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ABSTRACT

An investigation of the kinetic energy budget of a "minor breakdown" of the stratospheric polar night vortex is performed. The computation covers the period 15 November–15 December 1958, for the 100–50 mb layer north of 40N.

Vertical motions and mean meridional circulations are computed from the thermodynamic equation. The calculations show a two-cell pattern with descending motions in mid-latitudes and ascent over the polar cap. During the period of restabilization after the "minor breakdown," a small area of mean descent appears over the polar cap.

During the amplification stage, the internal energy conversions are acting to *increase* the eddy kinetic energy. The mean meridional circulation is *direct* at higher latitudes, opposite to that occurring during a major breakdown.

The restabilization period is characterized by a reversal in sign of the internal energy conversions and by large boundary fluxes of zonal kinetic energy.

Kinetic energy dissipation values are obtained as computational residuals. The values are large and probably unrealistic. It is shown that spuriously large computed kinetic energy dissipations can result from errors in the radiational estimates.

1. Introduction

With the advent of adequate sounding techniques, the circulation of the stratosphere has received a great deal of attention. In particular, much interest has been focused upon the "sudden warming" phenomenon associated with a major breakdown of the polar night vortex. (For a detailed review of the observational characteristics of the sudden warming, see e.g., Hare and Boville, 1965). The spectacular nature in which this vortex breaks down during such a period has led to numerous suggestions that the breakdown is attributable to an instability mechanism (Fleagle, 1957; Murray, 1960; Charney and Stern, 1962; Reed, 1963; Reed *et al.*, 1963; Sekiguchi, 1963; Van Mieghem, 1963; Hare and Boville, 1965; Mahlman, 1966, 1967). The general lack of agreement as to specifically what type of instability (if any) is acting during the sudden warming has prompted a number of diagnostic analyses of various phases of the energetics of this phenomenon (Boville, 1961; Hare and Boville, 1961; Miyakoda, 1963; Reed *et al.*, 1963; Sekiguchi, 1963; Teweles, 1963, 1965; Lateef, 1964; Julian and Labitzke, 1965; Muench, 1965a, b; Murakami, 1965; Perry, 1967). These studies have shed considerable light on the actual energetics of the stratosphere during a sudden warming period. In general, the analyses have shown that the energy conversions during sudden warming phenomena are not incompatible with many of the proposed instability hypotheses. However, these studies have also pointed out that the sudden warmings are characterized by strong energy interchanges with the troposphere, thus suggesting that the proposed instability mechanisms may not provide a complete explanation of the polar vortex breakdown (Miyakoda, 1963; Reed *et al.* 1963; Julian and Labitzke, 1965; Muench, 1965a, b; Perry, 1967).

One noteworthy characteristic of the polar night vortex is that it does not build up continuously over the winter season, but is subjected to a series of significant perturbations and departures from zonal symmetry (or "minor breakdowns") throughout the winter season (e.g., see Godson and Lee, 1958; Allington *et al.*, 1960; Mahlman, 1966). This fact leads to a number of questions which are presently unanswered. Are the so-called destabilizing mechanisms in cases of the minor breakdowns similar to those observed during major breakdowns of the polar night vortex? Are the energy sources for the minor breakdowns externally or internally produced? What mechanisms are acting to restabilize the polar night vortex during such cases that are not present during a major breakdown period? Why does the minor breakdown at times occur when the zonal mean temperature is still increasing northward, in sharp contrast to the very cold polar region which exists prior to a major breakdown? How does the mean meridional circulation compare with that associated with a major breakdown and what is its time variability?

The intent of this work is to provide answers for these questions and to provide a possible framework for a more thorough inquiry as to how the dynamics of these shorter period fluctuations tie in with the present knowledge of the stratospheric general circulation (e.g., see Oort, 1964). The approach will be to prepare a diagnostic analysis for the complete kinetic energy balance of the lower polar stratosphere treated as an open system in a manner similar to that outlined by Muench (1965a, b).

The case study chosen for analysis is the period 15 November–15 December 1958. This one month period was during the general buildup of the intensity of the polar night vortex. On 15 November, the circulation was dominated by a relatively weak polar vortex. By 25 November, a pronounced breakdown of the zonal character of the motion had begun with the Aleutian high pushing toward the pole and the Canadian low extending down to the United States. This breakdown subsequently reached its peak by 5 December. By 15 December, the nearly symmetric polar night vortex had returned to produce an intense mid-winter circulation as shown in Fig. 1.

The zonal mean temperature ($\bar{T}$) structure was nearly constant over this minor breakdown period (Fig. 2). In fact, the only noteworthy change occurred from 10–15 December, a period of very pronounced cooling of the polar region.

These features of the zonal mean temperature field are in marked contrast to those observed during a major breakdown or sudden warming shown in Fig. 3 which gives $\bar{T}$ as a function of latitude and time for the breakdown of January 1958, illustrating a very rapid reversal of the mean temperature gradient within the space of a few days. The dissimilarity of the $\bar{T}$ profiles between Figs. 2 and 3 indicates that the minor breakdown may not result from the same process that produces a major breakdown. It should be noted, however, that the structure of the polar vortex itself is similar in the two cases. The warm polar region seen in Fig. 3 is predominantly due to the large displacement of the cold polar vortex center from the north pole.

It might be noted that Julian (1967) suggested the term major warming be restricted to cases in which the mean temperature gradient reverses north of the mid-latitude warm belt. In view of the information given by Figs. 2 and 3, this definition seems to be a reasonable one.

The differences between Figs. 2 and 3 imply that the minor breakdown cannot be considered as being directly analogous to a major breakdown. However, enough similarities do exist overall that comparison of the probable mechanisms producing the two phenomena is justified.

2. Computational procedures

As noted earlier, the period chosen for study was 15 November–15 December 1958. There were several motivations for this choice. As is well known, the data coverage for the International Geophysical Year (IGY) was reasonably good. Also, the U. S. Weather Bureau (1963) prepared an excellent and detailed series of 100-, 50- and 30-mb charts for the Northern Hemisphere. Further, considerable effort has already been expended on the IGY data by the Massachusetts Institute of Technology Planetary Circulations Project to establish the energetics of the stratosphere on the climatic scale. These works serve as a valuable background for the study presented here.

