ELECTROMAGNETIC ENVIRONMENT DUE TO A PULSED MOVING CONDUCTOR

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Abstract

The space and time behavior of the magnetic field external to a system of moving pulsed finite conductivity conductors is determined. The moving conductor system is comprised of a continuous solid conductor in close proximity to a spatially periodic stationary conductor system that provides the source current. The relationship of the external magnetic field to time dependent image theory is critically examined. A numerical example predicts the magnitude and frequency of the magnetic field as a function of distance from the stationary conductor. Applications are discussed.

I. INTRODUCTION

There is continuing interest in the development of pulsed-power energy sources and accelerators based on the processes of transient magnetic field exclusion and diffusion. These devices typically involve a stationary source current and the generation of eddy currents in a moving finite conductivity conductor. Applications utilizing this process can be found in electromagnetic braking of large electromechanical systems, and the generation of high electrical power pulses [1]. These applications require 100's of kA with millisecond pulse-widths.

Additionally, these currents can produce external environmental magnetic fields of a few Tesla. These "environmental" magnetic fields may pose serious electromagnetic compatibility (EMC) problems if proper shielding methods are not used. Figure 1 shows the relationship between the spatially periodic source currents and the moving conductor and the geometry used for determining the time behavior of the magnetic field at point $P(x_o, y_o, z_o)$ due to the source currents. The periodic system is infinite in the $\pm x$-directions.

Our model quantifies the damped harmonic electromagnetic fields generated by a moving conductor in terms of the periodic spatial separation between the current sources, the conductivity and velocity of the moving conductor, and the location of the observation point relative to the moving conductor. The principal contributions to the far-field magnetic signature are due to the source current distribution and its time-dependent image.

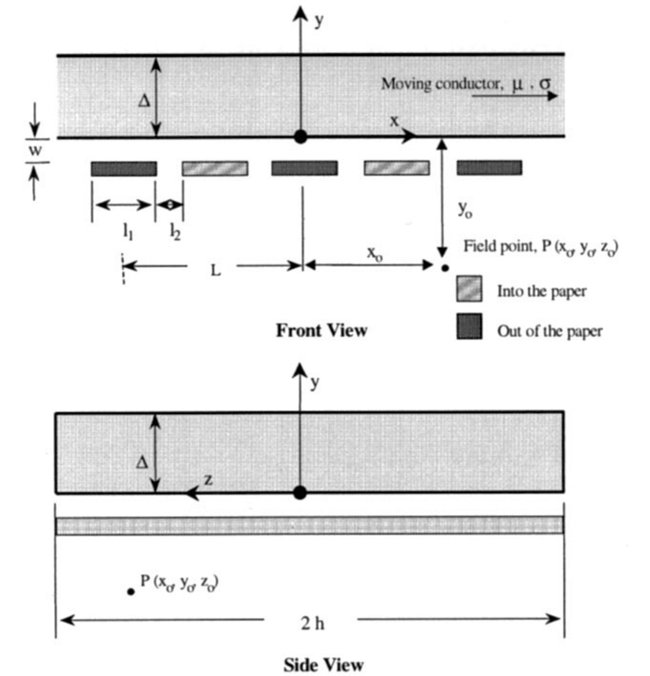

Figure 1. Geometric Considerations.

II. THEORETICAL CONSIDERATIONS

Much insight into the structure of the electromagnetic fields described in Fig. 1 can be gleaned by first considering the two-dimensional solution for a stationary case where the thickness, Δ, is suitably large.

A. *Zero Velocity, Infinite Conductivity Case.*

A close-up of the situation for a single conductor with $\sigma = \infty$ is shown in Fig. 2.

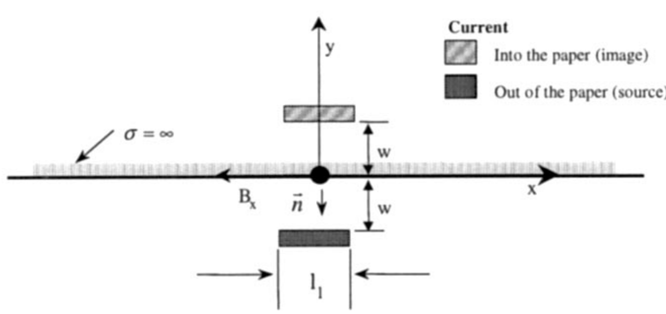

Figure 2. Single Current Source and Its Image.

