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**A New Fast, Accurate and Non-Oscillatory Numerical Approach for Wave Propagation Problems in Solids Application to High-frequency Pulse Propagation in the Hopkinson Pressure Bar**

Alexander Idesman  
TEXAS TECH UNIVERSITY SYSTEM

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<b>14. ABSTRACT</b> We have developed an effective two-stage time-integration technique for elastodynamics and acoustic wave propagation problems solved with explicit and implicit time-integration methods and different space-discretization methods. For the first time, we have quantified the range of spurious oscillations for different space-discretization methods and have effectively filtered the spurious oscillations at the filtering stage. We have developed new finite elements with reduced dispersion for explicit time-integration methods as well as an analytical procedure for the selection of the size of time increments for the stage of basic computations and the filtering stage for the new finite elements with reduced numerical dispersion. The solution of 1D, 2D and 3D benchmark wave propagation problems showed that the new technique yields accurate and non-oscillatory results without interaction between computer code and user and reduces the computation time by a factor of 10 – 1000 and more compared with the standard finite element approaches. A surprisingly good agreement between experiments by group of Dr. Foley from AFRL/RWMF, Eglin and the simulations with the new numerical technique has been obtained for wave propagation in the components of the Hopkinson Pressure Bar.					
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# **A New Fast, Accurate and Non-Oscillatory Numerical Approach for Wave Propagation Problems in Solids: Application to High-Frequency Pulse Propagation in the Hopkinson Pressure Bar**

Alexander Idesman  
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## **Final report**

The main accomplishments with the technical details have been reported in the archival publications. Therefore, below we will list the main findings with the corresponding figures from our presentation at the 2015 Program Review Meeting and short explanations. We also include the references to our publications for the derivations and the detailed explanations.

### **Main accomplishments**

#### **1. A two-stage time-integration technique for elastodynamics and acoustic wave propagation problems**

The application of the finite elements or any other space-discretization method to the partial differential equations of linear elastodynamics and acoustics leads to a system of ordinary differential equations in time, see Fig. 1.

**Semi-discrete formulation of linear elastodynamics**

Space discretization methods:

- a) Finite element method
- b) Spectral elements method
- c) Isogeometric elements
- d) Boundary element method
- e) smoothed particle hydrodynamics (SPH) method
- f) others

Semi-discrete system of equations 
$$\begin{cases} M\ddot{U} + C\dot{U} + KU = R & (1) \\ U(t=0) = U_0, \quad \dot{U}(t=0) = V_0 \end{cases}$$

1. The exact time integration of the semi-discrete system, Eq. (1), may yield very inaccurate solutions of the original (before the space discretization) system of elastodynamics equations due to large spurious high-frequency oscillations.
2. The goal of calculations is not an accurate integration of Eq. (1) but accurate solutions of original elastodynamics problems before the space discretization.
3. Accurate time integration of low modes and filtering spurious high modes are needed !

Fig. 1

## Issues with existing time-integration methods for wave propagation problems due to spurious high-frequency oscillations

- ❑ For wave propagation and impact problems, existing time-integration methods may yield locally divergent results at small time increments and mesh refinement due to large spurious high-frequency oscillations.
- ❑ There is no quantification of the range of spurious oscillations for semi-discrete elastodynamics equations
- ❑ There is no understanding how to select the amount of numerical dissipation (or artificial viscosity) and the size of time increments with existing time-integration methods for wave propagation problems
 

"In no way can today's computer program for wave propagation and impact be treated as 'black boxes'. A minimum 6 months to 2 years of experience is needed to be able to use such programs successfully" (Concept and applications of finite element analysis, R. D. Cook et al, 2002).
- ❑ Usually, accuracy analysis of existing time-integration methods with numerical dissipation (or artificial viscosity) does not include the effect of the number of time increments on the accuracy of final numerical results.
- Numerical dissipation used in existing time-integration methods for filtering the spurious high modes affects the low modes of a numerical solution resulting in inaccurate solutions, especially for long-term integrations (the spectral radius  $r^n$  should be close to 1 for actual frequencies; if  $r = 0.99$  for  $n = 1$ , then  $r^n = 0.99^{100} = 0.366$  for  $n = 100$ , or if  $r = 0.999$  for  $n = 1$ , then  $r^n = 0.999^{1000} = 0.3677$  for  $n = 1000$ )
- ❑ The selection of the most effective time-integration method for wave propagation problems among known methods

