER VALUES FOR EACH SECTION - UNSCALED

SECTION-END CONDN-SYSTEM     MASS      WATER INERTIA     DX/EI(N)       DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)     P(N)/Y
     1        1        1   5.1900E 01   -0.              1.5310E 01    3.2000E 01    3.2000E-06    5.7880E-03   -0.

                         TAU*CX(N,N+1) KA*DX(N,N+1)  KB*DX(N,N+1)       P(N)/Z
                           1.1580E-03   -0.            -0.            -0.


                             MASS     WATER INERTIA     DX/EI(N)       DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)     P(N)/Y
     2        0        1   5.1900E 01   -0.              1.0000E-15    3.2000E 01    3.2000E-06    7.2350E 00   -0.

                         TAU*DX(N,N+1) KA*DX(N,N+1)  KB*DX(N,N+1)       P(N)/Z
                           1.4470E 00   -0.            -0.            -0.


                             MASS     WATER INERTIA     DX/EI(N)       DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)     P(N)/Y
     3        0        1   5.1900E 01   -0.              1.0000E-15    3.2000E 01    1.9000E-06    7.2350E 00   -0.

                         TAU*DX(N,N+1) KA*DX(N,N+1)  KB*DX(N,N+1)       P(N)/Z
                           1.4470E 00   -0.            -0.            -0.
```

```
                         MASS         WATER INERTIA     DX/EI(N)       DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)        P(N)/Y
   4       0       1     1.9960E 00     -0.             1.2295E 00     6.0000E 00    2.2108E 00    5.7880E-03    -0.

                        TAU*DX(N,N+1) KA*DX(N,N+1)    KB*DX(N,N+1)       P(N)/Z
                         1.1580E-03    -0.            -0.              -0.

                         MASS         WATER INERTIA     DX/EI(N)       DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)        P(N)/Y
   5       0       1     1.1640E 01     -0.             7.1721E 00     3.5000E 01    1.2896E 01    1.4910E-02    -0.

                        TAU*DX(N,N+1) KA*DX(N,N+1)    KB*DX(N,N+1)       P(N)/Z
                         2.9820E-03    -0.            -0.              -0.

                         MASS         WATER INERTIA     DX/EI(N)       DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)        P(N)/Y
   6       0       1     1.1640E 01     -0.             7.1721E 00     3.5000E 01    1.2896E 01    1.4910E-02    -0.

                        TAU*DX(N,N+1) KA*DX(N,N+1)    KB*DX(N,N+1)       P(N)/Z
                         2.9820E-03    -0.            -0.              -0.

                         MASS         WATER INERTIA     DX/EI(N)       DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)        P(N)/Y
   7       0       1     1.1640E 01     -0.             7.1721E 00     3.5000E 01    1.2896E 01    1.4910E-02    -0.

                        TAU*DX(N,N+1) KA*DX(N,N+1)    KB*DX(N,N+1)       P(N)/Z
                         2.9820E-03    -0.            -0.              -0.

                         MASS         WATER INERTIA     DX/EI(N)       DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)        P(N)/Y
   8       0       1     4.9900E 00     -0.             3.0738E 00     1.5000E 01    5.5269E 00    2.2330E-02    -0.

                        TAU*DX(N,N+1) KA*DX(N,N+1)    KB*DX(N,N+1)       P(N)/Z
                         4.4660E-03    -0.            -0.              -0.

                         MASS         WATER INERTIA     DX/EI(N)       DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)        P(N)/Y
   9       0       1     4.9900E 00     -0.             3.0738E 00     1.5000E 01    5.5269E 00    2.2330E-02    -0.

                        TAU*DX(N,N+1) KA*DX(N,N+1)    KB*DX(N,N+1)       P(N)/Z
                         4.4660E-03    -0.            -0.              -0.

                         MASS         WATER INERTIA     DX/EI(N)       DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)        P(N)/Y
  10       0       1     4.9900E 00     -0.             3.0738E 00     1.5000E 01    5.5269E 00    2.2330E-02    -0.

                        TAU*DX(N,N+1) KA*DX(N,N+1)    KB*DX(N,N+1)       P(N)/Z
                         4.4660E-03    -0.            -0.              -0.

                         MASS         WATER INERTIA     DX/EI(N)       DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)        P(N)/Y
  11       0       1     1.5000E 01     -0.             9.2213E 00     4.5000E 01    1.6580E 01    2.9210E-01    -0.

                        TAU*DX(N,N+1) KA*DX(N,N+1)    KB*DX(N,N+1)       P(N)/Z
                         5.8420E-02    -0.            -0.              -0.

                         MASS         WATER INERTIA     DX/EI(N)       DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)        P(N)/Y
  12       0       1     1.0300E 01     -0.             6.3525E 00     3.1000E 01    1.1422E 01    1.0930E-01    -0.
```

```
                              TAU*DX(N,N+1) KA*DX(N,N+1)  KB*DX(N,N+1)      P(N)/Z
                               2.1860E-02    -0.          -0.            -0.

                               MASS         WATER INERTIA   DX/EI(N)        DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)      P(N)/Y
     13        0        1      1.0300E 01    -0.            6.3525E 00      3.1000E 01     1.1422E 01     1.0930E-01   -0.

                              TAU*DX(N,N+1) KA*DX(N,N+1)  KB*DX(N,N+1)      P(N)/Z
                               2.1860E-02    -0.          -0.            -0.

                               MASS         WATER INERTIA   DX/EI(N)        DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)      P(N)/Y
     14        0        1      1.0300E 01    -0.            6.3525E 00      3.1000E 01     1.1422E 01     1.0930E-01   -0.

                              TAU*DX(N,N+1) KA*DX(N,N+1)  KB*DX(N,N+1)      P(N)/Z
                               2.1860E-02    -0.          -0.            -0.

                               MASS         WATER INERTIA   DX/EI(N)        DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)      P(N)/Y
     15        0        1      1.0300E 01    -0.            6.3525E 00      3.1000E 01     1.1422E 01     1.0930E-01   -0.

                              TAU*DX(N,N+1) KA*DX(N,N+1)  KB*DX(N,N+1)      P(N)/Z
                               2.1860E-02    -0.          -0.            -0.

                               MASS         WATER INERTIA   DX/EI(N)        DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)      P(N)/Y
     16        0        1      1.0300E 01    -0.            6.3525E 00      3.1000E 01     1.1422E 01     1.0930E-01   -0.

                              TAU*DX(N,N+1) KA*DX(N,N+1)  KB*DX(N,N+1)      P(N)/Z
                               2.1860E-02    -0.          -0.            -0.

                               MASS         WATER INERTIA   DX/EI(N)        DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)      P(N)/Y
     17        0        1      1.0300E 01    -0.            6.3525E 00      3.1000E 01     1.1422E 01     1.0930E-01   -0.

                              TAU*DX(N,N+1) KA*DX(N,N+1)  KB*DX(N,N+1)      P(N)/Z
                               2.1860E-02    -0.          -0.            -0.

