

EFFECT OF TRANSFORMER LEAKAGE INDUCTANCE  
ON CAPACITIVE INPUT FILTERS

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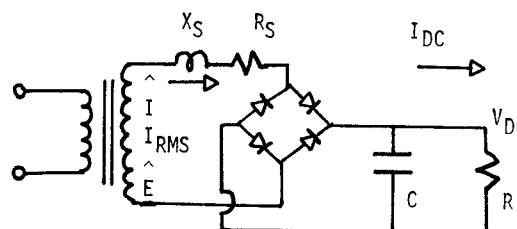
Summary

The effect of series inductance on the performance of the capacitive input power filter has been analyzed by computer and is presented in a set of design curves. The results show that the normally encountered leakage inductance of a transformer has a pronounced effect on reducing the peak and RMS currents in the capacitive input filter circuit. This reduction shows the capacitive input filter to be much more practical than previously believed, especially for high power applications.

Introduction

The detailed analysis of capacitive input filters including inductive reactance was first published in the doctoral thesis of R. L. Freeman in 1934<sup>1</sup>. The equations are awkward and calculations so time consuming they are not usable in practical design work nor were they ever reduced to design tables or charts. A simplified analysis neglecting the reactance was done by O. H. Schade and published as the well-known "Schade Curves" in 1943.<sup>2</sup> Schades Curves give misleading results in most practical cases; specifically the rectifier peak and RMS currents given are much higher than those actually encountered. In this paper computer generated curves are given which show that when the normally encountered transformer leakage inductance is accounted for the capacitive input filter is comparable to the choke input filter in terms of component utilization. In reality the capacitive input filter is much more practical and competitive at high power levels than was generally accepted. The circuit of a full wave single phase capacitive input filter (CIF) is shown in Fig. 1 and includes series inductance and resistance. Analysis of the circuit is more trouble than one would expect from such a simple schematic. The problem is one of lengthy awkward equations not one of mathematical difficulty. The current flows for only a portion of each cycle and the equations for the starting and ending angles of conduction are transcendental. The calculation of the parameters of interest, i.e. peak, RMS and average currents and voltages are very tedious and time consuming, and is the reason that before the coming of computers only the simplified case of zero inductance was published by O. H. Schade in 1943. In practice the zero inductance case is seldom realized; in fact any capacitive input rectifier fed by a transformer, of comparable power rating, will be dominated by the transformer leakage inductance. The leakage inductance strongly influences the circuit because of the high harmonic content of the current waveform and the fact that the reactance of the leakage is proportional to frequency. Only a few percent reactance at the fundamental frequency, which is normally inherent in a transformer, typically reduces the RMS current by a large factor compared to the zero inductance case. Historically CIF's have been avoided in high power circuits because it was wrongly assumed that the zero inductance curves were a reasonable approximation to the case of a few percent react-

ance. Consequently the RMS currents predicted gave a transformer kVA rating much higher than that actually required. This apparently poor transformer utilization factor would thus lead to the choice for a choke input filter. Furthermore the zero reactance curves are always on the conservative side, that is they always predict RMS and peak currents higher than those in a real circuit which contains a small amount of inductance. Therefore, any time a circuit was designed on the basis of the zero reactance curves, it never gave any trouble because it was actually over designed. Since few problems developed from over-stressed components the practice of assuming the zero reactance curves were a good approximation was not questioned.



CAPACITIVE INPUT FILTER (CIF) RECTIFIER  
Fig. 1

Method of Analysis

The circuit upon which the analysis is based is that shown in Fig. 1. Following the work of Schade, the basic parameter taken as the independent variable and abscissa for the curves is  $\omega RC$ . A computer program was written to calculate the peak, average and RMS currents and the peak to peak voltage ripple and average voltage as a function of  $\omega RC$  over the range of .1 to 1000. These computations were made for various series resistance ( $R_s$ ) and reactances ( $X_s$ ) expressed as percentages of the load resistance ( $R$ ). The reactance  $X_s$  is taken at the line frequency  $\omega$ . The results are plotted in Figs. 2 and 3 for series resistance values of 1% and 7% and reactance values of 0%, 1%, 3%, 7% and 10%. Space does not permit more extensive plots but the important point to be made is the effect of the reactance on the peak and the RMS currents and this is clearly shown in Figs. 2 and 3. For example at  $\omega RC = 100$ ,  $X_s = 0.0$  and  $R_s = 1\%$  the RMS current is 2.3 times the d.c. current; and with the same condition except  $X_s = 3\%$ , the RMS current drops to 1.65 times the d.c.. The corresponding change in peak current is 6.64 times the d.c. at  $X_s = 0\%$  to 3.51 times at  $X_s = 3\%$ .