Fig. 2

The main issues with the integration of this system of ordinary differential equations are listed in Fig. 2. The most important issue is related to divergent results at time and mesh refinements (e.g., below see Fig. 7 with no filtering). To resolve these issues we have developed a two-stage time-integration approach consisting of the stage of basic computations and the filtering stage. The idea of the two-stage time-integration procedure is very simple. Because for the ordinary differential equations of linear elastodynamics and acoustics there is no interaction between different modes during time integration (due to modal decomposition they are integrated independently and do not affect one another, see Fig.3), the most accurate time-integration method (without numerical dissipation or artificial viscosity) should be used for basic computations (one stage), especially for a long-term integration. This means that all modes (including high modes) are integrated very accurately, and the solution includes high-frequency oscillations. Then, for damping out high spurious (inaccurate) modes, a time-integration method with large numerical dissipation (or with artificial viscosity) can be used as a pre-, or post-processor (another stage). This method can be considered a filter of high modes; see Fig. 4. Usually, a small number of time increments (we always use 5 positive plus 5 negative time increments) is necessary for the pre-, or post-processing (filtering) stage, with no error accumulation at low modes. This technique yields no error accumulation due to numerical dissipation (or artificial viscosity) at the stage of basic computations and does not require any guess for the selection of numerical dissipation or artificial viscosity as in the existing approaches. One of the impotent components of the new numerical approach is the quantification of the range of spurious frequencies. Using the dimensionless analysis of the 1-D impact wave propagation problem (all frequencies are excited for this problem), defining the amplitudes of



spurious frequencies as the difference between the exact and numerical velocities before and after the wave front for the 1-D problem, we have