                               MASS         WATER INERTIA   DX/EI(N)        DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)      P(N)/Y
     18        0        1      1.0300E 01    -0.            6.3525E 00      3.1000E 01     1.1422E 01     1.0930E-01   -0.

                              TAU*DX(N,N+1) KA*DX(N,N+1)  KB*DX(N,N+1)      P(N)/Z
                               2.1860E-02    -0.          -0.            -0.

                               MASS         WATER INERTIA   DX/EI(N)        DX(N,N+1)  DX/KAG(N,N+1)  TAUD*DX(N,N+1)      P(N)/Y
     19        2        1      1.5000E 01    -0.            9.2213E 00     -0.            -0.             -0.          -0.

                              TAU*DX(N,N+1) KA*DX(N,N+1)  KB*DX(N,N+1)      P(N)/Z
                              -0.            -0.          -0.            -0.

REAL PARTS OF SCALING FACTORS

     N         M       SYSTEM   K(N,M)/Y       C(N,M)/Y      C(N,M)/W/Y     DX(N,M)       DX/KAG(N,M)   TAUD*DX(N,M)    Q(N,M)/Y
                          1     1.0000E 06    -0.           -0.            -0.            -0.            -0.             -0.

                              TAU*DX(N,M)    KA*DX(N,M)    KB*DX(N,M)      K(N,M)/Z       C(N,M)/Z      C(N,M)/W/Z      Q(N,M)/Z
```

```
                     -0.            -0.            -0.            1.0000E 09   -0.            -0.            -0.

                      K(N,M)/Y       C(N,M)/Y       C(N,M)/W/Y     DX(N,M)        DX/KAG(N,M)   TAUD*DX(N,M)   Q(N,M)/Y
                 2   1.0000E 16     -0.            -0.            -0.            -0.            -0.            -0.

                     TAU*DX(N,M)    KA*DX(N,M)     KB*DX(N,M)      K(N,M)/Z      C(N,M)/Z      C(N,M)/W/Z     Q(N,M)/Z
                     -0.            -0.            -0.            1.0000E 16   -0.            -0.            -0.

PARAMETER VALUES FOR SPECIAL CONNECTIONS

   N        M       SYSTEM   K(N,M)/Y       C(N,M)/Y       C(N,M)/W/Y     DX(N,M)        DX/KAG(N,M)   TAUD*DX(N,M)   Q(N,M)/Y
   6        0        1      9.7030E 00     -0.            -0.            -0.            -0.            -0.            -0.

                     TAU*DX(N,M)    KA*DX(N,M)     KB*DX(N,M)      K(N,M)/Z      C(N,M)/Z      C(N,M)/W/Z     Q(N,M)/Z
                     -0.            -0.            -0.            2.5680E 00   -0.            -0.            -0.

                      K(N,M)/Y       C(N,M)/Y       C(N,M)/W/Y     DX(N,M)        DX/KAG(N,M)   TAUD*DX(N,M)   Q(N,M)/Y
   8        0        1      4.3660E 00     -0.            -0.            -0.            -0.            -0.            -0.

                     TAU*DX(N,M)    KA*DX(N,M)     KB*DX(N,M)      K(N,M)/Z      C(N,M)/Z      C(N,M)/W/Z     Q(N,M)/Z
                     -0.            -0.            -0.            -0.            -0.            -0.            -0.

                      K(N,M)/Y       C(N,M)/Y       C(N,M)/W/Y     DX(N,M)        DX/KAG(N,M)   TAUD*DX(N,M)   Q(N,M)/Y
  12        0        1      1.3100E 00     -0.            -0.            -0.            -0.            -0.            -0.

                     TAU*DX(N,M)    KA*DX(N,M)     KB*DX(N,M)      K(N,M)/Z      C(N,M)/Z      C(N,M)/W/Z     Q(N,M)/Z
                     -0.            -0.            -0.            -0.            -0.            -0.            -0.

                      K(N,M)/Y       C(N,M)/Y       C(N,M)/W/Y     DX(N,M)        DX/KAG(N,M)   TAUD*DX(N,M)   Q(N,M)/Y
  19        0        2      1.0000E 00     -0.            -0.            -0.            -0.            -0.            -0.

                     TAU*DX(N,M)    KA*DX(N,M)     KB*DX(N,M)      K(N,M)/Z      C(N,M)/Z      C(N,M)/W/Z     Q(N,M)/Z
                     -0.            -0.            -0.            1.0000E 00   -0.            -0.            -0.

NFREQ= 3
 CHARACTRON PLOTTING CHARACTERS
CHAR(1)=
CHAR(2)=     +
CHAR(3)=     0
CHAR(4)=     X
CHAR(5)=     -
CHAR(6)=     .
```

```
GBRC3
          SAMPLE PROBLEM 2

          USS OBSERVATION ISLAND. WHIRLING FREQUENCIES.-AIR. H=1/4

FREQUENCY      6.50 CPS

                                                        XY-PLANE

SECTION                       DEFLECTIONS                                                    MOMENTS
           REAL PART   IMAG PART    ABS VALUE   PHASE ANGLE    REAL PART   IMAG PART    ABS VALUE   PHASE ANGLE
     1   1.0000E 00  0.           1.0000E 00  0.              7.4553E-06 -0.           7.4553E-06 -0.
     2   7.6143E-01 -0.           7.6143E-01 -0.             -2.8729E 06 -0.           2.8729E 06  0.
     3   5.2286E-01 -0.           5.2286E-01 -0.             -1.3959E 07 -0.           1.3959E 07  0.
     4   2.8429E-01  0.           2.8429E-01  0.             -2.6495E 07 -0.           2.6495E 07  0.
     5   2.4106E-01  0.           2.4106E-01  0.             -2.7711E 07 -0.           2.7711E 07  0.
     6   5.8424E-02  0.           5.8424E-02  0.             -3.4950E 07  0.           3.4950E 07  0.
     7  -2.9187E-02 -0.           2.9187E-02  0.             -2.2384E 07  0.           2.2384E 07  0.
     8  -6.0601E-02 -0.           6.0601E-02  0.             -9.7963E 06  0.           9.7963E 06  0.
     9  -7.1006E-02 -0.           7.1006E-02  0.             -8.3660E 06  0.           8.3660E 06  0.
    10  -7.7551E-02 -0.           7.7551E-02  0.             -6.9255E 06  0.           6.9255E 06  0.
    11  -8.0900E-02 -0.           8.0900E-02  0.             -5.4742E 06  0.           5.4742E 06  0.
    12  -6.8196E-02 -0.           6.8196E-02  0.             -1.0156E 06  0.           1.0156E 06  0.
    13  -5.8451E-02 -0.           5.8451E-02  0.             -6.7107E 05  0.           6.7107E 05  0.
    14  -4.7374E-02 -0.           4.7374E-02  0.             -2.8970E 05  0.           2.8970E 05  0.
    15  -3.5716E-02 -0.           3.5716E-02  0.              1.2056E 05  0.           1.2056E 05  0.
    16  -2.4289E-02 -0.           2.4289E-02  0.              5.5134E 05  0.           5.5134E 05  0.
    17  -1.3943E-02 -0.           1.3943E-02  0.              9.9430E 05  0.           9.9430E 05  0.
    18  -5.5527E-03 -0.           5.5527E-03  0.              1.4416E 06  0.           1.4416E 06  0.
    19  -1.3992E-12 -0.           1.3992E-12  0.              1.8866E 06  0.           1.8866E 06  0.

