

CHARACTERIZING THE EFFECTS OF LOW ORDER PERTURBATIONS ON GEODETIC SATELLITE PRECISION ORBIT DETERMINATION

Eric Eiler*, and John G. Warner†

Satellite operations often rely on the ability to precisely determine and accurately predict the satellite's orbit. Thus, there are numerous papers dedicated to developing methodologies for successful orbit determination. However, there are also lower order forces that act upon satellites that are not directly studied in detail. Two such phenomenon are studied here; perturbations due to the Lunar geopotential, and lower order relativistic corrections. The effects of both on orbit determination are studied with US Naval Research Laboratory's Orbit Covariance Estimation and ANalysis (OCEAN) tool. High precision laser ranging data of geodetic satellites are used as test cases to evaluate the solution accuracy and predictive capabilities. Orbit fit quality and prediction comparison metrics are generated for a number of lunar gravity field models, as well as including or excluding several lower order relativistic corrections. Recommendations are made based on the results.

INTRODUCTION

Satellite operations often rely on the ability to precisely determine and accurately predict the satellite's orbit. Thus, there are numerous papers dedicated to developing methodologies for successful orbit determination. However, there are numerous lower order forces that act upon satellites that are not directly studied in detail. Two such phenomenon are studied here; perturbations due to the Lunar geopotential, and lower order relativistic corrections. The effects of both on orbit determination are studied with US Naval Research Laboratory's Orbit Covariance Estimation and ANalysis (OCEAN) tool. The effects of these perturbations on orbit determination compliment the research of these effects found in references 1, 2, 3, 4, and 5. Several lunar gravity field models will be used at various orders in the precision orbit determination process to several geodetic satellites to build general recommendations. Further, the orbit determination process is run both including and excluding several lower order relativistic corrections to determine the methodologies most suitable. High precision laser ranging data to geodetic satellites are used as test cases to evaluate the solution accuracy and predictive capabilities. Results from these test cases are used to draw more general recommendations for orbit determination methodologies.

OCEAN is a highly configurable, database driven software tool that enables precision orbit determination for a range of satellite missions. OCEAN allows users to simulate data, propagate a spacecraft state, or solve for an orbit using a Kalman Filter-Smoother (KFS) or Weighted Least

* Aerospace Engineer Trainee, Mission Development Branch, US Naval Research Laboratory, 4555 Overlook Ave SW, Washington DC 20375

† Aerospace Engineer, Mission Development Branch, US Naval Research Laboratory, 4555 Overlook Ave SW, Washington DC 20375

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14. ABSTRACT Satellite operations often rely on the ability to precisely determine and accurately predict the satellite's orbit. Thus, there are numerous papers dedicated to developing methodologies for successful orbit determination. However, there are also lower order forces that act upon satellites that are not directly studied in detail. Two such phenomenon are studied here; perturbations due to the Lunar geopotential, and lower order relativistic corrections. The effects of both on orbit determination are studied with US Naval Research Laboratory's Orbit Covariance Estimation and Analysis (OCEAN) tool. High precision laser ranging data of geodetic satellites are used as test cases to evaluate the solution accuracy and predictive capabilities. Orbit fit quality and prediction comparison metrics are generated for a number of lunar gravity field models, as well as including or excluding several lower order relativistic corrections. Recommendations are made based on the results.					
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Squares Orbit Determination (WLS-OD) process. Early history of OCEAN is given in Reference 6, while references 7, 8, 9, 10, 11, and 12 discuss further developments. More recently OCEAN has been used to calculate orbits to support operations for the NRL UPPERSTAGE and TACSAT-4 satellite missions.

