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DESIGN CONSIDERATIONS FOR DIRECTIONAL COUPLERS

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DESIGN CONSIDERATIONS FOR DIRECTIONAL COUPLERS

Abstract

Directional couplers possess properties which make them useful for power monitoring purposes and for impedance measuring or matching. The action of a number of different general classes of directional couplers is explained qualitatively. Characteristics of specific types in both waveguide and coaxial line are given, together with information necessary for the design of these couplers to meet particular requirements. The emphasis in this report is upon directional couplers such as might be used as test points in radar systems at which one can monitor transmitter power and introduce signals of known amplitude to measure receiver sensitivity.

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Approved by:

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DESIGN CONSIDERATIONS FOR DIRECTIONAL COUPLERS

INTRODUCTION

When a directional coupler is inserted in a transmission line, it will couple more tightly to the wave travelling in one direction than to the wave travelling in the opposite direction. As a result of this property, directional couplers may perform various useful functions. On radar systems, they may be used as test points at which to monitor transmitter power and to introduce signals of known amplitude to test the receiver, in a manner that is relatively free from errors due to standing waves in the line. In test sets, they may be used to monitor the power output from a signal generator, avoiding the error which may result from the critical dependence upon impedance of the power split at a simple tee. They may be used as "reflectometers", to determine standing wave ratio by measuring the amplitude of the reflected wave relative to that of the wave in a transmission line, or to match a load by tuning the output of the reflectometer to zero.

The performance of a coupler in any of these uses may be predicted from a knowledge of its coupling coefficient and its directivity. Conversely, the requirements for the coupling and directivity are set by the specifications for the coupler.

There are many different kinds of directional couplers and quite a bit of information has been obtained about their design. The purpose of this report is to collect this information and make it available in the form of discussion, formulas, and graphs.

I. DIRECTIONAL COUPLERS - Their Use, Advantages, and Limitations

A. DEFINITIONS:

A DIRECTIONAL COUPLER is a device which, when inserted in a transmission line on which there exist waves travelling in both directions, delivers to a pair of terminals located in an AUXILIARY TRANSMISSION LINE a voltage which is largely a function of the amplitude of the wave going in one PREFERRED DIRECTION, and relatively independent of the wave going in the opposite direction.

That part of the directional coupler which forms a section of the transmission line in which the coupler is inserted will be referred to as the MAIN LINE. It possesses a set of terminals referred to as the MAIN LINE INPUT TERMINALS, at which the wave travelling in the preferred direction enters. The other set of main line terminals will be called the MAIN LINE OUTPUT TERMINALS.

The AUXILIARY LINE contains, in addition to the AUXILIARY LINE OUTPUT TERMINALS, a resistive TERMINATION. The latter serves to absorb the "wrong way" energy, and to provide a good match looking into the auxiliary line output terminals. The main and auxiliary lines are connected together by a COUPLING MECHANISM which gives the coupler its directional properties.

The COUPLING of the directional coupler specifies that fraction of the power proceeding in the preferred direction which is delivered to the auxiliary line output terminals. More precisely, the COUPLING, which may be expressed either as a voltage or power ratio,* is defined as the ratio of the voltage (or power) delivered to a matched detector at the auxiliary line output terminals to the voltage (or power) delivered to the coupler's main line input terminals, provided that the main line output terminals are terminated by a matched load. Coupling will be denoted by the letter C.

The DIRECTIVITY of a directional coupler is defined as the ratio of the voltage (or power) delivered to a matched detector at the auxiliary line output terminals under the two conditions: (1) power is fed in the main line input terminals, with a matched load attached to the main output terminals; and (2) the same amount of power is delivered to the main line output terminals, with a matched load attached to the main line input terminals. Directivity will be denoted by the letter D.

The MAIN LINE VSWR is the voltage standing wave ratio seen looking into the main line input terminals, with matched loads on the other sets of terminals. The AUXILIARY LINE VSWR is that VSWR seen looking into the auxiliary line output terminals, with matched loads on the other sets of terminals.

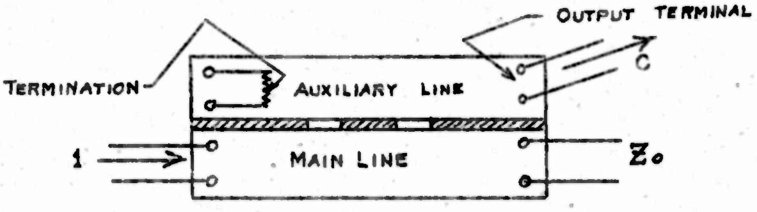

Figure 1

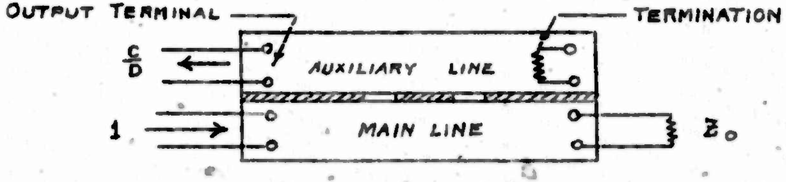

Figure 2

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* As defined here, coupling expressed as a ratio has a value less than unity, and thus when it is expressed in decibels it is a negative number. This negative sign is often omitted for convenience, although it is retained in this report.

Figures 1 and 2 are schematic diagrams, which may serve to clarify the definitions of coupling and directivity. The representation of the directional coupler here as a (dissipative) six terminal element is to emphasize the fact that the definitions apply to the entire coupler, and thus include the effect of the resistive termination.

The performance of the directional coupler, when it is used to introduce a signal into the main transmission line may be predicted from reciprocity considerations, and is indicated in Figure 3.

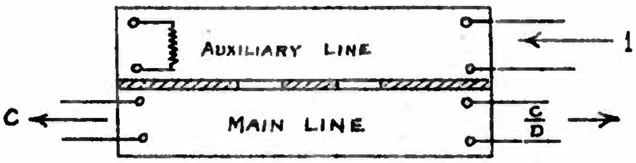


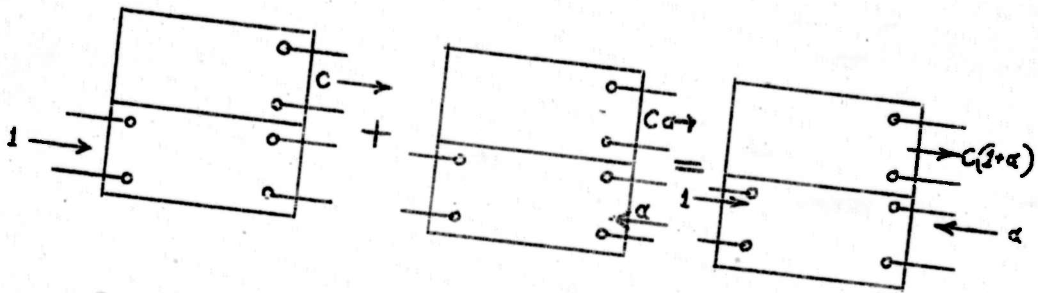
Figure 3

B. USE OF DIRECTIONAL COUPLERS IN POWER MONITORING:

A directional coupler, the coupling of which is known, may be used together with a power measuring device to obtain a quantitative measure of the power flowing down the transmission line in which the coupler is inserted. There are errors which arise in this measurement due to the presence of a reflected wave on the line, but, as will be shown, these errors are an order of magnitude less than those which would be encountered with the use of a non-directional coupling device under the same conditions.

The output of a non-directional coupling device, such as a simple probe, inserted in a transmission line on which there are standing waves will depend on the position of insertion in such a way that if a curve of power output versus position along the line is plotted, the familiar standing wave pattern is traced out. The ratio of the maximum power to the minimum power will be exactly equal to the square of the voltage-standing-wave ratio.

Thus, even for standing-wave-ratios for which the power carried in the reflected wave is negligible compared with that carried by the transmitted wave, the voltage amplitude of the reflected wave is not negligible compared with that of the direct wave. Since the voltage output of the non-directional coupler is simply proportional to the vector sum of these two amplitudes, the range of output voltage is between limits proportional to their arithmetical sum and difference, as shown in Figure 4.



C = Coupling
α = Reflection Coefficient } expressed as voltage ratios.

Figure 4

A directional coupler, on the other hand, may be inserted in the line so as to couple more tightly to the direct wave proceeding from the generator, than to the reflected wave. The variation of output voltage with the position of the standing wave pattern in the line is reduced to an extent which is determined by the directivity. In the extreme case of a coupler with an infinite directivity, the position of the standing wave pattern will not influence the output voltage at all. Since the coupler is in this case essentially a monitor of the direct wave alone, there will be some error introduced if we regard the power delivered to the output terminals as proportional to the amount of power being delivered to the load. However, this error is quite small for moderate VSWR's and is just equal to the power lost by reflection.

Figure 5 illustrates the fact that when a directional coupler is used for power monitoring, the variation in output voltage due to position of the standing wave pattern is between limits having a ratio of $1+|\alpha/D|$ to $1-|\alpha/D|$, where α is the voltage reflection coefficient of the load attached to the main line output terminals, and D is the directivity expressed as a voltage ratio.

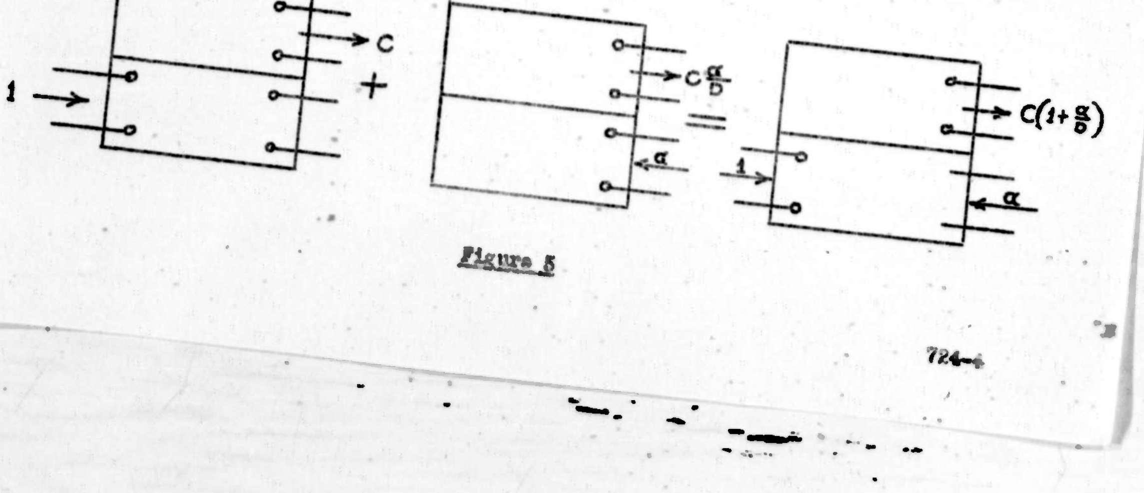


Figure 5

The values of this variation for various directivities and VSWR's are tabulated in Figure 6, and plotted in Figure 7. The following is a useful approximation for this variation.

$$\text{DB VARIATION IN POWER OUT COUPLER DUE TO STANDING WAVES} = \pm 8.7 \frac{a}{D},$$

where a and D are again expressed as voltage ratios.

Error	VSWR in Line							
	1.1	1.2	1.5	2.0	2.5	3.0	7.0	∞
± .1 dh	13 db	18 db	25 db	30 db	33 db	35 db	38 db	40 db
± .2	6.5	12	19	23	26	27	31	35
± .3	3	8.5	15.5	20	22	23	27	29
± .4	3	6	13	17	19	21	24	27
± .5	0	4	11	15	17.5	18.5	22.5	25
± .75	0	.5	7	12	14	15.5	19	22
± 1.0	0	0	5	9	12.5	13	16.5	19

Table lists Directivity (in db) required in order to reduce error in monitoring direct wave to specified amounts for various VSWR's in the main line.

Figure 6

A signal of known amplitude may be introduced through the directional coupler into the main transmission line, and one may desire to know the power which proceeds in one direction down that line. As may be seen from Figure 8 or by reciprocity from Figure 5, the same errors enter here as in the previous case. These are just those listed in Figures 6 and 7.

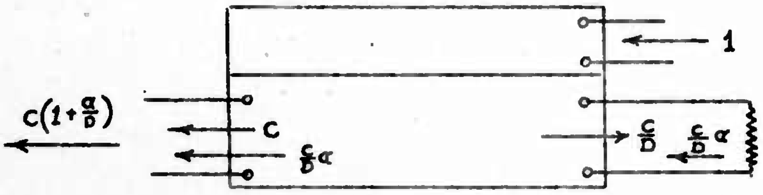


Figure 8

The specifications for the directivity required of a directional coupler for any given power monitoring application may be arrived at by the use of Figure 7, and by considering both the magnitude of the standing wave ratio likely to be encountered in the line, and the maximum error allowable in

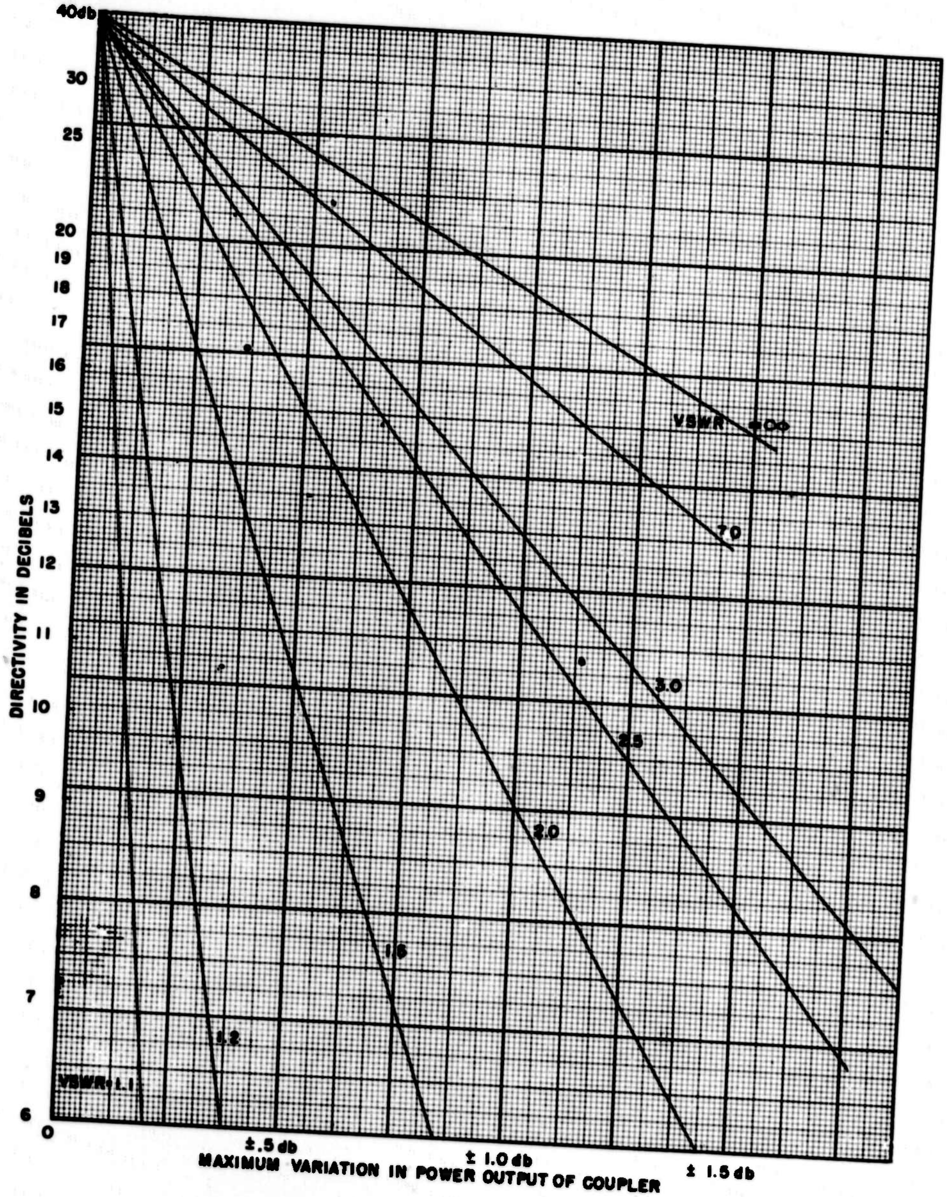


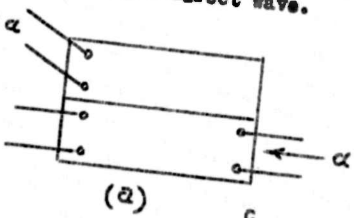
FIG. 7
EFFECT OF DIRECTIVITY UPON POWER MONITORING
ERRORS DUE TO STANDING WAVE RATIO IN MAIN LINE

the measurement of the transmitter power. Thus to reduce error to ± 1/2 db, in the presence of a VSWR as high as two, a minimum directivity of 15 db is required.

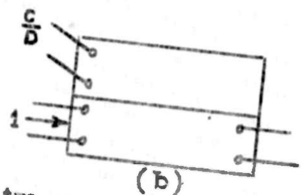
C. DIRECTIONAL COUPLERS USED TO MEASURE REFLECTED POWER:*

Directional couplers, inserted in a transmission line so as to couple preferentially to the reflected wave, may be used (1) as a null reading device for indicating when the main line is matched, and (2) in conjunction with a measurement of forward power, or by calibration with a known impedance, it may be used to determine the magnitude of the reflection coefficient.

For successful operation in these applications, the directivity has to be much higher than for direct wave monitoring. The reason for this is that the voltage coupled to the auxiliary line output from the small amplitude reflected wave must be made large compared to that coupled from the stronger direct wave.



(a) If there were just the reflected wave, α, then Cα would be delivered to the output.



(b) Because of the finite directivity an amount of C/D of the direct wave will be added.

(c) Since these two may be either in phase or out of phase, the output may vary between limits of $C(|\alpha| + |\frac{1}{D}|)$ and $C(|\alpha| - |\frac{1}{D}|)$.

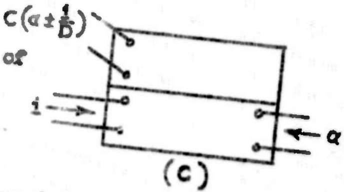


Figure 9

Figure 9 shows a directional coupler used to measure reflected power. It indicates that the voltage at the output terminals of the coupler, as a measure of the amplitude of the reflected wave, will vary between limits having a ratio of $(|\alpha| + |1/D|)$ to $(|\alpha| - |1/D|)$, where α and D are the reflection coefficient and the directivity respectively, both expressed as voltage ratios.

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* See RL Report No. 643, The Use of the Magic Tee Microwave Bridge in Measuring Impedance, by R. L. Kyhl, for a more complete discussion of the errors involved in measuring impedance by means of a magic tee (or a directional coupler), and a brief discussion of the relative merits of each when used in that connection.

Thus the problem here is to keep 1/D small compared with α, which is itself a small quantity; whereas to keep within the same error in monitoring the direct wave, we have to make α/D small compared with unity.

There is a special case which is of some interest. If power is being delivered to the main line output terminals, a tunable load attached to the main line input terminals may be adjusted to give zero output to the auxiliary line terminals. For a coupler with infinite directivity, the VSWR of the load after adjustment will be exactly unity. In general, the load VSWR will correspond to a reflection coefficient of magnitude $\alpha = 1/D$. We shall call the VSWR so defined, the DIRECTIVITY-STANDING-WAVE RATIO, or DSWR. Figure 10 shows a graph of directivity in decibels as a function of DSWR.

This experiment may be used as a convenient way of measuring the directivity of a directional coupler without the necessity of making actual attenuation measurements. The procedure is to feed power into the main line output terminals and adjust a tunable load at the main line input terminals to give zero power to a detector at the auxiliary terminals. Then measure the VSWR of the load. It will be equal to the DSWR of the coupler. It may be converted to decibels, if so desired, by referring to Figure 10.

The figure for DSWR also enables one to make a fairly handy approximation for the errors involved in measuring standing-wave-ratios with a directional coupler used as a reflectometer.

If the true standing wave ratio is denoted by (VSWR) true, then the VSWR indicated by the reflectometer, (VSWR) refl. may vary between limits which are approximately:

$$\mathrm{VSWR}_{\mathrm{refl.}} = (\mathrm{VSWR}_{\mathrm{true}})(\mathrm{DSWR})$$

and,

$$\mathrm{VSWR}_{\mathrm{refl.}} = \frac{\mathrm{VSWR}_{\mathrm{true}}}{\mathrm{DSWR}}\,, \text{ or } \frac{\mathrm{DSWR}}{\mathrm{VSWR}_{\mathrm{true}}}$$

whichever is greater than unity.

Thus a directivity of 38 db corresponds to DSWR = 1.03, and in measuring a VSWR of, say, 1.20, the reflectometer might read anywhere between 1.20/1.03 = 1.165, or (1.20)(1.03) = 1.235.

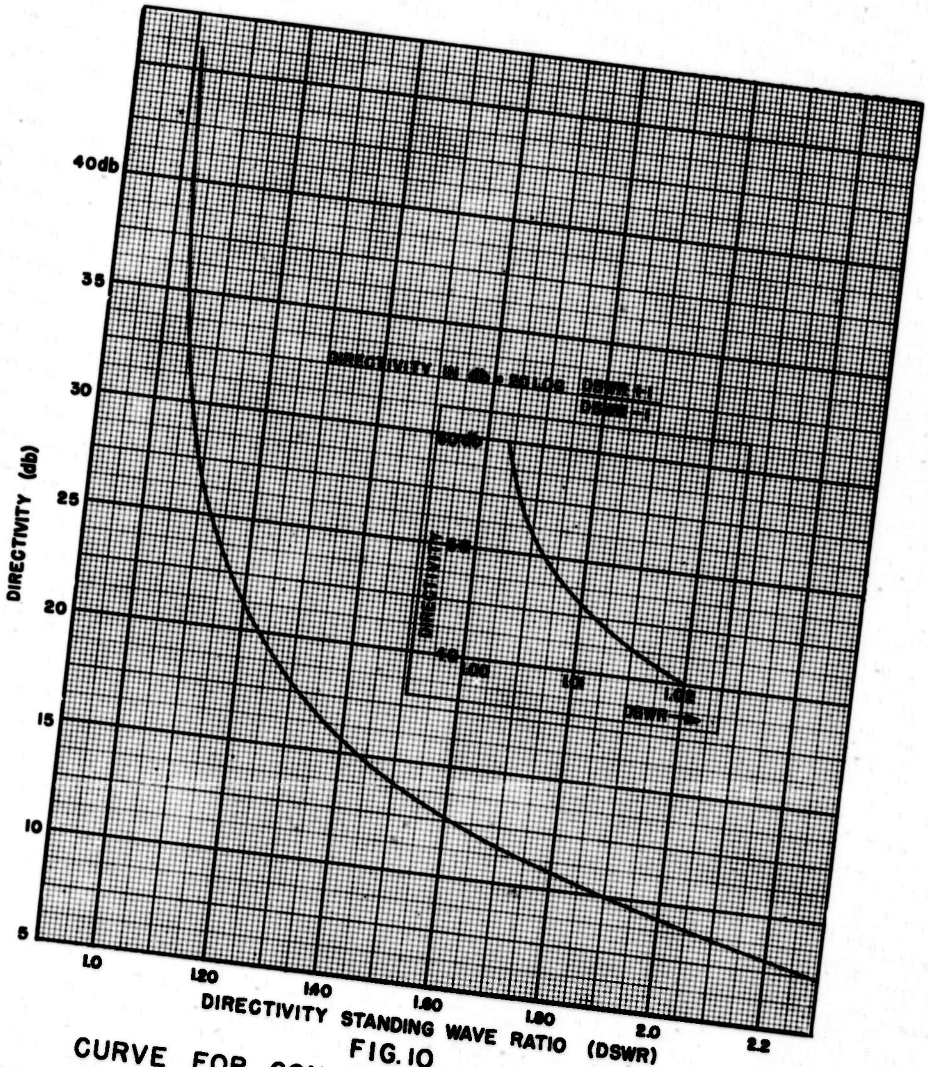


FIG. 10
CURVE FOR CONVERTING DIRECTIVITY IN DECIBELS TO DIRECTIVITY STANDING WAVE RATIO (DSWR)

II. DESIGN INFORMATION

The wide variety of applications to which directional couplers may be put, gives rise to a correspondingly wide variety of specifications. Thus, the considerations discussed in Part I of this report determine the minimum value of directivity consistent with the desired performance. In addition, there are specifications upon coupling and VSWR over a band of frequencies. The coupler must be made in a particular kind of transmission line, with the proper connectors. It must have an adequate power capacity. Finally, considerations of space, weight, and ease of manufacture must be taken into account.

To assist in designing a directional coupler meeting all these particulars, there will first be presented some general design considerations, and then more specific information regarding the various types of directional couplers. Finally, a summary will be given to serve as a brief index to this information.

A. GENERAL DESIGN CONSIDERATIONS:

The design problem for directional couplers is one of finding a coupling mechanism possessing the desired characteristics, and then combining it with a suitable resistive termination.

1. Coupling Mechanism.

Essentially, the coupling mechanism is the directional coupler, apart from the termination.* It contains sections of the main and auxiliary lines, including all regions in which there is electrical coupling between the two. It may be regarded as a linear, non-dissipative eight terminal network, and usually has at least one plane of symmetry.

The many different kinds of coupling mechanisms have been classified according to the method of obtaining their directional characteristics. The general classes are listed here; subsequent sections will discuss each of them in detail.

a. Multiple Path

The directivity of this class of coupler results from the interference of two or more waves which have been made to travel different paths, due to the relative phase shift along these paths. Examples of this type are the two hole coupler in waveguide and the slotted block coupler in coaxial line.

---

*Sometimes the term "directional coupler" is applied to the coupling mechanism alone, rather than to the over-all combination of coupling mechanism and termination as is done in this report.

b. Long Slot

This is similar to the multiple-path type since it depends on simple interference due to path differences and might be classed as a limiting case where the number of paths becomes infinite.

c. Reversed Phase

This differs from a two-path coupler in that there is an extra 180° phase shift at one point of coupling, which results in waves which have travelled equal path lengths being out of phase, and those with paths differing by half wavelengths being in phase. The two probe types developed by RCA and the two slot type in waveguide suggested by Schwinger are in this category.

d. Electromagnetic

The directivity of this class of couplers depends upon having two kinds of coupling at a point ("electric" and "magnetic", or "shunt" and "series"), which set up waves which interfere in one direction and reinforce in the other. The "Bethe hole" couplers in waveguide and coaxial line are members of this class.

2. Termination Design.

a. Effect of Termination Mismatch

The resistive termination is an essential component of a directional coupler.* Its functions, as have been stated, are to absorb the "wrong-way" energy which is coupled into the auxiliary line, and to present a good match looking into the auxiliary line output terminals. The VSWR of the termination has a large effect upon the directivity, but has only a small effect upon the coupling.

By definition, the directivity of a directional coupler is high when the voltage at the auxiliary line output terminals due to energy coupled from the wave entering the main line output terminals is small. This voltage may be considered to consist of two components: (1) a component due to the imperfect directional properties of the coupling mechanism itself; and (2) a component arising from the fraction of the wave which, after having originally been directed by the coupling mechanism toward the termination, is reflected back toward the auxiliary line output terminals.

Since component (2) may have an arbitrary phase and magnitude with respect to component (1), the measured directivity of the coupler may be either greater or less than the directivity it would have with a perfectly matched termination.

---

*It may be shown from network theory that no non-dissipative six-terminal network can have any directivity.

In order to separate the effects produced by the coupling mechanism and by the termination mismatch, it is convenient to define the *directivity of the coupling mechanism* as that directivity which would be measured if the termination were perfectly matched to the characteristic impedance of the auxiliary line. This will be denoted by D', to distinguish it from the previous definition of directivity denoted by D which is an over-all characteristic of the coupler, and includes the effect of the termination. A coupling C', referring to the coupling mechanism, may be defined in a similar manner.

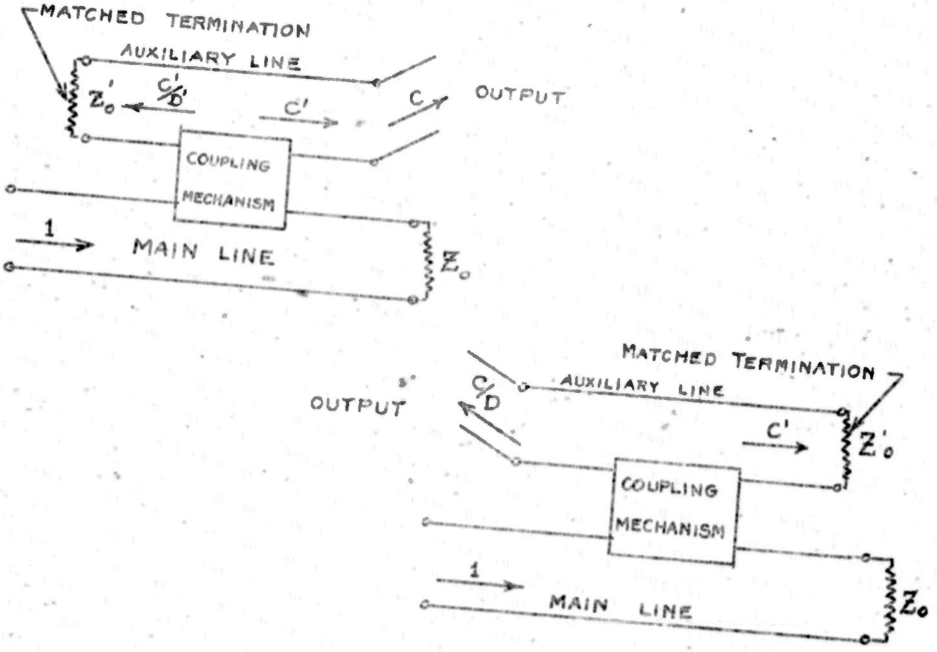
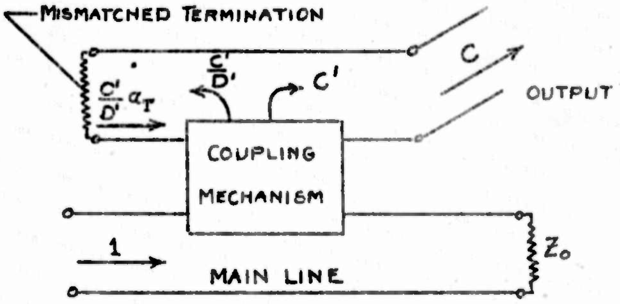


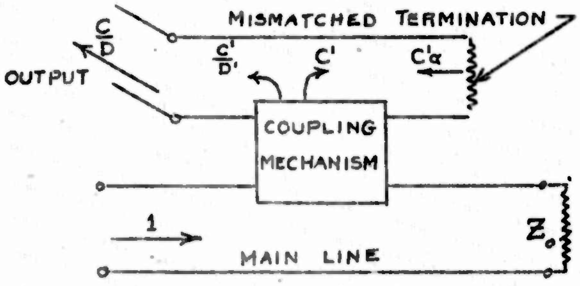
Figure 12

Figure 12 shows a coupler in which the termination is matched to the auxiliary line. In this case, the measured directivity D will be equal to D'.



C
C
D
D
α_T
} Expressed as voltage ratios

$$C = C'\left(1 + \frac{\alpha_T}{D'}\right)$$



$$\frac{C}{D} = C'\left(\frac{1}{D'} + \alpha_T\right)$$

$$\frac{1}{D} = \frac{C'}{C}\left(\frac{1}{D'} + \alpha_T\right) \approx \left(\frac{1}{D'} + \alpha_T\right)$$

Figure 13

Figure 13 shows a coupler in which the termination has a voltage reflection coefficient α_T. The measured coupling C will be only slightly different from the coupling C' of the coupling mechanism. The measured directivity D will be such that $1/D \cong 1/D' + \alpha_T$.

The maximum effect upon the directivity that a termination with arbitrary VSWR can have, may be obtained from the following approximate formulas. These are expressed in terms of the directivity standing-wave-ratio defined previously, D'SWR denoting the directivity standing-wave-ratio that the coupler would have with a matched termination, and DSWR representing the directivity standing-wave-ratio that the coupler actually has with the mismatched termination. $(VSWR)_T$ represents the standing-wave-ratio of the termination.

The limits on DSWR will be:

$$\mathrm{DSWR}_{max} \cong (\mathrm{D'SWR})(\mathrm{VSWR})_T$$

$$\mathrm{DSWR}_{min} \cong \frac{\mathrm{D'SWR}}{(\mathrm{VSWR})_T}, \text{ or } \frac{(\mathrm{VSWR})_T}{\mathrm{D'SWR}},$$

whichever is greater than unity.

The maximum effect upon the coupling that the termination VSWR may produce may be obtained by referring back to Figure 7, or it may be calculated approximately from the formula:

MAXIMUM CHANGE IN DECIBELS
COUPLING DUE TO MISMATCHED
TERMINATION $= \pm 8.7 \left| \frac{\alpha_T}{D'} \right|$,

with α_T and D' both expressed as voltage ratios.