                                                        XZ-PLANE

SECTION                       DEFLECTIONS                                                    MOMENTS
           REAL PART   IMAG PART    ABS VALUE   PHASE ANGLE    REAL PART   IMAG PART    ABS VALUE   PHASE ANGLE
     1  -0.           1.1274E 01  1.1274E 01  9.0000E 01 -0.            8.6213E-05  8.6213E-05  9.0000E 01
     2  -0.           8.5156E 00  8.5156E 00  9.0000E 01  0.           -3.1269E 07  3.1269E 07 -9.0000E 01
     3  -0.           5.7568E 00  5.7568E 00  9.0000E 01 -0.           -9.7003E 07  9.7003E 07 -9.0000E 01
     4  -0.           2.9980E 00  2.9980E 00  9.0000E 01 -0.           -1.7870E 08  1.7870E 08 -9.0000E 01
     5  -0.           2.4889E 00  2.4889E 00  9.0000E 01 -0.           -1.9205E 08  1.9205E 08 -9.0000E 01
     6   0.           1.1929E-03  1.1929E-03  9.0000E 01 -0.           -2.7158E 08  2.7158E 08 -9.0000E 01
     7   0.          -1.7653E 00  1.7653E 00 -9.0000E 01 -0.           -2.4390E 08  2.4390E 08 -9.0000E 01
     8   0.          -2.9192E 00  2.9192E 00 -9.0000E 01 -0.           -2.1500E 08  2.1500E 08 -9.0000E 01
     9   0.          -3.3144E 00  3.3144E 00 -9.0000E 01 -0.           -2.0226E 08  2.0226E 08 -9.0000E 01
    10   0.          -3.6162E 00  3.6162E 00 -9.0000E 01 -0.           -1.8911E 08  1.8911E 08 -9.0000E 01
    11   0.          -3.8307E 00  3.8307E 00 -9.0000E 01 -0.           -1.7549E 08  1.7549E 08 -9.0000E 01
    12   0.          -3.7443E 00  3.7443E 00 -9.0000E 01 -0.           -1.3032E 08  1.3032E 08 -9.0000E 01
    13   0.          -3.4274E 00  3.4274E 00 -9.0000E 01 -0.           -9.7191E 07  9.7191E 07 -9.0000E 01
    14   0.          -2.9184E 00  2.9184E 00 -9.0000E 01 -0.           -6.2224E 07  6.2224E 07 -9.0000E 01
    15   0.          -2.2863E 00  2.2863E 00 -9.0000E 01 -0.           -2.5693E 07  2.5693E 07 -9.0000E 01
    16   0.          -1.6032E 00  1.6032E 00 -9.0000E 01 -0.            1.2059E 07  1.2059E 07  9.0000E 01
    17   0.          -9.4351E-01  9.4351E-01 -9.0000E 01 -0.            5.0664E 07  5.0664E 07  9.0000E 01
    18   0.          -3.8341E-01  3.8341E-01 -9.0000E 01 -0.            8.9766E 07  8.9766E 07  9.0000E 01
    19   0.          -1.2669E-10  1.2669E-10 -9.0000E 01 -0.            1.2906E 08  1.2906E 08  9.0000E 01
```

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GBRC3
          SAMPLE PROBLEM 2

          USS OBSERVATION ISLAND, WHIRLING FREQUENCIES,-AIR, H=1/4

FREQUENCY       7.84 CPS

                                                       XY-PLANE

SECTION                 DEFLECTIONS                                             MOMENTS
        REAL PART IMAG PART    ABS VALUE  PHASE ANGLE   REAL PART  IMAG PART    ABS VALUE  PHASE ANGLE
   1   1.0000E 00  0.          1.0000E 00  0.           7.3091E-06 -0.          7.3091E-06 -0.
   2   7.6611E-01 -0.          7.6611E-01 -0.          -4.0372E 06 -0.          4.0372E 06  0.
   3   5.3222E-01 -0.          5.3222E-01 -0.          -1.2372E 07 -0.          1.2372E 07  0.
   4   2.9833E-01  0.          2.9833E-01  0.          -2.2854E 07 -0.          2.2854E 07  0.
   5   2.5551E-01  0.          2.5551E-01  0.          -2.4602E 07 -0.          2.4602E 07  0.
   6   6.7434E-02  0.          6.7434E-02  0.          -3.5052E 07  0.          3.5052E 07  0.
   7  -2.4243E-02 -0.          2.4243E-02  0.          -2.2667E 07  0.          2.2667E 07  0.
   8  -5.9013E-02 -0.          5.9013E-02  0.          -1.0257E 07  0.          1.0257E 07  0.
   9  -7.0604E-02 -0.          7.0604E-02  0.          -8.7927E 06  0.          8.7927E 06  0.
  10  -7.8137E-02 -0.          7.8137E-02  0.          -7.3155E 06  0.          7.3155E 06  0.
  11  -8.2292E-02 -0.          8.2292E-02  0.          -5.8239E 06  0.          5.8239E 06  0.
  12  -7.0541E-02 -0.          7.0541E-02  0.          -1.2123E 06  0.          1.2123E 06  0.
  13  -6.1093E-02 -0.          6.1093E-02  0.          -8.4575E 05  0.          8.4575E 05  0.
  14  -4.9962E-02 -0.          4.9962E-02  0.          -4.3177E 05  0.          4.3177E 05  0.
  15  -3.7967E-02 -0.          3.7967E-02  0.           2.0989E 04  0.          2.0989E 04  0.
  16  -2.6002E-02 -0.          2.6002E-02  0.           5.0317E 05  0.          5.0317E 05  0.
  17  -1.5021E-02 -0.          1.5021E-02  0.           1.0054E 06  0.          1.0054E 06  0.
  18  -6.0152E-03 -0.          6.0152E-03  0.           1.5192E 06  0.          1.5192E 06  0.
  19  -1.6709E-12 -0.          1.6709E-12  0.           2.0375E 06  0.          2.0375E 06  0.

                                                       XZ-PLANE

SECTION                 DEFLECTIONS                                             MOMENTS
      REAL PART     IMAG PART  ABS VALUE PHASE ANGLE REAL PART     IMAG PART  ABS VALUE  PHASE ANGLE
   1   0.         -1.1684E-01 1.1684E-01 -9.0000E 01  0.          -8.8340E-07 8.8340E-07 -9.0000E 01
   2   0.         -8.8573E-02 8.8573E-02 -9.0000E 01  0.           4.7105E 05 4.7105E 05  9.0000E 01
   3   0.         -6.0304E-02 6.0304E-02 -9.0000E 01 -0.           8.1279E 05 8.1279E 05  9.0000E 01
   4   0.         -3.2035E-02 3.2035E-02 -9.0000E 01 -0.           1.3978E 06 1.3978E 06  9.0000E 01
   5   0.         -2.6763E-02 2.6763E-02 -9.0000E 01 -0.           1.5994E 06 1.5994E 06  9.0000E 01
   6   0.         -1.6109E-05 1.6109E-05 -9.0000E 01 -0.           2.8033E 06 2.8033E 06  9.0000E 01
   7  -0.          1.9160E-02 1.9160E-02  9.0000E 01 -0.           2.5598E 06 2.5598E 06  9.0000E 01
   8  -0.          3.1904E-02 3.1904E-02  9.0000E 01 -0.           2.2975E 06 2.2975E 06  9.0000E 01
   9  -0.          3.6304E-02 3.6304E-02  9.0000E 01 -0.           2.1793E 06 2.1793E 06  9.0000E 01
