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A Ranging Comparison of Two Sonar Systems

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FOREWORD

This reprint report comprises two parts. The first part (beginning on pg. 1) reproduces a paper presented at the 7th European Conference on Underwater Acoustics on 5-8 July 2004. The second part (beginning on pg. 8) reproduces a related presentation made at the same conference.

Proceedings of the Seventh European Conference on Underwater Acoustics, ECUA 2004
Delft, The Netherlands
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A RANGING COMPARISON OF TWO SONAR SYSTEMS

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A comparison of the passive ranging performance between two hypothetical sonar systems is presented using theoretical methods. Each system is composed of eight hydrophones that are symmetrically located in a straight line spanning the same total array length. The systems differ in the way the hydrophones have been grouped, which significantly affects the associated range-estimator variance. The performance of each system has been analyzed using triangulation-based ranging as well as optimal processing that would asymptotically achieve the Cramér-Rao lower bound (CRLB). While this work compares two specific eight-sensor array configurations, the results have implications regarding the design and performance of any multi-aperture sonar system.

1. INTRODUCTION

The localization performance of a passive sonar system varies with the placement of the sensors comprising the sonar array. In many applications, the accurate estimation of range from an array to a radiating source relies on the system's ability to measure wave-front curvature. As a result, configurations designed optimally for ranging have a portion of the sensors placed in the middle of the array and others placed near the ends. Carter [1] showed, for a symmetric line array, that the variance of the maximum likelihood (ML) range estimate is minimized when the sensors are configured in groups of one quarter, one half, and one quarter. Other research and studies, consistent with this approach, supported the decision to install triple-aperture sonars on modern naval platforms. While performance has been proven both theoretically and at-sea, a three-site installation is not always economically feasible. Whether the sensors are mounted on the hull of a submarine, for example, or moored on the ocean floor, cost savings could be realized by eliminating the middle sensor grouping. The objective of this work is to quantify the theoretical loss in ranging performance due to the installation of two groups of sensors instead of three.

Two hypothetical sonar systems based on symmetric line arrays have been used to perform this analysis. Each system is composed of eight sensors that span the same overall array length and differ only in the way the sensors are grouped. The performance of each system has been analyzed using triangulation-based ranging as well as optimum processing that would asymptotically achieve the *Cramér-Rao lower bound* (CRLB) [2]. The choice of eight sensors allows for a comparison of systems with the same number of hydrophones and opportunity for either system to achieve the same maximize output signal-to-noise ratio (SNR). However, there is nothing special about the number eight, and this analysis generally applies to systems with an arbitrary number of sensors.

Fig. 1(a) illustrates the first sonar system, designated *System I*, which has its sensors grouped in two, four-element sub-arrays, each spanning length, L_S. It has been assumed that this sub-array length is much smaller than the total array length, or $L \gg L_S$. The range and bearing to the source referenced to the central origin is denoted by R and B, respectively. Using triangulation,