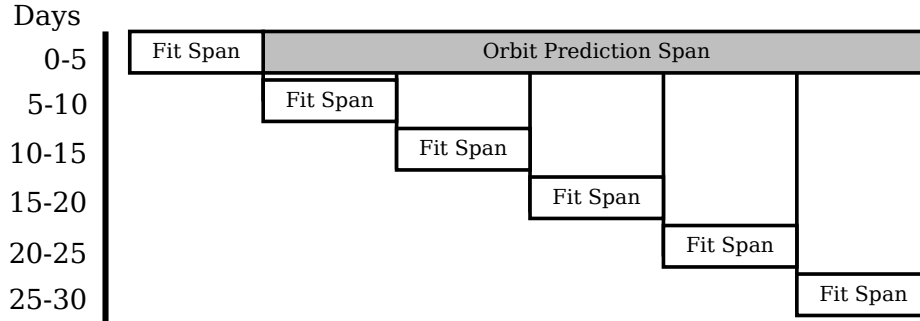
ORBIT DETERMINATION METHODOLOGY

The OCEAN Weighted Least Squares Orbit Determination (WLS-OD) capability employs extensive spacecraft, measurement, and force modeling to estimate the desired spacecraft state and parameters. Force models include solid Earth tides; pole tides; lunar and solar third body gravitational effects; indirect lunar oblateness; general relativistic effects; atmospheric variability; drag; and solar radiation pressure. OCEAN WLS-OD accounts for various time systems, including TAI and UTC time. The WLS-OD functionality also uses Earth Orientation Parameter (EOP) models from the International Earth Rotation and Reference Systems Service (IERS), which account for precession and nutation, Earth rotation, and polar motion. For this application, a ninth order multi-step predictor-corrector algorithm is used to perform the integration of the state variables and state transition matrix. OCEAN follows the standards described in the IERS 2010 Conventions.¹³ OCEAN also performs iterative data editing to prevent low quality data from biasing the orbit solution.

Historical orbital data is used to perform precision orbit determination. This high precision data is provided by the International Laser Ranging Service (ILRS) in the form of global satellite laser ranging (SLR) data. The ILRS exists to support geodetic research activities.¹⁴ The ILRS has cataloged SLR data for a large number of geodetic satellites since 1998. Satellites used by the ILRS contain laser retro-reflectors to facilitate laser ranging data collection and are typically designed to study Earth's gravity. The cataloged laser ranging data can then be used to perform high precision orbit determination. Normal point laser range measurements to LAGEOS-1 are precise to under one centimeter.¹⁵

Several geodetic satellites were chosen for test cases including LAGEOS-1, Etalon-1, Galileo-102, GLONASS-129, Starlette, and Stella. Precision orbits are calculated by OCEAN using the SLR data from these satellites. By comparing predictive orbits to fitted orbits for each satellite / force model combination, the predictive accuracy of the models may be calculated. The OCEAN WLS-OD methodology will be used to determine orbits using successive five day increments of laser ranging data. The first five day data arc will produce an orbit solution that will be propagated forward in time thirty days. The WLS-OD process will be repeated for successive five day data arcs. The resulting orbit solutions are then compared to the predicted orbit from the first data arc solution. 25 to 30 days were chosen as comparison time spans to demonstrate the longer term variation in predictive accuracy. The degree to which the orbit solutions agree will be used as a metric to evaluate the suitability of the lunar gravity field models and relativistic corrections to precision orbit determination. OCEAN's fit vs. prediction methodology can be seen in Figure 1.

Figure 1. Depiction of Orbit Solution Comparison Methodology



An orbit solution is then fit to 25 to 30 days of SLR data by OCEAN. The relative effectiveness of the lunar gravity field models and relativity term corrections are compared by using the RMS of the error residual as a comparison metric. This enables capturing of the long term variation in the satellite's orbit. While data editing methodologies and raw data quality often have a large impact on the RMS of the error residuals, it is nonetheless used as a metric to compare orbit solution fit quality across the various models.

GEODETIC SATELLITE TEST CASES

LAGEOS-1 and Etalon-1 were chosen as test cases for both studies due to the large amount of SLR data available for each satellite. Galileo-102 and GLONASS-129 were chosen for the lunar geopotential study due to their higher altitude closer to the moon in Medium Earth Orbit. Starlette and Stella were chosen for the Relativity study due to their closer proximity to the Earth.

LAGEOS-1

The LAGEOS-1 satellite was launched in 1976 by the National Aeronautics and Space Administration (NASA). The LAGEOS satellites are part of the Earth and Ocean Dynamics application program. LAGEOS-1 was designed to provide a long-lasting laser target in a well known orbit. The satellite enabled researchers to study a range of geophysical phenomena with improved accuracy, including the Earth's geopotential.¹⁶ The satellite's low ballistic coefficient combined with its spherical shape minimize the orbital uncertainty due to drag and solar radiation forces.¹⁷

The nominal orbital elements are given in Table 1.

