

HOW DO PLASMA FLOW SWITCHES SCALE WITH CURRENT? ISSUES IN THE 6 MA TO 30 MA REGIME

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ABSTRACT

Point mass calculations are used to model switched implosions on several pulsed power machines. The model includes a lumped circuit representation of the pulsed power source. A simple switching model is used to describe a standard plasma flow switch. Implosion kinetic energies are obtained at a convergence ratio of 20 to 1. Heuristic arguments are used to estimate the plasma temperature at pinch, the total x-ray output and the radiation pulse width. Switched models are presented for Pegasus II, Shiva Star, Procyon and Atlas.

I. INTRODUCTION

A simple point mass model is used to calculate switched implosions on a variety of pulsed power machines. The kinetic energy and velocity of the load at a convergence of 20:1 are used to estimate the plasma temperature of a pinch, the total x-ray output and the pulse width. The discussion includes proposed Atlas parameters, but results for Pegasus II and Shiva Star switched implosions, as well as for Procyon high explosive (HE) driven generator driven implosions are included. For some of these experimental data and extensive 2-D models exist and may be used to put the Atlas results in perspective. The point mass model couples a single loop, lumped external circuit equation for a capacitor bank or HE generator to dynamic equations describing the load implosion. A dynamic model is included for the motion of the switch down the coax and for the transfer of current to the load (switching). The equations are expressed in units of g, cm and μ s. In this system the natural units include: current in 10^7 amps (DMA); voltage in 10^4 volts (DkV); inductance in nH; capacitance in mF; and resistance in $m\Omega$. The unit of energy is the energy unit, $1 \text{ eu} = 10^{12} \text{ ergs} = 0.1 \text{ MJ}$. Computational results will be expressed in conventional units. The pressure gradient in the plasma which produces bounce is not included in the point mass model.¹ Therefore, the calculations are followed only to a radius of about 0.25 cm, corresponding to a convergence of 20:1. This gives a pinch radius typical of some two-dimensional perturbed implosion models.

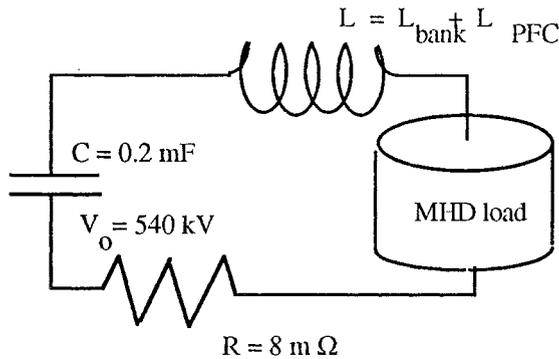


Figure 1. Single loop equivalent circuit model for capacitively driven pulsed power system.

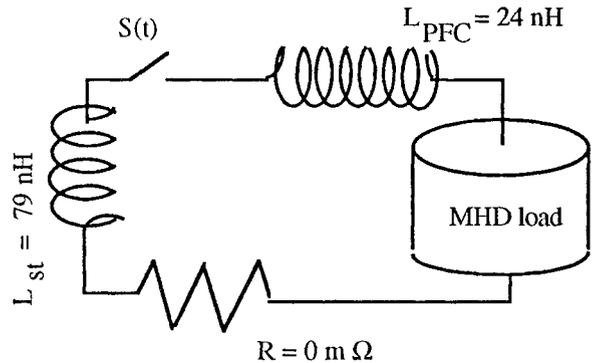


Figure 2. Single loop equivalent circuit model HE driven pulsed power system (Procyon).

Specific inductive store systems are represented by the equivalent circuits shown in Figure 1 (Atlas parameters for the capacitively driven system) and Figure 2 (Procyon parameters for the HE system). System specific values for the circuit parameters will be defined below. The bank or HE system is modeled by a single loop potential drop equation which relates the bank parameters or storage inductor parameters to the current and the change in inductance of the load. The inductance change of the switch is also included. The damping resistor has not been included,

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¹ Simple terms can be included in the model that would produce a bounce and pinch. The most physical is associated with an axial magnetic field seeded inside the load foil at the beginning of the implosion. Other, heuristic terms can also be constructed to represent the plasma pressure. These do not include the effects of shocks formed when the plasma stagnates on axis, and thus should not be considered reliable.