  10  -0.          3.9697E-02 3.9697E-02  9.0000E 01 -0.           2.0545E 06 2.0545E 06  9.0000E 01
  11  -0.          4.2140E-02 4.2140E-02  9.0000E 01 -0.           1.9225E 06 1.9225E 06  9.0000E 01
  12  -0.          4.1466E-02 4.1466E-02  9.0000E 01 -0.           1.4581E 06 1.4581E 06  9.0000E 01
  13   0.          3.8118E-02 3.8118E-02  9.0000E 01 -0.           1.1057E 06 1.1057E 06  9.0000E 01
  14   0.          3.2583E-02 3.2583E-02  9.0000E 01 -0.           7.2354E 05 7.2354E 05  9.0000E 01
  15   0.          2.5613E-02 2.5613E-02  9.0000E 01 -0.           3.1605E 05 3.1605E 05  9.0000E 01
  16   0.          1.8013E-02 1.8013E-02  9.0000E 01  0.          -1.1136E 05 1.1136E 05 -9.0000E 01
  17   0.          1.0627E-02 1.0627E-02  9.0000E 01  0.          -5.5277E 05 5.5277E 05 -9.0000E 01
  18   0.          4.3272E-03 4.3272E-03  9.0000E 01  0.          -1.0024E 06 1.0024E 06 -9.0000E 01
  19   0.          1.4608E-12 1.4608E-12  9.0000E 01  0.          -1.4554E 06 1.4554E 06 -9.0000E 01
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GBRC3
          SAMPLE PROBLEM 2

          USS OBSERVATION ISLAND. WHIRLING FREQUENCIES.-AIR. H=1/4

FREQUENCY      23.53 CPS

                                                   XY-PLANE

SECTION                        DEFLECTIONS                                                     MOMENTS
           REAL PART   IMAG PART   ABS VALUE    PHASE ANGLE  REAL PART   IMAG PART     ABS VALUE    PHASE ANGLE
     1    1.0000E 00   0.          1.0000E 00   0.          -4.6079E-07 -0.            4.6079E-07   0.
     2    1.0147E 00  -0.          1.0147E 00  -0.          -7.3702E 07 -0.            7.3702E 07   0.
     3    1.0295E 00  -0.          1.0295E 00  -0.           6.5737E 07  0.            6.5737E 07   0.
     4    1.0442E 00   0.          1.0442E 00   0.           1.6779E 08  0.            1.6779E 08   0.
     5    1.0243E 00   0.          1.0243E 00   0.           1.3990E 08  0.            1.3990E 08   0.
     6    5.5339E-01   0.          5.5339E-01   0.          -3.3151E 07  0.            3.3151E 07   0.
     7    2.3313E-01   0.          2.3313E-01   0.          -2.4091E 07  0.            2.4091E 07   0.
     8   -2.7416E-02  -0.          2.7416E-02   0.          -1.7550E 07  0.            1.7550E 07   0.
     9   -1.3163E-01  -0.          1.3163E-01   0.          -1.8264E 07  0.            1.8264E 07   0.
    10   -2.2735E-01  -0.          2.2735E-01   0.          -1.8666E 07  0.            1.8666E 07   0.
    11   -3.1432E-01  -0.          3.1432E-01   0.          -1.8499E 07  0.            1.8499E 07   0.
    12   -4.9677E-01  -0.          4.9677E-01   0.          -2.4081E 07  0.            2.4081E 07   0.
    13   -5.8118E-01  -0.          5.8118E-01   0.          -3.4797E 07  0.            3.4797E 07   0.
    14   -5.9558E-01  -0.          5.9558E-01   0.          -3.6707E 07  0.            3.6707E 07   0.
    15   -5.3616E-01   0.          5.3616E-01   0.          -3.0110E 07 -0.            3.0110E 07   0.
    16   -4.1606E-01   0.          4.1606E-01   0.          -1.6527E 07 -0.            1.6527E 07   0.
    17   -2.6235E-01   0.          2.6235E-01   0.           1.4953E 06  0.            1.4953E 06   0.
    18   -1.1091E-01   0.          1.1091E-01   0.           2.0724E 07  0.            2.0724E 07   0.
    19   -2.8399E-11  -0.          2.8399E-11   0.           3.7665E 07  0.            3.7665E 07   0.

                                                   XZ-PLANE

SECTION                        DEFLECTIONS                                                     MOMENTS
           REAL PART   IMAG PART   ABS VALUE    PHASE ANGLE  REAL PART   IMAG PART     ABS VALUE    PHASE ANGLE
     1     0.         -2.6959E 01  2.6959E 01 -9.0000E 01   0.          -3.1538E-04   3.1538E-04  -9.0000E 01
     2     0.         -1.6867E 01  1.6867E 01 -9.0000E 01   0.           9.7940E 08   9.7940E 08   9.0000E 01
     3     0.         -6.7749E 00  6.7749E 00 -9.0000E 01  -0.           3.0702E 09   3.0702E 09   9.0000E 01
     4    -0.          3.3174E 00  3.3174E 00  9.0000E 01  -0.           5.4070E 09   5.4070E 09   9.0000E 01
     5    -0.          4.9374E 00  4.9374E 00  9.0000E 01  -0.           5.7510E 09   5.7510E 09   9.0000E 01
     6     0.         -6.4890E-02  6.4890E-02 -9.0000E 01  -0.           7.7112E 09   7.7112E 09   9.0000E 01
     7     0.         -2.6573E 01  2.6573E 01 -9.0000E 01  -0.           3.8379E 09   3.8379E 09   9.0000E 01
     8     0.         -6.2628E 01  6.2628E 01 -9.0000E 01  -0.           2.0037E 08   2.0037E 08   9.0000E 01
     9     0.         -7.8135E 01  7.8135E 01 -9.0000E 01   0.          -1.2596E 09   1.2596E 09  -9.0000E 01
    10     0.         -9.3013E 01  9.3013E 01 -9.0000E 01   0.          -2.5914E 09   2.5914E 09  -9.0000E 01
    11     0.         -1.0664E 02  1.0664E 02 -9.0000E 01   0.          -3.7707E 09   3.7707E 09  -9.0000E 01
    12     0.         -1.3130E 02  1.3130E 02 -9.0000E 01  -0.          -5.7557E 09   5.7557E 09  -9.0000E 01
    13     0.         -1.3661E 02  1.3661E 02 -9.0000E 01  -0.          -6.1870E 09   6.1870E 09  -9.0000E 01
    14     0.         -1.2939E 02  1.2939E 02 -9.0000E 01  -0.          -5.6551E 09   5.6551E 09  -9.0000E 01
    15     0.         -1.1070E 02  1.1070E 02 -9.0000E 01  -0.          -4.2112E 09   4.2112E 09  -9.0000E 01
    16     0.         -8.3429E 01  8.3429E 01 -9.0000E 01  -0.          -1.9881E 09   1.9881E 09  -9.0000E 01
    17     0.         -5.2030E 01  5.2030E 01 -9.0000E 01  -0.           8.2055E 08   8.2055E 08   9.0000E 01
    18     0.         -2.2114E 01  2.2114E 01 -9.0000E 01  -0.           3.9912E 09   3.9912E 09   9.0000E 01
    19     0.         -1.0654E-08  1.0654E-08 -9.0000E 01  -0.           7.3101E 09   7.3101E 09   9.0000E 01
```

