(1)

LABORATORY

INDEXED

UNCLASSIFIED

FOR OFFICIAL USE ONLY

WATERTOWN ARSENAL LABORATORY

AD A951123

EXPERIMENTAL REPORT

NO. WAL 321/1-2

14

DTIC ELECTE SEP 1 4 1981

6 DEVELOPMENT OF PROJECTILE STEELS

Second Partial Report

DISTRIBUTION STATEMENT A

Approved for public release; Distribution Unlimited

FOR OFFICIAL USE ONLY
INFORMATION CONTAINED IN THIS REPORT
IS NOT TO BE RELEASED WITHOUT PRIOR
APPROVAL OF COMMANDING OFFICER,
WATERTOWN ARSENAL, WATERTOWN 72, MASS.

10 D. Van Winkle

C. Zener
Senior Physicist

UNCLASSIFIED

DATE 11 23 May 1944

WATERTOWN ARSENAL
WATERTOWN, MASS.

DISCLAIMER NOTICE

5 June 1944

Laboratory

Subject: Watertown Arsenal Report No. 321/4-1.

To: Chief of Ordnance, U.S.A.
Pentagon Building
Washington 25, D. C.

Attn: SPOTB - Tech. Reports

1. Inclosed are eighteen copies of Report No. WAL 321/4-1, entitled "Development of Projectile Steels, Second Partial Report". These reports are for distribution as follows:

1 copy - Ordnance Technical Library - Index cards are inclosed.
3 copies - SPOTB - Technical Reports.
1 copy - SPOTM
1 copy - SPOTS
1 copy - SPOIM
1 copy - SPOIS
5 copies - NDRC, Division 2, care of Dr. John Burchard.
5 copies - War Metallurgy Committee.

2. Copies of this report have also been sent to the Springfield Armory, Watervliet Arsenal, Rock Island Arsenal, Picatinny Arsenal, Tank Automotive Center, Frankford Arsenal, Ordnance Research Center, U. S. Naval Proving Ground and to certain other interested individuals.

3. Since it is believed that the British Army Staff would find this report of interest, three (3) copies are being retained in accordance with directive letter O.O. 400.112/7404 - Wtn. 350.05/607, 23 May 1944, pending further instructions from his office.

4. In the first report of this series it was pointed out that in projectile steels a high bend strength at high hardness levels is desirable. In this report the various factors are investigated which influence the bend strength of a given steel at high hardness levels.

For the Commanding Officer:

H. H. ZORNIG
Colonel, Ord. Dept.,
Assistant.

2 Incls.
Rpt. 321/4-1 (18 copies)
Index cards.

R[redacted]D

DISTRIBUTION **UNCLASSIFIED**

REPORT NO. WAL 721/... TOTAL NO. OF COPIES ...

DATE DISTRIBUTED June 8, 1944 EXTRA COPIES ~~27~~ 26

Author

Laboratory Report File - 1

Office, Chief of Ordnance - 2/ 1-COO-Naval Ord Lab-1/17/46

TECHNICAL DIVISION

Ordnance Technical Library 1
SPOTB - Tech. Reports
SPOTB - For British Army Staff 3- 7/5/44
SPOTR
SPOTM - 1 1-OO-Bureau of Ord-Navy Dept-10/5/44
SPOTT 1-F.A. 3/25/47
SPOTC
SPOTS - 1
Others

3 Ret'd from O.O.-10/2/44(C)

INDUSTRIAL DIVISION

SPOIR
SPOIM - 1
SPOIS - 1

DETROIT

SPOME - EE
SPOME - EM

1- Capt Hillman

1- F.A.-4/11/47

✓ Springfield Armory - 1

✓ Watervliet Arsenal - 1

✓ Rock Island Arsenal - 1

✓ Picatinny Arsenal - 1

Ordnance Research Center

Paint & Automotive Chemical Laboratory

Others

✓ The Proving Center ... 1- BRL, 1-Small Arms Branch

1-A.P.G. (Ammo & Ammunition Section) 11/9/44

1- ... Australian Military Mission ...

1-Crucible Steel Co. -1/4/45(u)

... by Regraded Review Board ... May 1944

UNCLASSIFIED

[redacted]

FOR OFFICIAL USE ONLY

Watertown Arsenal Laboratory
Report Number WAL 321/4-1
Problem Number J-2.3

23 May 1944

DEVELOPMENT OF PROJECTILE STEELS

Second Partial Report

OBJECT

To investigate the various methods by which a high bend strength may be obtained at high hardness levels.

SUMMARY

A study has been made of the effect various types of heat treatments have on the bend strength of FXS-318 steel. The standard heat treatment of straight quench and temper has been found to give nearly as good results as a more complicated heat treatment involving an interrupted quench. Several other heat treatments with various austenitizing and quenching conditions produced very poor bend properties. The only remaining variable in the standard quench and temper treatment is the time-temperature combination used in tempering. Compared at the same resulting hardness values the higher-temperature, short-time temper gave similar results to the lower-temperature longer-time temper. This fact suggests a rapid method for base tempering hardened projectiles. That is, to heat by induction the outer portion of the projectile back of the bourrelet for a time short compared to the time for equilibration of temperature. The equilibrium

[redacted]

temperature of the projectile would then be sufficiently low as to produce no softening of the ogive.

Two FXS-318 steels were studied, one with a higher carbon and alloy content than the other. The one with the lower carbon content (0.64%) could be heat treated to have a very high bend strength at the as quenched hardness level of 62/63 RC. The steel with the higher carbon (0.76%) had, as expected, a higher as quenched hardness of 65 RC, but at that hardness the bend strength was low under all conditions of heat treatment. When the hardness was reduced by tempering to the hardness level of the lower carbon steel, its bend strength increased but remained lower than that of the lower carbon steel. A theoretical interpretation of this reduced bend strength at higher carbon levels is given. The carbon content of British projectiles is over 85%. From the above discussion it appears that the slight gain in ogive hardness associated with this higher carbon content is obtained only at the expense of a lowered bend strength back of the bourrelet.

From the above results it may be concluded that the fracture stress of a hardened steel is determined primarily by two factors: the magnitude of micro-stresses, and the size of carbide particles. Another method, other than tempering, is studied which may reduce micro-stresses. This is plastic deformation. From a theoretical analysis it is concluded that these micro-stresses will be reduced exponentially by plastic deformation, being reduced to 1/e times the original value when the plastic strain becomes equal to the elastic strain. It is therefore anticipated that the bend strength of steel in the as quenched condition will be improved by a prior plastic deformation, such as a twist.

A theoretical analysis is also made of the possible effects of a surface layer under compression upon the bend strength. It is concluded that unless the surface has defects, such as surface cracks, which lower the fracture stress of the surface layers with respect to the inner core, no appreciable improvement will result from the presence of such a skin.

D Van Winkle
D. Van Winkle
Associate Physicist

C. Zener
C. Zener
Senior Physicist

APPROVED:

H. H. ZORNIG
Colonel, Ordnance Dept.
Director of Laboratory

CONTENTS

INTRODUCTION

In performing its primary function of penetrating armor, an AP projectile is subjected to high stresses of all types, compressive, shear and tensile. The range of conditions under which a projectile will be successful depends largely upon its ability to resist fracture or appreciable deformation by these stresses. No particular difficulty is presented in obtaining the maximum possible resistance to deformation of the steel. The steel is simply heat treated to its maximum hardness. The difficult metallurgical problem is in obtaining a sufficiently high hardness combined with a high fracture stress.

