

ANALYSIS OF THE PERFORMANCE OF GALLIUM ARSENIDE PHOTOAVALANCHE SWITCHES*

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Abstract

This work investigates how the dielectric relaxation that occurs after optical absorption can raise the electrical field from below avalanche threshold to above threshold inside a simple GaAs photoconductor. The process of optically raising the electrical field above its initial value we call "dynamic field enhancement." Trade-offs between optical intensity, doping, optical absorption depth, and sample thickness are discussed with respect to obtaining useful performance of a dynamically field enhanced photoavalanche switch. We trace the origin of various contributions to field enhancement and deduce certain bounds on the magnitude of the process. In this work, response time is not considered. From a one-dimensional analysis, we conclude that, in homogeneous photoconductors with ohmic contacts, dynamic field enhancement is limited at low fields to roughly a factor of two increase. We compare our analysis to one- and two-dimensional calculations obtained with computer codes based on a drift/diffusion model.

Introduction

In a recent patent disclosure, Ragle and Davis[1] describe a switch whose basic mode of operation purportedly is uniform avalanching induced by dynamic field enhancement. The initial field inside their switch is below avalanche threshold everywhere, but dynamic field enhancement makes the device avalanche uniformly, it is averred. Ragle and Davis assert that by adjusting the ratio of the physical thickness to the optical absorption depth and then illuminating the switch through either or both electrodes, one can dramatically lower the energy required for photoavalanching. Because Lawrence Livermore National Laboratory has a variety of applications for rapidly switched, high-voltage power, understanding how to design and use such switches is important to us.

In the simplest view, optical absorption enhances avalanching inside a semiconductor by reducing the effective spacing between two electrodes without reducing the voltage. The photo-injected carriers raise the electrical conductivity, killing the electric field wherever the conductivity is high. It is important that there be a small segment where the conductivity is not raised. In effect, shorting the electric field everywhere except across a small gap moves the electrodes closer together. Assuming that the time scale is too brief for the photocarriers to flow out of the device, the voltage across the semiconductor remains constant, and so the electric field in the gap rises as the effective separation between electrodes shrinks. If the field rises enough, avalanching occurs.

The preceding view is not rigorous. By Maxwell's equation of electrostatics (Poisson's Equation), it is not a change in conductivity *per se* that kills electrostatic fields but the subsequent redistribution of charge. That is, charge configuration not charge motion determines the magnitudes of both field cancellation and field enhancement. In a sense, the extent of switch closure (i.e., the peak switched out voltage) depends primarily on the position of photocarriers in the switch. The speed of operation, on the other hand, depends primarily on the velocities of the carriers. Although the velocities of the carriers depend nonlinearly upon the electric field, which depends in turn upon the position of the charges, it is convenient to examine the effects of charge velocity separately from those of charge position in order to elucidate the performance of photoconductive switches.

Bounds on Enhancement due to Charge Distribution

We have analyzed the performance of photoconductive switches by starting with Maxwell's equation for electrostatics for a uniformly doped semiconductor that has no traps. We consider a simple circuit comprised of a photoconductive slab in series with an ideal battery and resistor, as in Figure 1. We write the total electric field, E^{tot} , as a superposition of three terms, an "external" field, a "dipole" field, and an "offset" field:

$$\vec{E}^{tot} = \vec{E}^{ext} + \vec{E}^{dip} - \vec{E}^{off}, \quad (1)$$

where

$$\vec{E}^{ext} = \hat{z}V/L \quad (2)$$

$$\vec{E}^{dip} = \frac{q\hat{z}}{2\epsilon} \int_0^L (\Delta p(z',t) - \Delta n(z',t)) dz' - \frac{q\hat{z}}{2\epsilon} \int_z^L (\Delta p(z',t) - \Delta n(z',t)) dz' \quad (3)$$

$$\vec{E}^{off} = \frac{1}{L} \int_0^L \vec{E}^{dip} dz. \quad (4)$$

$\vec{E}^{ext}$ is the field that would be present if the photoconductor were in its initial charge configuration but with the instantaneous voltage V across the switch. $\vec{E}^{dip}$ is a dipole field that would be present if the excess charge carriers in the photoconductor (Δp and Δn) were standing immobile in otherwise free space. $\vec{E}^{off}$ is a constant offset field that arises by forcing $\vec{E}^{tot}$ to satisfy the constraint of Kirchhoff's Circuital Voltage Law. It equals the spatial average of the dipole field. The other terms in these expressions represent time (t), electronic charge ($-q$), static dielectric constant (ϵ), position (z and z'), a unit vector pointing towards the cathode ($\hat{z}$), and sample thickness (L).

