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EAGLE STOWAGE AND SHOCK MITIGATION STUDY

Technical Note TN 1-1624-B

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NAVAL SEA SYSTEMS COMMAND
Department of the Navy

30 November 1960

Launching Systems Department

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SILVER SPRING, MARYLAND

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EAGLE STOWAGE AND SHOCK MITIGATION STUDY

Technical Note TN 1-1624-B

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I. INTRODUCTION

A. Background

The EAGLE weapon system is being developed to provide aircraft carriers with long range defensive and offensive capabilities. The successful operation of this weapon system is dependent on a shipboard weapon installation that can continuously stow, handle, and check out EAGLE missiles in large quantities and at rapid rates, around the clock. Because of the large size and weight of the EAGLE missile and the required delivery rates, mechanization of the stowage and handling facilities will be required. Under adverse conditions of vibration and high shock input the handling and stowage facilities must protect the missiles and be capable of operating even though the ship may be severely damaged.

B. Assignment

Under Contract NOas 60-6052-c this Laboratory is conducting engineering studies and structural and dynamic analyses in the preparation of preliminary design concepts for the EAGLE shipboard weapon installations. Vitro's Technical Report No. 122, "Preliminary Design Concepts for Stowing and Handling Air Launched Guided Missiles", includes a handling and stowage system which provides for "overhead" mechanized handling, and mechanized magazines equipped with individual missile trays for stowage. As directed by RSWI, (BuWeps letter RSWI-24-RJM:lmb 024344), this Laboratory has investigated and analyzed the design of a system of shock mitigation for the "Tray Stowage System" based on shock conditions defined on the "damped sinusoidal velocity-time shock curve" included in NAVSHIPS 250-423-29.

C. Scope

The stowage tray is the basic element of the Tray Stowage System; therefore, the initial investigations and analyses of methods of shock mitigation have been limited primarily to the missile stowage tray. This report includes a brief description of the Tray Stowage System, dynamic and structural analyses of the trays and the supporting structure, and descriptions and analyses of several methods of providing shock mitigation. Conclusions and recommendations are based upon the information and data acquired from the overall study.

This study has been conducted utilizing the shock data presented on Figures 8, 9, and 10 of NAVSHIPS 250-423-29, and vibratory criteria specified in MIL-STD-167 (Ships). During conferences on shock mitigation, BuShips has stated that the shock inputs, derived from the previous data, should be considered as being applied at the deck supporting the magazine. It is felt that such high inputs are unrealistic design parameters because under such conditions the distortion and structural damage would prevent the operation of the complete handling and stowage system. Therefore, this analysis has been undertaken basically to determine the feasibility of providing protection for the missiles when stowed in the trays. Before an analysis of the magazine mechanisms and the overhead handling equipment can be conducted, more specific information on shock inputs and ship deflections at the locations affected will be required.

II. CONCLUSIONS

An adequate stowage concept requires integration into and compatibility with the entire shipboard missile handling system for all modes of operation, including replenishment and strikedown, delivery, checkout, and warhead exchange. It must provide adequate protection for the missiles from vibration and high shock inputs, such that as long as the system remains operational the magazine is operable. It must be fully automated to comply with the heavy demands anticipated for the EAGLE weapon system.

Preliminary analyses indicate that the tray stowage system meets these requirements as an element of the shipboard installation. From the standpoint of capability, reliability, compatibility, and simplicity, it is felt to provide the optimum configuration. For the stowage system, the suspension of the missiles in trays incorporating standard rubber shear mounts, aligned in a fore and aft direction, is felt to afford the best protection, and to require minimal pre-installation testing. This system is recommended for the EAGLE missile shipboard magazine installation only if the high shock input criteria specified on the Bureau of Ships curves for surface vessels in NAVSHIPS 250-523-29 are applicable to the CVA magazine areas. Otherwise, more compact tray stowage arrangements as possible.

Rubber shear mounts have been selected because they are inexpensive shelf items proven through years of related usage and are easily replaceable.

The resiliency of the aluminum trays contributes almost half of the total static deflection required to achieve the proper spring constant for mitigated suspension of the missiles. This enables use of compact, economical shock mounts. Rubber lining in the chocks accommodates deflection of the chocks with respect to the missile during tray excursion.

It should be noted that input criteria are felt to be extreme and possibly unrealistic. Should such be the case, reduction of the inputs could possibly permit use of resilient trays with no shock mounts. This would provide a more economical and less complex configuration.

III. TRAY STOWAGE SYSTEM

A. Basic Features

The tray stowage system provides ready service stowage for the entire ships allowance of EAGLE and other large air launched missiles. Delivery and strikedown are accomplished automatically, with the exception of aerodynamic surface installation and removal. There is no below decks manual handling of the missiles when the missiles are in the system. The system has the following advantages:

1. Ability to meet CNO arming rate requirements.

2. Rapid and efficient delivery and strikedown capabilities.

3. Positive restraint and positioning of the missiles throughout the system.

4. One hundred per cent selectivity from stowage.

5. Protection from high shock input and vibration throughout the system.

6. Maximum missile density in protected stowage.

7. Automatic transfer between stowage and checkout fixture.

8. Rapid and convenient warhead exchange.

9. Safety for ship, personnel, missiles, and aircraft.

B. Magazine Stowage

With the tray stowage and handling system, the missiles are stowed in four horizontal levels of trays in the magazine. The trays have rollers at the ends supported by rails attached to the magazine structure. The ends of the trays have interconnect fittings which connect each tray to that adjacent and provide horizontal positioning.

C. System Operation

Hoists, at the ends of the magazine, raise or lower missiles in trays; or empty trays. The outboard hoists include a horizontal tray shifting mechanism. This arrangement permits the automatic indexing of any missile from any position in the magazine to the position from which it is automatically transferred to a rammer cart which delivers it via the lower stage elevator to the second deck. From there, after aerodynamic surface installation, the missile is automatically transferred to the upper stage elevator for delivery to the flight deck upon skids latched to the elevator platform. Strikedown is similarly accomplished in the reverse order. Delivery to and from checkout is similarly automatic. For operation of a magazine tray shift cycle see Figure 1.

D. Tray Configuration

All trays analyzed in this report consist of a pair of fore and aft standard structural channels, attached to a doubled skinned bottom, with the necessary gussets and reinforcements for structural stability. The tray ends are forgings or castings. Both ends are identical. These end fittings provide the tray interconnects which secure any tray to any adjacent tray and act as guides and positioners for the trays on the end hoists, and pick up

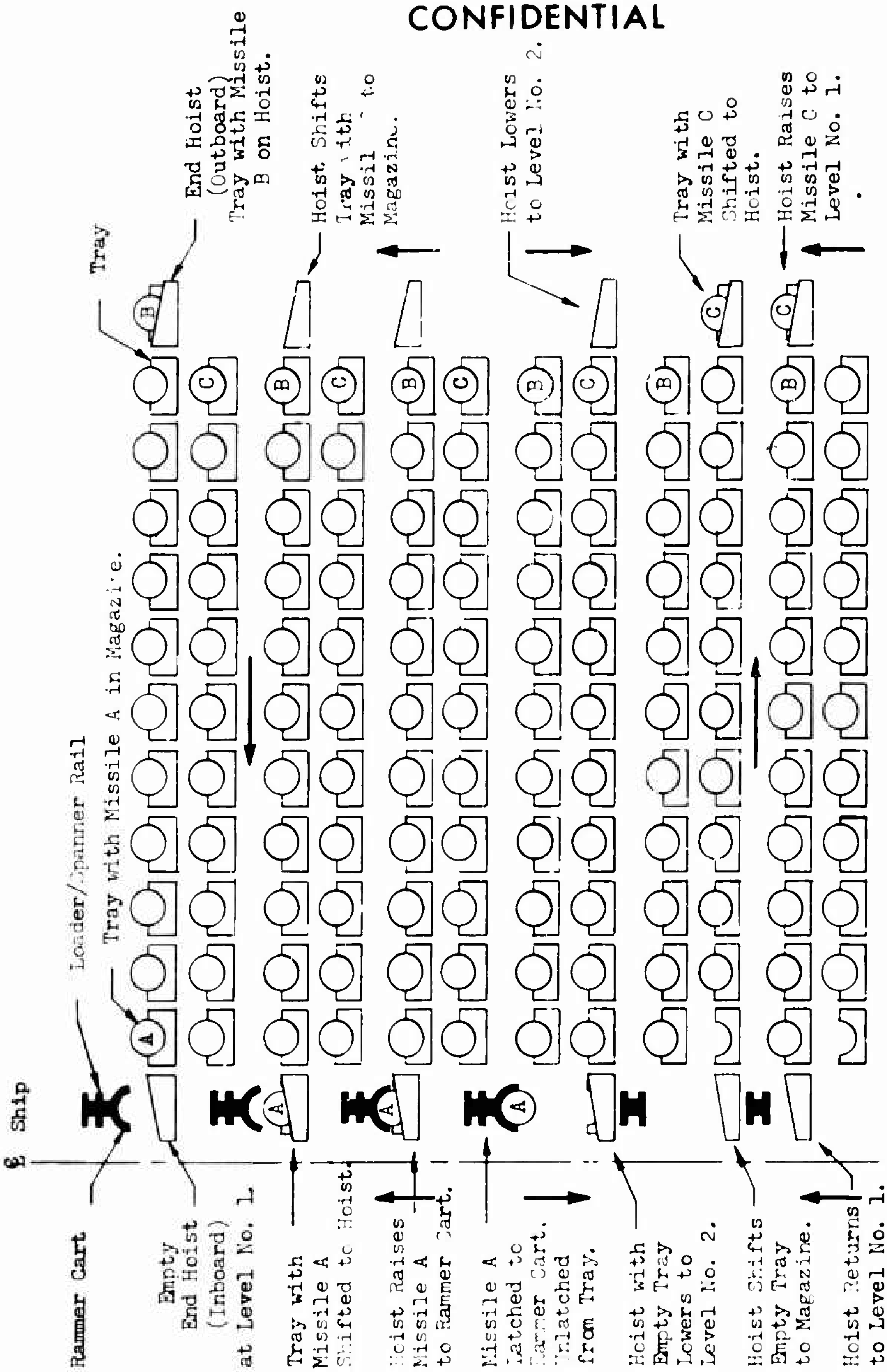

Figure 1 - Tray Shift Cycle, Levels No. 1 and No. 2

spring loaded locators on the magazine or end hoists. They also support the tray rollers which roll in the magazine rails. See Figure 2. All trays with the exception of those utilized in a resiliently supported magazine are of aluminum construction.

The trays are equipped with suitable chocks at the missile hard spots. These chocks contain latches which automatically engage the missile handling lugs and latch the missiles to the trays. They are lined with rubber to accommodate deflection of the tray with respect to the missile. The linkage operating the latches is actuated by the handling system as a missile is latched to or unlatched from a tray. The motion causing engagement of the rammer cart latches with the missile launch lugs causes the handling lugs to be unlatched. See Figure 3. The various configurations subsequently analyzed vary in the type of shock mounts provided, their locations, and the methods of attachment.

IV. DETERMINATION OF LOADS

A. Shock Environment

The applicable high-impact shipboard shock environments of NAVSHIPS 250-423-29, Figures 8, 9, and 10, were used in this study.

The data of Figure 8 were used in conjunction with Goodyear Aircraft Corporation Report GER-9367 to obtain critical input and response values of displacement and acceleration. Inputs, (not concurrent), from which responses are derived in the analyses are as follows:

Vertical	Athwartship	Fore-Aft
120.0 g peak	72.0 g peak	38.0 g peak
1.657 in. ship displacement	0.689 in. ship displacement	0.368 in. ship displacement

The data of Figure 9 indicate maximum shock inputs of the same order as Figure 8, but ship whipping characteristics could cause the responding system to develop unduly large displacements unless the proper fundamental response frequency is provided. A study by BuShips indicates that a fundamental natural response frequency of the loaded trays of approximately 5 cps, vertical, would not be objectionable, (study not included here), and this value, as applied below, was used.

The data of Figure 10 indicate conditions which are not critical.

The calculations which follow assume a response damping of 10 per cent of critical, which is considered representative of the characteristics of the rubber available for mounts and some inherent damping in the aluminum tray stowage structure.

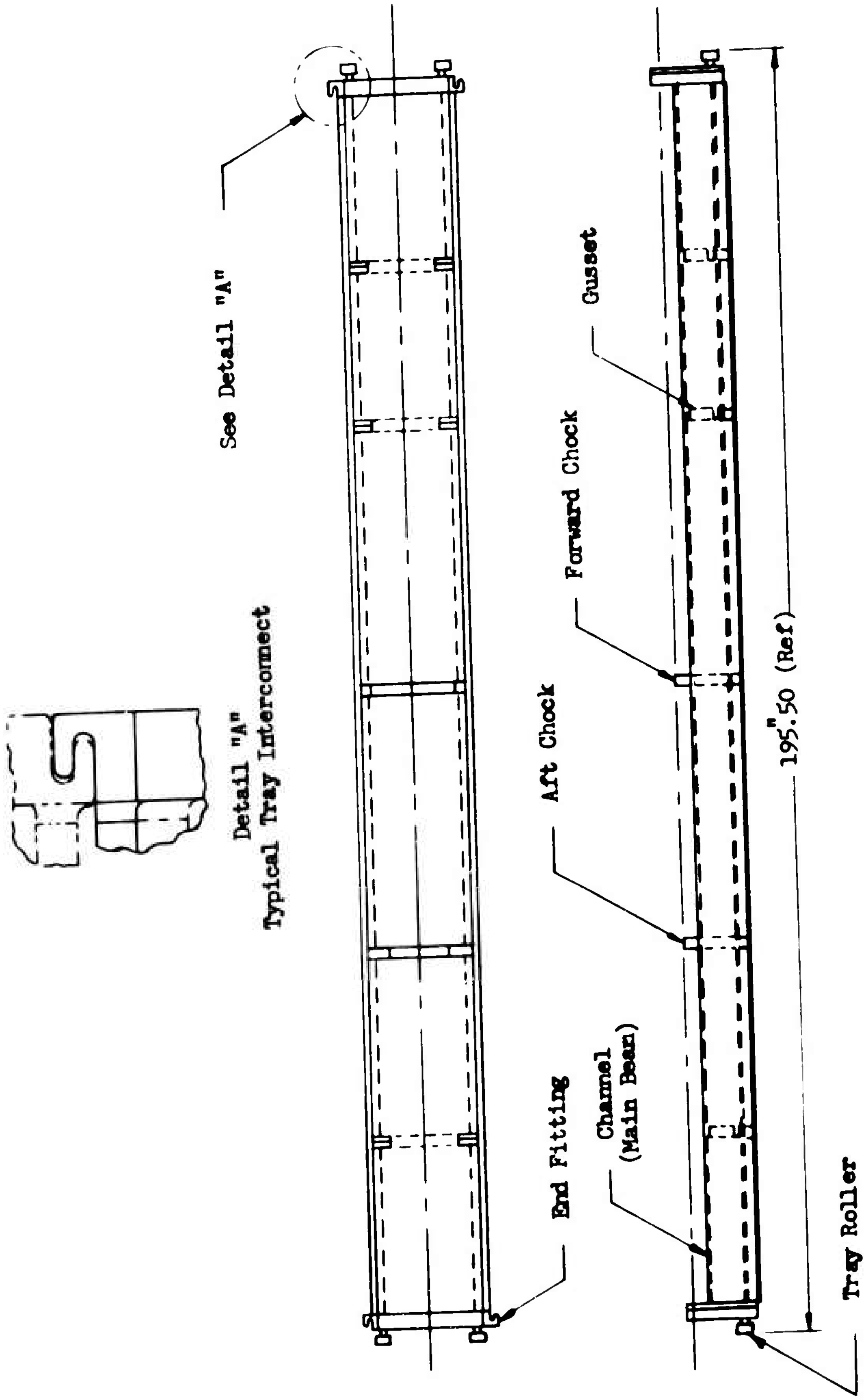

Figure 2 - Tray Configuration

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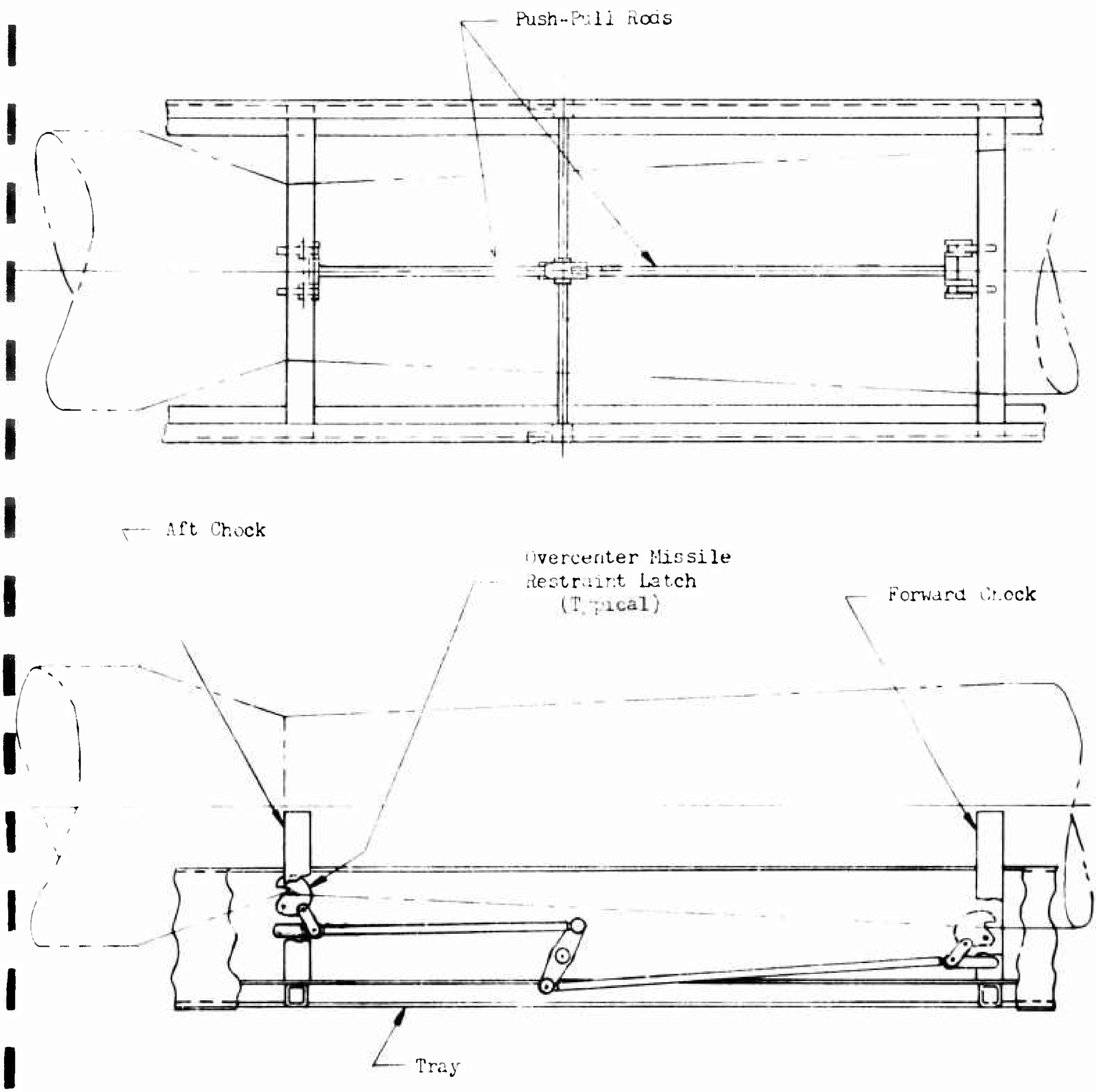

Figure 3 - [illegible] Latching Mechanism

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B. Steady-State Vibration Environment

BuShips data for the applicable CVA indicate propeller forcing frequencies of 14.2 cps at maximum speed, 7.75 cps at normal cruising speed, and a possible cruising speed frequency of 9.5 cps.

MIL-STD-167 (Ships) specifies the following input amplitudes:

Input Frequency	Input Half Amplitude
5-15 cps	± 0.03 in.
16-25 cps	± 0.02 in.
26-33 cps	± 0.01 in.

Vibration response damping is also assumed to be 10 per cent of critical.

Per BuShips Specifications, magnification factor should not exceed approximately 3.0. Factor is obtained from standard curves such as Figure 3 of David Taylor Model Basin Report R-189.

C. Input Derivation

Shock input characteristics per BuShips data for surface ships for second platform, CG, CLG, CG(N) or main deck, DD; considered applicable to CVA:

Direction	$V_o = V_{max.}$ fps	Frequency cps	Decay Decrement V_1/V_o (Ratio of peak velocities for consecutive half-cycles)
Vertical	15.0	25	1/3
Athwartship	7.5	30	1/3
Fore-Aft	4.0	30	1/3

As previously noted, it has been stated by the Bureau of Ships that these forces are to be applied locally. However, because these values represent the typical shock motion of decks for destroyer and cruiser types it is felt that they are overly conservative for use within the carriers armored box.

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Shock input accelerations and displacements determined per Goodyear Report, GER-9367:

For decay to 1/3 amplitude in 1/2 cycle, decay decrement $= 2.2 = 2\pi \tan\phi$; $\phi = 0.33677$ radians; $\sin\phi = \frac{C}{C_c}_{\text{Ship}} = 0.33045$. For $\phi = 0.33677$ radians, $K\alpha = 1.635$ and $K_d = 1.445$.

Vertical	Athwartship	Fore-Aft
$V_o = 15.0$ fps,	For $V_o = 7.5$ fps,	For $V_o = 4.0$ fps,
$f = 25$ cps:	$f = 30$ cps:	$f = 30$ cps:
$M_o = 73.14$	$M_o = 43.9$	$M_o = 23.4g$
$d_o = 1.147$ in.	$d_o = 0.477$ in.	$d_o = 0.255$ in.
$M_p = K\alpha M_o$ $= 1.635(73.14)$ $= 120.0g$ = peak input acceleration;	$M_p = K\alpha M_o$ $= 1.635(43.9)$ $= 72.0g$ = peak input acceleration;	$M_p = K\alpha M_o$ $= 1.635(23.4)$ $= 38.0g$ = peak input acceleration;
$d_r = K_d d_o$ $= 1.445(1.147)$ $= 1.657$ in. = ship displacement	$d_r = K_d d_o$ $= 1.445(.477)$ $= 0.689$ in. = ship displacement	$d_r = K_d d_o$ $= 1.445(.255)$ $= 0.368$ in. = ship displacement

Steady-state vibration input characteristics per BuShips data for CVA-63 class:

Shaft RPM = 170 maximum; 93 cruising; 5 blades

Forcing frequency, maximum; $= \frac{170(5)}{60} = 14.2$ cps

Forcing frequency, cruising; $= \frac{93(5)}{60} = 7.75$ cps

Per MIL-167:

Input Frequency	Input Half Amplitude
5-15 cps	$\pm$ 0.03 in.
16-25 cps	$\pm$ 0.02 in.
26-33 cps	$\pm$ 0.01 in.

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V. METHODS FOR PROVIDING SHOCK MITIGATION

A. General

The design approach comprises investigation of three fundamental methods of achieving the desired stowage density in conjunction with adequate shock mitigation. These methods were selected on the basis of practicability, economy, and compatibility. They comprise the following:

1. Shock mitigated trays - aluminum trays with steel support rails and aluminum columns; each tray is provided with shock mounts at the ends.

2. Resilient trays - aluminum trays with steel support rails and aluminum columns; no shock mounts provided except for thin pads at the missile chocks.

3. Resiliently supported magazine - steel trays with steel support rails and magazine structure.

B. Shock Mitigated Trays

1. General

With shock mitigation provided by the trays with rubber mounts, the potential vertical density is not as great as with a resiliently supported magazine. This is because clearance for vertical excursion must be provided such that there is no contact when the excursion between vertically adjacent units is 180 degrees out of phase. However, there are many advantages, including the following:

a. Mitigation of shock loads on stowed missiles is essentially constant, regardless of the magazine loading.

b. The design can utilize standard, off the shelf, proven shock mounts.

c. Required preliminary testing is minimal.

d. The installation weighs less than a resiliently supported magazine.

e. The resiliency of the aluminum trays contributes an appreciable amount of excursion to the total required, thus permitting the use of relatively compact rubber shock mounts.

C. Resilient Trays

This configuration utilizes only the resiliency of the tray for the mitigation of shock. Construction is economical and simple. Stowage density is high. However, inputs to the missile are excessive with shock criteria employed. Should subsequent reevaluation of applicable data result in reduced inputs to the magazine areas this design would be worthy of consideration.

D. Resiliently Supported Magazine

This arrangement suspends the whole magazine, including end hoists, on a resilient sub-structure, capable of sufficient deflection, below yield stress, to adequately mitigate the entire assembly when the deck is subjected to specified inputs. It permits the maximum possible stowage density because, under shock input, the missiles and the trays in the magazine excurse the maximum amount in unison, thus lessening required vertical clearances, one from another. The trays are relatively stiff, since they are made of steel, so that tray excursion itself, with respect to the magazine structure or to adjacent trays in the magazine is minimal. See Figure 4.

The weight, due to the use of steel trays, is greater than that of a magazine utilizing resilient aluminum trays. Therefore, any comparative evaluation should take into consideration the effect upon the ship of this additional weight versus the advantages of maximum stowage density. The differential is not felt to be excessive, being on the order of 15,000 pounds distributed throughout the greater portion of the magazine area.

E. Other Methods of Mitigation

While many other methods of shock mitigation in addition to those evaluated in this report are feasible, previous studies by this Laboratory have eliminated many from consideration for this type of installation. For example, yielding mounts provide no protection after an initial input. Coulomb damping devices require resetting to be effective. Steel springs in the trays become too heavy, bulky, and costly. With oil or pneumatic springs the possibility of leakage exists. Torsional arrangements require cumbersome linkages. None of these devices, in the ranges required, are available as standard commerical items. This would complicate the replacement and logistic problems should they be employed. (For details see Vitro Laboratories TN-1-1624-A.) Therefore, the analyses have been confined to those configurations previously described.