The region selected for analysis was the polar cap north of 40N and 100–50 mb layer. Most previous studies of this type have utilized a somewhat larger area (usually north of 10N). However, since the time changes of the zonal mean circulation in the area south of 40N are very small (Fig. 1), inclusion of this additional region may lead to inconclusive or even misleading results on the mechanics of the minor break-

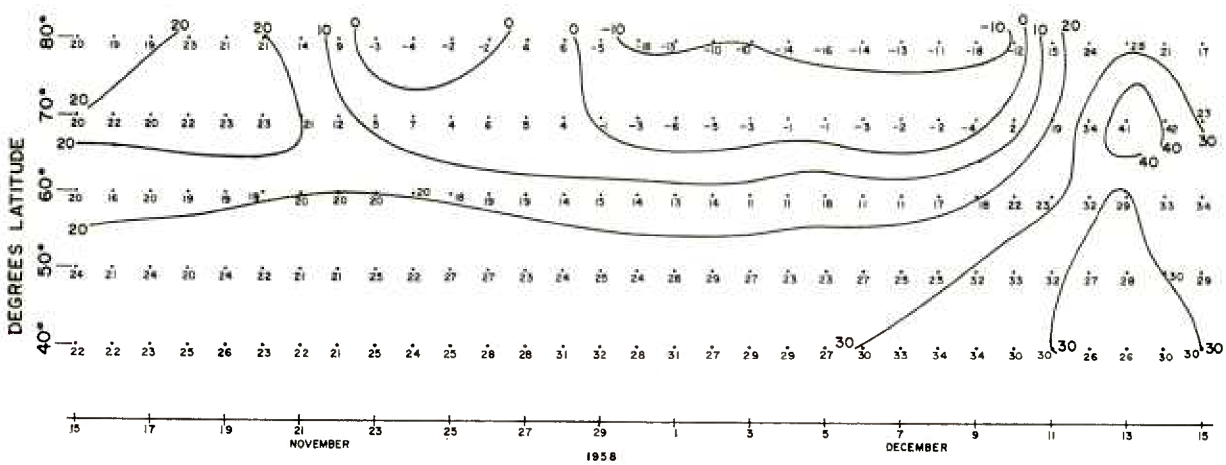

FIG. 1. Average zonal velocity component $\bar{u}(kt)$ as a function of latitude and time at 50 mb for the computation period including the minor breakdown.

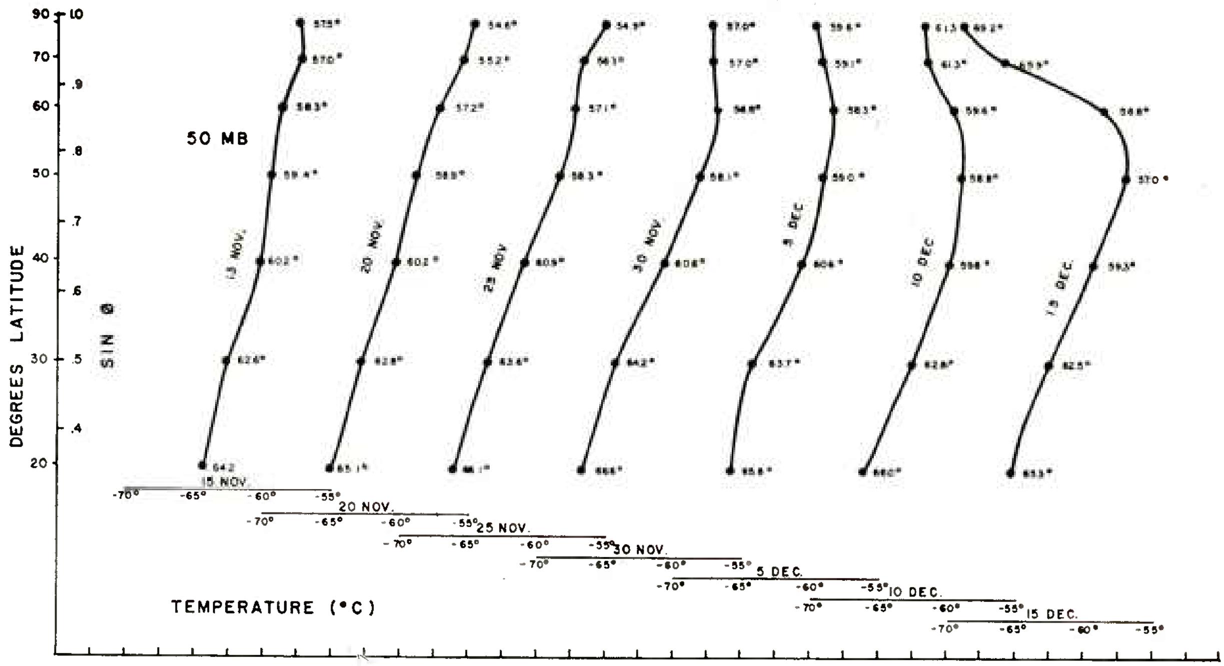

FIG. 2. Zonal mean temperatures plotted as a function of $\sin\phi$ at 50 mb for indicated dates over the computation period. Temperature scale is shifted 10C to the right for each successive 5-day interval. Numerical values of (negative) zonal mean temperature (°C) are given at each point to the right of the curve.

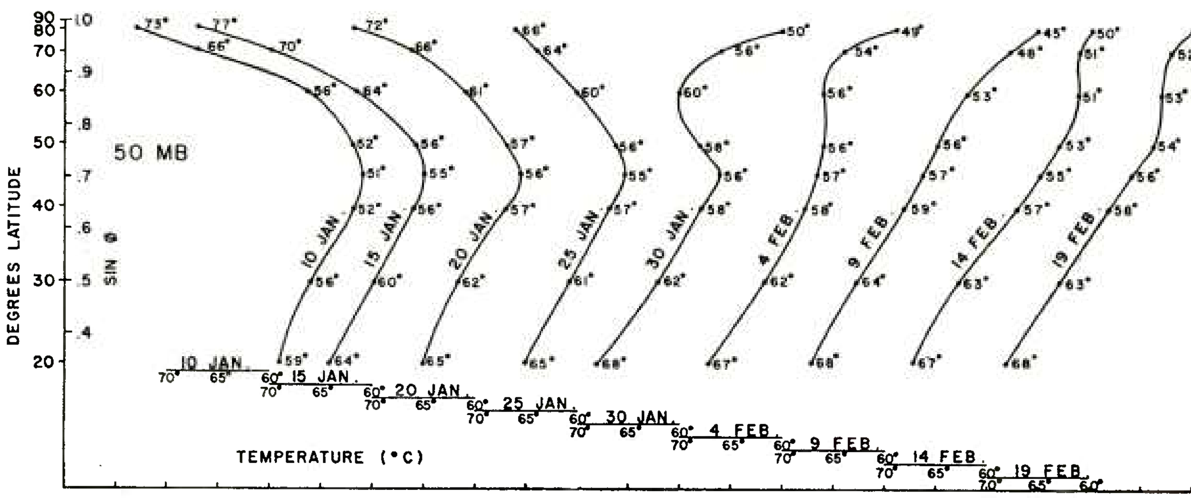

FIG. 3. Same as Fig. 2 except for period of major polar vortex breakdown of January–February 1958 (Mahlman, 1966).

down. The choice of the 100–50 mb layer was required in order to obtain calculations at daily intervals since the 30-mb charts are only available every 10 days.