In CIF circuits using transformers the effect increases with the power level of the circuit due to transformer characteristics. In general the per unit (or percent) resistance of a transformer decreases and

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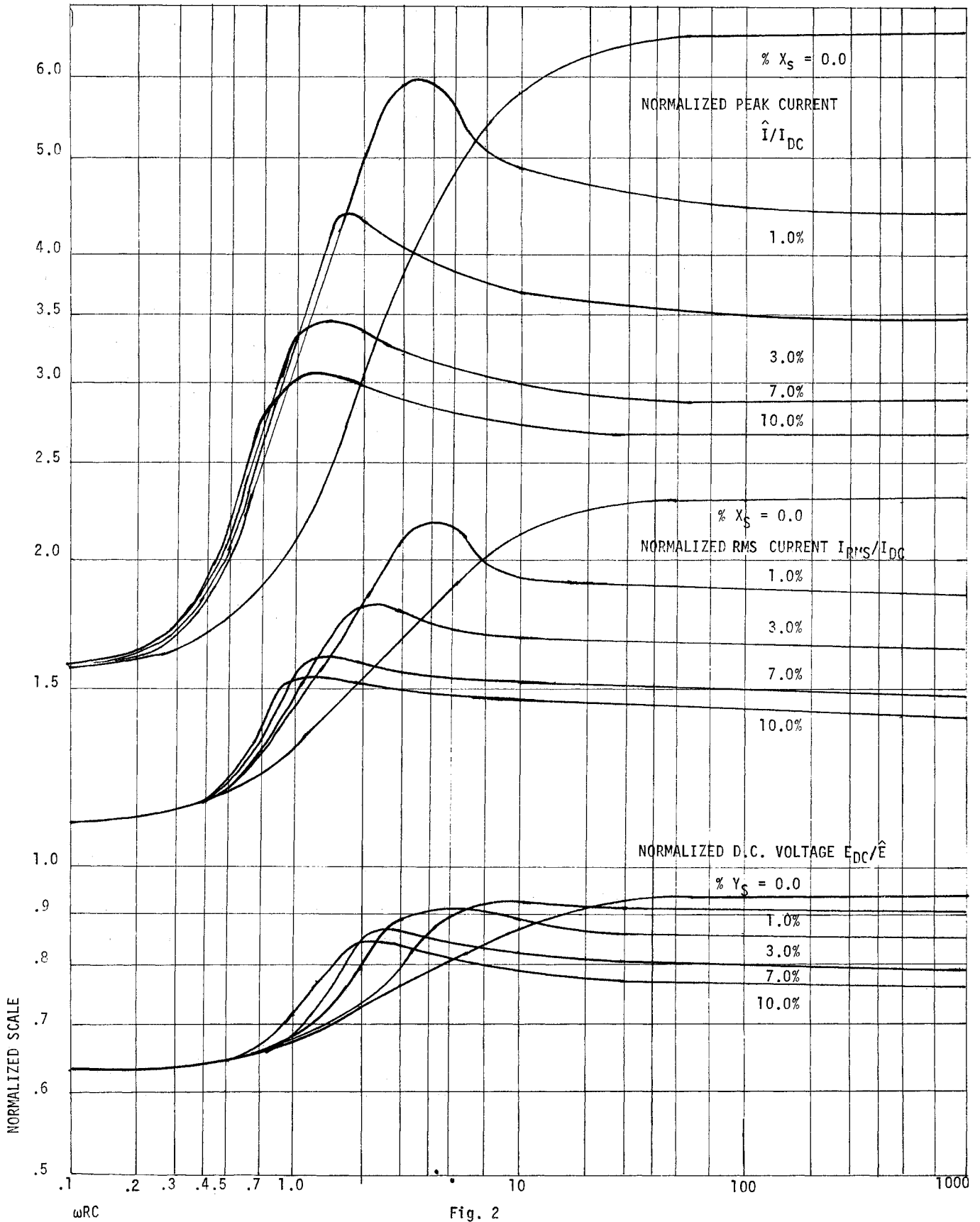


Fig. 2  
 Capacitive Input Filter Characteristics  
 Normalized Peak Current ( $\hat{I}/I_{DC}$ ), RMS Current ( $I_{RMS}/I_{DC}$ ) and DC Voltage ( $E_{DC}/\hat{E}$ )  
 vs  
 $\omega RC$  for 1% ( $R_S/R_L$ ) and  $X_S/R_L$  of 0%, 1%, 3%, 7% and 10%