Table 1. Nominal Orbital Elements for LAGEOS-1

Element	Nominal Value
Semi-major Axis	12, 240 km
Eccentricity	0.0045
Inclination	109.84°

Etalon-1

The Etalon-1 satellite was launched in 1989 by the former Soviet Union. It was launched along with a twin satellite (Etalon-2) and two GLObal'naya NAvigatsionnaya Sputnikovaya Sistema (GLONASS) satellites. The Etalon satellites were launched for gravity field studies through

the use of laser ranging.¹⁸ The Etalon satellites are passive satellites with only retro-reflectors as on-board instruments.¹⁹

The nominal orbital elements are given in Table 2.

Table 2. Nominal Orbital Elements for Etalon-1

Element	Nominal Value
Semi-major Axis	25,490 km
Eccentricity	0.00061
Inclination	64.9°

Galileo-102

The Galileo-102 satellite was launched in 2011 by the European Space Agency (ESA). It is a part of the Galileo global navigation satellite system and was launched along with Galileo-101. The Galileo satellite system was created to provide a global navigation system for civilian purposes.²⁰ Galileo-102 was the fourth Galileo satellite launched and the second used for signal validation.²¹

The nominal orbital elements are given in Table 3.

Table 3. Nominal Orbital Elements for Galileo-102

Element	Nominal Value
Semi-major Axis	29,590 km
Eccentricity	< 0.001
Inclination	56°

GLONASS-129

The GLONASS-129 satellite was launched in 2011 by the Russian Federation Ministry of Defense. It is a part of the GLONASS global navigation satellite system and was launched along with identical satellites GLONASS-127 and GLONASS-128. The GLONASS satellite system was created to identify users' positions and velocities.²² GLONASS-129 is a Type M GLONASS satellite.²³

The nominal orbital elements are given in Table 4.

Table 4. Nominal Orbital Elements for GLONASS-129

Element	Nominal Value
Semi-major Axis	25,476 km
Eccentricity	0.0031
Inclination	64.78°

Starlette

The Starlette satellite was launched in 1975 by Centre Nationale d'Etudes Spatiales (CNES). The spacecraft was designed to improve the geopotential model and to study solid Earth tides, ocean

tides, and polar motion.²⁴ Starlette was also the first spacecraft to be entirely covered by laser corner reflectors which allow passive SLR observation capabilities.²⁵ The Starlette orbit is highly sensitive to zonal variations in the gravity field.

The nominal orbital elements for Starlette are given in Table 5.

Table 5. Nominal Orbital Elements for Starlette

Element	Nominal Value
Semi-major Axis	7,190 km
Eccentricity	0.0206
Inclination	49.83°

Stella

The Stella satellite was launched in 1993 by CNES, and is virtually identical to the Starlette satellite. As with the LAGEOS satellites, both Stella and Starlette are spherically-shaped spacecraft with low ballistic coefficients to minimize orbital uncertainty caused by drag and solar radiation pressure forces.²⁶

The nominal orbital elements for the Stella satellite are given in Table 6.

Table 6. Nominal Orbital Elements for Stella

Element	Nominal Value
Semi-major Axis	7,178 km
Eccentricity	0.0206
Inclination	98.6°

LUNAR GRAVITY FIELD MODELS

A selection of lunar gravity field models are evaluated for suitability in precision orbit determination. This selection includes the Goddard Lunar Gravity Model 2 (GLGM-2), the Jet Propulsion Laboratory's Lunar Prospector Gravity Science Team's JGL165P1 model, and the Goddard Space Flight Center's GRGM660PRIM model. The chosen models use data from various lunar missions. The models include coefficients to various degrees. They are compared to a baseline degree 4 lunar gravity field model (LUN75A).

The GLGM-2 model was developed by Goddard Space Flight Center in 1995 based on data from Clementine, Lunar Orbiters 1-5, Apollo 15, and Apollo 16. It includes coefficients to degree 70. Results from the GLGM-2 model confirmed features of the lunar gravity field previously discovered as well as unveiled more detail. The resulting lunar gravity anomalies found by this model range from -294 to +358 mGal.²⁷ At degree 70, the GLGM-2 model is the least detailed model chosen for this study.