Image theory permits the field at all points in space to be computed by summing the fields due to the actual

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14. ABSTRACT

The space and time behavior of the magnetic field external to a system of moving pulsed finite conductivity conductors is determined. The moving conductor system is comprised of a continuous solid conductor in close proximity to a spatially periodic stationary conductor system that provides the source current. The relationship of the external magnetic field to time dependent image theory is critically examined. A numerical example predicts the magnitude and frequency of the magnetic field as a function of distance from the stationary conductor. Applications are discussed.

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physical real source and its image. The real source and its image produce a magnetic field determined from the Biot-Savart law,

$$\vec{B} = \frac{\mu_o}{4\pi}\iiint \frac{\vec{j}_c(\bar{x},\bar{y},\bar{z})\times \vec{r}}{|\vec{r}\,|^3} d\bar{x}d\bar{y}d\bar{z}, \tag{1}$$

where $\vec{j}_c$ is the current density. For our case the currents that flow only in the $\pm$ z-direction. The computation of the surface magnetic field, $\vec{H}_s$, and surface current density, $\vec{J}_s$ for the $\sigma=\infty$ case are straightforward although tedious [1]. The results are

$$\vec{H}_s = -\vec{i}J_o(t)\sum_{n=1}^{n=\infty}\Omega_n \cos\frac{2n\pi x}{L}, \tag{2}$$

$$\vec{J}_s = -\vec{k}J_o(t)\sum_{n=1}^{n=\infty}\Omega_n \cos\frac{2n\pi x}{L}, \tag{3}$$

where

$$\Omega_n = \frac{2}{L}\int_0^L \cos\frac{2n\pi x}{L}\left[\sum_{n'=-\infty}^{n'=+\infty}(-1)^{n'}\frac{w}{\pi}\int_{-l_1/2}^{+l_1/2}\frac{d\bar{x}}{(x-\frac{n'L}{2}-\bar{x})^2+w^2}\right]dx \tag{4}$$

$$J_o(t) = \frac{I(t)}{l_1}, \tag{5}$$

and $I(t)$ is the current in each stationary conductor.

B. Zero Velocity, Finite Conductivity Case.

The basic problem is that of a horizontal dipole over a conducting ground plane [2]. For this case approximate image theory can be used [3]. If w is the location of the image for $\sigma=\infty$, the total displacement, w_c, for $\sigma\neq\infty$ is given by [3]

$$w_c = w + d, \tag{6}$$

where d is the additional displacement due to the conductivity. In frequency space, d is actually a complex quantity, a consequence of fitting the rigorous theory with a mathematical approximation [3]. If we associate the magnitude of d with the physical displacement and let $\omega \approx 2\pi / t_p$, where t_p is the duration of a pulse, we obtain

$$|d| = \sqrt{\frac{2t_p}{\pi\mu_o\ \sigma}}, \quad \text{for } \sigma >> \omega\varepsilon. \tag{7}$$

Equation (7) is essentially the root-mean-square displacement determined from diffusion theory. Thus provides the physically intuitive result that, for short times, the location of the image is essentially that obtained from the $\sigma=\infty$ case.

The magnitude of the image current will be less than that for the infinite conducting case because there are energy losses in the system associated with finite conductivity. The rigorous solution of this problem requires the formulation in terms of a frequency-dependent horizontal dipole over a conducting half space [2].

III. ELECTROMAGNETIC FIELDS AND CURRENTS IN A MOVING CONDUCTOR

An exact prediction of the fields is possible, provided that we can precisely specify the magnetic field at the bottom surface for all time, $t\geq 0$. This is exactly where the difficulty arises; the magnetic field at y = 0 cannot precisely be defined for all time without simultaneously solving Maxwell's equations in the space below the moving conductor. An approximate solution to this problem can be obtained using perturbation theory concepts. The concept is as follows: we solve for the electromagnetic fields in the conductor by assuming that the magnetic field at y = 0 is given by image theory for $\sigma=\infty$. This approximation enables us to determine the first-order correction for the displacement of the image due to diffusion and the decrease in the magnitude of the image current due to energy losses associated with finite conductivity. Corrections to the surface magnetic field are then made.