It is straightforward to show several important facts about the electric field in a homogeneous photoconductor. First, field enhancement, if it occurs, is due primarily to the dipole field and its spatial average, the offset field. These fields depend only upon the distribution of excess charge, Δp and Δn . Second, according to Eq. (3), the values of the dipole field at opposite ends of the slab are equal and opposite. Moreover, the dipole fields at the ends of the device depend only on the dielectric constant and the net excess charge per unit area, Q/A , within the slab, where

$$Q/A = q \int_0^L (\Delta p - \Delta n) dz. \quad (5)$$

Third, if the net charge density in the photoconductor is everywhere positive or everywhere negative, then the total field has its extrema at the ends of the slab. Finally, when positive charge is bunched towards the positive electrode and negative charge is bunched towards the negative electrode, then the offset field is parallel to the external

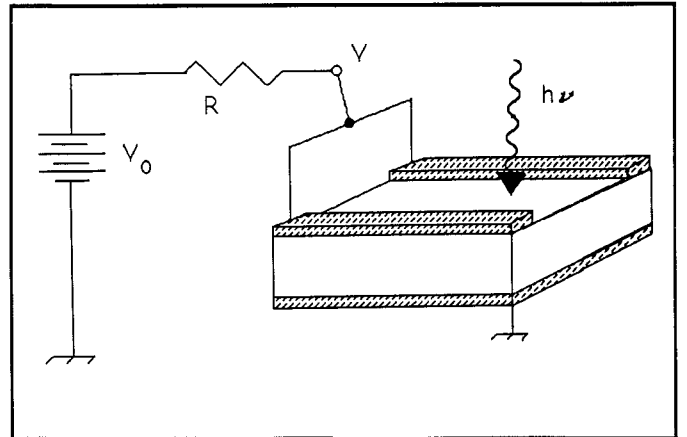


Figure 1. Photoconductive switch in a simple circuit. Photoinjected electrons and holes separate under the influence of an externally applied voltage, creating a dipole field. Where the dipole field is strongly negative (opposite to the external field), the device tends to short out. Where the dipole field is weakly negative or even positive, field enhancement tends to occur, which may lead to avalanche ionization.

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field, a condition that opposes field enhancement. Mathematically, the criterion that the offset field be parallel to the external field is stated as follows:

$$\int_0^z (\Delta p - \Delta n)(z - L/2) dz \leq 0. \quad (6)$$

This condition results by substituting Eq. (3) into Eq. (4) and integrating by parts.

We now show that the maximum field enhancement is limited to a factor of 2 under certain conditions. First, we assume that the total field cannot reverse sign as a function of position so long as the only charges in the circuit originate somewhere within the circuit. (For example, electrons from an external gun do not bombard the photoconductor.) Second, if the dipole field has its extrema at the boundaries of the photoconductor ($z = 0$ and $z = L$), then the maximum dipole field is equal and opposite to the minimum dipole field, according to the preceding paragraph. Further, if the offset field is parallel to the external field, then

$$E^{tot} \leq V/L + \max\{E^{dip}\} \quad (7)$$

and

$$0 \leq V/L + \min\{E^{dip}\}. \quad (8)$$

(In these expressions, vector signs have been omitted, since the geometry of interest is one dimensional.) Adding relationships (7) and (8), one obtains

$$E^{tot} \leq 2V/L. \quad (9)$$

Thus, field enhancement is limited to a factor of two when the minimum total field is zero, the dipole field has its extrema at the electrodes, and the offset field is parallel to the external field.