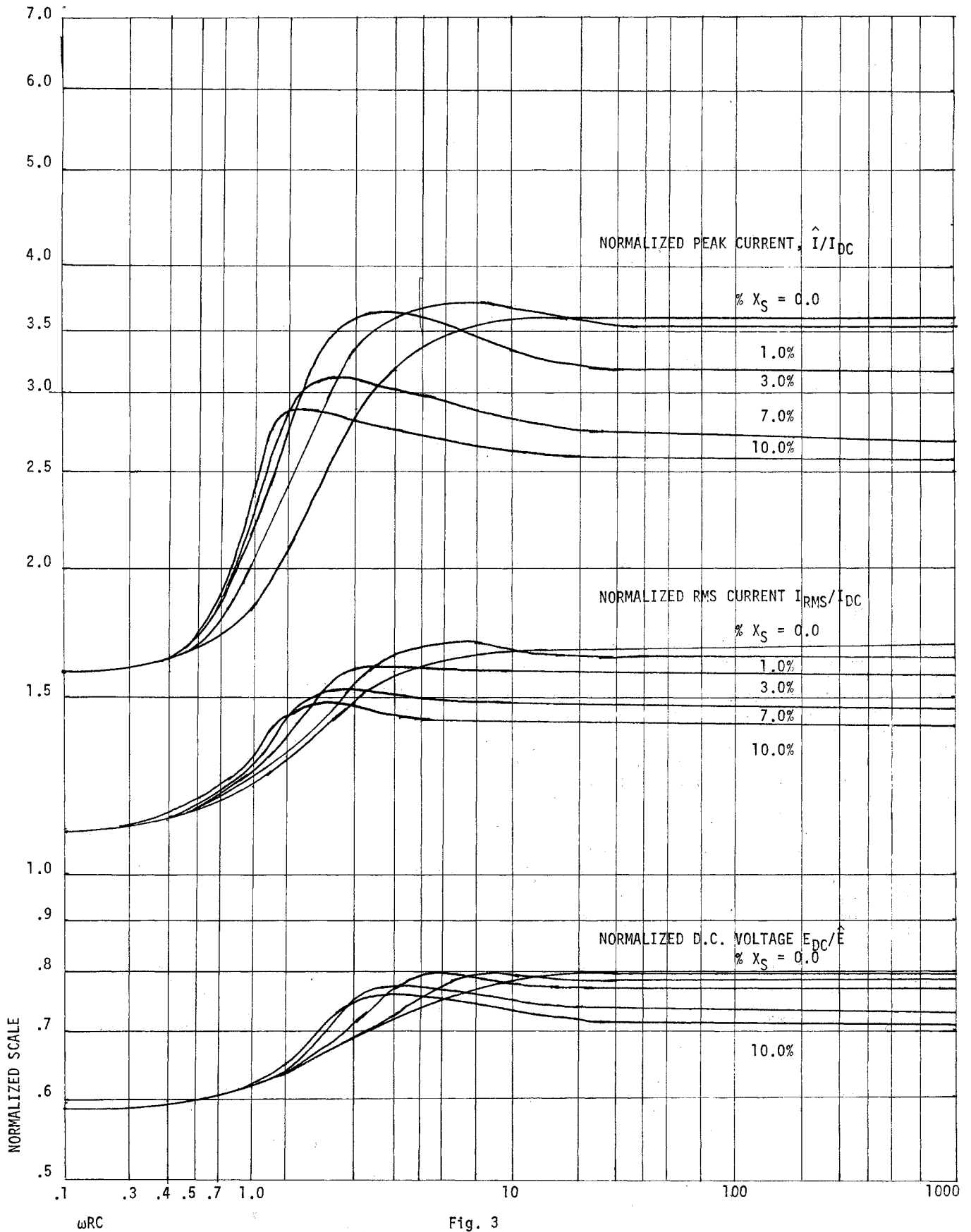


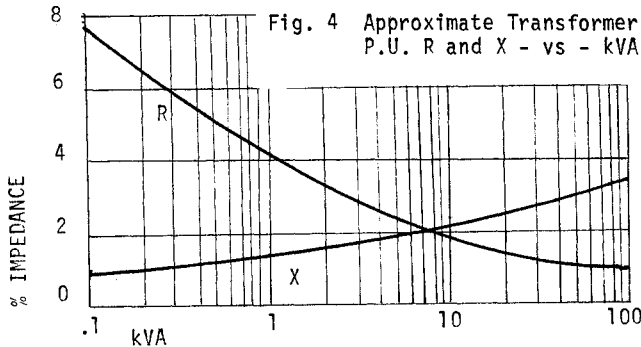
Fig. 3  
 Capacitive Input Filter Characteristics  
 Normalized Peak Current ( $\hat{I}/I_{DC}$ ), RMS Current ( $I_{RMS}/I_{DC}$ ) and DC Voltage ( $E_{DC}/\hat{E}$ )  
 vs  
 $\omega RC$  for 7% ( $R_S/R_L$ ) and  $X_S/R_L$  of 0%, 1%, 3%, 7% and 10%

the per unit reactance increases as the volt-amp rating of the transformer increases. An approximate empirical pair of equations for this behavior is:

$$(1) \%R = .75 \times VA^{-.292} \times f^{-.219} \times 100$$

$$(2) \%X = .000875 \times VA^{.161} \times f^{.438} \times 100$$

Equations (1) and (2) are plotted in Fig. 4, normalized for 60 Hz. Note that the P.U. R and X in equations (1) and (2) and Fig. 4 are on the base of the transformer impedance not the rectified load resistance. As the power level increases the increase in reactance tends to improve the transformer utilization in a CIF but it decreases the utilization in a choke input circuit because of the tendency to drag out the rectifier commutation<sup>3</sup>.