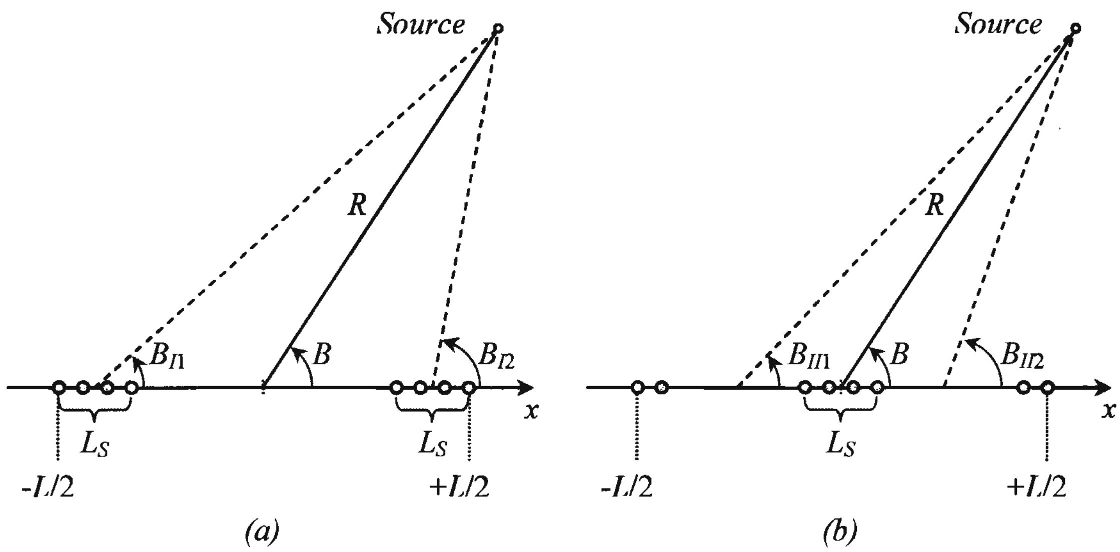

Fig.1: (a) System I, an eight-sensor, dual-aperture configuration. (b) System II, a triple-aperture configuration formed by grouping four sensors in the middle of the array.

range R can be expressed in terms of the System-I sub-array bearings, B_{I1} and B_{I2}, which are approximately separated by the total array length, L, given that $L \gg L_S$. Fig. 1(b) illustrates the second sonar system, designated *System II*, which has its sensors configured optimally in groups of one quarter, one half, and one quarter. The middle grouping of four sensors spans the same length, L_S, as each of the sub-arrays in System I. However, the System-II sub-arrays formed using two middle sensors and two end sensors span half the total array length and have phase centers located at $x = \pm L/4$. Thus, range can be expressed in terms of the bearings, B_{II1} and B_{II2}, which are approximately separated by $L/2$.

The analysis that follows assumes what Van Trees [2] refers to as SPLOT, i.e., stationary processes, long observation times. Furthermore, signals and noises are assumed Gaussian and mutually uncorrelated with the same arbitrary power spectra at each sensor. Thus, a direct-path, isotropic-noise model is implied, where the propagated acoustic energy to each sensor is the same. There is also an implicit assumption that the SNR is high enough for the system to be tracking the local peak in the correlation domain and, thereby, achieve performance that depends on the bandwidth and the square of the center frequency. This is classical CRLB theory that is difficult to achieve below a threshold SNR, where a tighter bound, the *Ziv-Zakai lower bound* (ZZLB) applies. Finally, there is an assumption that for the integration time, T, the source and receiver motion is negligible.

2. OPTIMUM PERFORMANCE

A system's performance is commonly measured by the mean-squared error (MSE) of the parameter, or parameters, it is used to estimate. Optimum performance in this sense is achieved when the MSE of the parameter estimate equals the Cramér-Rao lower bound (CRLB) [2]. If an estimator that achieves the CRLB exists, it is the *maximum likelihood* (ML) estimator. The ML estimator is asymptotically *efficient*; that is, its mean-squared error approaches the CRLB as more data are incorporated into its solution. Hence, for SPLOT conditions, ML estimates of either range or bearing may be considered optimum in the MSE sense. The statistics of the ML estimator were derived by Bangs and Schultheiss [3]. Under their assumptions of Gaussian random processes and spatially incoherent noise, it was shown that this estimator is unbiased, and thus MSE is equivalent to the variance of the parameter. Letting ξ denote the parameter of interest (range or bearing), the variance of the ML estimate for an M-sensor array with generic configuration "A", is given by the following [3]:

$$\sigma_A^2(\hat{\xi}) = K_A^{(\xi)} \left\{ \frac{T}{2\pi} \int_0^{\Omega} d\omega\, \omega^2 \frac{P^2(\omega)}{1 + M \cdot P(\omega)} \right\}^{-1}. \tag{1}$$