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 GBRC3 OCT 7, 1966
 SAMPLE PROBLEM 2

DATA CONTROL CARD

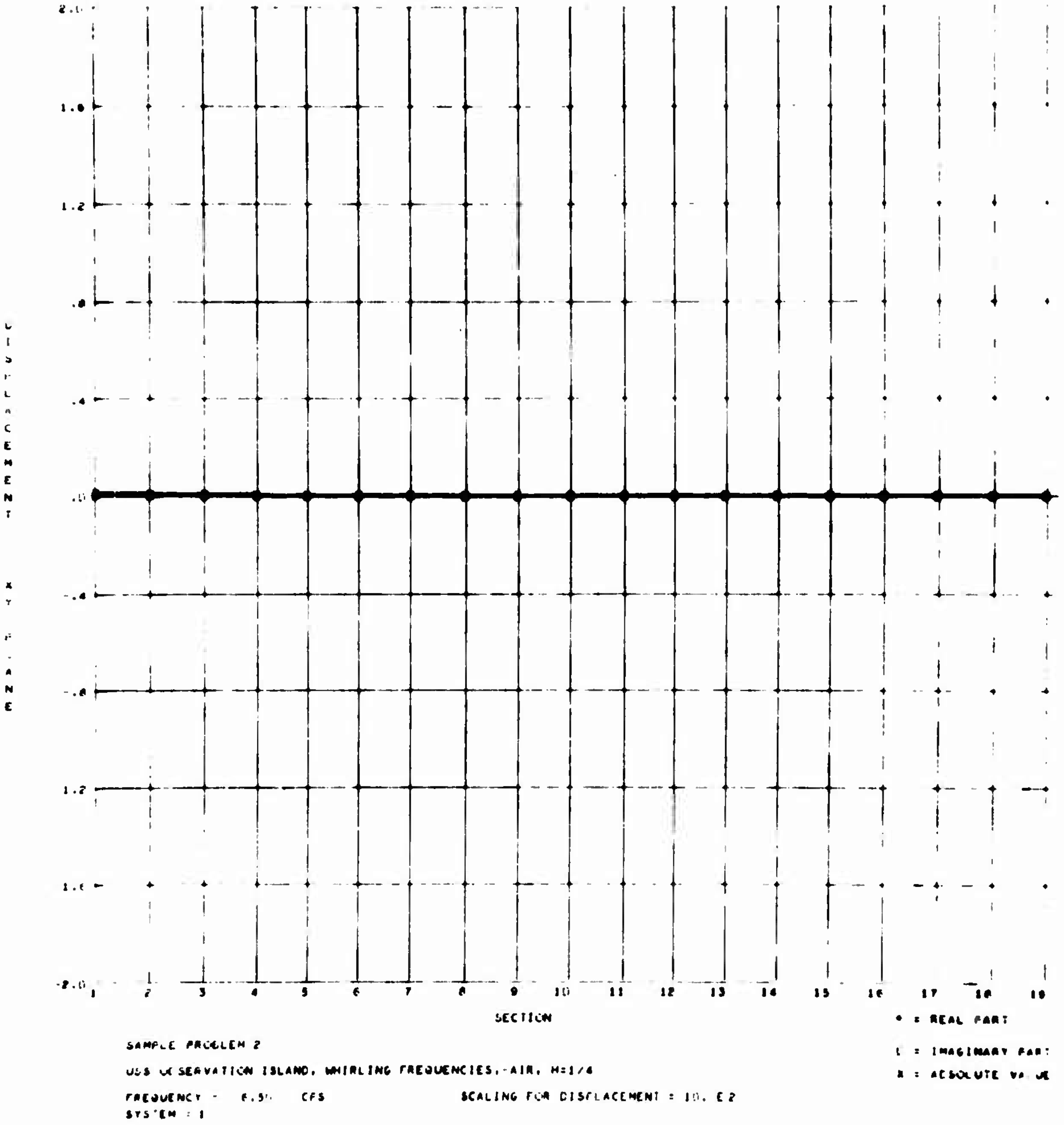
NO  TYPE
```

```
END GBRC3 RUN
```

Best

Available

Copy

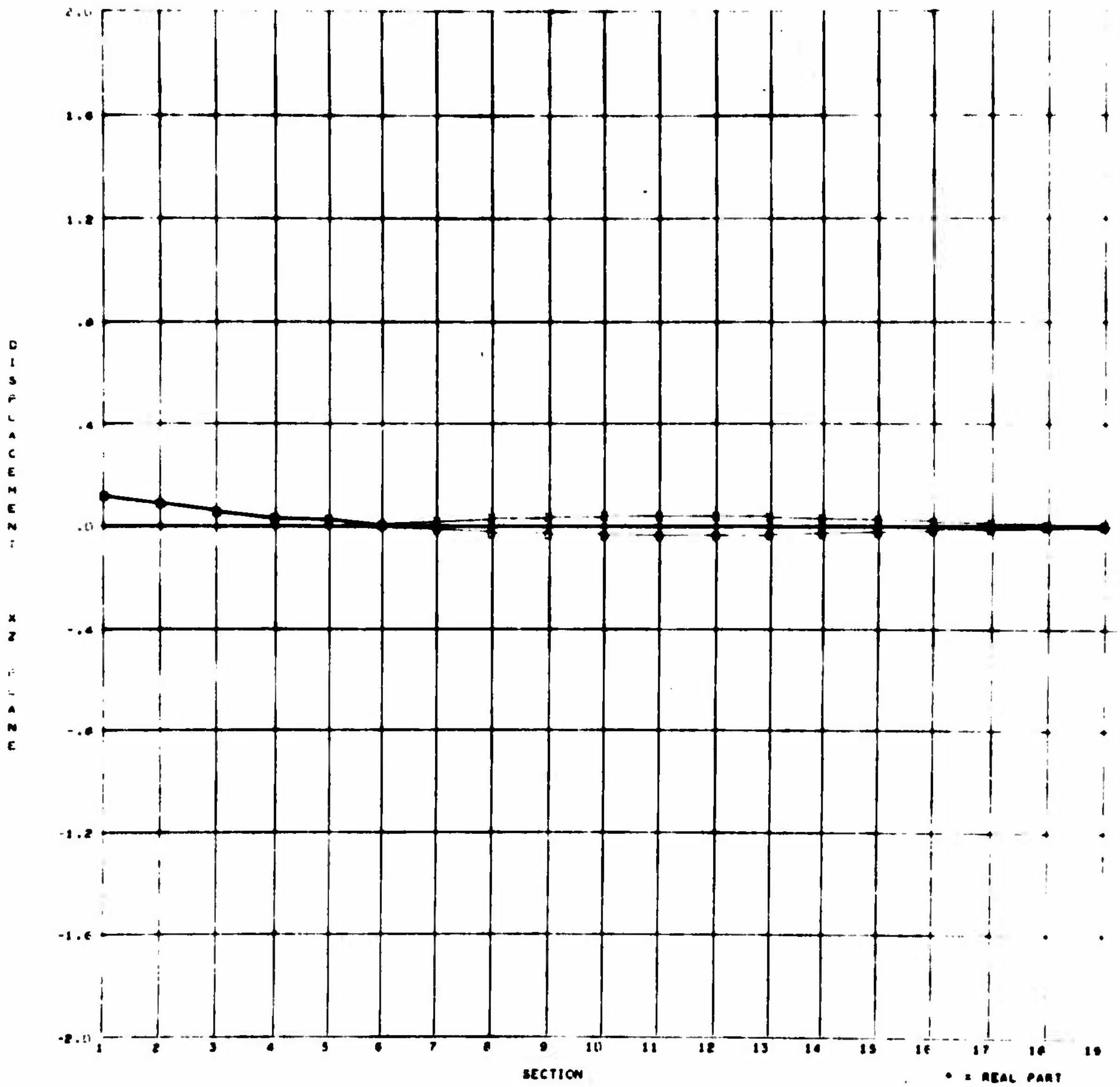


SAMPLE PROBLEM 2

USS OBSERVATION ISLAND, WHIRLING FREQUENCIES,-AIR, H=1/4

FREQUENCY = 6.50 CPS SCALING FOR DISPLACEMENT = 10. E 2
SYSTEM = 1

Copy available to DTIC does not permit fully legible reproduction



SAMPLE PROBLEM 2