A. Governing Equations.

From our geometry, both B_x and B_y satisfy the same equation:

$$\frac{1}{\mu\sigma}\left[\frac{\partial^2 B}{\partial x^2} + \frac{\partial^2 B}{\partial y^2}\right] = \frac{\partial B}{\partial t} + v\frac{\partial B}{\partial x}, \tag{8}$$

where v is the velocity of the moving conductor. The divergence equation, $\nabla\bullet\vec{B}=0$, provides the relationship between B_x and B_y. The boundary condition at y = Δ is greatly simplified for pulse lengths, t_p, that are short (i.e., $\Delta > |d|$). We then have the boundary condition $B_x(x, y=\Delta, t) = B_y(x, y=\Delta, t) = 0$, which is relevant only for those cases where the diffusion time, $t_D \approx \Delta^2\mu_o\sigma$, is larger than t_p. When t_p is significantly shorter than t_D, the top boundary of Fig. 1 is never reached during its duration and, for all practical purpose, it could be located at $y=\infty$.

The solution of the problem requires that we specify the time behavior of the *x-component* of the magnetic field at y=0, $B_x^o = B_x(x, y=0,\ t)$, for $t\geq 0$. From equations (2) and (3) we have the first approximation in a perturbation scheme.

$$B_x^o = -\mu_o J_o(t)\sum_{n=1}^{n=\infty}\Omega_n \cos\frac{2n\pi x}{L}, \tag{9}$$

which is the field due to the periodic source of Fig. 1 over a perfectly conducting plane. Our analysis for arbitrary $J_o(t)$ is executed using the impulse response theory. A Delta (impulse) function current density source, $\delta(t) = J_o(t)$, is specified at y = 0. B_x and B_y are now interpreted as the solutions to equation (8) for the impulse function. We have [1]:

$$B_x=-\mu_o\frac{\sqrt{\mu_o\sigma y^2}}{2\sqrt{\pi t^3}}e^{-\frac{\mu_o\sigma y^2}{4t}}\sum_{n=1}^{n=+\infty}\Omega_n e^{-\frac{\gamma_n^2 t}{\mu_o\sigma}}\cos(\gamma_n x-\gamma_n vt)\quad(10)$$

$$B_y=\sqrt{\frac{\mu_o}{\sigma}}\frac{1}{\sqrt{\pi t}}e^{-\frac{\mu_o\sigma y^2}{4t}}\sum_{n=1}^{n=+\infty}e^{-\frac{\gamma_n^2 t}{\mu_o\sigma}}\gamma_n\Omega_n\sin(\gamma_n x-\gamma_n vt).\quad(11)$$

B. Characterization of External Magnetic Field.

The two contributions to the environmental magnetic field come from the input current source elements and from the spatial distribution of current density within the moving conductor. The only existing *z-component* of current density in the moving conductor is given by

$$j_c=\frac{1}{\mu_o}\left(\frac{\partial B_y}{\partial x}-\frac{\partial B_x}{\partial y}\right).\quad(12)$$

The resulting expression for j_c is inserted in equation (1) to compute the environmental magnetic field. There is a more physically illuminating way to compute the environment magnetic field. The vertical integrated current (i.e., image) is

$$J^*=\int_0^\infty j_c(y)dy=\frac{1}{\mu_o}\left[\frac{\partial}{\partial x}\int_0^\infty B_y(x,y,t)dy-\int_0^\infty\frac{\partial B_x(x,y,t)}{\partial y}dy\right].(13)$$

From the infinite conductivity case, we know that the current in the conductor will act to compensate the magnetic field generated by the source current. For the $\sigma=\infty$ case, the integrated vertical current is 180° out of phase with the source current. This image is located a distance w above the $y=0$ plane. When $\sigma\neq\infty$, the image current cannot be placed any closer to $y=0$ than w. This is true irrespective of the velocity. In the other extreme, we can place the image current so far above the $y=0$ plane (this is only a mathematical abstraction) that the environmental magnetic field will be due to only the source current.

In summary, the upper bound on the environmental magnetic field will be determined by completely neglecting the current in the moving conductor. The lower bound will be computed by locating the image current a distance w above the $y=0$ plane. We choose to render the results in this format to provide enhanced insight the physical processes.