The computational grid interval was 10° longitude at 40, 50, and 60N, 20° at 70N, and 40° at 80N. In this study actual winds from analyzed isotach fields were used whenever possible. In data-poor regions the geostropic approximation was utilized. It should be noted here that virtually all previous studies on the energetics of the polar vortex breakdown utilized winds obtained either from the geostropic or the balance approximations.

The vertical motion (ω) fields were obtained from the thermodynamic equation in the form[1]

$$\omega=\frac{\dfrac{\partial T}{\partial t}+\mathbf{V}_2\cdot\nabla T-\bar{H}}{\dfrac{\alpha}{c_p}\dfrac{\partial T}{\partial p}}. \tag{1}$$

As will be seen later, the validity of an energy balance calculation depends very critically upon the reliability

[1] See the Appendix for a list of symbols.

of the computed ω fields. This method is recognized to be reasonably accurate in the stratosphere since the static stability is high and no heating by precipitation is present. However, two difficulties do arise when computing stratospheric ω values using Eq. (1). First, the diabatic heating in the stratosphere due to long- and shortwave radiation is not accurately known. In the study presented here, this problem was in part circumvented by using zonally averaged solar heating rates from Manabe and Strickler (1964) and longwave cooling rates from Davis (1963) for the applicable season (see Table 1). This procedure, however, does not take variations of diabatic heating along latitude circles into account and must be recognized as a possible source of error in the computations.

TABLE 1. Stratospheric diabatic heating rates [(°K) day^{-1}] used in the ω calculations (Davis, 1963; Manabe and Strickler, 1964).

Latitude N	Pressure levels (mb) 100	50
40	−0.40	−0.22
50	−0.71	−0.52
60	−0.85	−0.70
70	−0.90	−0.80
80	−1.02	−0.97

The second difficulty which appears in using Eq. (1) results from attempting to compute $\mathbf{V}_2\cdot\nabla T$ directly from the relatively coarse data grid used in this study. In fact, huge errors often result which can completely destroy the reliability of the ω computations. This

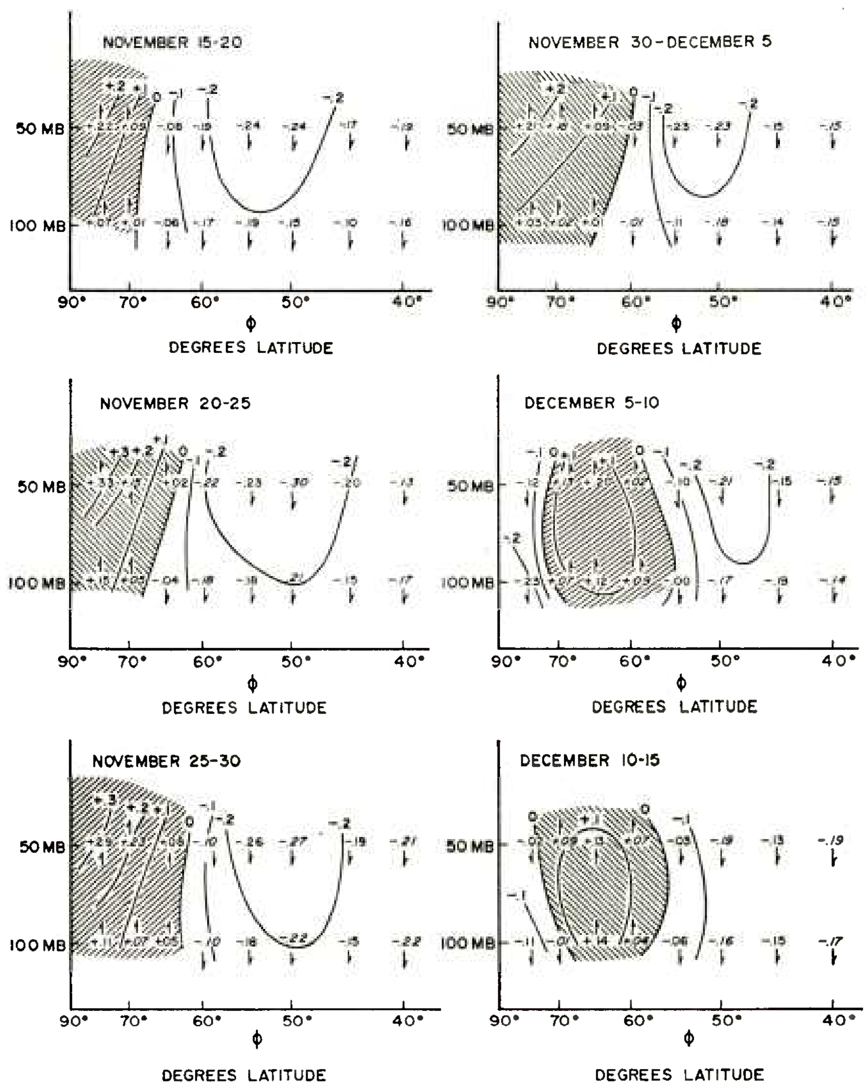

FIG. 4. Zonal mean vertical motion (km day^{-1}) computed for successive 5-day periods from 15 November to 15 December 1958. Italicized values are computed directly from Eq. (1). Values between indicated latitudes are computed from the heat flux method given by Mahlman (1967, 1969). The values show reasonable consistency between the two computational techniques.

problem was avoided by using the natural coordinate form, measuring the gradient of T along the streamlines directly from the analyzed charts and using a shorter grid distance. This procedure, although laborious, has been shown to produce stratospheric vertical motion fields which exhibit both space and time consistency (Mahlman, 1966, 1967).

In an energetical calculation such as this, a reliable determination of the mean meridional circulation is very important. This is especially true in the stratosphere, since the mean meridional circulation seems to play a more important role in the energy balance than in the troposphere (Reed *et al.*, 1963; Oort, 1964; Muench, 1965a, b; Julian and Labitzke, 1965).

In this study the mean cell was computed by two not entirely independent methods. The first technique was simply to obtain daily zonal averages of the ω values from Eq. (1). These were then averaged over 5-day intervals to increase the reliability of the computed values. The second method was to apply an area averaging operator to Eq. (1) for a given level over the polar cap bounded by an arbitrary southern latitude ϕ_s (for details see Mahlman, 1967, 1969). The 5-day average $\bar{\omega}$ values obtained from this technique were found to be comparable to the first method, thus lending some confidence to the values used (Fig. 4).

The mean meridional circulation given in Fig. 4 shows a two-cell pattern with rising motion at the pole and descent in mid-latitudes. In view of the northward increasing zonal mean temperature shown in Fig. 2, this is a *direct* circulation in higher latitudes. However, toward the end of the computation period, a small region of descending motion appears over the polar cap.