In Eq. (1), $P(\omega)$ denotes the frequency dependent SNR, $S(\omega)/N(\omega)$, where $S(\omega)$ and $N(\omega)$ represent the known power spectra of the signal and noise, respectively, and ω is radian frequency. M is equal to the number of sensors or hydrophones in the array, Ω is the processing bandwidth in radians per second, and T is the observation time in seconds. $K_A^{(\xi)}$ depends on the parameter being estimated, as well as the placement of the array's sensors. Let $\xi = R$, where R denotes the range from the origin to the source, and let B denote the bearing angle relative to the array's axis, as illustrated for both systems in Fig.1. Eq. (1) represents the range variance when $K_A^{(\xi=R)}$ is given by [3]

$$K_A^{(R)} = \frac{4c^2R^4}{\sin^4 B} \left\{ \sum_{i=1}^{M} \sum_{j=1}^{M} \left(x_i^2 - x_j^2\right)^2 \right\}^{-1}, \tag{2}$$

where x_m is the coordinate of sensor m along the axis of the array in meters, and c is the speed of sound in water in meters per second. Eq. (2) is valid when the sensors are placed symmetrically about the array's origin, which is consistent with the analysis presented here. Analyses of asymmetric arrays with arbitrary origin placement require an additional degradation factor, which would account for the coupling of range and bearing uncertainty [3]. Eqs. (1) and (2) hold only for the estimation of a single parameter (range), given that all other parameters are known. Note that the contribution of Eq. (2) to the range variance is purely geometrical. Thus, Eq. (1) is the product of two distinct components: a *signal dependent* factor and a *geometrically dependent* factor. The former includes quantities such as frequency-dependent SNRs at the sensors, bandwidth, and observation time. Thus, if the same optimum processing is applied to two M-sensor array configurations under the same signal and noise conditions, then the difference in ranging performance depends only on the placement of the array's sensors.

Eq. (2) can be computed for each sonar system and substituted into Eq. (1) to yield the range variance in terms of sub-array length, L_S, and total array length, L. For System I ($A=I$) and System II ($A=II$), respectively,

$$\sigma_I^2\left(\hat{R}\right)=\frac{c^2R^4}{4L_S^2L^2\sin^4B}\left\{\frac{T}{2\pi}\int_0^\Omega d\omega\,\omega^2\frac{P^2(\omega)}{1+8P(\omega)}\right\}^{-1}, \tag{3}$$

$$\sigma_{II}^2\left(\hat{R}\right)=\frac{2c^2R^4}{L^4\sin^4B}\left\{\frac{T}{2\pi}\int_0^\Omega d\omega\,\omega^2\frac{P^2(\omega)}{1+8P(\omega)}\right\}^{-1}. \tag{4}$$

Hence, the range variance, or MSE, for System II is independent of L_S under the assumption of $L>>L_S$ and is inversely proportional to the fourth power of the total array length, L. This is consistent with Carter's result in [1] for the optimal ranging array with quarter, half, quarter sensor groupings. In contrast, the range variance for the dual-aperture configuration of System I is inversely proportional to the squared product of L and L_S. The relative performance is expressed by taking the square root of the ratio of Eqs. (3) and (4):

$$\frac{\sigma_I\left(\hat{R}\right)}{\sigma_{II}\left(\hat{R}\right)}=\frac{\sqrt{2}}{4}\frac{L}{L_S}\approx 0.35\frac{L}{L_S}. \tag{5}$$

Eq. (5) shows that the range error of System I is proportional to the range error of System II and grows linearly with the ratio of L to L_S. When this ratio is equal to thirty, for example, the range error for the dual-aperture system, System I, would be ten times the error expected for the triple-aperture system, System II.

3. TRIANGULATION

The result summarized by Eq. (5) states that System II theoretically outperforms System I if optimum processing were attainable. A more practical comparison would be one that accounts for a potential loss in performance due to sub-optimal processing. Such a comparison can be made for triangulation-based ranging, which relies on independent relative bearing estimates from two sub-arrays and a simple geometrical model to triangulate the range from the array to the acoustic source. The range error variance due to triangulation can be expressed approximately by [4]:

$$\sigma_{\underline{A}}^2(\hat{R}) \approx \frac{R^4}{L_E^2}\left[\sigma^2(\hat{B}_{A1}) + \sigma^2(\hat{B}_{A2})\right], \tag{6}$$

where $\sigma^2(\hat{B}_{A1})$ and $\sigma^2(\hat{B}_{A2})$ are the variances of the two bearings used to estimate range, R is the actual range to the source, and L_E is the effective phase-center separation of the sub-arrays that independently produce the bearing estimates. System I generates bearing estimates $\hat{B}_{I1}$ and $\hat{B}_{I2}$ using sub-arrays that are effectively separated by $L_E = L\sin B$, as shown by Fig. 1(a). System II generates bearing estimates $\hat{B}_{II1}$ and $\hat{B}_{II2}$ using sub-arrays that are effectively separated by $L_E = L/2\sin B$, as shown by Fig. 1(b). Note that in Eq. (6), and in what follows, the under-bar notation, $\underline{A}$, identifies that two independent sub-array processes are performed using half the sensors in generic array configuration "A" to produce a final range estimate via triangulation.