From this discussion we see that the ideal termination would be one which would have exactly the right phase and magnitude of reflection coefficient to correct for the imperfections of the coupling mechanism. While it is not usually possible to arrange this over a wide band of frequencies, for narrow band, or at spot frequencies, by using a termination which may be adjusted in both phase and magnitude, a coupler may be tuned to have extremely high directivity.

Sometimes, for use in measuring standing waves, it may be desirable to have two auxiliary line outputs on one coupler as in Figure 14; one output to monitor the direct wave on the main line, and the other at which to monitor the reflected wave.

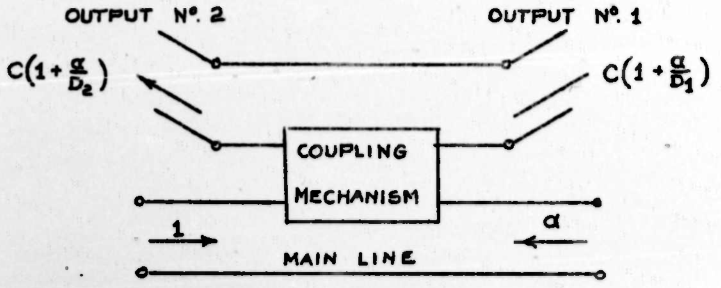


Figure 14

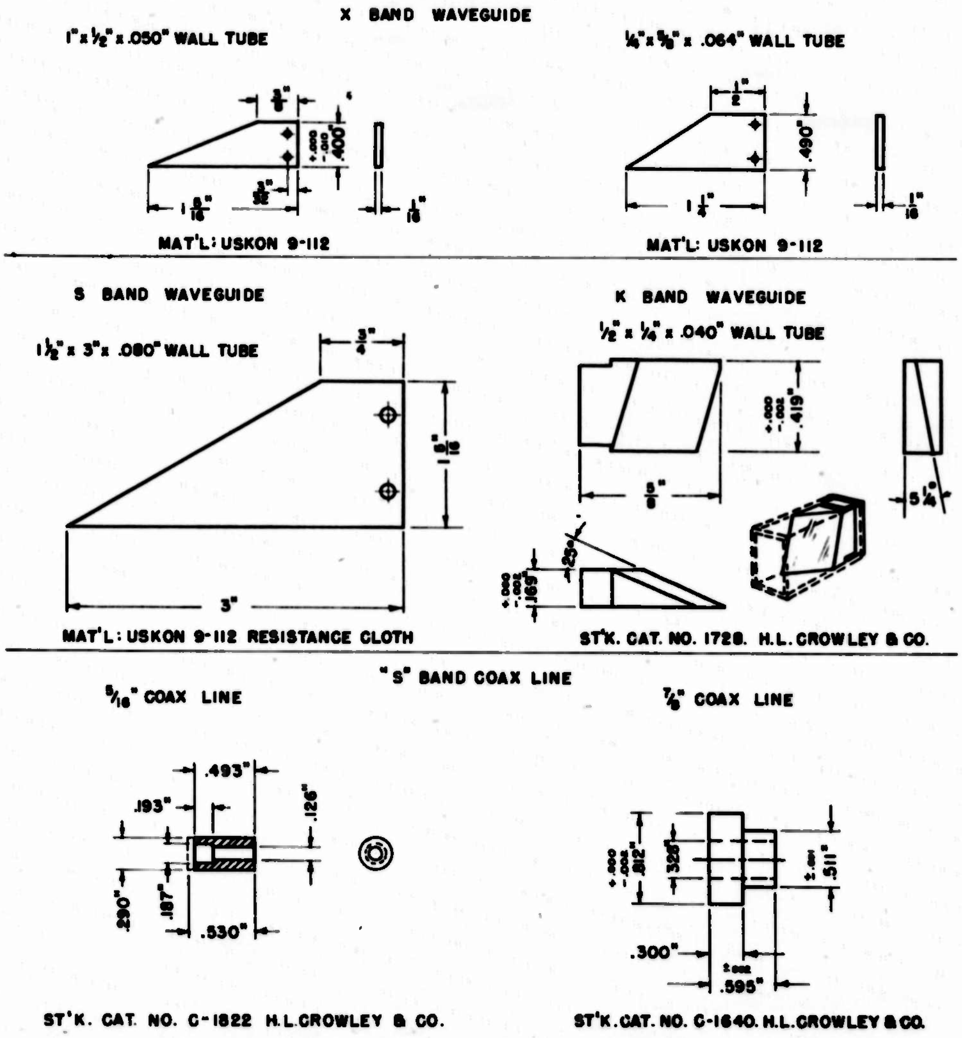


FIG. 15

TERMINATIONS USED IN DIRECTIONAL COUPLERS

This arrangement may be regarded as two different directional couplers, each with its own directivity value. The coupler which is used for direct wave monitoring has for its termination the power measurer which is connected to output (2). The power measuring device which is connected to output (1), is, in turn, the termination of the coupler used to measure the reflected wave. Thus the high directivity which is needed in the coupler for reflected wave measurements sets stringent VSWR requirements on the power measuring device used at output (1) for direct wave monitoring. This difficulty may be minimised by using a matched pad ahead of output (1), or avoided by the use of two separate directional couplers.

b. <u>Specific Termination Designs</u>

The termination to be used in a directional coupler must be chosen such that its maximum VSWR is consistent with the directivity requirements. Other considerations are the type of transmission line that must be matched, the kind of lossy material available, and mechanical and space limitations.

Figure 12 gives drawings of a number of termination designs which are generally satisfactory for directional couplers used for <u>power monitoring</u>. The procedure in designing these has been largely a matter of cut and try. The materials used in these designs; namely, USKON fabric and polyiron, are readily adapted to this method.

A taper cut out of a strip of USKON material, which consists of six layers of carbon-impregnated cloth bonded together with bakelite, is used as a termination in both X and S-band waveguides. If cut long enough, it may be made to match as well as may be desired. Those shown in the drawings are matched to a VSWR which averages less than 1.10, and may be used over their entire bands. They are mounted in the center of the waveguide and backed by a short-circuiting plate.

The tapered polyiron termination used in 1/2" x 1/4" O.D. waveguide has a VSWR below 1.05 over a ± 2% band.

The polyiron terminations shown for use with coaxial line at S-band give VSWR's of 1.10 or better. They are remarkably broadband in spite of their stepped character, and some individual ones have been made to give VSWR's of less than 1.10 from 8 to 12 cms., and less than 1.05 from 9 to 11 cms.

The characteristics of polyiron which make this broadband characteristic possible, are its high loss, with relatively low reflection. To achieve this, some consideration must be given to the choice of polyiron used (with respect to such factors as particle size and binder used). Although a specific statement cannot be given describing the optimum composition for the polyiron to be used in any particular application, generally speaking a satisfactory stepped-type termination may be made by using the H. L. Crowley material designated by MP-1826. For tapered terminations, MP-2875-D is preferable.

B. MULTIPLE PATH DIRECTIONAL COUPLERS:

1. General Properties.

The directional properties of the coupling mechanism of a multiple-path directional coupler depend upon simple interference--destructive in the back direction, and constructive in the forward direction--caused by the relative phase shift of waves which have been made to travel along different paths.

There is a close correspondence between the multiple-path type of directional coupler, and end-fire antenna arrays. Both may be considered as arrays of radiators whose spacing is equal to their time phasing, this equality being achieved by feeding the array from one end. There are various kinds of each which differ among themselves as to number, spacing, relative amplitudes, and types of radiating element.

The principle of operation of the multiple-path type coupler may best be explained in terms of the waves which are excited in the auxiliary line at the coupling elements due to a wave travelling in one direction in the main line. Each coupling element, in the sense that Huygens' principle is used in optics, is regarded as the source of secondary waves which are sent in the two directions in the auxiliary line.

The resulting amplitude of the induced wave in the auxiliary line travelling in the same direction as the exciting wave in the main line may be determined by summing, over all the coupling elements with proper regard to phasing, the amplitudes of the secondary waves sent out in this same direction. By making use of the fact that the relative time phasing of the radiation from the coupling elements is determined by the phase of the exciting wave at each element, it may be shown that all the secondary waves sent out in this direction are in phase. This result is independent of the spacing of the elements, provided that the phase velocity in the auxiliary line is the same as it is in the main line.

In similar manner, the amplitude of the induced wave in the auxiliary line travelling in the direction opposite to that of the wave in the main line may be determined. Here it can be shown that the secondary waves from the various coupling elements tend to interfere destructively, to an extent which depends upon the relative amplitudes and spacing of the elements.

Thus, we may consider the two-element multiple-path coupler, which is by definition the simplest variety of multiple-path directional couplers.

In this, the coupling mechanism consists of two separate but similar coupling elements which join together the main and auxiliary transmission lines, and which are spaced a quarter-wavelength apart from each other (wavelength measured along the transmission line), as in Figure 16 .

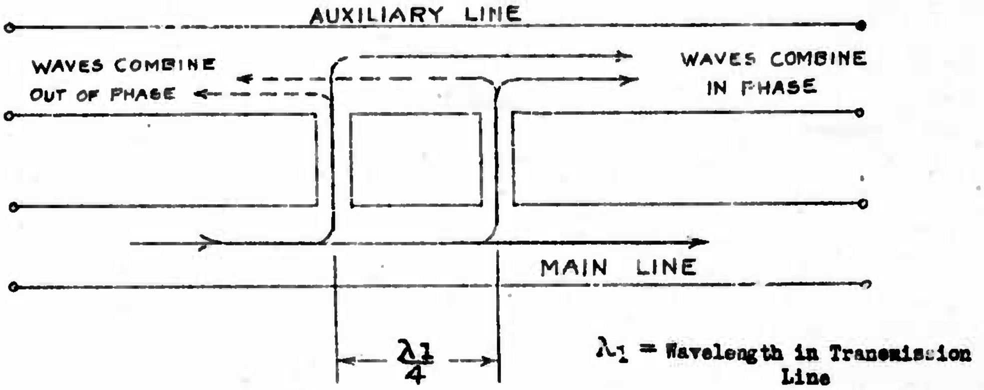


Figure 16

Consider a wave moving towards the right in the main line. The fields set up by this wave couple a small fraction of the energy into the auxiliary line, in such a way that at each of the coupling elements, secondary waves travelling in the two directions are excited. The relative amplitudes of these waves are determined by the directional properties of the coupling element.

The two secondary waves going toward the right will combine *in* phase, since the phase shift of one in travelling the quarter wavelength distance from the first element to the second element in the auxiliary line is the same as the phase shift of the exciting field in going the same distance in the main line.

When the secondary waves travelling toward the left combine, they will have experienced a relative phase shift of 180°. This is the sum of the 90° phase shift of the exciting wave in the main line in getting to the second element, plus the 90° phase shift in the auxiliary line that the secondary wave set up at this second element undergoes in getting back to the first element. If the amplitudes of these two secondary waves are equal, they will cancel.

The net result is that a wave going towards the right in the main line will only excite a wave in the auxiliary line going in the same direction.

In this discussion we have ignored effects such as the fact that the coupling elements cause a wave going by them to suffer a small phase shift, and a decrease in amplitude. The result may be that the directivity will be a maximum when the physical distance between elements is somewhat different from the nominal quarter-wavelength, with this maximum value of directivity far from infinite. The magnitude of these effects will in general decrease with looser values of coupling.

An exact analysis of multiple-path couplers, valid even for tight couplings, may be obtained by an extension of the method which has been used. Another approach involves the application of circuit theory to the situation.*

A coupling element connecting two transmission lines may be represented by an eight-terminal network. The lengths of transmission lines between elements may be represented as simple tee or π networks. Thus, the complete circuit, which reduces to a single eight-terminal network representing the directional coupling mechanism, may be drawn and analyzed by conventional circuit theory. An example of such an equivalent circuit representation will be given in the discussion of the double-stub (branched guide) variety of multiple-path couplers.

a. Frequency Sensitivity of Two-Element Multiple-Path Directional Coupler

1. Coupling: The frequency sensitivity of coupling is almost wholly dependent upon the choice of coupling elements, since the addition of waves in the forward direction only depends upon the equality of path in the primary and secondary lines, and thus does not depend on frequency.

2. Directivity: For two-element couplers in which the individual coupling elements are not themselves directional, the directivity is substantially independent of the particular design. It will be frequency sensitive, since the destructive interference of the waves excited in the wrong direction depends on the fact that the difference in path of the two waves is an integral number of half wavelengths.

For this case the directivity expressed as a voltage ratio is given by:

$$|D| = \frac{1}{\sin \frac{\pi}{2} \frac{\Delta\lambda_\ell}{\lambda_\ell}} ,$$

where λ_ℓ is the wavelength measured along the transmission line (equal to the free space wavelength (λ_o) for air-filled coaxial line propagating the principal mode and equal to λ_g for waveguide). $\Delta\lambda_1$ is equal to the difference between

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*Equivalent Circuit Analysis of Directional Couplers; B. A. Lippmann; Report 41-1/3/45.

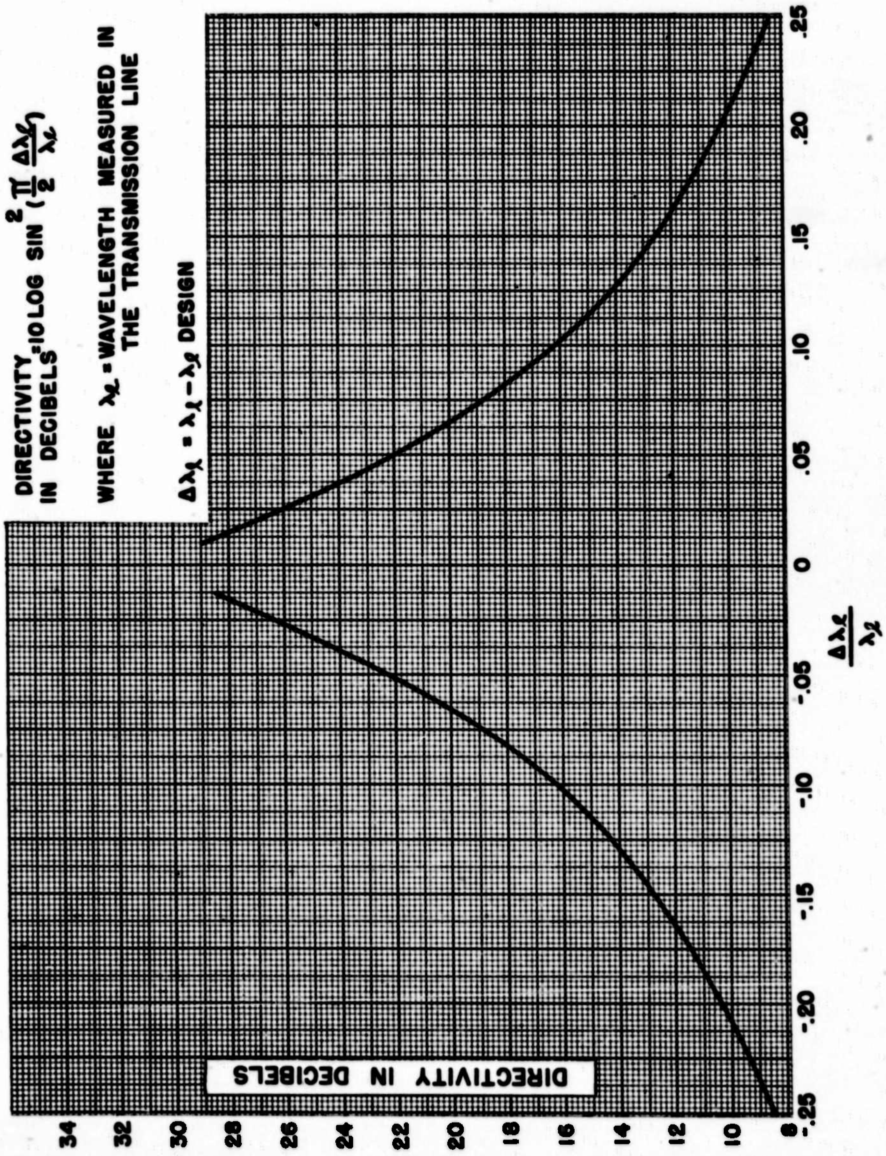

FIG. 17
DIRECTIVITY IN DECIBELS VS. BANDWIDTH FOR TWO ELEMENT MULTIPLE PATH DIRECTIONAL COUPLERS

BINOMIAL ARRAY TYPE MULTIPLE PATH COUPLER

	DEVELOPMENT	CONFIGURATION 1)		DIRECTIVITY (VOLTAGE RATIO)
2 ELEMENT		① ① 0.0 0.0	← M(db) 2)	$\frac{1}{\sin\frac{\pi}{2}\frac{\Delta\lambda_g}{\lambda_g}}$
3 ELEMENT	① ① + ① ① =	① ② ① 6.0 0.0 6.0	← M(db)	$\frac{1}{\sin^2\frac{\pi}{2}\frac{\Delta\lambda_g}{\lambda_g}}$
4 ELEMENT	① ② ① + ① ② ① =	① ③ ③ ① 12.0 2.5 2.5 12.0	← M(db)	$\frac{1}{\sin^3\frac{\pi}{2}\frac{\Delta\lambda_g}{\lambda_g}}$
5 ELEMENT	① ③ ③ ① + ① ③ ③ ① =	① ④ ⑥ ④ ① 18.1 6.0 2.5 6.0 18.1	← M(db)	$\frac{1}{\sin^4\frac{\pi}{2}\frac{\Delta\lambda_g}{\lambda_g}}$
n ELEMENT		① (n-1) $\frac{(n-1)(n-1)}{2}$ ◯ ..... (n-1) ① $20 \log 2^{n-2}$, $20 \log \frac{2^{n-2}}{n-1}$	← M(db)	$\frac{1}{\sin^{n-1}\frac{\pi}{2}\frac{\Delta\lambda_g}{\lambda_g}}$

NOTES –

1) NUMBERS INSIDE CIRCLES REPRESENT RELATIVE AMPLITUDES COUPLED AT EACH ELEMENT.

2) TO DESIGN THESE COUPLERS USING DESIGN CURVES SHOWING DIMENSION VS. COUPLING FOR TWO ELEMENT COUPLERS:

CHOOSE DIMENSION FOR EACH ELEMENT CORRESPONDING TO A VALUE OF COUPLING WHICH IS M db LOOSER THAN THE COUPLING FINALLY DESIRED FROM THE COMPLETE COUPLER.

FIG. 18

ADAPTED FROM REPORT MM-44-170-6, R.S. JULIAN. "DIRECTIONAL TRANSMISSION LINE TAPS"

UNIFORM COUPLING OVER LENGTH L, $D = \frac{\beta L}{\sin \beta L}$

N EQUAL ELEMENTS-SPACED S, $D = \frac{N \sin \beta s}{\sin N\beta s}$

N BINOMIAL ELEMENTS-SPACED S, $D = \frac{1}{\cos^{n-1} \beta s}$

$\beta = \frac{2\pi}{\lambda_\ell}$

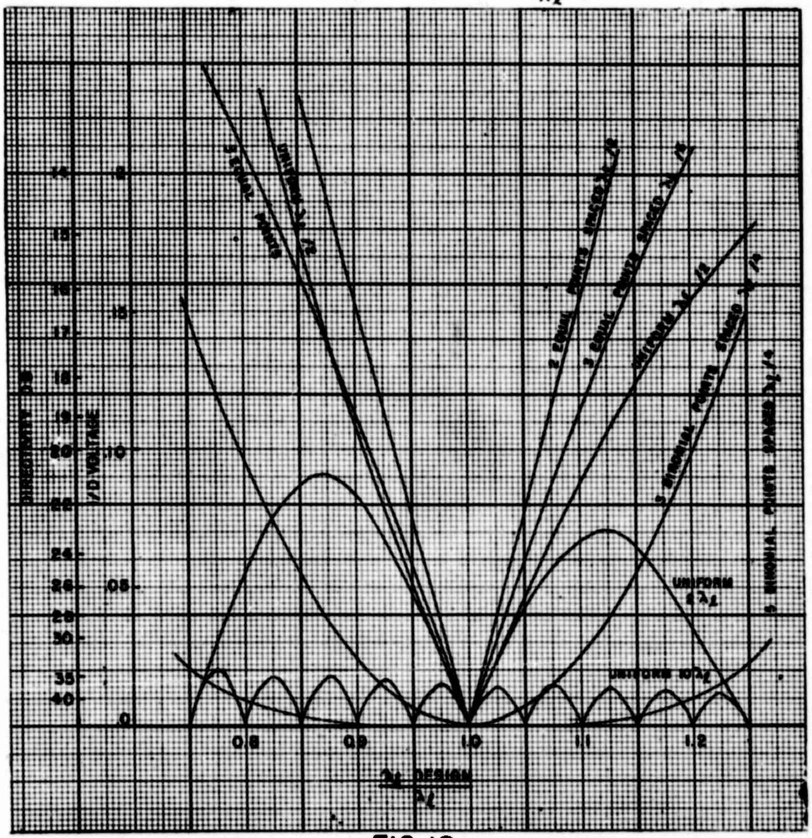

FIG. 19
FREQUENCY CHARACTERISTICS OF DIRECTIVITY
FOR VARIOUS TYPES OF MULTIPLE PATH-COUPLER

of the different types of multiple-path couplers. It shows the extent to which high directivity may be maintained over a broad frequency band. Although theoretically the directivity over a given frequency band should become greater and greater with an increase in the number of elements, since conditions in an actual coupler are not exactly theoretical for both mechanical and electrical reasons, a limitation is soon reached.

The rest of this section on multiple-path couplers will be devoted to discussing a number of different types of coupling elements which may be used, together with experimental results on couplers made using these elements.

2. Multiple-Path Directional Couplers in Waveguide.

Elements for coupling power between two waveguides may take the form of probes, loops, tees, or slots, or holes of different shape located anywhere on the guides. The choice of a particular type is governed by electrical and mechanical considerations, and by the state of one's knowledge regarding these considerations.

Theoretical results concerning the behavior of small holes used as coupling elements between two waveguides are available* and are of considerable assistance in connection with the design of directional couplers and predictions of their characteristics.

These results express the coupling in terms of the two sets of three possible field components (normal E and tangential H) that a travelling wave in each of the two guides would have at the position of the hole were the hole replaced by a conducting wall, and three constants characteristic of the geometry of the hole (one electric and two magnetic "polarizabilities").

Figure 20 shows a number of conceivable designs for single coupling elements; two parallel waveguides joined so as to have a hole in a common wall. A and C are the most satisfactory for multiple-path coupler elements. In these two cases, a single hole will have completely non-directional properties, the coupling being proportional to the longitudinal magnetic field in the guides. Since this field component is strongest at the side of the guide, the coupling of C will be looser than that of A for the same size hole, and will increase with the distance of the hole from the central position, being zero at the center (Case B).

A single hole as in D, E, or F will possess a "wrong-way" directional property,--the amplitude of the wave in the auxiliary guide excited in the direction opposite to that of the incident wave in the main guide will be less than the amplitude of the wave in the auxiliary guide excited in the same direction. This is due to the fact that there is both electric and magnetic coupling between the guides in these cases.

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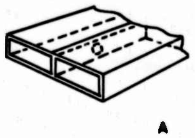
*Radiation Laboratory Reports, 43-27, 43-22 by H. A. Bethe, which deal with coupling through small holes.



A. GUIDES JOINED TOGETHER WITH HOLE IN NARROW SIDE, THE LONGITUDINAL COMPONENTS OF THE MAGNETIC FIELD IN EACH GUIDE COUPLE TO EACH OTHER.

B. NARROW SIDE OF GUIDE JOINED TO CENTER OF WIDE SIDE OF OTHER. NEITHER ELECTRIC NOR MAGNETIC COUPLING FOR HOLE IN CENTRAL POSITION.

C. NARROW SIDE OF ONE GUIDE JOINED TO WIDE SIDE OF OTHER OFF CENTER COUPLING BY MEANS OF LONGITUDINAL COMPONENTS OF MAGNETIC FIELD.

D) GUIDES JOINED TOGETHER WITH
E) HOLE IN WIDE SIDES BOTH
F) ELECTRIC AND MAGNETIC COUPLING.

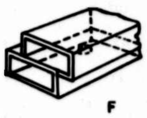


FIG. 20

POSSIBLE WAYS OF COUPLING TWO PARALLEL WAVEGUIDES USING HOLES IN THEIR COMMON WALL

a. <u>Couplers Using Small Holes in the Side of the Guide</u>.

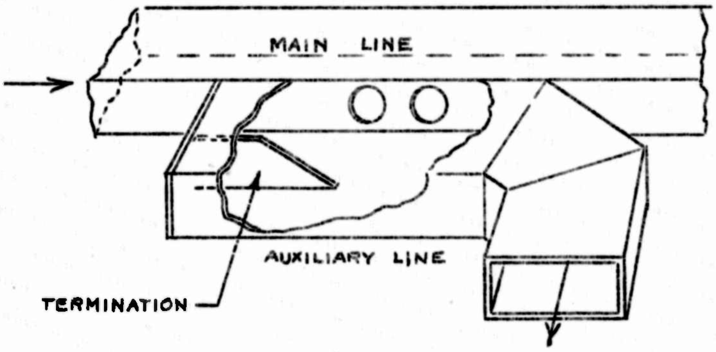


Figure 21

The coupling of a two element multiple-path coupler using round holes joining the narrow sides of two waveguides, as in Figure 21, is given by the following theoretical formula, derived on the basis of the "small hole" theory.

$$\text{Coupling (in decibels)} = 20 \log_{10} \frac{\pi d^3 \lambda g}{6a^3 b} + \text{att } H_{db},$$

where d is hole diameter, a, b, are guide dimensions, with a > b, λg is guide wavelength and att H_{db} is a term representing the cutoff attenuation introduced by the finite wall thickness, t, at the hole; given by:

$$\text{Att } H_{db} = -32.0 \frac{t}{d} \sqrt{1 - \left(\frac{1.71\, d}{\lambda_o}\right)^2} \text{ dbs.}$$

The theoretical formula given above is plotted for X-band 1" x 1/2" O.D. x .050" wall waveguide in Figure 22. In view of experimental results on a number of two-hole couplers of this type at X-band, it is advised that in using Figure 22 or the formula to design a coupler, a hole diameter should be chosen corresponding to a theoretical coupling which is .4 db closer than the desired coupling.

Experimentally, it is found that the directivity is best when the actual spacing is somewhat less than the nominal quarter guide wavelength. For a coupler in 1" x 1/2" waveguide having a coupling of -20 db, the optimum spacing was determined to be .973 λg/4. For other values of coupling, we have assumed that the departure of optimum spacing from λg/4 is proportional to the voltage coupling.

