3.2

HIGH POWER PULSE MODELING OF COAXIAL TRANSMISSION LINES

JAMES P. O'LOUGHLIN

AIR FORCE WEAPONS LABORATORY
KIRTLAND AFB, NM 87117

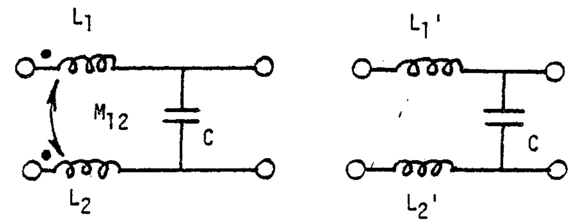
ABSTRACT

When coaxial cable is used for high voltage pulse transmission, a voltage transient appears on the outer sheath conductor. Although the magnitude of the transient is in the order of only a few per cent, this amounts to several kilovolts in many cases and must be carefully considered in terms of its effect on instrumentation, control and safety. To a first approximation, theoretically a coaxial cable should not develop any voltage on the outer sheath. A more refined analysis and model shows that the complete cancellation depends upon the self inductance of the sheath being exactly equal to the mutual inductance between the sheath and the center conductor. This condition is never exactly satisfied due to current distribution effects, even when the distribution is uniform and radially symmetric. The situation becomes worse when proximity effects are accounted for. The predicted sheath voltage agrees with experimental data within reasonable limits.

INTRODUCTION

The analysis of coaxial transmission lines is commonly based upon the incremental section model as shown in Fig 1. The self inductance of the center conductor is L_1 the outer sheath L_2 and the mutual is M_{12}. The lumped equivalent capacitance of the element is C. Also shown in Fig 1 is the equivalent model using uncoupled inductors with the corresponding relations between circuit valves. Note that if $L_2 = M_{12}$ the effective inductance of the outer sheath is zero (short circuit) and all the loop inductance is associated with the inner conductor. In reality, $L_2 \approx M_{12}$ to within a few percent, however, there is a multiplicative effect such that a given percentage unbalance between L_2 and M_{12} leads to several times that percentage unbalance in the division of voltage between the inner conductor and sheath. This simple mechanism is the basis for explaining the existance of the voltage transient on the outer sheath. The equation relating the voltage on the sheath to the circuit values is plotted in Fig 2. and reads:

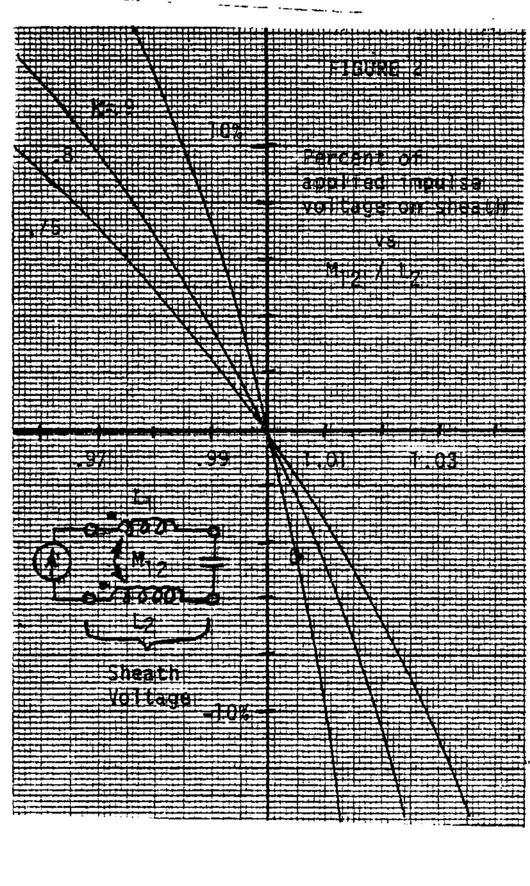
(1) $V_2/V = K*(1-a) / (1+K*(1-2*a))$



$L_1 > L_2$ $\qquad$ $L_1' = L_1 - M_{12}$

$M_{12} <$ SQRT (L_1*L_2) $\qquad$ $L_2' = L_2 - M_{12}$

FIGURE 1
MODEL OF INCREMENTAL SECTION
OF TRANSMISSION LINE



Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE	2. REPORT TYPE	3. DATES COVERED
JUN 1979	**N/A**	-