VI. STRUCTURAL AND DYNAMIC ANALYSES

A. Shock Mitigated Trays

1. Load Summary

Analyses were performed on shock mitigated trays 8 and 9 inches deep, respectively. The results are summarized in the following tables. Magnification factors shown are based on cruising or maximum ship steady-state

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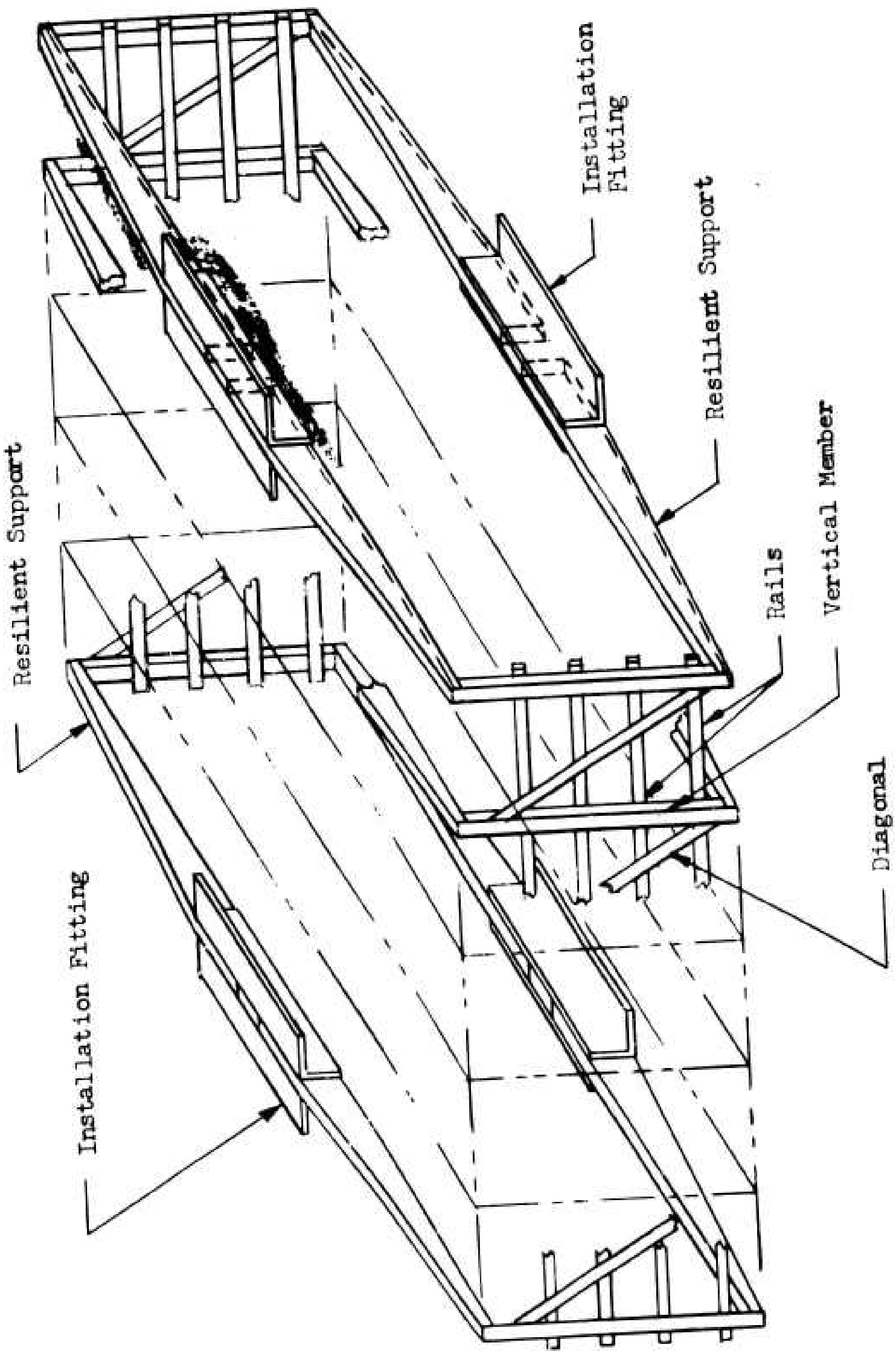


Figure 4 - Resiliently Supported **Magazine**

forcing frequencies - whichever produce the most critical condition. Any resonant conditions are at frequencies other than cruising or maximum, and are considered to be transient. These summaries are derived from the arrangement utilizing sandwich shear mounts in the fore and aft plane.

Summary - Shock Mitigated Tray, 8" Deep

	Tray Loaded			Tray Empty		
	Vertical	Athwartship	Fore-Aft	Vertical	Athwartship	Fore-Aft
Steady-State Vibration Maximum Magnification Factor	0.5	1.5	2.75	2.4	1.0	2.5
Shock Maximum g at Chocks	4.2	-	2.1	-	-	-
Maximum Stress on Tray, psi	33,000	Not Critical	Not Critical	Not Critical	Not Critical	Not Critical

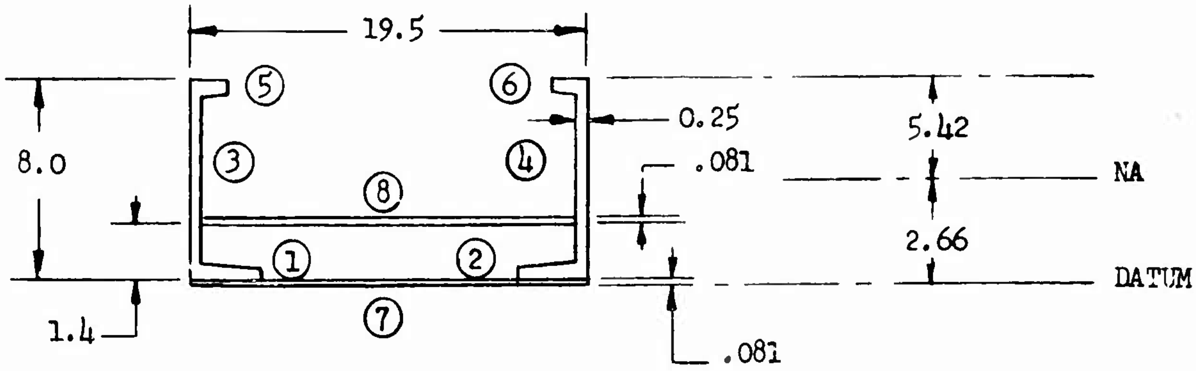
Summary - Shock Mitigated Tray, 9" Deep

	Tray Loaded			Tray Empty		
	Vertical	Athwartship	Fore-Aft	Vertical	Athwartship	Fore-Aft
Steady-State Vibration Maximum Magnification Factor	0.7	1.5	2.75	2.8	1.0	2.8
Shock Maximum g at Chocks	4.8	-	2.1	-	-	-
Maximum Stress on Tray, psi	25,000	Not Critical	Not Critical	Not Critical	Not Critical	Not Critical

2. General Analysis

a. Tray Properties

(1) Aluminum Tray 8" Deep



Side beams 8" x 4.38#.
Top flange cut to 1.0" width.

Moment of Inertia - Vertical:

Item		Area	x	Arm	=	Moment
1 2	.75(2)	1.5		.45		.68
3 4	2.0(2)	4.0		4.08		16.32
5 6	.33(2)	.66		7.86		5.24
		6.16		3.6		22.24
7	19.5(.081)	1.57		.04		.06
8	19.0(.081)	1.53		1.52		2.32
		9.26		2.66		24.62

1 2	$Ad^2 = 1.5(2.21)^2$	7.36
3 4	$Ad^2 = 4.0(1.42)^2$	8.08
5 6	$Ad^2 = .66(5.2)^2$	17.80
7	$Ad^2 = 1.57(2.62)^2$	10.70
8	$Ad^2 = 1.53(1.14)^2$	2.00
3 4	$Io = 2(.25)(8)^3/12$	21.33

Total I Vertical = 67.27 in.^{4}

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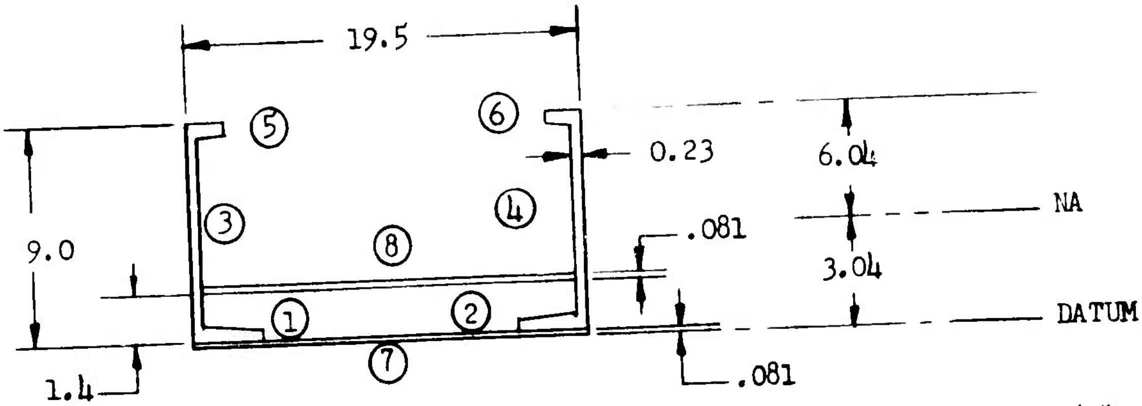
Moment of Inertia - Lateral:

7 8	$I_o = .081 \left(\overline{19.5}^3/12 + \overline{19}^3/12\right)$	= 97.0
1 - 6 incl.	$Ad^2 = 6.16\ (9.19)^2$	= 519.0
	Total I Horizontal	= 616.0 in.^{4}

(2) Aluminum Tray 9" Deep



Side beams 9" x 4.74#.
Top flange cut to 1.0" width.

Moment of Inertia - Vertical:

Item		Area	x	Arm	=	Moment
1 2	.945(2)	1.89		0.51		.96
3 4	2.07(2)	4.14		4.58		19.00
5 6	.5(2)	1.00		8.43		8.43
		7.03		4.04		28.39
7	19.5(.081)	1.57		.04		.06
8	19.0(.081)	1.53		1.52		2.32
		10.13		3.04		30.77

1 2	$Ad^2 = 1.89(2.53)^2$	12.14
3 4	$Ad^2 = 4.14(1.54)^2$	9.80
5 6	$Ad^2 = 1.0(5.39)^2$	29.00
7	$Ad^2 = 1.57(3.0)^2$	14.13

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8 $Ad^2 = 1.53(1.52)^2$ 3.54

3 4 $Io = 2(.23)(9)^3/12$ 27.94

Total I Vertical = 96.55 in.^{4}

Moment of Inertia - Lateral:

7 8 $Io = .081\ (\overline{19.5}^3/12 + \overline{19}^3/12)$ 97.0

1 - 6 incl. $Ad^2 = 7.03(9.15)^2$ 588.0

Total I Horizontal = 685.0 in.^{4}

(3) Weight Estimate

	8" Deep Tray	9" Deep Tray
Crossbeam 3" x 1.46# (11) Required; 19" long-2.3# each	25#	25#
Main Side Beams (2) Required; 192" long each	118#	135#
Skins (2) Required; 192" long each	60#	60#
Rollers (4) Required	8#	8#
Gussets - except at Chocks (18) Required	12#	13#
Gussets at Chocks (4) Required	4#	4#
Bulkheads at Chocks (2) Required	10#	11#
End Castings (2) Required	42#	44#
Shafts (4) Required	11#	11#
Latching Mechanism (1) Required	20#	20#
Contingency	10#	10#
Total Weight =	320#	341#

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(4) Weight Distribution on Tray

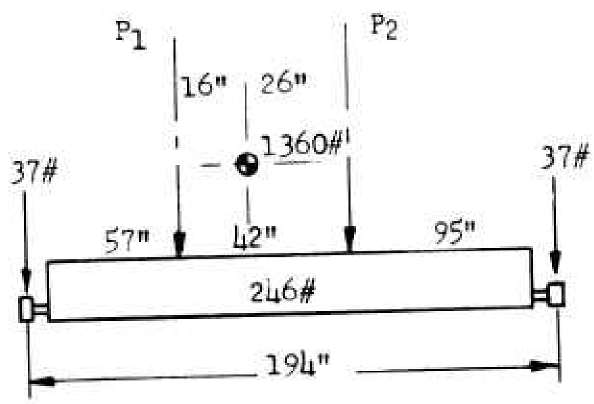
Each End	8" Deep Tray	9" Deep Tray
End Castings	21.0#	22.0#
Shafts	5.5#	5.5#
Rollers	4.0#	4.0#
Crossbeams	2.3#	2.3#
Gussets	0.5#	0.5#
Skins $\left(60 - \frac{60 \times 186}{192}\right)(.5)$	1.0#	1.0#
Main Side Beams		
$\left(118 - \frac{118 \times 186}{192}\right)(.5)$	2.0#	
$\left(135 - \frac{135 \times 186}{192}\right)(.5)$		2.5#
Contingency	0.5#	0.5#
Total =	37.0# Per End	38.0# Per End

Distributed Along Tray Span	8" Deep Tray	9" Deep Tray
Crossbeams (2.3) (9)	20.5#	20.5#
Gussets	15.0#	16.0#
Skins	58.0#	58.0#
Main Side Beams	114.0#	130.0#
Contingency	9.0#	9.0#
Bulkheads at Chocks	10.0#	11.0#
Latching Mechanism	20.0#	20.0#
Total =	246.0#	264.0#

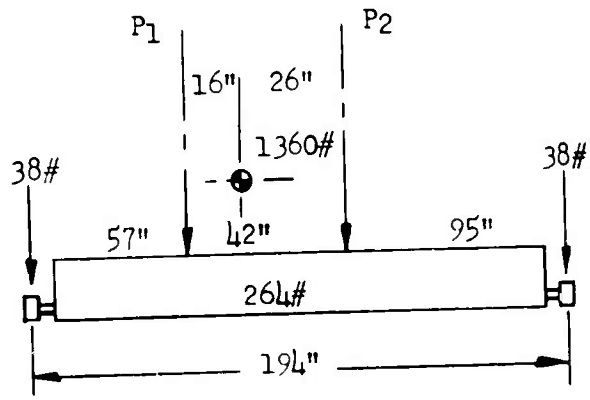
b. Tray Analyses

Vertical Direction - Tray Loaded:

8" Deep Tray



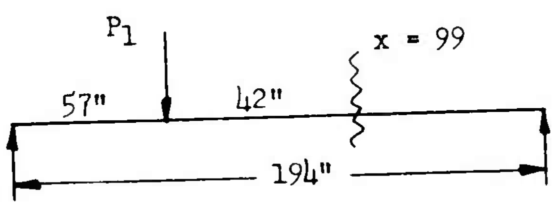
9" Deep Tray

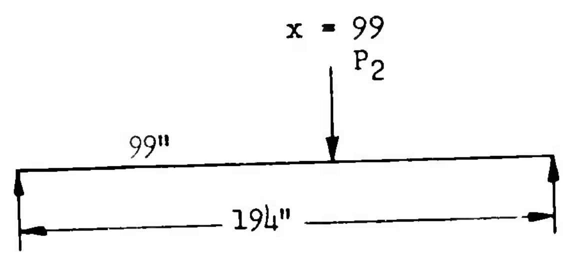


Weight of missile = 1344 lbs. plus 16 lbs. contingency = 1360 lbs.

$P_1 = 1360 \times \frac{26}{42} = 842$ lbs.; $P_2 = 1360 \times \frac{16}{42} = 518$ lbs.

Tray Deflections - 8" or 9" Deep Trays:





Deflection at x = 99 due to P_1

$$= \frac{P_1 a\ (L-x)\ \left[2Lb - b^2 - (L-x)^2\right]}{6EIL}$$

where a = 57, b = 137, x = 99; E = 10.3×10^6, P_1 = 842, L = 194

$$= \frac{9.65}{I}$$

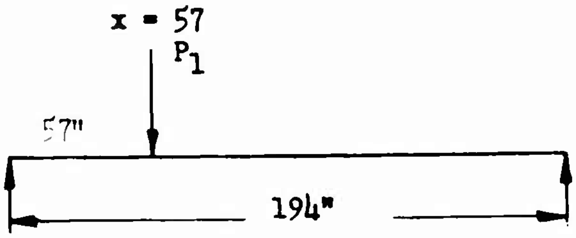
Deflection at x = 99 due to P_2

$$= \frac{P_2 a\ (L-x)\ \left[2Lb - b^2 - (L-x)^2\right]}{6EIL}$$

where a = 99, b = 95, x = 99; E = 10.3×10^6, P_2 = 518, L = 194

$$= \frac{7.65}{I}$$

Total deflection at $P_2 = \frac{17.3}{I}$

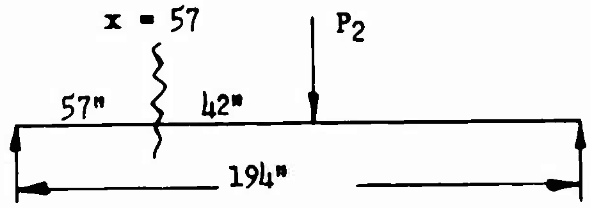


Deflection at x = 57 due to P_1

$$= \frac{P_1 bx\left[2L(L-x)-b^2-(L-x)^2\right]}{6EIL}$$

where b = 137, x = 57, L = 194, E = 10.3×10^6, P_1 = 842

$$= \frac{8.55}{I}$$



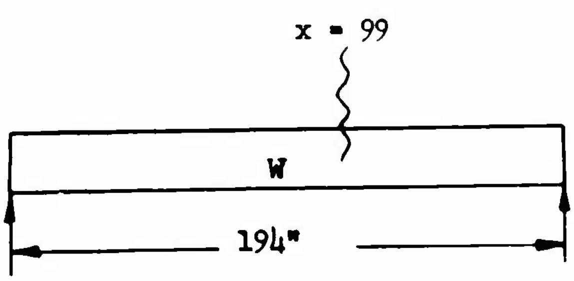
Deflection at x = 57 due to P_2

$$= \frac{P_2 bx\left[2L(L-x)-b^2-(L-x)^2\right]}{6EIL}$$

where b = 95, x = 57, L = 194, E = 10.3×10^6, P_2 = 518

$$= \frac{5.95}{I}$$

Total deflection at $P_1 = \frac{14.5}{I}$

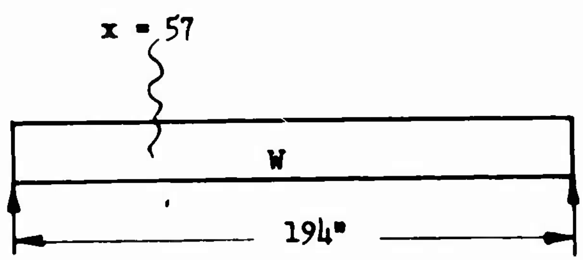


Deflection at x = 99 due to distributed load.

$$= \frac{Wx(L^3-2Lx^2+x^3)}{24EIL}$$

where x = 99, L = 194, E = 10.3×10^6

$$= \frac{.0092W}{I} \text{ at } P_2$$



Deflection at x = 57 due to distributed load.

$$= \frac{Wx(L^3-2Lx^2+x^3)}{24EIL}$$

where x = 57, L = 194, E = 10.3×10^6

$$= \frac{.0074W}{I} \text{ at } P_1$$

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8" Deep Tray

Deflection at P_2

$$\frac{17.3}{I} = \frac{17.3}{67}$$

$= .258$ in.

$$\frac{.0092\ W}{I}$$

$$= \frac{.0092(246)}{67}$$

$= .0338$ in.
$.2918$ in.

Deflection at P_1

$$\frac{14.5}{I} = \frac{14.5}{67}$$

$= .216$ in.

$$\frac{.0074\ W}{I}$$

$$= \frac{.0074(246)}{67}$$

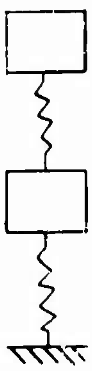
$= .0272$ in.
$.2432$ in.

Mean $= .2675$ in.

Spring constant $K = \dfrac{P_1+P_2+W}{.2675}$

$$K = \frac{842+518+246}{.2675}$$

$= 6000$ lbs. per in. for tray.



$$M_1 = \frac{842+518+246}{386.4} = 4.15$$

$K_3 = 6000$ - tray

$$M_2 = \frac{37(2)}{386.4} = .191$$

$K_2 = 6640$ (vibration)

or 7550 (shock) - mounts

9" Deep Tray

Deflection at P_2

$$\frac{17.3}{I} = \frac{17.3}{97}$$

$= .1785$ in.

$$\frac{.0092\ W}{I}$$

$$= \frac{.0092(264)}{97}$$

$= .025$ in.
$.2035$ in.

Deflection at P_1

$$\frac{14.5}{I} = \frac{14.5}{97}$$

$= .1495$ in.

$$\frac{.0074\ W}{I}$$

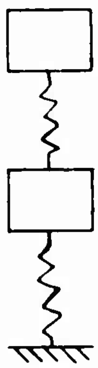
$$= \frac{.0074(264)}{97}$$

$= .0202$ in.
$.1697$ in.

Mean $= .1866$ in.

$$K = \frac{842+518+264}{.1866}$$

$= 8700$ lbs. per in. for tray.



$$M_1 = \frac{842+518+264}{386.4} = 4.2$$

$K_3 = 8700$ - tray

$$M_2 = \frac{38(2)}{386.4} = .197$$

$K_2 = 6640$ (vibration)

or 7550 (shock) - mounts

$$\omega^4 - \omega^2\left(\frac{K_3}{M_1} + \frac{K_2+K_3}{M_2}\right) + \frac{K_2K_3}{M_1M_2} = 0$$

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Steady-State Vibration:

8" Deep Tray

$$\omega^4 - \omega^2\left(\frac{6000}{4.15} + \frac{6640+6000}{.191}\right) + \frac{6640(6000)}{4.15(.191)} = 0$$

Solving,

$\omega = 27.4$; fn = 4.4 cps

$\omega = 258.0$; fn = 41.0 cps

9" Deep Tray

$$\omega^4 - \omega^2\left(\frac{8700}{4.2} + \frac{6640+8700}{.197}\right) + \frac{6640(8700)}{4.2(.197)} = 0$$

Solving,

$= 29.6$; fn = 4.7 cps

$\omega = 282.0$; fn = 45.0 cps

Check on fn at mounts (tray ends only):

fn = 6.2 cps for 8 in. or 9 in. trays (reference athwartship calculations).

Shock:

8" Deep Tray

$$\omega^4 - \omega^2\left(\frac{6000}{4.15} + \frac{7550+6000}{.191}\right) + \frac{7550(6000)}{4.15(.191)} = 0$$

$\omega = 28.2$; fn = 4.5 cps

$\omega = 267.0$; fn = 42.0 cps

Reference Goodyear Report GER-9367:

$$\lambda = \frac{25}{4.5} = 5.55$$

For $\phi = .33$, $\theta = .10$,

A = .035

Z = 1.13

g on missile = 120(.035) = 4.2
displacement at midpoint of tray = 1.657(1.13) = ±1.87 in. (approx. 0.9 in. each for tray and mounts)

9" Deep Tray

$$\omega^4 - \omega^2\left(\frac{8700}{4.2} + \frac{7550+8700}{.197}\right) + \frac{7550(8700)}{4.2(.197)} = 0$$

$\omega = 30.8$; fn = 4.9 cps

$\omega = 289.0$; fn = 46.0 cps

$$\lambda = \frac{25}{4.9} = 5.1$$

For $\phi = .33$, $\theta = .10$,

A = .040

Z = 1.127

g on missile = 120(.04) = 4.8
displacement at midpoint of tray = 1.657(1.127) = ±1.86 in. (approx. 0.9 in. each for tray and mounts)

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8" Deep Tray

Stress:

Maximum stress would be result of mitigation from mounts only.

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K}{M}} = .159\sqrt{\frac{7550}{4.15+.191}}$$

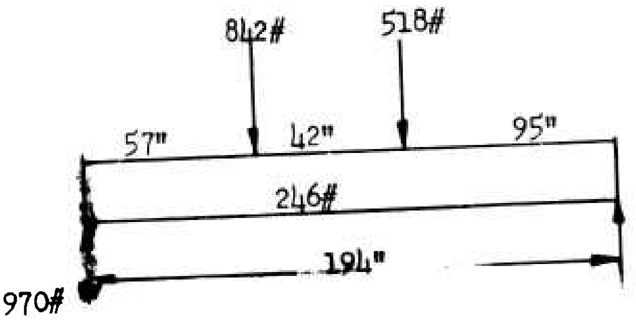
$= 6.6$ cps

Reference Goodyear Report GER-9367:

$$\lambda = \frac{25}{6.6} = 3.8$$

For $\phi = .33$, $\theta = .10$, $A = .073$

Maximum g on tray (loaded) = 120(.073) = 8.75



M maximum is at x = 99,

and = 54,800 in. lbs. at 1.0g

M 8.75g = 480,000 in. lbs.

$$f_b = \frac{My}{I} = \frac{480{,}000(5.42)}{67}$$

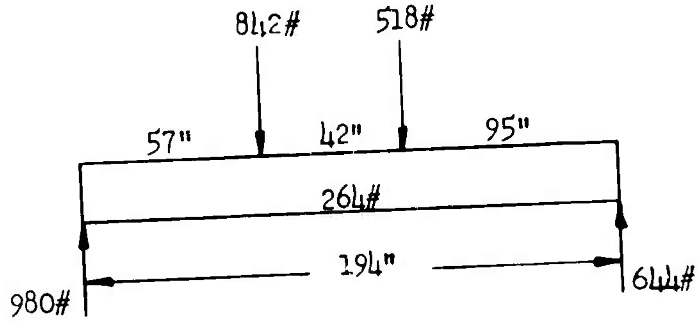
= 38,800 psi applied.

If 1.33 dynamic conversion factor is applicable (Reference NAVSHIPS 250-660-30), $f_b = \frac{38{,}800}{1.33} = 29{,}000$ psi applied compared to 46,000 psi allowable in yield for 2024-T4 aluminum.

9" Deep Tray

$$= .159\sqrt{\frac{7550}{4.2+.197}}$$

$= 6.6$ cps



M maximum is at x = 99,

and = 54,900 in. lbs. at 1.0g

M 8.75g = 480,000 in. lbs.

$$f_b = \frac{My}{I} = \frac{480{,}000(6.04)}{97}$$

= 30,000 psi applied.

$$f_b = \frac{30{,}000}{1.33} = 22{,}500 \text{ psi applied}$$

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Vertical Direction - Tray Empty:

8" Deep Tray

Deflection at mid-span

$$= \frac{5WL^3}{384EI} = \frac{5(246)(194)^3}{384(10.3x10^6)(67)} = .034 \text{ in.}$$

$$K = \frac{246}{.034} = 7250$$

$$M_1 = \frac{246}{386.4} = .637$$

$$K_3 = 7250$$

$$M_2 = \frac{37(2)}{386.4} = .191$$

$K_2 = 5740$ (vibration or shock when empty)

9" Deep Tray

$$= \frac{5(264)(194)^3}{384(10.3x10^6)(97)} = .025 \text{ in.}$$

$$K = \frac{264}{.025} = 10,600$$

$$M_1 = \frac{264}{386.4} = .683$$

$$K_3 = 10,600$$

$$M_2 = \frac{38(2)}{386.4} = .197$$

$K_2 = 5740$ (vibration or shock when empty)

$$\omega^4 - \omega^2 \left(\frac{K_3}{M_1} + \frac{K_2+K_3}{M_1} \right) + \frac{K_2K_3}{M_1M_2} = 0$$

8" Deep Tray:

$$\omega^4 - \omega^2 \left(\frac{7250}{.637} + \frac{5740+7250}{.191} \right) + \frac{5740(7250)}{.637(.191)} = 0$$

$\omega = 67.7$; fn = 10.8 cps

$\omega = 273.0$; fn = 43.0 cps

9" Deep Tray:

$$\omega^4 - \omega^2 \left(\frac{10,600}{.683} + \frac{5740+10,600}{.197} \right) + \frac{5740(10,600)}{.683(.197)} = 0$$

$\omega = 69.5$; fn = 11.1 cps

$\omega = 306$; fn = 49.0 cps

Stress:

Maximum stress due to mitigation of mounts only.