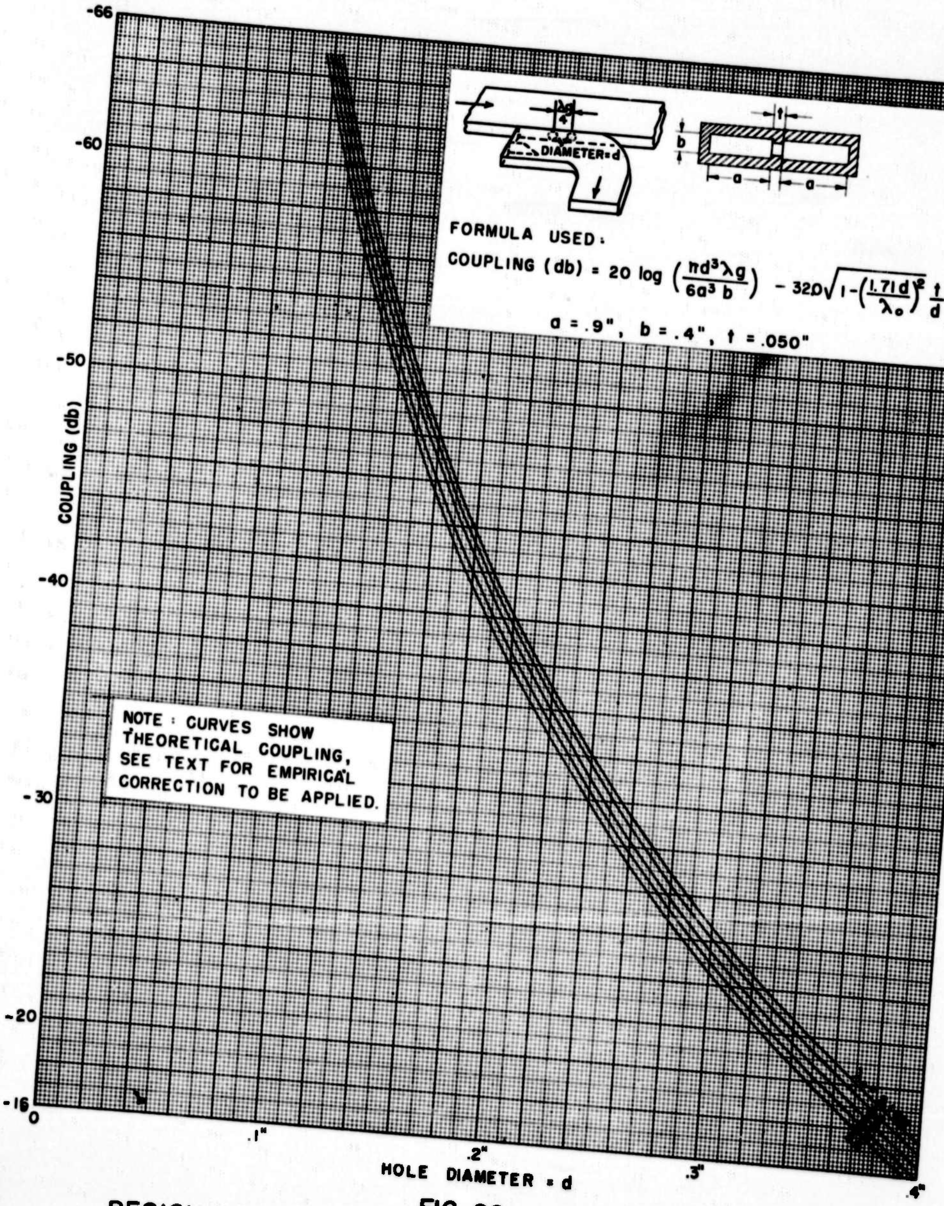


FIG. 22
DESIGN CURVES FOR TWO ELEMENT COUPLERS USING ROUND HOLES IN SIDE OF 1"X 1/2"X .050" WALL WAVEGUIDE

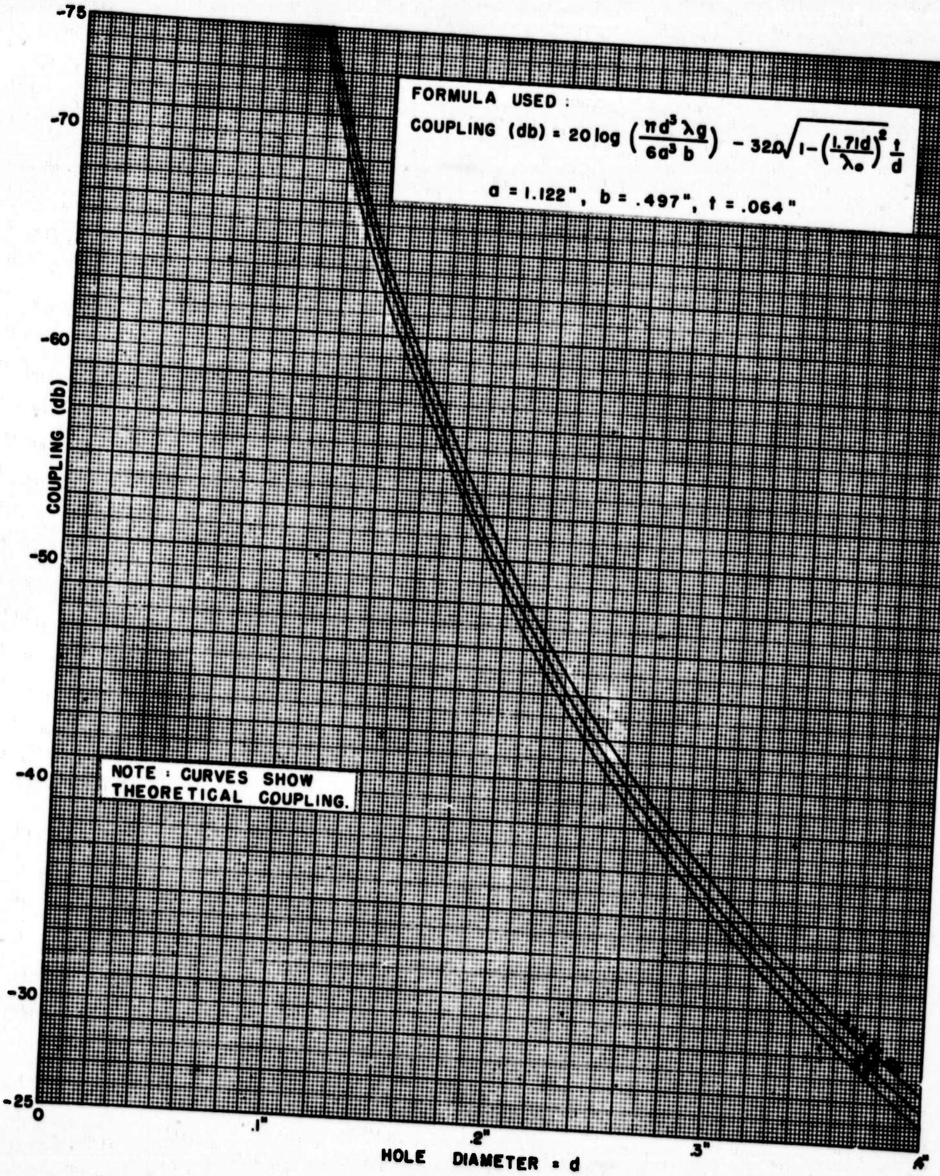


FIG. 23

DESIGN CURVES FOR TWO ELEMENT COUPLERS USING ROUND HOLES IN SIDE OF 1¼" X ⅝" X .064" WALL WAVEGUIDE

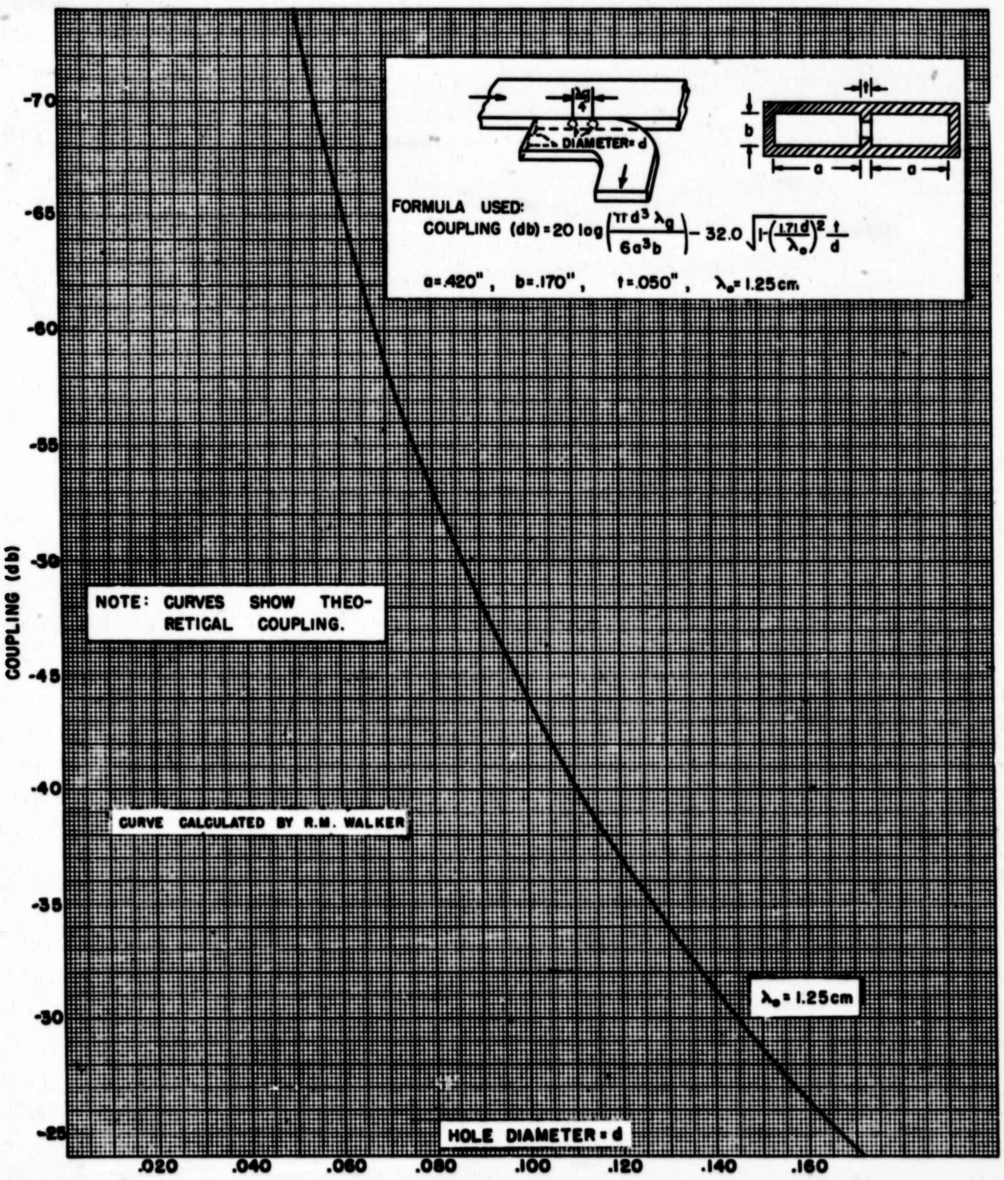
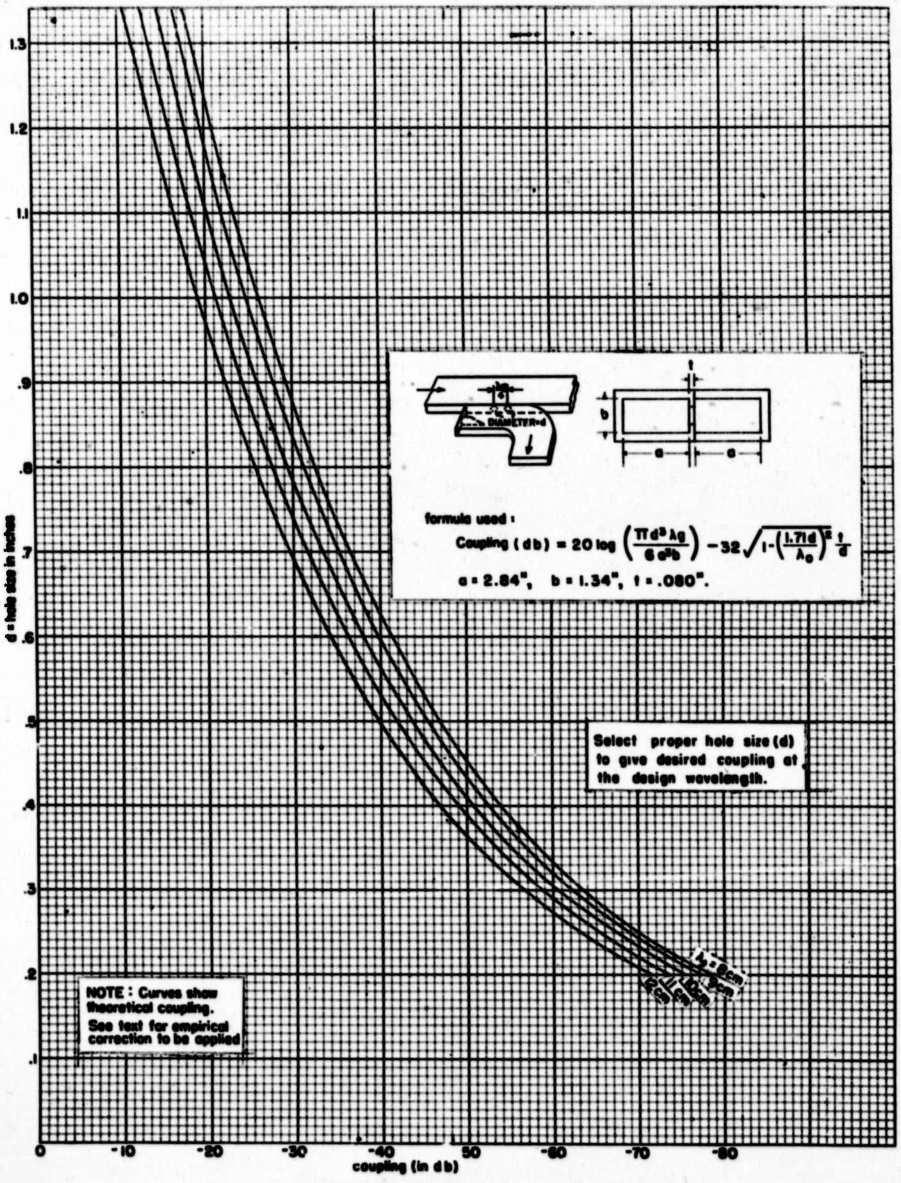


FIG. 24

DESIGN CURVES FOR TWO ELEMENT COUPLERS USING ROUND HOLES IN SIDE OF ½" X ¼" X .040 WALL WAVEGUIDE



formula used :

$$\text{Coupling (db)} = 20 \log \left(\frac{\pi d^3 \lambda g}{6 a^2 b}\right) - 32 \sqrt{1 - \left(\frac{1.71 d}{\lambda_0}\right)^2} \frac{t}{d}$$

$a = 2.84"$, $b = 1.34"$, $t = .080"$.

Select proper hole size (d) to give desired coupling at the design wavelength.

NOTE : Curves show theoretical coupling. See text for empirical correction to be applied.

FIGURE 25

DESIGN CURVES FOR TWO ELEMENT COUPLERS USING ROUND HOLES IN SIDE OF 3" X 1-1/2" X .080" WALL WAVE GUIDE

The tightness of coupling that may be obtained using two round holes is limited by the obvious fact that the diameter of the holes must be both less than the height of the guide and less than a quarter guide wavelength. In 1" x 1/2" guide, the former limitation gives about -16 db as the tightest coupling which can be obtained. In 1 1/4" x 5/8" guide, the shorter guide wavelength limits one to a coupling of -28 db, and in 1/2" x 1/4" guide to approximately -30 db. Figures 23, 24, and 25 show theoretical curves plotted for other guide sizes and wavelengths. Couplers made in 1 1/2" x 3" guide had measured couplings .8 db looser than theoretical. In the 1/2" x 1/4" guide the difference was within experimental error.

It must be remembered that all these give the theoretical coupling for a two-hole coupler. To design other types of multiple-path couplers, hole dimensions corresponding to appropriate coupling values must be used.

Any shape of hole joining the narrow sides of two waveguides possesses characteristics which are the same as for a round hole, as long as the dimensions are small enough so that there are no near resonances (the near resonant case will be discussed in a following section). For example, the coupling of a two-element coupler using small rectangular slots in the side of the guide is given by the formula:

$$\text{Coupling}_{db} = 20 \log \frac{\pi l w^2 \lambda g}{16\, a^3 b} + \text{att } H_{db}$$

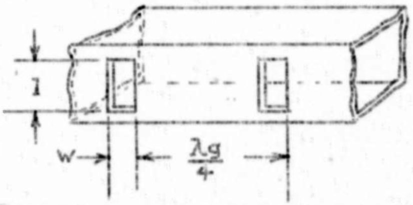


Figure 26.

where a, b, with a > b are the inside guide dimensions, w is the width of the slot, l is the slot length, t is the wall thickness, and att H_{db} for this case is given by the formula:

$$\text{Att } H_{db} = -27.3 \frac{t}{w} \sqrt{1 - \left(\frac{2w}{\lambda_o}\right)^2} \text{ dbs.}$$

Group 55 has not designed couplers using these rectangular holes, because of the difficulty in construction. Edwards* reports

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* C. F. Edwards; Measurements on Directional Couplers in 1/2 x 1 inch Waveguide; BTL-588 (MM-44-160-111).

measurements on rectangular slots cut the entire height of the waveguide. Another construction used by BTL approximating rectangular shaped slots involves the use of wires as partitions in a cutaway section of wall between the two waveguides.

Frequency Sensitivity of Coupling: The theoretical formulae for the coupling of both round and rectangular holes in the side of the guide, show that the coupling expressed as a power ratio is proportional to the square of the guide wavelength for thin walled guides, and is only modified very slightly by the frequency sensitivity of the outoff attenuation term.* Figure 27 shows the theoretical variation of coupling in db with wavelength of two-hole couplers in X-band 1"x 1/2"x .050" wall waveguide. Experimentally, it was found that the variation in coupling over the 12% band for a 20 db coupler was ± .7 db. For the wider 1 1/4"x 5/8"x .064" guide, the theoretical variation is about 25% less.

An approximate expression for the theoretical frequency sensitivity of coupling is:

$$\Delta \text{ coupling}_{db} = 8.7 \left(\frac{\lambda g}{\lambda_o}\right)^2 \frac{\Delta\lambda_o}{\lambda_o} ,$$

$$\Delta\lambda_o = \lambda_o - \lambda_o \quad \text{(at design wavelength)}$$

with $\left(\frac{\lambda g}{\lambda_o}\right)^2$ $\sim$ 2 for 1" x 1/2" guide.
$\sim$ 1.5 for 1 1/4" x 5/8" guide.
$\sim$ 1.5 for 1/2" x 1/4" guide.

Frequency Sensitivity of Directivity: The theoretical directivity curve for loose coupling two-element couplers, in X-band 1"x 1/2" O.D. guide, peaked at λ_o = 3.2 cm, and λ_o = 3.33 cm respectively, are shown in Figure 28. This is to be compared with an experimentally determined directivity curve, also plotted.

The theoretical directivity curves for a "binomial" array of n holes may be obtained from the curves for two holes by multiplying the directivity figures in db by (n-1).

Experimental curves for three and four hole couplers have shown increased directivity at the band edges, although the curve of directivity versus λ has usually shown "humps" not predicted by theory.

The tolerances which must be held in order to maintain the coupling at the desired figure, and to keep the directivity sufficiently high, may be estimated theoretically.

Tolerances for Coupling: The theoretical coupling formula shows to what extent the coupling coefficient is affected, not only

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*Since the frequency sensitivity of the cutoff attenuation term opposes the λg^2 term, with the use of a large hole (near cutoff), and a very thick wall, overall frequency insensitivity may be achieved. However, the couplings obtainable with this method are of the order of -60 db.

FORMULA USED: COUPLING IN db. $= 10 \log\left(\frac{\pi d^3 \lambda_g}{6a^3 b}\right) + \text{ATT. } H_{db}$

WHERE ATT. $H_{db} = -32\sqrt{1-\left(\frac{1.71d}{\lambda_o}\right)^2}\,\frac{t}{d}$
HOLE DIAMETER $= d$
WALL THICKNESS AT HOLE $= t = .050"$
INSIDE GUIDE DIMENSIONS: $a = .900"$; $b = .400"$

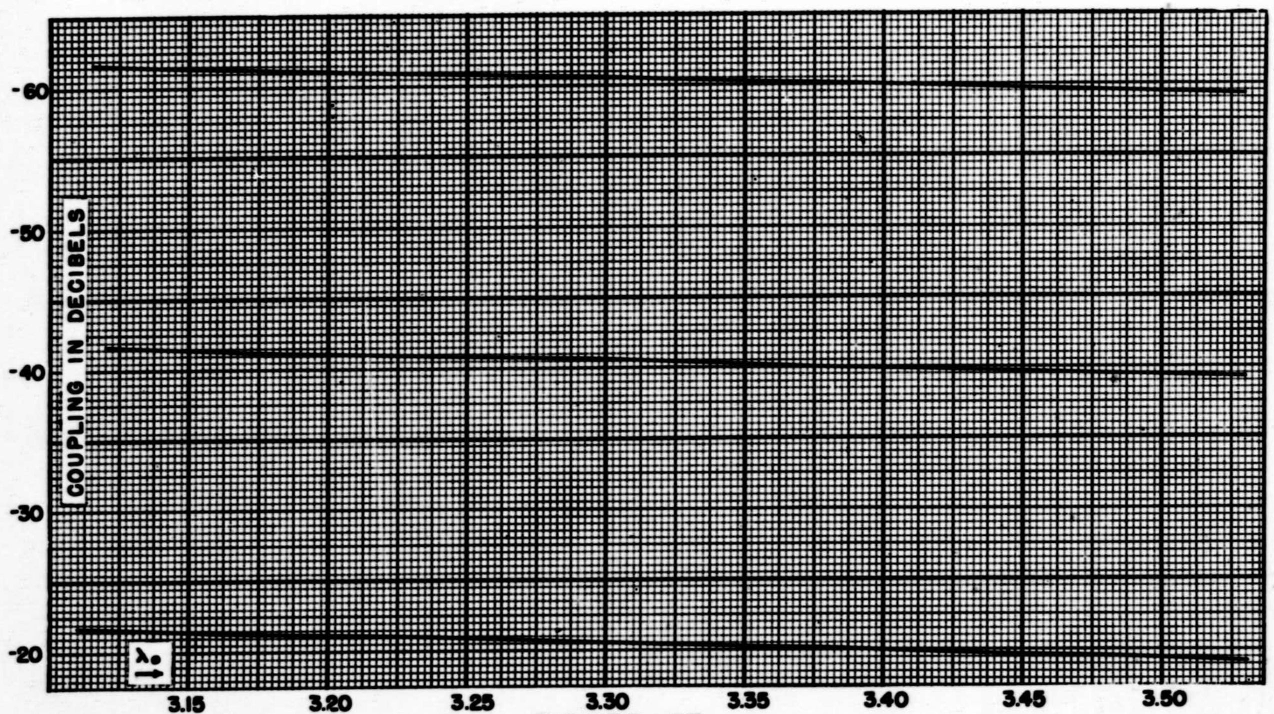


FIGURE 27
THEORETICAL COUPLING VS. WAVELENGTH FOR
TWO HOLE IN SIDE OF GUIDE DIRECTIONAL COUPLER

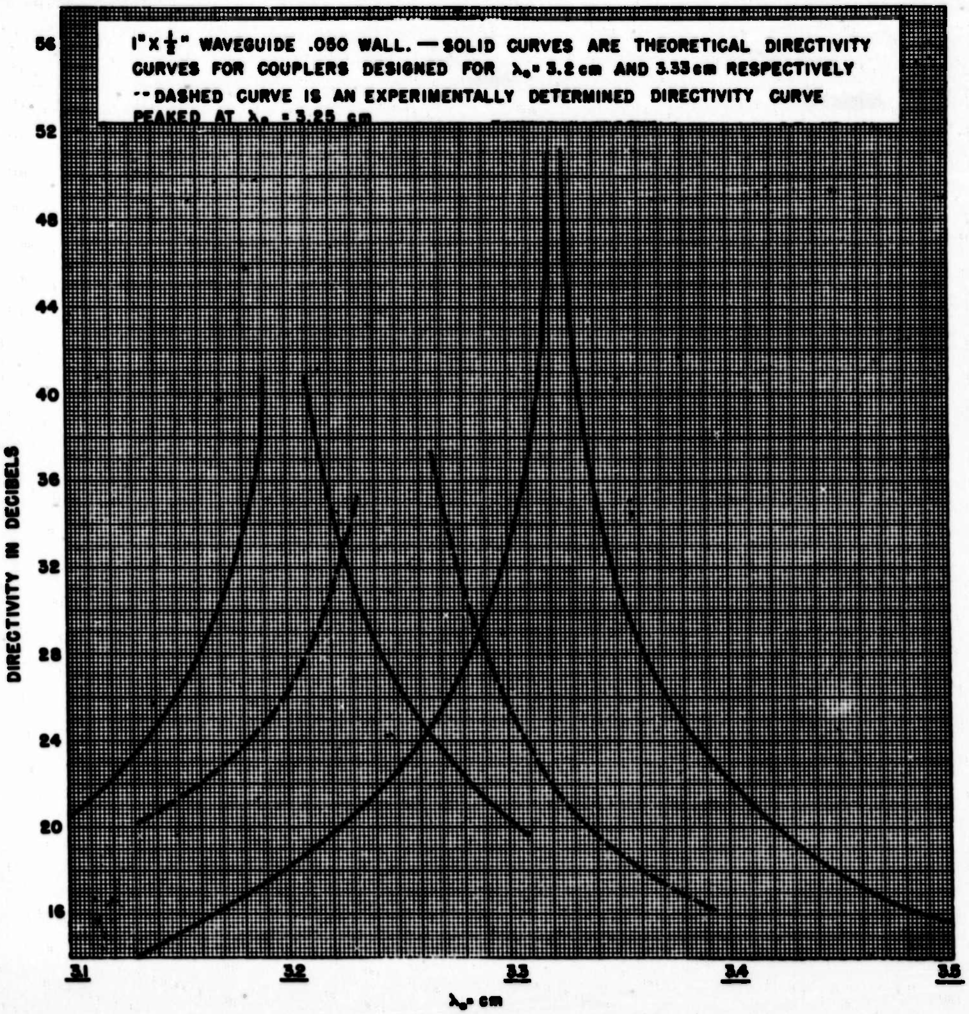


FIG. 28
DIRECTIVITY VS. λo FOR TWO
HOLE IN SIDE OF GUIDE DIRECTIONAL COUPLERS

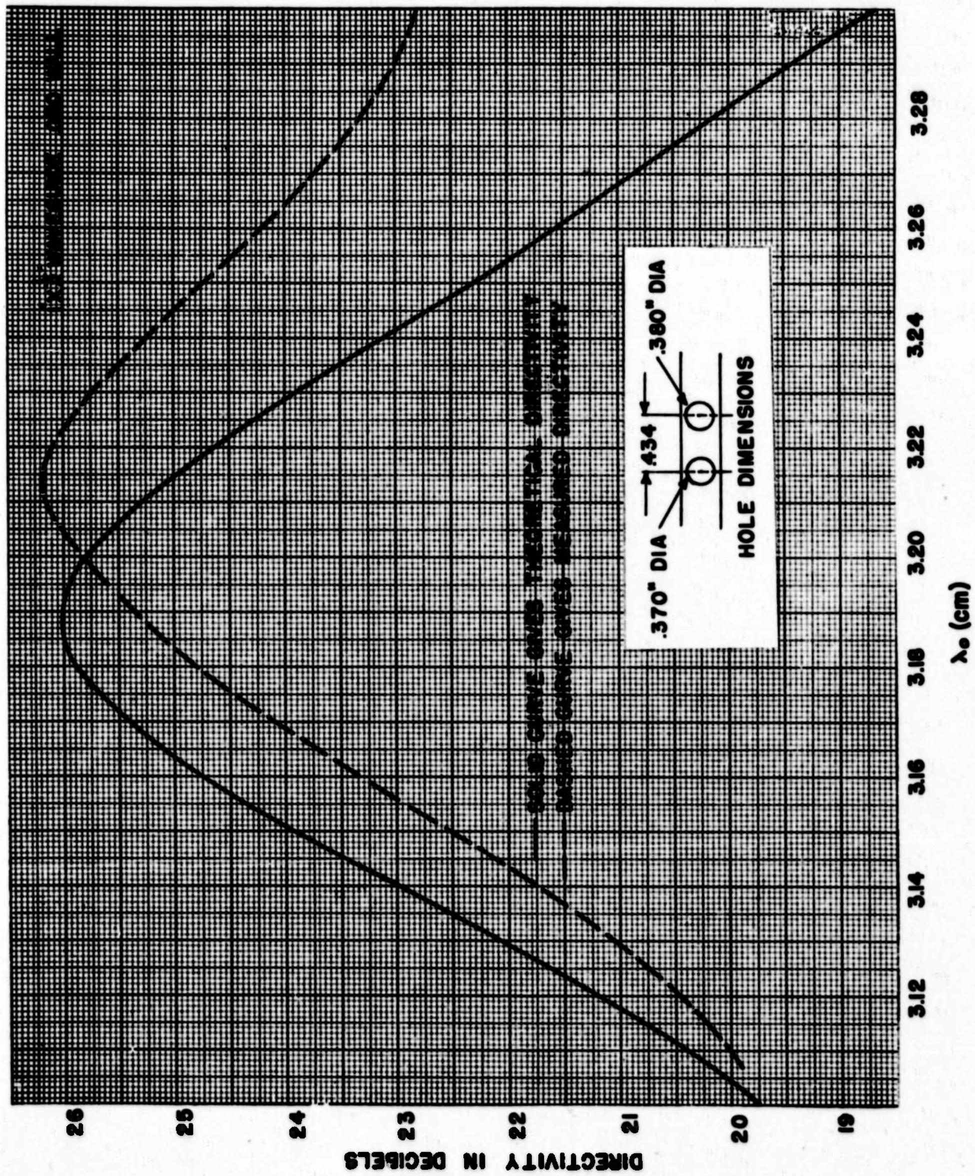


FIG. 29
EFFECT UPON DIRECTIVITY OF HAVING UNEQUAL HOLE SIZES IN TWO HOLE COUPLER

by the hole dimensions, but by the waveguide inner dimensions and wall thickness as well. The effect of a change in hole diameter upon coupling may also be determined from the graphs of coupling versus hole size. The "a" dimension (wide side) of the waveguide is an important one in this connection. Not only does it enter directly into the formula raised to the third power, but it also enters into the λg term.

A formula for the change in decibels coupling due to a change in any or all dimensions; namely, hole diameter d, guide dimensions a and b, and wall thickness t, is:

$$\Delta C_{db} \cong 26\left(1+1.1\frac{t}{d}\right)\frac{\Delta d}{d} - 8.7\left[3+\left(\frac{\lambda g}{2a}\right)^2\right]\frac{\Delta a}{a} - 8.7\frac{\Delta b}{b} - \frac{30t}{d}\frac{\Delta t}{t},$$

Thus, the coupling would be changed .3 db by a 1% change in a, and .1 db by a 1% change in b.

Tolerances for Directivity: The effect of an incorrect separation of the holes is to peak the directivity at other than the design wavelength, and so the broadness of the directivity curve will show to what extent the directivity will be reduced at the band edges. The guide width will also have an effect upon the directivity because of the relationship between it and guide wavelength.

An experimental curve of directivity versus λ shown in Figure 29 was obtained upon a coupler made with one hole .010" larger than the other. This agrees favorably with a calculated curve for this case.

b. Use of Near-Resonant Slots in Side of Guide.

In order to get closer couplings than are attainable with round holes, slots of the form shown in Figure 30 were tried.

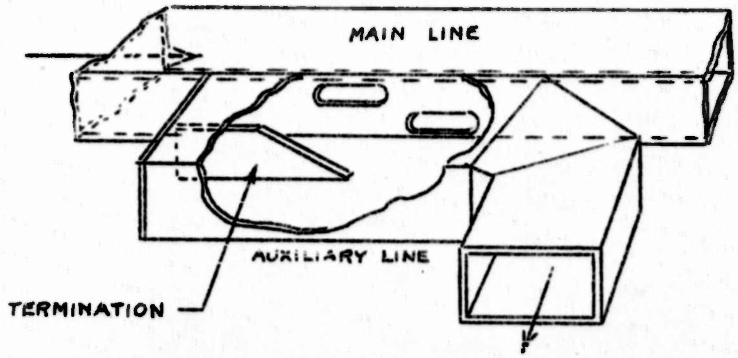
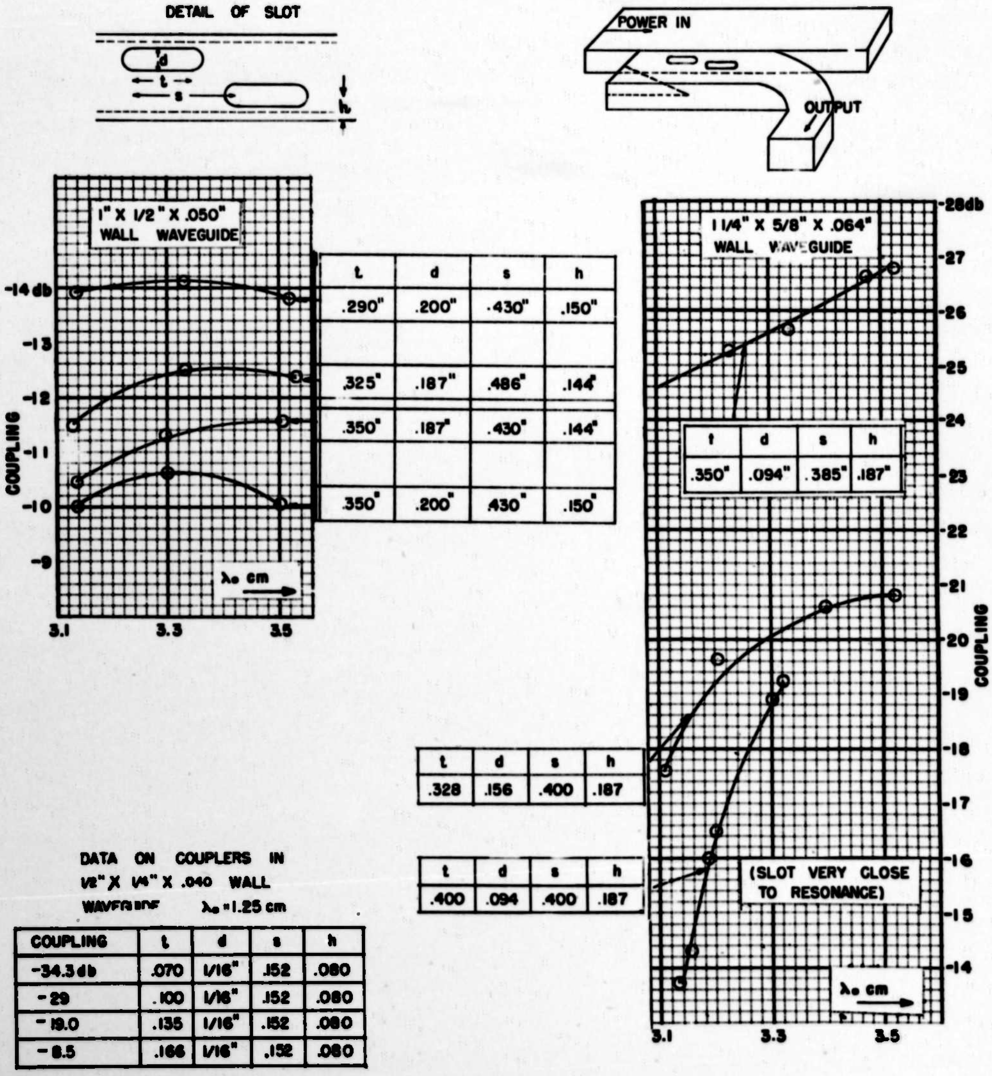


Figure 30



t	d	s	h
.290"	.200"	.430"	.150"
.325"	.187"	.486"	.144"
.350"	.187"	.430"	.144"
.350"	.200"	.430"	.150"

t	d	s	h
.350"	.094"	.385"	.187"

t	d	s	h
.328	.156	.400	.187

t	d	s	h
.400	.094	.400	.187

DATA ON COUPLERS IN 1/2" X 1/4" X .040 WALL WAVEGUIDE λ_o = 1.25 cm

COUPLING	t	d	s	h
-34.3 db	.070	1/16"	.152	.080
-29	.100	1/16"	.152	.080
-19.0	.135	1/16"	.152	.080
-8.5	.166	1/16"	.152	.080

FIG. 31

EXPERIMENTAL RESULTS ON TWO ELEMENT COUPLERS USING NEAR-RESONANT SLOTS IN THE SIDE OF GUIDE.

The length of the slots was such that a resonance effect was noticed. The coupling increases rapidly as the resonant length is approached. This effect tends to give the slot a frequency sensitivity of coupling which is opposite to the normal behavior of a small hole in the side of the waveguide.

This feature makes it possible to design a coupler which is broadband in coupling, by proper choice of slot geometry.

Some experimental results for this type coupler are shown in Figure 31 for both 1" x 1/2" and 1 1/4" x 5/8" guide. For a given slot width, a length can be chosen so that at a particular frequency, the small hole effect and the resonance effect are just equal and opposite. When the slot is somewhat shorter, the small hole effect is dominant; when the slot is longer, the resonance effect is dominant.

This technique of broadbanding coupling has also been applied to the "Schwinger" type of reversed phase coupler to be discussed later.

c. Branched Guide (Double Stub) Directional Coupler

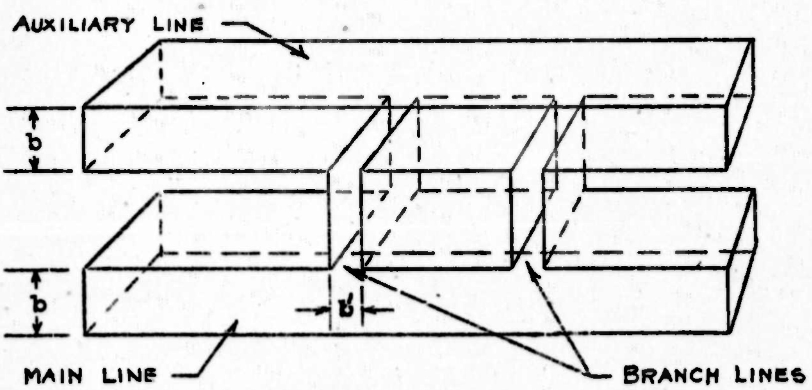


Figure 32

The branched guide directional coupler, suggested by Purcell, is a multiple-path coupler which utilizes low impedance ("series") E-plane branches as coupling elements between the main and auxiliary guide. (Figure 32.) This type coupler is characterized by extreme frequency insensitivity of coupling, and by the fact that very close couplings may be obtained without setting up unduly high standing-wave-ratios in the main line.