4. TITLE AND SUBTITLE
High Power Pulse Modeling Of Coaxial Transmission Lines

5a. CONTRACT NUMBER
5b. GRANT NUMBER
5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)

5d. PROJECT NUMBER
5e. TASK NUMBER
5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Air Force Weapons Laboratory Kirtland Afb, Nm 87117

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release, distribution unlimited

13. SUPPLEMENTARY NOTES
See also ADM002371. 2013 IEEE Pulsed Power Conference, Digest of Technical Papers 1976-2013, and Abstracts of the 2013 IEEE International Conference on Plasma Science. Held in San Francisco, CA on 16-21 June 2013. U.S. Government or Federal Purpose Rights License.

14. ABSTRACT
When coaxial cable is used for high voltage pulse transmission, a voltage transient appears on the outer sheath conductor. Although the magnitude of the transient is in the order of only a few per cent, this amounts to several kilovolts in many cases and must be carefully considered in terms of its effect on instrumentation, control and safety. To a first approximation, theoretically a coaxial cable should not develop any voltage on the outer sheath. A more refined analysis and model shows that the complete cancellation depends upon the self inductance of the sheath being exactly equal to the mutual inductance between the sheath and the center conductor. This condition is never exactly satisfied due to current distribution effects, even when the distribution is uniform and radially symmetric. The situation becomes worse when proximity effects are accounted for. The predicted sheath voltage agrees with experimental data within reasonable limits.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE			
unclassified	**unclassified**	**unclassified**	**SAR**	**4**	

where: V_2 = voltage on the sheath
V = impressed voltage
$K = L_2/L_1$
$a = M_{12}/L_2$
L_1 = Inner conductor inductance
L_2 = Sheath inductance
M_{12} = Mutual inductance

Note that as a changes from $a<1$ to $a>1$, the polarity on the sheath reverses.

FACTORS AFFECTING MUTUAL INDUCTANCE

Two factors affecting mutual inductance are the distribution of flux within the finite thickness of the sheath current, and the current distribution in the cable as determined by the proximity effect of other current carrying conditions such as ground plane images, etc. Consider first the simple case illustrated in Fig 3., that of a coax cross-section with a uniform current distribution and thus a flux field which is perfectly concentric. By fundamental definition, mutual inductance is measured by the flux coupling the inner conductor due to a unit current in the outer conductor. Thus, the mutual is measured by all of the flux. Also by definition, the self inductance of the sheath is measured by the flux coupling the sheath current due to a unit sheath current. The sheath current is uniformly distributed over the thickness T and the flux varies linearly from zero at the inner surface to maximum at the outer surface thus the flux internal to the sheath doesn't effectively couple all the sheath current, so L_2 will be less than M_{12}. Modifying the inductance equation for cylindrical conductors given by Grover[1] to account for the uncoupled flux internal to the sheath one obtains the expression in equation (2) for the ratio M_{12}/L_2:

(2) $M_{12}/L_2 = 1+(1/2)*LN(1/(1-\delta))/2*(LN(B/R_2-1))$

where: R_2 = Mean radius of sheath (cm)
B = Length (cm)
T = Sheath thickness (cm)
$\delta = T/R_2$

Equation (2) is plotted in Fig 4.

Consider now the effect of a non-uniform current distribution, the radially symmetric flux of Fig 3 will no longer exist, in fact, the flux between the sheath and center conductor will no longer be zero. The simple evaluations of self and mutual inductance as above are no longer possible.

