Evolution of Internal Waves in Hydrostatic Models

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LONG-TERM GOALS

My long-term goal is improving our understanding of interactions between sub-grid scale models and numerical errors, which affect simulation skill for physical and biological processes in estuaries and coastal ocean environments. For prognostic simulations, slow accumulation of truncation-level errors will progressively distort model results. Present sub-grid scale models selectively address certain processes (e.g. turbulence) while neglecting other processes (e.g. small-scale internal waves, particulate nature of algae). As small-scale nonlinear errors can accumulate over time, understanding the errors and modeling the neglected processes are critical issues in improving our prognostic skill.

OBJECTIVES

I am examining the un-physical behavior of internal waves modeled under the hydrostatic approximation. This knowledge is a precursor to developing improved modeling techniques that account for transfer of energy from resolved internal waves into sub-grid scale internal waves. These higher frequency waves can break on sloping boundaries or lead to enhanced wave-wave interaction, thereby amplifying vertical mixing and changing the spatial and temporal evolution of the oceanic density structure.

APPROACH

Nonlinear terms in the momentum equations slowly cause an initially-linear internal wave to steepen, resulting in nonlinear and, what should be, non-hydrostatic evolution. However, in a hydrostatic model, the non-hydrostatic dispersion is missing, therefore modeled internal waves steepen until numerical diffusion, dispersion, or dissipation provides balance. Thus, steepening internal waves in a hydrostatic model will artificially diffuse the density gradients, disperse the waves or dissipate their energy. Alternatively, a theoretically error-free hydrostatic model must result in waves that always steepen until breaking – thus artificially diffusing the density gradient in breaking-induced mixing. We are examining approaches for modeling the correct steepening/dispersion balance without resorting to the computational expense of a full non-hydrostatic solution. I am working with two University of Texas (UT) graduate students, Ms. B. Wadzuk and Ms. S. Delevan, applying hydrostatic and non-hydrostatic models of the Navier-Stokes equations to simulate internal waves in simple geometries. These results will be compared to laboratory experiments and the KdV wave model of Horn et al 2000. After validating the models, we will examine the divergence of results between the hydrostatic and

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14. ABSTRACT
My long-term goal is improving our understanding of interactions between sub-grid scale models and numerical errors, which affect simulation skill for physical and biological processes in estuaries and coastal ocean environments. For prognostic simulations, slow accumulation of truncation-level errors will progressively distort model results. Present sub-grid scale models selectively address certain processes (e.g. turbulence) while neglecting other processes (e.g. small-scale internal waves, particulate nature of algae). As small-scale nonlinear errors can accumulate over time, understanding the errors and modeling the neglected processes are critical issues in improving our prognostic skill.

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non-hydrostatic models under varying model conditions (e.g. grid resolution, time step, stratification). By separately quantifying errors in diffusion of mass and momentum, we will look for a means of predicting required model parameters for adequately resolving internal waves with a hydrostatic code. In the past year, work on separating and analyzing numerical diffusion has been done in collaboration with Asst. Prof. B. Laval, University of British Columbia (UBC) and Prof. J. Imberger, University of Western Australia (UWA). Collaboration with Prof. Imberger also involves development of nested grid methods, which are necessary for higher resolution where internal waves are breaking. One of my UT graduate students, Mr. J. Furnans, is presently at UWA on a Fulbright Scholarship and is conducting this work.

WORK COMPLETED

This first full year of the project has been focused on completing preliminary work for quantifying numerical diffusion, while simultaneously building student expertise and involvement. This project's first graduate student, Ms. Wadzuk, has completed her M.S. degree and has begun Ph.D. studies. Her M.S research report, entitled *Evolution of Internal Waves in Hydrostatic Models: A Study of Dynamic Pressure,* is available as a web download at the project web site listed above. Ms. Wadzuk is in the midst of developing a practical approach for implementing the solution of non-hydrostatic pressure in the framework of a hydrostatic numerical model. We have completed the literature review and analysis of prior approaches. In the past year, Ms. Wadzuk has developed proficiency in Fortran 95 coding and investigated Poisson methods for the solution of a fractional-step equation as part of the non-hydrostatic solution. A second graduate student, Ms. Delavan, joined this project over the past summer. Her focus is on conducting and analyzing simulations to quantify the performance of the hydrostatic model over a range of internal wave characteristics. During the summer, Ms. Delavan has developed the non-dimensional formulation of the numerical experiments and become familiar with applying the hydrostatic model.