Tensile stresses in projectiles arise from the bending moments associated with the transverse component of the force acting upon the ogive.[1] The maximum transverse component which the projectile can withstand without fracturing is proportional to its bend strength. The bend strength is in turn proportional to the fracture stress only when this stress is below the yield stress. As the fracture stress is raised above the yield stress the bend strength increases in a discontinuous manner, as is illustrated in Figure 1. The origin of this sudden

1. C. Zener and R. E. Peterson: "Principles of Projectile Design for Penetration, First Partial Report", WAL 762/231.

rise in bend strength was analysed in the First Partial Report[1] of this series. The sudden rise is due to the redistribution of stress caused by a slight plastic deformation. Thus an increase of fracture stress by only 15,000 psi over the yield stress is sufficient to allow the outer fibres to flow plastically 1%, resulting in a 50% rise in bend strength. A primary problem in the making of AP projectiles is therefore the raising of the fracture stress somewhat above the yield stress. The purpose of the present report is to analyse the various factors to be considered in this problem.

The first factor considered is the heat treatment of a given composition, then the effect of composition with the optimum heat treatment. The possibility is theoretically investigated of raising the fracture stress by relaxing the microscopic stresses through plastic deformation. Finally, the possible influence of putting the outer skin into compression is discussed.

If, at the highest hardness level obtainable in the steel the fracture stress cannot be raised above the yield stress, the yield stress and therefore the hardness must be reduced or the projectile must be given a differential heat treatment. In such a treatment the forward portion of the projectile, which is subjected to

1. D. Van Winkle and C. Zener: "Development of Projectile Steels, First Partial Report", WAL 321/4.

the greatest compressive and shear stresses, is given the maximum hardness. The portion just back of the bourrelet, which is subjected to the maximum bending moment, is tempered to such a hardness that the fracture stress lies above the yield stress. Even when, as is usually the case, such a differential heat treatment is resorted to, it is still desirable that the portion just back of the bourrelet be as hard as is consistent with a high bend strength for the following two reasons. Firstly, the harder the steel, the smaller is the deformation required to reach a given bend strength. Secondly, the higher the hardness at which a high bend strength may be obtained the less is the drop in hardness necessary between the ogive and the high bend strength region, and consequently the less difficult becomes the differential temper.

RESULTS AND DISCUSSION

1. General Considerations

Factors which will apply in general to all development work on bend strength tests may serve to give a clearer picture of the problem at hand and therefore will be considered first. The analysis of the bend test given in the First Partial Report of this series demonstrated that the bend strength of a round bar, assuming no strain hardening, can be only 70 per cent higher than the yield strength even with unlimited ductility. Calculations show that for a steel of 66 Rockwell "C" or a yield stress of 352,000 psi. the bend strength could only reach a value of approximately 600,000 psi. Any higher values would be due to strain hardening. Since maximum strains of about 5 per cent in bending are all that can be allowed in a projectile without lessening its penetration ability, the strain hardening will not contribute a great deal to the bend strength. Therefore bend strengths of between 600,000 and 700,000 psi. represent a reasonable upper limit. Once bend strengths of such magnitude are obtained effort should be placed upon maintaining that bend strength at higher yield strength levels rather than increasing the bend strength at the present yield strength level.

Emphasis is usually placed upon the hardness of a projectile rather than its yield strength simply because the hardness is the easier quantity to measure. Yield strength is, however, a more appropriate criterion of performance and hence materials should be compared in terms of equal yield strengths rather than hardnesses. The yield strengths of low alloy steels, in the fully quenched and tempered conditions increase with hardness at approximately the same rate, so a plot of bend strengths vs. hardnesses of several low alloy steels will place them in the same relative position as if the yield strength were plotted instead of the hardness. The yield strengths of high-speed tool steels, however, increase at a greater rate with hardness than do the low alloy steels,[1] and for comparing low alloy and high-speed tool steels, a truer picture of relative bend strength and resistance to deformation is obtained if the yield strength rather than hardness is plotted against bend strength.[2]

Assuming a low alloy steel and a high-speed tool steel of the same hardness and comparable bend strengths, the latter would be a much superior projectile steel from the point of view of resistance to deformation and shatter. Since however, the high-speed tool steel obtains its bend strength by high yield strengths and not by being ductile, it is capable of absorbing very little energy before fracturing. At the present time there is insufficient data available to say whether high bend strength alone will prevent fracture of the projectile base or whether the projectile base must be able to absorb energy in order to withstand oblique impact without fracturing.

---

1. Ibid. and from additional bend tests made at Watertown Arsenal on several high-speed tool steels, to be reported on soon.
2. Figure 2 gives a graphic illustration of possible stress-strain curves of two steels, both having the same hardness but different yield strengths. The hardness value is a measure of the flow stress at some appreciable strain. (In low hardness steels this flow stress corresponds to the T.S.)

2. Heat Treatments and Compositions Investigated

In order to make the study of the effects of various heat treatments as complete as possible only one type of steel, FXS-318, was studied. Two shipments of steels were received from the Frankford Arsenal and several chemical analyses were taken from each lot. The compositions varied only by negligible amounts within lot shipments but the composition was quite different for the two lots. The compositions are as follows:

	C	Mn.	Si.	S	P	Ni.	Cr.	Cu.	Mo.	Va.
Lot 1	.64	.73	.23	.022	.019	Tr.	.09	---	.67	Nil.
Lot 2	.76	.95	.27	.014	.010	Tr.	.01	.08	.97	Nil.

This difference in composition provided an opportunity to examine what differences existed in the physical properties of two steels whose compositions lie at the two extremities of the specified compositions.

In the first set of experiments an attempt was made to give the specimens as good a heat treatment as possible. An austenitizing temperature was therefore chosen sufficiently high so that no visible carbides remained undissolved, but not so high as to cause appreciable grain growth. After several trials 1650° F was found suitable. The grain size of specimens austenitized for one half hour at this temperature was not greater than A.S.T.M. #6, and no visible carbides were present at magnifications of 1,000.*

A quenching procedure was chosen to satisfy the following two requirements. Firstly, it must be sufficiently severe as to obtain a homogeneous martensitic structure. Secondly, it must be sufficiently slow through the martensitic transition range so that macroscopic stresses are not developed by a non-uniform transformation, and so that the unavoidable micro-stresses become relaxed as much as possible. The quench procedure finally adopted was to quench in oil at 400° F, to hold at that temperature for one minute, to air cool to room temperature, and finally to cool to -110° F by immersion in an alcohol and dry ice mixture.

The drawing operation allowed considerable flexibility in time and temperature of draw. Thus in drawing to a given hardness, one could heat to a high temperature for a short time, or to a lower temperature for a long time. Since no information was available in the literature as to which extreme is preferable, several combinations of time and temperature were tried.

The observed values of bend strengths are given in Figure 3 for the lower carbon (0.64%) steel, in Figure 4 for the higher carbon (0.64%) steel. An increase in drawing temperature by 200° F, together with a decrease

* Both the grain size determinations and the inspections for carbides were made by Miss M. R. Norton.

in drawing time by a factor of about 100, is seen to have no effect within experimental error of the bend strength for a given hardness. The maximum hardness of the higher carbon steel is nearly 3 Rockwell "C" points higher than for the lower carbon steel. This higher maximum hardness is attained, however, only at the expense of a lower maximum bend strength.

A second set of experiments was undertaken to determine whether the comparatively complicated heat treatment described above gives noticeably better results than a simpler quench and temper treatment. Several quenching procedures were tried, including: austenitizing at 1650° F and quenching into oil at 80° F; austenitizing at 1650° F, transferring to a furnace at 1525° F and quenching into oil at 80° F; austenitizing at 1650° F, holding in air for 1/2 to 3 minutes and quenching in oil at 80° F. All bars which had been heated to 1650° F during the heat treatment and quenched in oil at 80° F broke brittlely even after tempering. If oil at around 80° F is to be used as the quenching medium, a low austenitizing temperature will have to be used. An austenitizing temperature of 1525° F gave essentially the same bend strength as when the carbides were fully dissolved. The observed values of bend strengths for this last heat treatment are given in Figure 5 for the lower carbon steel, in Figure 6 for the higher carbon steel.