An evaluation of the proximity effect on mutual inductance for simple geometrical cases was done by computer using the model shown in Fig 5. The inner and outer conductors and their images were modeled using 100 independent current filaments, 50 for

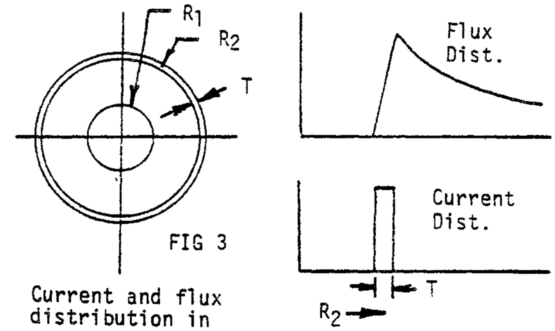
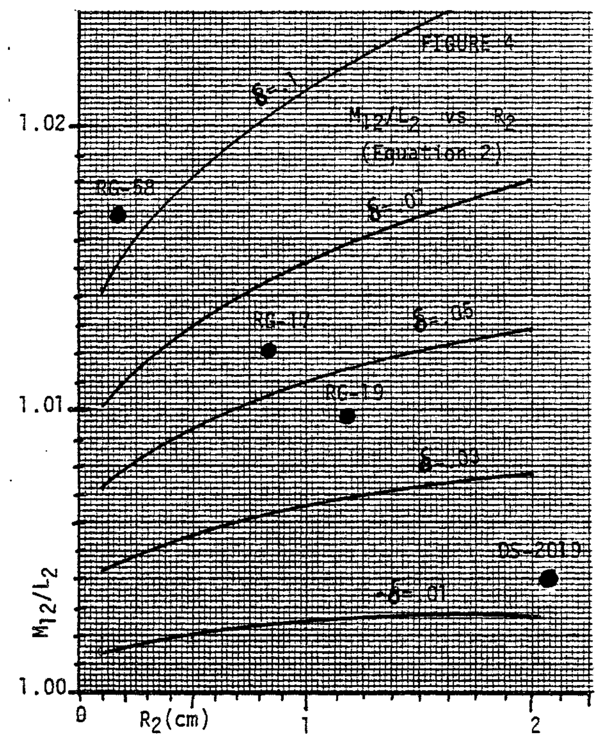


FIG 3

Current and flux distribution in Coax outer sheath



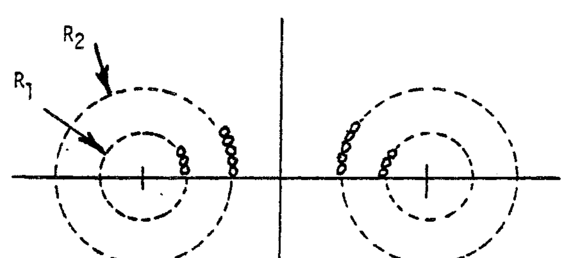


FIGURE 5

Filamentry model of coax and ground plane image each. By symmetry, the total number of filaments in the model is 400. Using expressions for the self and mutual inductances in terms of the geometry, a solution for the 100 independent currents was obtained using Creamers rule to solve the loop equations on a CYBER 176 computer. The ASPLIB library program DECOMP was used to evaluate the 100 x 100 determinants. This model was used to evaluate only the proximity effect, thus in free space, i.e. no images, it was calibrated to give zero voltage on the sheath. This was accomplished by adjusting the diameter of the filaments to null the sheath voltage to less than one part in 10^{10} per unit of impressed voltage. The diameter used to accomplish this was 1.07596 times the circumference of the conductor being modeled divided by 100.