USS OBSERVATION ISLAND, WHIRLING FREQUENCIES,-AIR, H=1/4

FREQUENCY : 6.50 CPS SCALING FOR DISPLACEMENT = 10. E2
SYSTEM : 1

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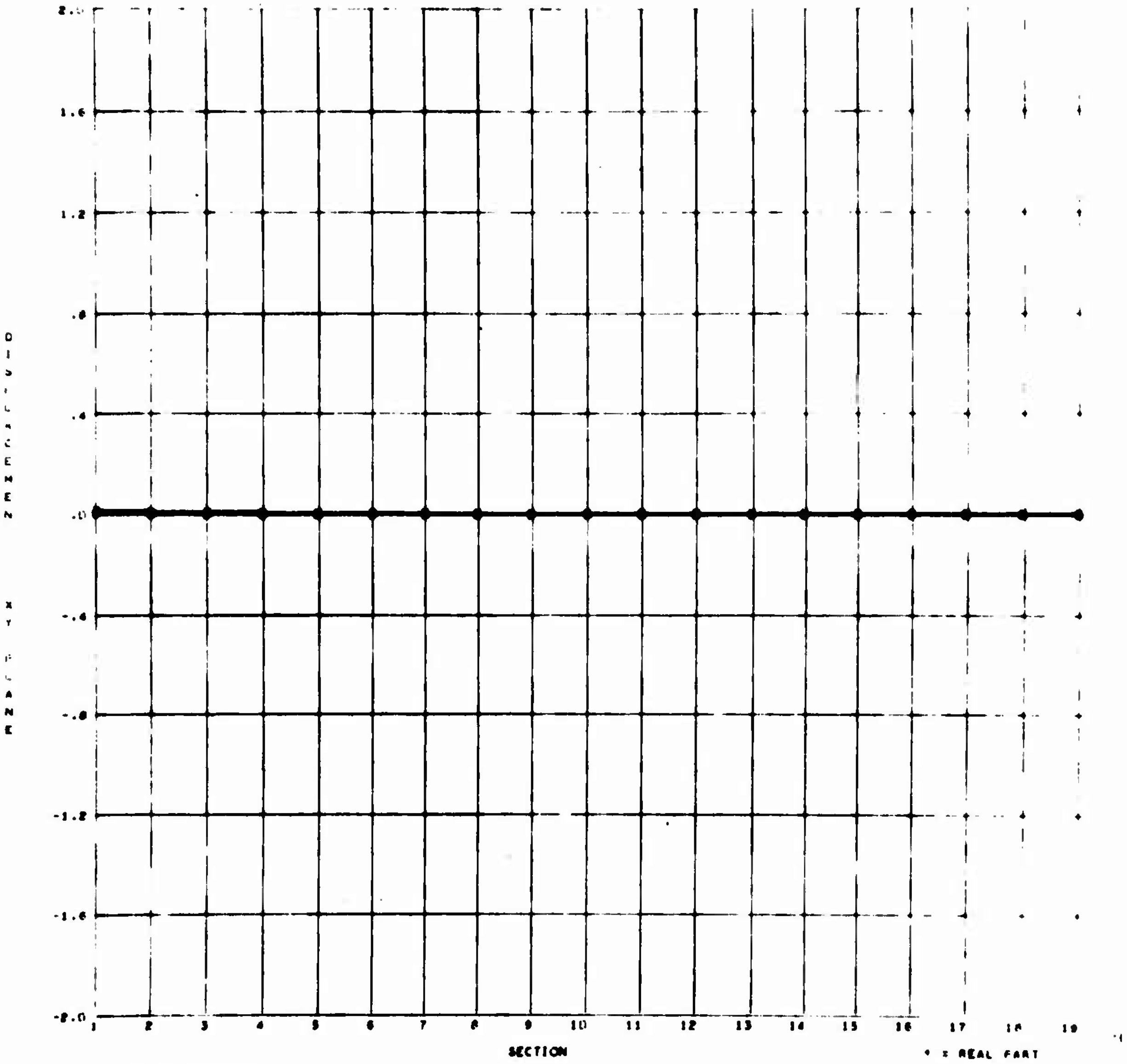


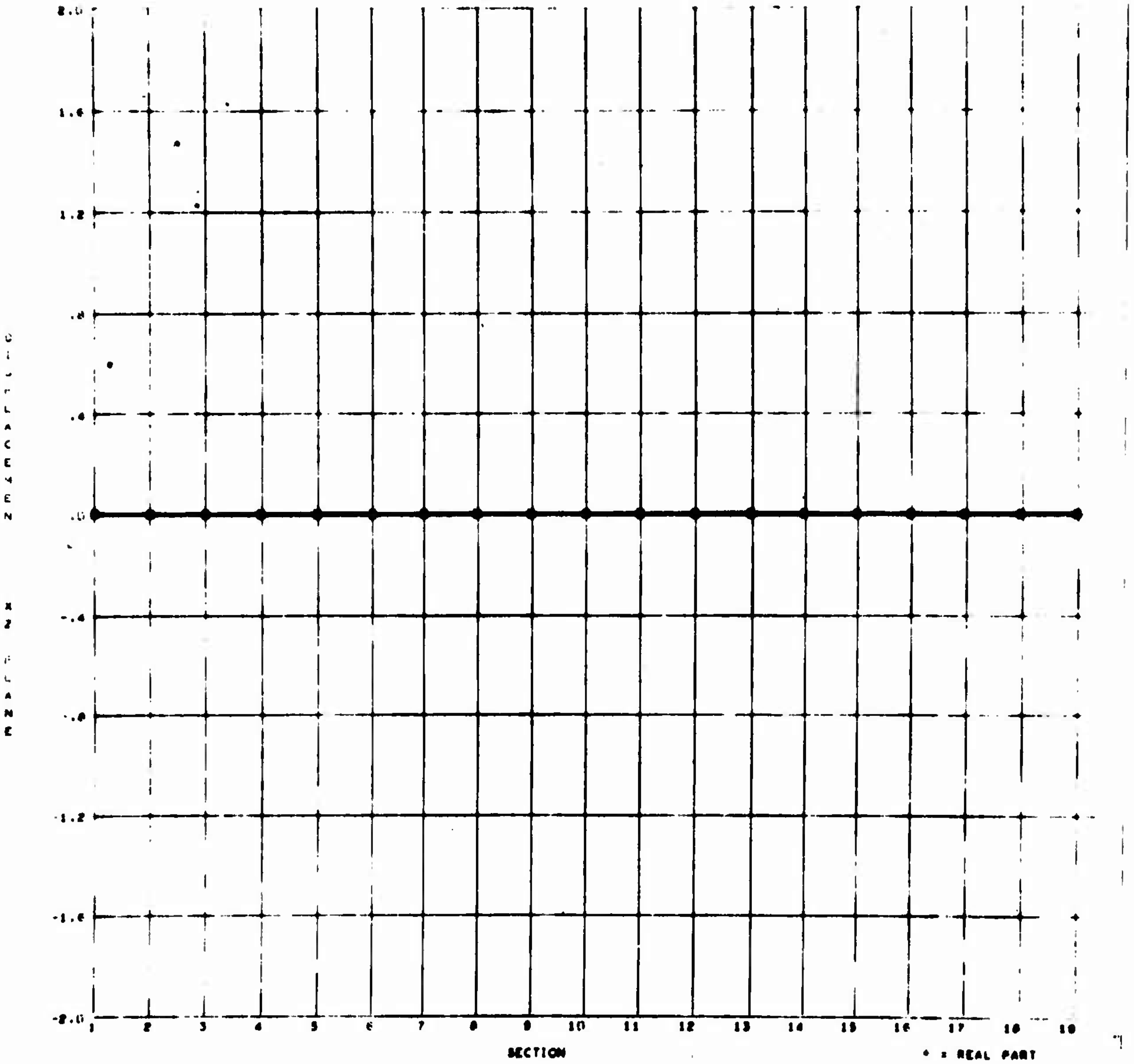
SAMPLE PROBLEM 2

USS OBSERVATION ISLAND, WHIRLING FREQUENCIES,-AIR, M=1/4

FREQUENCY = 7.04 CPS SCALING FOR DISPLACEMENT = 10. E 2

SYSTEM = 1

Copy available to DDC does not permit fully legible reproduction



+ = REAL PART