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although the equations below could be easily modified to include it. The current predicted without the damping resistor will therefore be an overestimate.

The point mass model discussed below is useful in understanding the coupling between prime power source, switch and load and the overall performance characteristics of the implosions. Extensive numerical models of these phenomena using 1-D and 2-D codes, as well as data from different pulsed power regimes, has shown that the behavior of the implosion is strongly dominated by instability effects. We have used the results of these studies as a guide in picking, for example, the convergence ratio at which to evaluate the radiation output from the model. In addition to their heuristic value, a comparison of the models below may be useful in studying trends.

Section II describes the point mass model for switched implosions in general. Section III discusses the results from switched implosions on Pegasus II, Shiva Star, Procyon and Atlas. Section IV summarizes the results.

II. POINT MASS MODEL

The point mass model² consists of a lumped circuit equation for the pulsed power system and the dynamic equations for the switch and the load. We consider a capacitively driven system first. The circuit equation is

$$\frac{Q}{C} + \frac{d(LI)}{dt} + IR = 0 \quad (1)$$

where Q is the instantaneous charge on the bank, C is its capacitance, I is the current, R is the bank resistance and L is the total inductance in the system. The latter is given by

$$L(t) = L_o + L_{PFS}(t) + f(z) L_{load}(t) \quad (2)$$

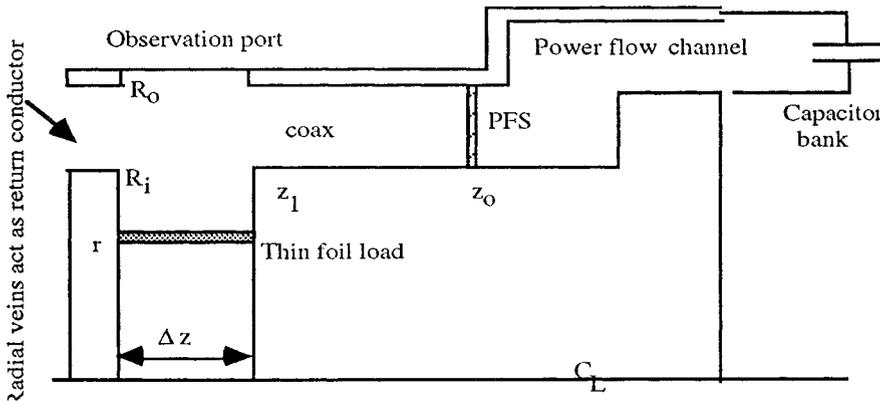


Figure 3. Schematic of a typical power flow channel and coax containing a PFS and a load foil. The PFS is located initially at z_o and moves to the left.

inductive change associated with switching into the load region. The inductance L_{PFS} depends on the switch's motion, and is given by (see Figure 2)

$$L_{PFS} = 2(z_o - z(t)) \ln\left(\frac{R_o}{R_i}\right) \quad (3)$$

where z_o is the initial position of the PFS, $z(t)$ is its instantaneous position in the coax, and R_i and R_o are the inner and the outer radii of the coax, respectively. In defining (3), the motion of the coax is understood to be from right to left with respect to its original position. For all of the examples below $R_i = 7.62$ cm and $R_o = 10.16$ cm.

II.A. Switching

Two-dimensional models of a standard PFS indicates approximately planar motion of the switch plasma down the coax. When the switch moves off the up-stream edge of the load region, magnetic flux is transferred to the load on

²We assume a "standard" PFS, as described, for example, in P.J. Turchi, M.L. Alme, F. Bird, C.N. Boyer, S.K. Coffey, D. Conte, J.F. Davis III, and S.W. Seiler, IEEE Transactions on Plasma Science, Vol. PS-15, No. 6, 1987, pg. 747.