The net proximity effect on M_{12} as a function of the distance of a RG-19 coax above a ground plane is shown in Fig 6. In Fig 7 are current distributions due to various proximity effects. The cases shown are for a RG-19 cable spaced 1.04 sheath radii from a ground plane. Case 1 is the distribution in the outer conductor with the coax center conductor used as a return in the normal manner. Case 2 is with the center conductor removed and an infinite ground plane carrying the return and Case 3 is with the center conductor removed and the image carrying the return (two wire open line). Notice the remarkable insensitivity to the proximity effect a coax has (1.5%) compared to the other cases, The effects of various geometrical distortions are shown in Fig 8. The initial geometry of the three cases shown is an RG-19 spaced 1.04 radii from ground. Case 1 is for the center conductor moved off center along the X axis by ± .25 sheath radii. Case 2 is for the center conductor moved along the Y axis by ± .25 sheath radii. Case 3 is for an eliptial distortion of the sheath, elongated along the Y axis by 0 to .25 sheath radii.

Comparing the data of Figures 2, 6, and 8 it is obvious that the ratio of mutual to self inductance M_{12}/L_2 is predominantly determined by the thickness of the outer sheath and the proximity and mechanical distortion effects can be neglected in most cases.

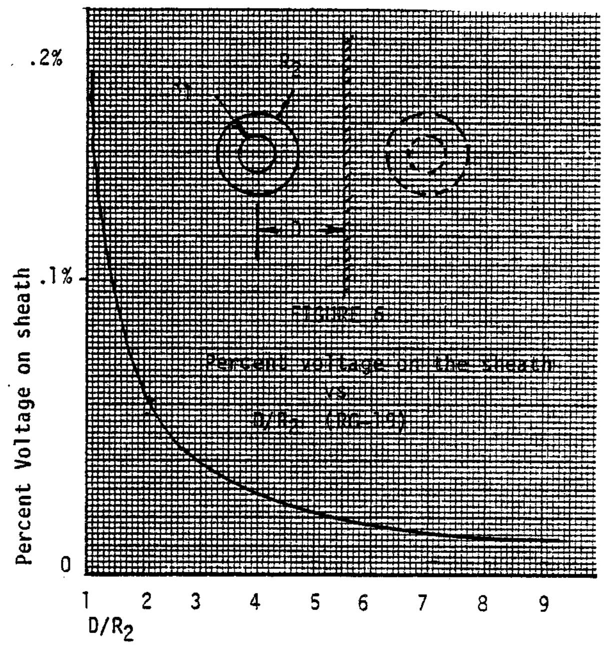


FIGURE 6

Percent voltage on the sheath vs D/R_2 (RG-19)

FIGURE 7

Current Distribution

vs

Ø

CASE 1, Coax above ground plane

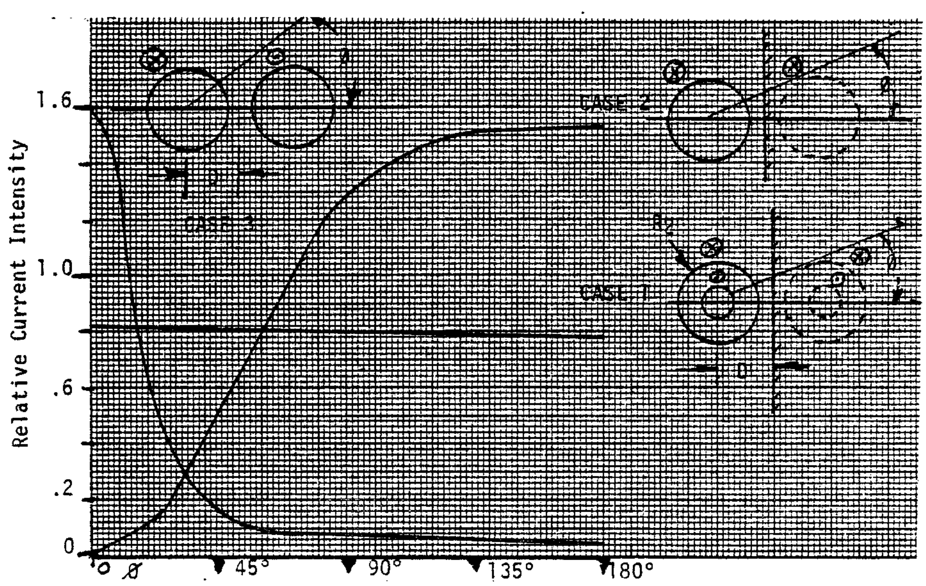
CASE 2, Round conductor above ground plane