The JGL165P1 is one of several models developed by the Jet Propulsion Laboratory in 2000 based on data from the Lunar Orbiter Missions, Apollo 15 and 16, Clementine and the Lunar Prospector. It includes coefficients to degree 165. The Lunar Prospector data was the primary source for this model and allowed for higher resolution gravity field mapping for the lunar nearside. The Lunar

Prospector data allowed for identification of new mascons and improved resolution of multiple craters. Lunar gravity anomalies in this model range from -489 to +380 mGal.²⁸

The GRGM660PRIM model was developed by the Goddard Space Flight Center in 2013 based on tracking data from the GRAIL mission. It includes coefficients to degree 660. This model resulted in a much smaller root-mean-square error in lunar gravity anomalies than previous models. The resulting lunar gravity anomalies found by the GRGM660PRIM model range from -538.30 to +336.99 mGal.²⁹ At degree 660, this model is the most detailed of the chosen models.

Each of these models are implemented in OCEAN in calculating lunar perturbations on geodetic satellites. The relative effects of these models are quantified and compared. Conclusions on the overall effect of lunar geopotential perturbations are presented.

LOWER ORDER RELATIVISTIC CORRECTIONS

There are several relativistic corrections to a satellite's equations of motion when an Earth-centered inertial frame is used. The relativistic corrections comprise of Schwarzschild terms, Lense-Thirring precession (frame-dragging), and de Sitter (geodesic) precession. The Schwarzschild, Lense-Thirring, and de Sitter terms can be seen on lines 1, 2, and 3 respectively in Equation 1.¹³

$$\begin{aligned} \Delta \vec{r} = & \frac{GM_E}{c^2 r^3} \left\{ \left[2(\beta + \gamma) \frac{GM_E}{r} - \gamma \vec{r} \cdot \vec{r} \right] \vec{r} + 2(1 + \gamma)(\vec{r} \cdot \vec{r}) \vec{r} \right\} + \\ & (1 + \gamma) \frac{GM_E}{c^2 r^3} \left[\frac{3}{r^2} (\vec{r} \times \vec{r})(\vec{r} \cdot \vec{J}) + (\vec{r} \times \vec{J}) \right] + \\ & \left\{ (1 + 2\gamma) \left[\vec{R} \times \left(\frac{-GM_S \vec{R}}{c^2 R^3} \right) \right] \times \vec{r} \right\} \end{aligned} \quad (1)$$

The Schwarzschild terms are considered to be the baseline slow-motion approximation of the relativistic effects on the gravitational field near Earth.³⁰ The Lense-Thirring precession terms account for relativistic corrections due to the rotation of the Earth and the resulting change in the rotational axis orientation.³¹ The de Sitter precession terms account for relativistic effects similar to Lense-Thirring precession, but only those due to Earth's presence as a central body, not its rotation.³² The largest magnitude correction comes from the Schwarzschild terms. However, each term will be examined individually and in combination in each of the orbit determination cases to determine the cumulative effect on the quality of the orbit solution.

TESTING RESULTS

Results for each satellite case are presented side by side. For both the lunar geopotential study and the relativity terms study, RSS position differences between the initial interval orbit prediction and the current interval orbit solution are given to show the predictive accuracy of the orbit solution for model being examined. Last, plots of average daily average RSS position difference are given for each case as a metric to evaluate the suitability of the underlying model for precision orbit determination.