Under what circumstances might the three preceding conditions be met? First, regarding the assumption that the total field cannot change sign as a function of position: in the range of validity of the drift/diffusion model of current, current ceases when the field vanishes. Therefore, charges cannot pile up so heavily as to more than cancel the external voltage unless the charges are forced into the circuit from a totally foreign source such as an electron gun.

Second, regarding the assumption that the dipole field has its extrema at the electrodes of the photoconductor: a sufficient (but not necessary) condition is that the net charge density be either non-positive or non-negative everywhere. We argue on heuristic grounds that the exponentially decaying (as a function of position) optical absorption leads to an exponentially decaying net charge density, and that the spatial monotonicity is maintained by charge injection that occurs at ohmic contacts — at least at low fields. (For high fields, instabilities can develop, as mentioned later, thereby permitting the sign of the net charge density to vary with z .) We implicitly assume that illumination is from one electrode towards another and that the photoconductor is homogeneous. If the photoconductor is not homogeneous or if illumination is not through an electrode, then charge may pile up in the middle of the device, creating a sort of parallel-plate capacitor with a high field away from the actual electrodes.

Third, regarding the assumption that the offset field is parallel to the external field: the offset field tends naturally to oppose field enhancement when illumination is through an electrode, at least for early times and homogeneous photoconductors. The reason is that the net excess charge density is distributed with its barycenter nearest the electrode of optical incidence, and the sign of the net excess charge is the same as the sign of the nearby electrode (negative for the cathode and positive for the anode). This, according to Eq. (6), is sufficient to cause the offset field to be parallel to the external field.

Assuming that the preceding conditions are met and that the total electric field can no more than double, one would conclude that photoavalanche devices must be operated near avalanche breakdown in order to work. If true, then users may not achieve arbitrary amounts of field enhancement by cleverly adjusting the optical absorption depth in homogeneous photoconductive switches.

Concepts for Improving Dynamic Field Enhancement

Equation (1) suggests three things to do to maximize the total field inside a photoconductive switch: minimize circuit load losses so that a maximum voltage falls across the switch for a given power sup-

ply voltage; maximize the dipole field; and create an offset field that is as large as possible and antiparallel to the external field. To minimize circuit load losses, one must keep the current as small as possible during the switching process. This might be feasible by placing barriers such as junctions near one or both electrodes. To maximize the dipole field, one must separate excess electrons and holes as much as possible in such a way that holes pile up towards the cathode and electrons pile up towards the anode. One way to achieve this might be to use side illumination and partially mask the switch. Perhaps by carefully positioning the light between the electrodes, one could cause the electrons to reach the anode at the same time that the holes reach the cathode, thus achieving maximum separation between the excess carriers. Alternatively, one could use a heterojunction to raise the dipole field at a point not near the electrodes. In such a case, there is no restriction that the maximum dipole field must be equal and opposite to the minimum dipole field (unlike the situation when the dipole field has its extrema at the electrodes). Consequently, when one adds relations (7) and (8), the maximum and minimum dipole fields do not cancel, which invalidates relation (9). That is, if the dipole field has its maximum not at the electrodes, fields can more than double. Finally, to maximize the offset field in a direction that is antiparallel to the external field, one desires to push all excess electrons towards the cathode and all excess holes towards the anode. This is opposite to the natural flow of the excess carriers, and so a reasonable alternative is to place appropriate traps near one electrode (say, the cathode), allowing one type of charge to flow (holes) but capturing the other type of charge (electrons). Bombarding the switch with charge (electrons, ions, or protons, for example) might also improve the field enhancement by creating an optimum offset field. A difficulty with this last approach is that it may require fast generation of current, which is part of the purpose of the photoconductive switch. Thus, this concept may require one fast switch just to activate another, resulting in a possible inefficiency. However, nonlinear effects may enable a photoconductive switch to make a fast device even faster. Figure 2 illustrates some concepts for possibly improving field enhancement.