CASE 3, Two wire pair

R_1 = .33 (cm)

R_2 = 1.1938 (cm)

D = 1.2438 (cm)



MODEL OF A REAL CIRCUIT

Shown in Fig. 9 is the circuit model of a pulse transmission coax including the ground plane. The cable is a Dielectric Sciences DS-2019, 61 meters long and modeled at an average of 15 cm above the ground plane. The distributed circuit of the cable and ground plane is modeled by 100 finite elements. The driving source is 330 KV with a one microsecond rise time and a 27.68 ohm source resistance. A FORTRAN computer code was used to solve the circuit by coventional loop current techniques. The result of the analysis giving the voltage from sheath to ground at the sending end is plotted in Fig. 10, also shown is the measured voltage. The cable was driven through a pulse transformer, CCG is the secondary to ground capacitance and RA is a 60 ohm resistor used to monitor the voltage via a current transformer.

CONCLUSION

It is concluded that the transient voltage which develops on the sheath of a coaxial cable under pulse conditions may be explained, analyzed and reasonably well predicted based upon the difference between the mutual inductance and the sheath inductance of the cable.

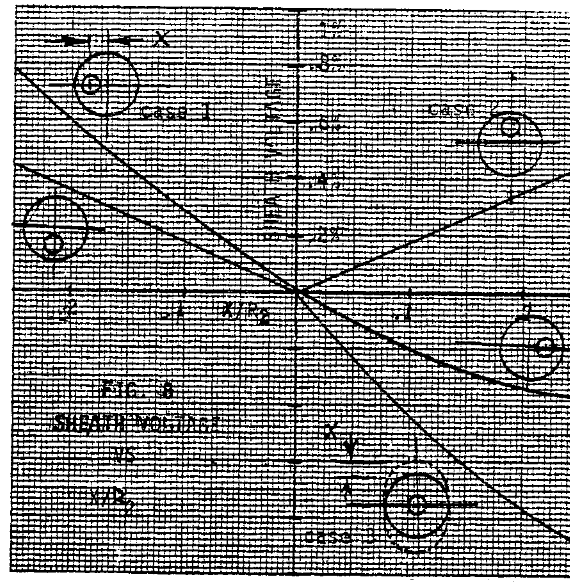
REFERENCES

1. John D. Ryder, Networkds Lines and Fields, 2nd Ed, Chapt 6, Prentice Hall, 1955.

2. Frederick W. Grover, Inductance Calculations, Working Formulas and Tables, P 271, Dover, 1962.

ACKNOWLEDGEMENTS

Appreciation is expressed for the assistance and cooperation in providing experimental data to J. J. Moriarty, P. A. Corbier, and Dr F. Donald Angelo of Raytheon Missile Systems Division.



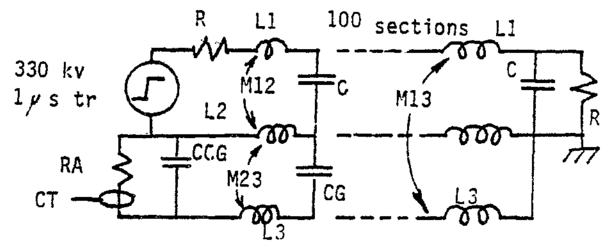


FIG. 9

MODEL OF DS 2019 CABLE 15 cm ABOVE GROUND

R	22.68	RA	60.0		
L1	1.65E-8	M12	8.03E-8	C	9.39E-11
L2	5.77E-8	M13	3.23E-8	CG	7.98E-12
L3	4.30E-8	M23	4.08E-8	CCG	2.00E-9