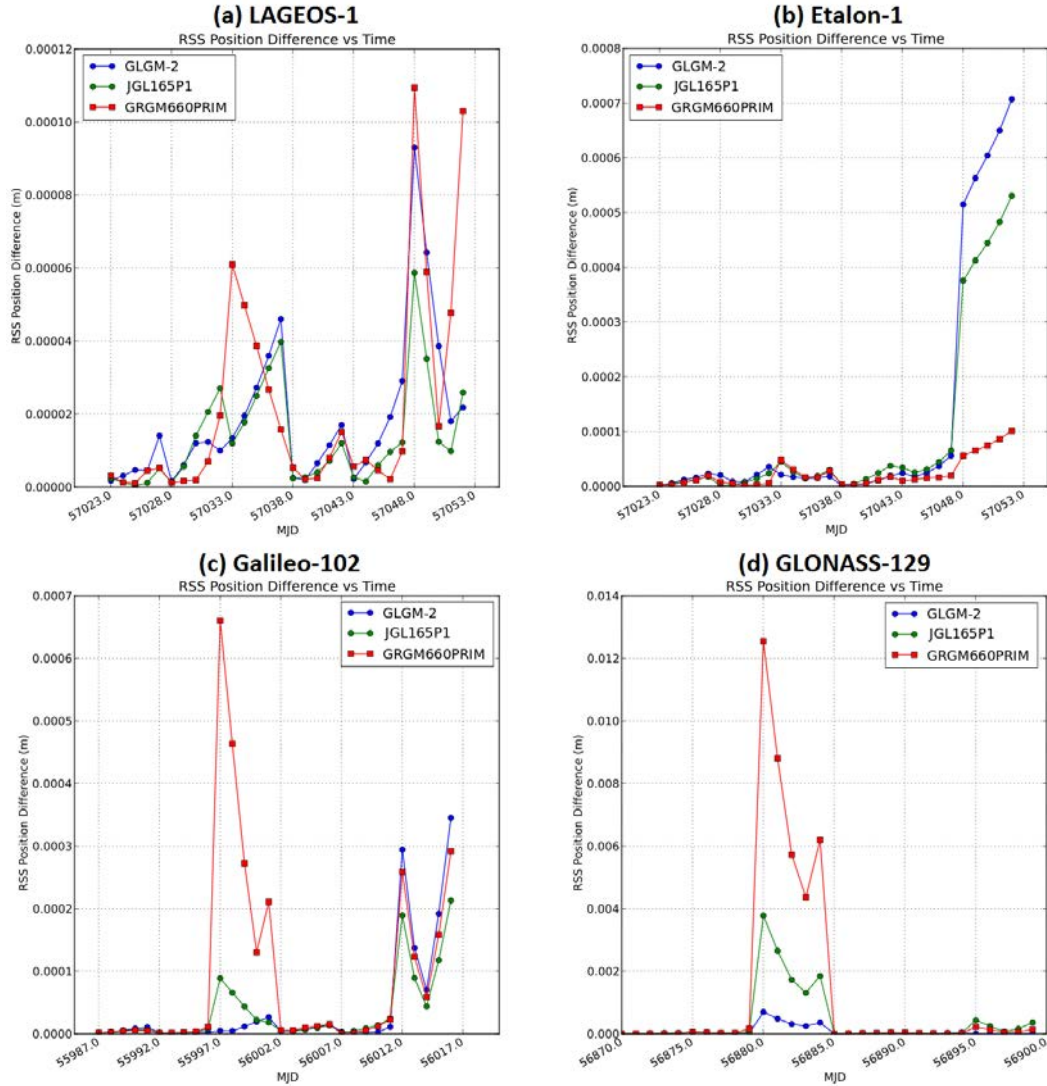
Lunar Gravity Field Model Results

Each satellite's orbit was predicted over a 30-day time span and was compared to an orbit fitted to its available ILRS data. A version of OCEAN was compiled using each lunar gravity field model.

Each model was truncated to include 30 coefficients since the use of any more coefficients caused negligible differences. Results for each model were produced for each satellite test case and were compared.

Differences between the definitive orbit solution for each lunar gravity field model and the definitive orbit solution for the 4th order baseline model were taken. These results show the small magnitude effects caused by the different models and can be examined in Figure 2. These effects cause position differences typically under 1 millimeter.

Figure 2. RSS Position Difference Comparison Between Solutions from Various Lunar Gravity Field Models and Solution from a Baseline 4th Degree Lunar Gravity Field Model