essentially the Alfvén time scale³. Assuming that the switch remains open during the implosion, the only effect of the PFS after switching is to produce a slight additional inductance as it moves across the load slot and the field is stripped off by the down-stream vanes (see Figure 3). Thus switching may be modeled in the following way. First, when the PFS open into the load region there is an increase in inductance which is associated with the volume between the PFS and the initial position of the load. This will occur over some time scale, which is input for this model. We assume that the inductance change is given by the volume bounded by the inner radius of the electrode and the instantaneous position of the load:

$$L_{load} = 2 \Delta z \ln\left(\frac{R_i}{r(t)}\right) \quad (4)$$

Here Δz is the width of the load slot (Figure 2), and $r(0)$ is the initial position of the load foil.⁴ To model switching we multiply this inductance by a form factor, $f(z(t))$, which depends on the instantaneous location of the PFS, $z(t)$, and on a length scale⁵ Δz_s over which complete transfer of current to the load region occurs:

$$f(z(t)) = \xi \left\{ 1 - \frac{1}{e^{[(z_1 - z(t))/\Delta z_s] + 1}} \right\} \quad (5)$$

Here ξ is an efficiency factor⁶ (unity for complete switching). The parameter z_1 represents the axial point at which the switch begins to open. Typically this will be taken as the up-stream edge of the load region. Thus $f(z)$ is essentially zero for z far from z_1 , and then transitions to unity as the PFS moves a characteristic distance Δz_s . The form of the switching function (5) has been chosen for convenience. Other functions which vary smoothly from zero to one could also be used.⁷ The switching time is then determined by this distance and the velocity of the PFS as it crosses the load slot. Since the load does not move significantly during this time,⁸ the radius $r(t)$ in (4) is essentially constant until switching is nearly complete⁹. Note that the time derivative of $f(z)$ is non-zero during switching, so that dL_{load} / dt is non-zero even if the load radius r is a constant.

As the PFS moves down the coax the inductance behind it increases, and this reduces the current that can be supplied by the bank. We assume that the bank inductance corresponds to all of the volume up to the back of the PFS. When switching begins there will be a further decrease in current. Finally, as the implosion proceeds, the load inductance will increase as the radius of the load decreases, and a further decrease in current will occur.

Once the bank current has been obtained the load current is obtained as the product of the switching function and the bank current:

$$I_L(t) = f(z(t)) I(t) \quad (6)$$

Thus the load current follows from a consistent solution of the circuit equation and the dynamic equation for the switch mass. Note that the latter also depends on the bank current. As the bank current decreases, so will the load current.

II.B. Circuit Equation

Substituting the results above into the circuit equation (1), and expressing the current in terms of the charge on the bank, gives the equation for the circuit coupled to the inductive changes associated with the PFS and the load:

$$0 = \frac{Q}{C} + \{L_o + L_{PFS}(t) + f(z) L_{load}\} \frac{d^2 Q}{dt^2} + \frac{dQ}{dt} \left\{ 2\Delta z \frac{df}{dt} \ln\left(\frac{R_i}{r}\right) - 2 \frac{dz}{dt} \ln\left(\frac{R_o}{R_i}\right) - f(z) \left(\frac{2\Delta z}{r}\right) \frac{dr}{dt} + R \right\} \quad (7)$$

³This aspect of the point mass switching model is contained in the parameters appearing in $f(z)$ defined in (5) below.

⁴In the model $r(0) = r(t_s)$, where t_s is the time at which switching begins.

⁵This should not be confused with the height of the load slot, which is denoted by Δz .

⁶Typically we expect $0.9 < \xi < 1.0$. We also assume that once the switch has opened it remains open. This aspect of the model could be relaxed.

⁷A physically based model for the switch function, based for example on a time varying resistivity, could be developed for use here.

⁸Vaporization of the load foil occurs during this switching period, and actual inward radial motion tends to occur only after most of the current has been transferred to the load.

⁹The model actually does allow the load to move during switching. Typical calculations show that for load masses of interest very little motion begins before switching is complete.

This represents the complete circuit equation. It depends on the dependent variables $z(t)$, describing the motion of the PFS down the coax, the switch function $f(z)$, and $r(t)$ describing the motion of the load during the implosion. The resistance of the system could be extended to include time dependent effects, such as a fuse or the variable resistance of the load during the implosion. For the systems considered below the load resistance is approximately $R = \eta l / A$, where $l = 2$ cm is the load height, and $A = 2\pi r \Delta r_s$ is the cross section area of the current carrying region. When the resistivity η is largest, $A \sim 5$ cm² and R is less than about 0.05 m Ω . The actual value varies during the implosion, but is generally small compared with the bank resistance. Dynamic equations for the variables appearing in (7) are described next.