The "stub" arms or branch guides, of the E-plane tees are waveguides having the same wide dimension as the main waveguide, but which have a smaller narrow dimension so chosen to give the desired coupling. The lengths of the branch guides are chosen, so that with corrections for edge effects taken into account, they are equivalent to an odd number of quarter guide wavelengths. By this choice, the reflections set up by the tees are minimized, and the least frequency sensitivity of coupling is achieved.

The branched guide coupler has been rather completely analysed by a method based on equivalent circuit concepts.* In the circuit, Figure 33, the E-plane tee is represented essentially as a series connection between the low impedance branch guide and the higher impedance main waveguide. The various lengths of transmission line are represented by appropriate tee networks.

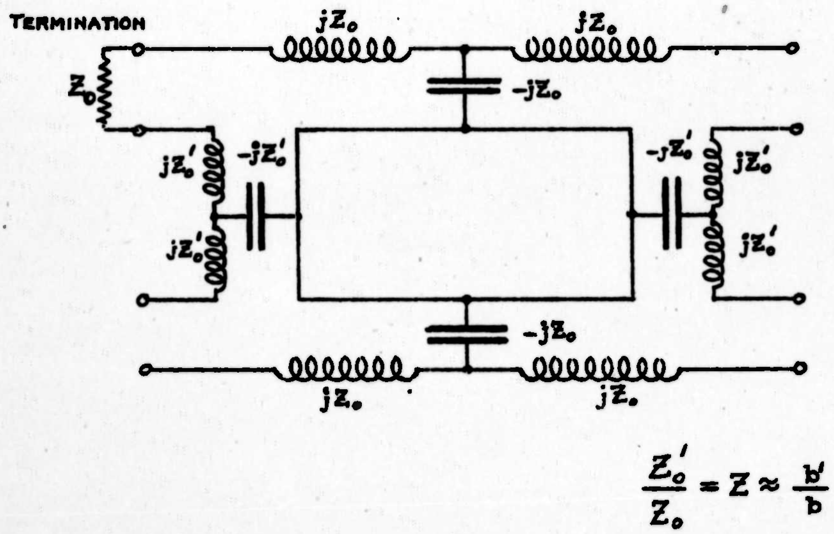


Figure 33

---

[*] Equivalent Circuit Analysis of Directional Couplers; B. A. Lippmann, Internal Group Report No. 41-1/3/45.

Design Information: The edge effects associated with the tee junction require that the actual length of the branch guides must be made somewhat less than $\lambda g/4$ or $3\lambda g/4$, and the actual spacing between branches somewhat greater than $\lambda g/4$. Lippmann has calculated these corrections for K-band using Schwinger's results for the E-plane tee. The design information given in Figures 34 and 35 embodies these corrections.

In the corresponding design curves for X and S-band waveguides more approximate corrections have been used. In particular, the approximate formula for the length of the branch guides is:

$$L = \frac{\lambda g}{4} - \left[\frac{2b}{\pi}\left(1 + \ln\frac{b}{2b'}\right)\right]$$

Coupling: If the lengths of the branch guides and the distance between them is chosen correctly, the coupling is given by the formula:

$$\text{Coupling}_{db} = 10 \log \frac{Z^2}{1 + Z^4 + Z^6} \simeq 10 \log Z^2,$$

where Z is the ratio of the characteristic impedance of the branch guide to that of the main guide corrected for end effects. For $b'/b \ll 1$, $Z = b'/b$, where b' and b represent the heights of the branch guide and the main guide respectively. With this approximation,

$$\text{Coupling}_{db} \approx 10 \log\left(\frac{b'}{b}\right)^2 .$$

Figures 34 - 39 are design curves giving coupler dimensions, various guide sizes, and frequency bands. Except for 1/2" x 1/4" guide, where Lippmann's results have been used, Z has been assumed to be given by $Z = n\, b'/b$, where n varies linearly with b'/b, taking on a value of unity for $b'/b = 0$, and a limiting value for $b'/b = 1$ given the British Report (Moe 2681) dealing with waveguide tees.

Frequency Sensitivity of Coupling: The equivalent circuit analysis predicts that the coupling is rather frequency insensitive in the vicinity of the design wavelength (coupling becomes tighter off design frequency). Experimental verification of this for X-band couplers in 1" x 1/2" guide,--the coupling varying only 1/4 db in a ± 6% X-band) is shown in Figure 40.

Sensitivity of Directivity: The frequency dependence of the directivity of this type coupler is similar to that of other types of two-element couplers. However, the exact circuit analysis which has been carried out for this coupler is of interest in that it predicts values of the directivity which are valid even for couplers having relatively close couplings. Thus, the extent to which the approximate analysis, assuming loose couplings breaks down, becomes clearly evident.

It turns out that the directivity never quite becomes infinite at the design wavelength, even theoretically, and is decreased to some extent depending upon the coupling coefficient, at other wavelengths. Figure 41 shows this more clearly. At design wavelength the directivity is

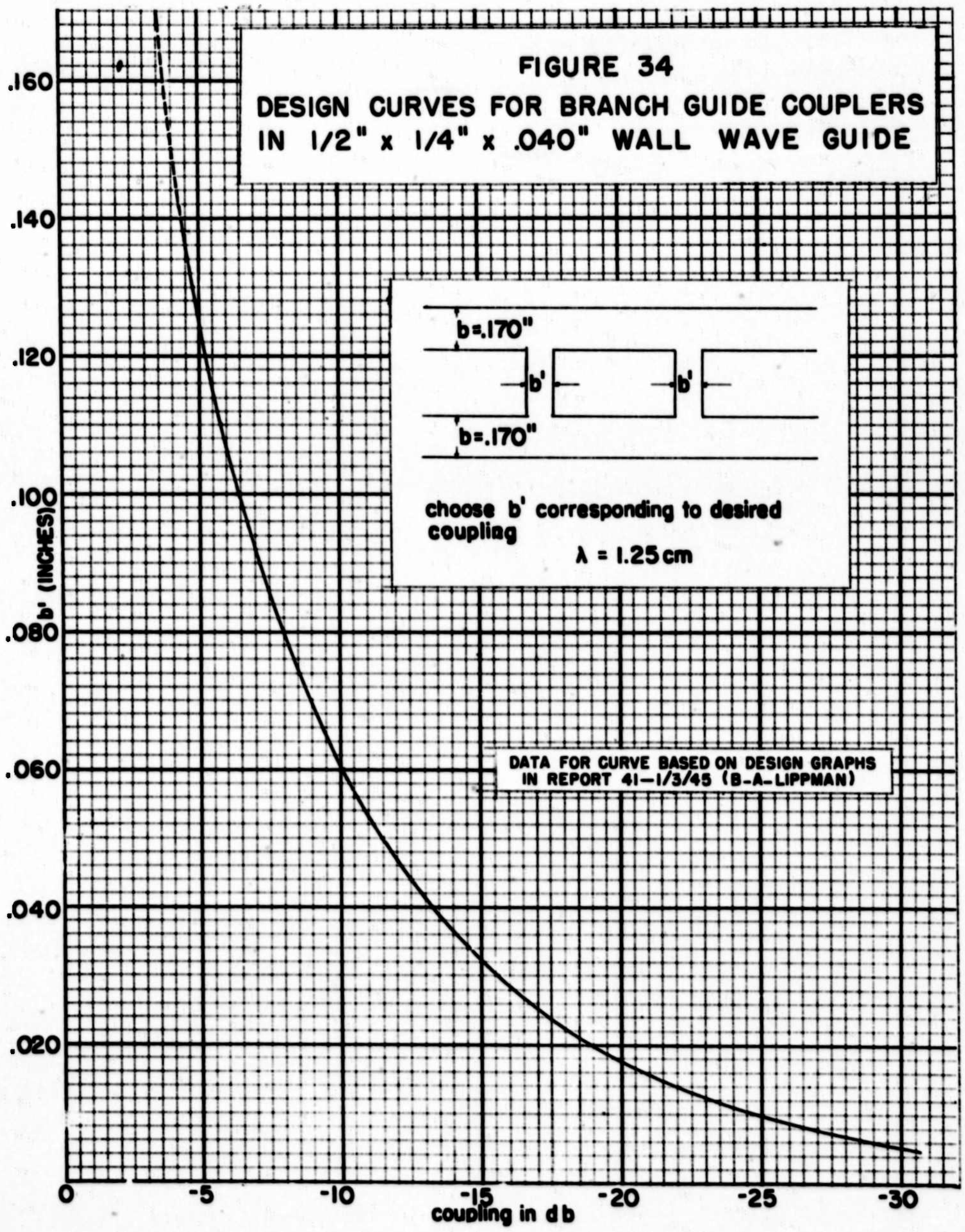


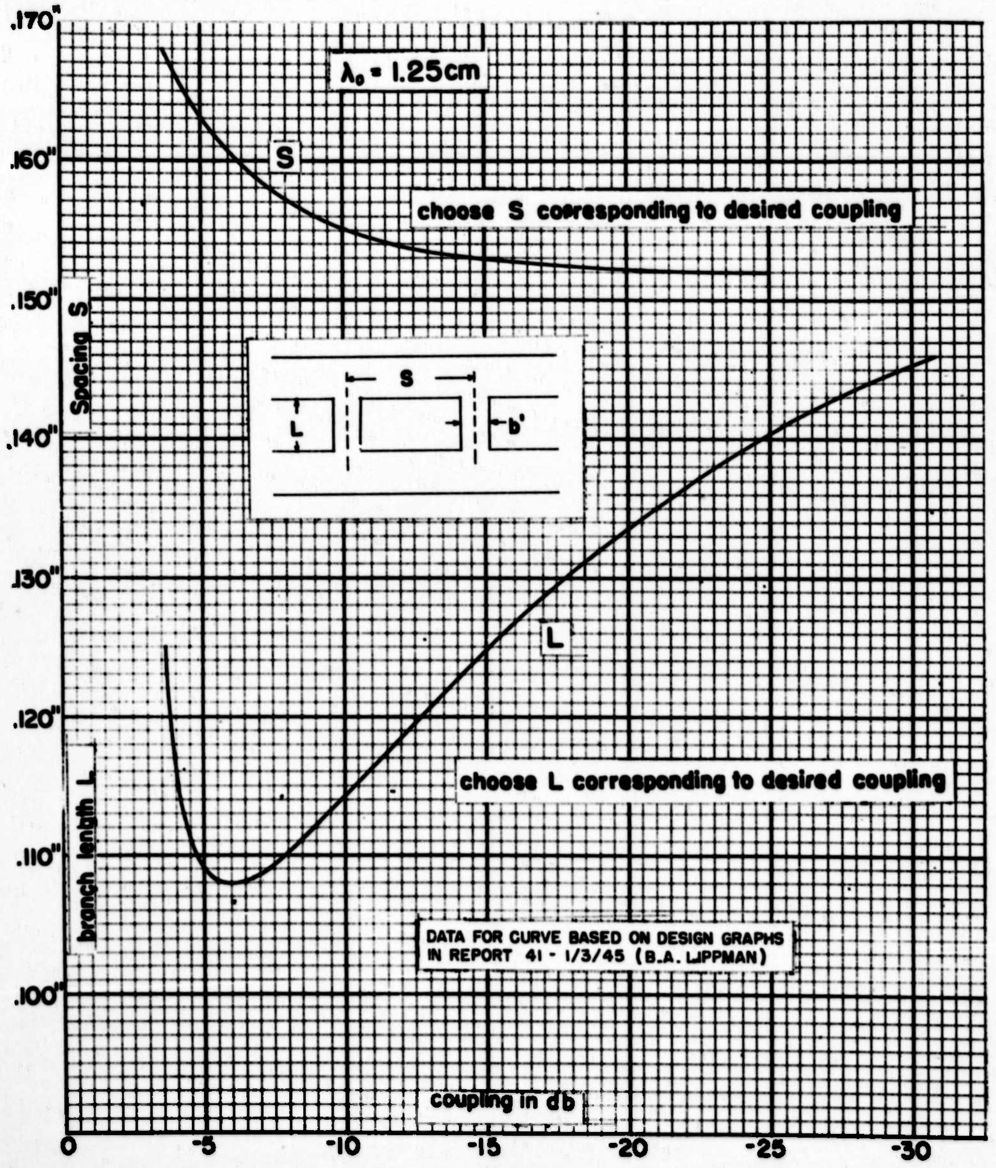
FIGURE 34

DESIGN CURVES FOR BRANCH GUIDE COUPLERS IN 1/2" x 1/4" x .040" WALL WAVE GUIDE

choose b' corresponding to desired coupling

λ = 1.25 cm

DATA FOR CURVE BASED ON DESIGN GRAPHS IN REPORT 41–1/3/45 (B-A-LIPPMAN)



choose S corresponding to desired coupling

choose L corresponding to desired coupling

DATA FOR CURVE BASED ON DESIGN GRAPHS IN REPORT 41 - 1/3/45 (B.A. LIPPMAN)

FIG. 35 DESIGN CURVES FOR BRANCH GUIDE COUPLERS IN $\frac{1}{2}$ X $\frac{1}{4}$ X .040" WALL WAVEGUIDE

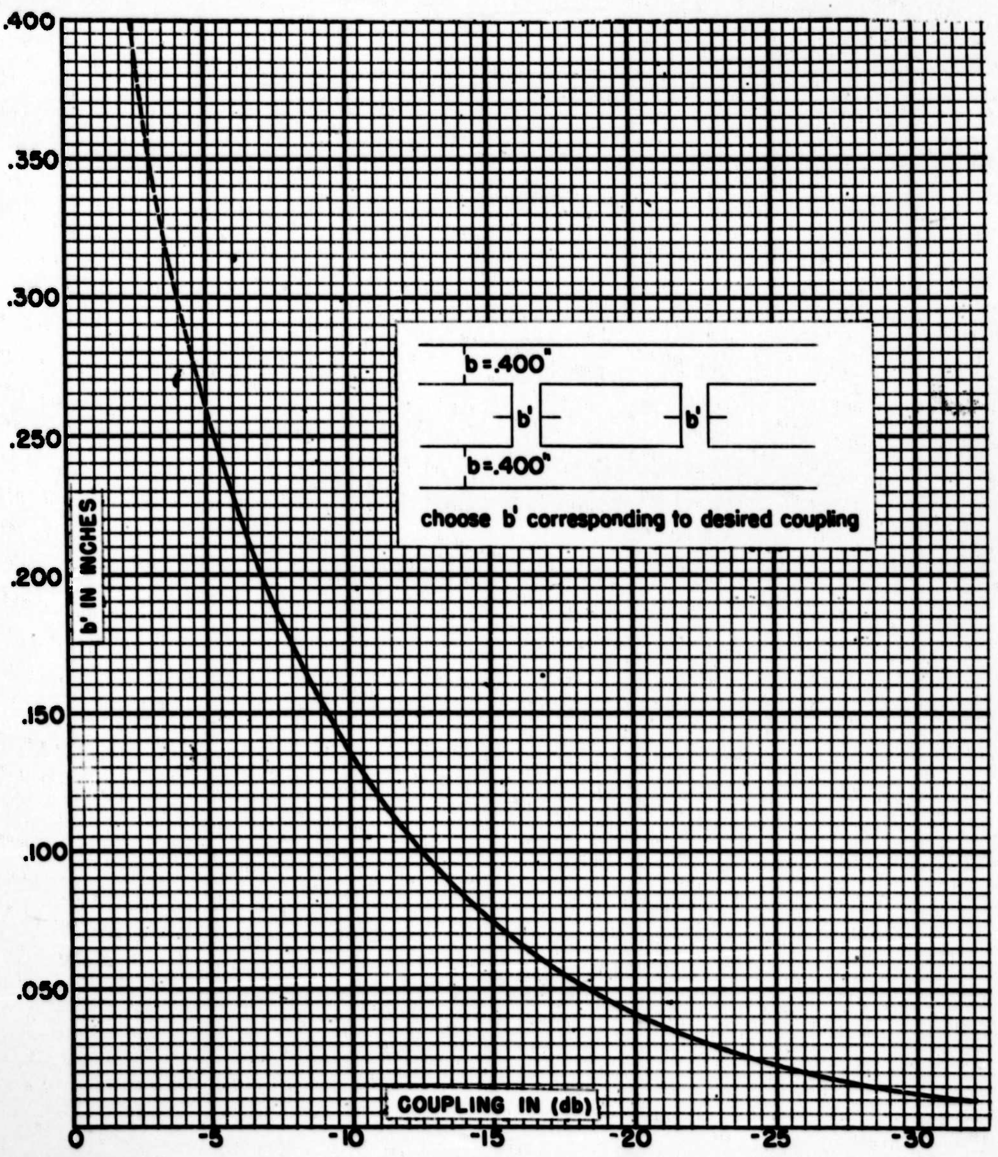
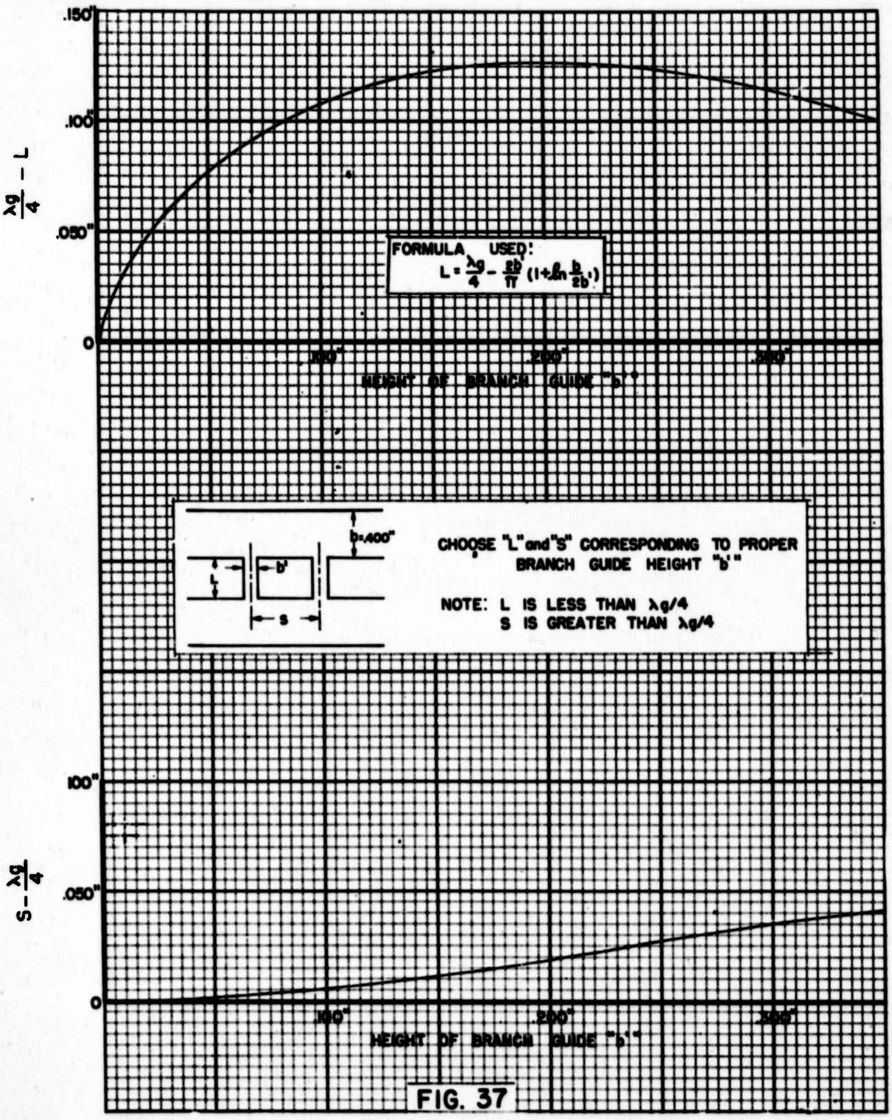


FIG.36 DESIGN CURVE FOR BRANCHED GUIDE COUPLER IN 1" x 1/2" x.050" WALL WAVEGUIDE



FORMULA USED:

$$L = \frac{\lambda g}{4} - \frac{2b'}{\pi}\left(1 + \ln \frac{b}{2b'}\right)$$

CHOOSE "L" and "S" CORRESPONDING TO PROPER BRANCH GUIDE HEIGHT "b'"

NOTE: L IS LESS THAN $\lambda g/4$
S IS GREATER THAN $\lambda g/4$

FIG. 37

DESIGN CURVES FOR BRANCHED GUIDE COUPLERS IN IN 1" X $\frac{1}{2}$" X .050" WALL WAVEGUIDE

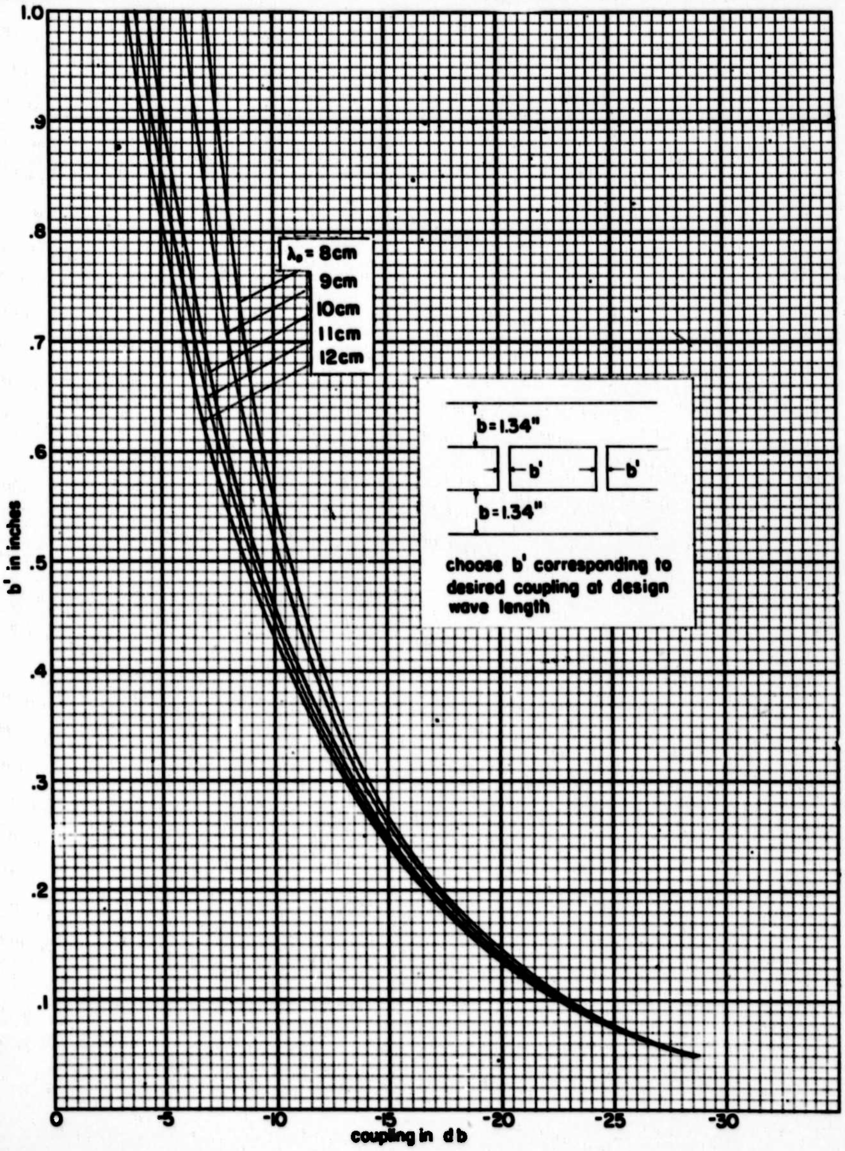
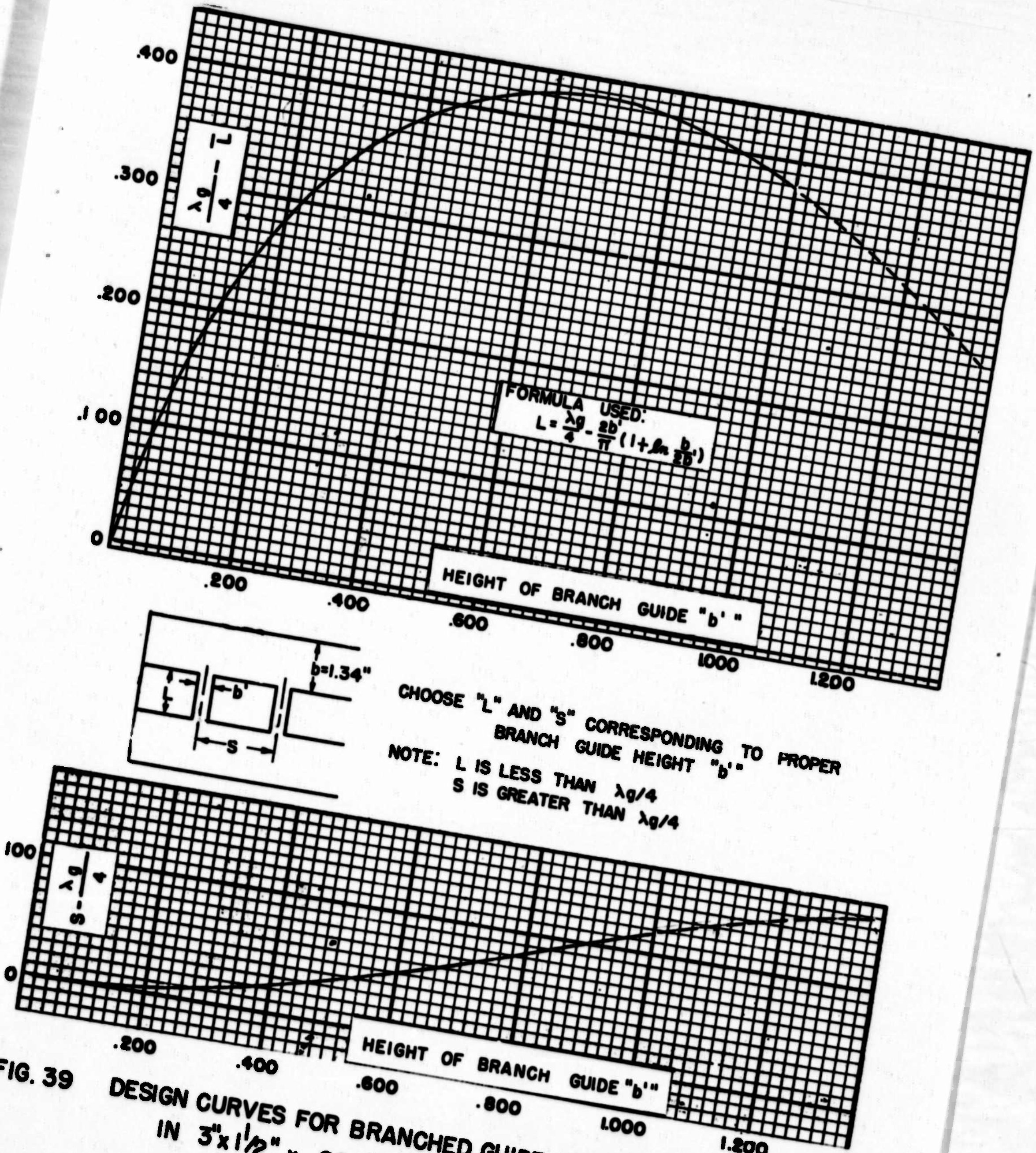


FIG.38

DESIGN CURVES FOR BRANCHED GUIDE COUPLERS IN 3" X 1½" X .080" WALL WAVEGUIDE



CHOOSE "L" AND "S" CORRESPONDING TO PROPER BRANCH GUIDE HEIGHT "b' "

NOTE: L IS LESS THAN λg/4
S IS GREATER THAN λg/4

FIG. 39 DESIGN CURVES FOR BRANCHED GUIDE COUPLERS IN 3"x1½" x .080" WALL WAVEGUIDE

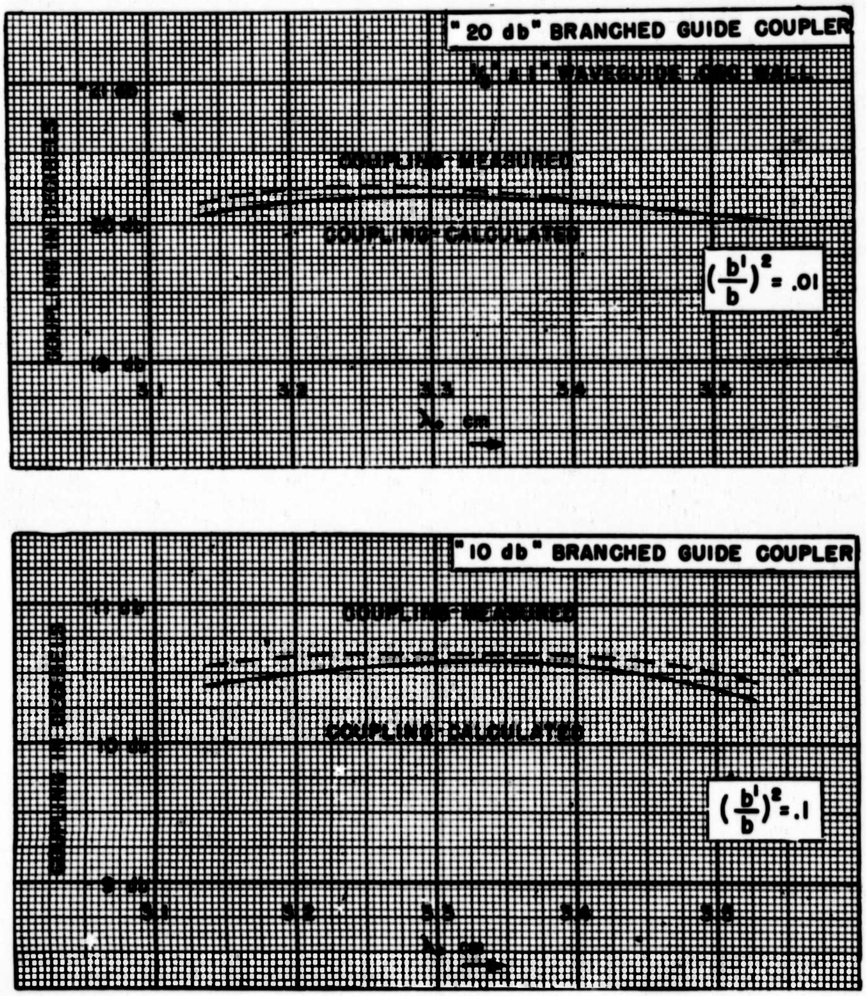


FIG. 40

FREQUENCY DEPENDENCE OF COUPLING
FOR BRANCHED GUIDE DIRECTIONAL COUPLER

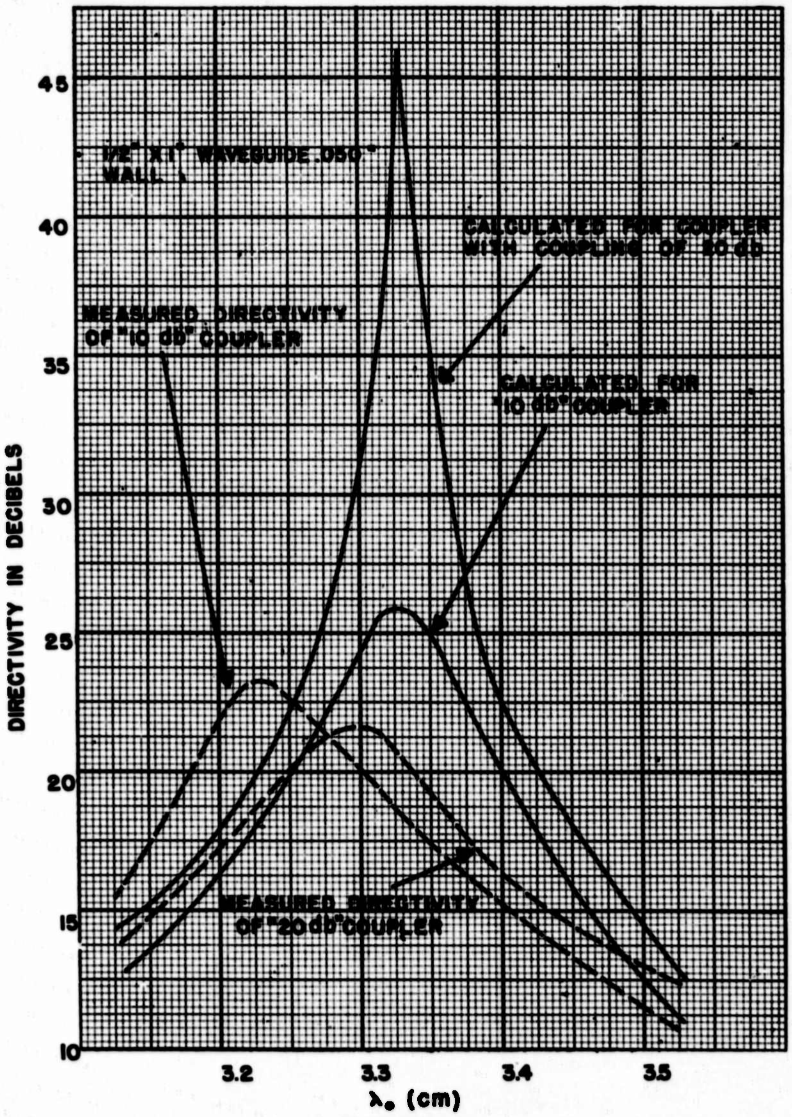


FIG. 41

FREQUENCY DEPENDENCE OF DIRECTIVITY FOR BRANCHED GUIDE DIRECTIONAL COUPLER theoretically given by:

$$\text{Directivity}_{db} = 20 \log \frac{x}{2}^{2} = 2 \times (\text{coupling in db}) - 6\text{db}.$$

It has been shown by Lippmann that if the characteristic impedance of the guide in the main line in between the coupling elements is adjusted properly, the branch guide couplers can be made with theoretically infinite directivity and zero reflection, even for very tight couplings.