With Asst. Prof. Laval (UBC) and Prof. Imberger (UWA), we have completed a study on quantifying and reducing the effects of numerical diffusion in models of internal waves (Laval et al. 2002). In related collaborative work with Dr. Chris Dallimore (also of the UWA), we have shown that 2D and 3D models can be linked to improve the representation of the propagation of density currents down a slope, which has resulted in two more papers accepted to the *Journal of Hydraulic Engineering* this past year. This linking of models of different spatial dimensions provides another avenue for investigating improvements to internal wave modeling. We have developed a hypothesis that an inviscid wave model for a 2D density surface could be linked to a 3D hydrodynamic model as a means of improving the internal wave representation.

RESULTS

The first difficulty in examining the evolution of internal waves in numerical models is separating the effects of numerical diffusion of mass, numerical dissipation of energy, and numerical dispersion of waves. We have learned that we can quantify and compensate for numerical diffusion by computing changes in background potential energy occurring during advection. This builds on concepts from prior ONR-funded studies of Winters et al. (1995). It has become clear that quantifying and controlling numerical diffusion is a necessary precursor to evaluating the steepening of internal waves under the hydrostatic approximation. Unless the numerical diffusion of mass is limited, the steepening of waves will be controlled by numerical diffusion rather than by the governing equations. The result is degradation of the thermocline gradients and distortion of wave evolution. In a paper recently accepted by the *Journal of Hydraulic Engineering*, we demonstrate an approach applying a sharpening filter to the density field. This creates an *a posteriori* adjustment that compensates for numerical diffusion's effect on the density structure. This has been demonstrated (Figure 1) to provide a better representation of the evolution of the sharp density structure as seen in field data. In other words, standard application of a numerical model leads to slow numerical diffusion of the thermocline, which distorts the internal wave evolution. By quantifying and compensating for this diffusion with sharpening filter, we can retain a better model of the thermocline gradient and improve the model skill. The major difficulty with the approach is that it is inherently ad hoc – i.e. it is an artificial method of countering error after it has occurred rather than preventing it during advection. The method cannot predict precisely where numerical diffusion has occurred, but only the net amount over the whole domain, which then is removed by artificially sharpening the thermocline throughout the system. Thus, this has some parallels to "global fill" approaches that are used to globally redistribute the net mass error in models with non-conservative transport routines. However, instead of being a global redistribution of mass, this is a global readjustment of density gradients (with weighting at the thermocline). This filtering method has been successful in controlling increases in numerical diffusion over the course of a simulation (Figure 2).

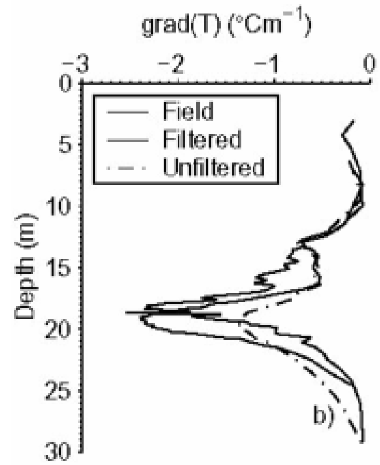

Figure 1. Temperature gradient v. depth for field data and model results with and without filter. [graph: Field data temperature gradient and filtered model results show peak at –2.5 C°cm^{-1} while unfiltered model results has peak at –1.2 C°cm^{-1}.]