For both steels the maximum hardness is slightly less, about 1 Rockwell "C" point, and the maximum bend strength is slightly less about 50,000 psi, than in the case of the more complicated heat treatment. These deviations are, however, within the range of scatter of either heat treatment. It must therefore be concluded that only slight improvement in bend strength at high hardness levels is to be obtained by a simple modification of the conventional heat treatment.

The steels of different carbon content respond in quite distinct manners with respect to tempering. They may be compared most readily by means of Figures 5 and 6. As the lower carbon steel is tempered, its bend strength rises to nearly its maximum value without any perceptible lowering of hardness. On the other hand, as the higher carbon steel is tempered, its bend strength rises only with a reduction in hardness. This difference in behavior is illustrated by Figure 7 which combines Figures 5 and 6. The as quenched hardness of the higher carbon steel is higher than that of the lower carbon steel by 1.5 Rockwell "C" points, but its maximum bend strength is less by about 100,000 psi. The fact that the higher carbon steel has a lower bend strength than the lower carbon steel at the same hardness is interpretable in terms of two current concepts. The first is that the yield stress is a function only of the mean ferritic path.[1] The second is that the fracture stress is determined by the size of the carbide particles,[2] the larger their size, the lower the fracture stress. If the mean ferritic path is identical for two steels of different carbon content, the one of higher carbon content must have the larger carbide particle size, and therefore the lower fracture stress.

3. Relaxation of Residual Stresses

The brittle behavior of steels in the as quenched condition is popularly attributed to the presence of residual micro-stresses. This popular concept of the embrittling effect of micro-stresses is in accord with the formal theories of plasticity. In some regions of the specimen the stresses transverse to the axis are all compressive, in other regions part are compressive part are tensile, in other regions all are tensile. The regions of the third type are apt to be brittle, for in them the yield stress has been raised, by an amount equal to at least the smaller of the transverse tensile stresses. If the yield stress has been raised above the fracture stress, the region will fracture with no

---

1. M. Gensamer, E. B. Pearsall, W. S. Pellini and J. R. Low: "The Tensile Properties of Pearlite, Bainite and Spheroidite", Trans. A. S. M. 30 983 (42).
2. C. Zener and J. H. Hollomon: "Plastic Flow and Rupture of Metals, First Partial Report", WAL 732/10.

plastic deformation whatever. The effects of tempering upon the bend strength of quenched steels, described in the first part of this report, finds a ready interpretation only in terms of these micro-stresses and their relaxation with temperature. The rate of relaxation of stresses in steels has not been investigated sufficiently so that one can tell whether the micro-stresses are completely relaxed at times and temperatures below those which soften the steel. It is therefore of importance to investigate in what ways, other than by tempering, the micro-stresses may be relieved.

In the present section an analysis is made of the relaxation of micro-stresses by plastic deformation. The analysis may be carried through to completion for the important case where the micro-stresses are small compared to the flow stress (i.e., smaller than 1/2 the flow stress). The result may be stated in simple terms. The micro-stress pattern, that is, the stress pattern which remains when the macro-stresses are removed, is unaltered in type but is gradually reduced in magnitude as plastic deformation proceeds. Thus at every point in the solid, the principal axes of the micro-stresses remain unaltered in direction, and the relative magnitudes of the three principal micro-stresses remain unchanged. The rate of decrease of the micro-stresses is given by an exponential function, thus

$$\text{Micro-stresses} \sim e^{-\alpha E/E_o}$$

where E is the macro-strain past yielding, E_o the strain at yielding, and α is a numerical constant nearly equal to unity, its precise value depending upon the type of macro-strain. A plastic deformation of several per cent is therefore very effective in relieving the micro-stresses. This theoretical conclusion is in agreement with the observed relaxation of residual macro-stresses by plastic deformation.[1]

The essential contribution of the authors is the integration, under the appropriate conditions, of a certain differential tensor equation obtained by Hohenemser and Prager. In order to clarify the assumptions involved in the final solution, a brief review is given of the ideas leading to this equation (Sections a - c).

a. <u>Von Mises Theory of Plastic Flow</u>

Von Mises has postulated a concise mathematical relation between the strain rates and the stresses. The relation is given by the following set of equations:

$$\dot{E}_{xx} = f \cdot \left\{ X_x - (Y_y + Z_z)/2 \right\}$$

1. J. T. Norton and D. Rosenthal: "An Investigation of the Behavior of Residual Stresses Under External Load and Their Effect on Safety", The Welding Journal, February 1943.

with two similar equations for $\dot{E}_{yy}$ and $\dot{E}_{zz}$, and

$$E_{yz} = 3f \cdot Y_z \qquad (1)$$

with two similar equations for $\dot{E}_{zx}$ and $\dot{E}_{xy}$.

The fundamental characteristic of this relation is that it is invariant with respect to a rotation of coordinate axes. The equations therefore rest upon the assumption that the material remains isotropic during deformation.

The quantity f is a function of the invariant stress quadratic Φ^2 defined by

$$\Phi^2 = (1/3)\left\{ (Y_y-Z_z)^2 + (Z_z-X_x)^2 + (X_x-Y_y)^2 \right\} + 2(Y_z{}^2 + Z_x{}^2 + X_y{}^2)$$

When Φ^2 is less than a critical value A^2, f is negligibly small. On the other hand, if Φ^2 is only slightly greater than A^2, f is very large. Therefore for ordinary rates of flow S_I may be taken as given by

$$\Phi^2 = A^2 \, . \qquad (3)$$

In the absence of strain hardening, A is regarded as a constant. When, as in all metals, strain-hardening occurs, the quantity A must be taken to be a function of the quadratic strain invariant[1] $e_I{}^2$ as well as of Φ^2.

---

1. C. Zener and J. H. Hollomon: "Plastic Flow and Rupture of Metals, First Partial Report", WAL 732/10, App. A.

It is to be particularly noted that since neither the quadratic stress invariant nor the quadratic strain invariant are constant throughout the specimen, the quantity f cannot be taken as a constant of the spatial coordinates. It is this indeterminancy of f which renders the plastic equations of flow particularly difficult of solution.

b. Reuss-Prandtl Generalized Flow Equations

A strain may be separated uniquely into two parts, elastic and plastic. The elastic strain is that which is recovered upon removal of all stresses, the plastic is the residual strain. The Von Mises equations relate professedly only to the plastic strains. They are therefore inapplicable whenever the elastic strains are changing, as they always do whenever the principal axes of the stress change their orientation. Reuss and Prandtl have suggested that this limitation be removed by writing the equations for the total strain rate, the elastic plus the plastic. These equations are

$$\dot{E}_{xx} = Y^{-1}\left\{ \dot{X}_x - \sigma(\dot{Y}_y + \dot{Z}_z)\right\} + f\cdot\left\{X_x - (1/2)\,(Y_y + Z_z)\right\},$$

with two similar equations for $\dot{E}_{yy}$ and $\dot{E}_{zz}$, and (4)

$$\dot{E}_{yz} = 2(1 + \sigma)\, Y^{-1}\, \dot{Y}_z + 3f\, Y_z\ ,$$

with two similar equations for $\dot{E}_{zx}$ and $\dot{E}_{xy}$. In the above equations Y is Young's Modulus.

c. Hohenemser and Prager's Formulation of Plastic Flow Equation

Eqs. (4) suffer from the same handicap as do Eqs. (1) in that they contain the quantity f which is an unknown function of position. In the particular case where plastic deformation is occurring Hohenemser and Prager[1] have been able to transform Eqs. (4) into a new set in which this function is not present. They have given their result in terms of the "reduced stress tensor", Σ_o, and the "reduced strain increment tensor", dE_o. These are defined by the following equations:

$$E_o = \begin{Bmatrix} E_{xx}-\Delta & (1/2)E_{xy} & (1/2)E_{yz} \\ (1/2)E_{xy} & E_{yy}-\Delta & (1/2)E_{zx} \\ (1/2)E_{yz} & (1/2)E_{zx} & E_{zz}-\Delta \end{Bmatrix} \qquad (5)$$

and

$$\Sigma_o = \begin{Bmatrix} X_x + P & Y_z & Z_x \\ Y_z & Y_y + P & X_y \\ Z_x & X_y & Z_z + P \end{Bmatrix} \qquad (6)$$

where

$$\Delta = E_{xx} + E_{yy} + E_{zz}$$

and

$$P = -(X_x + Y_y + Z_z)/3 \; .$$

1. K. Hohenemser and W. Prager, "Beitrag zur Mechanik des bildsamen Verhaltens Von Flussstahl," Zeits f. Aug: Math. und Mech. 12, 1 (32).