<u>Tolerances</u>: The dimensions directly affecting the coupling are b' and b dimensions. (The tolerances on length of branch guides and spacing affect the coupling only to the same degree that an equivalent change in wavelength would.) For couplers with loose couplings, the b' dimension becomes very small so that a small linear error becomes equivalent to a large percentage error, and hence causes a large error in coupling. This error may be estimated from the approximate formula:

$$\Delta \text{Coupling}_{db} \approx 8.7 \frac{\Delta b'}{b'}$$

3. Multiple-Path Couplers in Coaxial Line.

As is the case for waveguide elements, the coupling elements between coaxial lines may take many different forms. The mechanical problems for coaxial lines are somewhat more involved because of their cylindrical shape and because of the complication introduced by the center conductor which must be maintained concentric with the outer conductor. The following sections describe some coaxial line multiple-path directional couplers which have been made using various types of coupling elements.

a. Slotted Block Coaxial Line Directional Coupler

A design developed by Julian* at BTL couples two parallel coaxial lines by means of slots cut in metal which contains the two lines. Transverse slots are used in preference to round holes, which would have a tendency to send power in the "wrong" direction.** A modified mechanical construction, shown in Figure 43, due to L. J. Neelands has been used to obtain the design data of Figure 44 giving coupling as a function of slot dimension.

By cutting the slot even wider than the diameter of the coaxial line, it was found possible to obtain couplings as close as -25 db at S-band, using a two-element coupler of this type.

---

*Directional Transmission Line Taps, R. S. Julian - 1/26/44-MM-44-170-6.

**This apparently poor directional property is used to advantage in the electromagnetic type coaxial line directional coupler described in a later section.

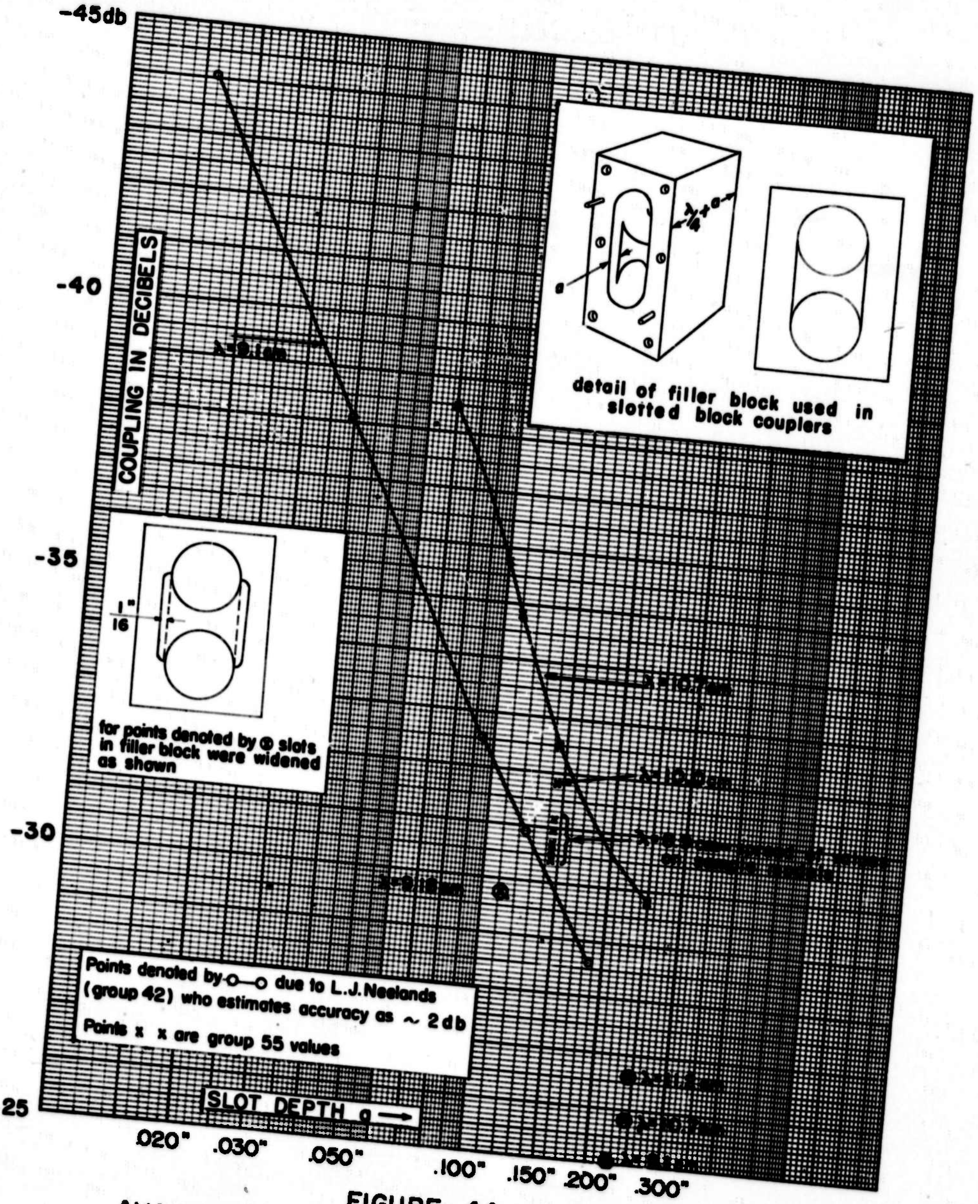
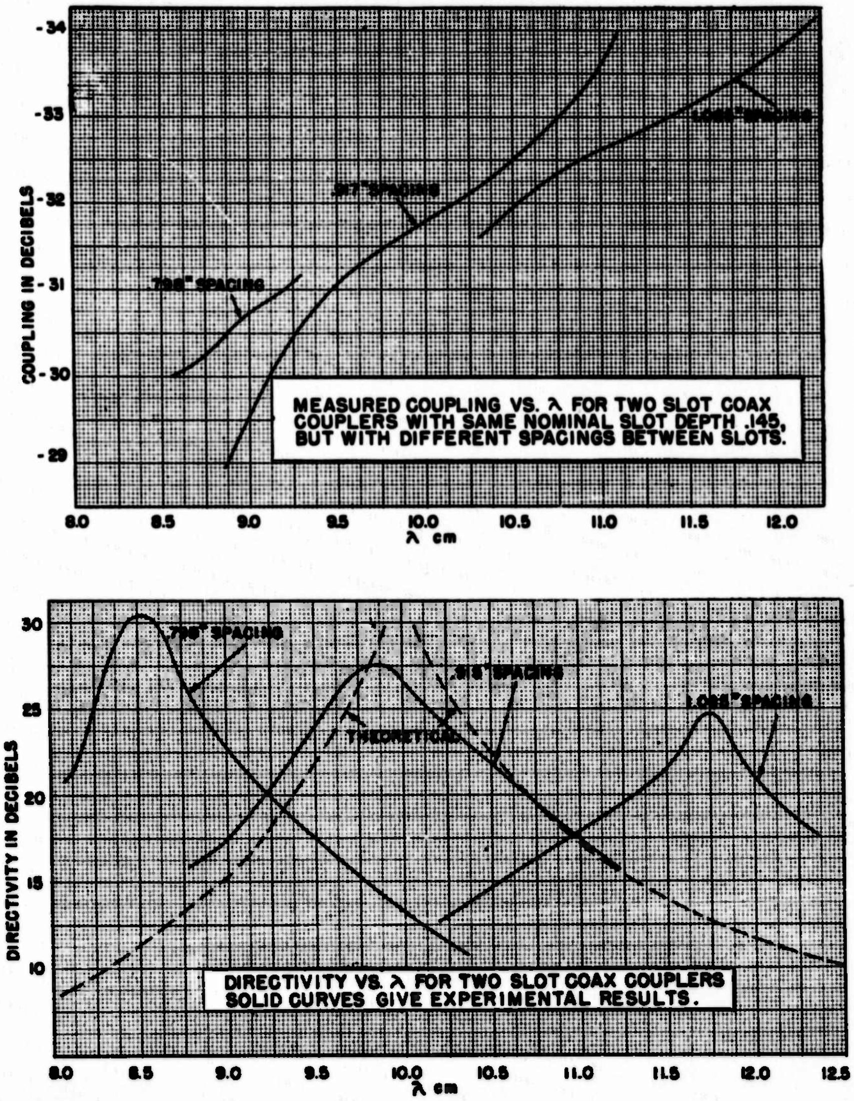


FIGURE 44
AVAILABLE DESIGN DATA ON SLOTTED BLOCK COAXIAL COUPLERS IN 7/8" LINE

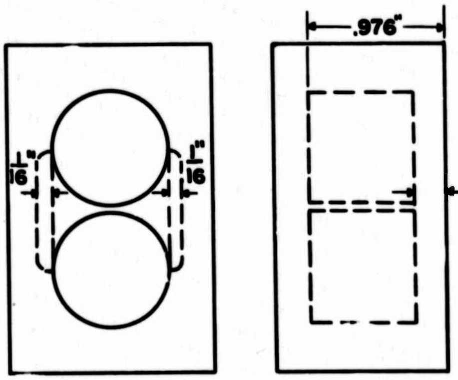


MEASURED COUPLING VS. λ FOR TWO SLOT COAX COUPLERS WITH SAME NOMINAL SLOT DEPTH .145, BUT WITH DIFFERENT SPACINGS BETWEEN SLOTS.

DIRECTIVITY VS. λ FOR TWO SLOT COAX COUPLERS SOLID CURVES GIVE EXPERIMENTAL RESULTS.

FIG. 46

EXPERIMENTAL RESULTS ON SLOTTED BLOCK TYPE DIRECTIONAL COUPLER IN ⅞" COAXIAL LINE



DIMENSIONS OF FILLER BLOCK

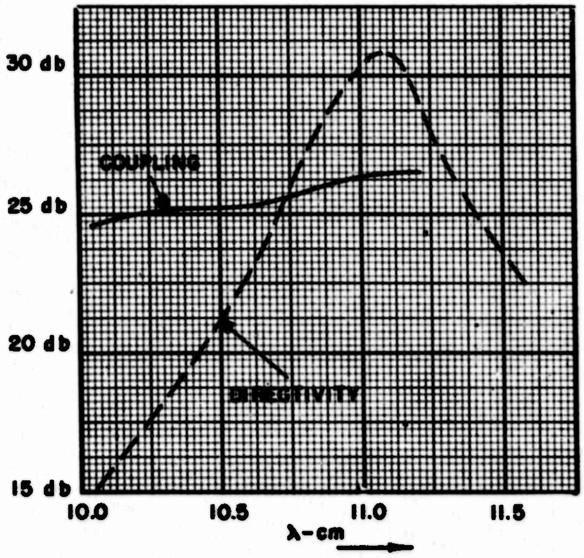


FIG. 47

EXPERIMENTAL RESULTS ON SLOTTED BLOCK TYPE 7/8" COAX DIRECTIONAL COUPLER-SLOT CUT OUT WIDER THAN DIAMETER OF COAX LINE

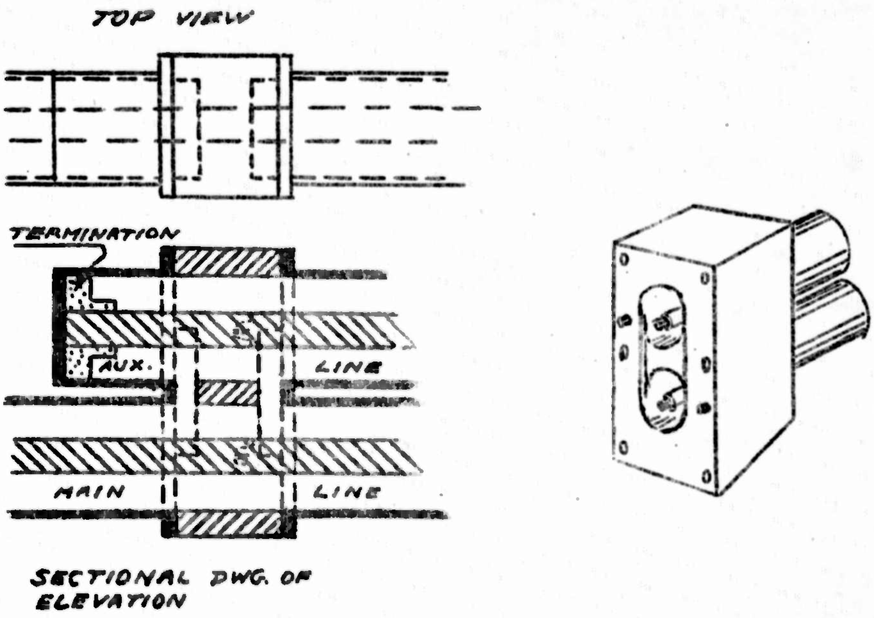


Figure 43

The geometry of the slotted block coupler is complicated by the non-uniform wall thickness at the slots, and theoretical design information is not available. Experimental data on two-element couplers of this type are summarized in Figures 44, 46 and 47. From these figures, it may be seen that the directivities of these couplers are maximum for slot spacings somewhat less than λ/4. The frequency sensitivity of directivity is similar to that of other two-element couplers. The coupling changes rather rapidly with wavelength, with the slope of the coupling versus wavelength curve approximately 2 db/cm. The curves of Figure 46 show a somewhat smaller slope in the neighborhood of the design wavelength, but it is not clear whether this is a real effect or due to inaccuracies in measurement.

b. Concentric Coax Two-Slot Directional Coupler

Figure 48

By putting one coaxial line inside another coaxial line, using the inner conductor of the larger coax line as the outer conductor of the smaller coax, and cutting slots in the common wall between the two lines, it is possible to achieve close coupling (as close as -10 db for S-band) and compactness. There are two variations on this type - the "inside out" and the "outside in". In the former, the main line is on the inside, and power is coupled out into the auxiliary line as in Figure 49. In the other, Figure 50, the main line is on the outside and couples power inwards to the auxiliary line. In both, the auxiliary line has a right angle bend before the output and a resistive termination.

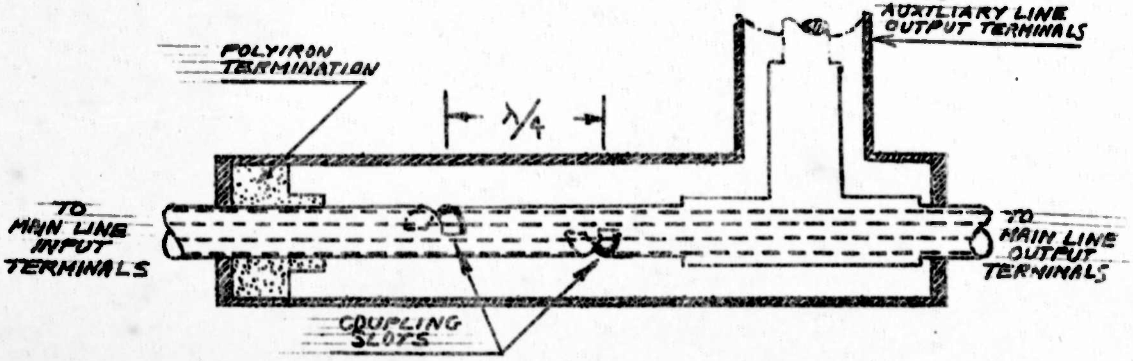
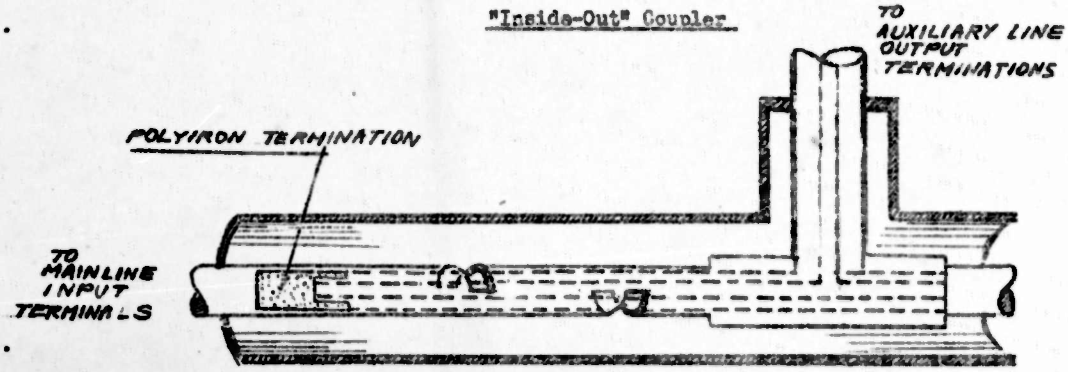


Figure 49

"Inside-Out" Coupler



"Outside-In" Coupler

Figure 50.

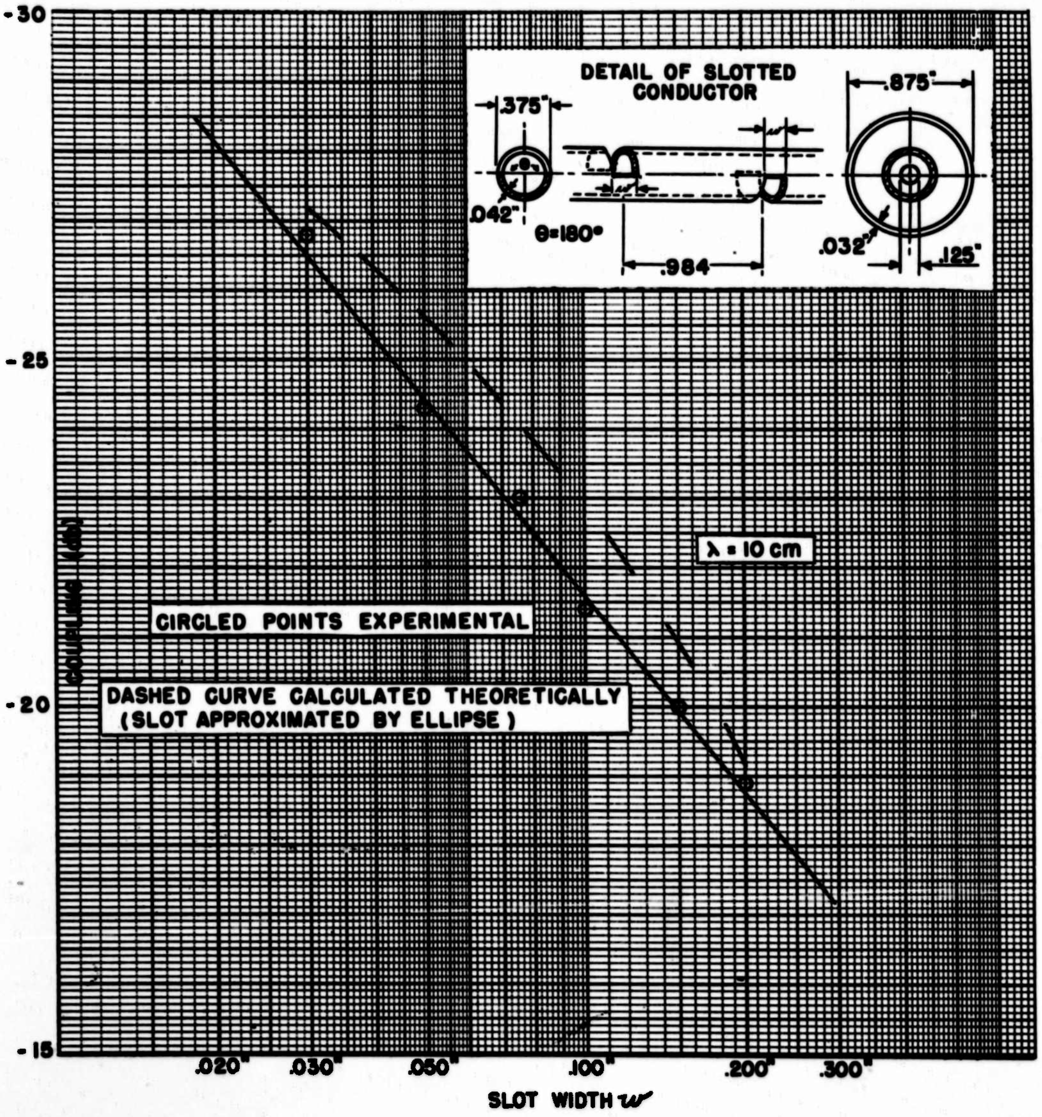


FIG. 51
DESIGN DATA FOR CONCENTRIC COAX
COUPLER BETWEEN $\frac{7}{8}$" O.D. AND $\frac{3}{8}$" O.D. COAXIAL LINES

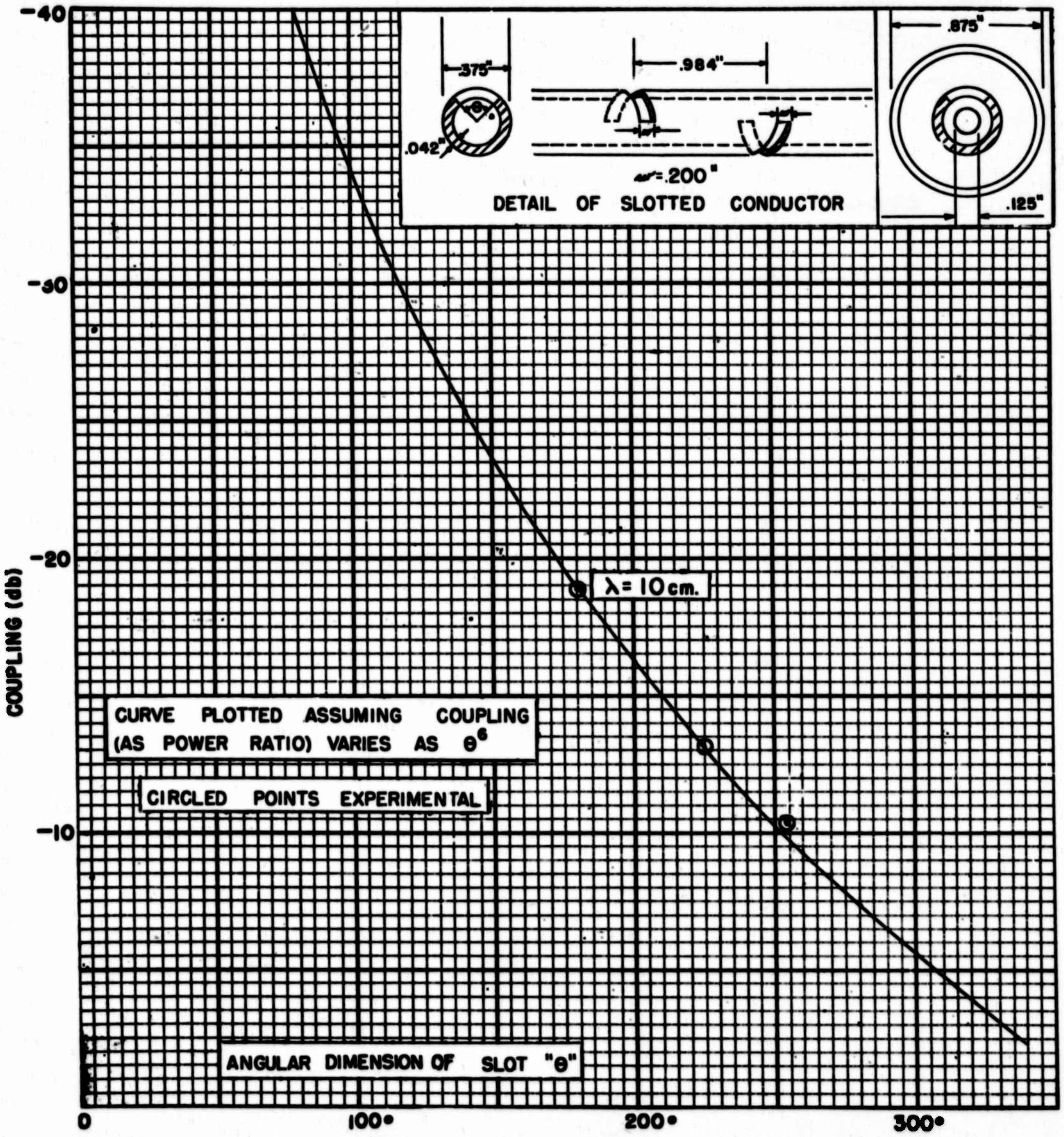


FIG. 52 DESIGN DATA FOR CONCENTRIC COAX COUPLER BETWEEN $7/8$" O.D. AND $3/8$" O.D. COAXIAL LINES

Coupling: Experimentally it has been found that for a concentric coax directional coupler the power coupled into the auxiliary transmission line is proportional to the first power of the slot width w, and to the sixth power of the angular aperture θ of the slot. Figure 51 shows these experimental results giving coupling as a function of slot width, for slots cut half way around (θ = 180°). Coupling as a function of θ is shown in Figure 52. In Figure 52 there is also a curve calculated theoretically using an approximation treating the slot as a small elliptical aperture in a plane wall. This analysis predicts the coupling to be given by the formula:

$$\text{Coupling}_{db} \simeq 10 \log \left[\frac{5\pi r_2 \theta^3}{2Z_o \lambda \left(\ln \frac{4r_2\theta}{w} - 1\right)}\right]^2 + \text{att } H_{db} ,$$

where θ (see Figure 53) is the angular aperture of the slot (use radians in formula), w is the slot width, r_2 is the radius of the outer conductor of the inner line (or the inner conductor of the outer line), Z_o is the characteristic impedance of the line, λ is the wavelength, and att $H_{db} \simeq 27\ t/\theta r_2$ is the cut-off attenuation through the slot, with t the wall thickness.

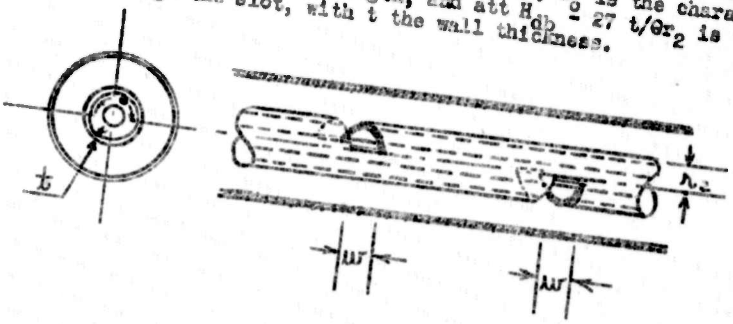


Figure 53

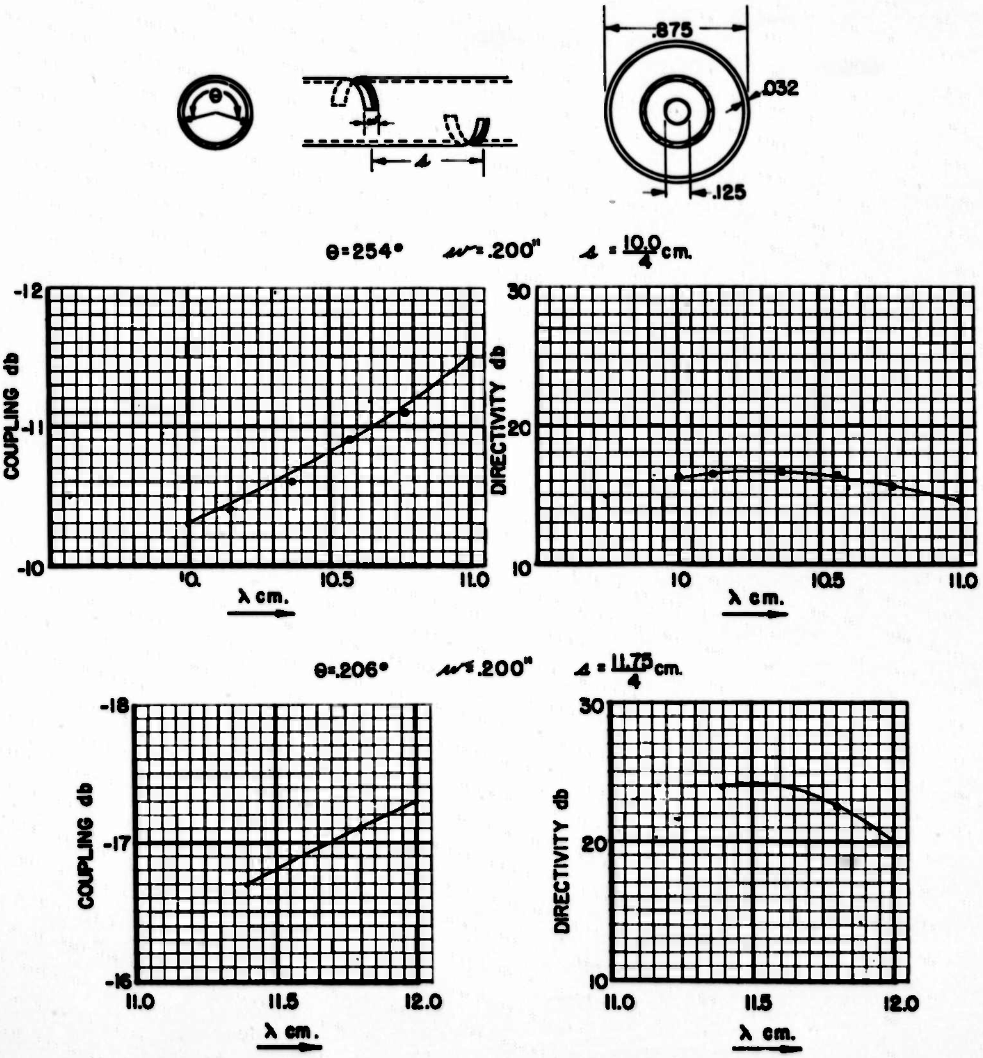
The theoretical formula predicts a variation of coupling with wavelength proportional to $1/\lambda^2$. The experimental results of Figure 54 exhibit an appreciably greater variation.

Effect of Eccentricity of Outer Conductor: If the center conductor of the smaller coaxial line is not concentric with the outer conductor, the coupling through the slot will be either increased or decreased, depending upon whether the center conductor comes closer to it, or further away. By cutting the two slots on opposite sides, rather than both on the same side, the effect of the variation of the coupling coefficiency due to eccentricity is minimized.

Directivity: The experimentally determined directivities of several S-band models of two-element concentric coax directional couplers are shown in Figure 54.

FIGURE 54

SOME EXPERIMENTAL RESULTS ON CONCENTRIC COAX COUPLERS



c. Branched Coax (Double Stub) Directional Coupler

The analogue in coaxial line to the branched waveguide coupler, using low impedance series tees as coupling elements, is a coaxial coupler using low admittance shunt coax tees--the branched coax coupler.

Models have been made in 7/8" coaxial line, using as the branch lines, coaxes having the same dimension of outer conductor as standard 7/8" line, but having a very small diameter center conductor (Figure 55).

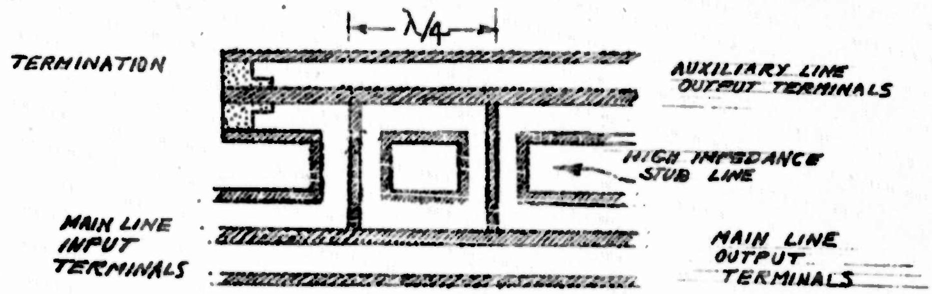


Figure 55

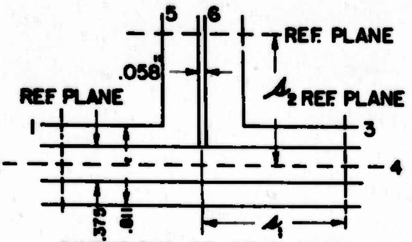
Since characteristic admittances of coaxial lines vary logarithmically with the line dimensions rather than directly, as for waveguides, the range of possible coupling values is restricted, since for coupling values much closer than 10 db, the center conductor of the branch line would have to be impractically thin.