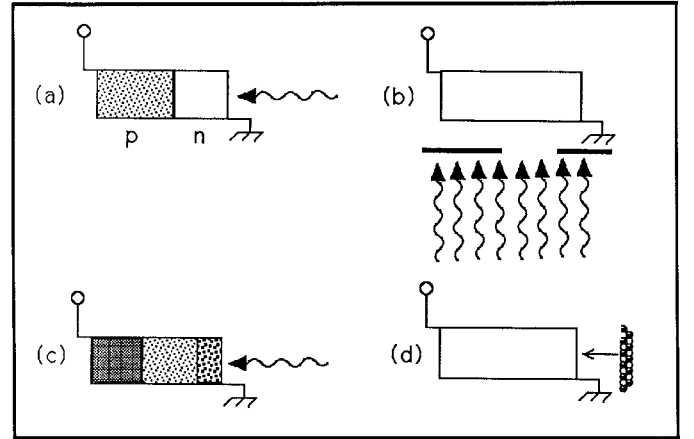


Figure 2. Possible schemes for improving dynamic field enhancement in photoconductors: (a) p-n junction, (b) masked lateral illumination, (c) heavy non-uniform trap density, (d) externally injected charges.

The common element in all of the schemes just mentioned is that homogeneous devices should not be used if one wishes to maximize field enhancement. Figure 3 illustrates the point by comparing computed fields in a homogeneous GaAs slab to computed fields in an inhomogeneous slab. The inhomogeneous slab has 10^{15} EL2-type electron traps per cubic centimeter in the one-quarter of the slab nearest the cathode and zero traps elsewhere. In the particular homogeneous slab used in this test, we achieve no more than a factor of 1.8 increase in the total electric field anywhere at any time by adjusting the optical intensity. In the trap-laden switch, however, field enhancement exceeds a factor of 4 for the same conditions.

Numerical Calculations

We have performed calculations to examine the tradeoffs among photon fluence, optical depth, and doping density in homogeneous samples. At low fields, we find that more field enhancement occurs

for p-type doping than for n-type. Further, illumination through the cathode yields slightly more enhancement than illumination through the anode. Best enhancement occurs with acceptor-ion densities on the order of 10^{13} cm^{-3} , incident fluences on the order of 10^{14} photons/ cm^2 , and optical absorption coefficients on the order of $8/L$, where L is sample thickness. Table 1 shows the field enhancement for a variety of doping levels and fluences with the initial field and optical thickness held fixed.

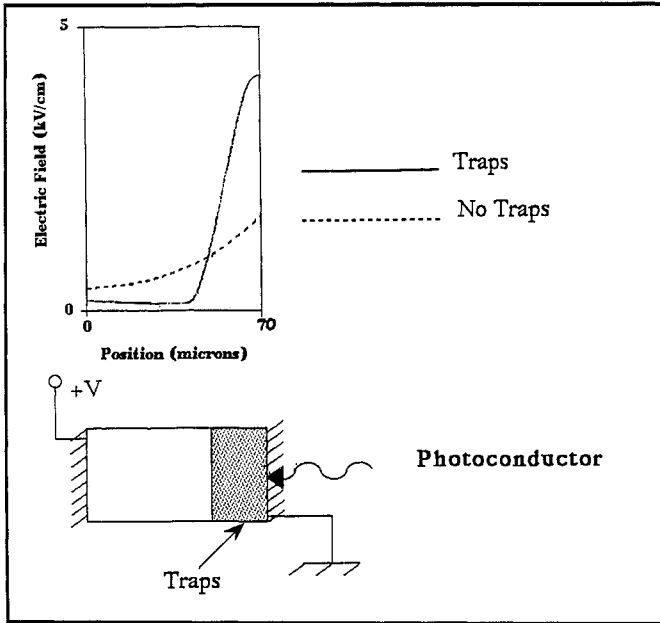


Figure 3. Effect on dynamic field enhancement of placing traps near an electrode. The presence of 10^{15} donor-type traps placed near the cathode of a photoconductor causes the electric field to rise by over a factor of four relative to its uniform, steady-state value of 1 kV/cm. Without traps, field enhancement is limited to about a factor of two. Importantly, the field enhancement is greatest near the traps, regardless of whether illumination is through the cathode or through the anode. In the absence of the traps, the field enhancement occurs at the electrode away from the illumination.