In terms of these tensors, their equations become

$$d\Sigma_o = -(2G/A^2)\ (\Sigma_o \cdot dE_o)\ \Sigma_o + 2G\ dE_o \tag{7}$$

and

$$P = -K\Delta\ .$$

In these equations, G and K are the shear and the bulk elastic moduli, respectively, and the constant A is the same as that introduced in Section a. The product $\Sigma_o \cdot dE_o$ refers to a scalar multiplication.

d. Solution for Micro-stresses

The macro-stress tensor in the region under consideration will be denoted by Σ_I. The micro-stress tensor Σ_{II} will then be the difference between Σ_o and Σ_I, namely

$$\Sigma_{II} = \Sigma_o - \Sigma_I\ . \tag{8}$$

The strains, referred to the state of zero macro-stresses, will be taken as constant on a microscopic scale. Although this assumption of constant microscopic strains is not strictly valid, the total strain is certainly much more nearly constant than either of its two components, the elastic and the plastic strains. In certain special cases of residual micro-stresses, such as in the case of a pre-stressed surface skin, the assumption of a constant strain is strictly valid. We may therefore set

$$E_o = E_I \,, \tag{9}$$

where Σ_I and E_I satisfy Eq. (7), namely

$$d\Sigma_I = -(2G/A^2)\ (\Sigma_I \cdot dE_I)\ \Sigma_I + 2G\ dE_I \tag{10}$$

In order to obtain an equation for the micro-stress tensor Σ_{II}, one subtracts Eq. (10) from Eq. (7), and uses Eq. (8) and (9). The result is

$$d\Sigma_{II} = -(2G/A^2) \left\{ (\Sigma_o \cdot dE_I)\ \Sigma_o - (\Sigma_I \cdot dE_I)\ \Sigma_I \right\} \tag{11}$$

The above equation may be considerably simplified if one assumes that the micro-stress tensor is small compared to the macro-stress tensor, i.e.,

$$\Sigma_{II}{}^2 \ll \Sigma_I{}^2 \,. \tag{12}$$

Prager has shown that if Σ is the stress pattern existing when plastic flow is taking place, then

$$\Sigma^2 = A^2 \,.$$

Therefore both $\Sigma_o{}^2$ and $\Sigma_I{}^2$ are equal to A^2. Their difference is therefore zero, and hence, in view of Eqs. (8) and (12)

$$\Sigma_I \cdot \Sigma_{II} = 0 \,. \tag{13}$$

Upon using this equation, and upon observing that the macro-stress tensor does not change with deformation, i.e., $d\Sigma_I = 0$, one obtains from Eq. (10) that

$$\Sigma_{II} \cdot dE_I = 0 \, . \qquad (14)$$

In virtue of this last equation, the product $(\Sigma_o \cdot dE_I)$ in Eq. (11) may be replaced by $(\Sigma_I \cdot dE_I)$. One may therefore rewrite Eq. (11) as

$$d\Sigma_{II} = -(2G/A^2)\,(\Sigma_I \cdot dE_I)\Sigma_{II} \, . \qquad (15)$$

The formal solution of the above equation is

$$\Sigma_{II} = (\Sigma_{II})_o \; e^{-(2G/A^2) \int (\Sigma_I \cdot dE_I)} \qquad (16)$$

The quantity $(\Sigma_{II})_o$ is to be interpreted as the micro-stress tensor at the initiation of plastic deformation. The integral in the exponent is to extend only over the plastic strains.

As an important example of Eq. (16), the case of a uniaxial tensile macro-stress will be considered. If σ is the uniaxial stress, E the corresponding strain, then from Eqs. (5) and (6), $\Sigma_I \cdot dE_I = \sigma dE$. In this case A^2, namely $\Sigma_I{}^2$, is equal to $(2/3)\sigma^2$. Therefore

$$\Sigma_{II} = (\Sigma_{II})_o \; e^{-\frac{3G}{Y}\frac{E'}{E_o}}$$

where E' is the plastic strain, E_o the elastic strain at yield, namely σ/Y.

4. Effect of Thin Layer Under Compression Upon Bend Strength

When a homogeneous bar is broken in bending, fracture appears always to start at the surface. The possibility therefore exists that a thin layer under compression, by delaying the initiation of fracture, may appreciably increase the bend strength.

Such a thin compressive skin can appreciably raise the bend strength only if a smaller tensile strain is required to initiate fracture at the surface than would be required to initiate fracture in the interior. Thus suppose that the same tensile strain is necessary to initiate fracture in the interior as at the surface. Then if initiation of fracture at the surface is prevented by a layer under compression, only a small percentage increase of the curvature of the bar will raise the strain, just under this layer, to a value which will lead to fracture. From Figure 8 it may be seen that this relative increase in curvature is given by 2W/D, where W is the thickness of the skin, D is the diameter of the bar.

If a smaller strain is necessary to initiate fracture at the surface than in the interior, then putting the surface under compression may appreciably raise the bend strength. Such a skin would have two distinct effects. Firstly, since the outer layer starts in a longitudinal compression, a higher strain is necessary for the outer layers to reach a given stress level. Secondly, the circumferential compressive stress lowers the longitudinal stress necessary to initiate plastic deformation. Once plastic deformation starts, the longitudinal stress rises much more slowly with tensile strain than in the elastic region. The outer layers may therefore undergo a considerably greater strain before the stress reaches the fracture value than in the absence of the compressive layer. The circumferential stress will relax rapidly as the plastic deformation continues, as pointed out in the previous section, but only a slight increase in strain is necessary to considerably increase the bend strength.

The possible increase in bend strength occurs primarily through a lowering of the yield stress of the outer layers with respect to the fracture stress. Therefore any method of introducing a surface layer under compression which at the same time lowers the fracture stress cannot have a beneficial effect. In fact, if the fracture stress were lowered further than the yield stress, the bend strength would be impaired.