3. The energy equations

As noted earlier, the energy equations for an open system used here are similar to those derived by Muench (1965a). Since the basic motivation is to examine the processes producing observed kinetic energy changes, only the budget for the zonal and eddy kinetic energies will be considered. The justification of this approach was noted in the original formulation by Lorenz (1955). Also, this is in part necessitated by the choice of the thermal equation for computing the vertical motion fields. Further, as Muench (1965a, b) has shown, the boundary fluxes of zonal and eddy available potential energy are negligible in comparison to the generation and conversion terms. As a result, little additional information can be obtained by evaluating the available potential energy budgets.

The kinetic energy equations for an open system may be expressed symbolically in the form

$$\frac{\partial K_Z}{\partial t}=C_Z+C_K+BK_Z+B\Phi_Z-D_Z, \tag{2}$$

$$\frac{\partial K_E}{\partial t}=C_E-C_K+BK_E+B\Phi_E-D_E. \tag{3}$$

The mathematical forms for these symbolic expressions can be written as follows:

$$K_Z=\int_{p_1}^{p_2}\left\langle\frac{\bar{u}^2+\bar{v}^2}{2}\right\rangle\frac{dp}{g},\quad K_E=\int_{p_1}^{p_2}\left\langle\overline{\frac{u'^2+v'^2}{2}}\right\rangle\frac{dp}{g}, \tag{4}$$

$$C_Z=-\int_{p_1}^{p_2}\langle\bar{\omega}^*\bar{\alpha}^*\rangle\frac{dp}{g}, \tag{5}$$

$$C_K=\int_{p_1}^{p_2}\left(\left\langle\frac{\overline{u'v'}}{a}\frac{\partial\bar{u}}{\partial\phi}\right\rangle+\left\langle\overline{u'\omega'}\frac{\partial\bar{u}}{\partial p}\right\rangle+\left\langle\frac{\overline{v'^2}}{a}\frac{\partial\bar{v}}{\partial\phi}\right\rangle+\left\langle\overline{v'\omega'}\frac{\partial\bar{v}}{\partial p}\right\rangle-\left\langle\overline{u'^2}\frac{\bar{v}}{a}\tan\phi\right\rangle+\left\langle\overline{u'v'}\frac{\bar{u}}{a}\tan\phi\right\rangle\right)\frac{dp}{g}, \tag{6}$$

$$BK_Z=\int_{p_1}^{p_2}\frac{1}{A}\oint_{\phi_s}v\left(\frac{\bar{u}^2+\bar{v}^2}{2}+\bar{u}u'+\bar{v}v'\right)a\cos\phi_s d\lambda\frac{dp}{g}+\left\langle\overline{\frac{\omega}{g}\left(\frac{\bar{u}^2+\bar{v}^2}{2}+\bar{u}u'+\bar{v}v'\right)}\right\rangle_{p_1}-\left\langle\overline{\frac{\omega}{g}\left(\frac{\bar{u}^2+\bar{v}^2}{2}+\bar{u}u'+\bar{v}v'\right)}\right\rangle_{p_2}, \tag{7}$$

$$B\Phi_Z=\int_{p_1}^{p_2}\frac{1}{A}\oint_{\phi_s}v(\Phi-\langle\bar{\Phi}\rangle)a\cos\phi_s d\lambda\frac{dp}{g}+\left\langle\frac{\bar{\omega}}{g}(\bar{\Phi}-\langle\bar{\Phi}\rangle)\right\rangle_{p_1}-\left\langle\frac{\bar{\omega}}{g}(\bar{\Phi}-\langle\bar{\Phi}\rangle)\right\rangle_{p_2}, \tag{8}$$

$$D_Z=\int_{p_1}^{p_2}(\langle\bar{u}\bar{F}_\lambda\rangle+\langle\bar{v}\bar{F}_\phi\rangle)\frac{dp}{g}, \tag{9}$$

$$C_E=-\int_{p_1}^{p_2}\langle\overline{\omega'\alpha'}\rangle\frac{dp}{g}, \tag{10}$$

$$BK_E=\int_{p_1}^{p_2}\frac{1}{A}\oint_{\phi_s}v\left(\frac{u'^2+v'^2}{2}\right)a\cos\phi_s d\lambda\frac{dp}{g}+\left\langle\overline{\frac{\omega}{g}\left(\frac{u'^2+v'^2}{2}\right)}\right\rangle_{p_1}-\left\langle\overline{\frac{\omega}{g}\left(\frac{u'^2+v'^2}{2}\right)}\right\rangle_{p_2}, \tag{11}$$

$$B\Phi_E=\int_{p_1}^{p_2}\frac{1}{A}\oint_{\phi_s}v'\Phi'a\cos\phi_s d\lambda\frac{dp}{g}+\frac{1}{g}\langle\overline{\omega'\Phi'}\rangle_{p_1}-\frac{1}{g}\langle\overline{\omega'\Phi'}\rangle_{p_2}, \tag{12}$$

$$D_E=\int_{p_1}^{p_2}(\langle\overline{u'F'_\lambda}\rangle+\langle\overline{v'F'_\phi}\rangle)\frac{dp}{g}. \tag{13}$$

In the actual computations, all above expressions are divided by $p_2 - p_1 = 50$ mb to recover the units in the form ergs cm^{-2} mb^{-1} sec^{-1}. The D_Z and D_E terms are not directly evaluated, but are inferred as a computational residual. All remaining terms in all expressions are evaluated with the exception of the very small $\langle \overline{v'\omega'(\partial \bar{v}/\partial p)} \rangle$ term in (5).

All calculations involving $\bar{v}$ or $\bar{\omega}$ are computed from 5-day averages. This has been found to be necessary to give a reliable estimate for the mean meridional circulations. These $\bar{v}$ and $\bar{\omega}$ terms are assumed to apply over the entire 5-day period while the eddy product terms are evaluated daily. This procedure could lead to some error, but in view of the relatively small time variability of the 5-day averages of $\bar{\omega}$ shown in Fig. 4, the error produced is probably not serious.

4. Computational results

As stated earlier, one of the basic motivations for this study is to explain the observed kinetic energy changes during a minor breakdown phenomenon. It is also of interest to compare the similarities and differences between this phenomenon and examples of major polar vortex breakdowns.

Fig. 5 shows a time sequence of K, K_Z and K_E over 5-day intervals. The period 15–20 November is a relatively inactive period. From 20 November to 5 December, K_E increases by more than a factor of 2 while K_Z remains nearly constant. This shall be hereafter called the minor breakdown period. It may be seen that kinetic energy changes during this minor breakdown are roughy analogous to those observed prior to the zonal mean temperature gradient reversal during a major warming (see Miyakoda, 1963; Reed *et al.*, 1963; Sekiguchi, 1963; Teweles, 1963; Julian and Labitzke, 1965; Muench, 1965a, b; Murakami, 1965; Perry, 1967). However, an important difference may be noted. During the beginning of a major breakdown, K_E increases rapidly and K_Z decreases, while K remains approximately constant. For the minor breakdown the K_E increase is *not* accompanied by a K_Z decrease (Fig. 5). In fact, the total kinetic energy increases markedly during the period. It thus is of importance to determine whether this net K increase is due to boundary flux processes or to internal conversions.

