

Coherent Effects in the Wave Chaos and in the Chaotic Transmission of Waves

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LONG-TERM GOALS

Our goal is to develop an analytical and numerical approach for description of chaotic sound wave fields at long range propagation in the ocean on the basis of the most advanced methods of dynamics that include ray dynamics, wave dynamics reconstruction on the basis of ray dynamics, specific asymptotic solutions in the short-wave approximation, and specific diagnostic codes adjusted to the wave-chaos analysis.

OBJECTIVES

Long-range sound propagation in the ocean can be studied using contemporary methods of nonlinear dynamics, resonance theory, theory of chaos and, particularly, quantum chaos. Our research is directed towards developing and understanding of new phenomenon: wave-chaos. The results can be applied to wave propagation in plasma layers and toroidal plasma devices, to mesoscopic systems (electron transport and conductivity of quantum wires and dots).

APPROACH

In our research we use:

1. Geometrical optics in terms of the Hamiltonian formalism
2. Method of parabolic equation
3. Topological analysis of chaotic orbits in the phase space
4. Kinetic theory and fractional kinetics
5. High-performance simulations

WORK COMPLETED

A few years ago it has been discovered numerically that even at very long ranges travel times of chaotic rays form compact and rather stable clusters. We have derived a theory of this phenomenon. Our approach provides description of different characteristics of chaotic sound rays including their travel times. Qualitative and quantitative explanation has been given to the stability of the early part of arrival pattern observed in both numerical and field experiments on long range sound propagation in the ocean.

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RESULTS

Properties of chaotic and regular rays have been investigated with an emphasis on studying ray travel times representing arrival times of sound pulses coming to the receiver through different ray paths.

It is shown that the ray travel time as a function of the initial momentum and propagation range in the unperturbed waveguide displays a scaling law. Some properties of the ray travel time predicted by this law still persist in periodically nonuniform waveguides with chaotic ray trajectories. As examples, we have considered few models with special attention to the underwater acoustic waveguide. It is demonstrated for a deep ocean propagation model that, even under conditions of ray chaos, the ray travel time is determined, to a considerable extent, by the coordinates of the ray endpoints and the number of turning points, i.e., by a topology of the ray path. We show how the closeness of travel times for rays with equal numbers of turning points reveals itself in ray travel time dependencies on the starting momentum and on the depth of the observation point. It has been shown that the same effect is associated with the appearance of the gap between travel times of chaotic and regular rays [1].

The Hamiltonian formalism in terms of the action-angle variables is applied to study ray travel times in a waveguide with a smooth sound speed profile perturbed by a weak range-dependent inhomogeneity. A simple approximate formula relating the differences in ray travel times to range variations of action variables is derived. This relation is applied to study range variations of the timefront (representing ray arrivals in the time-depth plane). Widening and bias of timefront segments in the presence of perturbations are considered. Qualitative and quantitative explanations are given to the stability of early portions of timefronts observed in both numerical simulations and field experiments. By ray tracing in a realistic deep water environment with an internal-wave-induced perturbation it has been demonstrated that our approach can be used at ranges up to, at least, 3000 km [2].

Using a parabolic equation, we consider ray propagation in a waveguide with the sound speed profile that corresponds to the dynamics of a nonlinear oscillator. An analytical consideration of the dependence of the travel time on the initial conditions is presented. Using an exactly solvable model and the path integral representation of the travel time, we explain the step-like behavior of the travel time as a function of the starting momentum of the ray. A periodic perturbation of the waveguide along the range leads to wave and ray chaos. We explain an inhomogeneity of distribution of the chaotic ray travel times, which has obvious maxima. These maxima lead to the clustering of rays and each maximum relates to a ray identifier, i.e. to the number of ray semi-cycles along the ray path [3]. Chaotic ray dynamics in the deep ocean has been studied using the Hamiltonian formalism taken in terms of the action-angle canonical variables. A realistic propagation model with an internal wave induced perturbation imposed on the smooth background sound speed field is considered. It is shown that the action variable can be approximated by a Wiener process representing the simplest mathematical model of diffusion. Stochastic ray theory based on this approximation has been applied for analysis of ray travel times. Our attention has been focused on studying the timefront. Estimates for the bias and widening of the timefront segments in the presence of ray chaos have been obtained [4].

IMPACT/APPLICATIONS

Our results contribute to general theory of wave propagation in waveguides. They also provide theoretical background for developing methods of acoustic thermometry of the ocean climate.

REFERENCES

1. “Sensitivity of ray travel times” by I.P. Smirnov, A.L. Virovlyansky, and G.M. Zaslavsky (Published in Chaos, Vol. 12, pp. 617-635 (Sep. 2002))
2. “Ray travel times at long ranges in acoustic waveguides”, by A.L. Virovlyansky (Published in JASA, Vol. 113, pp. 2523-2532 (May 2003))
3. “Sensitivity of ray paths to initial conditions” by A. Iomin and G.M. Zaslavsky (Published in Communications in Nonlinear Science and Numerical Simulation Vol. 8, pp. 401-413 (Sep.-Dec. 2003))
4. “Description of stochastic ray motion of action-angle variables” by A.L. Virovlyansky (Submitted to JASA)

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