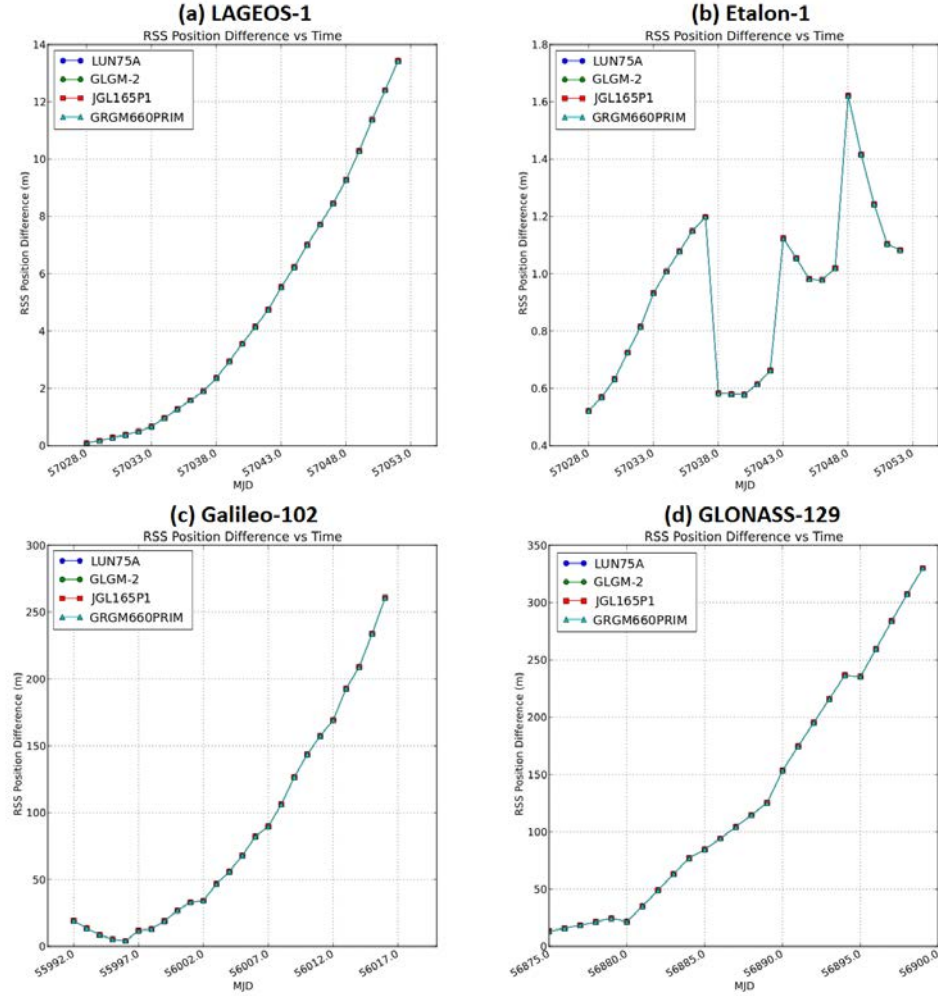


The magnitudes of the differences between each model's results and the baseline model's results for LAGEOS-1, Etalon-1, and Galileo-102 seen in Figure 2 (a), (b), and (c) respectively are all below 1 millimeter. Only differences in the GLONASS-129 results found in Figure 2 (d) reach a magnitude over 1 centimeter. It is believed that this is actually a result of estimation error occurring

around the time with the worst RMS residual error and is not a direct result of differences in the lunar gravity field models.

Figure 3 shows the RSS position differences over time for the three examined lunar gravity field models along with the baseline 4th order lunar gravity field model (LUN75A). The errors over time are presented for each satellite.

Figure 3. RSS Position Differences for Various Lunar Gravity Field Model Solutions



It can be seen that all four lines representing each RSS position difference lay almost directly on top of one another. The satellite with the smallest RSS position differences over time is Etalon-1. Its errors over time stay below 2 meters and can be seen in Figure 3 (b). Here, the differences caused by the various lunar gravity field models are not apparent even on an almost sub-meter scale.

With their closer proximity to the moon in Medium Earth Orbit, it was expected that the Galileo-102 and GLONASS-129 test cases, seen in Figure 3 (c) and (d) respectively, would be the most affected by differences in lunar gravity field models. However, modeling error over time grew too quickly to see any differences between the models' RSS positions. This error growth was determined to be caused by lower SLR data sample sizes and OCEAN solving accuracy errors.

It was concluded that the use of the three higher-order lunar gravity field models did not result in a significant improvement in precision orbit determination with differences with magnitudes at the sub-millimeter level over time. It was predicted that test cases involving satellites closer to the moon would experience a larger effect than satellites at lower altitudes, but, higher altitude satellites showed poor RMS residual errors as a result of a bad fit; thus, this conclusion can't be made.

Lower Order Relativistic Correction Results

A similar method to the lunar gravity field model testing was used for testing the relativity correction terms. A version of OCEAN compiled with the use of the Schwarzschild terms only was considered a baseline solution. Another version of OCEAN was compiled with the addition of the Lense-Thirring terms. Finally, a version including the Schwarzschild, Lense-Thirring, and the even less effectual de Sitter terms was compiled. All three versions were tested and compared using the four chosen satellite test cases. Plots showing the differences taken between results for the lower order terms and results for the baseline Schwarzschild terms show the small magnitude of each term's effect and can be found in Figure 4.