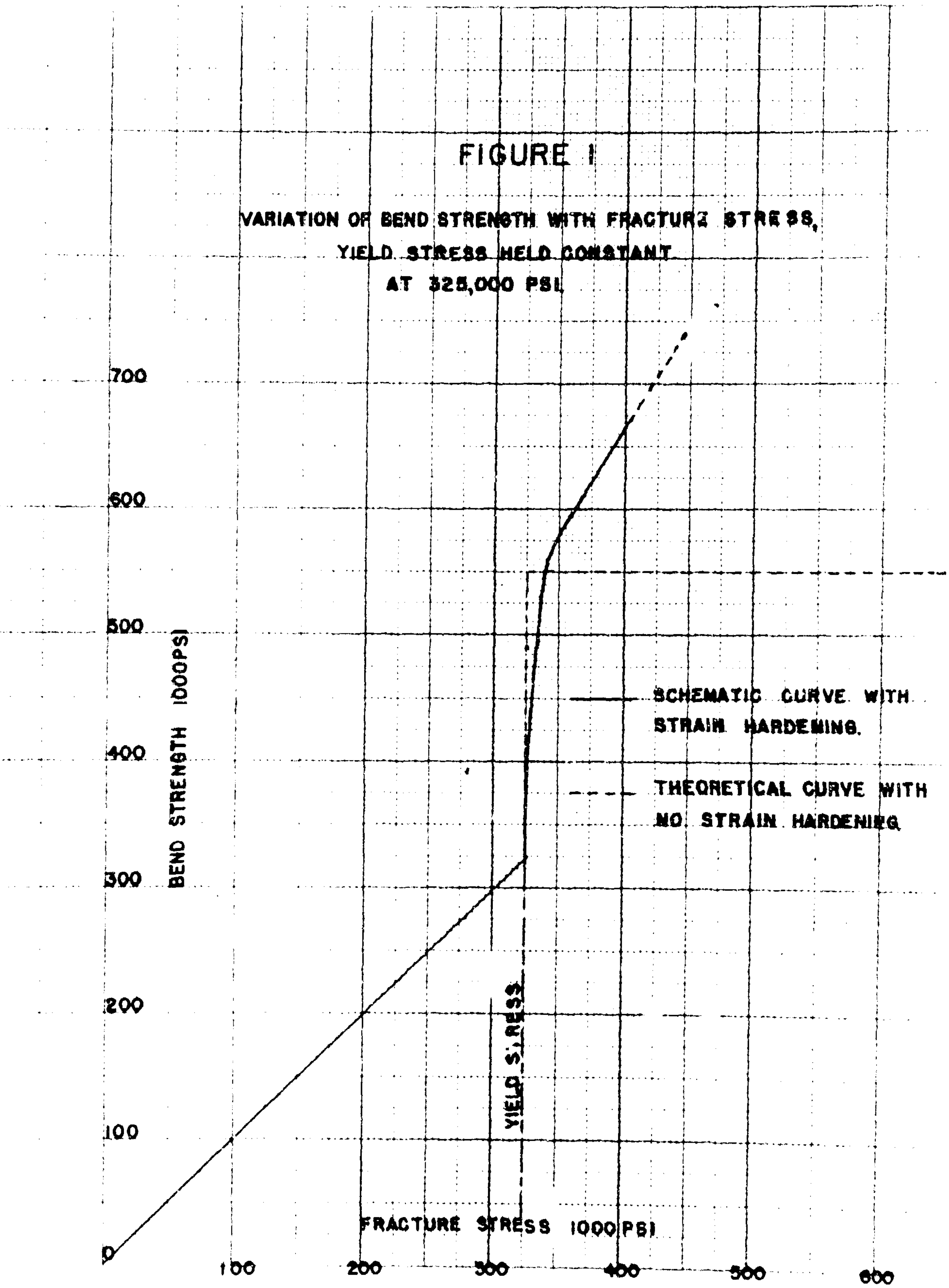


FIGURE I

VARIATION OF BEND STRENGTH WITH FRACTURE STRESS, YIELD STRESS HELD CONSTANT AT 325,000 PSI

FIGURE 2

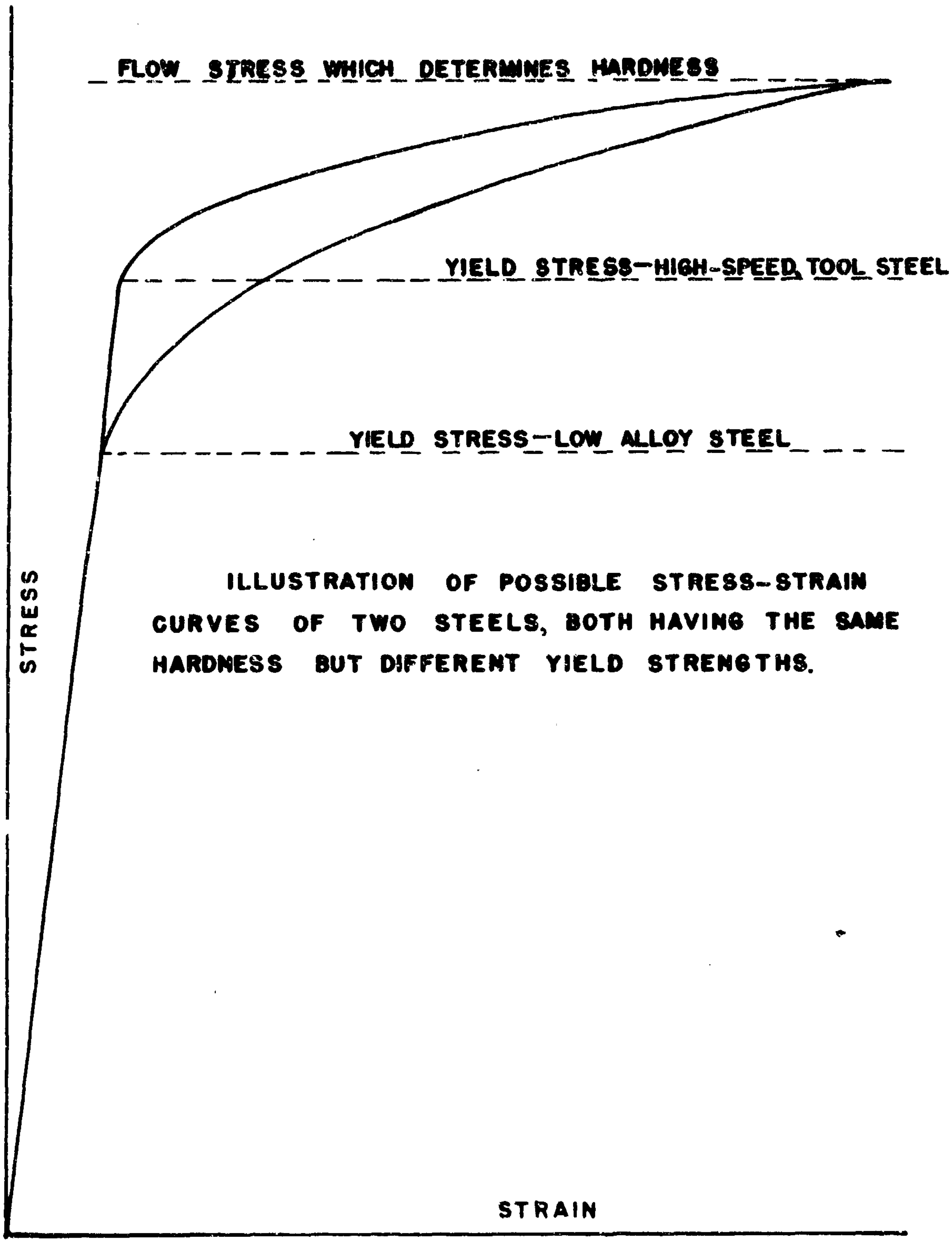

FLOW STRESS WHICH DETERMINES HARDNESS
YIELD STRESS—HIGH-SPEED TOOL STEEL
YIELD STRESS—LOW ALLOY STEEL
ILLUSTRATION OF POSSIBLE STRESS-STRAIN CURVES OF TWO STEELS, BOTH HAVING THE SAME HARDNESS BUT DIFFERENT YIELD STRENGTHS.
STRESS
STRAIN

FIGURE 3

Bend strength of 0.64 carbon steel hardened by interrupted quench, and tempered to various hardnesses.