Equation (12) can be integrated to yield [1]:

$$J^*=\frac{G(x,t)}{\mu_o}-\delta(t)\sum_{n=1}^{n=\infty}\Omega_n\cos\ \gamma_n x,\quad(14)$$

$$G(x,t)=\frac{1}{\sigma}\sum_{n=1}^{n=+\infty}e^{-\frac{\gamma_n^2 t}{\mu_o\sigma}}\gamma_n^2\Omega_n\cos(\ \gamma_n x-\gamma_n vt).(15)$$

The step function response for the total current per unit length is

$$J^*_{step}=A_J\int_0^t J^*(t-\tau)d\tau=A_J\int_0^t J^*(\tau)d\tau,\quad(16)$$

where A_J is the magnitude of the step. When equation (14) is inserted into equation (16), we get

$$J^*_{step}=J^*_{DM}-J^*_I.\quad(17)$$

The image contribution is given by

$$J^*_I=A_J\int_0^t\delta(\tau)d\tau\sum_{n=1}^{n=\infty}\Omega_n\cos\gamma_n x=A_J\sum_{n=1}^{n=\infty}\Omega_n\cos\gamma_n x,\quad(18)$$

and the diffusion-motion contribution is

$$J^*_{DM}=\frac{A_J}{\mu_o}\int_0^t G(x,\tau)d\tau.\quad(19)$$

The general solution for J^*_{DM} is

$$J^*_{DM}=\frac{A_J}{\mu_o\sigma}\sum_{n=1}^{n=\infty}\gamma_n^2\Omega_n\cos(\gamma_n x)\Psi_{nc}(t)+\frac{A_J}{\mu_o\sigma}\sum_{n=1}^{n=\infty}\gamma_n^2\Omega_n\sin(\gamma_n x)\Psi_{ns}(t),\quad(20)$$

$$\Psi_{nc}=\frac{1}{\left((\gamma_n^2/\mu_o\sigma)^2+(\gamma_n v)^2\right)}\left(\frac{\gamma_n^2}{\mu_o\sigma}-e^{-\frac{\gamma_n^2 t}{\mu_o\sigma}}\left(\frac{\gamma_n^2}{\mu_o\sigma}\cos(\gamma_n vt)-\gamma_n v\sin(\gamma_n vt)\right)\right),\quad(21)$$

$$\Psi_{ns}=\frac{1}{\left((\gamma_n^2/\mu_o\sigma)^2+(\gamma_n v)^2\right)}\left(\gamma_n v-e^{-\frac{\gamma_n^2 t}{\mu_o\sigma}}\left(\frac{\gamma_n^2}{\mu_o\sigma}\sin(\gamma_n vt)+\gamma_n v\cos(\gamma_n vt)\right)\right).\quad(22)$$

The foregoing expressions show that the conductor current contains harmonics with angular frequencies $\omega_n=\gamma_n v=\frac{2n\pi v}{L}$. Using the orthogonal property of the cosine and sine functions we compute the spatial averages defined in equations (23) to (26):

$$\left|J^*_{DM}\right|_{ave}=\left(\frac{A_J^2}{2}\sum_{n=1}^{n=\infty}\mathrm{H}_n^2\Omega_n^2\right)^{1/2},\quad(23)$$

$$\left|J^*_I\right|_{ave}=\left(\frac{A_J^2}{2}\sum_{n=1}^{n=\infty}\Omega_n^2\right)^{1/2},\quad(24)$$

$$\mathrm{H}_n^2=\frac{n^2}{n^2+(\ t_{dn}\omega_n\ )^2}h_n(t),\qquad t_{dn}=\frac{\mu\sigma}{\gamma_n^{\ 2}},\quad(25)$$

$$h_n(t)=1+e^{-\frac{2t}{t_{dn}}}-2e^{-\frac{t}{t_{dn}}}\cos\omega_n t.\quad(26)$$

Figures 3 and 4 show $h_n(t)$ and $\mathrm{H}_n^2(t)$, respectively, as a function of time with mode number, n, as a parameter for $\mu_o=4\pi\times10^{-7}$ H/m, $\sigma=2.2\times10^7$ mhos/m, L= 0.5 m, and v = 200 m/s.

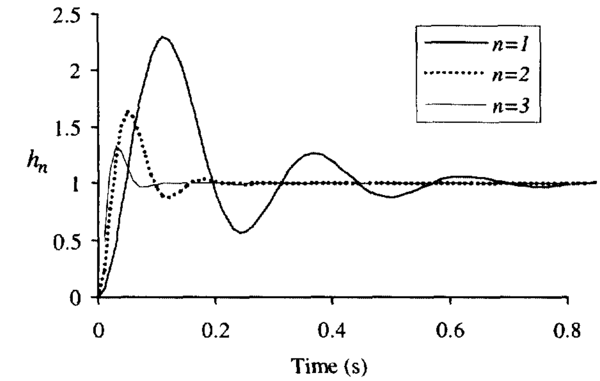

Figure 3. $h_n(t)$ as a Function of Time With Mode Index.