The 5–15 December period is characterized by a strong decrease in K_E, a strong increase in K_Z, and by nearly constant K. Hereafter, this shall be denoted as the "restabilization" period. It is this period which departs radically from the comparable time for a major breakdown case. Following a major breakdown, K_Z and K_E simultaneously decrease very rapidly, producing a pronounced loss of K in a very short time.

Thus, the kinetic energy changes of the minor breakdown appear at the onset very much like a major breakdown, but at the apparent peak of the amplification

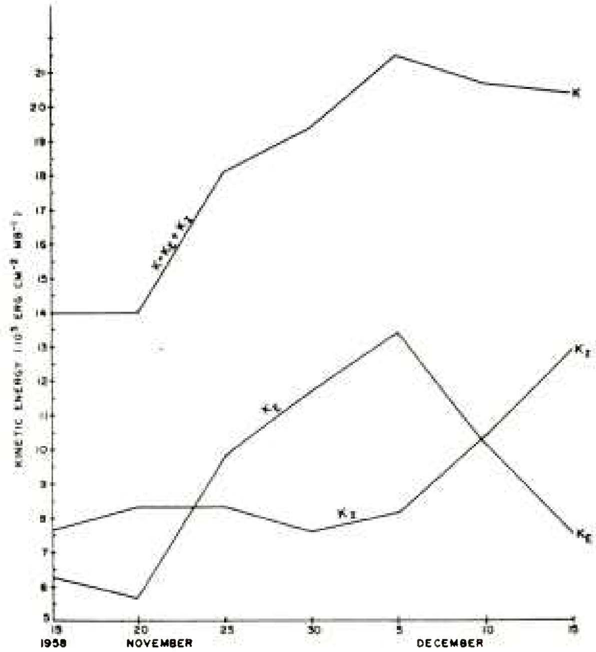

FIG. 5. K, K_Z and K_E for the 100–50 mb layer north of 40N for period between 15 November and 15 December 1958. The period 20 November–5 December is designated as the minor breakdown and 5–15 December is called the restabilization period.

stage the two phenomena are very much different; the former stabilizes in a manner similar to tropospheric wave developments, while the latter is completely irreversible.

In view of the previously mentioned instability hypotheses attempting to explain polar night vortex breakdowns, the internal energy conversions leading to changes in K_E are of special interest. For the onset period of major breakdowns, the sign and magnitude of the C_K conversion has been the subject of some controversy. For the entire warming period period C_K is generally found to be positive, thus supplying kinetic energy from the eddies to the zonal current. However, some investigators (Miyakoda, 1963; Sekiguchi, 1963; Murakami, 1965) have measured *negative* values of C_K just prior to the reversal of the zonal mean temperature gradient.

The calculations of C_K for this case are given in Fig. 6. The lower graph is a plot of 5-day mean C_K values (solid lines) vs daily values (dashed lines). This figure shows a rather consistent daily variability in C_K, and also that C_K becomes *negative* during the minor breakdown period. The upper part of Fig. 6 gives 5-day means of C_K with and without the mean cell terms included. This shows that the effect of including the mean cell terms is to *decrease* the computed value of C_K.

Since many previous studies concentrated on the internal energy conversion terms rather than the boundary fluxes, a comparison of observed kinetic energy changes against internal conversions is given in Fig. 7. The left side of this figure shows a comparatively strong

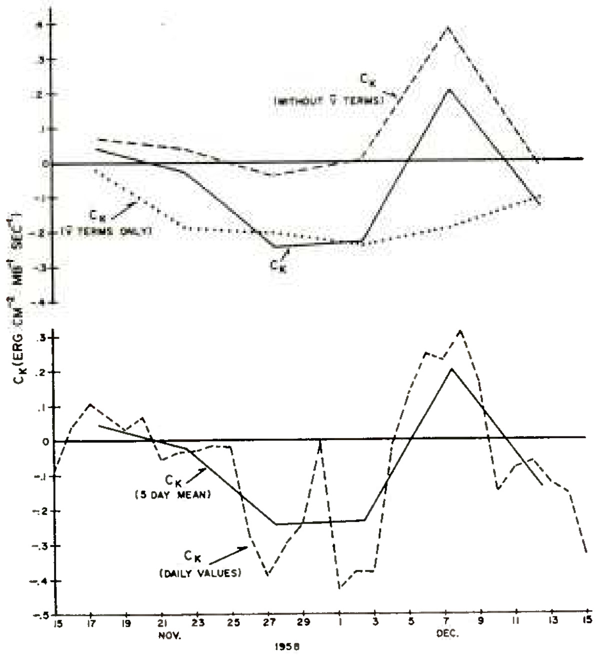

FIG. 6. Results of C_K computations from 15 November–15 December 1958. Lower graph shows daily values (dashed lines) against 5-day means (solid lines) of C_K. Upper graph compares C_K without mean cell terms (dashed lines), C_K with only mean cell terms (dotted lines), and C_K including all terms (solid lines).

positive C_Z conversion (direct cell) during the computation period. However, for 10–15 December C_Z becomes negative, thus reflecting the reversal of the zonal mean temperature gradient seen in Fig. 2. It is clear from Fig. 7 that the observed changes of K_Z are quite unrelated to the internal conversion terms.

The right side of Fig. 7 is somewhat more encouraging in that the internal conversion terms are at least qualitatively related to the changes in K_E. This figure does show, however, that the observed K_E changes are larger than the sum of the internal conversions alone.

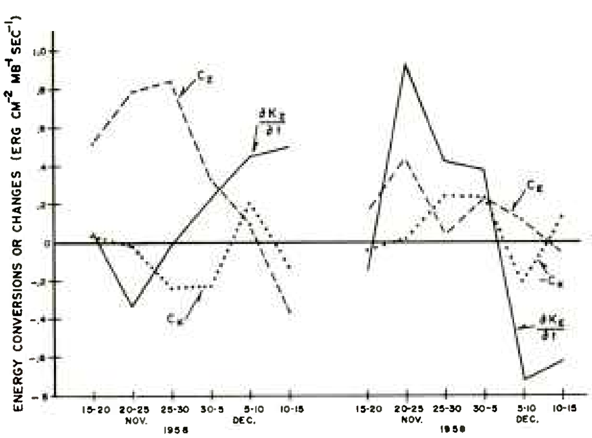

FIG. 7. Comparison of observed kinetic energy changes vs internal conversions for indicated periods. Left graph is for the zonal kinetic budget, right graph for the eddy kinetic energy budget.