STEEL-FXS-318, LOT 1, Ø=.430" DIA. ROUNDS – 1675°F FOR 1/2 HR. – QUENCHED IN OIL AT 350°F FOR 1 MIN. – AIR COOLED TO RM. TEMP. – COOLED IN ALCOHOL TO –110°F

POINT NO.	TEMPER TEMP.	TEMPER TIME.	POINT NO.	TEMPER TEMP.	TEMPER TIME.	POINT NO.	TEMPER TEMP.	TEMPER TIME.
1	350°F	10 SEC	12	350°F	1000 SEC	11	550°F	10 SEC
3	"	"	13	"	"	7	"	"
2	"	"	15	"	10000 SEC	16	"	100 SEC
6	"	"	19	"	"	18	"	"
10	"	100 SEC	14	"	"	20	"	"
5	"	"	8	550°F	10 SEC	17	"	"
4	"	"	9	"	"	21	"	1000 SEC
						22	"	"

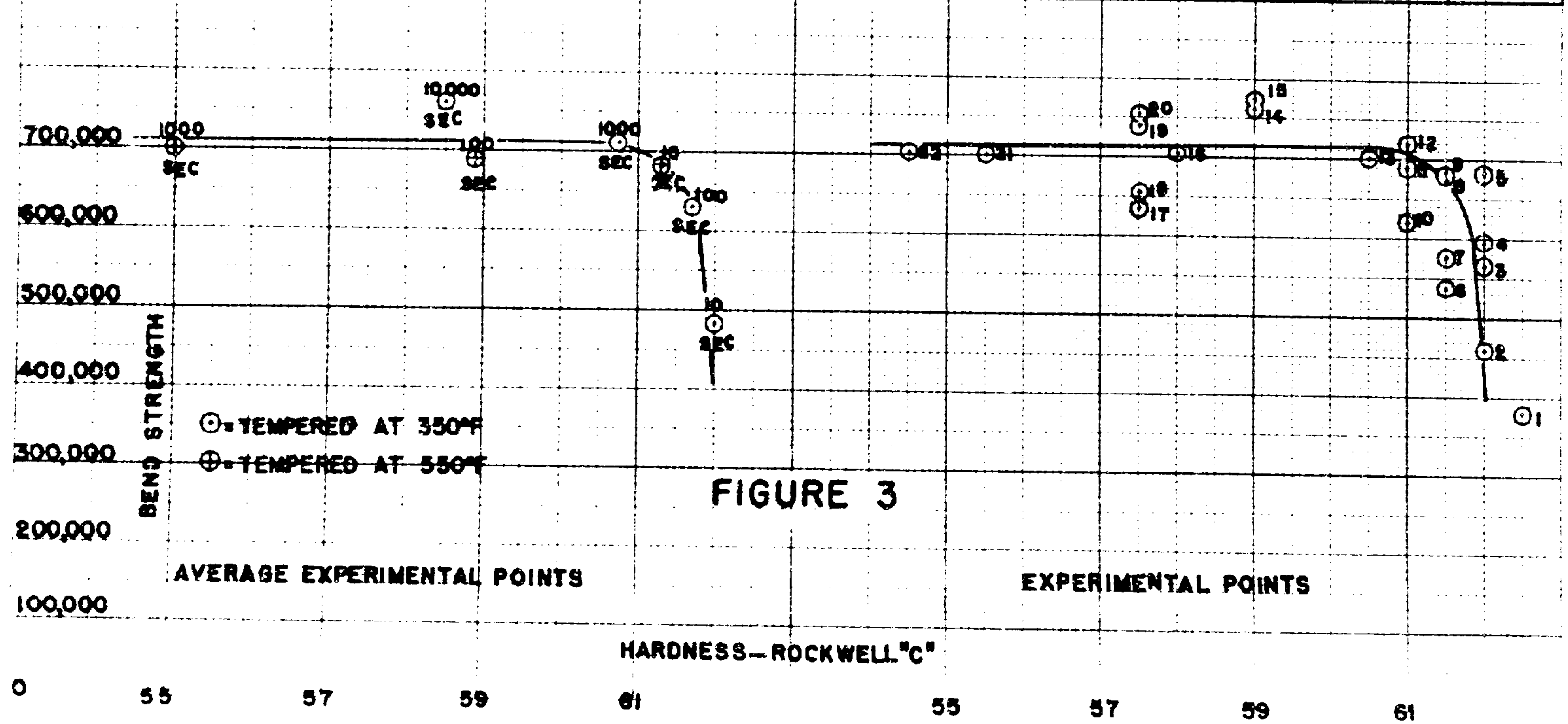


FIGURE 3

FIGURE 4

Bend strength of 0.76 carbon steel hardened by interrupted quench, and tempered to various hardnesses.

STEEL-FXS-318, LOT 2, 0-.430" DIA. ROUNDS-1675°F. FOR 1/2 HR, QUENCHED IN OIL AT 350°F. FOR 1 MIN, AIR COOLED TO RM. TEMP-COOLED IN ALCOHOL TO -110°F

POINT NO	TEMPER TEMP	TEMPER TIME	POINT NO	TEMPER TEMP	TEMPER TIME
15	450°F	5MIN	3	350°F	8MIN
16	"	"	10	"	"
17	"	"	6	"	"
4	350°F	2MIN	12	"	"
1	"	"	7	"	"
9	"	"	13	"	16MIN
5	"	4MIN	11	"	"
2	"	"	8	"	"
			14	"	30MIN

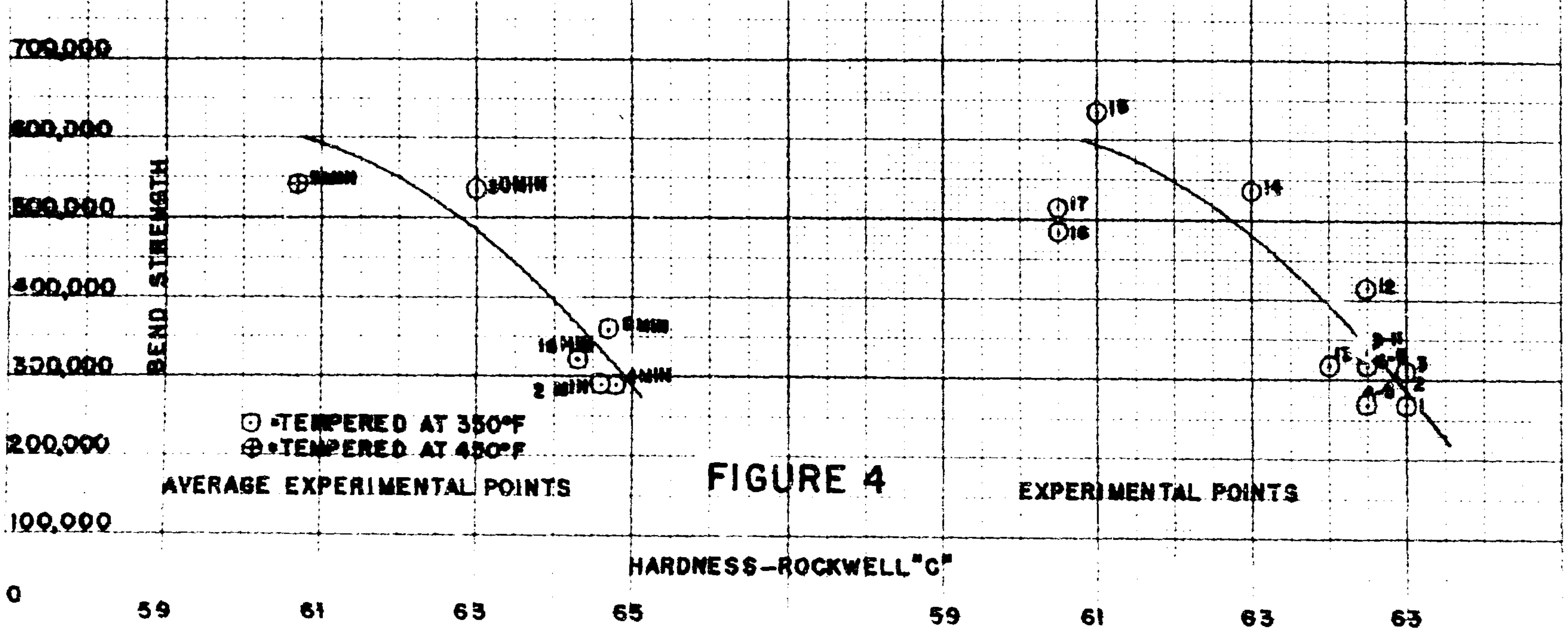


FIGURE 4

FIGURE 5

Bend strength of 0.64 carbon steel hardened by conventional method, and tempered to various hardnesses.