Reference athwartship - tray empty - calculations fn for either 8" or 9" deep tray $\cong$ 13 cps.

$$\lambda = \frac{25}{13} = 1.9$$

$$A = .26$$

$$g \text{ tray} = 120(.26) = 31.0$$

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8" Deep Tray	9" Deep Tray
$M\ 31.0g = \frac{WL(31)}{8} = \frac{246(194)(31)}{8}$	$\frac{264(194)(31)}{8}$
$= 185{,}000$ in. lb.	$= 198{,}000$ in. lb.
$fb = \frac{My}{I} = \frac{185{,}000(5.42)}{67(1.33)} = 11{,}200$ psi	$fb = \frac{198{,}000(6.04)}{97(1.33)} = 9200$ psi

Stress not critical.

Athwartship Direction - Tray Loaded:

Torsion in tray due to missile attachments (adjacent end castings of trays assumed clamped).

Vertical C.G. estimate exclusive of tray ends:

8" Deep Tray

Item	Weight lbs. x	Arm in. =	Moment in. lbs.
Main Side Beams	114.0	3.60	410.0
Crossbeams	20.5	.52	10.6
Gussets	15.0	3.75	56.2
Skins	58.0	.78	45.2
Chocks	10.0	4.00	40.0
Latching Mechanism	20.0	8.50	170.0
Contingency-Tray	9.0	2.70	24.3
	246.0	3.1"	756.0
		Above datum (tray bottom)	
Missile and Contingency	1360.0	10.33	14,050.0
	1606.0 lbs.	9.2 in.	14,806.0 in. lbs.
		Above datum (tray bottom)	

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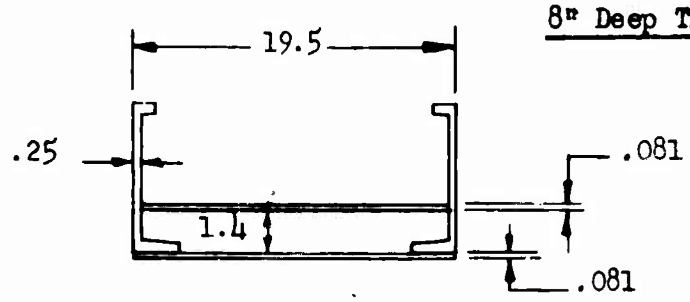
Torsional Stiffness, J:

8" Deep Tray



$t = .25$

$t_1 = .081$

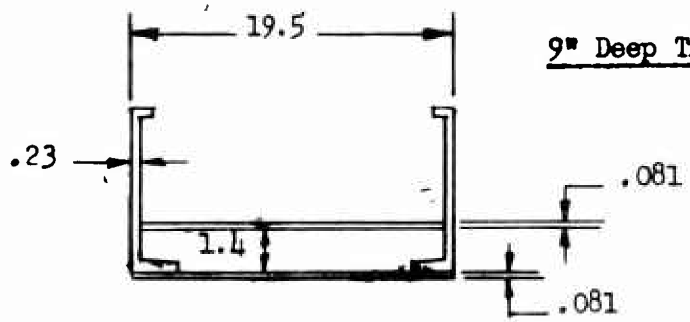
$a = 19.5$

$b = 1.562$

J of individual side beams = 0.17 each. Add 0.25 for effect of crossbeams and gussets.

$$J = \frac{2tt_1(a-t)^2(b-t_1)^2}{at+bt_1-t^2-t_1^2} + 2(.17) + 0.25 = 7.23$$

9" Deep Tray



$t = .23$

$t_1 = .081$

$a = 19.5$

$b = 1.562$

J of individual side beams = 0.20 each. Add 0.25 for effect of crossbeams and gussets.

$$J = \frac{2tt_1(a-t)^2(t-t_1)^2}{at+bt_1-t^2-t_1^2} + 2(.2) + 0.25 = 7.33$$

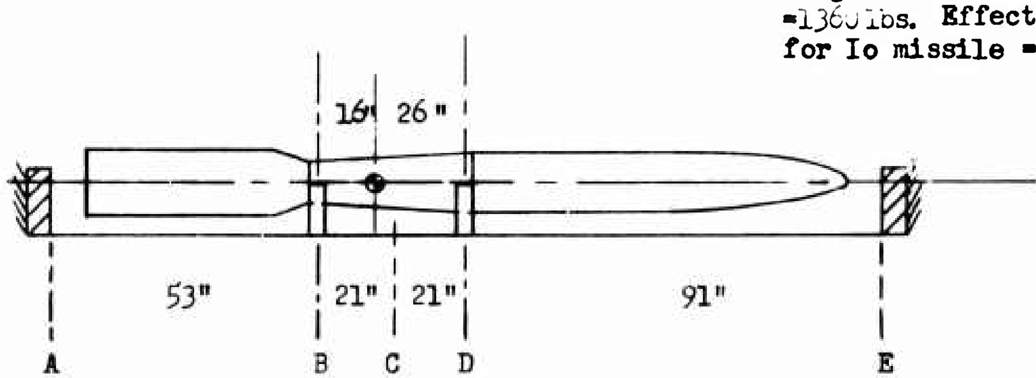
Mass moment of inertia and radius of gyration about combined C. G.

8" Deep Tray

Item	M=W/386.4	r in.	Mr^2	Io	$Im=Io+Mr^2$
Main Side Beam	.295	$\sqrt{\overline{9.2}^2 + \overline{5.6}^2} = 10.75$	34.2	$\frac{.295}{12}\left(\overline{8}^2 + \overline{2.3}^2\right) = 1.7$	35.9
Crossbeams	.053	8.7	4.03	$\frac{.053}{12}\left(\overline{19}^2 + \overline{1.4}^2\right) = 1.59$	5.6
Gussets	.039	$\sqrt{\overline{8.4}^2 + \overline{5.5}^2} = 10.0$	3.9	Negligible	3.9
Skins	.150	8.4	10.58	$\frac{.15}{12}(19.25)^2 = 4.65$	15.2

Item	M=W/386.4	r. in.	Mr^2	Io	$Im=Io+Mr^2$
Chocks	.026	8.25	1.77	$\frac{.026}{12}\left(\overline{19}^2 + \overline{5}^2\right)$ =.84	2.6
Latching Mechanism	.052	0.7	.025	$.052(6)^2 = 1.87$	1.9
Contingency-Tray	.023	10.0	2.3	Negligible	2.3
Totals	.638				67.4

Weight missile and missile contingency =1360 lbs. Effective missile radius for Io missile = 6.65."



fn (rocking at B):

	M	r	Mr^2	Io	Im
Tray Total x $\frac{74}{186}$	.254				26.8
Missile x $\frac{26}{42}$	2.180	1.13	2.79	$.5(2.18)(6.65)^2 = 48.2$	51.0
Totals	2.43				77.8

radius of gyration $= \sqrt{\frac{Im}{m}} \quad = \sqrt{\frac{77.8}{2.43}} \quad = 5.65$ in.

natural frequency, fn $= \frac{1}{2\pi}\sqrt{\frac{GJ}{L\ IM}} \quad = .159\sqrt{\frac{3.85\times10^6(7.23)}{53(77.8)}} = 13.1$ cps torque, $T = \frac{Im\ g}{5.65} = \frac{77.8(386.4)}{5.65} = 5340$ in. lb.

fn (rocking) at D:

	M	r	Mr^2	Io	Im
Tray Total x $\frac{112}{186}$	.384				40.55
Missile x $\frac{16}{42}$	1.340	1.13	1.71	$.5(1.34)(6.65)^2 = 29.6$	31.30
Totals	1.724				71.85

radius of gyration $= \sqrt{\frac{71.85}{1.724}} = 6.45$ in.

natural frequency, $fn = .159\sqrt{\frac{3.85x10^6(7.23)}{91(71.85)}} = 10.4$ cps

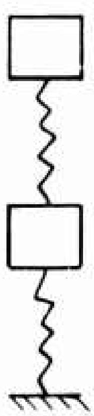
torque, $T = \frac{71.85(386.4)}{6.45} = 4300$ in. lb.

Athwartship Direction - Tray Loaded:

8" Deep Tray

Combined frequencies: torsion in tray, and athwartship translational effects in mounts.

Point B



$M_1 = 2.43$

$K_3 = 4\pi^2 fn^2 M = 4\pi^2(13.1)^2(2.43)$

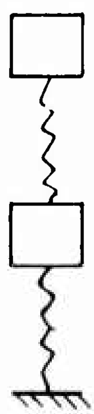
$= 16450$

$M_2 = \frac{37}{386.4} = .0955$

$K_2 = \frac{6640}{2} = 3320$ (vibration)

$\frac{7550}{2} = 3775$ (shock)

Point D



$M_1 = 1.724$

$K_3 = 4\pi^2(10.4)^2(1.724)$

$= 7350$

$M_2 = .0955$

$K_2 = 3320$ (vibration)

or 3775 (shock)

$$\omega^4 - \omega^2 \left(\frac{K_3}{M_1} + \frac{K_2+K_3}{M_2} \right) + \frac{K_2K_3}{M_1M_2} = 0$$

Vibration:

$$\omega^4 - \omega^2 \left(\frac{16{,}450}{2.43} + \frac{3320+16{,}450}{.0955} \right) + \frac{3320(16{,}450)}{2.43(.0955)} = 0$$

$$\omega^4 - \omega^2 \left(\frac{7350}{1.724} + \frac{3320+7350}{.0955} \right) + \frac{3320(7350)}{1.724(.0955)} = 0$$

Solving,

$\omega = 33.3;$ $fn = \frac{W}{2\pi} = 5.3$ cps

$\omega = 400.0;$ $fn = 73.0$ cps

$\omega = 36.0;$ $fn = 5.7$ cps

$\omega = 338.0;$ $fn = 54.0$ cps

Check on fn at mounts (tray ends only):

$$\text{deflection} = \frac{246+37+37+1360}{6640} = .253 \text{ in.}$$

$$fn = \frac{3.13}{\sqrt{.253}} = 6.2 \text{ cps}$$

Shock:

$$\omega^4 - \omega^2 \left(\frac{16{,}450}{2.43} + \frac{3775+16{,}450}{.0955} \right) + \frac{3775(16{,}450)}{2.43(.0955)} = 0$$

$$\omega^4 - \omega^2 \left(\frac{7350}{1.724} + \frac{3775+7350}{.0955} \right) + \frac{3775(7350)}{1.724(.0955)} = 0$$

Solving,

$\omega = 35;$ $fn = 5.55$ cps

$\omega = 466;$ $fn = 74.0$ cps

$\omega = 37.5;$ $fn = 5.98$ cps

$\omega = 345;$ $fn = 55.0$ cps

Reference Goodyear Report:

$$\lambda \cong \frac{30}{(5.98+5.55)(.5)} = 5.2$$

For $\phi = .33$, $\theta = .10$, $A = .04$, $Z = 1.127$

g on missile = .04(72) = 2.9

Total displacement of missile = 1.127(.689) = ± .777 in.

(trays are interlocked and move together at ends)

Tray displacement ≅ 0.2 in.

Mounts displacement ≅ 0.6 in.

Stress on Tray.

Maximum stress would be result of mitigation from mounts only.

$$fn = \frac{1}{2\pi}\sqrt{\frac{K}{M}} = .159\sqrt{\frac{7550}{\frac{246+37+37+1360}{386.4}}} = 6.6 \text{ cps}$$

$$\lambda = \frac{30}{6.6} = 4.5$$

For $\phi = .33$, $\theta = .10$, $A = .056$

g tray = .056 (72) = 4.0 maximum

$$\text{Maximum stress (torsion)} = \frac{T}{2t_1(a-t)(b-t_1)}$$

$$= \frac{5340(4)}{2(.081)(19.5-.25)(1.562-.081)} = 4600 \text{ psi}$$

Maximum stress (bending), (M 1.0g bending = 55,250 (ref.))

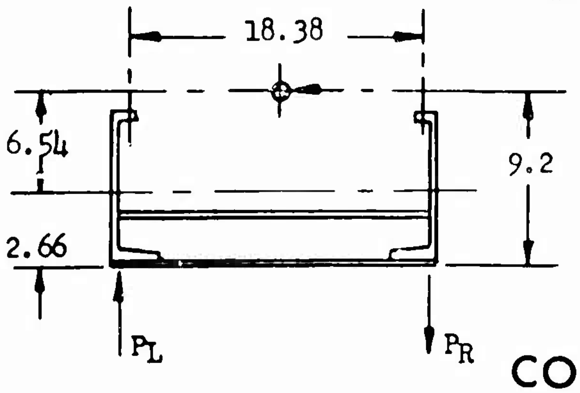
$$fb = \frac{My}{I} = \frac{55,250(4)(19.5)(.5)}{616} = 3500 \text{ psi}$$

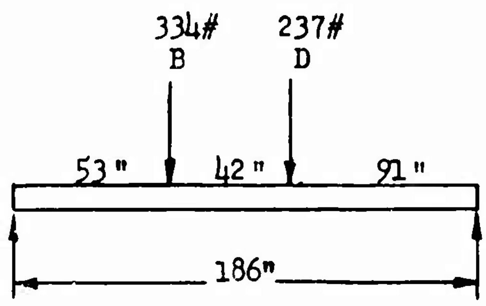
Combined stress ≅ 4600 (.707) + 3500 = 6700 psi

Stress athwartship not critical.

Alternate Check on Athwartship Direction - Tray Loaded.

8" Deep Tray





Load at B (per side beam)

$$= 246 \times \frac{74}{186} \times \frac{6.54}{18.38} + 1360 \times \frac{26}{42} \times \frac{6.54}{18.38} = 334 \text{ lb.}$$

Load at D (per side beam)

$$= 246 \times \frac{112}{186} \times \frac{6.54}{18.38} + 1360 \times \frac{16}{42} \times \frac{6.54}{18.38} = 237 \text{ lb.}$$

Deflection at x = 95:

Deflection due to B

$$= \frac{Ba(L-x)\left[2Lb-b^2-(L-x)^2\right]}{6EIL}$$

$$= \frac{334(53)(91)\left[2(186)(133)-(133)^2-(91)^2\right]}{6(10.3\text{x}10^6)(67)(.5)(186)}$$

= .098 in.

Deflection due to D

$$= \frac{Da(L-x)\left[2Lb-b^2-(L-x)^2\right]}{6EIL}$$

$$= \frac{237(95)(91)\left[2(186)(91)-(91)^2-(91)^2\right]}{6(10.3\text{x}10^6)(67)(.5)(186)}$$

= .092 in.

Total deflection at D = .098 + .092 = 0.19 in.

$$\text{fn at D} = \frac{3.13}{\sqrt{0.19}} = 7.2 \text{ cps}$$

Deflection at x = 53:

Deflection due to B

$$= \frac{Bbx\left[2L(L-x)-b^2-(L-x)^2\right]}{6EIL}$$

$$= \frac{334(133)(53)\left[2(186)(133)-(133)^2-(133)^2\right]}{6(10.3\text{x}10^6)(67)(.5)(186)}$$

= .086 in.

Deflection due to D

$$= \frac{Dbx\left[2L(L-x)-b^2-(L-x)^2\right]}{6EIL}$$

$$= \frac{237(91)(53)\left[2(186)(133)-(91)^2-(133)^2\right]}{6\ (10.3\text{x}10^6)(67)(.5)(186)}$$

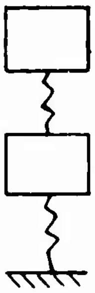
= .070 in.

Total deflection at B = .086 + .070 = .156 in.

$$\text{fn at B} = \frac{3.13}{\sqrt{.156}} = 7.9 \text{ cps}$$

Combined frequencies: couple on tray, and athwartship translational effects in mounts.

Point B



$M_1 = 2.43$

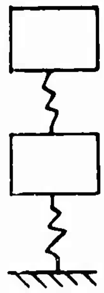
$K_3 = 4\pi^2(7.9)^2(2.43) = 6000$

$M_2 = .0955$

$K_2 = 3320$ (vibration)

or 3775 (shock)

Point D



$M_1 = 1.724$

$K_3 = 4\pi^2(7.2)^2(1.724) = 3520$

$M_2 = .0955$

$K_2 = 3320$ (vibration)

or 3775 (shock)

Vibration:

$$\omega^4 - \omega^2\left(\frac{6000}{2.43} + \frac{3320+6000}{.0955}\right) + \frac{3320(6000)}{2.43(.0955)} = 0 \qquad \omega^4 - \omega^2\left(\frac{3520}{1.724} + \frac{3320+3520}{.0955}\right) \frac{3320(3520)}{1.724(.0955)} = 0$$

Solving,

$\omega = 29.5$; fn = 4.7 cps $\omega = 31.2$; fn = 5.0 cps

$\omega = 315.0$; fn = 50.0 cps $\omega = 270.0$; fn = 43.0 cps

Shock:

$$\omega^4 - \omega^2\left(\frac{6000}{2.43} + \frac{3775+6000}{.0955}\right) + \frac{3775(6000)}{2.43(.0955)} = 0 \qquad \omega^4 - \omega^2\left(\frac{3520}{1.724} + \frac{3775+3520}{.0955}\right) + \frac{3775(3520)}{1.724(.0955)} = 0$$

Solving,

$\omega = 30.6$; fn = 4.9 cps $\omega = 33.3$; fn = 5.3 cps

$\omega = 322.0$; fn = 51.0 cps $\omega = 280.0$; fn = 45.0 cps

Reference Goodyear Report:

$$\lambda \cong \frac{30}{(5.3+4.9)(.5)} = 5.9$$

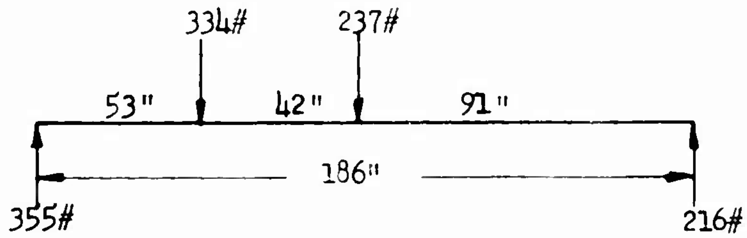
For $\phi = .33$, $\theta = .10$, $A = .0307$, $Z = 1.139$

g on missile = .0307 (72) = 2.2

Total displacement of missile = 1.139 (.689) = $\pm$0.783 in.

Stress on Tray (Bending):

From analysis above, maximum stress would be result of mitigation from mounts only, with g = 4.0.



M maximum is at x = 95, and = 19,700 in. lb. at 1.0g.

Vertical component $\quad f_b = \frac{My}{I} = \frac{19,700(4)(5.42)}{67(.5)} = 12,750$ psi

Side component $\quad f_b = \frac{My}{I} = 3500$ psi (reference calculations above)

Combined stress = 12,750+3500 = 16,200 psi.

Stress athwartship not critical.

Athwartship Direction - Tray Loaded:

Analysis similar to 8" deep tray above.

9" Deep Tray

Item	Weight lbs.	x	Arm in.	=	Moment in. lbs.
Main Side Beams	130.0		4.04		525.0
Crossbeams	20.5		.52		10.6
Gussets	16.0		4.18		67.0
Skins	58.0		.78		45.2
Chocks	11.0		4.1		45.0
Latching Mechanism	20.0		8.5		170.0
Contingency-Tray	9.0		3.0		27.0
	264.0 lbs.		3.4 in.		890.0 in. lbs.

Above datum
(tray bottom)

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Item	Weight	x	Arm	=	Moment
Missile and Contingency	1360.0 1624.0 lbs.		10.33 9.2 in.		14050.0 14940.0 in. lbs.

Above datum
(tray bottom)

9" Deep Tray

Item	M=W/386.4	r in.	Mr^2	Io	Im=Io +Mr^2
Main Side Beams	.337	$\sqrt{9.15^2+5.16^2}$ = 10.5	37.4	$\frac{.337}{12}(9^2+2.4^2)$ = 2.43	39.8
Crossbeams	.053	8.7	4.03	$\frac{.053}{12}(19^2+1.4^2)$ = 1.59	5.6
Gussets	.041	9.6	3.8	Negl.	3.8
Skins	.150	8.4	10.58	$\frac{.15}{12}(19.25)^2$ = 4.65	15.2
Chocks	.028	8.2	1.9	$\frac{.028}{12}(19^2+5^2)$ = .9	2.8
Latching Mechanism	.052	0.7	.025	$.052(6)^2$=1.87	1.9
Contingency-Tray	.023	10.0	2.3	Negligible	2.3
Totals	.684				71.4

fn (rocking) at B:

	m	r	Mr^2	Io	Im
Tray Total x $\frac{74}{186}$	.272				28.4
Missile x $\frac{26}{42}$	2.180	1.13	2.79	$.5(2.18)(6.65)^2$=48.2	51.0
Totals	2.452				79.4

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radius of gyration $= \sqrt{\dfrac{79.4}{2.452}} = 5.68$ in.

natural frequency, fn $= .159 \sqrt{\dfrac{3.85 \times 10^6 (7.33)}{53(79.4)}} = 13.0$ cps

torque, $T = \dfrac{79.4(386.4)}{5.68} = 5400$ in. lb.

fn (rocking) at D:

	M	r	Mr^2	Io	Im
Tray Total x $\frac{112}{186}$	.412				43.0
Missile x $\frac{16}{42}$	1.340	1.13	1.71	$.5(1.34)(6.65)^2 = 29.6$	31.3
Totals	1.752				74.3

radius of gyration $= \sqrt{\dfrac{74.3}{1.752}} = 6.5$ in.

natural frequency, fn $= .159 \sqrt{\dfrac{3.85 \times 10^6 (7.33)}{91(74.3)}} = 10.3$ cps

torque, $T = \dfrac{74.3(386.4)}{6.5} = 4420$ in. lb.

Combined frequencies: torsion in tray, and athwartship translational effects in mounts.

Point B

$M_1 = 2.452$

$K_3 = 4\pi^2(13)^2(2.452) = 16300$

$M_2 = \dfrac{38}{386.4} = .0985$

$K_2 = 3320$ (vibration) or 3775 (shock)

Vibration

$$\omega^4 - \omega^2 \left(\frac{16300}{2.452} + \frac{3320+16300}{.0985} \right)$$

Point D

$M_1 = 1.752$

$K_3 = 4\pi^2(10.3)^2(1.752) = 7340$

$M_2 = .0985$

$K_2 = 3320$ (vibration) or 3775 (shock)

Vibration

$$\omega^4 - \omega^2 \left(\frac{7340}{1.752} + \frac{3320+7340}{.0985} \right)$$

$$+\frac{3320(16300)}{2.452(.0985)} = 0 \qquad\qquad + \frac{3320(7340)}{1.752(.0985)} = 0$$

$\omega = 33.1$; fn = 5.3 cps $\qquad$ $\omega = 35.5$; fn = 5.65 cps

$\omega = 450.0$; fn = 72.0 cps $\qquad$ $\omega = 334.0$; fn = 53.0 cps

Check on fn at mounts (tray ends only):

$$\text{deflection} = \frac{264+38+38+1360}{6640} = .256 \text{ in.}$$

$$\text{fn} = \frac{3.13}{\sqrt{.256}} = 6.2 \text{ cps}$$

Shock:

$$\omega^4 - \omega^2\left(\frac{16300}{2.452} + \frac{3775+16300}{.0985}\right) \qquad\qquad \omega^4 - \omega^2\left(\frac{7340}{1.752} + \frac{3775+7340}{.0985}\right)$$

$$+\frac{3775(16300)}{2.452(.0985)} = 0 \qquad\qquad + \frac{3775(7340)}{1.752(.0985)} = 0$$

$\omega = 34.9$; fn = 5.55 cps $\qquad$ $\omega = 37.3$; fn = 5.93 cps

$\omega = 457.0$; fn = 73.0 cps $\qquad$ $\omega = 340.0$; fn = 54.0 cps

Reference Goodyear Report:

$$\lambda \cong \frac{30}{(5.55+5.93)(.5)} = 5.23$$

For $\phi = .33$, $\theta = .10$, A = .041, Z = 1.126

g on missile = (.041)(72) = 2.95

Total displacement of missile = (1.126)(.689) = ±.775 in.

(conditions very nearly the same as for 8" deep tray)

Stress on Tray:

Maximum stress based on mitigation from mounts only.

$$\text{fn} = .159\sqrt{\frac{7550}{\frac{264+38+38+1360}{386.4}}} = 6.6 \text{ cps}$$

$$\lambda = \frac{30}{6.6} = 4.5$$

For $\phi = .33$, $\theta = .10$, $A = 0.56$

g tray = .056 (72) = 4.0 maximum

$$\text{Maximum stress (torsion)} = \frac{5400\ (4)}{2(.081)(19.5-.23)(1.562-.081)} = 4700 \text{ psi}$$

Maximum stress (bending), (M 1.0g = 55,950 (ref.))

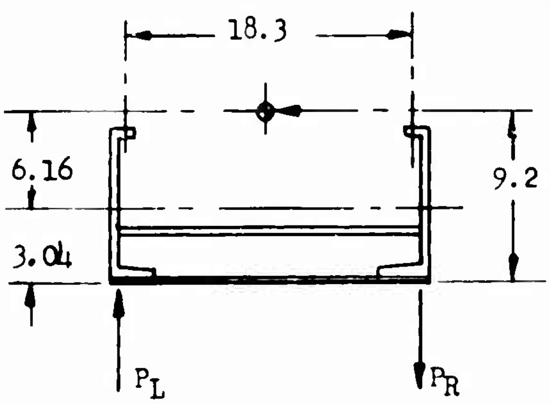
$$= \frac{55,950(4)(19.5)(.5)}{685} = 3200 \text{ psi}$$

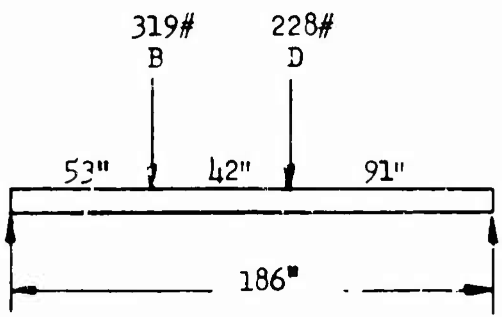
Combined stress $\cong$ 4700(.707)+3200 = 6500 psi

Stress athwartship not critical.

<u>Alternate Check on Athwartship Direction - Tray Loaded:</u>

<u>9" Deep Tray</u>





Load at B (per side beam)

$$= 264 \times \frac{74}{186} \times \frac{6.16}{18.3} + 1360 \times \frac{26}{42} \times \frac{6.16}{18.3} = 319 \text{ lb.}$$

Load at D (per side beam)

$$= 264 \times \frac{112}{186} \times \frac{6.16}{18.3} + 1360 \times \frac{16}{42} \times \frac{6.16}{18.3} = 228 \text{ lb.}$$

Deflection at $x = 95$:

Deflection due to B
(Ref. 8" Tray)

$= .098 \times \frac{319}{334} \times \frac{67}{97} = .065$ in.