During a major breakdown, the C_Z conversion is *negative* up to the point of the reversal of the zonal mean temperature gradient, then becomes positive thereafter (Reed *et al.*, 1963; Julian and Labitzke, 1965; Mahlman, 1966). For this minor breakdown almost the exact opposite occurs, with an initially positive C_Z becoming negative during the restabilization period.

The C_E conversion term is very similar in behavior for both major and minor breakdowns. During the amplification stage C_E is positive, but becomes negative during the later stages.

It should be noted that the approximate magnitude of the internal conversions is about a factor of 5 larger for major breakdowns than for this minor breakdown. This suggests that the processes occurring in this minor breakdown may be so weak as to be near the limit of the ability of this type of computational approach to delineate them. In anticipation of this difficulty, all data tabulations were performed daily rather than on selected days as was done in most major breakdown studies (Miyakoda, 1963; Reed *et al.*, 1963; Julian and Labitzke, 1963; Muench, 1965a, b; Perry, 1967). Also, the observed time changes in K_E and K_Z in Figs. 7 and 8 and the boundary flux terms in Figs. 8 and 9 are as large here as for the major breakdown studies. Further, the mean meridional circulations shown in Fig. 4

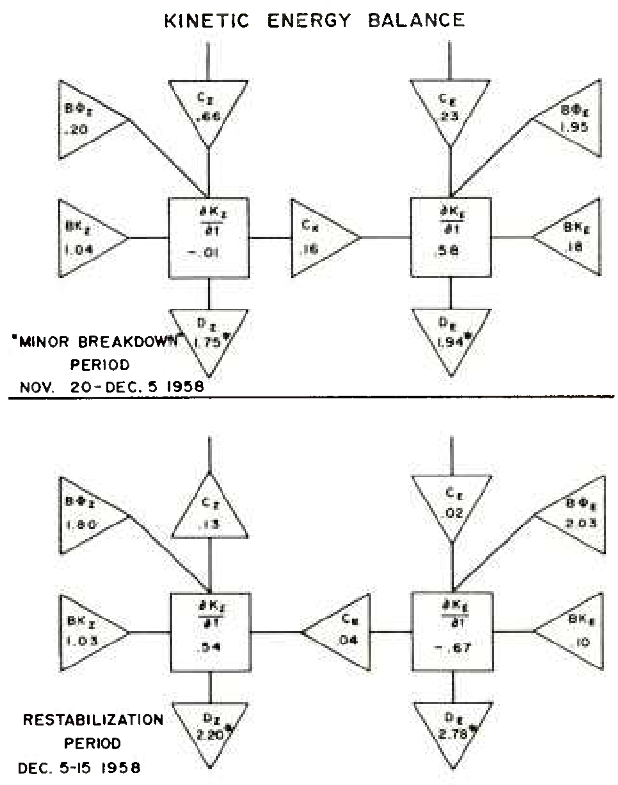

FIG. 8. Kinetic energy balances (ergs cm^{-2} mb^{-1} sec^{-1}) for the minor breakdown and restabilization periods. D_Z and D_E are inferred from computational residuals in the K_Z and K_E budgets and do not necessarily represent true kinetic energy dissipations.

are nearly as intense as during a major breakdown (Miyakoda, 1963; Reed, *et al.*, 1963; Julian and Labitzke, 1965; Mahlman, 1966, 1969; Perry, 1967). Consequently, such a diagnostic analysis of this somewhat weaker phenomenon appears justified.

To aid in clarification of the results obtained from this approach, the kinetic energy balance calculations have been separated into two periods, minor breakdown and restabilization. Fig. 8 shows the kinetic energy balances for these two distinct periods.

The minor breakdown is characterized by positive C_Z, positive C_E, and negative C_K. These three internal conversions are all opposite in sign to the annual mean values typical of the lower stratosphere (Oort, 1964). The $B\Phi_Z$ term is relatively small while BK_Z is a large term in the K_Z budget. This is in marked disagreement with the contention by Jensen (1961) that the BK terms are negligibly small relative to the $B\Phi$ terms. On the other hand, $B\Phi_E$ is considerably larger than BK_E in the K_E budget. In fact, the $B\Phi_E$ term remains large throughout the month. However, the increase of K_E during the minor breakdown is probably largely due to a direct input from the conversion terms C_K and C_E. This result is compatible with the hypotheses of Charney and Stern (1962) and Mahlman (1966) that the major polar vortex breakdown is attributable to a combined barotropic-baroclinic instability phenomenon.

Estimates of the dissipation terms D_Z and D_E are obtained as computational residuals from the sum of all the other terms. This point will be discussed in more detail later.

For the restabilization period Fig. 8 shows a considerably different result. Now C_K is positive, C_Z negative, and C_E has diminished to near zero. The pronounced increase in K_Z is apparently due to the large increase in $B\Phi_Z$. This increase is sufficient to overcome the effect of the sign reversal which has occurred in C_Z. The decrease in K_E apparently is in part due to the small C_E term and the positive C_K term. Note that the implied dissipations are larger for the restabilization period than for the minor breakdown.

Because of the generally large contribution of the boundary flux terms and also because of the large implied dissipation, it is very difficult to explain small changes in kinetic energy from the computations. Although the $B\Phi_E$ terms appear relatively consistent, their daily values tend to fluctuate rapidly. Consequently, this is a source of considerable uncertainty in the computations, particularly for periods of a few days or less. However, one can state with some certainty that the stabilization in this case results from a large input of zonal kinetic energy from the C_K, $B\Phi_Z$ and BK_Z terms. In a major breakdown these boundary flux terms become smaller or quite possibly even negative after the reversal of the zonal mean temperature gradient (Miyakoda, 1963; Julian and Labitzke, 1965). Because of the difficulty in computing $B\Phi_Z$, this inference is somewhat uncertain. If this is correct, however, one may hypothesize that the irreversibility of a major polar vortex breakdown results from a combination of outside influences and internal energy conversions during the amplification stage. On the other hand, the restabilization after a minor breakdown appears to be a somewhat more "normal" process due to the increased efficiency of the nonlinear stabilizing effects resulting from the original amplification.

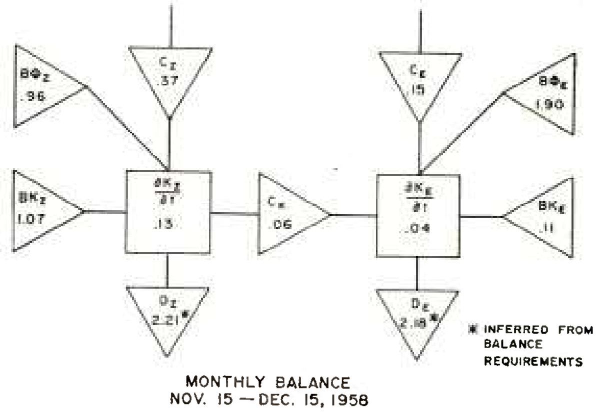

FIG. 9. Kinetic energy balances for the period 15 November–15 December 1958. D_Z and D_E are inferred as residuals in the K_Z and K_E budgets and do not necessarily represent true kinetic energy dissipations.