STEEL—FXS-318, LOT 1, Ø= .430" DIA. ROUNDS
1525°F. FOR 1/2 HR, QUENCHED IN OIL AT 80°F.
TEMPERED IN SALT

POINT NO	TEMPER TEMP.	TEMPER TIME	POINT NO	TEMPER TEMP	TEMPER TIME
1	Q *	———	3	350°F.	10 MIN
4	Q	———	10	"	"
5	Q	———	11	"	"
2	330°F	5 MIN	12	"	"
6	"	"	13	"	"
7	"	"	14	"	30 MIN
8	"	"	15	"	"
9	"	"	16	"	"

* Q = AS QUENCHED

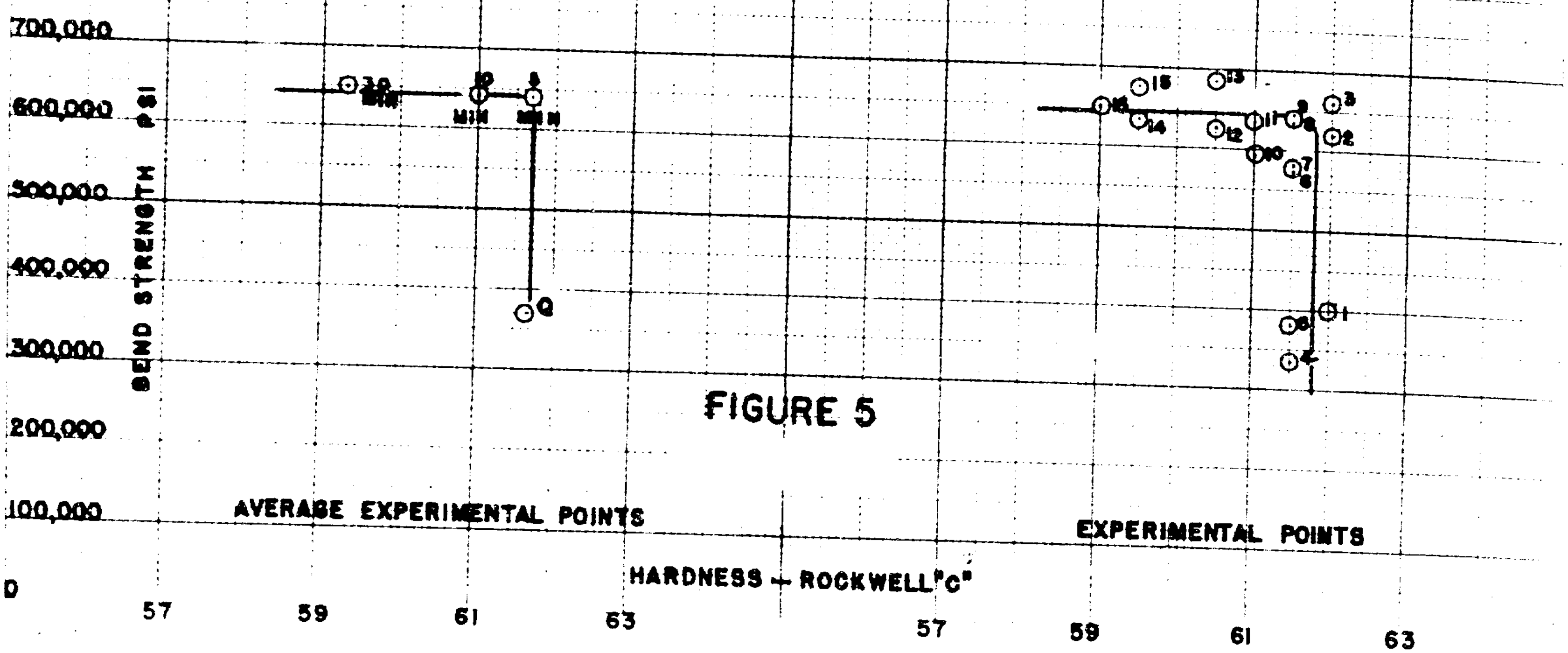


FIGURE 5

FIGURE 6

Bend strength of 0.76 carbon steel hardened by conventional method, and tempered to various hardnesses.

STEEL-FXS-318, LOT 2, Θ=.430" DIA. ROUNDS
1525°F FOR 1/2 HR. QUENCHED IN OIL AT 80°F.

POINT NO	TEMPER TEMP	TEMPER TIME	POINT NO.	TEMPER TEMP	TEMPER TIME
12	Q *	———	14	350°F	8 MIN
6	Q	———	9	"	"
1	Q	———	8	"	"
2	Q	———	11	"	16 MIN
10	350°F	2 MIN	13	"	"
4	"	"	17	"	"
5	"	"	18	"	32 MIN
3	"	"	16	"	"
7	"	4 MIN	15	"	"