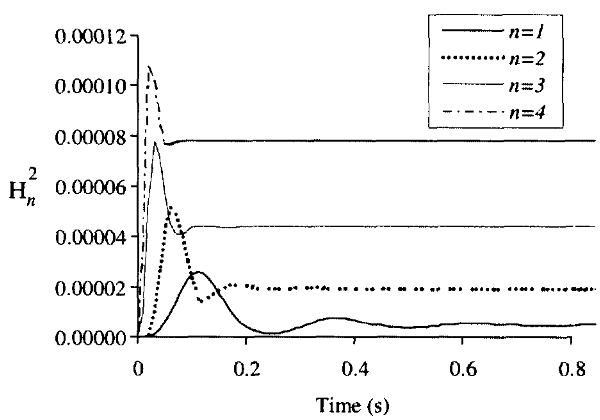

Figure 4. $H_n^2(t)$ as a Function of Time With Mode Index.

In the first few modes, $\left|J^*_{DM}\right|_{ave} << \left|J^*_l\right|_{ave}$ .The motion of the conductor does not appreciatively affect the environmental magnetic field.

The magnetic field at point $\vec{r}_o = \vec{i}x_o + \vec{j}y_o + \vec{k}z_o$ is computed using the Biot-Savart law. The z-dimension of the system is set at $2L$. The coordinates of the field point for which a specific calculation is rendered are $x_o = 0$, $z_o = L/2$, and arbitrary y_o. The magnitude of the total magnetic field is B_{net}. We have

$$B_{net} = B_R(\eta) - B_R(\eta - \frac{2w}{L}), \qquad \eta = \frac{y_o + w}{L}, \tag{27}$$

$$B_R = -\frac{\mu_o}{4\pi} A_J \sum_{n=1}^{n=\infty} a_n P_n(\eta), \tag{28}$$

$$a_n = \frac{2}{n\pi}(1 - \cos n\pi)\sin\frac{n\pi l_1}{L}, \tag{29}$$

$$P_n = 4\eta \int_0^\infty \frac{\cos 2n\pi\rho}{(\eta^2 + \rho^2)(1 + \eta^2 + \rho^2)^{1/2}} d\rho. \tag{30}$$

B_R is the contribution to the magnetic field from the source current system and y_o is negative for all cases. Figure 5 shows the magnetic field for: l_1 = 175 mm, l_2 = 75 mm, $L = 2(l_1 + l_2) =$ 0.5 m, $w =$ 2.5 mm, and. $I =$ 600 kA.

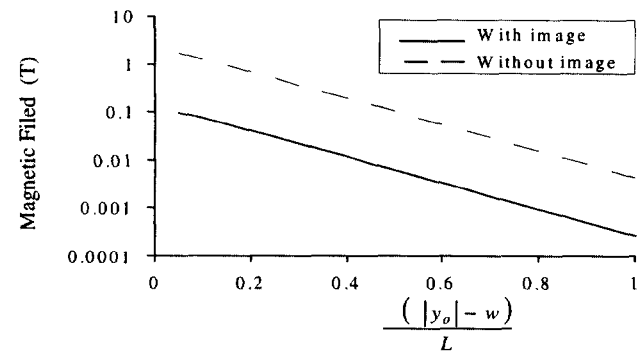

Figure 5. Magnetic Field vs. Dimensionless Distance.

As observed, the magnetic field with the contribution from the image current reduces the environmental field by more than an order of magnitude. However, in both cases, the field has decayed roughly 3 orders of magnitude at one conductor spacing (L) from the source.

IV. SUMMARY AND CONCLUSIONS

In this analysis we show that the induced currents are nearly equal in magnitude to those in the source. The eddy currents in the moving conductor delay the penetration of the magnetic flux. The environmental magnetic field decays rapidly to negligible levels at most distances of interest.

In order to estimate the environmental magnetic field produced by a moving pulsed conductor, we have assumed a step function current for the current source elements. In an actual alternator system, these currents would be produced as a result of voltage generation due to a spatially varying direct current excitation field established in the air gap between the moving conductor and the armature windings. This excitation source has not been considered in this analysis but can readily be computed using the techniques developed in this study.

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