O = IMAGINARY PART
X = ABSOLUTE VALUE

SAMPLE PROBLEM 2

USS OBSERVATION ISLAND, WHIRLING FREQUENCIES,-AIR, H=1/4

FREQUENCY = 7.04 CPS SCALING FOR DISPLACEMENT = 10. E 2
SYSTEM # 1

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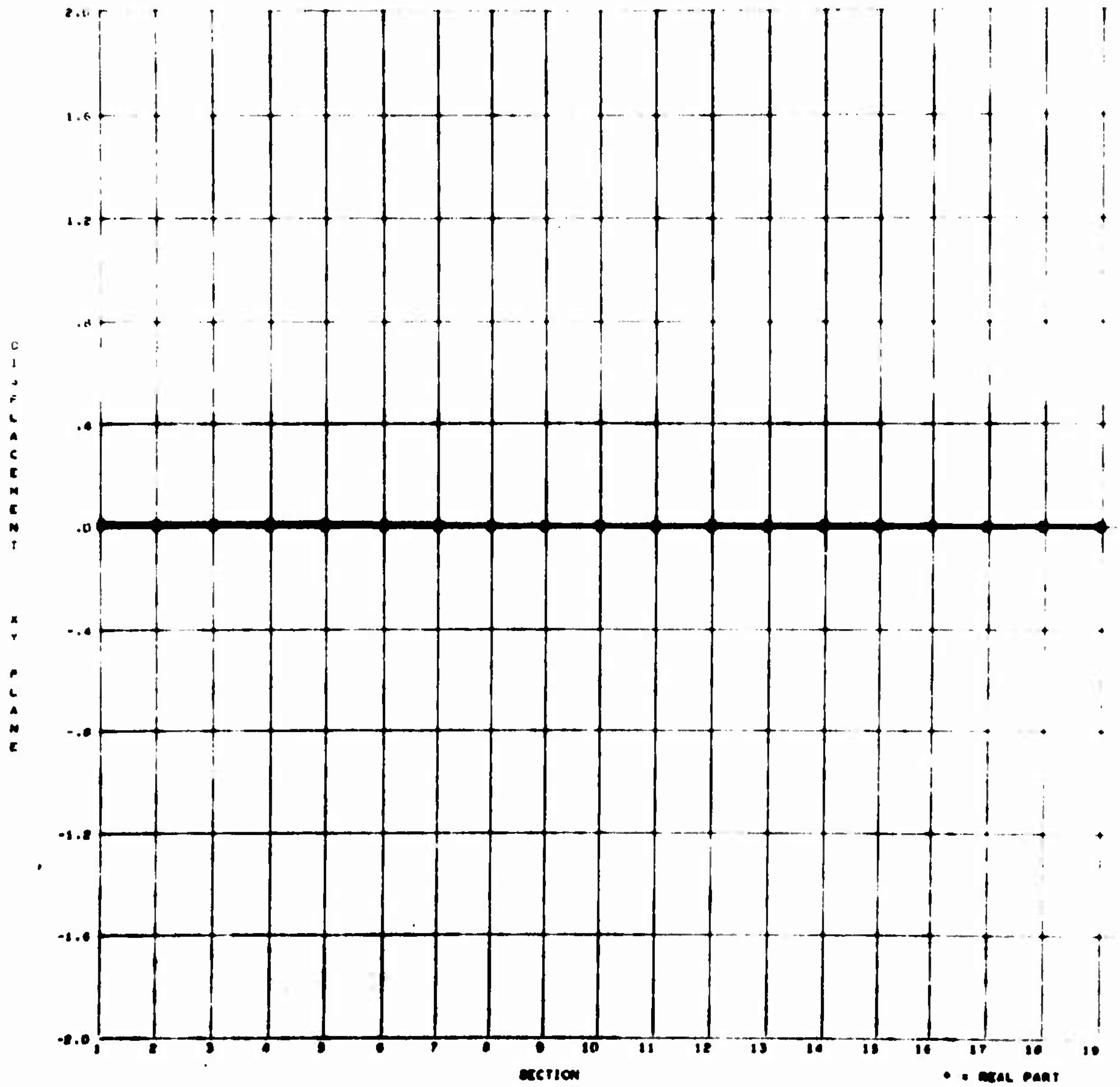

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SAMPLE PROBLEM 2

USS OBSERVATION ISLAND, WHIRLING FREQUENCIES,-AIR, N=1/4

FREQUENCY = 23.33    CPS                  SCALING FOR DISPLACEMENT = 10. E 2
SYSTEM = 1
```