Deflection due to D
(Ref. 8" Tray)

$= .092 \times \frac{228}{237} \times \frac{67}{97} = .061$ in.

Total deflection at D = .065 + .061 = 0.126 in.

$$fn \text{ at } D = \frac{3.13}{\sqrt{0.126}} = 8.8 \text{ cps}$$

Deflection at $x = 53$:

Deflection due to B

$= .086 \times \frac{319}{334} \times \frac{67}{97} = .057$ in.

Deflection due to D

$= .070 \times \frac{228}{237} \times \frac{67}{97} = .046$ in.

Total deflection at B = .057 + .046 = 0.103 in.

$$fn \text{ at } B = \frac{3.13}{\sqrt{0.103}} = 9.8 \text{ cps}$$

Combined frequencies: couple on tray, and athwartship translational effects in mounts.

Point B

$M_1 = 2.452$

$K_3 = 4\pi^2(9.8)^2(2.452) = 9300$

$M_2 = .0985$

$K_2 = 3320$ (vibration) or 3775 (shock)

Point D

$M_1 = 1.752$

$K_3 = 4\pi^2(8.8)^2(1.752) = 5400$

$M_2 = .0985$

$K_2 = 3320$ (vibration) or 3775 (shock)

Vibration:

Point B:

$$\omega^4 - \omega^2\left(\frac{9300}{2.452} + \frac{3320+9300}{.0985}\right) + \frac{3320(9300)}{2.452(.0985)} = 0$$

$\omega = 31.2$; $fn = 5.0$ cps

$\omega = 362.0$; $fn = 57.0$ cps

Point D:

$$\omega^4 - \omega^2\left(\frac{5400}{1.752} + \frac{3320+5400}{.0985}\right) + \frac{3320(5400)}{1.752(.0985)} = 0$$

$\omega = 33.3$; $fn = 5.3$ cps

$\omega = 300.0$; $fn = 48.0$ cps

Shock:

$$\omega^4 - \omega^2\left(\frac{9300}{2.452} + \frac{3775+9300}{.0985}\right) + \frac{3775(9300)}{2.452(.0985)} = 0$$

$\omega = 32.7$; fn = 5.2 cps

$\omega = 370.0$; fn = 59.0 cps

$$\omega^4 - \omega^2\left(\frac{5400}{1.752} + \frac{3775+5400}{.0985}\right) + \frac{3775(5400)}{1.752(.0985)} = 0$$

$\omega = 35.2$; fn = 5.6 cps

$\omega = 308.0$; fn = 49.0 cps

Reference Goodyear Report:

$$\lambda \cong \frac{30}{(5.6+5.2)(.5)} = 5.5$$

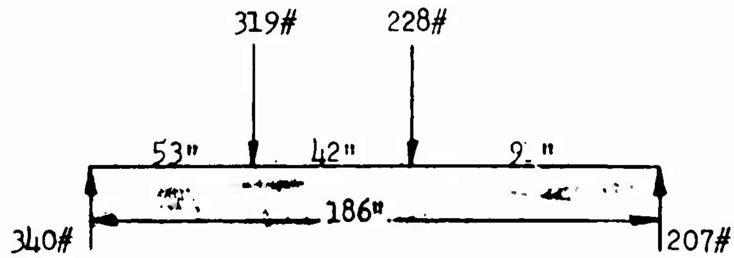
For $\phi = .33$, $\theta = .10$, A = .036, Z = 1.126

g on missile = .036 (72) = 2.6

Total displacement of missile = 1.126(.689) = $\pm$ 0.775 in.

Stress on Tray (bending):

From analysis above, maximum stress is from mitigation of mounts only, with g = 4.0.



M maximum is at x = 95, and = 18,800 in. lb. at 1.0g.

Vertical component $\quad fb = \frac{My}{I} = \frac{18,800(4)(6.04)}{97(.5)} = 9350$ psi

Side component $\quad fb = \frac{My}{I} = 3200$ (reference calculations above)

Combined stress = 9350+3200 = 12,500 psi.

Stress athwartship not critical.

Athwartship Direction - Tray Empty:

8" Deep Tray

Deflection at mid-span

$$= \frac{5WL^3}{384EI}$$

$$= \frac{5(246)(186)^3}{384(10.3x10^6)(616)} = 0.00324 \text{ in.}$$

$$K = \frac{246}{.00324} = 76{,}000$$

$$M_1 = \frac{246}{386.4} = .637$$

$$K_3 = 76{,}000$$

$$M_2 = \frac{37(2)}{386.4} = .191$$

K_2 = 5740 vibration or shock when empty

$$\omega^4 - \omega^2\left(\frac{76000}{.637} + \frac{5740+76000}{.191}\right) + \frac{5740(76000)}{.637(.191)} = 0$$

$\omega = 81.5$; fn = 13.0 cps

$\omega = 735.0$; fn = 117.0 cps

9" Deep Tray

$$\frac{5(264)(186)^3}{384(10.3x10^6)(685)} = 0.00313 \text{ in.}$$

$$K = \frac{264}{.00313} = 84{,}000$$

$$M_1 = \frac{264}{386.4} = .683$$

$$K_3 = 84{,}000$$

$$M_2 = \frac{38(2)}{386.4} = .197$$

K_2 = 5740 vibration or shock when empty

$$\omega^4 - \omega^2\left(\frac{84000}{.683} + \frac{5740+84000}{.197}\right) + \frac{5740(84000)}{.683(.197)} = 0$$

$\omega = 79.0$; fn = 12.6 cps

$\omega = 755.0$; fn = 120.0 cps

Stress:

Maximum stress due to mitigation of mounts only.

8" Deep Tray

$$fn = \frac{1}{2\pi}\sqrt{\frac{K}{M}}$$

$$= .159\sqrt{\frac{5740}{\frac{246+37(2)}{386.4}}}$$

= 13.2 cps

9" Deep Tray

$$.159\sqrt{\frac{5740}{\frac{264+38(2)}{386.4}}}$$

= 12.9 cps

For either 8" or 9" tray,

$fn \cong 13$ cps

$\lambda = \frac{30}{13} = 2.3$

$A = 0.17$

g tray $= 72\ (.17) = 12.0$

8" Deep Tray

$$M\ 16.0g = \frac{WL(12)}{8}$$

$$= \frac{246(186)(12)}{8} = 68{,}500 \text{ in. lb.}$$

$$fb = \frac{My}{I} = \frac{68{,}500(19.5)(.5)}{616(1.33)}$$

= negligible psi

9" Deep Tray

$$= \frac{264(186)(12)}{8} = 73{,}500 \text{ in. lb.}$$

$$fb = \frac{73{,}500(19.5)(.5)}{685(1.33)}$$

= negligible psi

Fore-Aft Direction - Tray Loaded:

Assume tray rollers held fore-aft direction

K mounts fore-aft = 4430

K tray fore-aft - very high

fn tray loaded fore-aft $= \frac{1}{2\pi}\sqrt{\frac{K}{M}}$

8" Deep Tray

$$fn = .159\sqrt{\frac{4430}{4.15 + .1915}} = 5.0 \text{ cps}$$

9" Deep Tray

$$fn = .159\sqrt{\frac{4430}{4.2 + .1965}} = 5.0 \text{ cps}$$

Reference Goodyear Report:

$\lambda = \frac{30}{5} = 6.0$

For $\phi = .33$, $\theta = .10$, $A = .0285$, $Z = 1.139$

g tray and missile = 38 (.0285) = 1.1g

mount displacement = .368 (1.139) = ± 0.42 in.

A study shows that snubbers in fore-aft direction would be of little or no help. They would increase frequency, and if effective in reducing the 0.42 in. value of displacement, an fn sufficiently above 14 cps would be required. This would increase g to order of 15.0 with actual little reduction in displacement.

If tray rollers are assumed to be held in fore-aft direction at only one end so that K is 4430/2, study shows that fn would be 3.6 cps (ship whipping not involved here).

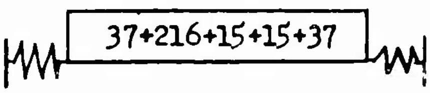
g missile tray would be 0.6

mount displacement would be ± 0.45 in.

Stress not critical.

Empty Tray - Fore-Aft Direction

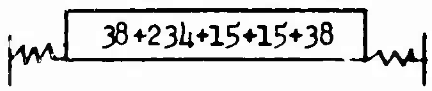
8" Deep Tray



$$fn = \frac{1}{2\pi}\sqrt{\frac{K}{M}} = .159\sqrt{\frac{4430}{320/386.4}}$$

= 11.7 cps

9" Deep Tray



$$fn = .159\sqrt{\frac{4430}{340/386.4}}$$

= 11.3 cps

Shock

Reference Goodyear Report:

$$\lambda = \frac{30}{11.7} = 2.56$$

A = .15, Z = .97

g tray = 38 (.15) = 5.7

$$\frac{P}{A} = \frac{340(5.7)}{9.26\text{-}8\text{" tray}} = \text{negligible psi}$$

or 10.13-9" tray

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Stress not critical.

Displacement = .368 (.97) = ± 0.36 in.

Preliminary Design of Vertical Support Columns for Tray Assembly:

Assume an assembly of 36 trays - 9 across, 4 high; 6 columns at each end. Column loading critical for loaded trays - equivalent of 3 per column (reference calculations above) at 8.75 g for shock mitigated tray and missile assembly; and at 120g for rollers and shafts (assuming mounts within tray).

Rails-steel-(8) for entire assembly; each column supports 4/6 x weight of rail at 120g.

Column - 8.04# alum. 6"H-beam-8 feet high.

Loading critical for 9" deep tray.

Missile	1360		
Tray Span	264		
Tray Ends	76		
	1700		
Less Shafts, Rollers	20		
	1680 x 8.75 =	'00 x 3 =	44,100 lb.
Rails	255 x 4/6 x 120.0 =		20,500
Column	8.04 x 8 x 120.0 =		7,700
Shafts, Rollers	20 x 120 =	2400 x 3 =	7,200
Total dynamic load per column =			79,500 lb.

Assume load per column = 100,000 lb. to provide for mechanism, etc. at 120g. 6"-H beam, 8.04#: A = 6.64 in.^{2}, r min. = 1.46 in.

$$\frac{P}{A} \text{ applied} = \frac{100{,}000}{6.64} = 15{,}100 \text{ psi}$$

$$\frac{P}{A} \text{ allowable} = 59{,}500-553 \left(\frac{L}{r}\right) \text{ for } \frac{L}{r} < 71$$

$$\frac{L}{r} = \frac{96}{1.46} = 66$$

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$\frac{P}{A}$ allowable = 59,500-553 (66) = 23,000 psi

$$MS = \frac{23,000}{15,100} - 1 = +52\%$$

If 1.33 dynamic conversion factor is applicable (reference NAVSHIPS 250-660-30),

$$\frac{P}{A} \text{ applied} = \frac{15,100}{1.33} = 11,300 \text{ psi}$$

$$\text{and } MS = \frac{23,000}{11,300} - 1 = \text{ample}$$

Weight Estimate of Typical 36-Missile Tray and Support Structure Assembly

Assume 9" Deep Trays

(36) Loaded Trays @ 1700	=	61,200
(8) Rails @ 255	=	2,040
(12) Columns @ 64	=	800
Mechanisms and Contingency	=	2,000
Total - Loaded System	=	66,000 lb.

Assume 9" Deep Trays - Empty

System Loaded	=	66,000
Less (36) Missiles @ 1360	=	49,000
Total - Empty System	=	17,000 lb.

3. Detailed Analyses

a. Rubber Shear Mounts

(1) General

This arrangement utilizes standard rubber shear type mounts which are affixed to the ends of the tray fore and aft structural channels and picked up by ears on the end fittings. The mounts are standard shelf items and require a minimum of testing for validation of characteristics. Two arrangements utilizing this type of mount were evaluated; one with the mounts aligned in an athwartship direction with respect to the missile; the other with the mounts disposed in a fore and aft direction. Descriptions of these configurations follow:

(2) Athwartship Arrangement

(a) Description

The tray assembly, Figure 5, includes the basic 8.0" or 9.0" aluminum tray with integral end transverse bulkheads and independent end members. One side of each of the four standard sandwich shear mounts is attached to vertical transverse bulkheads adjacent to the side channels. The other sides of the mounts are attached to the end members. An arrangement of gibs is provided to maintain vertical alignment between the tray and the end members. The gibs also provide a fail-safe feature to maintain attachment of tray and end members in the event of shock mount failure. This tray configuration with the shear mounts located in the athwartships plane, perpendicular to the missile centerline, requires an exceedingly long or wide tray to accommodate the mounts either to the rear of the booster nozzles or on each side of the booster.

(b) Comments

While structurally and dynamically efficient the arrangement is poor from the standpoint of stowage density. Insertion of the mounts plus required clearance for excursion at the booster section of the missile requires excessive width of the assembly. If the mounts are moved to the rear beyond the end of the booster the length of the tray becomes excessive because of magazine space limitations. This arrangement was therefore eliminated from consideration and no analysis has been included.

(3) Fore and Aft Arrangement

(a) Description

The tray assembly, Figure 6, includes the basic 8." or 9.0" aluminum tray with independent end members. One side of each of the four standard sandwich shear mounts is attached to the vertical web at the ends of both side channels. The other sides of the mounts are attached to ears protruding from the end members. An arrangement of gibs is provided to maintain vertical alignment between the tray and the end members. The gibs also provide a fail-safe feature to maintain attachment of tray and end member in the event of shock mount failure. This tray configuration with the shear mounts located in the fore and aft plane, parallel to the missile centerline, permits the tray assembly to have a minimum length and width.

Normally the interconnected trays are positively restrained in the athwartship direction by the locking devices on the support rails engaging the end members of the outboard tray at each level. However, with this type of tray the athwartship locking devices are spring mounted to absorb the athwartship shock. Most of the impact of an athwartship shock load into the support rails is not transmitted to the trays but is absorbed by the spring mounted locking devices and the tray rollers permit the support rails to be displaced relative to the trays. The design of the locking device would not permit relative motion between the support rails and the trays under normal ship motions.

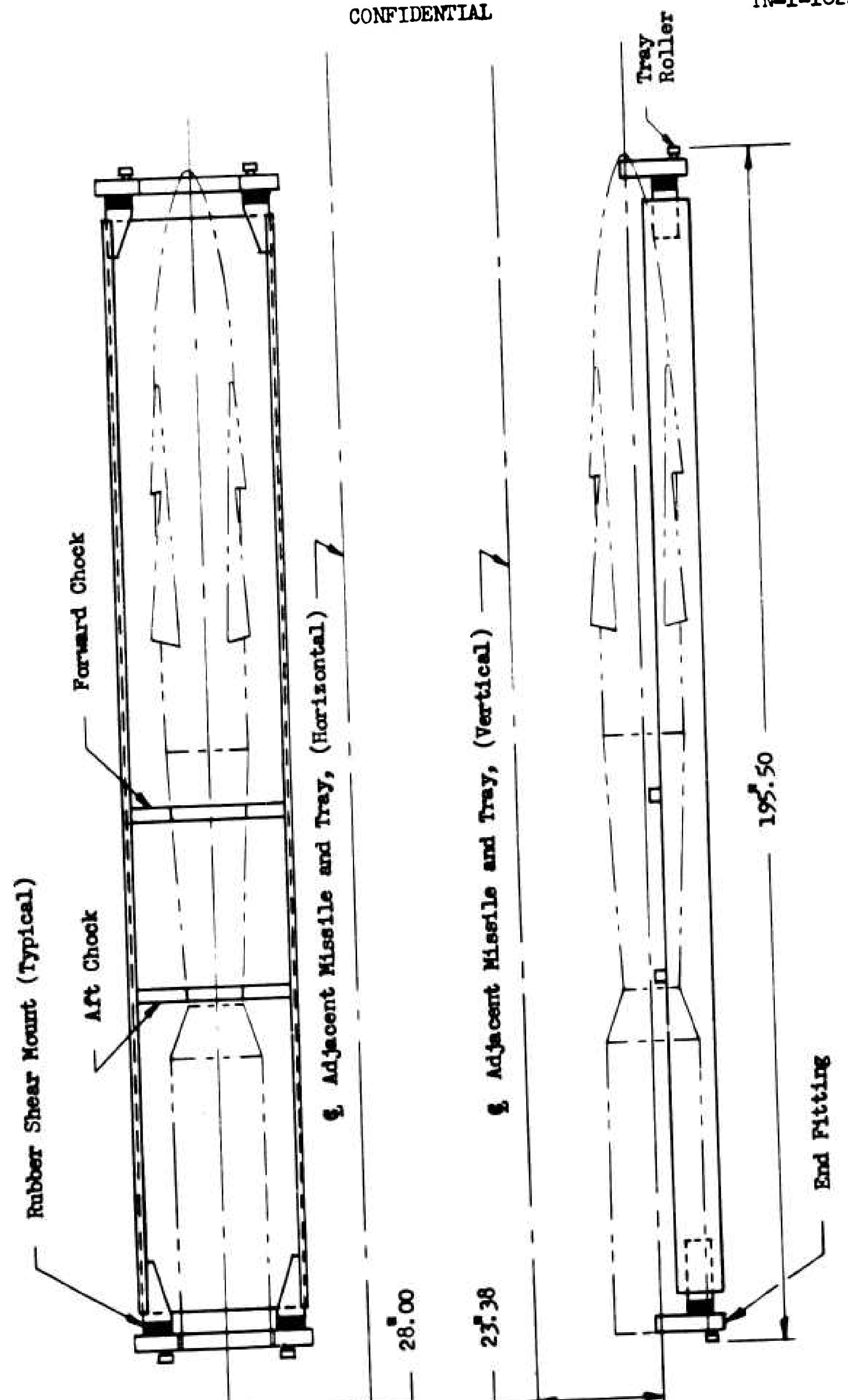


Figure 5 - Mitigated Tray - Athwartship Shear Mounts

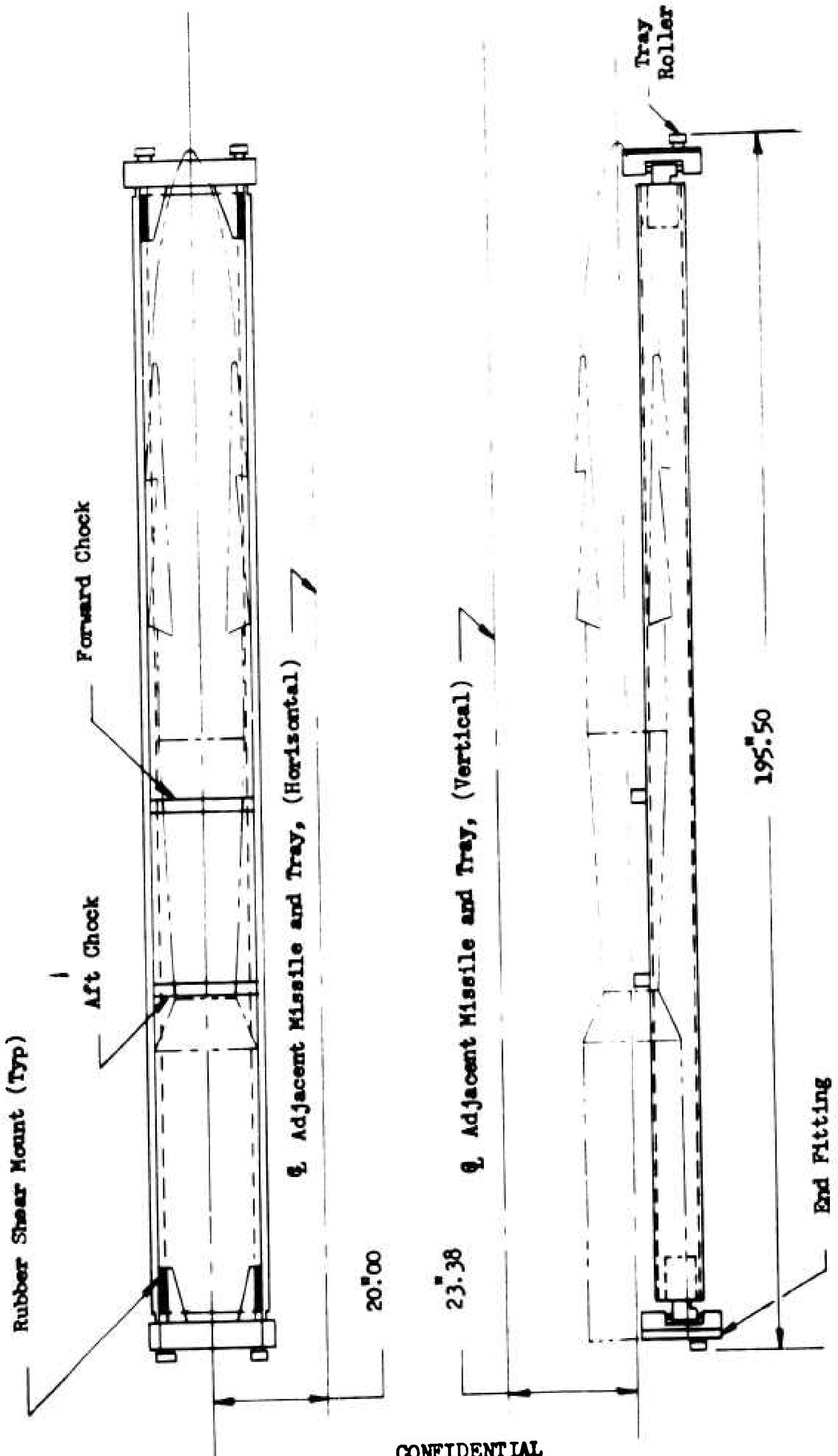


Figure 6 - Mitigated Tray - Fore/Aft Shear Mounts

(b) Detailed Analysis

As shown in the design studies of the tray structure and of the roller type shock mounts, a total spring constant, K, of approximately 7550 lbs. per in. for shock mitigation from the mounts is desired for the vertical direction. This gives the optimum natural frequency response for the resilient tray mount combination. Based on the working range of mount deflection a standard sandwich type mount such as M. B. Co. No. 7221009 would be acceptable.

$$K \text{ required per mount} = \frac{7550}{4} = 1890 \text{ lbs. per in.}$$

K of M. B. Co. mount in deflection range required = approximately 2000 lbs. per in.

This mount is of a stiffness between Nos. 7221007 and -1008, thus permitting a change of the rubber durometer if required, with no change in dimensions.

For steady state vibration conditions, K per mount would also be approximately 2000 lbs. per in.

Load - deflection data is per M. B. Co. curves.

Vertical Direction:

Steady-State Vibration - Tray Loaded:

8" Deep Tray	9" Deep Tray
K tray = 6000 lbs. per in.	K tray = 8700 lbs. per in.
K mounts = 8000 lbs. per in.	K mounts = 8000 lbs. per in.
m = 4.2	m = 4.25

m_1 and m_2 type analysis, used in roller type mounts, not required, since in in this design m_2 is but 0.04 compared to total of 4.2 or 4.25, respectively, for 8" and 9" trays.

$$K \text{ combined} = \frac{K_1 K_2}{K_1 + K_2}$$

8" Deep Tray	9" Deep Tray
$= \dfrac{6000(8000)}{6000+8000}$	$= \dfrac{8700(8000)}{8700+8000}$
= 3430 lbs. per in.	= 4150 lbs. per in.

8" Deep Tray

$$fn = \frac{1}{2\Pi}\sqrt{\frac{K}{m}}$$

$$= .159\sqrt{\frac{3430}{4.2}}$$

$$= 4.5 \text{ cps}$$

Critical magnification factor, M.F., (based on 7.75 cps forcing frequency and $\frac{c}{cc}$ of 10% for response) = 0.5

9" Deep Tray

$$= .159\sqrt{\frac{4150}{4.25}}$$

$$= 4.9 \text{ cps}$$

Critical M.F. (same basis as for 8" tray) = 0.65

Shock - Tray Loaded:

Reference Goodyear Report GER-9367.

8" Deep Tray

$$\lambda = \frac{25}{4.5} = 5.55$$

For $\phi = .33$, $\theta = .10$, $A = .035$, $Z = 1.13$

g on missile = 120(.035) = 4.2g;

displacement at midpoint of tray

= 1.657 (1.13) = ± 1.87 in.

(approximately 0.9 in. each for tray and mounts).

9" Deep Tray

$$\lambda = \frac{25}{4.9} = 5.1$$

For $\phi = .33$, $\theta = .10$, $A = .040$, $Z = 1.127$

g on missile = 120(.04) = 4.8g;

displacement at midpoint of tray

= 1.657(1.127) = ± 1.86 in.

(approximately 0.9 in. each for tray and mounts).

Stress:

Maximum stress would be result of mitigation from mounts only.

8" Deep Tray

$$fn = .159\sqrt{\frac{8000}{4.2}}$$

$$\cong 7 \text{ cps}$$

9" Deep Tray

$$fn = .159\sqrt{\frac{8000}{4.25}}$$

$$\cong 7 \text{ cps}$$

Reference Goodyear Report:

$$\lambda = \frac{25}{7} = 3.6$$

For $\phi = .33$, $\theta = .10$, $A = .083$

g tray = 120(.083) = 10.0g

8" Deep Tray	9" Deep Tray

Reference tray analysis, page 22, pro-rating for 10.0g vs. 8.75g:

8" Deep Tray

Maximum stress

$= 38,800 \left(\frac{10}{8.75}\right)$

$= 44,000$ psi

If 1.33 dynamic conversion factor (reference above) is applicable,

stress $= \frac{44,000}{1.33}$

$= 33,000$ psi applied compared to 46,000 psi yield for 2024-T4 aluminum.

9" Deep Tray

Maximum stress

$= 30,000 \left(\frac{10}{8.75}\right)$

$= 34,000$ psi

If 1.33 dynamic applied,

stress $= \frac{34,000}{1.33}$

$= 25,000$ psi applied compared to 46,000 psi yield.

Steady-State Vibration - Tray Empty:

8" Deep Tray

K tray = 7250

K mounts = 8000

m = .68

K combined $= \frac{7250(8000)}{7250+8000}$

$= 3800$

fn $= .159 \sqrt{\frac{3800}{.68}}$

$= 11.9$ cps

Critical M.F. (based on forcing frequency of 9.5 cps) = 2.4

9" Deep Tray

K tray = 10,600

K mounts = 8000

m = .73

K combined $= \frac{10,600(8000)}{10,600+8000}$

$= 4550$

fn $= .159 \sqrt{\frac{4550}{.73}}$

$= 12.6$ cps

Critical M.F. (based on forcing frequency of 14.2 cps) = 2.8

Shock - Tray Empty:

Maximum stress due to mitigation from mounts only.

8" Deep Tray	9" Deep Tray
$fn = .159\sqrt{\frac{8000}{.68}}$	$fn = .159\sqrt{\frac{8000}{.73}}$
$\cong 17$ cps	$\cong 17$ cps

For either 8" or 9" tray:

$$\lambda = \frac{25}{17} = 1.47$$

Reference Goodyear Report:

For $\phi = .33$, $\theta = .10$, $A = .46$

g tray = 120(.46) = 55.0

Reference tray analysis, page 23, pro-rating for 55g vs. 31g:

8" Deep Tray	9" Deep Tray
Maximum stress	Maximum stress
$= 11,200\left(\frac{55}{31}\right)$	$= 9200\left(\frac{55}{31}\right)$
= 20,000 psi	= 16,000 psi

Stress not critical.