An equivalent circuit analysis carried out for this shunt case gives exactly the same results as were obtained for the series case, provided admittances are substituted for impedances. Thus the coupling formula becomes:

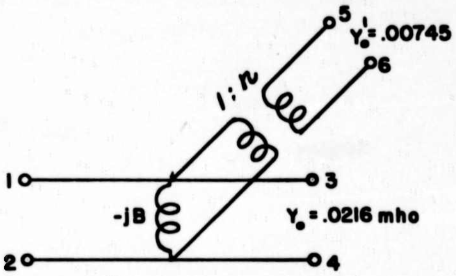
$$\text{Coupling}_{db} = 10 \log \frac{Y^2}{1 + Y^4 + Y^6} \approx 10 \log Y^2,$$

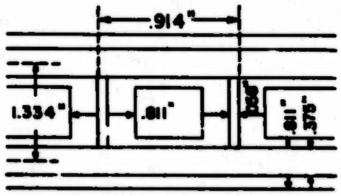
where Y is the ratio of the effective (corrected for end effects) characteristic admittance of the stub line to that of the main line.

The performance of a branched coaxial coupler built in 7/8" line, designed for 10 cm is shown in Figure 56. No theoretical information was available from which one could estimate the errors involved in the assumption that a low admittance coaxial tee stub of the type used in this coupler could be represented as a pure shunt element. Therefore, the constants for the equivalent circuit shown in Figure 57, were determined by measurements upon an experimental model.



DIMENSIONS OF STUB USED





COUPLER DIMENSIONS

MEASURED EQUIVALENT CIRCUIT
STUB FOR $\lambda = 10.0$ cm.

$\ell_1 = 4.86$ cm.
$\ell_2 = 5.45$ cm.
$-jB = -.27\ Y_o$
$n^2 = .84$

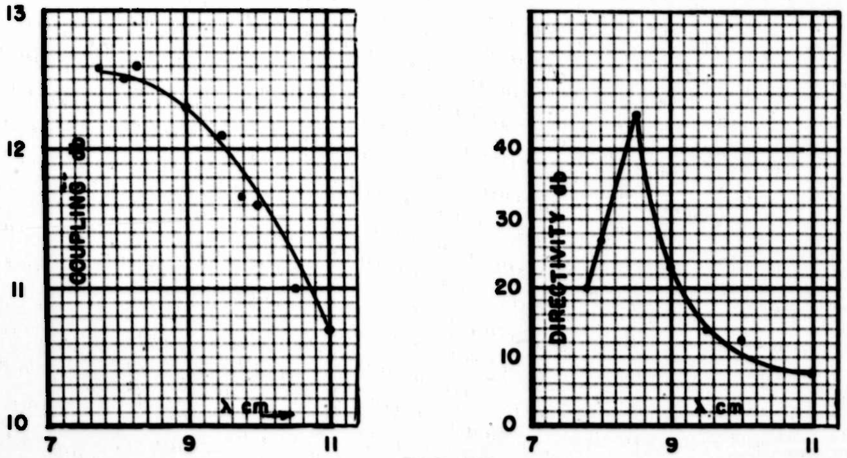


FIG. 56

EXPERIMENTAL RESULTS FOR BRANCHED COAX (DOUBLE STUB) COUPLER IN $\frac{7}{8}$" O.D. COAXIAL LINE

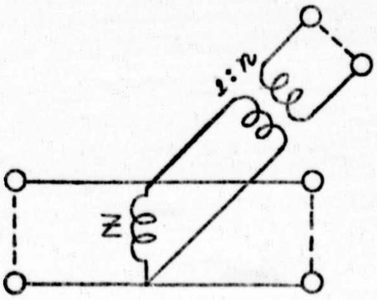


Figure 56a

The reference planes at which this circuit is valid were first determined by utilizing the following facts. A shorting plunger placed at the proper reference plane in the stub will produce an infinite standing-wave-ratio in the main line. The positions of zero voltage in the main line under this condition will correspond to the reference planes there. Two additional measurements, one of the admittance at the reference plane, with Z_o terminating the main line; and the other, a determination of the position of a short circuit in the stub for which the stub is matched looking by it, were sufficient to determine the turns ratio n , and the shunt susceptance Z.

Results of these measurements are included in Figure 56. These values are to be taken as an indication of the order of magnitude of the quantities involved, rather than as precise values.

C. LONG SLOT DIRECTIONAL COUPLER

The long slot directional coupler may be regarded as a limiting case of a multiple-path coupler consisting of n equal elements spaced l/n apart (l being an integral number of half wavelengths for best directivity); the limit being approached as the number of elements is increased keeping l constant. Figure 57 shows the phasing of the secondary waves sent in the backward direction which interfere destructively, for $l = \lambda_g/2$.

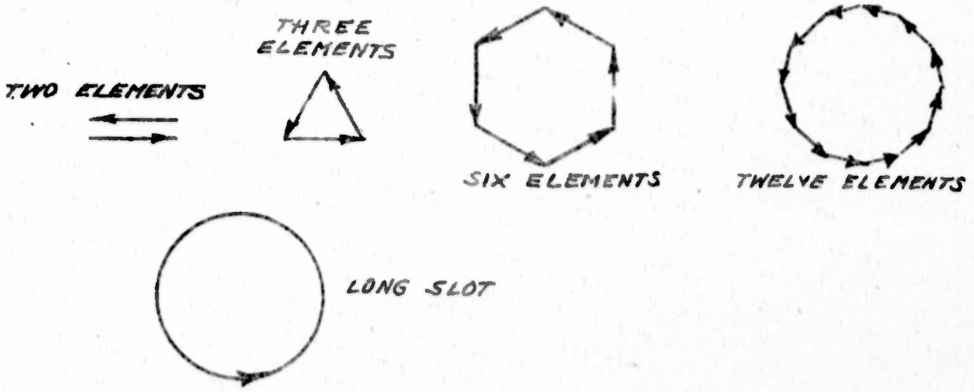
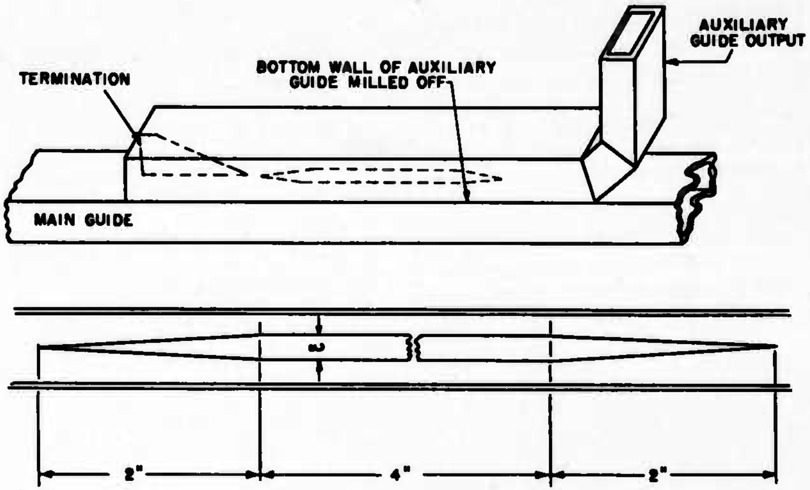
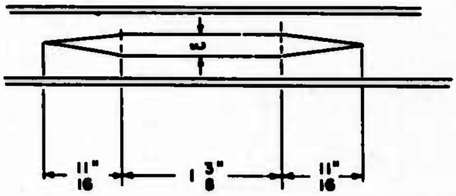


Figure 57



DETAIL OF SLOT IN MAIN GUIDE
X BAND

GUIDE DIMENSIONS- ½" x 1" x .050" WALL

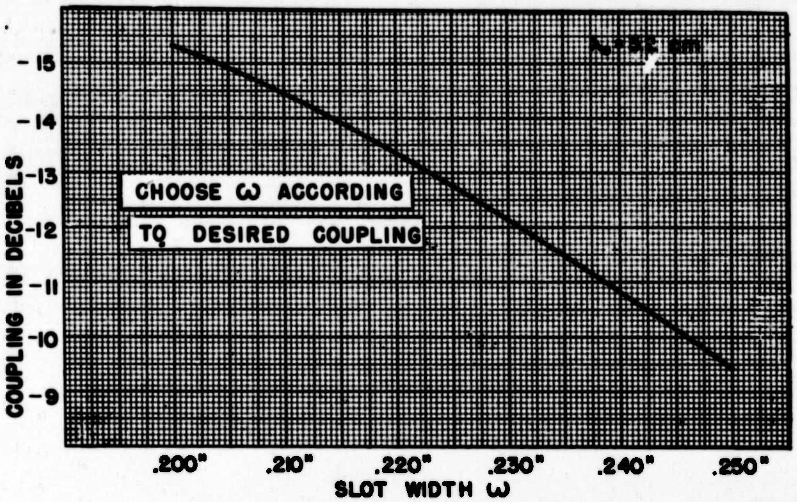


DETAIL OF SLOT IN MAIN GUIDE
K BAND

GUIDE DIMENSIONS- ½" x ¼" x .040" WALL

FIG.58

LONG SLOT DIRECTIONAL COUPLER - CONSTRUCTION



DESIGN DATA FOR LONG
SLOT COUPLERS IN 1" X ½ "X .050" WALL WAVEGUIDE

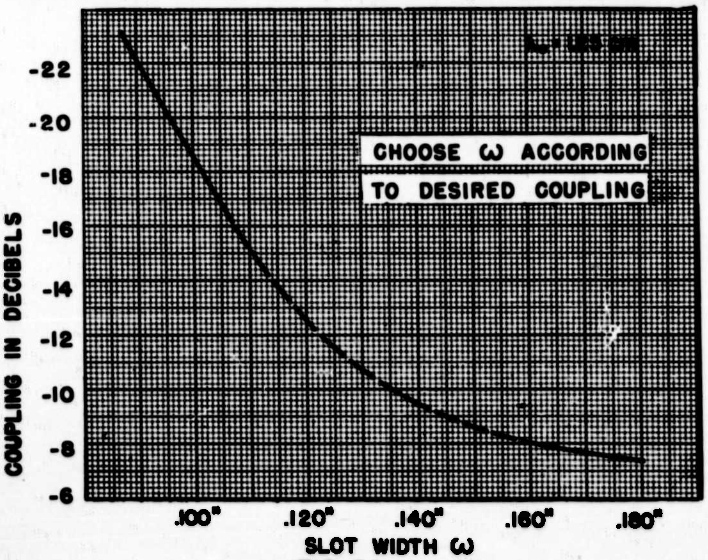
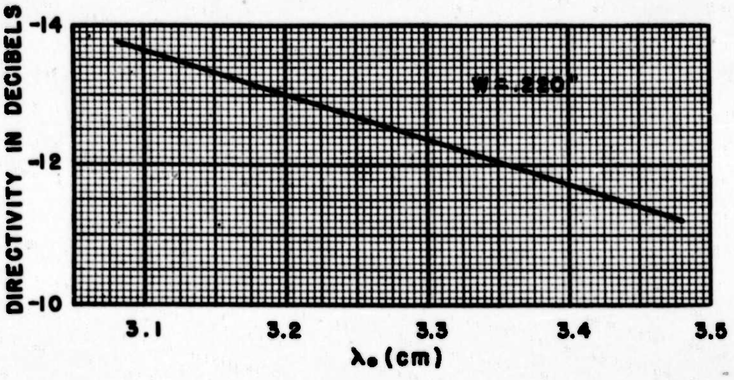


FIG. 59
DESIGN DATA FOR LONG
SLOT COUPLERS IN ½"X¼" X 040" WALL WAVEGUIDE



FREQUENCY DEPENDENCE OF COUPLING FOR LONG SLOT COUPLER IN 1" X $\frac{1}{2}$" X .050" WALL WAVEGUIDE

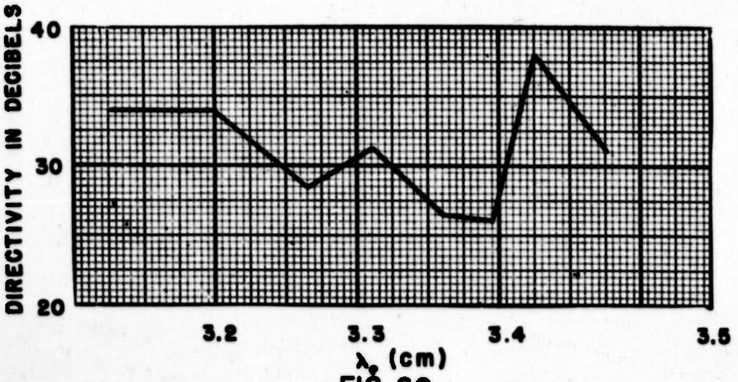


FIG. 60

FREQUENCY DEPENDENCE OF DIRECTIVITY FOR LONG SLOT COUPLER IN 1" X $\frac{1}{2}$" X .050" WALL WAVEGUIDE

The directivity (assuming loose coupling) for a coupler using a slot of length l, in which there is uniform coupling along the length of the slot, is given by:

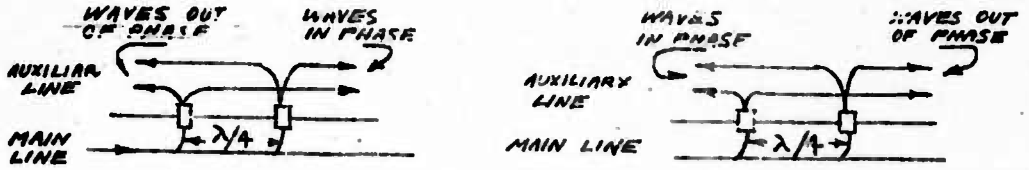
$$\text{Directivity}_{db} = 20 \log \frac{\beta l}{\sin \beta l}$$

where β is the propagation constant along the main and auxiliary transmission lines. The formula shows that the directivity will be peaked for slots an integral number of half wavelengths long (wavelength measured in the transmission line), and moreover, it will always be high if the slot is very long.

Research has been done on long slot couplers by a number of different companies, particularly with a view toward reflectometer use. Figures 58, 59, and 60 present data on a few waveguide models at X and K-band, with slots in the center of the broad side of the waveguide, made by this group primarily for use in power monitoring.

D. REVERSED PHASE DIRECTIONAL COUPLER

The two-element reversed phase directional coupler is similar to the two-element multiple-path coupler, but whereas the latter uses two identical elements spaced a quarter wavelength apart, the former operates on the principle that the phase shift caused by one element in coupling between the primary and secondary transmission lines will be exactly 180° different from the phase shift caused by the second element.



Multiple-Path Coupler
Same phase shift through each element

Reverse Phase Coupler
Relative phase shift of 180° through elements

Figure 61

Thus, in the elementary analysis, for a wave travelling toward the right in the main line, because of the 180° difference in phase shift at the two coupling elements, the secondary waves excited toward the right in the auxiliary line will interfere destructively - even though there is no difference in path length. The secondary waves coupled by the two elements toward the left will have experienced a 180° relative phase shift because of the λ/2 path difference. This, combined with the 180° phase shift caused by the elements themselves, will cause the two waves to interfere constructively.

Thus, the distinctive feature of this type coupler are that the directivity is inherently broadband, since it does not depend upon the separation of the elements, and that the power coupled out is travelling in the opposite direction from the power in the main line.

Means of Obtaining Phase Reversal: A number of different methods of obtaining the necessary phase reversal have been suggested. Loops oriented in opposite senses with respect to the magnetic field will induce currents 180° out of phase with each other. A scheme used by RCA for coupling two waveguides is illustrated in Figure 62.

SECTION THROUGH 1st ELEMENT

SECTION THROUGH 2nd ELEMENT

Figure 62

With the orientation of the probes as shown, the induced currents will be 180° out of phase with each other.

Probe-fed slots offer another possibility of achieving a phase reversal.* These have been used in the design of antennas, but owing to the greater precision needed in individual coupling elements for directional couplers, no work has as yet been done in the Radiation Laboratory to adapt them to the latter use. The slot is placed in such an orientation on the wall of the guide that it cuts no lines of current flow, and then the current is drawn across the slot by use of a suitable probe extending into the guide. (Figure 63).

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*Probe-Fed Slots as Radiating Elements in Linear Arrays; Roger E. Clapp, RL Report 455.

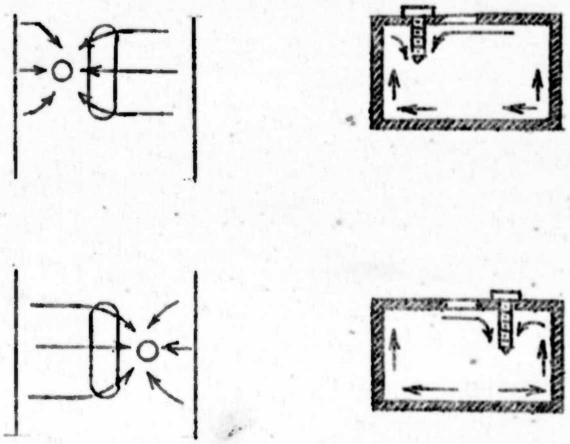


Figure 63

The direction of the current lines across the slot, and hence phase of coupling, will depend upon which side of the slot the probe is inserted.

"Schwinger Type" Reversed Phase Coupler: A method suggested by Schwinger is to join two parallel waveguides as in Figure 64, so that the narrow side of one falls on the center of the wide side of the other.

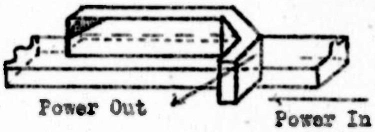
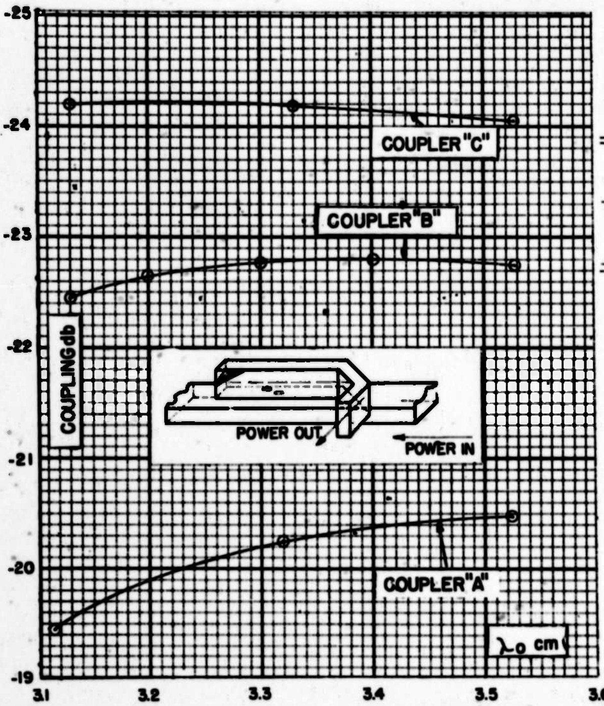
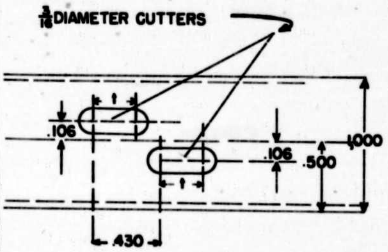


Figure 64

Two slots are milled on opposite sides of the center line, and are spaced $\lambda g/4$ apart along the guide. The necessary phase reversal occurs since the slots in the side of the upper guide will only couple to a longitudinal magnetic field, and this longitudinal magnetic field in the lower guide has opposite signs for the two slots.

The coupling obtained with this type coupler may be estimated from the coupling that the same two slots would give in an ordinary two-slot-in-the-side-of-the-guide-coupler. As far as each slot by itself is concerned, the main difference between the two cases is that near the center of the guide the slot couples to a weaker longitudinal magnetic field (which increases sinusoidally from 0 at the center to a maximum at the side) than exists at the side of the guide. Consequently, the coupling is smaller. Therefore, the frequency sensitivity of coupling for the two cases may be expected to be





SLOT DIMENSIONS OF COUPLERS

COUPLER	CUTTER TRAVEL "t"
A	.330"
B	.290"
C	.270"

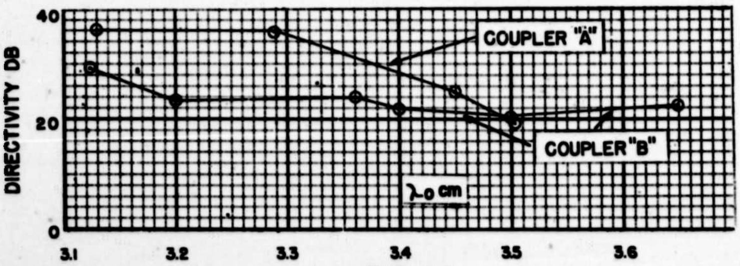
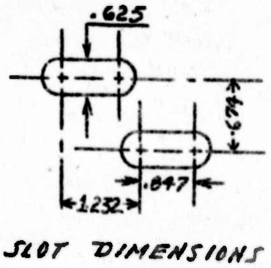


FIG. 65

"SCHWINGER" TYPE REVERSED PHASE DIRECTIONAL COUPLER
IN 1" x 1/2" x .050" WALL WAVEGUIDE — EXPERIMENTAL RESULTS

similar, and therefore the same broadbanding technique may be used. Figure 65 shows some results obtained in the design of a coupler of this type in 1" x 1/2" waveguide. Figure 66 tabulates results on an S-band model, built in 3" x 1 1/2" waveguide.



λ	Coupling	Directivity
8.9 cm	-19.6 db	22 db
9.24	-20.8	24
10.0	-22.1	30
10.55	-22.7	30
11.1	-22.9	26
11.55	-22.7	22

Figure 66

E. ELECTROMAGNETIC TYPE OF DIRECTIONAL COUPLER

In the electromagnetic type of directional coupler, at a single point in the main transmission line, there is both electric and magnetic coupling into the auxiliary line. The waves excited in the auxiliary line due to the electric (voltage) coupling interfere destructively in one direction, and constructively in the other direction with the corresponding waves excited by magnetic (current) coupling.

From a circuit standpoint, an electromagnetic type coupler is characterized by having both series and shunt coupling at one point, between the main and auxiliary lines. The circuit shown in Figure 67

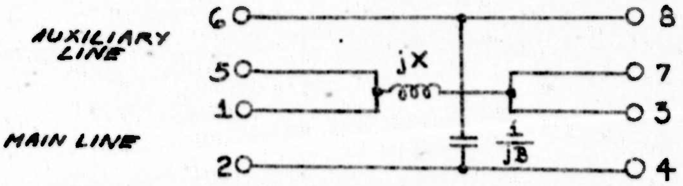


Figure 67

fulfills the requirement* in the simplest possible fashion, and moreover, with proper choice of values for the series and shunt impedances possesses all the properties of a directional coupler. This may be shown more clearly if the circuit is redrawn in the form of a bridge network. Figure 68 shows this for the case of power input at the terminal pair 1-2, and matched loads on the other terminal pairs.

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* In general, any directional coupler may, at appropriate reference planes, be represented by an equivalent circuit which is essentially of this same form. This representation arises most naturally for the electromagnetic type coupler having all the coupling take place at a single point.

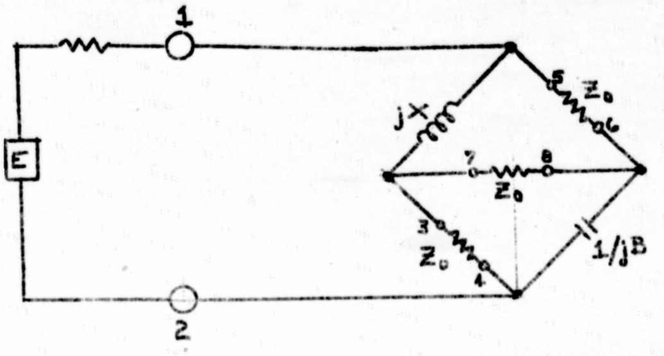


Figure 68

The condition for zero voltage delivered across terminals 7-8 corresponding to an infinite directivity, is $jX/Z_o = Z_o/(1/jB)$, or $jX/Z_o = jB/Y_o$. With this condition satisfied, it may easily be verified that the coupler is perfectly matched as seen from the input terminals 1-2, and moreover, the coupling is given by the formula:

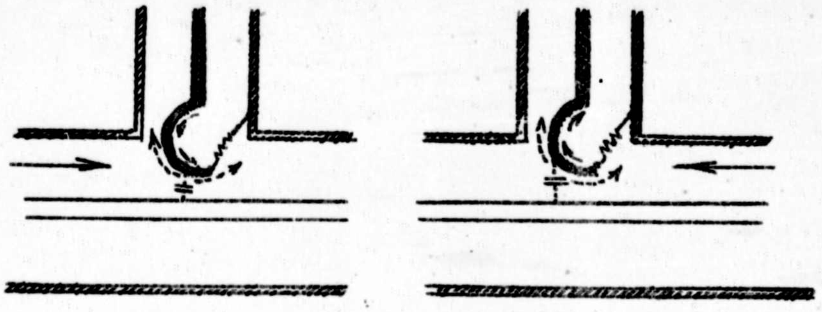
$$\text{Coupling (as a power ratio)} = \frac{X^2}{X^2 + Z_o^2} \approx \left(\frac{X}{Z_o}\right)^2, \text{ for small}$$

coupling.

The circuit representation just given may make the action of electromagnetic type couplers seem more plausible. In the Bethe hole coupler, for example, the series or current coupling may be thought of as arising from the fact that the longitudinal currents associated with the transverse magnetic field in the guide or coax are interrupted at the hole, and so induce a voltage across the hole which is proportional to the current which would have existed in the absence of the hole. Similarly, the electric field lines which would otherwise have been terminated on the metallic wall, due to the presence of the hole, are responsible for voltage or shunt coupling into the auxiliary line. More details and design information concerning the Bethe hole directional coupler will be given in a subsequent section.

1. Kinds of Electromagnetic Couplers.

Resistive Loop Directional Coupler: The "resistive loop" variety electromagnetic type couplers (described in Report Navy 521) is worth mentioning. Figure 69 sketches the construction and action of this coupler. There is loop coupling into the main coaxial line with both electric and magnetic coupling. The magnetic coupling is adjusted for optimum directivity by rotation of the loop. A small resistor, which acts as termination for the auxiliary line, is an essential feature of this coupler.



⟶ current induced by magnetic coupling.
--→ current induced by electric coupling.

(a) Shows wave travelling toward right in the main line. The relative phasing of the two kinds of coupling is such as to interfere destructively at the output and send all coupled power into termination.

(b) Shows wave travelling toward left in the main line. Here the two kinds of coupling are in phase at output.

Figure 69

2. Bethe Hole Directional Couplers.*

Bethe hole directional couplers in waveguide consist of two waveguides crossed at an angle which are coupled by means of a hole in their common wall (this wall forming a part of the broad dimensions of both waveguides). This type coupler couples out power "backwards" as is indicated in Figure 70.

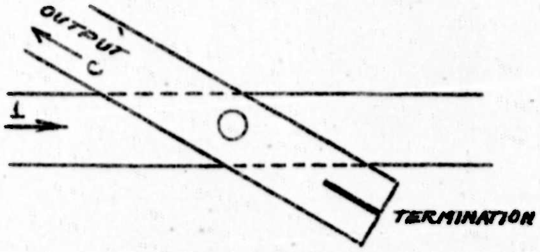


Figure 70

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*Group 41 has done a great deal of work on Bethe hole couplers, especially at K-band, and the theory in what follows has been taken from a memo prepared by B. A. Lippmann of that Group, as well as from Bethe's original reports.

In an analysis developed by H. A. Bethe in RL Reports 43-22, 43-26, and 43-27, it is shown that, for two waveguides joined by a hole through their broad sides, there is both magnetic and electric coupling between the two. The two waves (which may be termed the H-wave and the E-wave respectively) which are set up in the auxiliary guide by the two kinds of coupling tend to interfere on one side of the hole and reinforce on the other. This action is such that the stronger wave in the second guide proceeds in a direction opposite that of the incident wave in the exciting guide.

The amplitude of the weaker wave in the auxiliary guide may be reduced to zero by adjusting such factors as the shape of the hole, the dimensions of the guides, and the angle of crossing of the two guides. These adjustments have the effect of controlling the relative strength of the electric and magnetic coupling.

The experience of this Group has been with round holes centrally located between standard size waveguides, with the angle of crossing adjusted for optimum directivity. A great help in design has been the fact that proper hole size and angle may be predicted reasonably accurately from theory.

In designing these directional couplers, Bethe's results for a small hole in an infinitely thin wall have been used, with corrections applied for finite wall thicknesses. These corrections assume that the hole may be regarded as a short section of circular waveguide beyond cutoff, and that the attenuation of the E and H-waves is given by the standard waveguide beyond cutoff attenuation formulas for the TM_{01} and TE_{11} modes respectively. These results from the "small hole" theory are expressed here as formulas in which the coupling, and angle of crossing "θ", are given as functions of design wavelength (λ_o = free space wavelength, λ_g = guide wavelength), hole diameter "d", and waveguide dimensions (a = larger inner dimension, b = smaller inner dimension, and t = total wall thickness at the hole).

The coupling through a hole in the center of the wide dimension of two waveguides crossed at an angle θ is given by:

(1) $$\text{Coupling}_{db} = 20 \log \frac{\pi d^3}{3ab\lambda_g}\left[\cos\theta - 1/2\left(\frac{\lambda_g}{\lambda_o}\right)^2\left(\frac{\text{att } E}{\text{att } H}\right)_{volt}\right] - 32.0\,\frac{t}{d}\sqrt{1-\left(\frac{1.71d}{\lambda_o}\right)^2}$$

For optimum directivity, the angle θ should be chosen so that

(2) $$\cos\theta = 1/2\left(\frac{\lambda_g}{\lambda_o}\right)^2\left(\frac{\text{att } E}{\text{att } H}\right)_{volt.}$$

With this choice (1) becomes,

(1') $$\text{Coupling}_{db} = 20 \log \frac{2\pi d^3 \cos\theta}{3ab\lambda_g} - 32.0\,\frac{t}{d}\sqrt{1-\left(\frac{1.71d}{\lambda_o}\right)^2}$$

The second term in formulas (1) and (1') gives the added attenuation in db (for a TE_{11} mode) because of the finite thickness of the hole.

$(\text{att } E/\text{att } H)_{\text{volt}}$ is another term correcting for the thickness of the hole and represents the relative attenuation, expressed as a voltage ratio for waves in the TM_{01} and TE_{11} modes respectively in going through the hole. It is given by the formula:

$$(3) \qquad \left(\frac{\text{att } E}{\text{att } H}\right)_{\text{volt}} = \frac{10^{\left[\frac{-41.8}{20} \frac{t}{d}\sqrt{1-\left(\frac{1.31d}{\lambda_o}\right)^2}\right]}}{10^{\left[\frac{-32}{20} \frac{t}{d}\sqrt{1-\left(\frac{1.71d}{\lambda_o}\right)^2}\right]}}$$

or, more approximately, for $t/d \ll 1$, by:

$$(3') \qquad \left(\frac{\text{att } E}{\text{att } H}\right)_{\text{volt}} \cong 1 - 1.1\frac{t}{d}$$

In order to find d and θ when the other quantities are known, either a graphical method, or the method of successive approximations may be used. Design charts have been plotted using the theoretical formulas for both 3" x 1 1/2" waveguide and 1/2" x 1/4" waveguide. These are discussed in a following section.

Directivity: The theoretical formula for the directivity of a Bethe hole coupler (expressed as a voltage ratio) is:

$$(4) \qquad \text{Directivity} = \frac{\cos\theta + 1/2\left(\frac{\lambda_g}{\lambda_o}\right)^2 \frac{\text{att } E}{\text{att } H}}{\cos\theta - 1/2\left(\frac{\lambda_g}{\lambda_o}\right)^2 \frac{\text{att } E}{\text{att } H}}$$

The angle θ is chosen so as to make the denominator of this expression equal to zero at the design wavelength.

For very thin holes the factor att E/att H is close to unity, and the condition for optimum directivity is satisfied for parallel waveguides when the ratio $(\lambda_g/\lambda_o)^2$ is close to two. For waveguides operating far from the cutoff wavelength, or holes thick compared to their diameter, the optimum angle gets larger. On the other hand, for waveguides operating very close to the region of cutoff it may become impossible, without resorting to other than circular holes, to obtain high directivity since even at 0° angle, the "E" coupling may be stronger than the "H" coupling.

The bandwidth with respect to directivity obtained with actual models of Bethe hole couplers in 3" x 1 1/2" waveguide have not measured up to that given by formula (4), as may be seen, for example, in Figure 71.

The second term in formulas (1) and (1') gives the added attenuation in db (for a TE_{11} mode) because of the finite thickness of the hole.