Triangulation performance can be related to the underlying array properties and SNR conditions using Eq. (1) to obtain the ML bearing variance for the sub-arrays in each system. For $\xi = B$, the geometrically dependent factor in Eq. (1) for an $M/2$-element sub-array is defined as follows [3]:

$$K_{\underline{A}}^{(B)} = \frac{c^2}{\sin^2 B}\left\{\sum_{i=1}^{M/2}\sum_{j=1}^{M/2}\left(x_i - x_j\right)^2\right\}^{-1}. \tag{7}$$

Unlike its range counterpart, bearing error variance is virtually independent of the defined origin since Eq. (7) depends on the differences in sensor coordinates instead of the differences of the squared coordinates as given by Eq. (2). Therefore, there is no need to redefine the origin placement for each sub-array analysis. Using the appropriate sensor separations, Eq. (7) can be used to compute $K_{\underline{I}}^{(B)}$ and $K_{\underline{II}}^{(B)}$, the geometrically dependent factors of the sub-array bearing variances for System I and System II, respectively. Due to the symmetry of the arrays, $K_{\underline{I}}^{(B)}$ applies to both sub-arrays in System I, and $K_{\underline{II}}^{(B)}$ applies to both sub-arrays in System II. This symmetry, in conjunction with the assumptions made regarding signal and noise at each sensor, yields equal bearing variances for the sub-arrays in System I, $\sigma^2(\hat{B}_{I1}) = \sigma^2(\hat{B}_{I2}) = \sigma^2(\hat{B}_{\underline{I}})$, and in System II, $\sigma^2(\hat{B}_{II1}) = \sigma^2(\hat{B}_{II2}) = \sigma^2(\hat{B}_{\underline{II}})$.

By substituting the appropriate bearing variances and effective sub-array separations, as defined above, into Eq. (6), the triangulation range variance for each system is obtained:

$$\sigma_{\underline{I}}^2(\hat{R}) = \frac{c^2 R^4}{2L_S^2 L^2 \sin^4 B}\left\{\frac{T}{2\pi}\int_0^{\Omega} d\omega\, \omega^2 \frac{P^2(\omega)}{1+4P(\omega)}\right\}^{-1}, \tag{8}$$

$$\sigma_{\underline{II}}^2(\hat{R}) = \frac{4c^2R^4}{L^4\sin^4 B}\left\{\frac{T}{2\pi}\int_0^{\Omega} d\omega\,\omega^2 \frac{P^2(\omega)}{1+4P(\omega)}\right\}^{-1}. \tag{9}$$

Comparing Eqs. (8) and (9) with Eqs. (3) and (4) shows that triangulation with ML bearing estimates theoretically achieves the CRLB when the output SNR is high, or $M{\cdot}P(\omega)>>1$. When the output SNR is low, the efficiency of triangulation degrades by seventy percent due to the independent processing of each $M/2$-element sub-array (the triangulation range variance is twice the CRLB). However, as long as both systems operate under the same input SNR conditions, their relative triangulation performance is equivalent to the relative performance seen for optimum processing, i.e. the ratio of Eqs. (8) and (9) is equal to Eq. (5). This is an intuitively satisfying result: System II has an inherent ability to outperform System I regardless of the range estimation methods evaluated here. While this comparison is based on optimum bearing estimation, the efficiency of triangulation generally scales with the efficiency of the underlying bearing estimator. It can be shown by replacing the ML bearing estimator with a *split-beam tracker* [3], for example, that the range variances given by Eqs. (8) and (9) both increase by a factor of two. The resulting ratio comparing System I to System II remains unchanged and is equal to Eq. (5).