Figure 4. RSS Position Difference Comparison Between Solutions from Low Order Relativistic Correction Terms and Baseline Schwarzschild Terms Only Solution

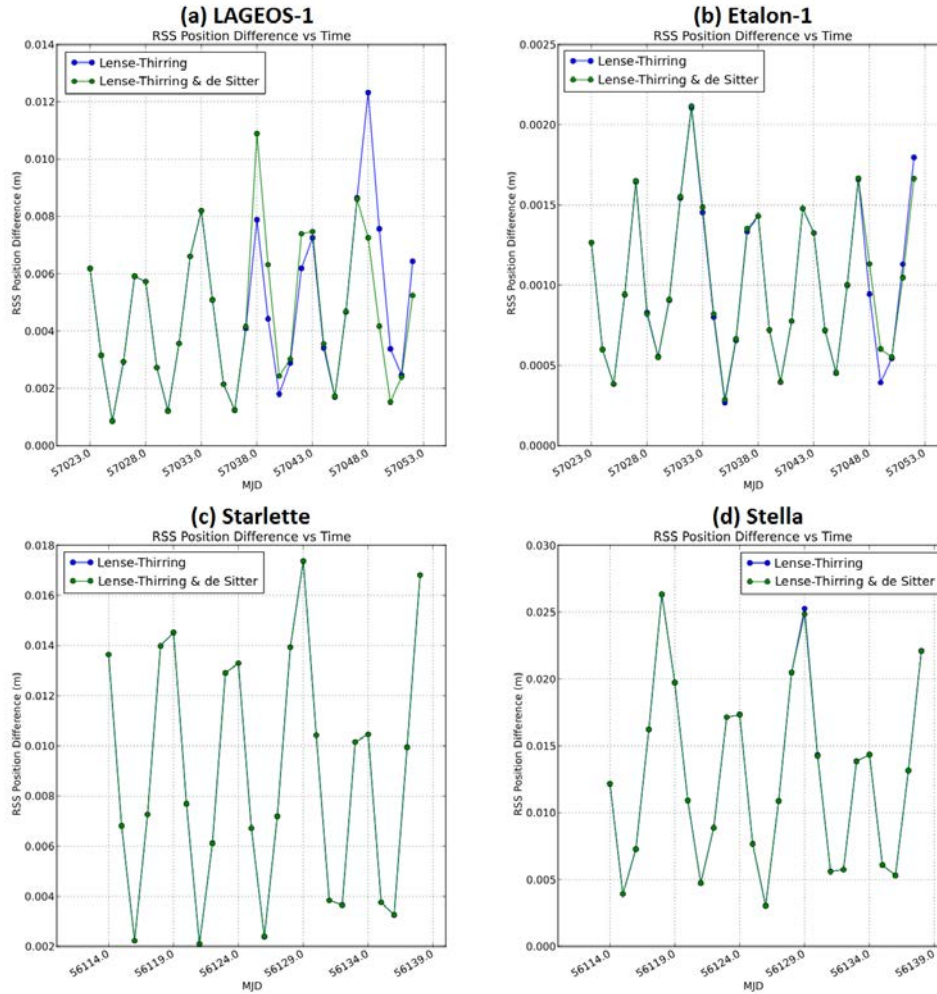
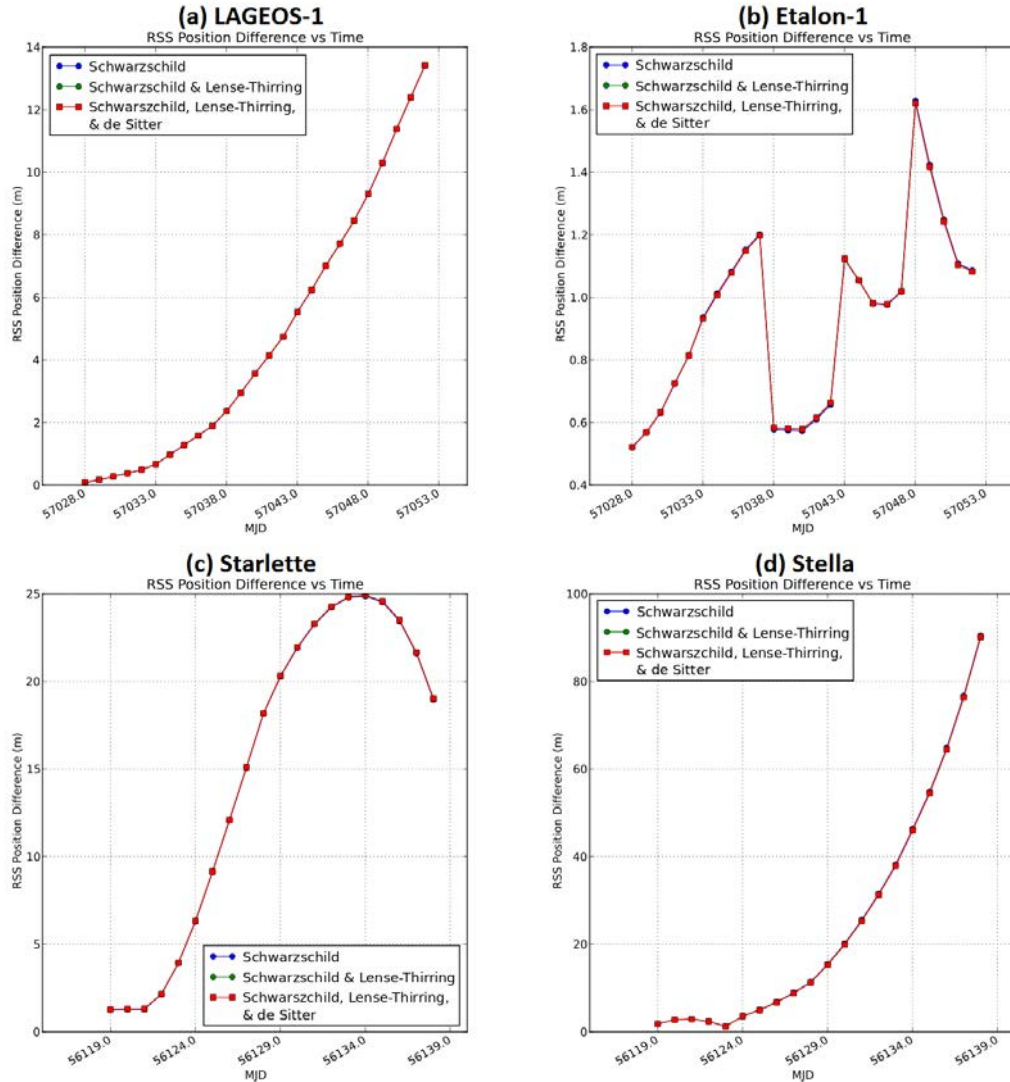