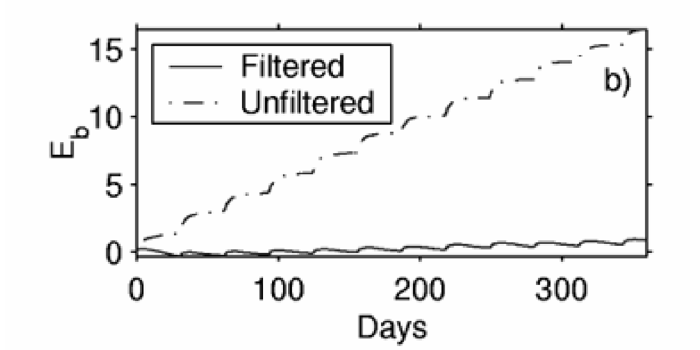

Figure 2. The change in background potential energy (E_b) for filtered and unfiltered model results (normalized by the change in the filtered result). These are simulations of adiabatic, periodically wind-forced, stratified basins for which a perfect hydrostatic model would show zero change in E_b. [graph: The unfiltered E_b results show an increase 15 times greater than the increase in the filtered results over the course of a year.]

IMPACT/APPLICATIONS

Improvements in our ability to correctly represent internal waves in hydrostatic models will provide improved representation of the spatial and temporal evolution of the density field in the oceans. This will improve our ability to use model results to predict thermocline position, thickness and density gradients – especially in coastal regions where steepening and breaking of internal waves along the slope may be an important source of mixing. The approach developed in Laval et al. (2002) provides a practical method of evaluating the magnitude of numerical diffusion and reducing its effects.

TRANSITIONS

The background potential energy algorithm and density-filtering algorithm of Laval et al. (2002) has been implemented in distribution version 1.5.2 of the ELCOM model (Hodges, et al. 2000). In addition to the present research program, the model is being used at Stanford University, University of Stuttgart, University of Western Australia, University of British Columbia, Instituto Nacional Del Agua Y Del Ambiente (Argentina), Kinneret Limnological Laboratory (Israel), University of Ioannina (Greece).

RELATED PROJECTS

The long-term improvement of models for sub-grid scale effects is the subject of ongoing collaborative efforts of the PI with Prof. Jörg Imberger (Stockholm Water Prize Laureate, 1997) at University of Western Australia. These efforts include development of a benthic boundary layer underflow model, development of particle tracking algorithm, and implementation of nested grids for increased local model resolution. A University of Texas graduate student, Mr. Jordan Furnans, who is under my supervision but is presently studying at the University of Western Australia with a Fulbright scholarship, is accomplishing the work on nested grids. This work is directly related to the study of internal wave evolution, as we would like to apply higher grid resolution in regions where waves are rapidly evolving or breaking.

REFERENCES

Hodges, B.R., J. Imberger, A. Saggio, and K. Winters, (2000), Modeling basin-scale internal waves in a stratified lake, *Limnol. Oceanogr.* **45**:1603-1620.

Horn, D.A., L.G. Redekopp, J. Imberger and G.N. Ivey (2000), Internal wave evolution in a space-time varying field, *J. Fluid Mech.* **424**:279-301.

Winters, K. B.; Lombard, P. N.; Riley, J. J.; D'Asaro, E. A. (1995), Available potential energy and mixing in density-stratified fluids. *J. Fluid Mech.* **289**:115-124

PUBLICATIONS

Dallimore, C.J., B.R. Hodges, and J. Imberger, (2002), Coupling an underflow model to a 3D hydrodynamic model, *J. Hydraul. Engrg,* (accepted, May 2002).

Dallimore, C.J., J. Imberger, and B.R. Hodges, (2002), Modeling a plunging underflow, *J. Hydraul. Engrg,* (accepted, Sept. 2002)

Laval, B., B.R. Hodges and J. Imberger, (2002), Amelioration of numerical diffusion in 3D lake models, *J. Hydraul. Engrg*, (accepted, Sept. 2002).

Wadzuk, B. (2002). *Evolution of Internal Waves in Hydrostatic Models: A Study of Dynamic Pressure,* M.S. Report, Department of Civil Engineering, University of Texas, Austin, May, 2002.