4. DISCUSSION AND CONCLUSIONS

Under the assumptions made and for the examples cited, it has been shown that the triple-aperture system, System II, shown in Fig. 1(b), significantly outperforms the dual-aperture system, System I, shown in Fig. 1(a). In particular, the noise-induced mean squared range error of System I can be reduced by a factor proportional to the ratio of total array length to sub-aperture length by simply repositioning half the sensors in the middle of the array, as in System II. While this comparison was performed using specific eight-element arrays, the results reported here apply generally to any two M-element symmetric line arrays where one in configured with $M/2$, $M/2$ sensor groupings, and the other is configured with $M/4$, $M/2$, $M/4$ groupings. In fact, by extending the triangulation-based analysis in Section 3, the ranging performance comparison can be generalized to yield the following:

$$\frac{\sigma_I(\hat{R})}{\sigma_{II}(\hat{R})} = \frac{\sqrt{3}}{4}\frac{L}{L_S} \approx 0.42\frac{L}{L_S} \qquad \text{for } M>>1. \tag{10}$$

Eq. (10) is very similar to Eq. (5); both are linearly related to the ratio of L to L_S. This more general result shows that increasing the total number of sensors does not affect relative performance. Moreover, for dual-aperture systems with sub-arrays separated by tens or hundreds of times their sub-aperture lengths, the installation of a third sensor site could reduce the range estimation error by one or two orders of magnitude. Hence, the cost saved by installing two sensor sites instead of three must be carefully weighed against the significantly increased ranging performance of a three-site system composed of the same number of individual hydrophone elements.

5. ACKNOWLEDGMENTS

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REFERENCES

[1] **G. C. Carter**, Variance bounds for passively locating an acoustic source with a symmetric line array, *JASA*, 62(4), pp. 922-926, October 1977.

[2] **H. L. Van Trees**, *Detection, Estimation, and Modulation Theory, Parts III & IV*, New York: John Wiley & Sons, 1971, 2002.

[3] **W. J. Bangs and P. M. Schultheiss**, Space-time processing for optimal parameter estimation, *Signal Processing*, J. W. R. Griffiths, P. L. Stocklin, and C. van Schoonveld, eds., New York: Academic, pp. 577-590, 1973.

[4] **G. C. Carter**, Passive ranging errors due to receiving hydrophone position uncertainty, *JASA*, 65(2), pp. 528-530, February 1979.

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Problem Statement and Assumptions

- Compare the underwater acoustic ranging performance of two systems: System I uses dual apertures; System II uses triple apertures. (Keep key system, processing, and underwater acoustic parameters constant.)
- Assumptions:
  - Stationary Processes, Long Observation Times (SPLOT)
  - Signals and noises are Gaussian and mutually uncorrelated with the same arbitrary power spectra at each sensor
  - Isotropic-noise, direct-path propagation, propagated acoustic energy to each sensor is the same (SNR at the sensor is key and propagation decouples for direct path.)

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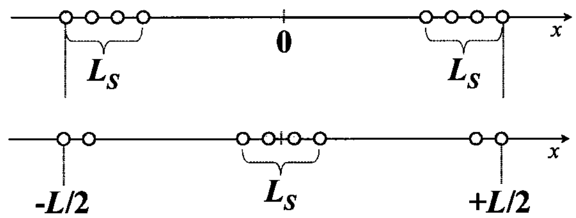
Two Sonar Systems Considered

System I
Dual Aperture

System II
Triple Aperture



- Two eight-sensor, symmetric line arrays with total length L
- System I is a dual-aperture array with sub-aperture length L_S
- System II is a triple-aperture array which is formed by grouping four sensors in the middle of the array and has sub-aperture lengths L_S and approximately $L_S/2$