Athwartship Direction:

Steady-State Vibration - Tray Loaded:

Applying the general approach derived on pages 27, 34, and alternate on 31 , 37 : (twisting of tray combined with rocking effect on mounts in vertical direction)

8" Deep Tray	9" Deep Tray
K tray = 16,450 + 7350 = 23,800	K tray = 16,300 + 7340 = 23,640
K mounts = 8000	K mounts = 8000
m = 4.2	m = 4.25
K combined $= \frac{K_1K_2}{K_1+K_2}$	K combined
$= \frac{23,800(8000)}{23,800+8000}$	$= \frac{23,640(8000)}{23,640+8000}$
= 6000	= 5975

8" Deep Tray

$$fn = \frac{1}{2\Pi}\sqrt{\frac{K}{m}}$$

$$= .159\sqrt{\frac{6000}{4.2}}$$

$= 6.0$ cps

9" Deep Tray

$$fn = .159\sqrt{\frac{5975}{4.25}}$$

$= 6.0$ cps

For both 8" and 9" deep trays, critical M.F. (based on 7.75 cps forcing frequency and $\frac{c}{cc}$ of 10% for response) = 1.5.

Employing the alternate method:

8" Deep Tray:

K tray = 6000 + 3520 = 9520

K mounts = 8000

m = 4.2

K combined

$$= \frac{9520(8000)}{9520+8000}$$

= 4340

$$fn = .159\sqrt{\frac{4340}{4.2}}$$

= 5.1 cps

Critical M.F. = 0.8

9" Deep Tray:

K tray = 9300 + 5400 = 14,700

K mounts = 8000

m = 4.25

K combined

$$= \frac{14,700(8000)}{14,700+8000}$$

= 5175

$$fn = .159\sqrt{\frac{5175}{4.25}}$$

= 5.5 cps

Critical M.F. = 1.0

Shock - Tray Loaded or Empty:

Athwartship shock not transmitted into tray in this design.

Steady-State Vibration - Tray Empty:

K tray = 76,000 (Ref. page 39)

(mounts not acting athwartship due to gib arrangement; no rocking for empty tray)

m = .68

K tray = 84,000

m = .73

8" Deep Tray

$$fn = .159 \sqrt{\frac{76,000}{.68}}$$

= approximately 53 cps

M.F. = 1.0 maximum

9" Deep Tray

$$fn = .159 \sqrt{\frac{84,000}{.73}}$$

= approximately 57 cps

M.F. = 1.0 maximum

Stress, by inspection, is not critical.

Fore-Aft Direction:

Tray structure can be assumed to be essentially rigid in this direction.

Steady-State Vibration - Tray Loaded:

$$fn = \frac{1}{2\pi} \sqrt{\frac{K}{m}}$$

m = shock mounted mass

$$fn = .159 \sqrt{\frac{2000(4)}{4.2}}$$

= 6.9 cps

$$fn = .159 \sqrt{\frac{2000(4)}{4.25}}$$

= 6.9 cps

For either 8" or 9" deep tray: critical magnification factor, M.F., (based on 7.75 cps forcing frequency) = 2.75. Factor is less for higher forcing frequencies. In these cases, $\frac{c}{cc}$ is assumed as 10% for response; reference magnification curves, Figure 3 of David Taylor Model Basin Report R-189.

Shock - Tray Loaded:

For either 8" or 9" deep tray:

Reference Goodyear Report:

$$\lambda = \frac{30}{6.9} = 4.35$$

For $\phi = .33$, $\theta = .10$, A = 0.056, Z = 1.11

g tray and missile = 38(.056) = 2.1g

mount displacement = .368(1.11) = $\pm$ 0.4 in.

Stress not critical, by inspection.

Steady-State Vibration - Tray Empty:

8" Deep Tray

$$fn = .159 \sqrt{\frac{8000}{.68}}$$

$$= 17.2 \text{ cps}$$

Critical M.F. (based on 14.2 cps forcing frequency) = 2.5

9" Deep Tray

$$fn = .159 \sqrt{\frac{8000}{.73}}$$

$$= 16.7 \text{ cps}$$

Critical M.F. (based on 14.2 cps forcing frequency) = 2.8

Shock - Tray Empty:

For either 8" or 9" deep tray:

Reference Goodyear Report:

$$\lambda = \text{approximately } \frac{30}{17} = 1.75$$

For $\phi = .33$, $\theta = .10$, $A = .36$, $Z = 1.07$

$$g \text{ tray} = 38(.36) = 13.5g$$

$$\text{displacement tray} = .368(1.07) = \pm 0.4 \text{ in.}$$

$$\text{Stress: } \frac{P}{A} \simeq \frac{13.5(340)}{10} = \text{negligible psi.}$$

(c) Comments

This configuration is the one recommended by this Laboratory. It provides adequate protection, stowage density and practicability. It should be kept in mind, however, that should future studies and investigations result in derivation of less severe input criteria, the possibility of elimination of the rubber remains. This would provide an even more simple and economical arrangement.

(4) Roller and Bushing Type Shock Mounts

(a) Description

An investiagtion was conducted into the possibility of incorporating a portion of the required resiliency into the rollers or roller supports at the tray ends. Two arrangements were evaluated wherein this was accomplished by including rubber in compression as a basic part of the roller assemblies. A third included rubber bushings in the tray end fittings. See Figures 7 and 8.

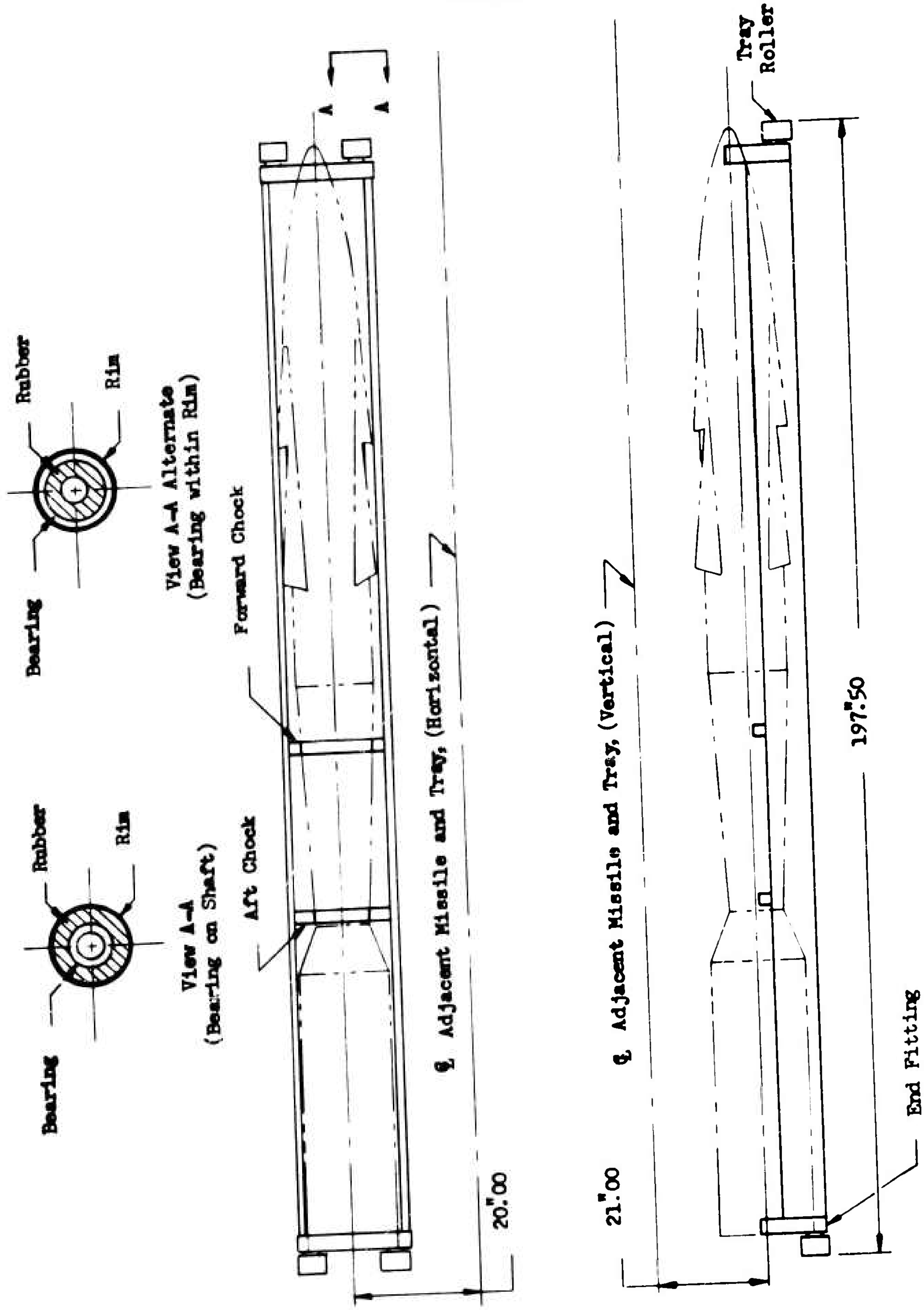


Figure 7 - Tray with Mitigation in Rollers

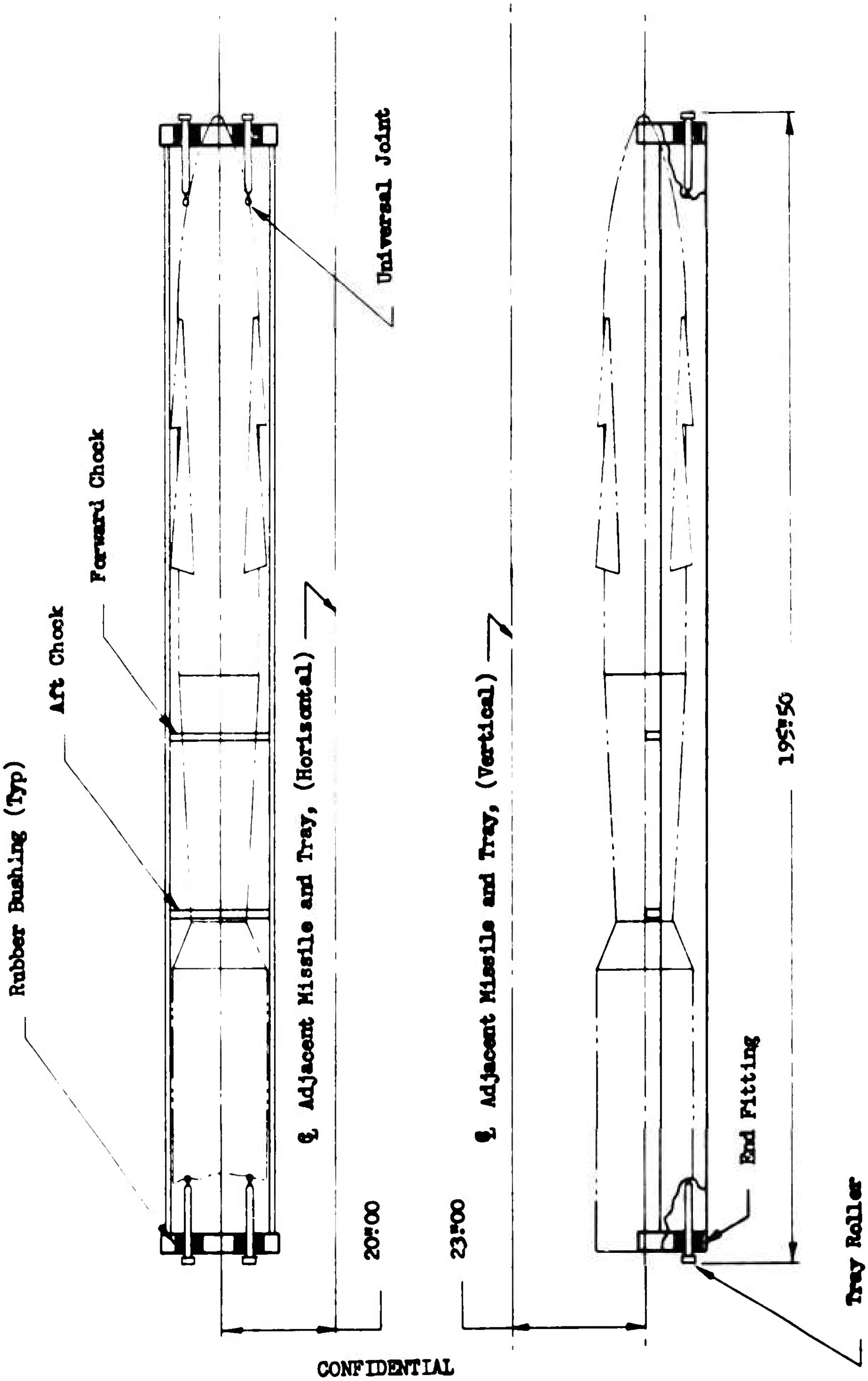


Figure 8 - Resilient Tray with Rubber Bushings

With these designs the tray end fittings are rigidly attached to the tray bodies and a portion of the excursion required for shock mitigation occurs within the rollers or bushings. The concept affords a compact configuration permitting a dense stowage arrangement.

Two basic roller designs were considered. One consisted of rollers with the rubber installed between the roller rim and the outer race of the bearing in the roller. The other utilized rubber located between the shaft and the inner race of the bearing in the roller.

The resilient bushing design incorporates the rubber in a shaft support bushing installed in the tray end casting. Criteria that follow are based upon information contained in "Vibration and Shock Isolation" by Charles E. Crede, published by John Wiley and Sons, Inc.

While the concepts are novel, none of the configurations required exist as stock items. Rubber bushings are available but not with the necessary strength and deflection parameters.

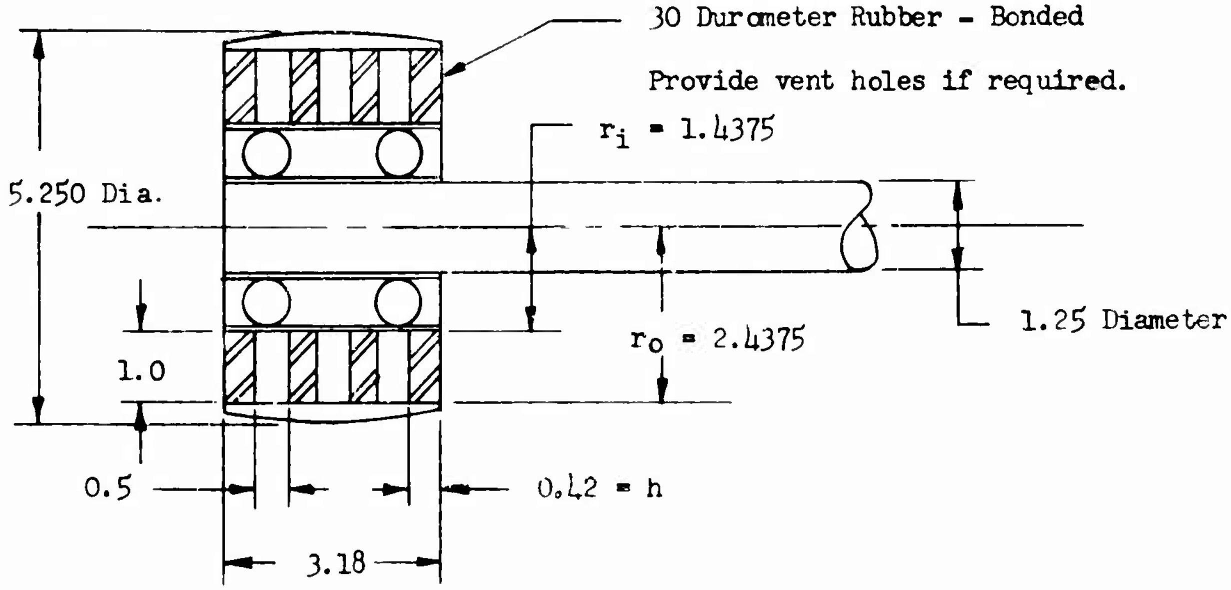
The subsequent analyses indicate the possibility of development of suitable items, with, however, the necessity for a protracted test program to establish reliability.

Because the analyses were performed as a part of the study they are included in this report. They are intended to serve as an illustration of a valid approach to the solution of the problem. But, without a protracted developmental program, recommendation of such configurations is held in abeyance.

It should be pointed out that these and other configurations that at this time appear to be borderline could well prove to be optimum should the input criteria be adjusted to what are felt to be more reasonable levels for the ships in question.

(b) Detailed Analyses

(1) Resilient Roller - Rubber at Rim



<u>Stiffness in Shear (Fore-Aft; Along Shaft):</u>

$$K = \frac{2\pi\, h\, G}{\log_e \left(\frac{r_o}{r_i}\right)}$$

$$K \text{ per bushing} = \frac{6.28(.42)(50)}{\log_e \left(\frac{2.4375}{1.4375}\right)} = 277 \text{ lbs./in.}$$

K for (4) bushings = 277 (4) = 1108 lbs./in. = K per mount

K for tray = 1108 (4) = 4432 lbs./in.

<u>Radial Stiffness (Vertical or Athwartship):</u>

(Bonded Pads)

$L = r_o + r_i > h = 2.4375 + 1.4375 = 3.875$ = length

$l = h < r_o + r_i = 0.42$ = width

$h_i = r_o - r_i = 1.0$ = thickness

Reference Crede: pages 231, 232, 225, 221.

From Figure 5.6, page 225,

$$\frac{L}{I} = \frac{3.875}{.42} = 9.23$$

$$\frac{l}{h_i} = \frac{.42}{1.0} = .42$$

$$\frac{A}{h_i} = 0.3$$

$$B = 1.06$$

Defl. % Thickness	Unit Force, psi	Total Force, lbs.	K, lbs./in.
		= 3.875(0.42)(psi) = 1.63 (psi)	= (total force) / (% thick)(hi) where h_i = 1.0
5	11	1.63(11) = 17.9	$\frac{17.9}{.05} = 358$
10	22	1.63(22) = 35.9	$\frac{35.9}{.10} = 359$
20	50	1.63(50) = 81.5	$\frac{81.5}{.2} = 408$
30	78	1.63(78) = 127.0	$\frac{127}{.3} = 423$
40	125	1.63(125) = 204.0	$\frac{204}{.4} = 510$
50	160	1.63(160) = 261.0	$\frac{261}{.5} = 522$

At approximately 25% deflection, K = 415 lbs./in. per bushing

(for vibration)

K for (4) bushings = K per mount = 415(4) = 1660 lbs./in.

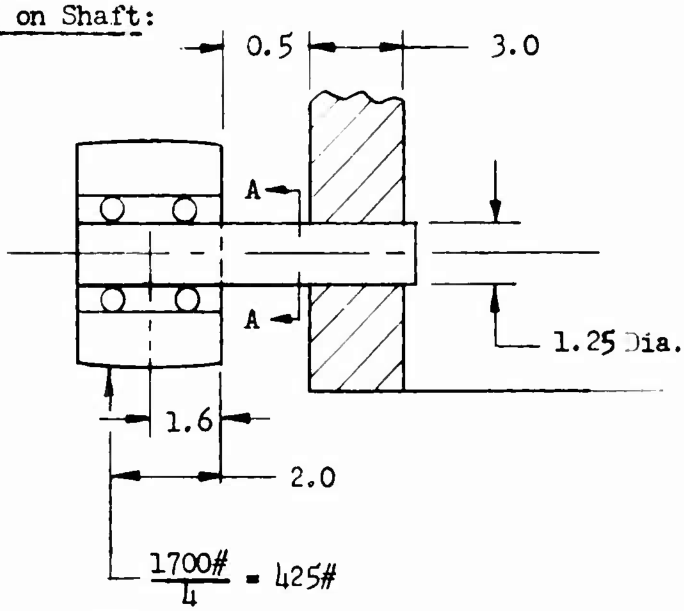
K for tray = 1660 (4) = 6640 lbs./in. (would be 9250 for 40 durom).

K for shock calculations:

for tray ≅ (415 + 530) (.5) (16) = 7550 lbs./in.

K for tray empty condition = 16 (358) = 5740 lbs./in.

Check on Shaft:



$M_{A\text{-}A\ 1.0g} \cong 2.5\ (425) = 1060$ in. lbs.

$I_{shaft} = \frac{\pi d^4}{64} = \frac{3.14(1.25)^4}{64} = 0.12$ in.^{4}

$f_{b\ 1.0g} = \frac{My}{I} = \frac{1060(.625)}{.12} = 5500$ psi

= approximately 61,000 psi under shock.

Check on Bearings:

Radial

Load on bearing (dynamic)

= 150 g (roller) + 10 g (approx.) x 418

= 150 (5) + 10 (418) = 4900 lbs.

Thrust

Load on bearing (dynamic)

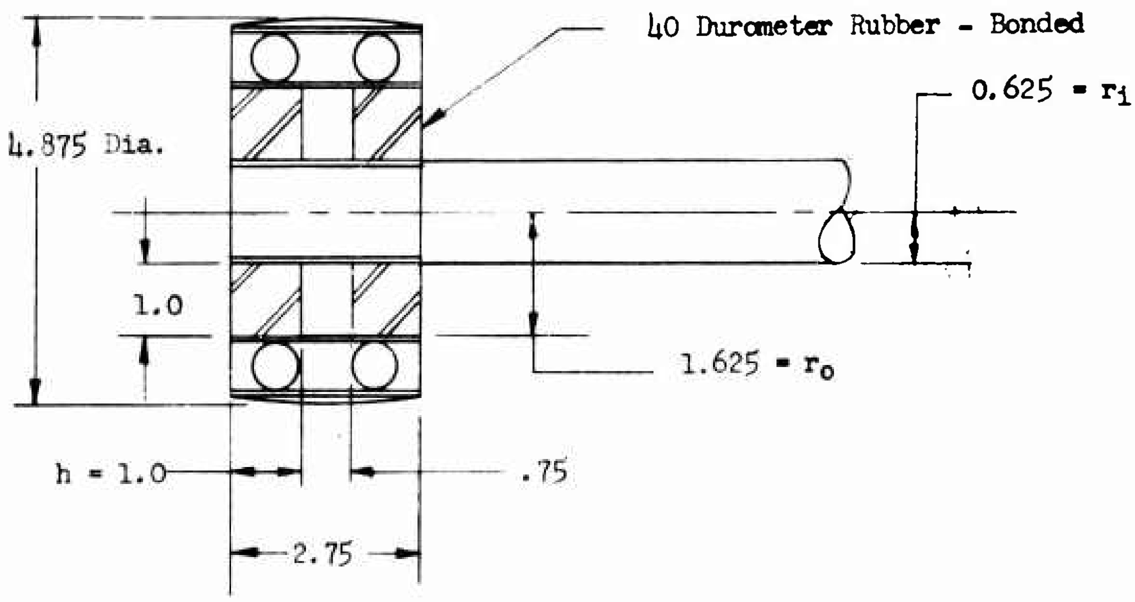
= 50 g (roller) + 10 g (approx.) x 418

= 50 (5) + 10 (418) = 4400 lbs.

Bore of bearing = approximately 1.25 in.

Bearings are available for this condition.

(2) Resilient Roller - Rubber at Shaft



Stiffness in Shear (Fore-Aft; Along Shaft):

$$K = \frac{2\pi\, h\, G}{\log e\left(\frac{r_o}{r_i}\right)}$$

$$K \text{ per bushing} = \frac{6.28(1)(65)}{\log e\left(\frac{1.625}{0.625}\right)} = 435 \text{ lbs./in.}$$

K for (2) bushings = K per mount = 435 (2) = 870 lbs./in.

K for tray = 870 (4) = 3480 lbs./in.

Radial Stiffness (Vertical or Athwartship):

(Bonded Pads)

$L = r_o + r_i > h = 1.625 + 0.625 = 2.25$ = length

$l = h < r_o + r_i = 1.0$ = width

$h_i = r_o - r_i = 1.0$ = thickness

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Reference Crede: pages 231, 232, 225, 221

From Figure 5.6, page 225,

$$\frac{L}{1} = \frac{2.25}{1.0} = 2.25$$

$$\frac{1}{h_i} = \frac{1.0}{1.0} = 1.00$$

$$\frac{A}{h_i} = 1.25$$

$$B = 2.0$$

Defl. % Thickness	Unit Force, psi	Total Force, lbs.	K, lbs./in.
		= 2.25(1)(psi) = 2.25 (psi)	= $\frac{\text{total force}}{\text{\% thick } (h_i)}$ where h_i = 1.0
5	20	2.25(20) = 45	$\frac{45}{.05}$ = 900
10	38	2.25(38) = 85	$\frac{85}{.1}$ = 850
20	80	2.25(80) = 180	$\frac{180}{.2}$ = 900
30	135	2.25(135) = 304	$\frac{304}{.3}$ = 1013
40	200	2.25(200) = 450	$\frac{450}{.4}$ = 1125
50	250	2.25(250) = 562	$\frac{562}{.5}$ = 1124

At approximately 23% deflection, K for (2) bushings

(for vibration)

≅ 930 (2) = 1860 lbs./in.

K for tray = 1860 (4) = 7440 lbs./in.

K for shock calculations:

for tray ≅ 1015 (8) = 8100 lbs./in.

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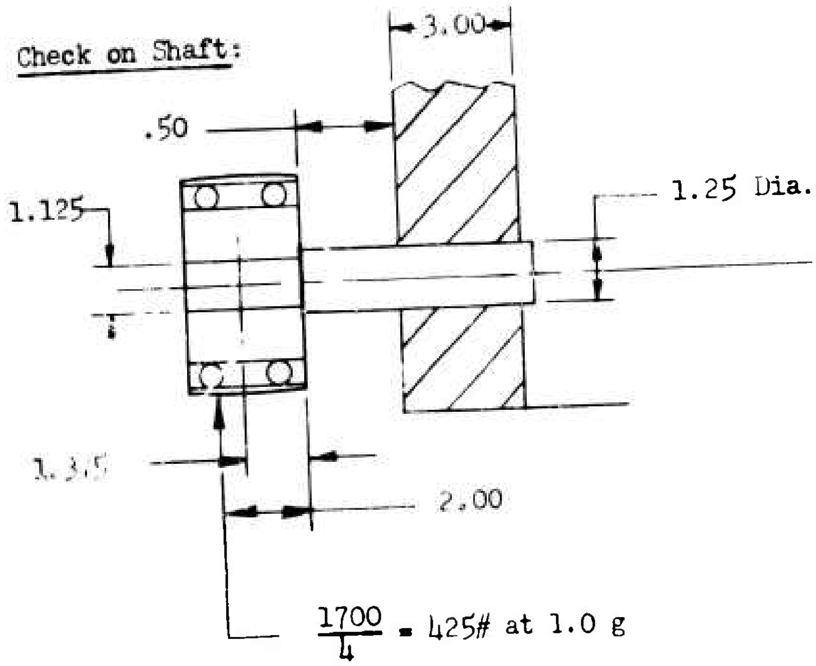
Check on Shaft:



$$\frac{1700}{4} = 425\# \text{ at } 1.0 \text{ g}$$

$$I_{1.125D} = \frac{\pi d^4}{64} = \frac{3.14(1.125)^4}{64} = 0.07854 \text{ in.}^4$$

$M = 425\ (2) = 850$ in.lbs.

$$f_{b\ 1.125D} = \frac{My}{I} = \frac{850(.5625)}{.07854} = 6100 \text{ psi at } 1.0 \text{ g}$$

deflection = negl.

$$I_{1.25D} = \frac{\pi d^4}{64} = \frac{3.14(1.25)^4}{64} \quad 0.12 \text{ in.}^4$$

$M = 425\ (2.5) \cdot 1060$ in. lbs.

$$f_{b\ 1.25D} = \frac{My}{I} = \frac{1060(.625)}{.12} = 5{,}500 \text{ psi at } 1.0 \text{ g}$$ (7600 psi if D is 1.125 all along)

= approximately 61,000 psi under shock.