II.C. Equations of Motion

For planar motion of an ideal PFS down the coax, the z component of the acceleration will be independent of radius and the radial acceleration will vanish. Using the relation for a toroidal magnetic field $2 I = r B_\theta$, the point mass axial equation of motion is

$$m_{PFS} \frac{d^2 z}{dt^2} = -I^2 \ln\left(\frac{R_o}{R_i}\right) \quad (8)$$

where m_{PFS} is the total mass of the PFS. This assumes that the mass distribution is close to $1/r^2$, so that the axial motion is essentially planar. The dependent variables are the current, $I(t)$, and the axial position, $z(t)$. No provision has been made to model flux stripping as the plasma moves past the vanes and out of the coax.¹⁰ In the models below the PFS usually has not moved far enough during the implosion of the load for this to be necessary.

The radial equation of motion for the load, whose mass is denoted by m_L , is

$$m_L \frac{d^2 r}{dt^2} = -I_L^2 \frac{\Delta z}{r} \quad (9)$$

where Δz is the height of the load foil, and $I_L(t)$ is the current in the load. The dependent variables are the radius $r(t)$, and the current $I_L(t)$. As noted previously, the model does not include a plasma pressure gradient which would be needed to simulate the bounce at pinch.

II.D. High Explosive Generator Driven Systems

The Procyon system, which has been described elsewhere¹¹, is modeled by a single loop circuit containing a storage inductance, L_{st} , a time dependent closing switch $S(t)$, and a power flow inductance L_o (Figure 1). The models above are used for the PFS and the load. An initial current, I_o , is set in the storage inductor and the switch S is open. The problem starts by ramping up a closing switch function $S(t)$, which delivers current to the PFS with a 10% to 90% rise-time of about 1.6 μ s as observed in the PC3 experiment. The closing switch function, $f_s(t)$ is chosen to be of the form

$$f_s(t) = \xi_s \left\{ 1 - \frac{I}{e^{[(t-t_o)/\Delta t]} + I} \right\} \quad (10)$$

The switch begins to close at t_o , which we set to zero, and Δt represents the time scale for closing. The factor ξ_s represents efficiency of closing. For simplicity we set the resistance of the system, $R = 0$. The equivalent circuit equation is then

$$L_{st} \frac{dI}{dt} + \frac{d(L(t)I)}{dt} = 0 \quad (11)$$

For the HE system, $L(t)$ includes the inductance of the PFS, the power flow channel, the load, and the effects of the closing switch $S(t)$. The current in the PFS is given by $f_s(t) I$, and the current in the load by $f_s(t) f(z) I$. In evaluating (11) we include the rate of change of both $f_s(t)$ and $f(z)$.

Finally, we note that the models above can be easily modified to describe capacitively driven or HE driven

¹⁰A simple stripping model could be included using a form factor, $f_s(z)$, which would supply a right-ward acting force on the PFS starting at a specified point in the coax, and bringing the plasma to a halt over some length scale. Such a term would be included on the right side of (9) to reduce the axial acceleration.

¹¹J. Goforth, et. al., *Megagauss Magnetic Field Generation and Pulsed Power Applications*, Ed. by M. Cohen and R.B. Spielman, (Nova Science Publishers: New York; 1994), pg. 841; J. Goforth et. al, *Proceedings of the Ninth IEEE International Pulsed Power Conference*, Albuquerque, June 1993, pg. 36; R.L. Bowers, A.E. Greene, D.L. Peterson and N.F. Roderick, *Proceedings of the Ninth IEEE International Pulsed Power Conference*, Albuquerque, June 1993, pg. 538.

include: interaction of the switch plasma and the electrodes as it moves down the coaxial barrel; the effects of distributed, background plasma density on the switch's efficiency and its ability to remain open during the implosion and pinch; effects of magnetically driven Rayleigh-Taylor and wall induced instabilities on the load plasma; instabilities induced by the switch plasma during switching; and absorption of pinch radiation by switch plasma.