Holding the sample thickness, applied voltage, and incident fluence constant, we varied the numerical value of the optical absorption coefficient. The most field enhancement that we obtained for a homogeneous sample was a factor of 2.6, which occurred when the optical absorption depth was approximately $L/8$. We attribute the result that field enhancement exceeds a factor of 2 to weak violation of the conditions cited earlier. As a function of optical absorption depth, field enhancement rolls off slowly, as shown in Figure 4.

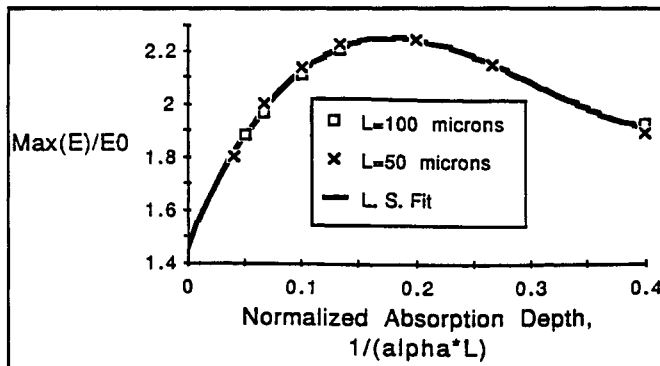


Figure 4. Dynamic field enhancement versus normalized optical absorption depth in homogeneous samples at low fields. Results are shown for two different sample lengths, 50 microns and 100 microns. A cubic polynomial connects the data. In this figure, the best possible field enhancement occurs when the sample is about 5.6 optical absorption lengths thick and has a value of about 2.25. By adjusting the doping and the optical intensity, we can obtain only slightly better enhancement, namely a value of 2.6. These computations do not include the effect of traps.

p-Type GaAs				
	Initial density of photocarrier pairs, Δn_0 (cm^{-3})			
$N_A \text{ cm}^{-3}$	5×10^{12}	5×10^{13}	5×10^{14}	5×10^{15}
10^9	—	—	1.08	1.19
10^{11}	1.46	1.46	1.42	—
10^{12}	1.46	1.82	1.72	1.00
10^{13}	1.45	1.89	1.87	—
10^{14}	1.24	1.70	1.82	1.00

n-Type GaAs				
	Initial density of photocarrier pairs, Δn_0 (cm^{-3})			
$N_D \text{ cm}^{-3}$	5×10^{12}	5×10^{13}	5×10^{14}	5×10^{15}
10^9	—	1.01	1.06	1.19
10^{11}	1.17	1.41	1.43	—
10^{12}	1.20	1.70	1.71	—
10^{13}	1.09	1.50	1.70	1.00

Table 1. Maximum dynamic field enhancement computed for n- and p-type GaAs slabs at low fields. The initial photocarrier density is defined as $\Delta n_0 = \alpha \Gamma$, where α is the optical absorption coefficient (cm^{-1}) and Γ is the optical fluence entering the sample (photons per cm^2). In these calculations, the sample thickness was $L = 50 \mu\text{m}$, the absorption coefficient was $\alpha = 0.05 \mu\text{m}^{-1}$, the electric field was about 1000 kV/cm, and the optical pulse was a square pulse with a width of 10ps. The optical absorption used to get these results was not necessarily optimal. By increasing α , we obtain slightly more enhancement than shown in the table.

Figure 4 also illustrates another result: at low fields the magnitude of the field enhancement depends upon the ratio of optical absorption depth to sample thickness but not on sample thickness alone. Results for two samples, $50 \mu\text{m}$ and $100 \mu\text{m}$ thick respectively, are shown. As can be seen, the field enhancement as a function of the dimensionless parameter $1/\alpha L$, is essentially the same in both cases. (Here α is the inverse of the optical absorption depth.)