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or, more approximately, for $t/d \ll 1$, by:

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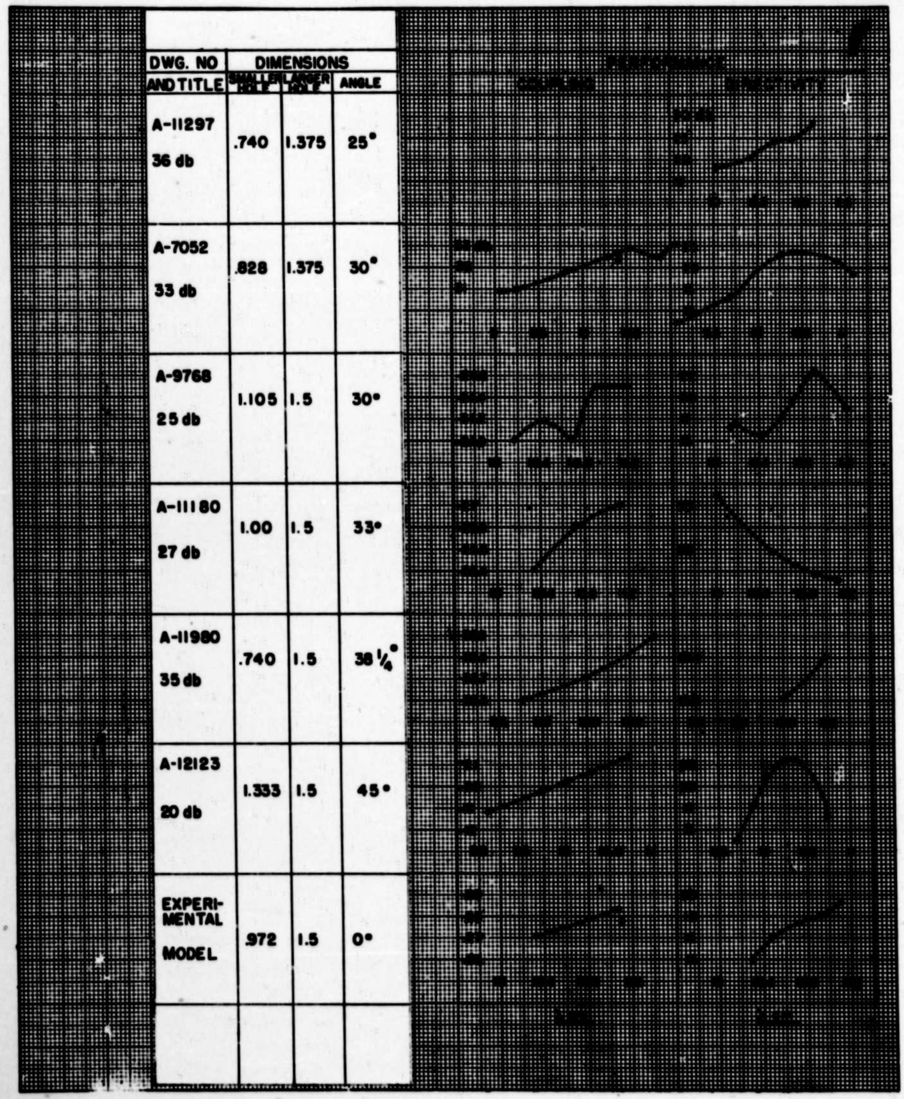


DWG. NO AND TITLE	DIMENSIONS		
	SMALLER HOLE	LARGER HOLE	ANGLE
A-11297 36 db	.740	1.375	25°
A-7052 33 db	.828	1.375	30°
A-9768 25 db	1.105	1.5	30°
A-11180 27 db	1.00	1.5	33°
A-11980 35 db	.740	1.5	38 ¼°
A-12123 20 db	1.333	1.5	45°
EXPERIMENTAL MODEL	.972	1.5	0°

FIGURE 71
EXPERIMENTAL RESULTS ON BETHE HOLE DIRECTIONAL COUPLERS IN 3" X 1½" X .081" WALL WAVEGUIDE

The theoretical frequency sensitivity of coupling may be obtained from formula (1). The major contribution to frequency sensitivity arises from the term involving guide wavelength, λ_g; the coupling, expressed as a power ratio, is inversely proportional to λ_g^2.

Thus, for couplers in 3" x 1 1/2" guide designed for $\lambda_o = 10$ cm, the theoretical rate of change of coupling is approximately 1 db per centimeter change in wavelength. (This may be compared with the experimental results shown in Figure 71.)

Experimental Results: A number of Bethe hole directional couplers have been made in 1 1/2" x 3" x .081" wall rectangular waveguide. The construction of these couplers is shown in Figure 72.

Two holes are drilled,--one of a size determined by the desired coupling in the broad face of the main waveguide, and another larger hole in the broad face of the auxiliary guide. The two guides are then soldered together (keeping their holes concentric) at that angle which makes the coupler most directive. The standing wave ratio which the hole sets up in the main waveguide is matched out by an inductive window approximately $\lambda_g/4$ toward the generator. In the auxiliary guide, the output is in the form of a guide-coax adaptor which is on one side of the hole; on the other side of the hole there is a termination made of tapered resistive cloth (USKOH-9-112) extending along the length of the guide.

This design, with one large hole and one small hole, was adopted as a compromise between making the common wall between the two guides as thin as possible, with a single hole in that wall; and the method of not milling either wall and having a hole of two wall thicknesses. The latter method is not desirable electrically since increased attenuation of thick holes requires an increase in hole size, and hence causes larger reflection if the coupling is to be kept constant. The former type is perhaps harder to make. The large hole in the secondary guide causes little additional attenuation, although the reflection set up by it in the secondary guide might be objectionable for some applications.

Figure 71 summarizes experimental information regarding a number of Bethe hole couplers in 3" x 1 1/2" waveguide that have been made, and lists Radiation Laboratory drawing numbers where these are available.

Charts calculated on the basis of the theoretical design formulas which may be used in the design of S-band Bethe hole couplers of this type are given in Figures 73 and 74. The larger hole in the auxiliary waveguide has been standardized at 1 1/2". The diameter "d" of the hole in the main waveguide and the angle of crossing θ are plotted against coupling for different wavelengths.

Figure 75 presents some data for this type coupler in which a comparison is made between experimental values of coupling, and values calculated from the same theoretical formulas used for plotting Figures 73 and 74. It can be seen that the experimentally measured coupling is consistently about 1 1/2 db looser than theoretical. In view of the fact that the theory is only strictly true for small holes and also in view of the somewhat doubtful validity

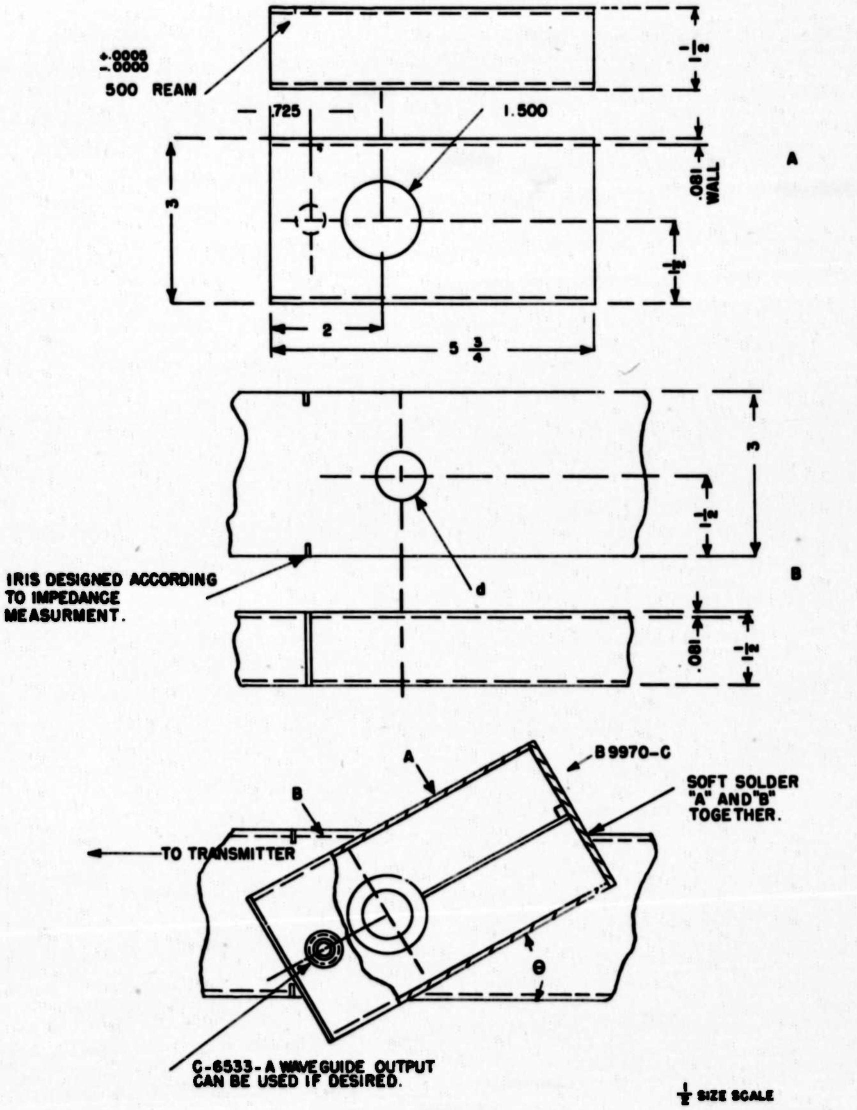
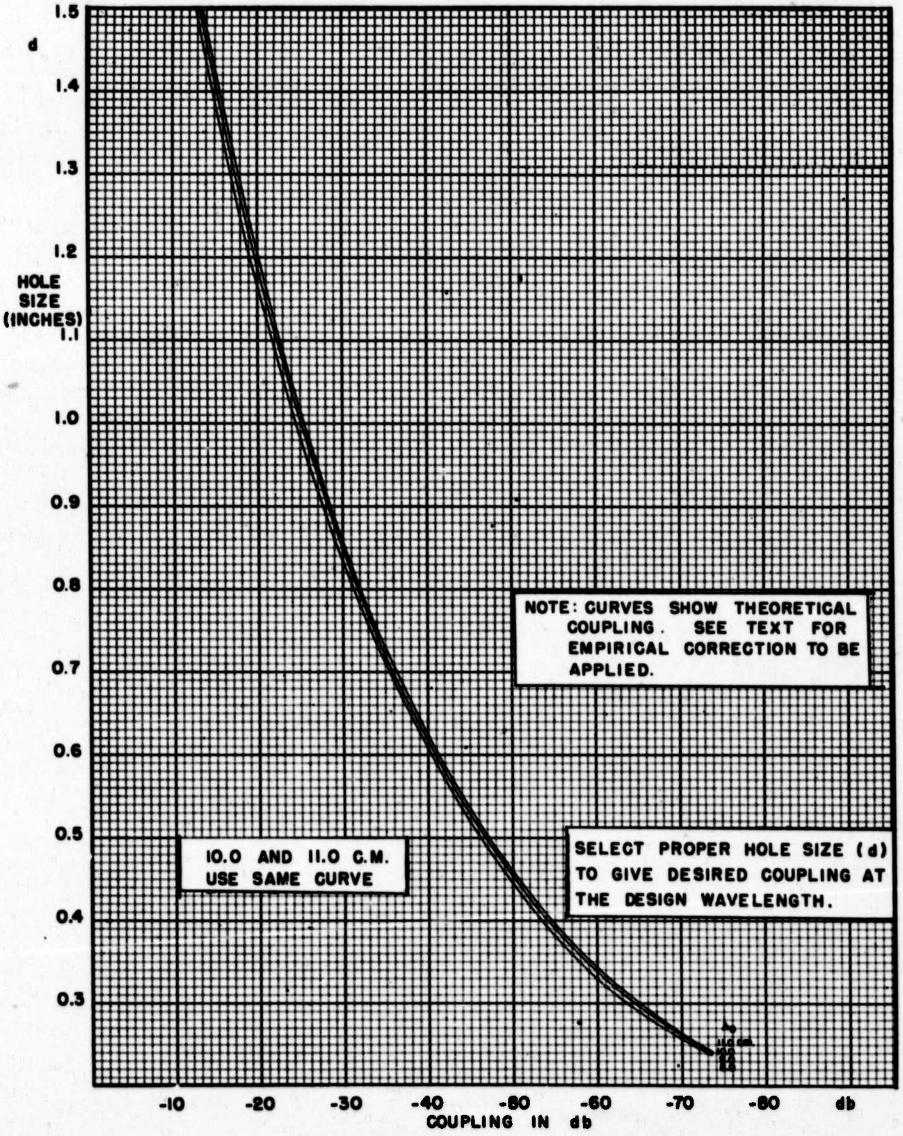


FIG. 72

BETHE HOLE COUPLERS IN 3" x 1½" x .081 WALL WAVEGUIDE
CHOOSE d AND θ ACCORDING TO DESIGN CURVES



NOTE: CURVES SHOW THEORETICAL COUPLING. SEE TEXT FOR EMPIRICAL CORRECTION TO BE APPLIED.

10.0 AND 11.0 C.M. USE SAME CURVE

SELECT PROPER HOLE SIZE (d) TO GIVE DESIRED COUPLING AT THE DESIGN WAVELENGTH.

FIGURE 73
DESIGN CURVES FOR BETHE HOLE COUPLERS
IN 3" X 1½" X .081" WALL WAVE GUIDE

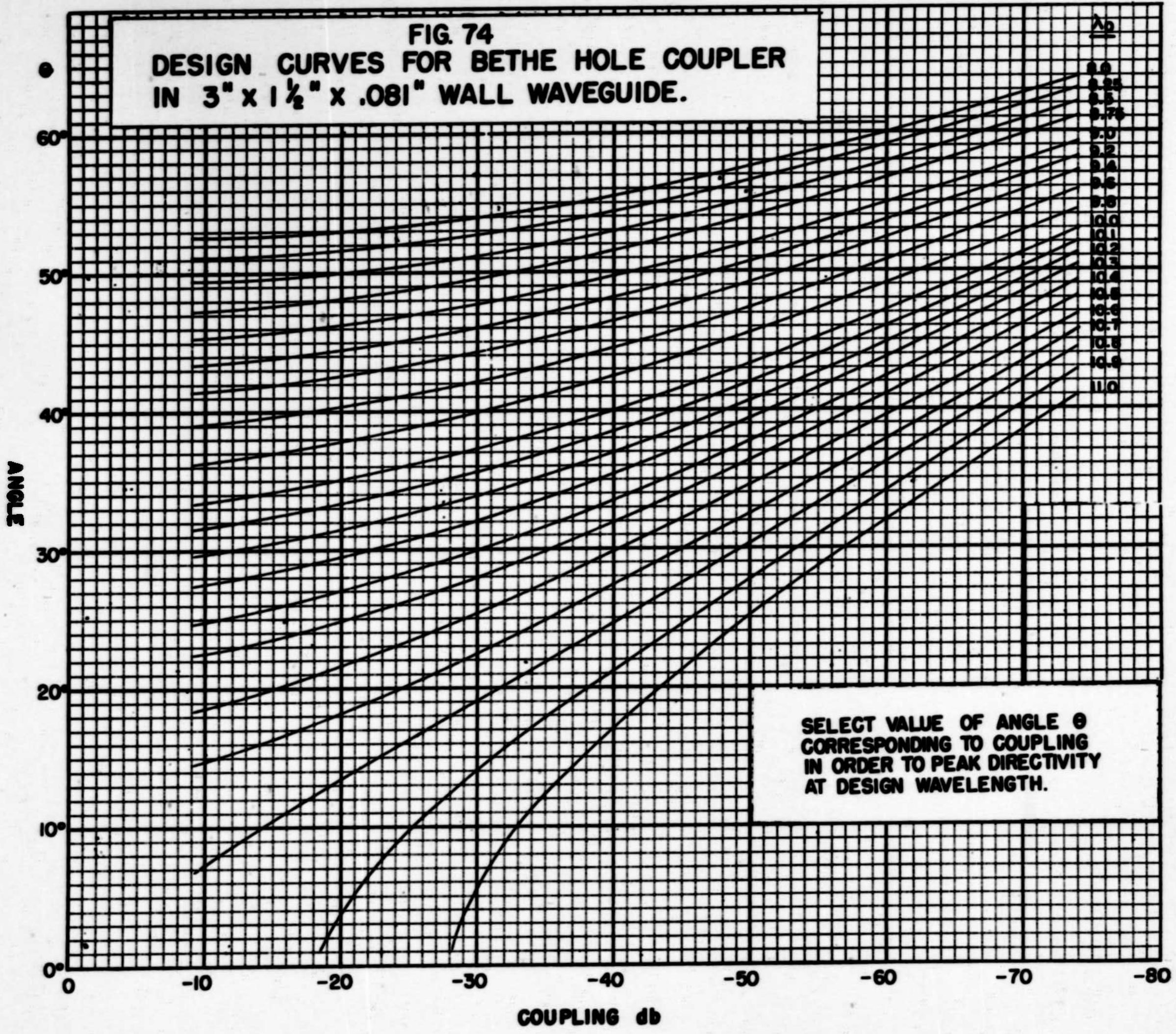


FIG. 74
DESIGN CURVES FOR BETHE HOLE COUPLER
IN 3" x 1½" x .081" WALL WAVEGUIDE.

SELECT VALUE OF ANGLE θ
CORRESPONDING TO COUPLING
IN ORDER TO PEAK DIRECTIVITY
AT DESIGN WAVELENGTH.

of the corrections applied for the attenuation through the double wall thickness, this deviation from theoretical is not too surprising.

Comparison of Experimental and Theoretical Results on S-Band Bethe Hole Couplers.

Dimensions				Coupling (db)		Theor. Minus	Directivity	
Small Hole	Large Hole	Angle	Wavelength	Theor.	Exper.	Exper. (db)	Theor.	Exper. (db)
.740	1.375	52°	8.5	-34.13	35.7	1.6	30.01	15.
.740	1.375	25°	10.7	-34.73	36.1	1.4	30.9	24.0
.740	1.5	38 1/2°	10.0	-34.6	35.7	1.1		21.
.828	1.375	30°	10.7	-31.54	-32.6	1.1	30.2	23.
1.105	1.5	30°	10.7	-22.84	-24.4	1.6	28.9	20.
1.3	1.5	34 1/2°	10.0	-17.8	-10.5	1.7		15.
.406	1.5	35°	10.7	-54.2	-54.7	.5		19.

Figure 75

Although the measured directivity is well below the theoretical directivity, it is still sufficiently high for power monitoring requirements. That there is greater deviation from theoretical results for the directivity than for the coupling might be expected, since the directivity is a more sensitive function of the relative amplitudes of the E and H coupling. Also, it is not clear just what effect the reflection introduced by the Bethe hole itself, and in particular by the large hole in the secondary guide, has on directivity.

Summary of Design Information for Bethe Hole Couplers in 3" x 1 1/2" Waveguide: Bearing in mind the experimental results, the design procedure may be summarized as follows:

1. The value of coupling coefficient to be used in determining d, and θ from Figures 73 and 74 should be approximately 1.5 db closer than the desired coupling.

2. d and θ are then determined from the Figures 73 and 74 as functions of the design wavelength and this empirical coupling value.

3. Once the coupler is built using these values of d and θ, the phase and magnitude of the reflection set up by the coupler may be measured, and a matching iris put in, if necessary.

4. Coupling and directivity over the band may then be measured.

5. If the coupler does not meet specifications, a redesign may be called for. It is usually possible, using the first four steps of this procedure to obtain a directional coupler whose coupling is within 1/2 to one db of the desired coupling and whose directivity is greater than 15 db at the design wavelength. By a small change in hole size and angle (step 5), the coupling and directivity specifications may be met more exactly.

It would seem desirable to illustrate this procedure especially with regard to Step 5 by tracing these steps as applied to the design of a directional coupler which had to meet the following specifications:

Coupling coefficient: -20 db.

Directivity: greater than 15 db.

Band: 9.7 cm to 10.35 cm.

The relatively close coupling that was required with this coupler suggested at once that the formulas used in drawing up the charts might not be valid for this range. However, as a first try, a hole diameter of 1.300" was chosen which corresponds on Figure 73 to -18 db coupling, and an angle of 34 1/2° was chosen from Figure 74 to peak the directivity at 10.0 cm.

A model was made with these dimensions, and measurements made on VSWR and the phase of the reflection set up in the main line. The measured VSWR was 1.53. Using the phase data and a Smith chart, the admittance referred to the position of the hole was determined to be .92 - .4j, the hole appearing mainly inductive.

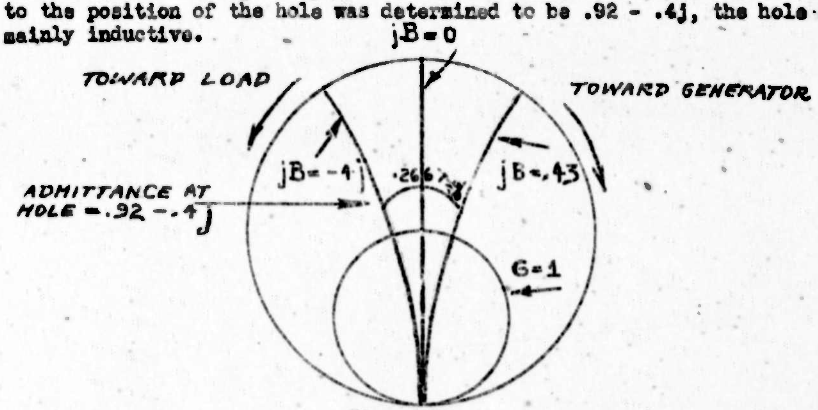


Figure 76

At a point .266 λ_g toward the generator (Figure 76), this admittance appears to be 1 + .43j. A shunt reactance of -.4j placed at this point would theoretically match out this reflection. A symmetrical inductive "iris" was designed (using the design curve given in Figure 27 of Microwave Techniques, NavShips 900,028) which gave the desired result.

Measurements were then made of coupling and directivity of the coupler over the band. The results were as follows:

Wavelength	Coupling	Directivity
9.72 cm	-18.5 db	12.4 db
10.0	-19.5	14.9
10.34	-19.8	28.

The fact that the directivity was peaked near the long wavelength end of the band indicated that the angle should be increased. The magnitude of this change was estimated semi-empirically from the fact the Figure 74 says that a coupler which has a 34 1/2° angle and which is peaked in directivity at 10.7 cm, could be made to have optimum directivity at 10.0 cm by increasing the angle to 43°.

The coupler was unsoldered and the angle changed to 45°. The coupling and directivity were remeasured.

λ_0	C	D
9.68	-20.1 db	18.5 db
10.0	-20.8	24.
10.35	-21.4	20

The model now had acceptable directivity, but the change in angle had changed the coupling by -1.3 db, or is approximately 10 log cos $[45°/\cos/(34.51°/2)]^2$. In order to increase coupling to -20 db, the hole diameter was increased to 1.33". This change was determined by reference to Figure 73 and seeing what change in hole diameter was necessary to produce a change of .8 db for a 20 db coupler. The final model was then tested. It was found that the original matching iris still was satisfactory. The results follow in Figure 77.

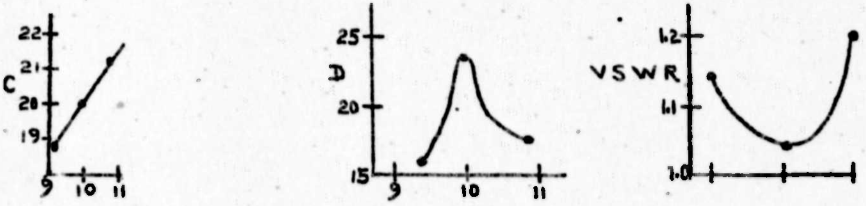


Figure 77

This model met specifications.

The use of Bethe hole directional couplers is not restricted to 3" x 1 1/2" waveguide. In fact, one of the first of the Bethe hole type made was in 1" x 1/2" waveguide. (No curves have been plotted for designing couplers in this size guide, and, therefore, the formulas must be used.)

Bethe hole couplers in 1/2" x 1/4" waveguide have been made, and Figures 78 and 79 give graphically the design data for this type. These charts are adapted from similar ones prepared by B. A. Lippmann. As can be seen from Figure 78, these couplers are constructed by drilling a hole of the proper size in the wall of the primary guide and milling off the wall of the secondary guide at the proper angle. As with the S-band couplers, for close couplings, the hole sets up an appreciable standing-wave-ratio which may require correction. The coupling of a "-45 db" coupler made this way agreed with the design curve within 1/2 db.

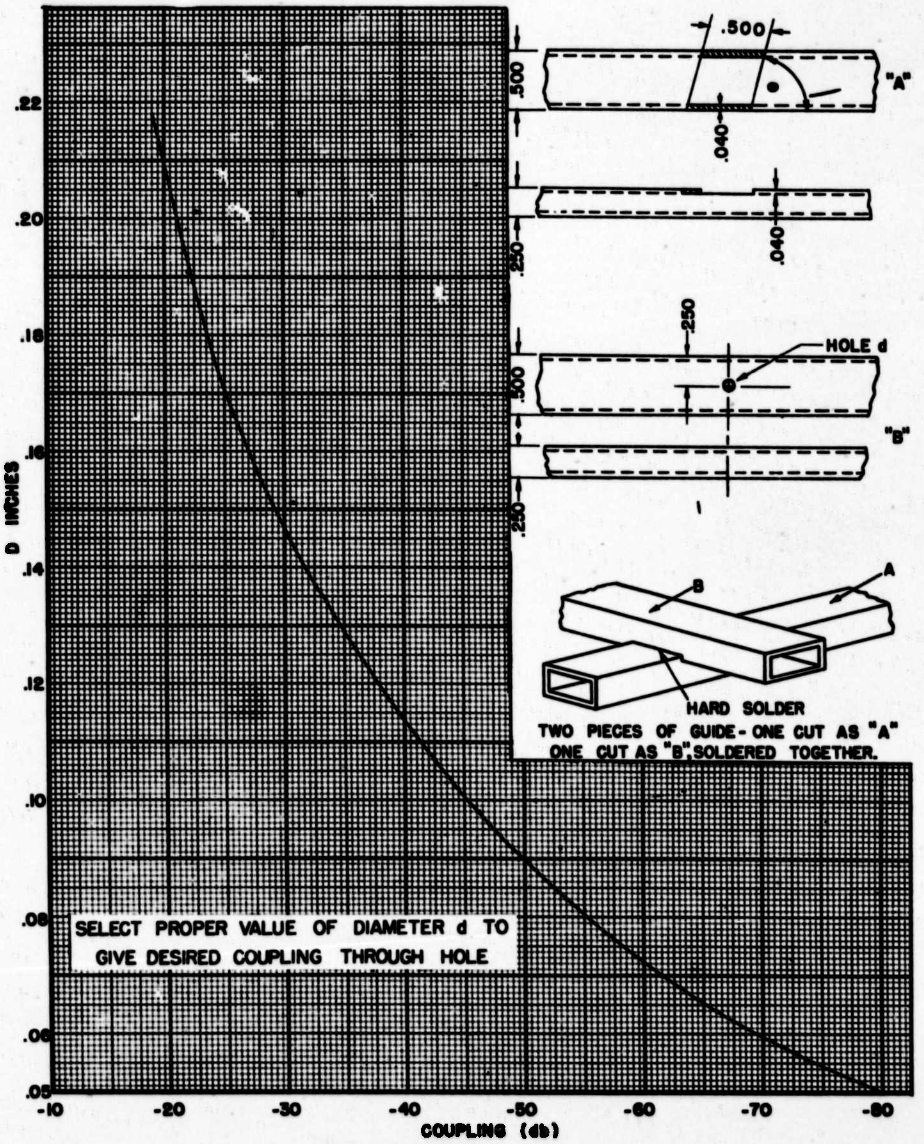
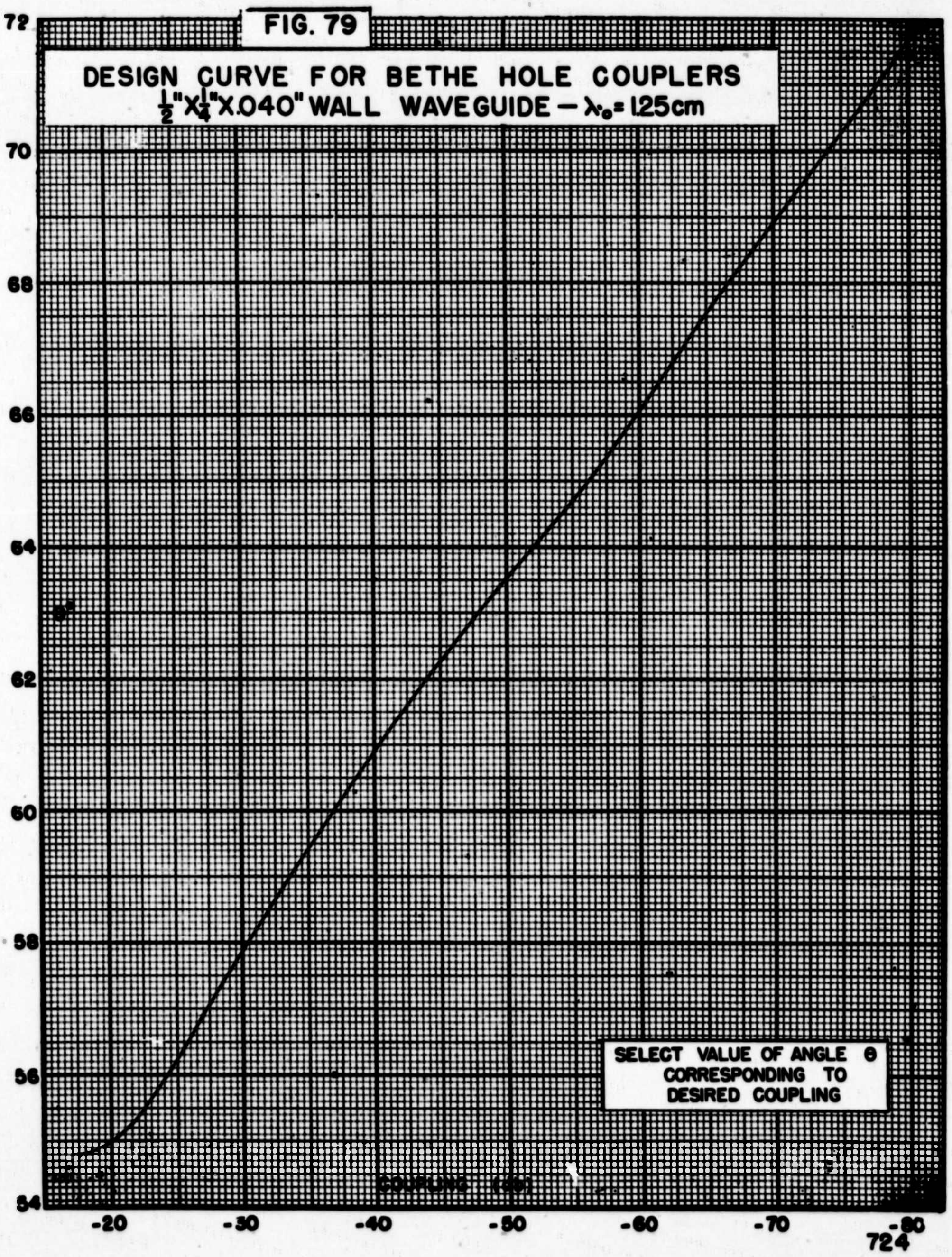


FIG. 78

DESIGN CURVE FOR BETHE HOLE COUPLERS IN ½" X ¼" X.040" WALL WAVEGUIDE — $\lambda = 1.25$ cm.

FIG. 79



DESIGN CURVE FOR BETHE HOLE COUPLERS
$\frac{1}{2}$"X$\frac{1}{4}$"X.040" WALL WAVEGUIDE — λ_0= 1.25cm

SELECT VALUE OF ANGLE θ CORRESPONDING TO DESIRED COUPLING

<u>Bethe Hole Directional Couplers in Coaxial Line</u>: Directional couplers of the single hole type may also be made in coaxial line. The principle of operation is essentially the same as for waveguide. The coaxial Bethe hole couplers are inherently much broader band with respect to directivity. This is related to the fact that the ratio of E to H for coaxial lines is independent of frequency. In fact, for thin round holes between two coaxial lines transmitting the principal mode only, the theoretical angle of crossing for optimum directivity is 60°, regardless of wavelength or line dimensions.

More exactly, the angle for optimum directivity is given by:

$\cos \theta = 1/2\ (\text{att } E/\text{att } H)$ volt., where (att E/att H) has the meaning as discussed previously for the case of waveguide couplers.

Theoretically, the coupling, expressed as a power ratio should vary inversely as the square of the wavelength (or directly as the square of the frequency).