Table 1. Input Parameters for Switched Implosion Models

Regime	V_0^a (kV)	C^b (mF)	z_l (cm)	Δz (cm)	mpFS (mg)	m_L (mg)	R (m Ω)	LPFC (nH)	Δz_s (cm)	ξ_s
Pegasus II	84	0.85	4.45	4.45	60	12	0.5	23	0.20	0.95
Shiva Star	90 ^c	1.313	6.3	6.3	120	20	1.0	19	0.10	0.95
	90 ^d	1.313	6.3	6.3	120	12.7	1.0	19	0.28	0.95
Procyon	21.7	79	3.5	6.3	200	30	0.	28	0.30	0.95
Atlas	540	0.2	6.3	6.3	200	14	8.0	23	0.05	0.95

For Procyon a) is initial current (MA) in storage inductor; b) storage inductance (nH); c) 20 mg load; d) 13 mg load.

Table 2. Comparison of Switched Implosion results.

Regime	E_{rad}^a (MJ)	dE/dt^a (MJ/ μ s)	T_c^b (eV)	I_{max} (MA)	τ^c (ns)	τ_s (ns)	T_{eff} (eV)
Pegasus II (12 mg)	0.45	22	190 - 320	9.2	20	50	116
	0.35	18	145 - 250	9.2	20	200	110
Shiva Star (20 mg)	0.63	16	170 - 280	12.2	39	70	108
(13 mg)	0.67	22	260 - 440	12.2	31	70	116
Procyon (30 mg)	1.2	67	200 - 330	13.8	18	350	152
Atlas (14 mg)	4.5	750	1600 - 2700	32.6	6	20	280

a) Assumes 50 % of implosion KE goes into radiation; b) Uses (12), with $C_V = 1200 - 2000 \text{ eu g}^{-1} \text{keV}^{-1}$; c) Calculated as twice $0.25 \text{ cm} / v_r$, where v_r is the load velocity at 0.25 cm.

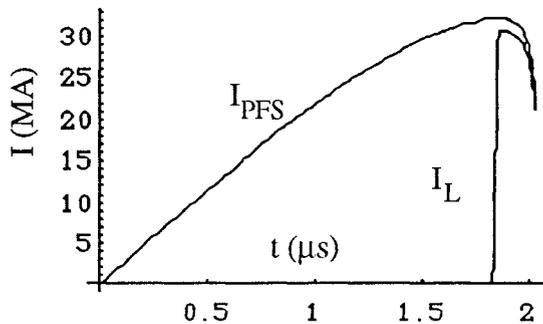


Figure 4e. Current in the PFS and in the load versus time. The onset of switching is shown by the switch function $f(t)$ in Figure 4d above.

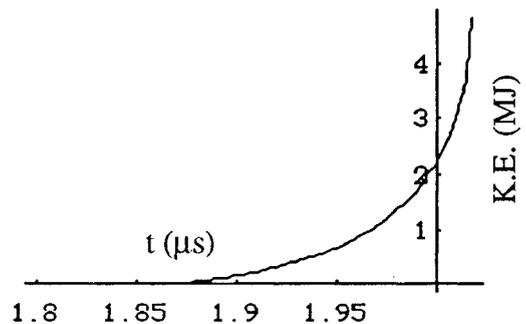


Figure 4f. Implosion kinetic energy in MJ, versus time. The kinetic energy at 20:1 convergence is about 4.5 MJ.

direct drive experiments.

III. OUTPUT FROM SWITCHED IMPLOSIONS

Although the point mass model does not include radiative losses, we can make order of magnitude estimates for quantities of interest from the results above and several simple assumptions¹². Starting with the relationship between the specific kinetic energy of the imploding load and its thermal energy per gram

$$C_V T = (1/2) v^2 \quad (12)$$

we can estimate the central temperature of the pinch. Here C_V is a pseudo-heat capacity, whose value is known from detailed calculations. We use values for the pseudo-heat capacity in the range 1200 - 2000 eu $g^{-1} keV^{-1}$ ($10^8 - 1.7 \times 10^8 \text{ erg } g^{-1} K^{-1}$), and the load velocity at 20:1 convergence. Results of two-dimensional calculations indicate that the amount of energy radiated often equals or exceeds the kinetic energy near pinch. We shall assume that all of the kinetic energy at 20:1 convergence goes into radiation, and set $\eta = 1$.