Fig. 9 gives the kinetic energy balances for the entire period 15 November–15 December 1958. For this period C_Z is positive, C_K negative and C_E positive, again opposite to Oort's (1964) climatic values. This indicates that the stratosphere during this period of the year is in part, at least, a self-sustaining system. On the other hand, Fig. 9 also indicates that the boundary fluxes are still of somewhat larger importance than the internal conversions for maintaining the stratospheric kinetic energy during this period of the year.

5. Computational imbalances and kinetic energy dissipation

The inferred kinetic energy dissipation terms for the entire period are surprisingly large (Fig. 9). Since these dissipations are inferred from computational residuals, they are subject to considerable uncertainty. However, the values of D_Z and D_E obtained are in each case larger than the sum of the extremes of the computed 95% confidence limits for the terms comprising the K_Z and K_E budgets, respectively (see Table 2). Further, large positive values of D_Z and D_E result from each 5-day calculation, thus making this result difficult to dismiss as *random* numerical error.

Other investigators have attempted to compute kinetic energy dissipation values for this region of the atmosphere. Kung (1967) obtained a D of 4.58 ergs cm^{-2} mb^{-1} sec^{-1} for the 100–50 mb layer over North America for the winter season using about two years of data. Jensen (1961) arrived at a total dissipation

TABLE 2. Monthly mean values of all terms in kinetic energy balances (ergs cm^{-2} mb^{-1} sec^{-1}). Confidence limits at the 95% level are given for the computed values.

Term	Monthly mean value	95% Confidence limit
BK_Z	1.07	±0.19
$B\Phi_Z$	0.96	±0.80
C_Z	0.37	±0.37
$\partial K_Z/\partial t$	0.13	±0.28
D_Z	2.21	±0.44
C_K	−0.06	±0.13
BK_E	0.11	±0.17
$B\Phi_E$	1.90	±1.00
C_E	0.15	±0.14
$\partial K_E/\partial t$	0.04	±0.52
D_E	2.18	±1.19

value of 2.12 ergs cm^{-2} mb^{-1} sec^{-1} for the 100–50 mb layer during January 1958 for the region north of 20N. Jensen's computations, however, employed the adiabatic assumption and did not include an evaluation of the $B\Phi$ or BK terms at the southern boundary. Both of these computed values compare reasonably well with the D of 4.39 ergs cm^{-2} mb^{-1} sec^{-1} obtained here.

However, because the 100–50 mb layer is characterized by relatively high static stabilities and is usually bordered by a region of higher kinetic energies, it is physically difficult to make a strong case for large values of kinetic energy dissipation in this volume. Thus, before these large computed values of kinetic energy dissipation are accepted, one should determine the possible effect of systematic errors on the calculations.

In the K_Z budget the $B\Phi_Z$ term was the one which behaved most erratically, and probably was the most unreliable. There are definite reasons why this is the case. First, the mean meridional circulation ($\bar{v}$ and $\bar{\omega}$) is quite difficult to determine reliably with present meteorological data. Second, this term is quite sensitive to the assumptions utilized for the zonal mean diabatic heating rates. The calculated monthly mean $B\Phi_Z$ using the heating rates in Table 1 is 0.96 erg cm^{-2} mb^{-1} sec^{-1}. However, utilizing Kennedy's (1964) diabatic heating rates, the same calculation gives a monthly mean $B\Phi_Z$ of −0.01 erg cm^{-2} mb^{-1} sec^{-1}. On the other hand, an assumption of a constant 1K day^{-1} diabatic cooling rate over the entire volume leads to a computed monthly mean $B\Phi_Z$ of 2.53 ergs cm^{-2} mb^{-1} sec^{-1}. Thus, the calculation is extremely sensitive to comparatively small differences in the radiation estimates. This fact alone implies a considerable uncertainty in the calculated values of D_Z.

For a major breakdown study poorly known zonal mean diabatic heating rates could produce a comparable degree of uncertainty in the C_Z term. For this calculation, however, this is not the case because of the comparatively weak gradient of zonal mean temperature (Fig. 2).

A possible source of error in the eddy kinetic energy balance and in the implied dissipation values arises from the neglect of longitudinal variations in radiative cooling. As pointed out by Winn-Nielsen (1964) and by Muench (1965a, b), use of an ω from the adiabatic assumption in the C_E term actually amounts to computing C_E-G_E. This is also the case if longitudinal variations in H are omitted. Further, a similar difficulty arises in the $B\Phi_E$ term. Since no longitudinal variations of H were employed here, it is desirable to obtain an estimate of this effect upon the calculations.

One straightforward way to estimate the contribution of this effect is to assume a very simple eddy cooling which is proportional to the eddy temperature $T'=T-\bar{T}$, i.e.,

$$H'\approx CT', \tag{13}$$

where C is a proportionality constant (which may vary with pressure).

Further, one can write $\omega=\omega_{\text{calc}}+\omega_{\text{Erad}}$, where ω_{calc} is given by Eq. (1) and

$$\omega_{\text{Erad}}=-\frac{H'}{\dfrac{\alpha}{c_p}-\dfrac{\partial T}{\partial p}}. \tag{14}$$

Combination of (13) and (14) and assuming $\alpha/c_p \gg |\partial T/\partial p|$ yields

$$\omega_{\text{Erad}}\approx-\frac{c_pCT'}{\alpha}. \tag{15}$$

The definition $\omega=\omega_{\text{calc}}+\omega_{\text{Erad}}$ suggests the relationships

$$\overline{\langle\omega'\Phi'\rangle}=\overline{\langle\omega'_{\text{calc}}\Phi'\rangle}+\overline{\langle\omega_{\text{Erad}}\Phi'\rangle}, \tag{16}$$

$$\overline{\langle\omega'\alpha'\rangle}=\overline{\langle\omega'_{\text{calc}}\alpha'\rangle}+\overline{\langle\omega_{\text{Erad}}\alpha'\rangle}, \tag{17}$$

where the first terms are the ones which have been evaluated, and the second terms represent the contribution by "eddy radiation."