The computer codes in this work use 1- and 2-D drift/diffusion models to solve simultaneous equations for circuit voltage and current, conservation of charge in the photoconductor, and Poisson's Equation for electrostatic potential[3,4]. Charge generation terms in the continuity equations include impact ionization of valence states and photon absorption (band-to-band transitions). Recombination terms include optical recombination, steady-state trap-assisted recombination (Shockley, Reed, Hall), and dynamic trap-assisted recombination (kinetic trapping). Omitted from the models are impact ionization of traps (field-dependent trapping and detrapping), Auger recombination, ionic optical absorption, and special boundary effects. The boundary conditions could be selected as Dirichlet, Von Neumann, or mixed. The condition that we most often used in our 1-D calculations was that the normal derivative of the total current vanish termwise at the electrodes.

High-field Results

At high fields, negative differential resistance (NDR) effects can improve dynamic field enhancement in a way similar to device inhomogeneities. NDR leads to instabilities in charge density that grow with time and tend to propagate in such a way that spikes in the electron density approach the anode while spikes in hole density approach the cathode. This can cause the offset field to be antiparallel to the external field and permits large field enhancements. NDR can occur in GaAs as a result of a rollover in the electronic velocity-versus-field function at fields above 3.5 kV/cm, or it can happen as a result of field dependent trapping effects[2]. Let us suppose that photoinjected charge kills the electric field only within a few microns of the electrode of optical incidence. Then, the field increases as a function of position in the direction towards the shadowed electrode. As the field increases with position, so does the electron velocity, until the field exceeds the velocity rollover value. Beyond this position, electrons are slower even though the field continues to rise, leading to a buildup of negative charge in the vicinity of the point of maximum velocity. The resulting spike in the electron density exacerbates the discontinuity in

the electric field, as discussed in the literature[5]. Thus, if dynamic field enhancement is strong enough to raise the field above 3.5 kV/cm, a runaway instability can conceivably occur in which the field is further compressed as the charge density wave moves towards the anode. In terms of the earlier analysis, the instabilities that develop between the field and the current density can lead to locally high dipole fields away from the edges of the device, thereby reversing the direction of the offset field and permitting field enhancements greater than a factor of 2. Figure 5 shows the field enhancement in a 70 μm slab for a high field. The five-fold field enhancement shown in Figure 5 is the most enhancement that we have computed so far. It suggests that instabilities (NDR) do indeed assist the enhancement process.

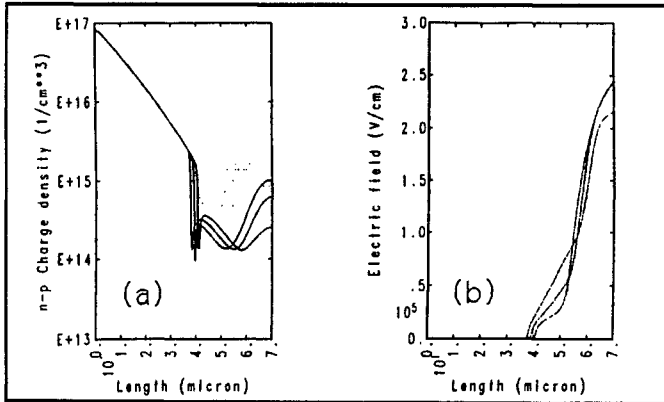


Figure 5. (a) Electron and hole density and (b) Electric field intensity versus position at high fields for three different times spaced 0.1 ns apart. In (a), solid lines represent electron density, n , and dotted lines represent hole density, p . The 70-micron slab has traps at one end, identically to the situation in Fig. 3. The initial field intensity here is 50 kV/cm. Dynamic field enhancement raises the field to almost 250 kV/cm, high enough for weak current avalanching to occur. The fact that enhancement is a factor of 5 at high fields suggests that instabilities may be assisting the process of field compression. The instabilities appear as sharp dips in carrier density that originate at the interface between the trap-free and the trap zones.

Because the nonlinear interaction between charge position and charge velocity is complex, it is difficult to obtain general analytical expressions for the growth of charge instabilities. Conventional descriptions of instabilities such as Gunn domains treat only the motion of electrons and are generally small-signal results[5]. To understand field enhancement thoroughly, one will need to understand better the growth and motion of charge accumulation and depletion zones that result from instabilities. Both electrons and holes must be considered, particularly in semi-insulating material.