#### Transformer Utilization Comparison

Based on the assumption of zero reactance one finds the transformer utilization of CIF to be much lower than a choke input filter. However, in reality this is not true, the utilization is typically about the same. To illustrate this the same 50 kVA transformer with 1.3% R and 3.0% X was analyzed in a choke input and capacitive input circuit. The circuits are shown in Fig. 5. The analysis was carried out by a computer program which adjusted the load resistance until the transformer was operating at rated kVA. The same value of capacitance, .46 farads, was used in both circuits, but the choke input circuit of course required a choke. The value used was minimum (critical) inductance of 180 microhenrys which has an equivalent kVA of 5.4. The program calculated current waveforms and their harmonic content as well as the voltages and currents. The results are tabulated in Figs. 6 and 7, and show that the utilization on a KW per kVA basis is higher for CIF. The most significant difference occurred in the harmonic content of the line current, being higher for the CIF.

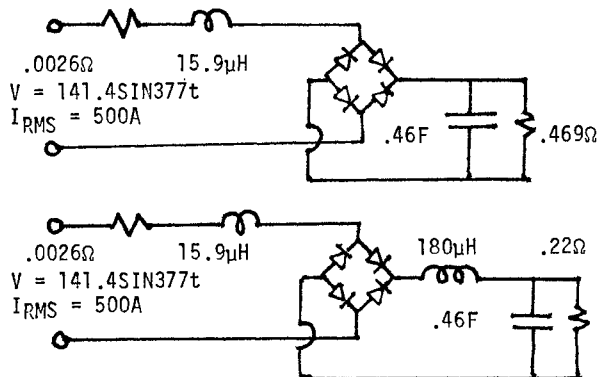


Fig. 5

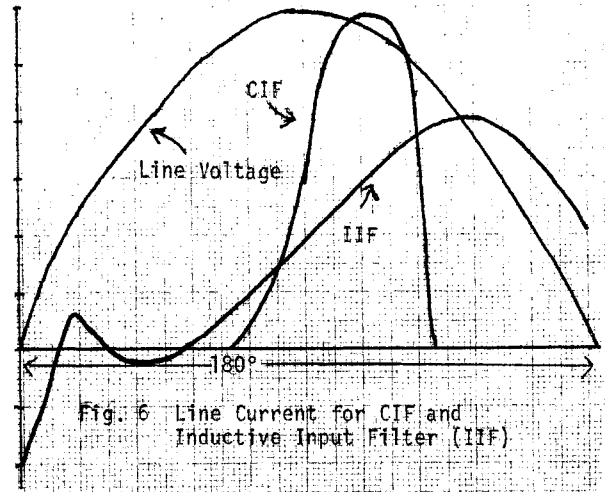


Fig. 7  
Comparative CIF and IIF Data

PARAMETER	CIF	IIF
Transformer kVA	50	50
Inductor kVA	0	5.4
RMS Line Voltage (V)	100	100
RMS Line Current (A)	500	500
% 3rd Line Current	76.1	28.0
% 5th Line Current	41.5	11.4
% 7th Line Current	14.2	6.7
% 9th Line Current	7.9	4.7
% 11th Line Current	6.7	3.5
% 13th Line Current	3.5	2.8
% 15th Line Current	3.3	2.3
D.C. Voltage (V)	128.3	87.8
D.C. Current (A)	273.5	398.1
Power Out (kW)	35.1	35.0
Utilization kW/kVA	.702	.632
% P-P Ripple	2.52	2.84

#### Conclusion

The leakage reactance normally encountered in a power transformer has a significant effect on the performance of CIF circuits. When this effect is accounted for, the utilization factor (KW per kVA) is about as good in the CIF circuit as if it is in the choke input rectifier circuit. This is particularly true in the higher power cases.

#### References

- (1) R. L. Freeman, Analysis of Rectifier-Filter Circuits, Doctoral Thesis, Stanford Univ. Library, 1934.
- (2) O. H. Schade, Analysis of Rectifier Operation, Proceedings of the IRE, July 1943, pp. 341-361.
- (3) R. L. Witzke, J. V. Kresser and J. K. Dellard, Influence of A-C Reactance on Voltage Regulation of 6-Phase Rectifiers, Transactions of AIEE, July 1953, pp. 244-252.