Figure 4 shows the very small effects caused by the addition of the lower order relativity correction terms. With the addition of the Lense-Thirring terms, the differences in the results are below 3 centimeters for Stella seen in Figure 4 (d), below 2 centimeters for LAGEOS-1 and Starlette in Figures 4 (a) and (c), and below 3 millimeters for Etalon-1 in Figure 4 (b) with Etalon-1 being the satellite whose long term dynamics were modeled most accurately.

Figure 5 shows the RSS position differences over time for the lower order relativity terms along with the baseline Schwarzschild-only solution. The errors over the time spans are presented for each satellite.

Figure 5. RSS Position Differences for Solutions with Various Low Order Relativistic Term Combinations



Similar to the results for the lunar geopotential cases, each plot's lines lay on top of one another. The differences caused by the addition of each set of lower order correction terms are too small for the scales seen in these plots. Only in Figure 5 (b) a small difference of nearly 1 centimeter can barely be seen. It was concluded that the addition of the lower order relativity correction terms did

not substantially affect the accuracy of precision orbit determination over time.

CONCLUSION

A number of lunar gravity field models were examined to determine the impact of the perturbations on precision orbit determination. It was seen that there is little cumulative impact between the models on the orbit solution quality. With effects with magnitudes typically at the sub-millimeter level, it was determined the inclusion of lunar gravity field models of higher order does not prove to be effective in reducing error in precision orbit determination. No model examined proved to conclusively be regarded as the best model. For geodetic orbit determination applications, higher order lunar gravity field models do not need to be applied.

Relativistic correction terms were also examined. Although the magnitudes of the effects due to the addition of the lower order terms were slightly higher than the effects due to the lunar gravity field models, it was similarly concluded that the addition of the Lense-Thirring and de Sitter terms will not cause any substantial accuracy increase in orbit determination over time and can be neglected.

Although lunar gravity field models of higher order model the moon's geopotential accurately and the Lense-Thirring and de Sitter terms account for the effects of relativity near Earth accurately, their application to precision orbit determination do not produce significant effects and can be neglected.

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