A coupler of this type, (RL Drawing No. 11957) between two 7/8" coaxial lines, at 9.1 cm was measured to have a coupling of -50 db, and a directivity of greater than 25 db. Limitations on hole size keep couplings of much closer than -45 db from being attained with this type design.

<u>"Short" Slot Coupler</u>: An electromagnetic type directional coupler, utilizing a narrow-longitudinal slot slightly more than a quarter-wavelength long between two coaxail lines, was designed by Dr. Brown of CRG at NRL for use at L-band. The coupling is in the reverse direction, as is generally characteristic of electromagnetic type couplers. In addition, the coupling coefficient was found to be extremely broadband. The action of this coupler may be explained in the same fashion as was the action of the long slot coupler, except for the fact that the secondary waves sent in the "backward" direction have an amplitude large compared to the forward radiation. The secondary waves will combine to form the maximum "backward" amplitude for slots of length $\lambda/4$. For couplers in which the slots are somewhat larger than $\lambda/4$, there would therefore be a tendency for the coupling to get closer with longer wavelengths. This is opposed to the effect due to the frequency sensitivity of the individual coupling elements, which would be to produce looser couplings with longer wavelengths. If these two effects are equal, the coupling may be broadbanded.

F. INDEX TO COUPLING MECHANISM DESIGN INFORMATION.

This is a tabulation of the characteristics with respect only to coupling value and kind of transmission line of the various types of directional couplers which have been described in the previous sections. In view of the numerous other specifications which must be met in the design of a coupler, the tabulation can only serve as a partial indication of which type couplers should be investigated more carefully for details of their performance and design.

Figure 81 lists those couplers for which the design information given in this report is complete enough so that by following a simple and straightforward procedure a directional coupler may be built to have, at the design wavelength, the desired coupling coefficient, and a directivity high enough for most power monitoring requirements.

The second table, Figure 22, lists couplers which can be made at the required coupling value, but will generally require some amount of experimentation, involving the construction of a series of models before the final design is obtained.

By putting two couplers in series, a composite coupler is formed whose coupling value in db is the sum of the couplings in db of the individual coupler, and whose directivity is essentially determined only by the directivity of the first coupler. By combining two couplers having opposite frequency sensitivities of coupling, such as a Bethe hole, and a two-hole coupler in waveguide, an over-all frequency insensitivity of coupling may be obtained.

A fairly complete list of these couplers for which RL drawings are available may be found in Report 55-2/13/45, Index List of Directional Couplers.

Coupler Types--Abbreviations Used, and Summary of Characteristics:

Multiple-Path Couplers: The coupling versus frequency characteristic depends almost entirely upon choice of coupling elements. "Binomial" array of elements gives increased frequency insensitivity of directivity. Either these, or simply arrays of n equal elements spaced $(2m+1)\ \lambda_g/n$, where m and n are integrals, may be used to obtain closer couplings than could be gotten with only two elements.

b.g. = "branched guide" coupler. The coupling elements, when properly designed, are characteristically extremely frequency insensitive with respect to coupling.

r.h. = "round hole" in side of guide coupler. The power coupled through such holes is approximately proportional to λ_g^2.

r.s. = near - "resonant slot" in side of guide. The coupling may be made broadband for some slot geometries.

s.b. = "slotted block" coaxial coupler. Coupling increases rather rapidly with frequency.

cc. (o.i.) or (i.o.) = "concentric coax"--"outside-in" or "inside-out". Coupling increases with frequency.

b.c. = "branched coax" coupler. Coupling is fairly frequency insensitive.

Other Types:

B.h. = "Bethe hole" coupler. For waveguide couplers, coupling (power) varies as $1/\lambda_g^2$. For coax couplers, variation is as $1/\lambda_0^2$. Directivity of coax couplers broadband.

l.s. = "long slot" coupler. Coupling fairly frequency sensitive. Directivity fairly frequency insensitive.

Schw. = "Schwinger" type inverse coupling directional coupler. The coupling may be made broadband for some slot geometries.

Table

Coupling Values (db)

Kind of Transmission Line	(-)5-10	10-15	15-20	20-25	25-30	30-35	35-45	45-60
Waveguide								
1/2"x1/4"x.040"wall	b.g.	b.g.	b.g	r.h. b.g.	r.h.	r.h. B.h.	r.h. B.h.	r.h. B.h.
1"x1/2"x.050"wall	b.g.	b.g. r.h. l.s.	b.g. r.h. l.s.	r.h.	r.h. B.h.	r.h. B.h.	r.h. B.h.	r.h. B.h.
5/8"x 1 1/4"x.064"wall	b.g.	b.g.	b.g.	r.h. b.g. r.s.	r.s. r.h.	r.h.	r.h. B.h.	r.h. B.h.
3"x 1 1/2"x.080"wall	b.g.	b.g. r.h.	b.g. r.h.	r.h. b.g. B.h.	r.h. B.h.	B.h. r.h.	B.h. r.h.	B.h. r.h.
Coaxial Line								
7/8" O.D. 50 ohms		c.c. (o.i.)	c.c. (o.i.)	c.c. (o.i.)	c.c. (o.i.) s.b.	c.c. (o.i.) s.b.	s.b.	B.h. (50 db at 9.1)
5/16" O.D. 50 ohms		c.c. (o.i.)	c.c. (o.i.)	c.c. (o.i.)				

Figure 81

Table

Coupling Values (db)

Kind of Transmission Line	5-10	10-15	15-20	20-25	25-30	30-35	35-40	45-60
Waveguide								
1/2" x 1/4" x .040" wall	l.s.	l.s	l.s	B.h. l.s. r.s.	B.h. r.s.	r.s.	Schw.	Schw.
1" x 1/2" x .050" wall	l.s.	r.s.	r.s. B.h.	r.s. B.h. Schw.	r.s. Schw.	Schw.	Schw.	
5/8" x 1 1/4" x.064" wall	l.s.	l.s.	l.s.	B.h. l.s. r.s.	B.h. r.s.	r.s.	Schw.	
3"x 1 1/2"x.080" wall	r.h. l.s.	l.s	B.h. l.s.	r.s. Schw.	r.s. Schw.	r.s. Schw.	Schw.	Schw.
Coaxial Line								
7/8" O.D. 50 ohms.	b.c. c.c.	b.c.	b.c.			c.c. (o.i.)	B.h. c.c. (o.i.)	
5/16" O.D. 50 ohms.	c.c. (i.o.)				c.c. (i.o.)	c.c. (i.o.)	s.b.	s.b.
5/8" O.D. 50 ohms.	c.c. (i.o.)	c.c. (i.o.)	c.c. (i.o.)	c.c. (i.o.)		s.b.		B.h.
1 5/8" O. D.								

Figure 82

G. PROCEDURES FOR TESTING DIRECTIONAL COUPLERS

After a directional coupler has been designed and a prototype model built, measurements must be made to determine whether the model meets electrical specifications with regard to values of coupling, directivity, and both main and auxiliary line VSWR's. These measurements do not differ fundamentally from any other r-f measurements of attenuation and impedance.

Thus, standard slotted-line technique may be used to determine the magnitude and phase of the reflections which are set up by the coupler, both in the main line, and looking into the auxiliary line output terminals.*

The determination of coupling is a straight forward attenuation measurement between input and output terminals of the coupler as shown in Figure 1. A second "attenuation measurement" with the coupler inserted in the line "backwards", as in Figure 2, is then sufficient to determine the value of the directivity.**

The directivity may also be determined by various other methods. One of these involves the measurement of "Directivity-Standing-Wave-Ratio" and has been described on page 7 of this report. This procedure has an advantage over others in that it is a null method, and is therefore unaffected either by the generator or detector mismatches.*** Another procedure which is, however, more critically dependent upon generator match is to feed power in the main line input terminals and note the maximum variation in power out the auxiliary line output terminals when a shorting plunger or other known VSWR is moved up and down to vary phase at the main line output terminals. The directivity may then be determined from Figure 7.

Errors Involved in Measurements: Directional coupler measurements are subject to the same errors that usually arise in connection with the determination of attenuation and impedance. The magnitude of these errors depends

---

* If this reflection is objectionable, an iris or transformer may be designed to match it out according to the usual procedures (such as those described in Chapters III and IV of Microwave Transmission Design Data, Sperry Publication No. 23-80).

** It should be remembered that the match of the load used to terminate the main line in the latter measurement is critical insofar as the measured directivity is affected by mismatches in the main line to the same degree that the actual directivity is affected by a mismatched termination (see Section II-A of this report).

*** When using this method, consideration should be given as to whether the detector has sufficient sensitivity. Also, if a square wave modulated r-f signal is used, care must be taken to be sure that there is not excessive frequency modulation, or else there may be difficulty in reducing output to zero.

to a large extent upon how refined a technique is used. Thus, in impedance measurements, greater precision than is attainable with ordinary slotted-line methods may be obtained by the use of auxiliary equipment, such as impedance bridges calibrated by means of sliding mismatches.

It may be helpful to point out some of the major sources of error which arise in the usual procedures used in measurement of attenuation. These are:

1. Errors inherent in standards of attenuation used.

2. Errors due to various combinations of mismatches in the coupler and lines connected to the coupler. These errors may become especially serious where non-dissipative (cutoff) attenuator, or where type-N or other cable fittings are used.

3. Errors inherent in setup--arising because of loose connections or improperly constructed fittings, oscillator or detector instability, attenuation of cables changing when flexed and similar difficulties.

4. "Leakage Error.": This is especially troublesome in the measurement of loose couplings. It can only be avoided by fairly complete shielding. A leakage signal which is 20 db down from the signal to be measured, may still cause an error of ± .9 db.

Coupler Measurements in Group 55.1: In this Group we have been concerned primarily for power monitoring purposes, and consequently, have not used the refined techniques necessary to measure high directivities accurately. The procedures used in the measurement of directivity have either been the "turn-around" method of measuring backwards and forwards attenuation, using a load in the main line with VSWR < 1.04, or what is usually more convenient, the "DSWR" null method, measuring standing-wave-ratios with an ordinary slotted section.

See Report RL-643; The Use of the Magic Tee Microwave Bridge in Measuring Impedance by R. L. Kyhl, or 55-1/5/45, Side Outlet Tee Impedance Bridge Bench Techniques, by R. N. Griesheimer and R. L. Kyhl.

BTL Report MM-44-170-18; A Precision Impedance Comparator, by R. Julian (3-10-44) also contains a description of the use of a sliding mismatch in precision measurements.

The setups which have been used in this Group in connection with the measurement of coupling will be described in some detail in order to give more concreteness to the description of measurement techniques, and in addition, to point out the manner in which most of the experimental data described in this report were obtained. It should be remembered that these setups are just some of the possible ones that may be used, not necessarily the best, and were chosen in the light of equipment, space, and time available in Group 55.1.

The setup used at S-band for measuring waveguide directional couplers with type-N output is shown schematically in Figure 83.

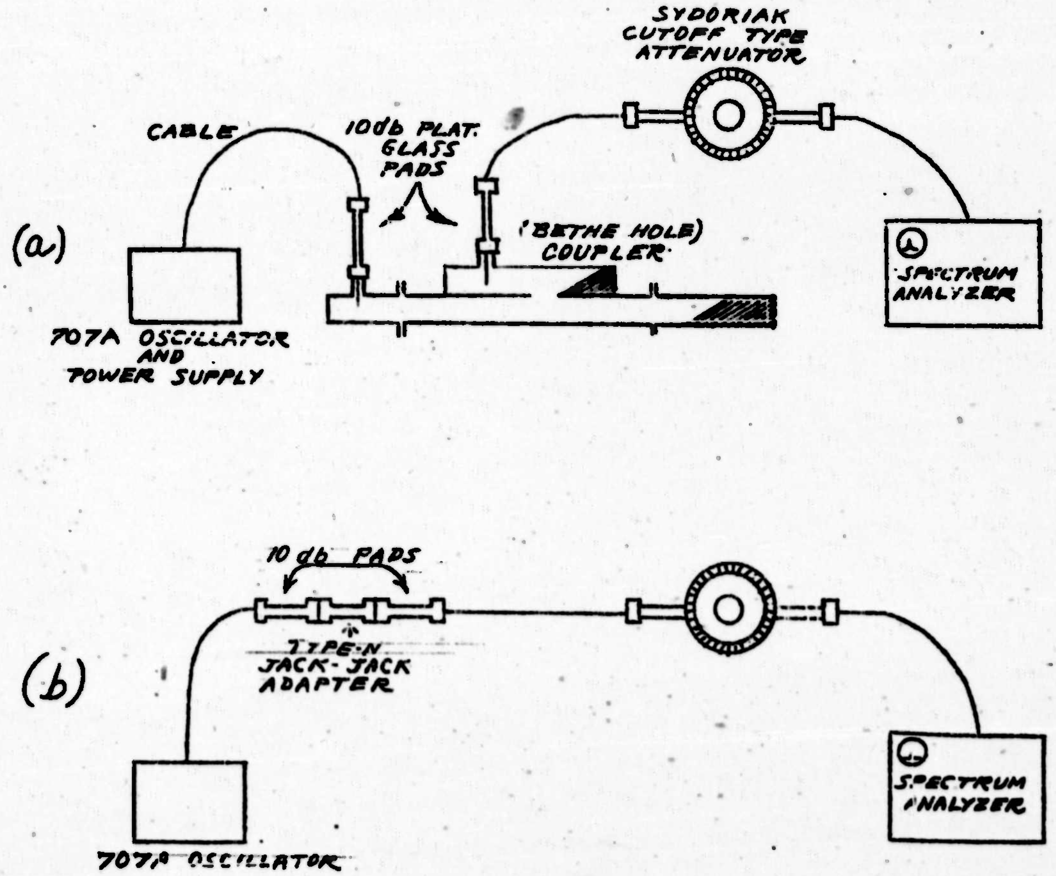


Figure 83

A series-substitution method is used. First, the power is fed through the coupler as in (a), with the signal height on the analyzer set to reference level. Next, a type-N jack-jack adapter is substituted for the combination of coupler and guide-coax adapter, and the standard cutoff attenuator, which is in series with the line, is readjusted until the signal is restored to its original setting. The difference between the original and final attenuator settings determines the db coupling value.

Three measurements are made, and an average is taken of these readings. With coupling of -35 db or closer, if care is taken in the measurement, these individual readings will agree to within ± .3 db, and the averages so obtained may be reproduced on the same coupler from day to day to within ± .2 db.

For much looser couplings, the day-to-day reproducibility is not better than about ± .5 db, because of the increased jitter of the pip on the analyzer at these lower signal levels.

In order to achieve these results, the following precautions are observed before making measurements:

1. All fittings are mechanically inspected to be sure that they will go together properly.

2. Checks are made to be sure that the guide-coax adapter and jack-jack adapter do not introduce loss in the line. Matches of these and of the 10 db pads should be checked.

3. When the cables are flexed to at least the same extent that they would be in performing the measurement, there should be no observable change in transmission through them.

4. Both the oscillator and the analyzer receiver should be sufficiently stable so that the pip height is resetable.

5. Evidences of leakage should be looked for--by setting the attenuator to maximum and then either looking for leakage signal with analyzer at maximum gain, or with signal adjusted to a fairly low level, try to detect leakage by phasing signal with respect to suspected source of leakage.

For measurement of couplers in 7/8" coax rigid line, almost the same setup is used (Figure 84), except that well-matched pads in 7/8" line are used rather than the type-N pads, so that the substitution may be made without errors due to mismatch of the 7/8" line to type-N adapters.

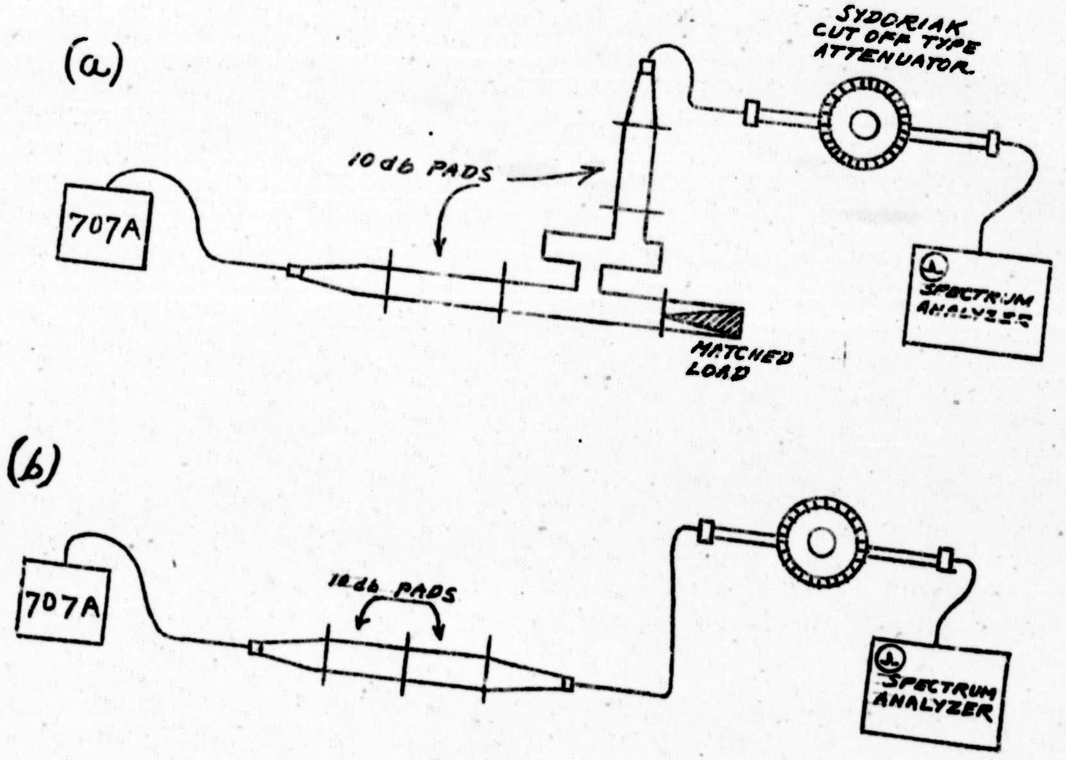
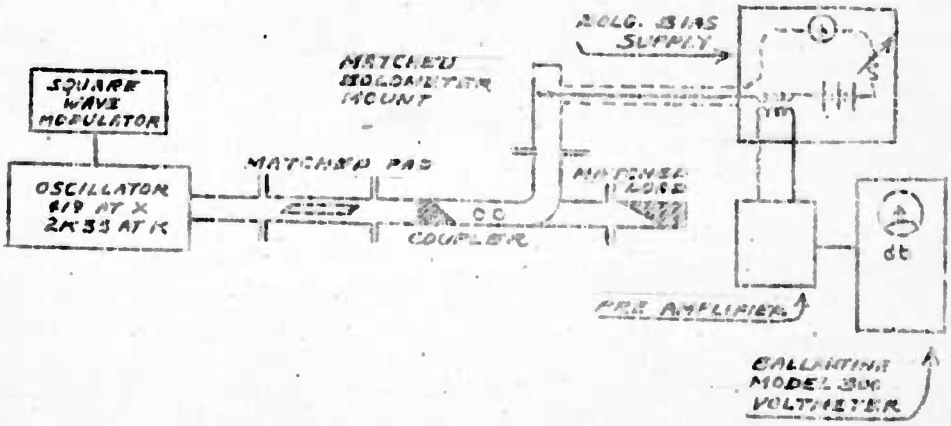


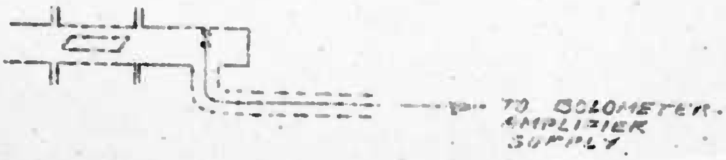
Figure 84

A "Sydoriak type" waveguide-beyond-cutoff attenuator has been used in these measurements at S-band as the primary standard of attenuation, assuming the theoretical slope (db/inch). The diameter of the cutoff tube, the rack and pinion drive and the dial were checked to be sure that they were right. In addition, a calibration of the cutoff attenuator, using a square-wave modulated source in connection with a bolometer-amplifier detector, assuming the bolometer to be square-law in its response, was made and agreement with theoretical slope obtained to within experimental error.

At X and K-bands, more use has been made of the bolometer-amplifier in the measurement of attenuation. Figure 85 shows the setup used in measuring couplers with both input and output terminals in waveguide.



(a) Power fed through coupler.



(b) Power fed directly to bolometer mount.

Figure 85

The bolometer is maintained at a constant d-c bias level, and the a-c variation superimposed upon this due to the modulated r-f signal is amplified by means of a linear preamplifier, the output of which is fed into a Ballantine (model 300) voltmeter, having a logarithmic scale. The measuring procedure used is still essentially a series-substitution method, power to the bolometer first being measured after being coupled through the directional coupler and then directly.

The relative accuracy with which measurements of coupling may be made with this system is quite high. For couplers with a coupling value of about -20 db, measurements can be repeated from day-to-day within ± .1 db. As far as absolute accuracy is concerned, this bolometer method has been checked against a cutoff attenuator at X-band, with agreement to .2 db in a 40 db range.

The increased reproducibility in readings which is attained here is due to both the increased accuracy in reading a steady pointer deflection on a meter scale over adjusting mechanically an attenuator line to bring a somewhat unsteady pip height to reference deflection on a scope and then reading the attenuator dial, and to the fact that the errors due to mismatches may be reduced so as to be negligible.

For measuring waveguide couplers at X-band having type-N outputs, the setup shown in Figure 86 has been used. Padding is utilized to reduce errors due to VSWR's at the type-N connectors.

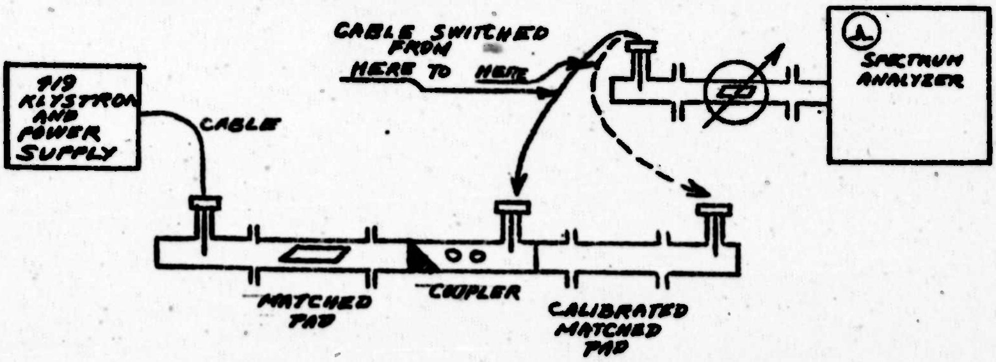


Figure 86

The difference in power level between the two type-N outputs is measured, and a correction made for the power lost in the calibrated pad, and also the fact that the power going by the coupler rather than into the coupler is being measured.

Reproducibility here (for -20 db couplers) is about ± .2 db for day-to-day measurements, and ± .5 db in individual readings.

Acknowledgement

The fundamental theory underlying the information given in this report was contributed almost entirely by people outside the Test Equipment Group. This report does however summarize experience gained by a number of individuals in this Group regarding the actual design and performance of directional couplers to be used in conjunction with other microwave test equipment in the measurement of system performance. Those in the Group who have contributed substantially to the work described in this report include W. C. Beckstrand and J. M. Wolf.

R. J. Harrison
November 5, 1945.

Appendix A

Application of Bethe Theory to Directional Coupler Design

The results given by Bethe in RL Reports 43-22, 43-26, and 43-27 for the case of coupling between two waveguides (or coaxial lines) by means of a hole which is small compared to the wavelength are extremely useful in directional coupler design. Some of these results are summarized below. It is assumed in Bethe's theory that the hole is very small, is in an infinitely thin plane wall, that it is far from corners, and that measurements are made in a region sufficiently far from the hole so that the higher modes there excited have been damped out. Agreement with experimental results is usually quite good, even for cases where these assumptions are not strictly valid.

Figure 87 shows schematically the situation of two waveguides coupled by means of a hole in a common wall. It is desired to determe the amplitudes of the waves A_a and B_a travelling in the two directions in the auxiliary guide, which are excited by a travelling wave of unit amplitude in the main guide.

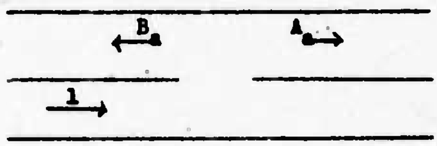


Figure 87

If H_{ol} and H_{om}, the tangential components of H in the direction of the principal axes l and m of the hole, and E_{on} the normal component of E represent the field components (leaving out the time and z dependence) that a travelling wave of unit amplitude in the exciting guide would have at the position of the hole were the hole replaced by a thin conducting wall; and if, similarly, H_{al}, H_{am}, and E_{an} represent the field components at the position of the hole corresponding to a travelling wave of unit amplitude in the auxiliary guide, then the amplitudes A_a, and B_a are given by the following formulas:

$$A_a = \frac{\pi i}{\lambda_o S_a}\left[-M_1 H_{ol} H_{al} - iM_2 H_{om} H_{am} + P E_{on} E_{an}\right] \qquad (1)$$

$$B_a = \frac{\pi i}{\lambda_o S_a}\left[+M_1 H_{ol} H_{al} - iM_2 H_{om} H_{am} + P E_{on} E_{an}\right], \qquad (2)$$

where S_a is a normalizing factor, equal to $\int \vec{E}_a \times \vec{H}_a . d\vec{A}$, the integral being taken over the waveguide cross section, and where M_1, M_2 and P, the two magnetic and one electric "polarizabilities", are constants characteristic of the hole dimensions only. Report 43-22 gives theoretical values of these polarizabilities for simple hole shapes and outlines, and a procedure which can be followed to determine them experimentally for other shapes. The following table lists values of M_1 M_2 and P copied from page 39 of Report 43-22, for the simple cases of round holes, ellipses, and slits.

Values of Polarizabilities for Simple Hole Shapes

Circle: diameter d

$$M_1 = M_2 = d^3/6, \quad P = 1/2\, M_1 = d^3/12.$$

Ellipse:

$$M_1 = \frac{\pi}{3} \frac{ab^2\varepsilon^2}{(1-\varepsilon^2)[F-E]} \quad \text{(for H parallel to semi-major axis, a).}$$

$$M_2 = \frac{\pi}{3} \frac{ab^2\varepsilon^2}{E-(1-\varepsilon^2)F} \quad \text{(for H parallel to semi-minor axis, b).}$$

$$P = \frac{\pi}{3} \frac{ab^2}{E}, \quad E(\varepsilon) \text{ and } F(\varepsilon) \text{ are complete elliptic integrals.}$$

$$\varepsilon = \sqrt{1-\left(\frac{b}{a}\right)^2}, \quad \varepsilon \text{ is the eccentricity.}$$

Long Narrow Ellipse (a b):

$$M_1 = \frac{\pi}{3} \frac{a^3}{\ln(4a/b)-1}, \quad \text{(H parallel to long axis).}$$

$$M_2 = P = \frac{\pi}{3}\, ab^2$$

Slit, Width w, Length l, field constant along slit:

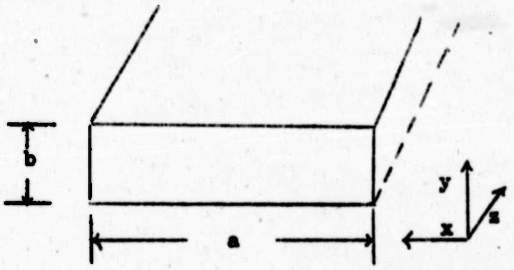
$$P = M_t = \frac{\pi}{16}\, lw^2, \quad (M_t \text{ refers to a magnetic field across the slit.})$$

Ditto, Field Varying as $e^{i\gamma y}$

P as before.

$$M_t = P(1-\gamma^2/k^2).$$

For rectangular waveguide, (TE_{o1} mode), the amplitudes of the field components, for a wave travelling toward +z in the x, y, and z directions (figure 88) are:



$$(3a)\ E_{ay} = \frac{\lambda g}{\lambda_o} \sin \pi \frac{x}{a}$$

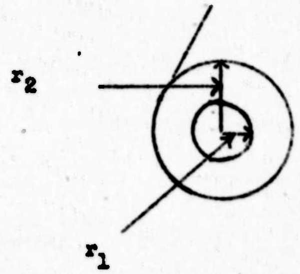
$$(3b)\ H_{ax} = -\sin \pi \frac{x}{a}$$

$$(3c)\ H_{az} = -1 \frac{\lambda g}{2a} \cos \pi \frac{x}{a},$$

$$(3d)\ S_a = \frac{\lambda g}{2\lambda_o} ab.$$

Figure 88

For coaxial line transmitting the principal mode, the amplitudes of the field components in the r, φ, z directions for a wave travelling towards +z, is given by:



$$(4a)\ H_{a\varphi} = E_{ar} = \frac{r_2}{r}$$

$$(4)\ H_{az} = 0$$

$$(4d)\ S = 2\pi r_2^2 \ln \frac{r_2}{r_1}$$

Figure 89

Approximate corrections to the theory to take the finite wall thickness into account are as follows:

The "exciting field" components H_{o1}, and H_{om}, and E_{on} are taken as those which would have existed for an infinitely thin wall, multiplied by the appropriate attenuation factor, considering the hole with thick walls as a small section of waveguide beyond cutoff for the lowest H and E modes respectively.

Application of Theory to Directional Coupler Design

Consider the case of coupling between two waveguides joined together by means of a hole in their narrow side (as in Figure 21). For this case:

$H_{o1} = H_{ox} = 0$, $H_{a1} = H_{ax} = 0$

$H_{om} = H_{oz} \cdot \text{att } H = -i \frac{\lambda g}{2a} \text{ att } H,$ $\qquad$ $H_{am} = H_{az} = -i \frac{\lambda g}{2a}$

$E_{on} = E_{oy} = 0$ $\qquad$ $E_{an} = E_{ay} = 0$

$$S_a = \frac{\lambda g}{2\lambda_o} ab$$

(1) becomes:

$$A_a = \frac{\pi i}{\lambda_o \frac{\lambda g}{2\lambda_o} ab} \left[-iM_2 \cdot -i \frac{\lambda g}{2a} \cdot \text{att } H \cdot -i \frac{\lambda g}{2a} \right]$$

$$A_a = - \frac{\pi \lambda g}{2a^3 b} M_2 \text{ att } H$$

Similarly,

$$B_a = - \frac{\pi \lambda g}{2a^3 b} M_2 \cdot \text{att } H .$$

For round holes, $M_2 = d^3/6$, and att H corresponds to the attenuation, expressed as a voltage ratio, for a wave in the $TE_{1,1}$ mode. For a two-element coupler, the forward amplitude becomes $2A_a$, and thus the coupling formula becomes:

$$\text{Coupling}_{db} = 10 \log \left(\frac{\pi \lambda g d^3}{6a^3 b} \right)^2 + \text{att } H_{db}$$

where $\text{att } H_{db} = 20 \log (\text{att } H)_{volt} = -32.0 \frac{t}{d} \sqrt{1 - \left(\frac{1.71d}{\lambda_o} \right)^2}$

Bethe Hole Waveguide Coupler

Consider two guides crossed at an angle θ, with a hole in the center of their common wall (broad side of guide), as in Figure 72.

Here:

$E_{on} = E_{oy} \cdot \text{att } E = \frac{\lambda g}{\lambda_o} \text{ att } E,$ $\qquad$ $E_{an} = E_{ay} = \frac{\lambda g}{\lambda_o}$

$H_{ol} = H_{or} \cdot \cos \theta \cdot \text{att } H_s = -\text{att } H \cdot \cos \theta,$ $\qquad$ $H_{al} = H_{ax} = -1$

$H_{om} = H_{oz} = 0,$ $\qquad$ $H_{am} = H_{az} = 0.$

$$S_a = \frac{\lambda g}{2\lambda_c} \; ab$$

Substituting these in (1) and (2), one obtains:

$$B_a = \frac{2xi}{ab\lambda g} \left[M_1 \cos \Theta \cdot \text{att } H + P \, \frac{\lambda g}{\lambda_o}^2 \, \text{att } E\right]$$

$$A_a = \frac{2xi}{ab\lambda g} \left[-M_1 \cos \Theta \; \text{att } H + P \, \frac{\lambda g}{\lambda_o}^2 \, \text{att } E\right]$$

For a round hole, $M_1 = d^3/6$, $P = 1/2 \, M_1$, giving:

$$B_a = \frac{2xid^3}{3ab\lambda g} \cdot \text{att } H \left[\frac{\cos \Theta + 1/2 \, \frac{\lambda g}{\lambda o}^2 \, \frac{\text{att } E}{\text{att } H}}{2}\right]$$

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ABSTRACT

Directional couplers posses properties which make them useful for power monitoring purposes and impedance measuring or matching. The ection of e number of different general classes is explained qualitatively. Characteristics of specific types in both wave-guide and coaxial lines ere given together with information necessary for the design of these couplers to meet particular requirements. Emphasis is upon directional couplers used aa test points in reder systems to monitor transmitter power and measure receiver sensitivity.

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