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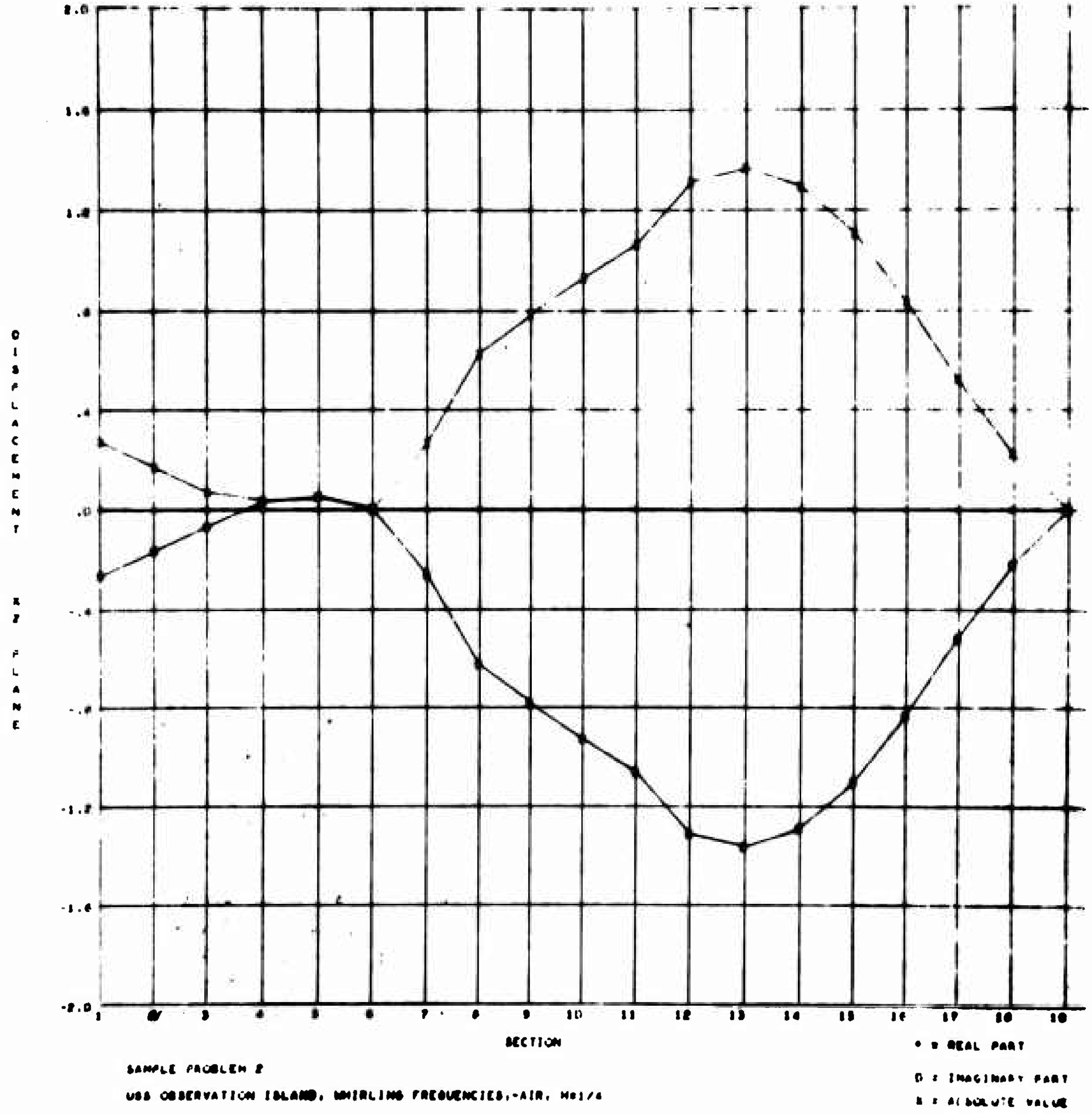

SAMPLE PROBLEM 2

USS OBSERVATION ISLAND, WHIRLING FREQUENCIES,-AIR, M=1/4

FREQUENCY = 23.33 CPS SCALING FOR DISPLACEMENT = 10. E 2
SYSTEM = 1

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TABLE 1

Comparison of Calculated Results

	Mode Number	Critical Whirling Frequencies in CPS	
		AML Computer Program 170	GBRC3*
Problem 1 h = 1	1	6.37	6.37
	2	8.08	8.08
	3	23.55	23.54
Problem 2 h = ¼	1	6.50	6.50
	2	7.85	7.84
	3	23.55	23.53

* IBM 7090 computer running time for each of these problems is approximately 8 minutes.

REFERENCES

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ACKNOWLEDGMENTS

The author wishes to express his thanks to Mr. Frederick W. Palmer Jr., Code 761, Surface Ship Structures Division who supplied data and results from earlier digital calculations for the USS OBSERVATION ISLAND shaft for use in checking and verifying GBRC3 output.

A special word of appreciation is due Mr. Stephen T. Liang, Code 850, Computer-Aided Ship Design Division whose active interest in the whirling shaft problem contributed significantly to the development of this program.

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