New Methods For Predictability Analysis

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LONG-TERM GOAL

The long-term goal of this work is to improve atmospheric and oceanic deterministic forecast capability by maximizing the accuracy of the initial state estimate from which a forecast is made while minimizing observational and computational costs required to obtain the initial state.

OBJECTIVES

The present objective of this work is to increase fundamental understanding of error dynamics and develop practical methods for computing error statistics. We intend to reduce the dimension of the error system so that error statistics can be practically obtained using forecast products. In addition from knowledge of the statistical structure of the time dependent error field we intend to determine the areas in space and time where observational resources can be most effectively applied.

APPROACH

Small error dynamics is governed by linear equations but because these equations are non-normal and time dependent theoretical tools for analyzing error growth have only recently been obtained. Our approach is to apply these recent theoretical advances in non-normal time-dependent stability analysis to the forecast error problem. Methods of modern control theory, specifically balanced Hankel operator truncation methods, will be used to reduce the dimension of the error system.

WORK COMPLETED

The fundamental theory of error growth in non normal time dependent forecast models has been developed.

The control theory approach of optimal balanced Hankel operator order reduction has been adapted and applied to model fluid error dynamics systems with encouraging results.

RESULTS

Methods for obtaining the structure of the error field in a time dependent flow have been developed which facilitate prediction of error field statistics. We have shown that the dominant error growth arises from destabilization of a set of most non-normal vectors of the time mean operator, and that a very good basis for calculating the time dependent error is the first few evolved optimals of the time

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mean propagator obtained over a period equal to the decorrelation time of the time varying flow (Fig. 1).

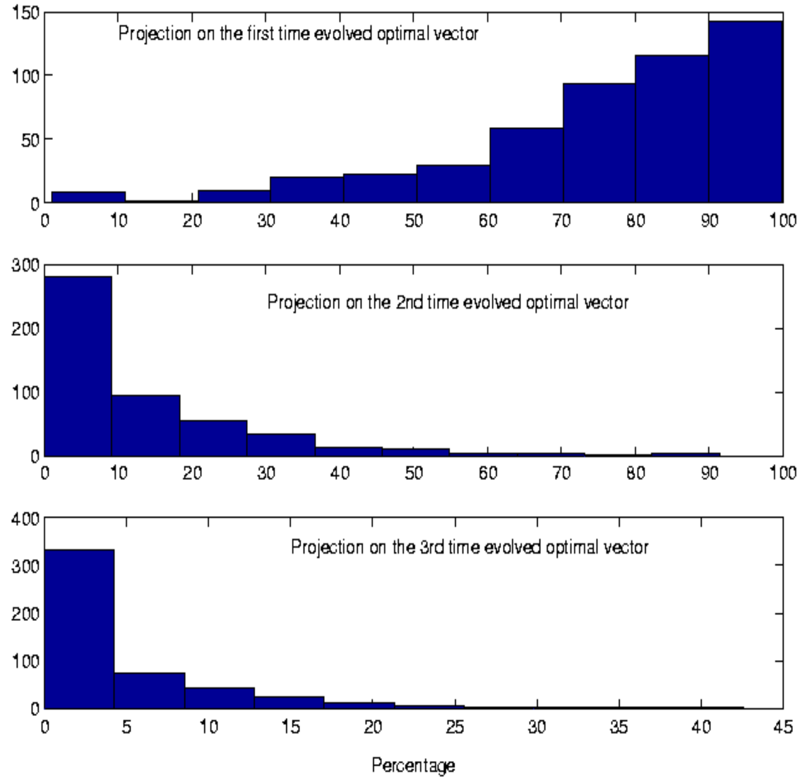

Fig. 1 : Error structure for a model of the time dependent midlatitude barotropic jet as revealed by a histogram of projection of 500 observations of the error on the first three evolved optimals of the time mean flow. The evolved optimals considered are the left singular vectors of the propagator $e^{\bar{A}T}$***, where*** $\bar{A}$ ***is the linearized operator associated with time mean flow, and in this example*** T ***is the decorrelation time of the time dependent flow. The error projects primarily on these evolved optimals; projection on the remaining evolved optimals is negligible.***

Methods for optimally reducing the dimension of the forecast error system have been applied to time independent error systems, as would be appropriate for forecast over a short enough interval that time dependence of the flow can be ignored (~24 hr in the atmosphere) . This optimal reduction of order is obtained by truncating the error dynamics using a balanced representation in the stochastic optimals and the empirical orthogonal functions of the system (Moore, 1981; Glover, 1984; Farrell and Ioannou, 1996).

Methods for quantifying the accuracy of the reduced low order error system have been developed. These methods have been applied to a number of model time independent forecast error systems in which a very satisfactory approximation to the error dynamics was obtained with dimension reduction by a factor of 10 (Fig. 2).

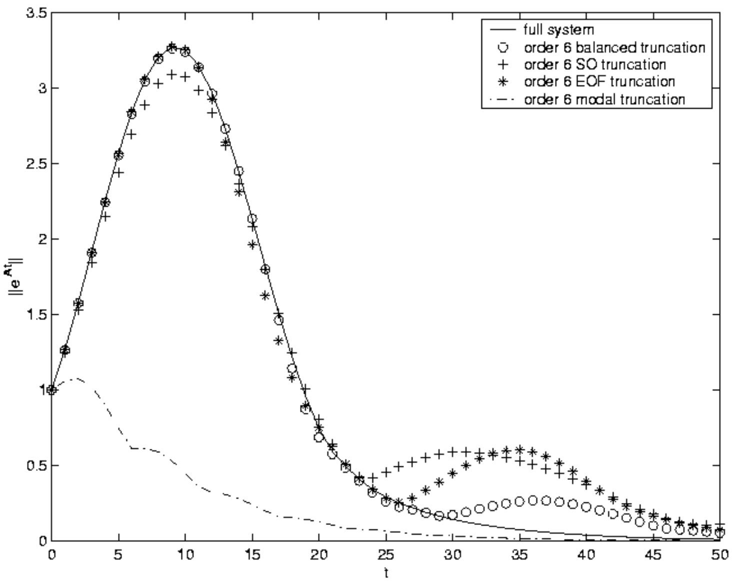

Fig. 2: Maximum error growth as a function of time for a model of the midlatitude barotropic mean jet. The original system (solid) has 100 degrees of freedom and various truncations of order 6 are shown. The best results are obtained for the balanced truncation (circles). Also shown are the optimal growth for an order 6 approximate system in which the top 6 stochastic optimals (SO's) have been retained (crosses); the optimal growth for an order 6 approximate system in which the top 6 empirical orthogonal functions (EOF's) have been retained (stars); and the optimal growth for an order 6 approximate system in which the top 6 least damped modes have been retained (dash-dot). The graph demonstrates that even with 6 degrees of freedom accurate error evolution can be obtained using balanced truncation.

It was determined that truncating the dynamical system in a balanced realization of the optimals and evolved optimals for a single time provides nearly optimal reduced order error system so long as the optimals are obtained for growth over an interval corresponding to the global optimal time. This is a significant result because it suggests that implementing the order reduction algorithm in an operational forecast mode will not be computationally difficult.

IMPACT/APPLICATION

The methods developed in this work can be directly extended to operational forecast models. Efficient calculation of error statistics will allow implementation of advanced methods of initial state estimation such as the Kalman filter.

TRANSITIONS

None

RELATED PROJECTS

None

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PUBLICATIONS

Farrell, B. F. and Ioannou, P. J. 1999: Perturbation growth and structure in time dependent flows. Journal of the Atmospheric Sciences. In Press.
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