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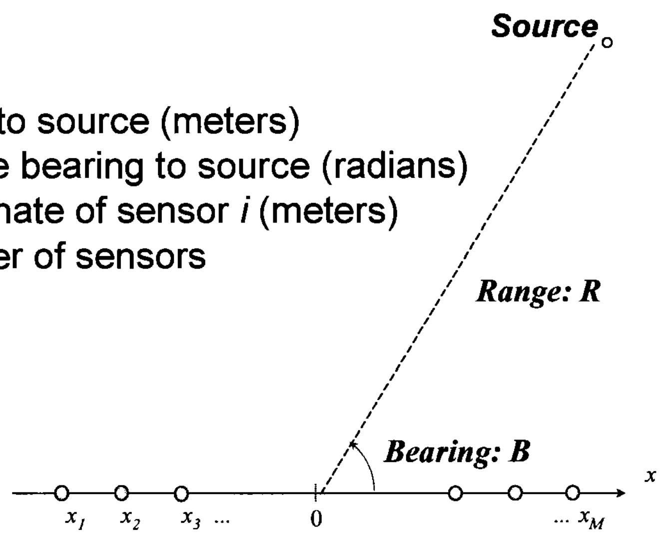
The Passive Ranging Geometry

R = range to source (meters)
B = relative bearing to source (radians)
x_i = coordinate of sensor i (meters)
M = number of sensors



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Minimum MSE Ranging Performance*

$$\sigma_A^2\left(\hat{R}\right) = \underbrace{K_A^{(R)}}_{\text{Geometrical Factor}} M^2 \underbrace{\left\{ \frac{T}{2\pi} \int_0^{\Omega} d\omega \; \omega^2 \; \frac{M^2 \cdot P^2(\omega)}{1 + M \cdot P(\omega)} \right\}^{-1}}_{\text{Signal-Dependent Factor}}$$

Geometrical Factor

- *Depends on relative placement of the array's sensors and acoustic source*
- *K is proportional to $1/M^2$*

Decreasing K is good

Signal-Dependent Factor

Increasing T and M is good

Where

T = observation time (seconds)

M = number of sensors

ω = frequency (radians/sec)

Ω = bandwidth (radians/sec)

$P(\omega) = S(\omega)/N(\omega)$, SNR vs. ω

$S(\omega)$ = signal power at each sensor

$N(\omega)$ = noise power at each sensor

*See references, including paper by Bangs and Schultheiss [1], 1973

Components of the Geometrical Factor

$$K_A^{(R)} = 4\underbrace{c^2}_{} \underbrace{\frac{R^4}{\sin^4 B}}_{} \underbrace{\left\{ \sum_{i=1}^{M} \sum_{j=1}^{M} \left(x_i^2 - x_j^2\right)^2 \right\}^{-1}}_{}$$

(Note K is proportional to $1/M^2$)

Depends on sound speed, c (meters/sec)

Depends on source location only

Depends on array configuration, i.e., placement of the sensors

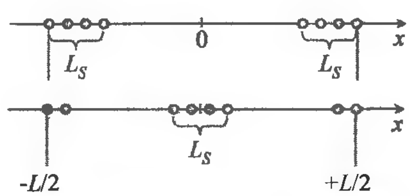
Decreasing K is good

The Configuration-Dependent Component

$$\left\{ \sum_{i=1}^{M} \sum_{j=1}^{M} \left(x_i^2 - x_j^2\right)^2 \right\}^{-1}$$

Approximations for $L >> L_S$:

- System I factor $\approx \dfrac{1}{16 L_S^2 L^2}$
- System II factor $\approx \dfrac{1}{2L^4}$



Note: System II ranging performance depends on L and this component is independent of sub-aperture length, L_S

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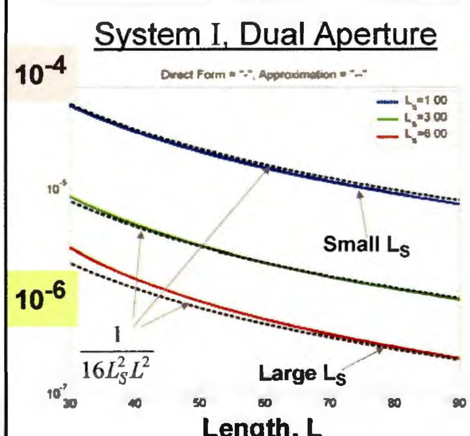
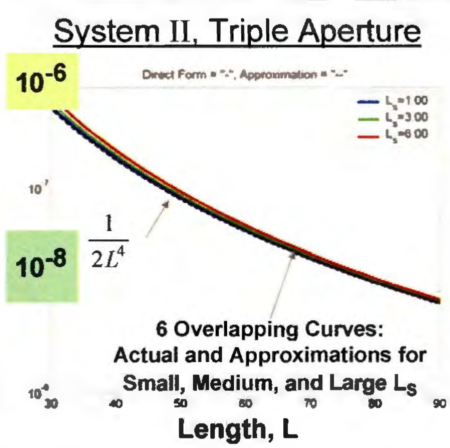
Performance Vs. Array Length, L