* Q = AS QUENCHED

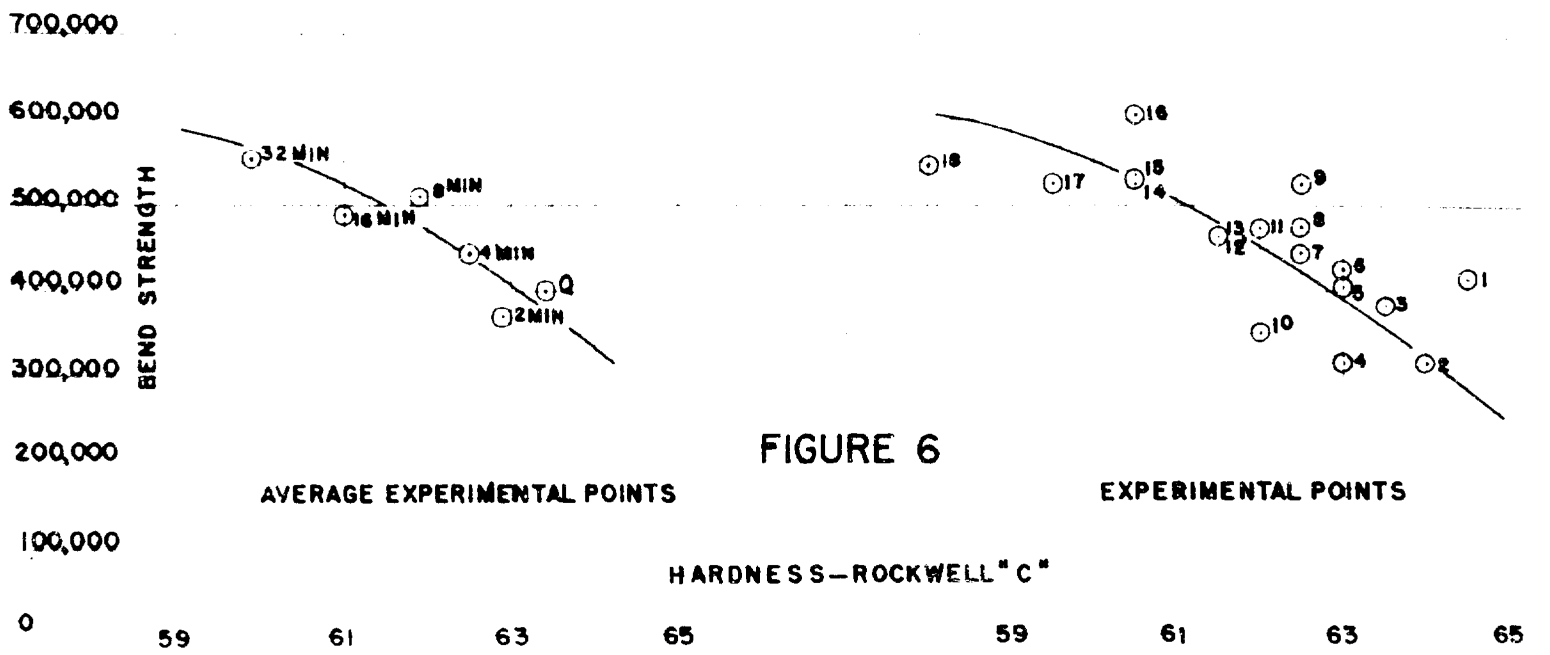


FIGURE 6

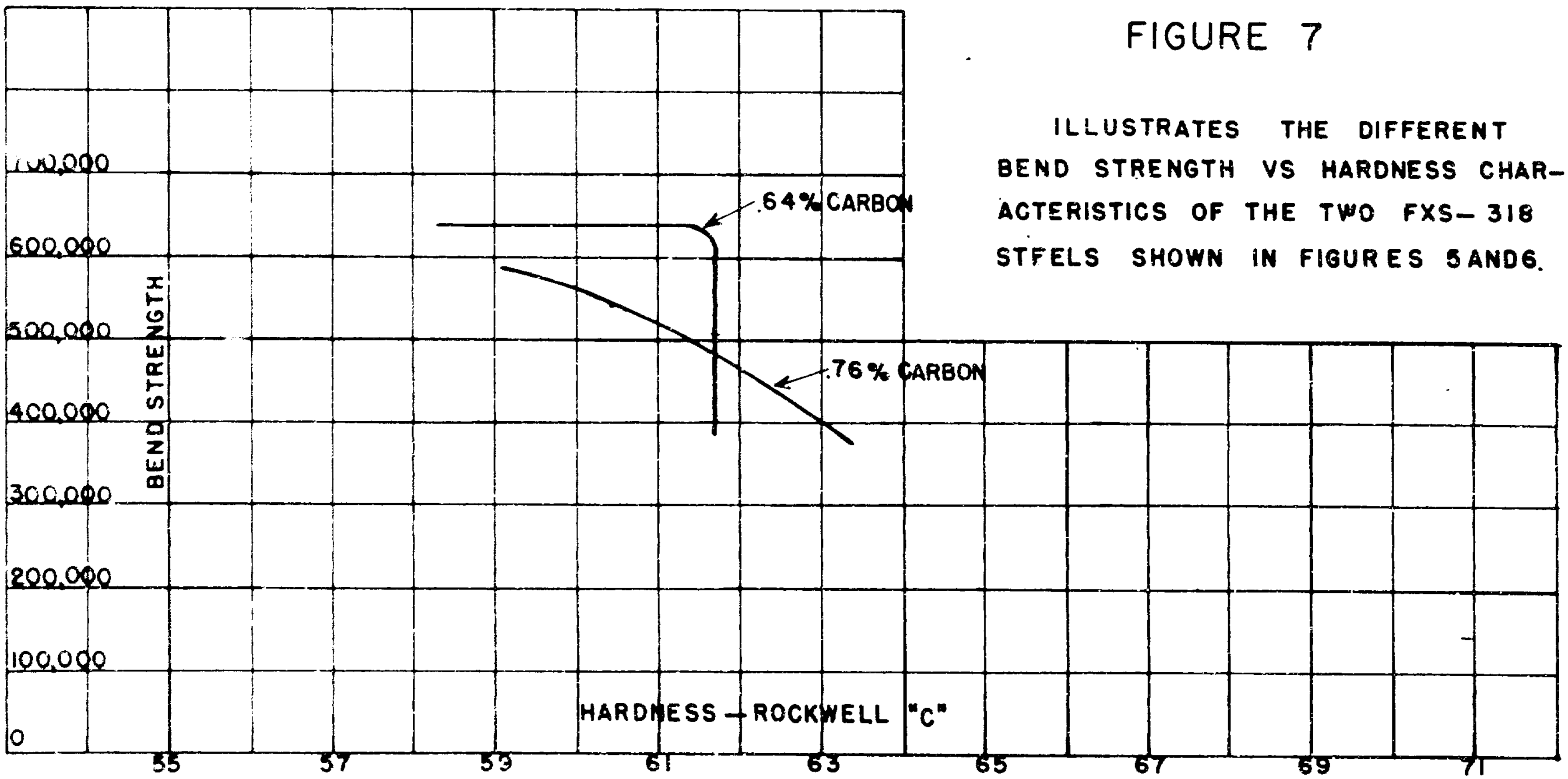
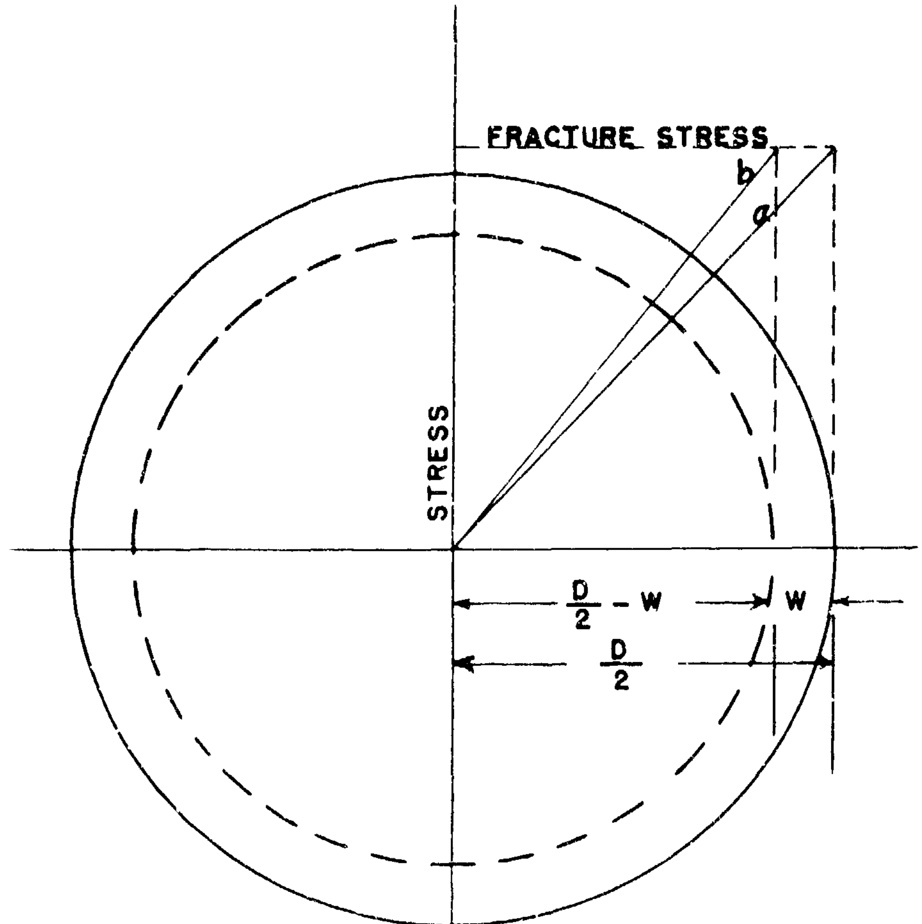


FIGURE 7

ILLUSTRATES THE DIFFERENT BEND STRENGTH VS HARDNESS CHARACTERISTICS OF THE TWO FXS-318 STFELS SHOWN IN FIGURES 5 AND 6.

FIGURE 8



CROSS SECTION OF BEND SPECIMEN

D = DIAMETER OF BAR.

R = RADIUS OF CURVATURE AT FRACTURE — BAR WITHOUT SURFACE LAYER UNDER COMPRESSION.

W = THICKNESS OF SURFACE LAYER UNDER COMPRESSION.

R' = RADIUS OF CURVATURE AT FRACTURE — BAR WITH SURFACE LAYER UNDER COMPRESSION.

$$\text{SLOPE } a \propto \frac{1}{D/2} \propto \frac{1}{R}$$

$$\text{SLOPE } b \propto \frac{1}{D/2 - W} \propto \frac{1}{R'}$$

$$\frac{R'}{R} = \frac{D/2 - W}{D/2} = 1 - \frac{2W}{D}$$