As a specific example, consider an Atlas switched implosion of a 2 cm high, 5 cm radius load with a mass of 14 mg. We assume a 200 mg switch plasma, initially located 6.3 cm to the right of the upstream edge of the load slot ($z_0 - z_1$ in Figure 2). The switch mass was arrived at by requiring that the PFS reach the load region with an axial velocity of 8 - 10 cm / μs , but before peak current delivery into a dead short. The initial bank voltage was $V_0 = 540 \text{ kV}$ and $C = 0.2 \text{ mF}$, corresponding to a bank energy of 29.2 MJ. The inductance of the bank and the power flow channel up to the back of the initial position of the PFS is assumed to be 23 nH. The switch moves to the left toward the load region as in Figure 2. The onset of the pinch is taken to be a load radius of 0.25 cm. The switch function (5) uses $\Delta z_s = 0.05 \text{ cm}$, $\xi = 0.95$ for 95% switching efficiency, $z_0 = 0.0 \text{ cm}$, and $z_1 = -6.3 \text{ cm}$. The point mass model predicts an implosion time to a 20:1 convergence of 2.017 μs . The current flowing in the PFS reaches a peak value of 32.3 MA prior to switching into the load region. Switching starts at $t_s = 1.82 \mu s$, and is complete in about 40 ns. The 10%-90% current rise time in the load is about 20 ns.

The actual implosion time, as measured to a radius of 0.25 cm, is about 160 ns. The load velocity reaches about 80 cm/ μs at 0.25 cm, which corresponds to a kinetic energy of 4.5 MJ. A stagnation time for the pinch may be defined as 2τ , where τ is 0.25 cm divided by the load velocity at that point. For Atlas $\tau = 2 \times 0.25 \text{ cm} / 80 \text{ cm}/\mu s = .006 \mu s$. The total power associated with the implosion is $\eta dE/dt = 4.5 \eta \text{ MJ} / 0.006 \mu s = 750 \eta \text{ MJ}/\mu s$. Assuming a conversion efficiency $\eta = 1$ we have $E_{rad} = 4.5 \text{ MJ}$ and $dE/dt = 750 \text{ MJ}/\mu s$. The x-ray output from the switched implosion represents about 15 % of the initial bank energy. Using (12) we can estimate the temperature of the pinch to be in the range 1.6 - 2.7 keV. In this model the peak current in the load is 30.6 MA. A summary of results from the point mass model is given in Figure 4.

The model has been used to evaluate the performance of other pulsed power systems. The parameters for each are listed in Table 1. Table 2 compares output and power for Atlas switched implosions with results for other pulsed power systems in Table 1. It also lists estimates of the pinch temperature, T_C , peak current in the load, I_{max} , stagnation time, τ_C , switching time, τ_s , and the surface temperature of the pinch. The latter is defined by

$$T_{eff} = \left(\frac{dE/dt}{4\pi r_{eff}\sigma} \right)^{1/4} = 30 (dE/dt)^{1/4} \text{ eV}$$

where the surface radius $r_{eff} = 0.25 \text{ cm}$, σ is the Stefan-Boltzmann constant and E is in eu.

IV. CONCLUSIONS

The point mass model for switched implosions has been applied to pulsed power systems whose stored energy ranges from 3.0 MJ to 29.2 MJ. For most of these systems 2-D calculations are available, and have been used to motivate the choice of parameters for the point mass models. The total radiation output predicted by the model is in reasonable agreement with experiments or calculations. The thermalization times are about 10 times too short, which results in estimated powers that are large by about the same factor. The absolute value of the plasma temperatures are also expected to be too high.

Two-dimensional radiation MHD models of switched implosions identify several features which can have a dramatic effect on the quality of a switched implosion that are not included in the point mass model.¹³ These

¹²These are based in part on experience with 2-D perturbed models of implosions. We cannot place too much credence in the quantitative nature of these estimates, but the trends exhibited between the systems modeled below may be useful.

¹³F.J. Wysocki, et. al., *Megagauss Magnetic Field Generation and Pulsed Power Applications*, Ed. by M. Cohen and R.B. Spielman, (Nova Science Publishers: New York; 1994), pg. 767; R.L. Bowers, J.H. Brownell, A.E. Greene, D.L. Peterson, N.F. Roderick and P. Turchi, *Megagauss Magnetic Field Generation and Pulsed Power*