Check on Bearings:

Radial

Load on bearing (dynamic)

= 150 g (roller) + 10 g (approx.) x 418

= 150 (5) + 10 (418) = 4900 lbs.

Thrust

Load on bearing (dynamic)

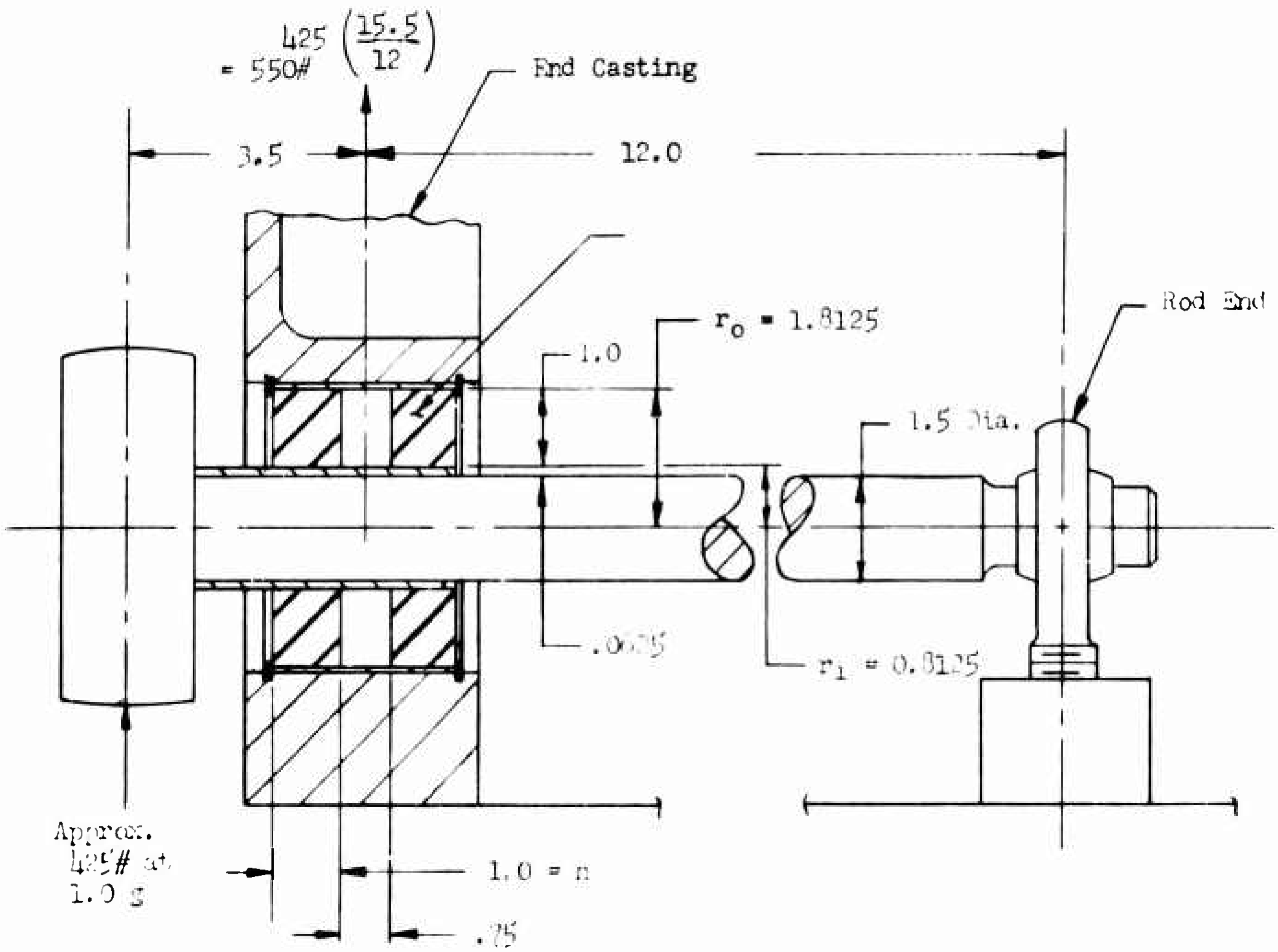
= 50 g (roller) + 10 g (approx.) x 418

= 50 (5) + 10 (418) = 4400 lbs.

Bore of bearing = approximately 3.5 in.

Rating is no problem.

(3) Rubber Bushing Type Shock Mount



Check on Shaft:

$$M_{1.0g} \cong 3.5\ (425) = 1480 \text{ in. lbs.}$$

$$I_{shaft} = \frac{\pi d^4}{64} = \frac{3.14(1.5)^4}{64} = 0.25 \text{ in.}^4$$

$$fb_{\ 1.0g} = \frac{My}{I} = \frac{1480(.75)}{.25} = 4440 \text{ psi}$$

= approximately 50,000 psi under shock

Stiffness in Shear (Fore-Aft; Along Shaft):

$$K = \frac{2\pi\ h\ G}{\log e \dfrac{r_o}{r_i}}$$

$$K \text{ per bushing} = \frac{6.28(1.0)(65)}{\log\ e \left(\dfrac{1.8125}{.8125}\right)} = 518 \text{ lbs./in.}$$

K for (2) bushings = K/mount = 518(2) = 1016 lbs./in.

K for tray = 1016 (4) = 4060 lbs./in.

Radial Stiffness (Vertical or Athwartship):

(Bonded Pads)

$L = r_o + r_i > h = 1.8125 + 0.8125 = 2.625$ = length

$l = h < r_o + r_i = 1.0$ = width

$h_i = r_o - r_i = 1.0$ = thickness

Reference Crede: pages 231, 232, 225, 221

From Figure 5.6, page 225,

$$\frac{L}{l} = \frac{2.625}{1.0} = 2.6$$

$$\frac{l}{h_i} = \frac{1.0}{1.0} = 1.0$$

$$\frac{A}{h_i} = 1.3$$

$$B = 2.0$$

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Defl. % Thickness	Unit Force, psi	Total Force, lbs.	K, lbs./in.
		$= 2.625(1.0)(psi)$ $= 2.625(psi)$	$= \frac{\text{total force}}{\%\ \text{thick}\ (h_i)}$ $= \frac{\text{total force}}{\%\ \text{thick}}$
5	20	2.625(20) = 53	$\frac{53}{.05} = 1060$
10	38	2.625(38) = 100	$\frac{100}{.10} = 1000$
20	80	2.625(80) = 210	$\frac{210}{.20} = 1050$
30	135	2.625(135) = 354	$\frac{354}{.30} = 1180$
40	200	2.625(200) = 525	$\frac{525}{.40} = 1310$
50	250	2.625(250) = 655	$\frac{655}{.50} = 1310$

At approximately 23% deflection, K for (2) bushings

(for vibration)

$\cong 1080\ (2) = 2160$ lbs./in.

K for tray = 2160 (4) = 8640 lbs./in.

Estimate of Required K: (for vibration)

for roller type mount - rubber at shaft:

$$\text{deflection} = \frac{\text{load}}{K} = \frac{425}{7440} = .058 \text{ in.}$$

for roller type mount - rubber at rim:

$$\text{deflection} = \frac{\text{load}}{K} = \frac{425}{6640} = .064 \text{ in.}$$

for mount within tray

$N.061 = \frac{550}{K}$ K req'd. = 9000 $\cong$ 8640

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K for shock calculations:

for roller type mount - rubber at shaft = 8100 lbs./in.

for roller type mount - rubber at rim = 7550 lbs./in.

K for mount (bushing type) within tray

$\cong 7850 \times \frac{550}{425} \cong 10,000$ lbs./in. req'd.

Actual $K \cong 1250\ (8) = 10,000$ lbs./in.

(c) Comments

The analyses indicate that the required excursion would impose excessive deformation of the rubber. The adequacy under shock conditions would have to be determined by extensive testing. Such mounts were therefore deemed impractical from the standpoint of reliability and eliminated from consideration for use in the preliminary design concept of the tray stowage shock mitigating system recommended by this Laboratory.

b. Resilient Trays

(1) Summary

Analyses were performed on the effectiveness of only tray resiliency for shock mitigation. A summary of critical loads is shown below.

Summary - Resilient Trays, 8" and 9" Deep

	Tray Loaded				Tray Empty		
	Tray Depth	Vertical	Athwart-ship	Fore-Aft	Vertical	Athwart-ship	Fore-Aft
Steady-State Vibration Maximum Magnification Factor	8"	1.4	4.9	1.6	3.3	1.0	1.0
	9"	3.5	4.9	1.6	1.8	1.0	1.0
Shock Maximum g at Chocks	8"	7.2	11.0	21.0	-	-	-
	9"	10.3	11.0	21.0	-	-	-
Maximum Stress on Tray, psi	8"	74,000	Not Critical	Not Critical	Not Critical	Not Critical	Not Critical
	9"	65,000					

(2) Detailed Analysis

In this arrangement, the primary source of shock mitigation (vertical or athwartship) is the resiliency of the aluminum tray structure, with the added effect of thin rubber pads at missile supporting bands (fore-aft). Mitigation in supporting columns is negligible. See Figure 9.

Fore-Aft Direction - Tray Loaded:

A study indicates insufficient deflection in tray structure to mitigate fore-aft shock load. Assume the addition of 2 in. wide missile hold-down straps or bands having 0.2 in. thick, 40 Durometer rubber liners covering entire missile circumference.

Area of rubber in shear = πdw = 3.14(10)(2) + 3.14(14)(2) = 150 sq. in.

Shear stress at 1.0g = $\frac{1360}{150}$ = 9 psi

Reference Crede: page 228, Figure 5.9,

shear strain = 0.1

deflection = (.1) (.2) = .02 in.

$f_n = \frac{3.13}{\sqrt{.02}} = 22.0$ cps

$\lambda = \frac{30}{22} = 1.36$

For $\phi = .33$, $\theta = .10$, A = .563, Z = 1.095

g on missile = .563 (38) = 21.0

displacement of missile = 1.095 (.368) = $\pm$ 0.4 in.

An inspection study shows that stress is not critical.

Fore-Aft Direction - Tray Empty:

An inspection study indicates that natural frequency is very high, but that stress is not critical.

Vertical Direction - Tray Loaded:

Area of rubber pads at missile, in compression = approximately

(10+4) (2) (.75) = 36 sq. in.

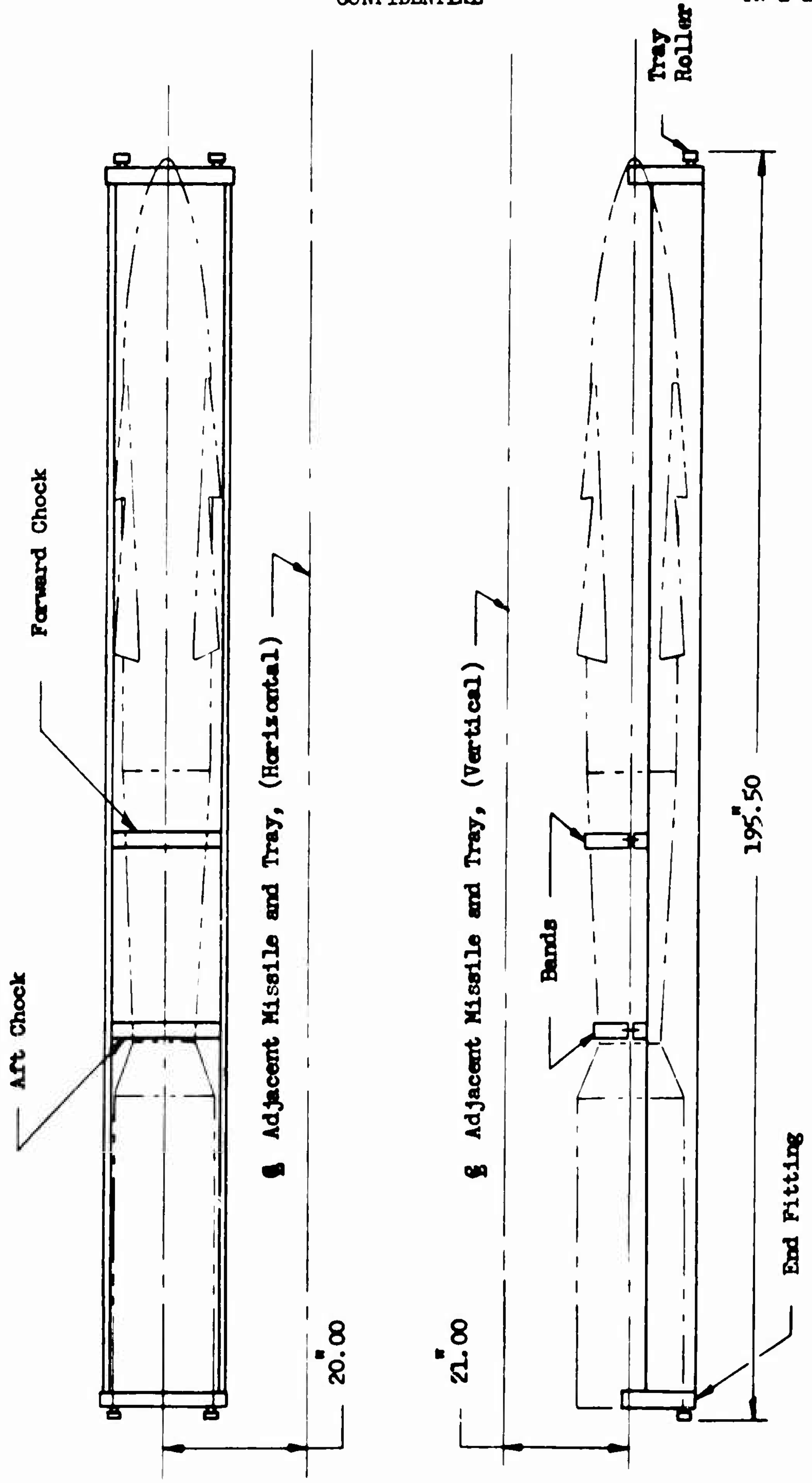


Figure 9 - Resilient Aluminum Tray - No Shock Mounts

For 40 Durometer rubber, K = approximately 950 lb. per in. per sq. in. per inch of thickness = 4750 lb. per in. per sq. in. for 0.2 in. t.

K total = 36 (4750) = 170,000 lb. per in.

8" Deep Tray

$M_1 = \frac{1360}{386.4} = 3.52$

$K_3 = 170,000$

$M_2 = \frac{246}{386.4} = .637$

$K_2 = 6,000$

$$\omega^4 - \omega^2\left(\frac{170000}{3.52} + \frac{6000+170000}{.637}\right) + \frac{6000(170000)}{3.52(.637)} = 0$$

$\omega = 37.5$; fn = 6.0 cps

$\omega = 565$; fn = 90.0 cps

$\lambda = \frac{25}{6} = 4.2$

For $\phi = .33$, $\theta = .10$,

A = .06,

Z = 1.114

g at mid-span of tray

= 120 (.06) = 7.2 on missile

displacement at mid-span of tray

= 1.657 (1.114) = ± 1.84 in.

From data above, M maximum is at
x = 99. and = 54800 in. lb. at 1.0g
M 7.2g = 395,000

9" Deep Tray

$M_1 = 3.52$

$K_3 = 170,000$

$M_2 = \frac{264}{386.4} = .683$

$K_2 = 8,700$

$$\omega^4 - \omega^2\left(\frac{170000}{3.52} + \frac{8700+170000}{.683}\right) + \frac{8700(170000)}{3.52(.683)} = 0$$

$\omega = 44.7$; fn = 7.1 cps

$\omega = 555.0$; fn = 88.0 cps

$\lambda = \frac{25}{7.1} = 3.52$

For $\phi = .33$, $\theta = .10$,

A = .086,

Z = 1.052

g at mid-span of tray

= 120 (.086) = 10.3 on missile

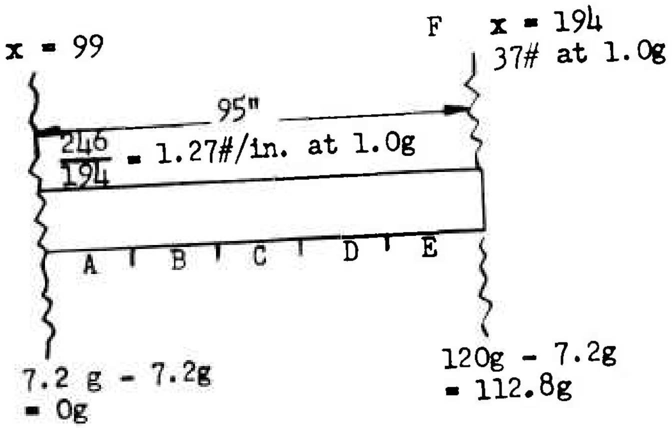
displacement at mid-span of tray

= 1.657 (1.052) = ± 1.74 in.

54900 in. lb.
M 10.3g = 565,000

8" Deep Tray

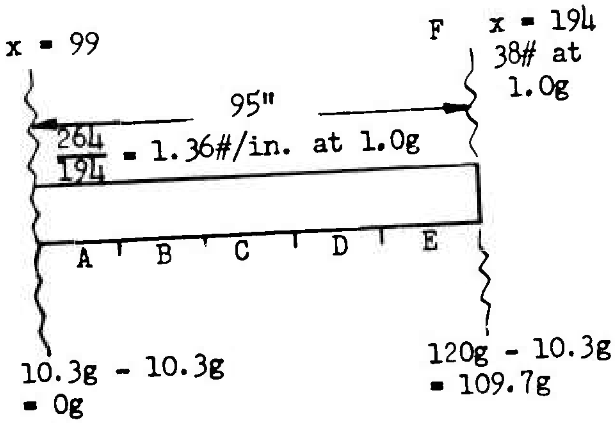
Superimposing possible effect of varying mitigation from none at tray ends to maximum at x = 99:



Zone A-E = $\frac{95}{5} \times 1.27 = 24.1$# each

F	$112.8 \times 37 \times 95 =$	395,000
E	$112.8 \times \frac{85.5}{95} \times 24.1 \times 85.5 =$	209,000
D	$112.8 \times \frac{66.5}{95} \times 24.1 \times 66.5 =$	126,500
C	$112.8 \times \frac{47.5}{95} \times 24.1 \times 47.5 =$	64,500
B	$112.8 \times \frac{28.5}{95} \times 24.1 \times 28.5 =$	23,200
A	$112.8 \times \frac{9.5}{95} \times 24.1 \times 9.5 =$	2,600
		820,800
		395,000
	M maximum =	1,215,800 in. lb.

9" Deep Tray



Zone A-E = $\frac{95}{5} \times 1.36 = 25.8$# each

F	$109.7 \times 38 \times 95 =$	395,000
E	$109.7 \times \frac{85.5}{95} \times 25.8 \times 85.5 =$	217,000
D	$109.7 \times \frac{66.5}{95} \times 25.8 \times 66.5 =$	131,000
C	$109.7 \times \frac{47.5}{95} \times 25.8 \times 47.5 =$	67,000
B	$109.7 \times \frac{28.5}{95} \times 25.8 \times 28.5 =$	24,000
A	$109.7 \times \frac{9.5}{95} \times 25.8 \times 9.5 =$	2,700
		836,700
		565,000
	M maximum =	1,401,700 in. lb.

8" Deep Tray	9" Deep Tray
Stress, $f_b = \frac{My}{I} = \frac{1,215,800(5.42)}{67}$	$\frac{1,401,700(6.04)}{97}$
$= 98,400$ psi	$= 87,000$ psi

If 1.33 dynamic conversion factor is applicable, (reference NAVSHIPS 250-660-30),

$f_b = \frac{98400}{1.33} = 74,000$ psi applied	$\frac{87000}{1.33} = 65,000$ psi applied

Various allowables - aluminum:

(yield points, tension and compression)

Sheet and Plate	2024-T86	66,000 psi
	7178-T6	73,000 psi
Extrusions	7075-T6	73,000 psi
(such as channels)	7178-T6	77,000 psi

Available MS $= \frac{73000}{74000} - 1 \cong -1\%$	Available MS $= \frac{73000}{65000} - 1 = +12\%$

Vertical Direction - Tray Empty:

Reference calculations above,

deflection empty tray = .034 in.	deflection = .025 in.
$f_n = \frac{3.13}{\sqrt{d}} = \frac{3.13}{\sqrt{.034}} = 17$ cps	$f_n = \frac{3.13}{\sqrt{.025}} = 20$ cps

Stress:

Conservatively,	Conservatively,
M maximum $= \frac{WL(120)}{8} = \frac{246(194)(120)}{8}$	M maximum $= \frac{WL(120)}{8} = \frac{264(194)(120)}{8}$
$= 700,000$ in. lb.	$= 770,000$ in. lb.
$f_b = \frac{My}{I} = \frac{700,000(5.42)}{67(1.33)} = 42,000$ psi	$f_b = \frac{My}{I} = \frac{770,000(6.04)}{97(1.33)} = 36,000$ psi
(not critical)	(not critical)

Athwartship Direction - Tray Loaded:

Reference page 31:

8" Deep Tray

$$fn = .159\sqrt{\frac{6000+3520}{2.43+1.724}} = 7.6 \text{ cps}$$

(effect of thin rubber pads in compression - negligible)

$$\lambda = \frac{30}{7.6} = 4$$

A = .065

g on missile = .065 (72) = 4.7

9" Deep Tray

$$fn = .159\sqrt{\frac{9300+5400}{2.452+1.752}} = 9.4 \text{ cps}$$

$$\lambda = \frac{30}{9.4} = 3.2$$

A = .096

g on missile = .096 (72) = 6.9

Note - By alternate analysis, fn not critical, but g on missile would be 11.0 for 8" and 9" deep trays.

Stress:

Combined stress

$$= 16200 \times \frac{11}{4} = \frac{44000}{1.33} = 33{,}000 \text{ psi}$$

(not critical)

Combined stress

$$= 12500 \times \frac{11}{4} = \frac{34000}{1.33} = 25{,}000 \text{ psi}$$

(not critical)

Athwartship Direction - Tray Empty:

Reference above calculations,

deflection at mid-span = 0.00324 in.

$$fn = \frac{3.13}{\sqrt{.00324}} = 55.0 \text{ cps}$$

Stress:

$$M \text{ maximum} = \frac{WL(72)}{8} = \frac{246(186)(72)}{8}$$

= 410,000 in. lb.

$$fb = \frac{My}{I} = \frac{410000(19.5)(.5)}{616} = 6{,}500 \text{ psi}$$

deflection = 0.00313 in.

$$fn = \frac{3.13}{\sqrt{.00313}} = 55.0 \text{ cps}$$

$$M \text{ maximum} = \frac{WL(72)}{8} = \frac{264(186)(72)}{8}$$

= 440,000 in. lb.

$$fb = \frac{440000(19.5)(.5)}{685} = 6{,}300 \text{ psi}$$

Stress not critical.

Preliminary Design of Vertical Support Columns for Tray Assembly:

Assume an assembly of 36 trays - 9 across, 4 levels; 6 columns at each end.

Column loading critical for loaded trays - equivalent of 3 per column at 10.3g for central part of tray at missile chocks and pro-rated to 120g at structure at outer ends of tray.

Rails, steel - (8) for entire assembly; each column supports 4/6 x weight of rail at 120g.

Column 9.4# alum., 6"H-beam-8 feet high.

Loading critical for 9" deep tray.

Missile	1360#
Tray Span	264#
Tray Ends	76#
	1700#
Less shafts, Rollers	20#
	1680 x 10.3 = 17300 x 3 = 51,900#
Shafts, Rollers	20 x 120 = 2400 x 3 = 7,200#

Tray Structure:

$38(2) \times 109.7 = 8300 \times 3$	=	24,900#
$2(25.8) \times 109.7 \times \frac{85.5}{95} = 5100 \times 3$	=	15,300#
$2(25.8) \times 109.7 \times \frac{66.5}{95} = 4000 \times 3$	=	12,000#
$2(25.8) \times 109.7 \times \frac{47.5}{95} = 2800 \times 3$	=	8,400#
$25.8 \times 109.7 \times \frac{28.5}{95} = 850 \times 3$	=	2,500#
$25.8 \times 109.7 \times \frac{9.5}{95} = 280 \times 3$	=	800#
Rails $255 \times \frac{4}{6} \times 120$	=	20,500#

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Column 9.4 x 8 x 120	=	9,000#
Add 20,000# to provide for mechanism, etc. at 120g	=	20,000#
Total dynamic load per column	=	172,000 lbs.

6"-9.4# H-beam, $A = 7.77$ in.^{2}, r min. = 1.42 in.

$$\frac{P}{A} \text{ applied} = \frac{172000}{7.77} = 22,000 \text{ psi}$$

$$\frac{P}{A} \text{ allowable} = 59500-553\left(\frac{L}{r}\right) \text{ for } \frac{L}{r} < 71 \quad \frac{L}{r} = \frac{96}{1.42} = 67.5$$

$$\frac{P}{A} \text{ allowable} = 59500-553\ (67.5) = 22,100 \text{ psi}$$

$$MS = \frac{22100}{22000} - 1 = +0\%$$

If 1.33 conversion factor (reference above) is applicable,

$$\frac{P}{A} \text{ applied} = \frac{22000}{1.33} = 16,500 \text{ psi}$$

$$MS = \frac{22100}{16500} - 1 = +34\%$$

Weight Estimate of Typical 36-Missile Tray and Support Structure Assembly:

Assume 9" Deep Tray

(36) Loaded Trays @ 1700 #	=	61,200 #
(8) Rails @ 255 #	=	2,040 #
(12) Columns @ 75 #	=	900 #
Mechanisms and Contingency	=	2,000 #
Total - Loaded System	=	66,100 lbs.

Assume 9" Deep Trays - Empty

System Loaded	=	66,100 #
Less (36) Missiles @ 1300#	=	49,000 #
Total - Empty System	=	17,100 lbs.

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(3) Conclusions

Inputs to the missile are too high for compatibility with the existing missile design. Application of shock inputs from the Bureau of Ships curves for surface vessels resulted in the following forces on the missile when supported by an 8 inch deep tray, as noted in the summary of the structural and dynamic analysis:

(a) 7.2 g's vertically.

(b) 11.0 g's athwartship.

(c) 21.0 g's fore and aft.