2-D Results

One issue in the performance of photoavalanche switches is, does avalanching occur in sheets (1-D approximation) or in filaments (2-D approximation)? If the avalanching is filamentary, then device burnout may be a problem in switching higher currents. This would prevent scaling up the switch size to handle arbitrary powers. Figure 6 shows some preliminary 2-D results for a small device (56x39 microns) with strip electrodes. Light enters through the hollow anode at the bottom of the device. The Gaussian optical pulse peaks at 0.4 ns, has a full $1/e$ width of 0.2 ns, and contains 0.9×10^{18} photons/cm². Optical absorption is $\alpha = 0.1\mu\text{m}^{-1}$. At the optical fluxes used, an initial surge of current fills the device during illumination. Shortly after the optical pulse, the current density recedes. The figure shows that switching tends to be filamentary near the anode but uniform near the cathode. Work is in progress to determine the effects of device dimensions on avalanching and switching performance.

Conclusion

In summary, we have investigated bounds on dynamic field enhancement and dynamically enhanced photoavalanching in low- and high-field regions. We have shown the benefit of using inhomogeneous

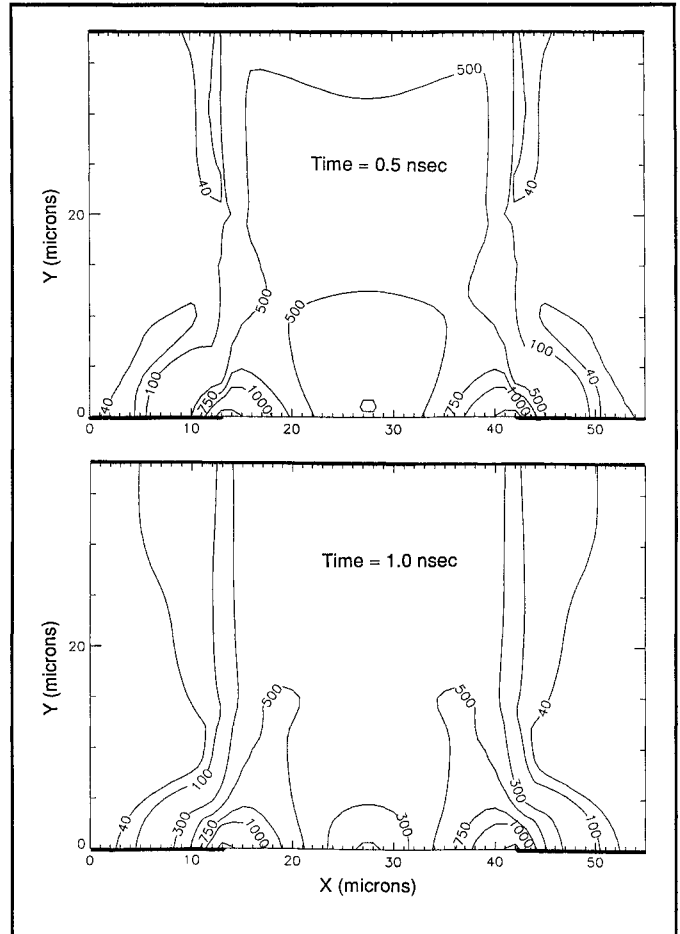


Figure 6. Current density contours across a 2-D photoconductive switch with a hollow anode. Labels on lines represent amps/cm². Initial voltage is 200V. Heavy black lines at the top and bottom of the borders represent the electrodes.

samples such as samples with hole barriers or donor-type traps near the cathode. We have put forth a heuristic argument to suggest that field enhancement is improved by negative differential resistance effects. We have considered the trade-offs between optical thickness, optical fluence, doping density, and applied voltage. What we have not done is demonstrate the relationship between dynamic field enhancement and switch performance, particularly timing considerations (rise time, fall time, or delays) and optical-to-electrical energy conversion efficiency. This work is in progress.

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