For Small, Medium, and Large L_S (with Approximations dashed)





- *Approximations hold for $L >> L_S$*
- *System I depends on sub-aperture length, L_S*
- *Range error variance decreases with increased L as shown*

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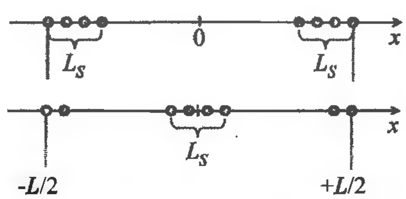
Ranging Performance Comparison

Range Error Variances for $L \gg L_S$:

- System I, $\sigma_I^2(\hat{R}) \propto \dfrac{1}{16 L_S^2 L^2}$
- System II, $\sigma_{II}^2(\hat{R}) \propto \dfrac{1}{2L^4}$



➢ Take the square root of the ratio of error variances:

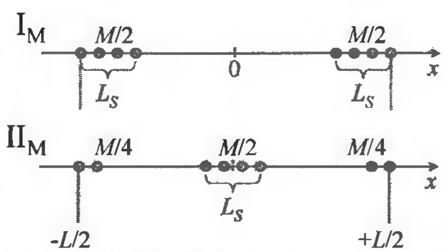
$$\boxed{\frac{\sigma_I(\hat{R})}{\sigma_{II}(\hat{R})} \approx \frac{\sqrt{2}}{4}\frac{L}{L_S} \approx 0.35\frac{L}{L_S}}$$

- ***Relative RMS range error is linear with the ratio of L to L_S***
- ***RMS error expected from System I is <u>35 times</u> that of System II for L = 100 L_S***

System II performs better than System I for all cases studied

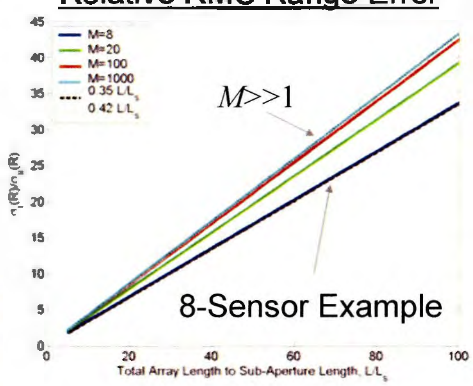
Effect of Additional Sensors

Two M-Sensor Configurations



$$\boxed{\frac{\sigma_I(\hat{R})}{\sigma_{II}(\hat{R})} \approx 0.42\frac{L}{L_S} \quad \text{for } M \gg 1}$$

Relative RMS Range Error



Relative results are insensitive to the number of sensors: M

Conclusions

- For the two sonar systems considered with the same aperture (length: L) and the same number of sensors: M, the relative ranging performance was calculated. The theory has various applications.
- For examples considered, improvement factors of 35 to 40 in RMS range error were observed.
- Any perceived cost advantage gained by installing sensors at only two sites instead of three must be weighed against the significantly improved ranging performance of a three-site system unless the two-site system meets the required performance.

References

1. **W. J. Bangs and P. M. Schultheiss**, Space-time processing for optimal parameter estimation, *Signal Processing*, J. W. R. Griffiths, P. L. Stocklin, and C. van Schoonveld, eds., New York: Academic, pp. 577-590, 1973.
2. **G. C. Carter**, Passive ranging errors due to receiving hydrophone position uncertainty, *JASA*, 65(2), pp. 528-530, February 1979.
3. **G. C. Carter**, Variance bounds for passively locating an acoustic source with a symmetric line array, *JASA*, 62(4), pp. 922-926, October 1977.
4. **H. L. Van Trees**, *Detection, Estimation, and Modulation Theory, Parts III & IV*, New York: John Wiley & Sons, 1971, 2002.

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