These forces are applied to the missile at the two launch lug hard spots. Even should the missile be capable of sustaining this loading it is felt that the band tension required to provide fore and aft restraint of the missile would induce excessive local loads on the missile, especially when the static and dynamic forces were additive.

The fore and aft loads could be reduced by spring loading the tray support rollers in a fore and aft direction. However, even should a tolerable load condition be realized, bands about the missile are incompatible with the automated tray stowage and handling system recommended by this Laboratory for the EAGLE missile. For these reasons the resilient tray configuration is not considered suitable for use with the tray stowage and handling system unless input parameters should become reduced.

c. Resiliently Supported Magazine

(1) Summary

The tabulation following represents a summary of the maximum missile loads derived in the structural and dynamic analysis of a resiliently supported magazine.

Summary - Resiliently Supported Magazine

	Trays Loaded			Trays Empty-Except One		
	Vertical	Athwartship	Fore-Aft	Vertical	Athwartship	Fore-Aft
fn (cps	4.33-33.6	2.34-125	4.1	7.97-32.2	3.66-136	7.22
Shock g's on Missile	3.22	1.06	.75	11.1	1.06	2.7
Relative Displacement Across Beam, in.	2.0	.92	.357	1.5	.66	.35
Relative Displacement Between Trays, in.	.7	.33	-	.60	Negligible	-
Rocking fn	-	13	-	-	-	-
Maximum Magnification Factor	1.3	.2	2.9	4	.4	5

Total weight of magazine and missiles = 82,119 lbs.

Total weight of missiles = 49,000 lbs.

(2) Detailed Analysis

Shock and Vibration Study of Resiliently Supported Magazine:

The following report presents an analysis to determine the feasibility of a resiliently mounted magazine. The sketch shown on page 12 indicates the general configuration of the system. The tray is shown on Figure 10.

The magazine is resiliently supported between the upper and lower decks by twelve (12) horizontal tapered beams. The fore and aft ends of the magazine are constructed to form a rigid structure; i.e., it is assumed there will be no relative motion between adjacent tapered beams.

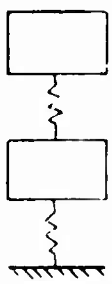
Although the support structure provides most of the resiliency, the missile trays also possess a certain degree of "spring." Advantage has been taken of the fact by assuming a 2-degree of freedom system in the vertical and athwartship directions.

Results indicate the feasibility of this proposed arrangement. However, due to the state of the art and absence of experimental data, it is felt that dynamic tests should be accomplished prior to the installation of such a system aboard ship.

It should be noted that although the support structure functions as a spring arrangement it does not fulfill a secondary function but is actually a part of the primary magazine structure and should be so considered. It includes the same margins and safety factors as the rest of the magazine structure.

Vertical Direction (Full Magazine):

The following mathematical model has been assumed in the vertical direction:



M_1 = mass of missiles and trays

K_3 = spring constant of the trays

M_2 = mass of the support structure

K_2 = spring constant of the support structure

Note: Since the most severe response of the magazine will occur when the motion of the decks are in phase, it will be assumed that this condition occurs for the purpose of analysis. Under such conditions the effects of the upper and lower tapered beams are additive.

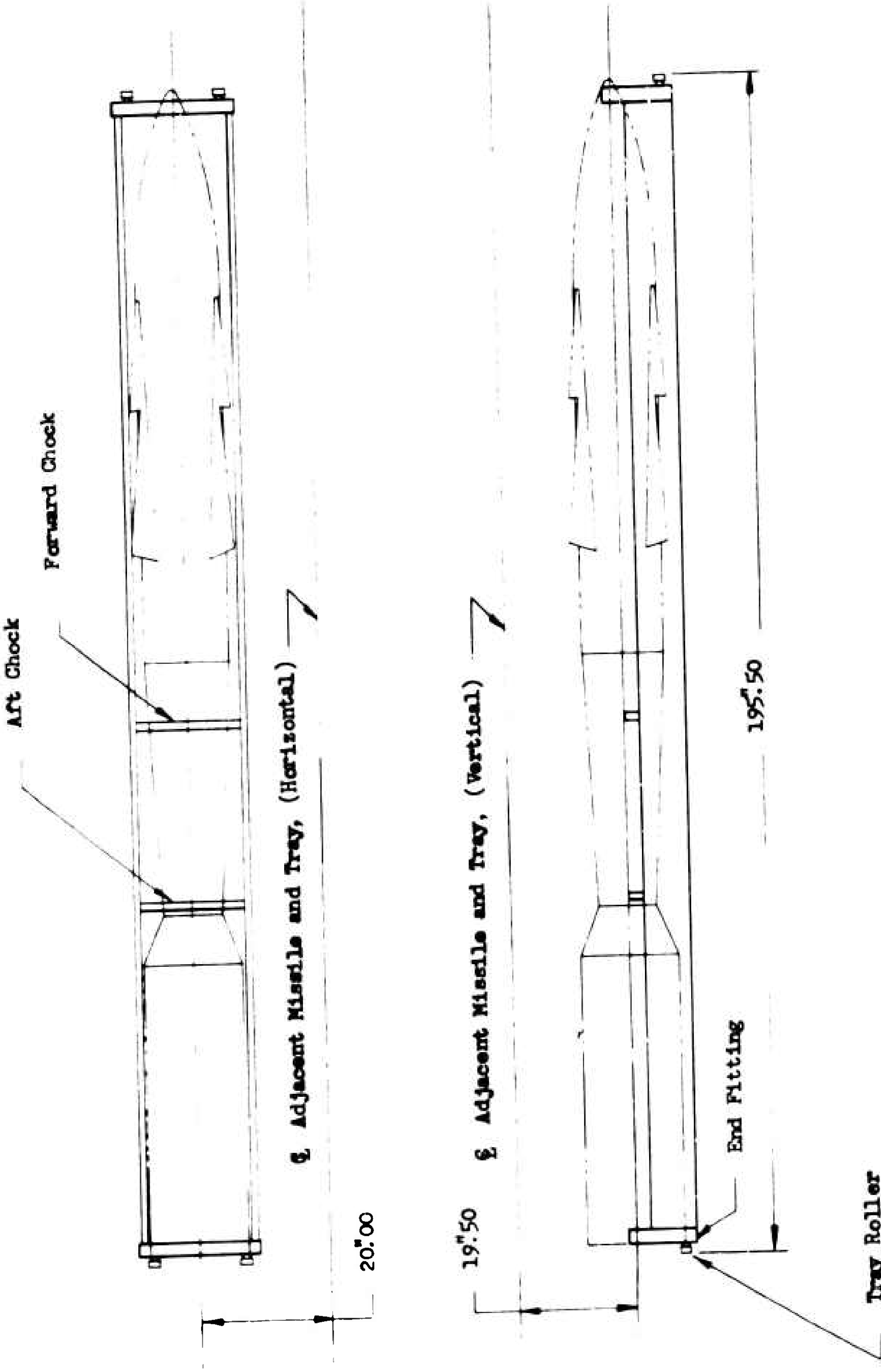
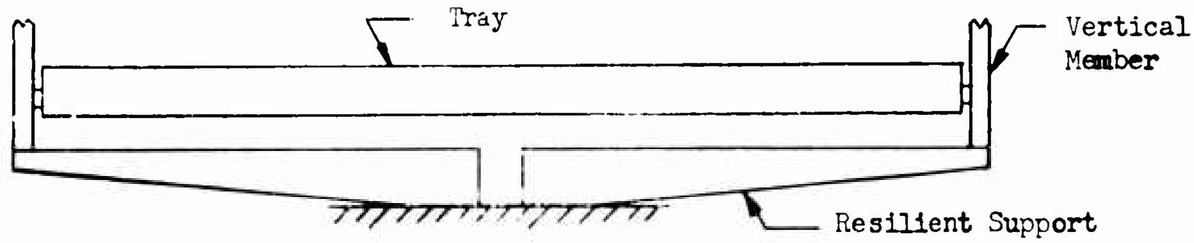


Figure 10 - Non-Mitigated Tray
For Resiliently Supported Magazine

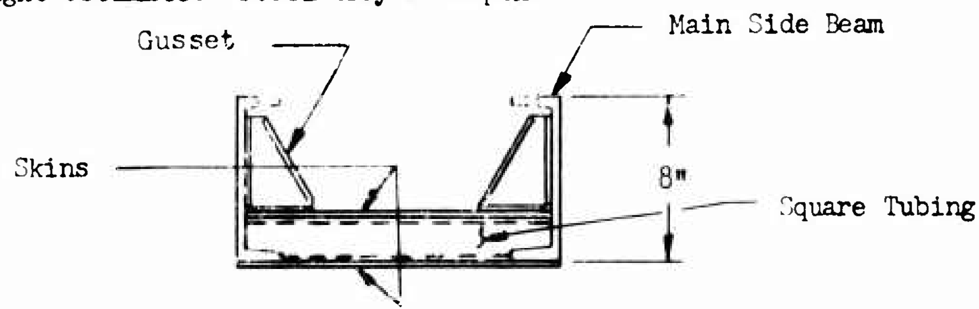
Furthermore, it shall be assumed that any static or dynamic loads on the structure will be distributed uniformly among the tapered beams.

With these assumptions in mind, a dynamically equivalent system can be obtained by subdividing the structure into smaller, more convenient units for purposes of calculation. This subdivision will be as shown below.



This portion of the structure will support 3 missile trays, 1 vertical member, 2/3 of a horizontal rail, and 12 tray rollers.

Weight estimate: Steel tray 8" depth.



Square tubing: 1 1/4 x 1 1/4 x .120 1.755 #/ft.

6 tubes, each 19" long

$$\text{wt.} = \frac{19 \times 6}{12} \times 1.755 = 17 \text{ lbs.}$$

Main side beams: Area/beam = .795 x 1.76 x .385 = 2.94 in.^{2}

2 beams each 192" long

wt. = 192 x 2 x 2.94 x .2836 = 320 lbs.

Skins: .05 in. thick, 192" long

one is 19" wide, the other 19.5" wide

wt. = (19+19.5) x 192 x .05 x .2836 = 105 lbs.

Gussets: Other than at missile supports.

8 required.

$$\text{wt.} = 8\left[\left(\frac{3.25+1}{2}\right)(x6) \times (.25) + 9\,(1.25)(.25)\quad .2836\right] = 14 \text{ lbs.}$$

Missile support gussets and bulkheads:

4 gussets required

2 bulkheads required

wt. gussets $4\,(3+3)(.375)(6)(.2836) = 15$ lbs.

wt. bulkheads $2\left[(18.5)(6.25) - 9\,(2.5)(.5)\right]\, .25 \times .2836 = 15$ lbs.

End castings: 2 required

$$\text{wt} = 2\left\{\left[10(20) - (16)^2(.7854)(.5)\right](.5)(.2836) + \left[(1.5)(.5)\left(20+10+10+\pi \times (16) \times (.5)\right)\right] .2836\right\} = 60 \text{ lbs.}$$

Total tray weight:

square tubing	17 lbs.
main beams	320 lbs.
skins	105 lbs.
gussets	14 lbs.
missile gussets and bulkheads	30 lbs.
end castings	60 lbs.
connections and contingency	30 lbs.
Total =	576 lbs. = wt. of 1 tray.

Each missile weighs 1360 lbs. Therefore,

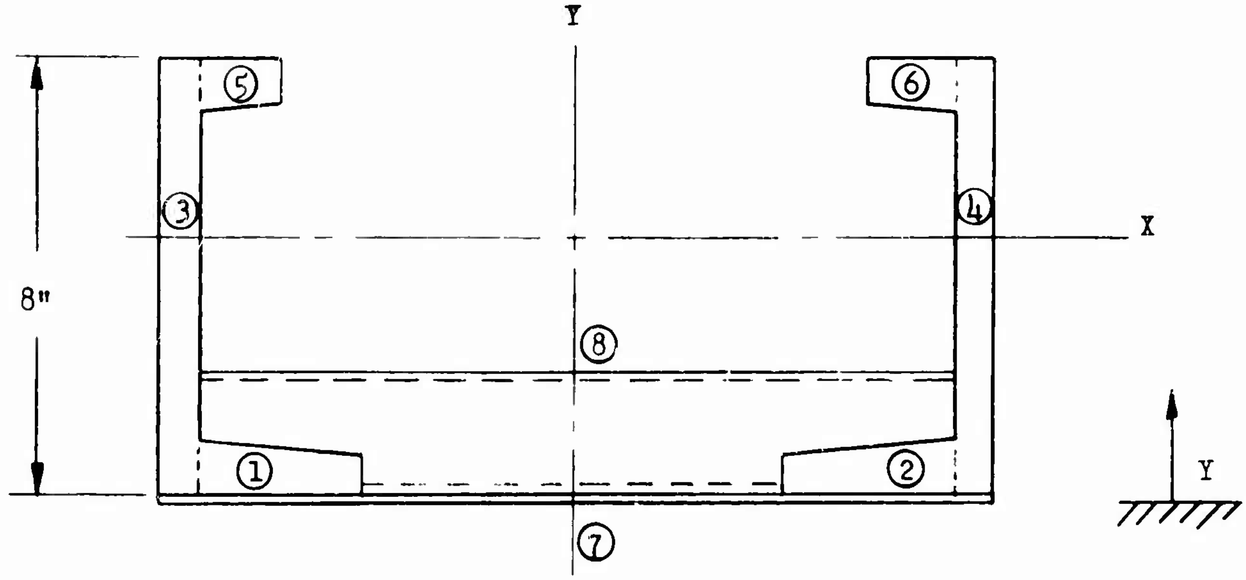
$$M_1 = \frac{(576+1360)\,3}{386} = 15.03$$

Determine K_3 = spring constant of the trays.

First must obtain neutral axis of the tray.

$$\bar{y} = \frac{\Sigma A.y}{\Sigma A}$$

The cross sectional area of the side channels are divided into smaller component areas.



			Area x Y		
1	2.04 x .39	=	.795 x .45	=	.36
2	2.04 x .39	=	.795 x .45	=	.36
3	.22 x 8	=	1.76 x 4.05	=	7.13
4	.22 x 8	=	1.76 x 4.05	=	7.13
5	$.795 - 1.26\left(\frac{.25+.40}{2}\right)$	=	.385 x 7.8	=	3.00
6	$.795 - 1.26\left(\frac{.25+.40}{2}\right)$	=	.385 x 7.8	=	3.00
7	19.5 (.05)	=	.98 x .025	=	.02
8	19 (.05)	=	.95 x 1.325	=	1.26
			7.81		22.26 = $\sum Ay$

$\bar{y} = \frac{22.26}{7.81} = 2.85$ in.

Calculate I of tray about neutral axis.

$I = I_o + \sum Ad^2$

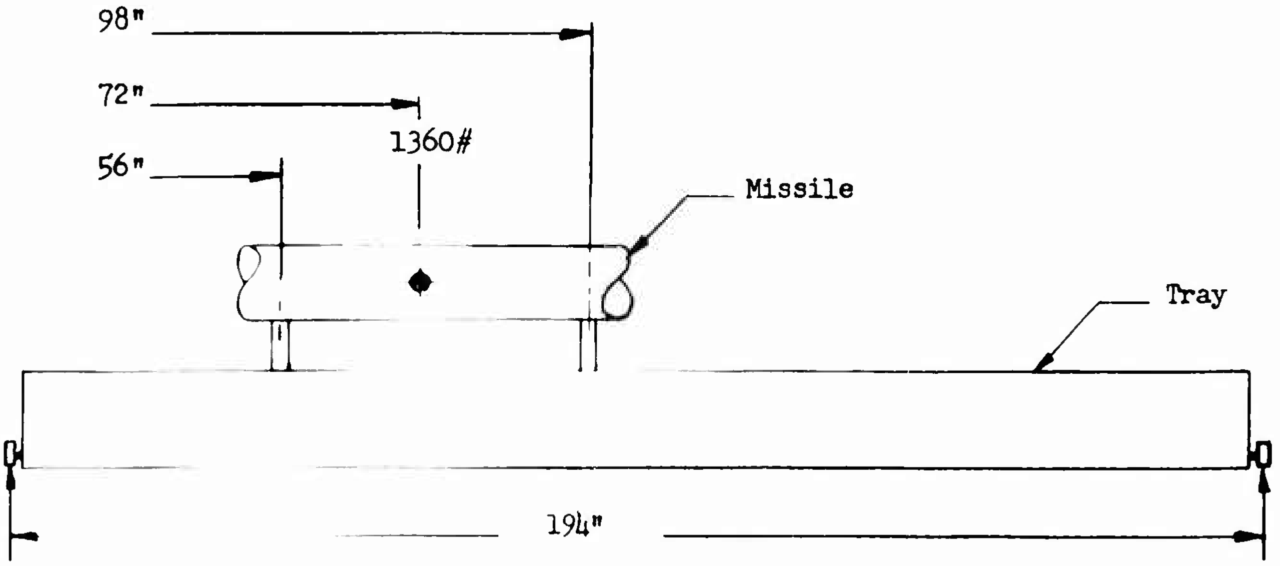
Sections 1, 2, 5, 6, 7, and 8 have negligible I_o.

1	Ad^2	=	$.795 \times (2.4)^2$	=	4.58
2		=	$.795 \times (2.4)^2$	=	4.58
3		=	$1.76 \times (1.2)^2$	=	2.54
4		=	$1.76 \times (1.2)^2$	=	2.54
5		=	$.39 \times (4.95)^2$	=	9.55
6		=	$.39 \times (4.95)^2$	=	9.55
7		=	$.98 \times (2.825)^2$	=	7.84
8		=	$.95 \times (1.525)^2$	=	2.21
3	$I_o = \frac{bd^3}{12}$	=	$.22 \times (8)^3/12$	=	9.39
4		=	$.22 \times (8)^3/12$	=	9.39
					$I_{NA} = 62 \text{ in.}^4$

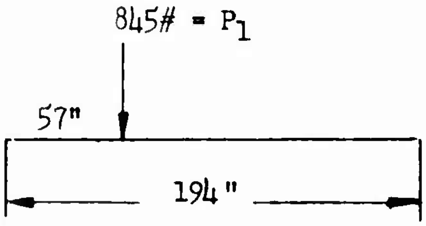
Determine K of tray:

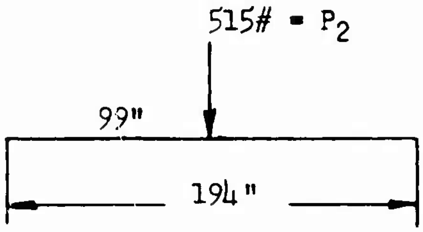
The missile is supported in the tray at two points. Thus, determine the deflection under each support, average the two and obtain the spring constant from the relation.

$K = \frac{F}{S}$ where S is the deflection unde supports and F is weight of missile and tray.



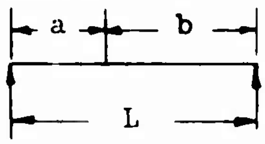
Deflections due to concentrated loads





Deflection @ x = 99 due to P_1 (assume simply supported tray).

$$y = \frac{P_1 a}{6EIL}(L-x)\left[aLb - b^2 - (L-x)^2\right] \quad \text{where}$$



$$= \frac{845 \times 57 \times 95}{6 \times 194 \times 29 \times 10^6 \times 62}\left[388(137) - (137)^2 - (95)^2\right]$$

$= .0553$ in.

Similarly @ x = 99 due to P_2, the deflection is found to be:

$y = .0436$

Total = .0989 in.

Deflection at x = 57" due to P_1.

$$y = \frac{P_1 bx}{6EIL}\left[2L(L-x) - b^2 - (L-x)^2\right] = \frac{845 \times 137 \times 57}{6 \times 194 \times 29 \times 10^6 \times 62}\left[388 \times 137 - 137^2 - 137^2\right]$$

$= .0492$ in.

Deflection at x = 57" due to P_2.

$y = .0337$

Total $y = .0829$ @ $x = 57"$

Deflection at x = 99" due to distributed load of tray.

$$y = \frac{WX}{24EIL}\left[L^3 - 2Lx^2 + x^3\right] \quad \text{where } W = 576 \text{ lbs.} = \frac{576 \times 99}{24 \times 29 \times 10^6 \times 62}\left[194^3 - 2 \times 194 \times 99^2 + 99^3\right]$$

$= .0306$ in.

Deflection at x = 57" due to distributed load of tray.

$y = .0245$ in.

Total deflection at $x = 57"$ due to P_1, P_2, and W.

$.0829 + .0245 = .1074$

Deflection @ $x = 99"$ due to P_1, P_2, and W.

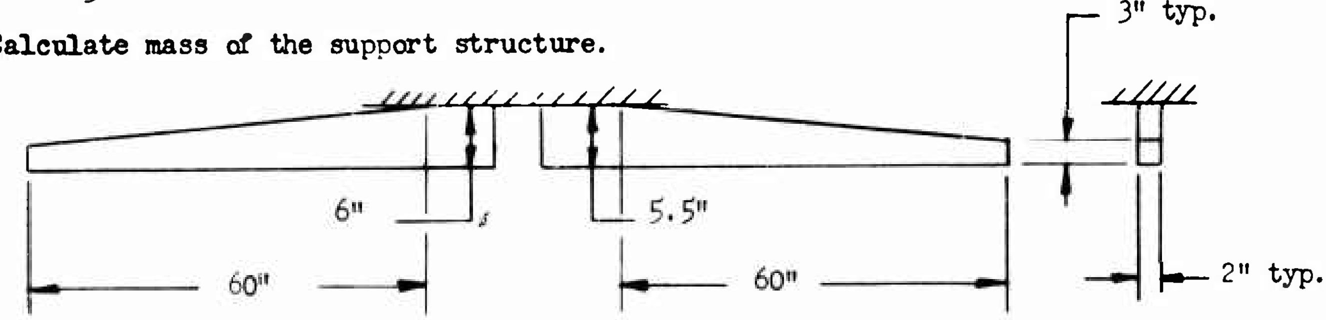
$.0989 + .0306 = .1295$ in.

Average $= (.1074 + .1295)/2 = .1185$ in.

$$K = \frac{F}{S} = \frac{P_1+P_2+W}{.1185} = \frac{1360+576}{.1185} = 16{,}337 \text{ lb./in.}$$

Thus $K_3 = 16337 \times 3 = 49{,}011$ lb./in.

Calculate mass of the support structure.



Constant width beams (2").

Left member: $(3x2x60x.283) + (\frac{1}{2}x3x60x2x.283) = 153$ lbs.

Right member: $(3x2x60x.283) + (\frac{1}{2}x2.5x60x2x.283) = 144.3$ lbs. $\cong 145$

Vertical members each 4 x 4 wide flange 7 ft. long -13 lbs.

weight per member $= 13 \times 7 = 91$ lbs.

Rails - each rail $6 \times 1\frac{1}{2}$ with 2 flanges $2 \times \frac{1}{2}$ 180 in. long

weight per rail $= 10 \times (.5)(180)(.2836) = 255$ lbs.

Roller - each roller = 2 lbs.

Thus M_2 =	153 lbs.	left member
	145 lbs.	right member
	91 lbs.	vertical members
	170 lbs.	rails
	24 lbs.	rollers
Approx.	580 lbs. total	

$M_2 = \frac{580}{386} = 1.512$

Determine spring constant of structure (K_2).

Find I of beam: In calculating deflections, use the average I for the cross section.

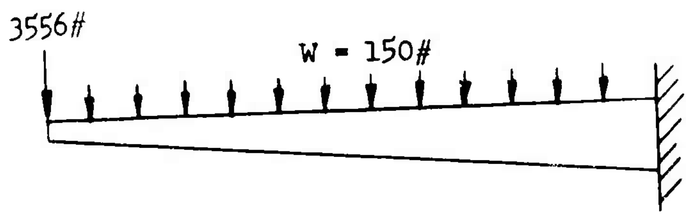
$I \text{ left} = \frac{1}{12} bd^3 = \frac{1}{12} \times 2 \times (3)^3 = 4.5 \quad \frac{1}{12} \times 2 \times (6)^3 = 36$

$I \text{ avg.} = 20.25 \text{ in.}^4$

$I \text{ right} = \frac{1}{12} \times 2 \times (3)^3 = 27.7 \quad \frac{1}{12} \times 2 \times (5.5)^3 = 4.5$

$I \text{ avg.} = 16.1 \text{ in.}^4$

$\bar{I} = 18.17 \text{ in.}^4$



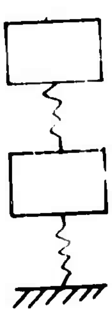
$$y = \frac{WL^3}{8EI} + \frac{PL^3}{3EI} = \frac{150x(60)^3}{8x29x10^6x18.17} + \frac{3556(60)^3}{3x29x10^6x18.17} = .4926 \text{ in.}$$

$$K = \frac{F}{Y} = \frac{3706}{.4926} = 7510 \text{ lbs./in.}$$

$$K_2 = 7510 \times 2 = 15,020 \text{ lbs./in.}$$

For the system on page 77, the natural frequencies can be obtained from the relation.

$$\omega^4 - \left(\frac{K_3}{M_1} + \frac{K_2+K_3}{M_2}\right)\omega^2 + \frac{K_2K_3}{M_1M_2} = 0$$



$M_1 = 15.03$

$K_3 = 49011$

$M_2 = 1.512$

$K_2 = 15020$

$$\omega^4 - \left(\frac{49011}{15.03} + \frac{15020+49011}{1.512}\right)\omega^2 + \frac{1.5020\times10^4 \times 4.9011\times10^4}{15.03\times1.512} = 0$$

$$\omega^4 - 45660\,\omega^2 + 32450000 = 0$$

This equation is quadratic in ω^2 and can be solved by the quadratic formula.

$$\omega^2 = \frac{45660 \pm \sqrt{1951200000}}{2} = \frac{45660 \pm 44172}{2}$$

$\omega^2 = 744$ | $\omega^2 = 44916$

$\omega = 27.2$ rad/sec. | $\omega = 211$ rad/sec.

$= 4.33$ cps | $= 33.6$ cps

Check vibration: At the two natural frequencies of the system, the masses oscillate at the same frequency. Approximate by an equivalent single degree of freedom system, the magnification factor of which is

$$M = \sqrt{\frac{1 + \left(\frac{2\omega C}{\omega_n C_{cr}}\right)^2}{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(\frac{2\omega C}{\omega_n C_{cr}}\right)^2}}$$

for small damping, use this form $\quad M = \dfrac{1}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(\frac{2\omega C}{\omega_n C_{cr}}\right)^2}}$

for $\dfrac{C}{C_{cr}} = .10$ and for $\lambda = \dfrac{\omega}{\omega_n} = \dfrac{7}{4.33} = 1.619$

$M = .7$

$\lambda = \dfrac{\omega}{\omega_n} = \dfrac{14}{4.33} = 3.25$ $\qquad M = .10$

$\dfrac{\omega}{\omega_n} = \dfrac{7}{33} = .212$ $\qquad M = 1.1$

$\dfrac{\omega}{\omega_n} = \dfrac{14}{33} = .424$ $\qquad M = 1.3$

All < 3 therefore O.K.