Multiplication of Eq. (15) by Φ' and α', respectively, and averaging over latitude and longitude gives to a good approximation

$$\overline{\langle\omega_{\text{Erad}}\Phi'\rangle}\approx-\frac{pC\langle\overline{T'\Phi'}\rangle}{\kappa\langle\bar{T}\rangle}, \tag{18}$$

$$\overline{\langle\omega_{\text{Erad}}\alpha'\rangle}\approx-\frac{c_p\langle\overline{CT'^2}\rangle}{\langle\bar{T}\rangle}. \tag{19}$$

Now if one assumes that a T' of 10K leads to an H' of −0.5K day^{-1}, this implies a C of −0.05 day^{-1} in Eq. (13). This is probably an overestimate for this case, but is very close to the 50-mb value estimated by

Hering *et al.* (1967) for the summer season. For this crude calculation the value of C is assumed to be independent of pressure. Typical values of the terms appearing in Eqs. (18) and (19) for this case study are $\langle\overline{T'\Phi'}\rangle$ (100 mb) $\approx 0.5\times10^4$ (°K) m² sec⁻², $\langle\overline{T'\Phi'}\rangle$ (100mb) $\approx 1.2\times10^4$ (°K) m² sec⁻², $\langle\overline{T'^2}\rangle$ (100 mb) ≈ 32 (°K)², $\langle\overline{T'^2}\rangle$ (50 mb) ≈ 39 (°K)², and $\langle\bar{T}\rangle\approx 215$K. Inserting these quantities and the above assumed value for C into Eqs. (18) and (19) and into Eqs. (12) and (10), respectively, gives

$$B\Phi_{E(\mathrm{Erad})}\approx +0.2 \text{ erg cm}^{-1}\text{ mb}^{-2}\text{ sec}^{-1},$$

$$C_{E(\mathrm{Erad})}\approx -1.0 \text{ erg cm}^{-2}\text{ mb}^{-1}\text{ sec}^{-1},$$

where (Erad) denotes the contribution to the $B\Phi_E$ and C_E terms by the assumed eddy radiative process. Thus, the inclusion of this effect strongly influences both the C_E and $B\Phi_E$ terms.

If the above assumptions are nearly correct, and also if Kennedy's (1964) zonal mean diabatic heating estimates are more nearly accurate than the values used here, this implies that the total dissipation D is about 2.6 ergs cm⁻² mb⁻¹ sec⁻¹ rather than the 4.39 value obtained from the original calculation. Substitution of this new value for D into the product KD^{-1} gives an estimated "dissipation time" of about 8 days for this volume. However, this value is highly uncertain in view of the computational difficulties outlined above.

Another difficulty in the K_E balance arises from the double integral term in (12), the expression for $B\Phi_E$. It can be readily shown that the contribution of this term depends upon a nonzero eddy covariance between the geopotential and the ageostrophic part of the northward wind component. Consequently, this term is extremely difficult to evaluate using present meteorological data. Because of the large expected uncertainty, this term was not included in the present analysis. However, it is desirable to include at least an estimate of its possible contribution.

Measurement of the observed velocity accelerations in the vicinity of 40N for this case gives an estimate of the rms northward component of the ageostropic wind of about 0.8 kt. At first glance this value appears small, but for this case the Rossby number is comparatively small ($\sim$0.02). Consequently, this does not appear to be an underestimate, at least for the measurable motion scales. Further, the rms geopotential height is about 120 m at 40N for this case. Now, if the eddy correlation coefficient between these two quantities is about $\pm$0.25, the estimated contribution of this term is

$$B\Phi_E(\phi_s)\approx \pm 0.4 \text{ erg cm}^{-2}\text{ mb}^{-1}\text{ sec}^{-1}.$$

If the above estimate is reasonable, then this term also can contribute significantly to the balance of eddy kinetic energy. However, the assumed eddy correlation coefficient of $\pm$0.25 for this calculation is probably an overestimate. This value was used here because it represents a value typical of such "well correlated" quantities as $v'T'$ and $u'v'$. As a result, the inability to calculate this term adequately probably does not seriously affect the K_E balance.

6. Summary

This investigation of the energetics of a "minor breakdown" of the polar night vortex has shown some similarities and some marked differences between this case and examples of major polar vortex breakdowns. The generally accepted two-cell mean meridional circulation pattern with rising motion over the pole and sinking in mid-latitudes is also present here. However, this cell is *direct* for most of the period, in contrast to a major breakdown. Also, a small area of mean sinking motion appears at the pole during the restabilization period.

The energy transfers during the minor breakdown period are quite similar to those observed for a major breakdown, although the C_Z conversion is reversed in sign. The restabilization period is characterized by large positive $B\Phi_Z$ and BK_Z terms which are small or even negative during the analogous period of a major breakdown. Thus, the destabilizing mechanisms acting in both major and minor breakdowns appear similar, but a strong restabilizing mechanism is present at the peak of the minor breakdown.

The C_E and C_K conversions suggest that the destabilization stages in both major and minor breakdowns can be related, in part at least, to a combined barotropic-baroclinic instability mechanism. However, the more advanced stages probably cannot be explained by any linear theories.

Calculations of the energy budget lead to an inference of rather high kinetic energy dissipation values for the region. These values agree rather well with Kung's (1967) estimates. However, further analysis reveals that the $B\Phi_Z$ term is quite sensitive to modest errors in the specification of the zonal mean diabatic heating. Also, the $B\Phi_E$ and C_E terms are found to be significantly altered by including estimates of the effect of longitudinal variations in the diabatic heating.

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APPENDIX

Table of Symbols

a radius of earth

A area of polar cap bounded by ϕ_s

C proportionality constant relating eddy diabatic heating to eddy temperatures
c_p specific heat of air at constant pressure
g acceleration of gravity
H diabatic heating rate
p pressure
R gas constant for dry air
t time
T temperature
u zonal velocity component$[=a\cos\phi(d\lambda/dt)]$
v meridional velocity component$[=a(d\phi/dt)]$
$\mathbf{V}_2$ horizontal velocity vector
α specific volume
κ R/c_p
λ longitude
ϕ latitude
ϕ_s latitude at southern boundary of computation (40N)
ω dp/dt
∇ horizontal del operator in pressure coordinates
$\bar{\gamma}$ zonal mean of arbitrary variable γ equal to

$$\frac{1}{2\pi}\oint \gamma d\lambda$$

γ' $\gamma-\bar{\gamma}$
$\langle\bar{\gamma}\rangle$ area average of γ equal to

$$\frac{1}{(1-\sin\phi_s)}\int_{\phi_s}^{\pi/2}\bar{\gamma}\cos\phi d\phi$$

γ^* $\gamma-\langle\bar{\gamma}\rangle$
$\bar{\gamma}^*$ $\bar{\gamma}-\langle\bar{\gamma}\rangle$
K_Z zonal kinetic energy
K_E eddy kinetic energy
K K_E+K_Z
C_Z conversion from zonal available potential energy to zonal kinetic energy
C_E conversion from eddy available potential energy to eddy kinetic energy
C_K conversion from eddy kinetic energy to zonal kinetic energy
BK_Z boundary flux of zonal kinetic energy
BK_E boundary flux of eddy kinetic energy
$B\Phi_Z$ boundary flux of zonal geopotential
$B\Phi_E$ boundary flux of eddy geopotential
D_Z dissipation of zonal kinetic energy by friction
D_E dissipation of eddy kinetic energy by friction
G_E generation of eddy available potential energy

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