Check Shock:

Reference: Goodyear Report GER-9680, Design Charts for Shipboard Weapon Stowage Systems.

$$\left.\begin{array}{l} V_{max} = 15 \text{ fps} \\ f = 25 \text{ cps} \end{array}\right\} \text{shock input}$$

from figure 1

$n_o = 72$ $\qquad K_\alpha = 1.64$ $\qquad \phi = .33$ input damping

$d_o = 1.2$ $\qquad K_d = 1.45$ $\qquad \theta = .10$ response damping

$n_p = 118g$

$d_r = 1.74$ in.

$$\lambda = \frac{25}{4.33} = 5.76$$

$A(+) = .032$ $\qquad \left.\begin{array}{l} Z(+) = .80 \\ Z(-) = 1.21 \end{array}\right\} \phi = .3$ $\qquad A(+) = .027$ $\qquad \left.\begin{array}{l} Z(+) = .77 \\ Z(-) = 1 \end{array}\right\} \phi = .4$

$A(-) = .025$ $\qquad A(-) = .022$

$A(+) = .0305$ $\qquad \left.\begin{array}{l} Z(+) = .791 \\ Z(-) = 1.147 \end{array}\right\} \phi = .33$

$A(-) = .0241$

$\bar{A} = .0273$

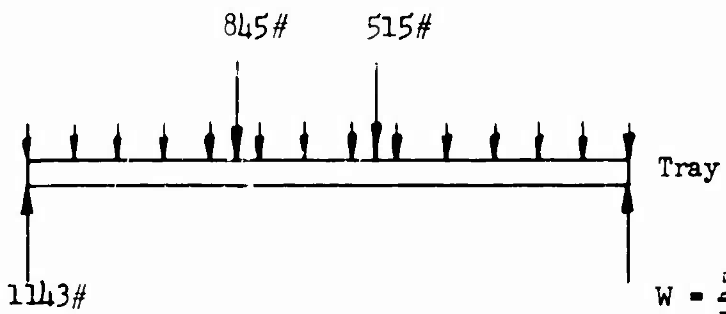
$\bar{Z} = .969$

Acceleration on missile = 118 x .0273 = 3.22 g

Relative displacement across beams = 1.74 x .969 = 1.688 in.

Stresses in Tray:

The bending stresses in the tray are given by $S = \frac{My}{I}$



$$W = \frac{576}{194} = 2.965$$

Find M maximum for $56 < X < 98$

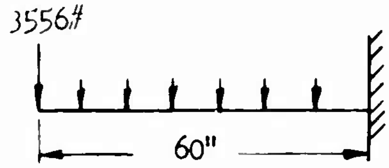
$$M = 1143X - \frac{[\text{illegible}]^2}{2} 845\ (X-56)$$

By trial and error, M maximum is found to occur at $X = 70"$

M max. = 66,000 in.-lbs.

$$S = \frac{66000 \times 5.2}{62} = 5530 \text{ under static load (1g) @ 3.22g}$$

$$S = 5530 \times 3.22 = 17,800 \text{ lbs/in.}^2$$



<u>Stresses in Support Structure:</u>

$$S = \frac{My}{I} \qquad M = 3556X + \frac{[\text{illegible}]^2}{2}$$

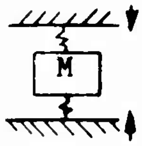
@ $X = 60"$ M = 217,860 in.-lbs.

$$W = \frac{150}{60} = 2.5$$

$$S = \frac{217860 \times 3}{36} = 18,110 \text{ lbs/in.}^2$$ under static deflection of .4926 in.

Under dynamic loading, stress is $18110 \times \frac{1.688}{.4926} = 62,000$ lbs/in.^{2}

Previous analysis considered decks moving in same direction (in phase). Consider now the decks being displaced in opposite directions (180° out of phase).

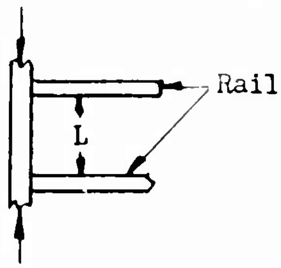


Shock conditions give 1.688 in. relative displacement. Assume 2".

3556 lbs. causes ½ in. displacement. Therefore, 2 in. displacement would cause $3556 \times 4 = 14,224$ lbs. This applied at top and bottom of vertical members.

$$S = \frac{P}{A} = \frac{14224}{3.82} = 3720 \text{ lbs/in.}^2 < \text{yield point}$$

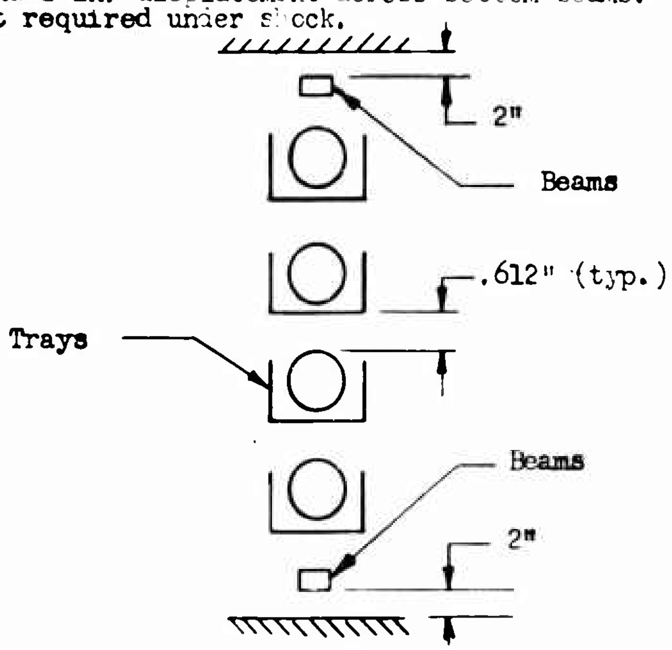
Check buckling: Consider distance along vertical members between horizontal rails.



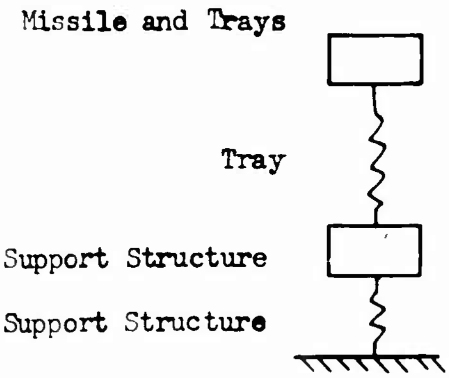
$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{(3.14)^2 x 29x10^6 x 3.4}{(84)^2} = 138{,}500 \text{ lbs.} > \text{any foreseeable loading.}$$

The relative displacement across resilient beams is $\cong$ 2 in. The displacement between trays will be $2 \times \frac{K\ beam}{K\ tray} = 2 \times \frac{15020}{49011} = .612$ in. = .7.

Total displacement required by magazine structure is 2 in. displacement across top beams. .612 x 5 = 3.06 in. displacement between each of 4 rows of trays, and 2 in. displacement across bottom beams. 7.5 in. additional displacement required under shock.



<u>Shock and Vibration When Magazine Empty or With One Missile:</u>



$M_1 = \frac{(576x3)+113}{386} = 4.76$

$K_3 = 49011$

$M_2 = 1.515$

$K_2 = 15020$

<u>Determine Natural Frequencies:</u>

$$\omega^4 - \left[\frac{K_3}{M_1} + \frac{K_2+K_3}{M_2}\right]\omega^2 + \frac{K_2K_3}{M_1M_2} = 0$$

$$\omega^4 - \left[\frac{49011}{4.76} + \frac{15020x49011}{1.515}\right]\omega^2 + \frac{15020x49011}{4.76x1.515} = 0$$

$$\omega^4 - 43329\,\omega^2 + 101900000 = 0$$

Solve by quadratic formula.

$$\omega^2 = \frac{43329 \pm \sqrt{1467400000}}{2}$$

$\omega^2 = 2551.5$

$\omega = 50.1$ rad/sec.

$= 7.97$ cps

$\omega^2 = 40817.5$

$\omega = 202$ rad/sec.

$= 32.2$ cps

Vibration Check:

$\lambda = \frac{\omega}{\omega_n} = \frac{7}{7.9} = .885$ $\quad M = 4$

$\frac{\omega}{\omega_n} = \frac{7}{32.2} = .217$ $\quad M = 1$

$\frac{\omega}{\omega_n} = \frac{14}{7.9} = 1.77$ $\quad M = .4$

$\frac{\omega}{\omega_n} = \frac{14}{32.2} = .434$ $\quad M = 1.3$

The magnification factor of 4 is high. However, there has been speculation that the lower ship forcing frequency is 9 cps.

Then $\frac{W}{W_n} = \frac{9}{7.9} = 1.14$ for which $M = 2.8$.

May propose to put soft padding between trays to prevent excessive amplitudes.

Shock:

$\lambda = \frac{25}{7.97} = 3.136$

$A(+) = .12$ $\quad Z(+) = .78$ $\quad \phi = .3$

$A(-) = .08$ $\quad Z(-) = 1.02$

$A(+) = .090$ $\quad Z(+) = .78$ $\quad \phi = .4$

$A(-) = .069$ $\quad Z(-) = .92$

$A_{(+)} = .111$ $\quad Z_{(+)} = .78$ $\quad \phi = .33$

$A_{(-)} = .077$ $\quad Z_{(-)} = .99$

$$\left.\begin{array}{l}\bar{A} = .094 \\ \bar{Z} = .885\end{array}\right\} \phi = .33$$

Acceleration on missile = 118 x .094 = 11.1 g

Relative displacement across beams = 1.74 x .885 = 1.54 in.

Stresses in Tray:

From previous work the stress was found to be 5530 lbs/in.^{2} under static load.

Stress under 11 g is 5530 x 11 = 60,800 lbs/in.^{2}

Stresses in Support Structure:

$S = \frac{My}{I}$ under static loading

$$M = 1572x + \frac{wx^2}{2}$$

$$= (1572 \times 60) + 150 \times \frac{60}{2}$$

$$= 99{,}000 \text{ in.-lbs.}$$

$S = \frac{99000 \times 3}{36} = 8250$ lbs/in.^{2} under static deflection.

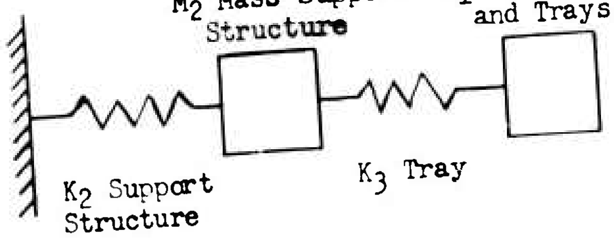
This deflection is $y = \frac{1}{3}\frac{PL^3}{EI} + \frac{1}{8}\frac{WL^3}{EI}$ where W = total distributed load.

$$y = \frac{1}{3} \times \frac{1572 \times (60)^3}{29 \times 10^6 \times 18.17} + \frac{1}{8} \times \frac{150 \times (60)^3}{29 \times 10^6 \times 18.17} = .2227 \text{ in.}$$

Under a deflection of 1.54 in., the stress will be $8250 \times \frac{1.54}{.2227} = 57{,}000$ lbs/in.^{2}

Athwartship Shock and Vibration (full):

Assume magazine full and consider decks to move in phase. Assume model to be:



Determine properties of tray athwartship. Reference page 81. Assume tray r[illegible]rs are locked to rails. This would be worst condition under shock.

$$I_y = I_o + \Sigma Ad^2$$

I_o	Ad^2
$\frac{8x.25^3}{12}$	$+ (8x.25)\ (9.63)^2$
$\frac{.220x2.04^3}{12}$	$+ (2.04x.22)\ (8.48)^2$
$\frac{.220x.75^3}{12}$	$+ (.220x.75)\ (9.125)^2$
$\frac{.05x19^3}{12}$	
$\frac{.05x19.5^3}{12}$	

Total = I = 1058.5 in.^{4}

Determine Spring Constant of Tray in Athwartship Direction:

$$K_{ath} = K_{vert}\left(\frac{I_{ath}}{I_{vert}}\right)$$

$$= 16337\left(\frac{1058}{62}\right)$$

$$= 278{,}500 \text{ lbs/in.}$$

Total K_3 = 278,500 x 3 = 835,500 lbs/in.

Determine Spring Constant of Beams in Athwartship Direction:

I_{ath} rear beam $= \frac{1}{12} bd^3 = \frac{1}{12} \times 6 \times (2)^3 = 4$ in.^{4}

and $\frac{1}{12} \times 3 \times (2)^3 = 2$ in.^{4}

Avg. I = 3 in.^{4}

I_{ath} fwd beam $= \frac{1}{12} \times 5.5 \times (2)^3 = 3.66$ in.^{4}

and $\frac{1}{12} \times 3 \times (2)^3 = 2$ in.^{4}

Avg. I = 2.83 in.^{4}

$\bar{I} = 2.91$ in.^{4} thus $K_{2ath} = K_{2vert}\left(\frac{I_{ath}}{I_{vert}}\right)$

$= 7510\left(\frac{2.91}{18.17}\right)$

$= 1204$ lbs/in.

Total $K_2 = 2408$ lbs/in. (2 beams)

Determine Natural Frequencies:

$$\omega^4 - \left[\frac{835000}{15.05} + \frac{2408+835000}{1.49}\right]\omega^2 + \frac{8.35\times10^8\times2.408}{15.05\times1.49} = 0$$

$$\omega^4 - 617400\,\omega^2 + 89300000 = 0$$

$$\omega^2 = \frac{617400 \pm \sqrt{38.06428\times10^{10}}}{2} = \frac{617400 \pm 616963}{2}$$

$\omega^2 = 218.5$ $\omega^2 = 617181.5$

$\omega = 14.7$ rad/sec. $\omega = 785.6$ rad/sec.

$= 2.34$ cps $= 125.1$ cps

If trays are empty:

$$\omega^4 - \left[\frac{835000}{4.76} + \frac{2408+835000}{1.49}\right]\omega^2 + \frac{8.35\times10^8\times2.408}{4.76\times1.49} = 0$$

$$\omega^4 - 736500\,W^2 + 282,500,000 = 0$$

$$\omega^2 = \frac{+736500 \pm\sqrt{540,870,000,000}}{2}$$

$\omega^2 = 531$

$\omega = 23$ rad/sec.

$= 3.66$ cps

$\omega^2 = 735969$

$\omega = 857$ rad/sec.

$= 136.5$ cps

It shoud be kept in mind that the above figures are based on the assumption that the ends of the rollers are rigidly attached to the rails. In practice the rollers are supposed to be free to roll under shock. This would tend to reduce the acceleration felt by the trays. However, based on the rigid attachments, the accelerations are obtained with the aid of the Goodyear Report.

$V_{max} = 7.5$ fps

$f = 30$ cps

From figure (1) $n_o = 43$ $K_\alpha = 1.64$

$d_o = .46$ $K_d = 1.45$

$n_p = 43 \times 1.64 = 70.5$ g

$d_r = .46 \times 1.45 = .667$ in.

$$\lambda = \frac{\omega}{\omega_n} = \frac{30}{3.66} = 8.19$$

$$\left.\begin{array}{ll} A(+) = .017 & Z(+) = .78 \\ A(-) = .014 & Z(-) = 1.3 \end{array}\right\} \phi = .3 \qquad \left.\begin{array}{ll} A(+) = .015 & Z(+) = .78 \\ A(-) = .013 & Z(-) = 1 \end{array}\right\} \phi = .4$$

$$\left.\begin{array}{ll} A(+) = .0164 & Z(+) = .78 \\ A(-) = .0137 & Z(-) = 1.2 \end{array}\right\} \phi = .33$$

$\bar{A} = .0150$

$\bar{Z} = .99$

Acceleration on missile $= 70.5 \times .0150 = 1.06$ g

Relative displacement across beams $= .66$ in.

Deflection of trays is $.66 \times \frac{K\ beam}{K\ tray} = .66 \times \frac{2408}{835500} = .00287$ in. (negligible)

<u>Bending Stresses in Tray:</u>

$$S = \frac{My}{I} \times 1.06 = \frac{66000 \times 10}{1058.5} \times 1.06 = 662 \text{ lbs/in.}^2$$

Stresses in Beam Structure:

3556 lbs. load in vertical direction yields deflection of .49 in. Apply same loading in athwartship direction. Deflection will be $.49 \times \frac{K_{vert}}{K_{ath}} = .49 \times \frac{7510}{1204} = 3.059$ in. This will cause stress of $S = \frac{My}{I} = \frac{217860 \times 1}{2.91} = 74{,}600$ lbs/in.^{2} for 3.059 in deflection.

However, for a .66 in. deflection, stress will be $74{,}600 \times \frac{.66}{3.059} = 16{,}100$ lbs/in.^{2} $<$ yield point.

Vibration (full):

$$\lambda = \frac{7}{3.66} = 1.91 \quad M = .4 \quad \frac{14}{3.66} = 3.82 \quad M \rightarrow 0$$

Empty:

$$\lambda = \frac{7}{2.44} = 2.865 \quad M = .2 \quad \frac{14}{2.44} = 5.74 \quad M \rightarrow 0$$

For the high natural frequency, $M \rightarrow 1$

In addition to the frequencies obtained above, there will be a rocking frequency of the tray due to torsion. This mode of vibration will be excited by the inertia force of the missile against the tray. An estimate of the frequency was obtained from the relation.

$$fn = \frac{1}{2\pi}\sqrt{\frac{KG}{IL}}$$

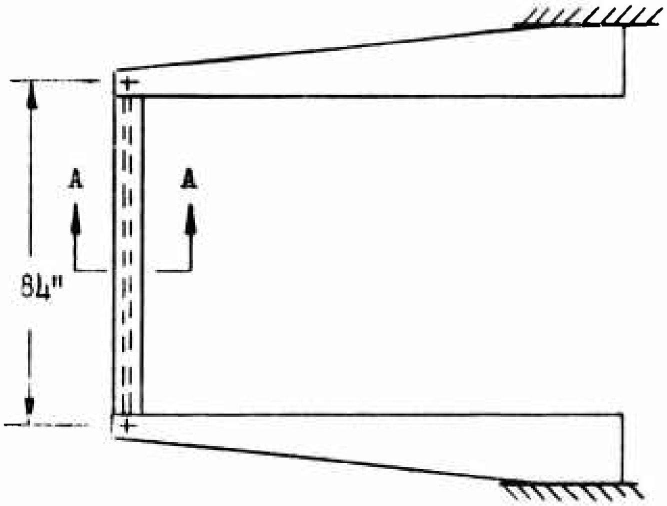
where K is a constant depending on the geometry of the section = 7.2

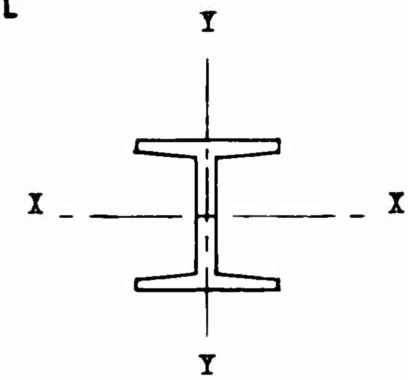
G modulus of rigidity = 12×10^6
I mass polar moment of inertia = 217
L length of beam = 53 in.

After calculating these constants for a loaded tray fn $\cong$ 13 cps in rotation.

Fore and Aft Shock and Vibration:

Consider vertical members as springs.





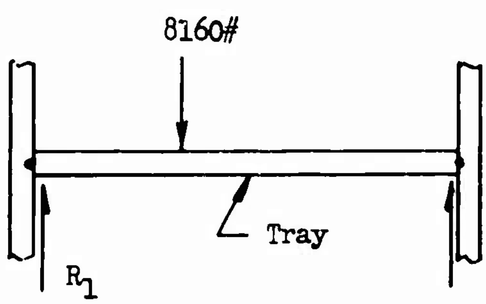
sec. A-A
4"x4" "H" beam
from tables.
$I_{y-y} = 3.4 \text{ in.}^4$

Determine strength of vertical members under static load.

The 2 vertical members of each of the 6 support structure assemblies carry $\frac{36}{6} = 6$ missiles and trays; 24 rollers, 1.66 rails, and their own weight.

Weight of constituents	8160	6 missiles
	3456	6 trays
	48	24 rollers
	340	1.66 rails
	182	dead weight
	12,186	≅12,200 lbs.

The load on each member will be:



$R_1 = 7143$ lbs. including weight of other constituents. Compression stress is $\frac{7143}{3.82} =$ 1870 lbs/in.^{2}.

However, load is applied eccentrically, therefore a moment of magnitude 7143 x 2 = 14286 in/lbs. results, where 2 in. is the distance of the point of application from the center of the vertical member.

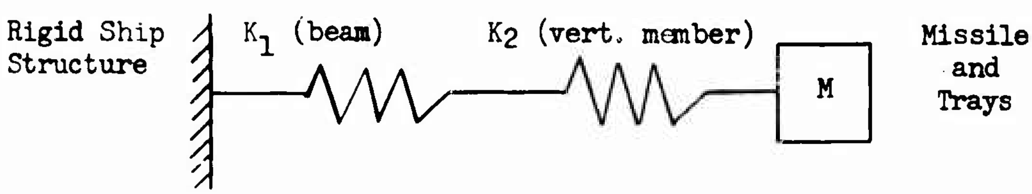
$$S = \frac{My}{I} = \frac{14286 \times 2}{3.4} = 8400 \text{ lbs/in.}^2$$

Total stress is $8400 + 1870 = 10{,}270$ lbs/in.^{2}

Under 2.4 g shock load, stress is $10270 \times 2.4 = 24{,}610$ lbs/in.^{2}
$<$ Yield Point therefore O.K.

Determine Shock Response:

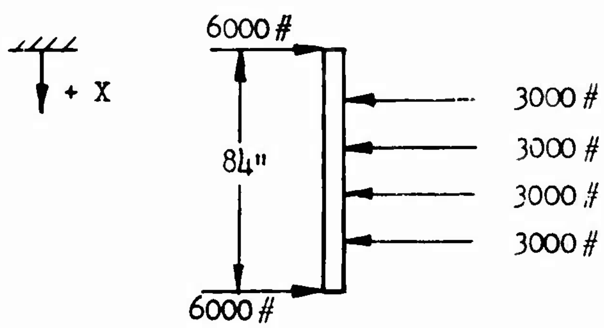
Assume model



$$\text{Effective } K = \frac{K_1K_2}{K_1+K_2}$$

Determine K_2 (Vertical Member):

Total weight is 12,200 lbs. Therefore, if trays experienced a 1 g load in fore and aft direction, a load of 12,200 would result. Thus, for a representative loading, assume 12,000 lbs. divided into 4 concentrated loads applied at the locations of the rails.



Applying the deflection formulas to find the deflection at $X = \frac{L}{2} = 42$ in., it is found that this deflection is $S = 1.119$ in.

$$K = \frac{F}{S} = \frac{12000}{1.119} = 10{,}720 \text{ lbs/in.}$$

Since there are 2 vertical members for each group of 6 trays, the effective K is

$$K = 10720 \times 2 = 21{,}440 \text{ lbs/in.}$$

The total effective K is from page 97.

$$K = \frac{21440 \times 30040}{21440 + 30040} = 12{,}540 \text{ lbs/in.}$$

These vertical members will be strengthened by diagonals which will also contribute to the K. Use K = 21,400 lbs/in.

Shock:

$V_{max} = 4$ fps

$f = 30$ cps

$n_o = 24$ $\quad K_\alpha = 1.65$

$d_o = .25$ $\quad K_d = 1.45$

$n_p = 39.6$ g $\qquad fn = \frac{1}{6.28}\sqrt{\frac{21400}{31.6}} = 4.1$ cps

$d_r = .362$

$$\lambda = \frac{\omega}{\omega_n} = \frac{30}{4.1} = 7.25$$

$A(+) = .022$ $\quad Z(+) = .78$, $Z(-) = 1.27$ $\quad \phi = .3$ $\qquad A(+) = .018$ $\quad Z(+) = .78$, $Z(-) = 1$ $\quad \phi = .4$

$A(-) = .017$ $\qquad A(-) = .016$

$A(+) = .0208$ $\quad Z(+) = .78$, $Z(-) = 1.19$ $\quad \phi = .33$

$A(-) = .0167$

$\bar{A} = .0188$

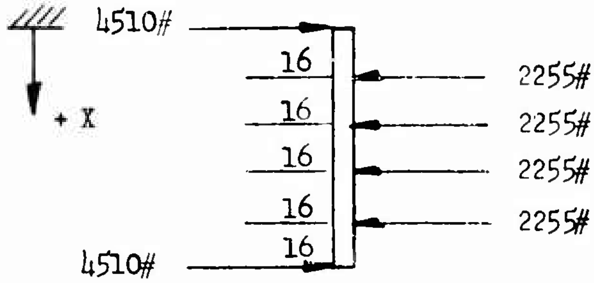
$\bar{Z} = .985$

Acceleration on missile = 39.6 x .0188 = .75 g

Relative displacement = .362 x .985 = .357 in.

Determine Stresses in Vertical Members:

At .74 g, the loading will be:



$@\ X = \frac{L}{2} = 42$ in.

$M = 4510X - 2255\,(X-16) - 2255\,(X-32) = 108{,}400$ in.-lbs.

$S = \frac{My}{I} = \frac{108400 \times 2}{3.4} = 63{,}700$ lbs/in.^{2}

Fore and aft with trays empty.

$$fn = \frac{1}{6.28}\sqrt{\frac{21440}{11.2}} = 6.99 \text{ cps} \cong 7.0 \text{ cps}$$

Acceleration on missile $\cong$ 2.85 g, slightly high estimate.

Relative displacement $\cong$ .434 in., slightly high estimate.

(3) Conclusions

While the arrangement appears feasible the approach is quite a contrast to methods in current usage. It is felt that an extended test program would be required for the development of a proven system prior to its availability for shipboard installation. For these reasons, the resiliently supported magazine is not recommended at this time.

VII. RECOMMENDATIONS

The missiles in the system must be protected from high shock and vibration. The CNO approved characteristics, Attack Aircraft Carrier, (CVA-66) states that "Equipment and machinery required for combat operations shall be capable of withstanding shock loading of a severity approaching that which will cause significant hull damage." While this is rather vague, a major requirement, as specified, is for the mitigation of shock inputs defined on the Bureau of Ships curve for surface vessels, in NAVSHIPS 250-423-29. These curves are applicable to 2nd platform CG, CLG, CG(N), or main deck DD. For a larger ship the inputs should be less. The protection must be such that no excessive vibratory or shock forces are impressed upon the missile along any of its three major axes when the local primary structure is subjected to such inputs.

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While vertical accelerations may reach forces on the order of 120 g's due to a 33 per cent damping coefficient it is felt that such inputs applied to the aircraft carrier magazine assembly are unrealistic, because:

1. Although the force due to high damping is felt by the ships primary structure, it is not necessarily an input by that structure to an assembly attached to it in one plane only and not itself intervening as primary structure.

2. The application of such forces by the bulkheads to the longitudinals would not result in such an output by them to any object supported upon them at a distance from the bulkheads, due to yielding of those members.

3. The weight of an installation itself provides a certain amount of shock mitigation, as implied in SI-10.

The net result of the necessary use of data available is reflected in excessive weight and complexity of installations. Definite requirements, with realistic values, as a part of the ships specifications, could probably result in a saving of weight, space, and cost at many locations.

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