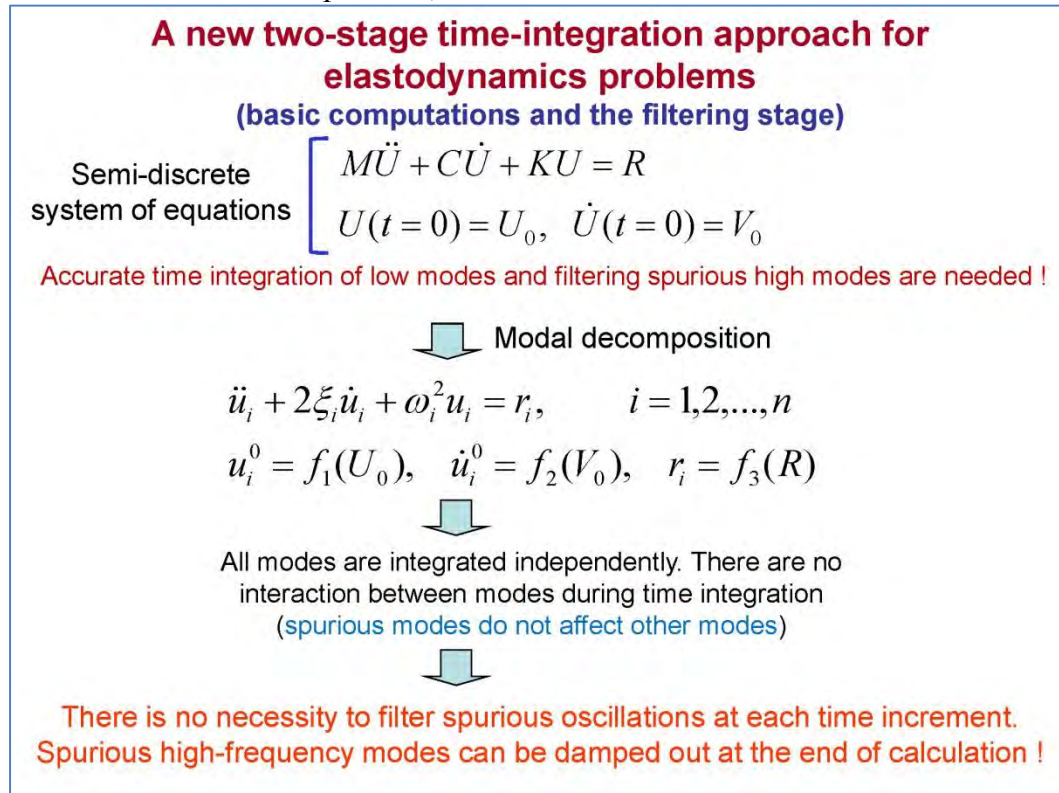


Fig. 3

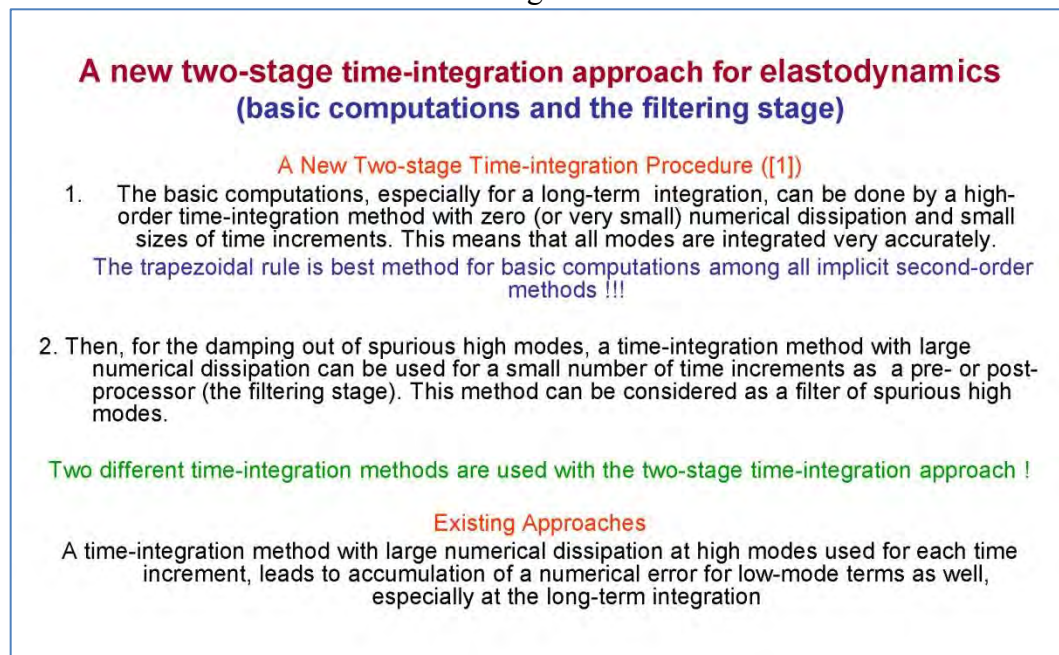


Fig. 4

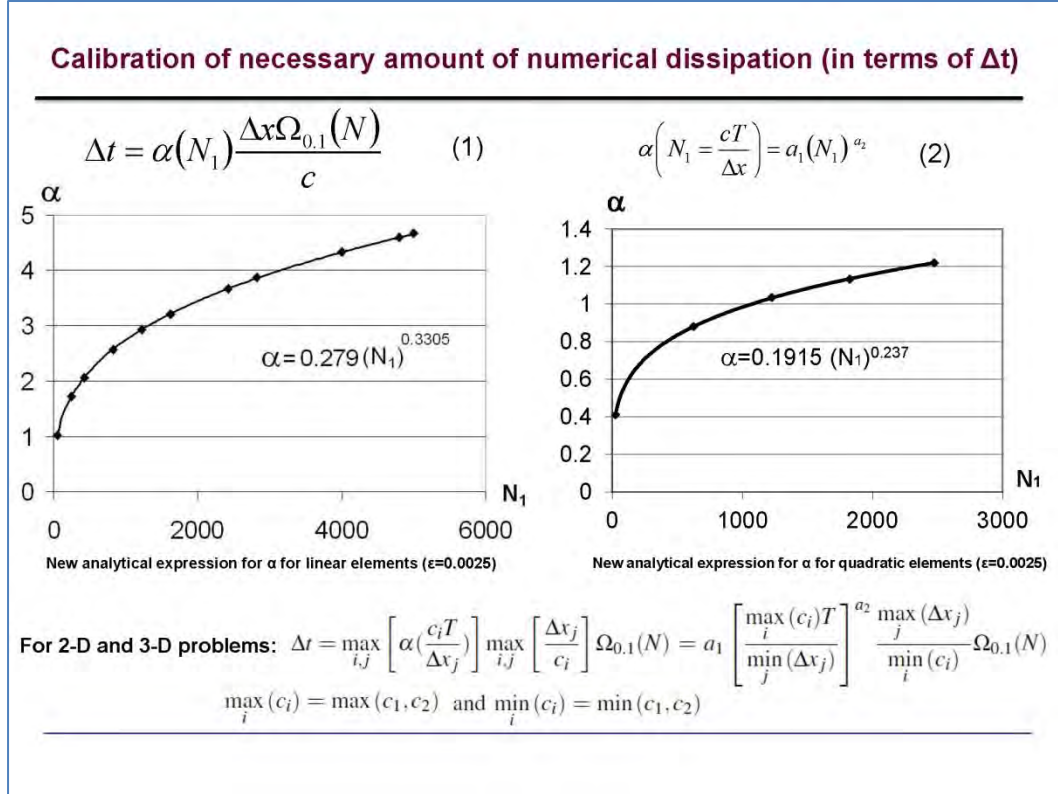


Fig. 5

found the analytical expression for the range of spurious frequencies in terms of the size of time increments at the filtering stage. Because the range of spurious frequencies depends on the mesh size and the observation time and is independent of the initial and boundary conditions, the analytical expression for the range of spurious frequencies is applicable to all elastodynamics and acoustics problems. This expression includes two parameters  $a_1$  and  $a_2$  that depend on the type and the order of the space discretization method; see Fig. 5. The values of the parameters  $a_1$  and  $a_2$  for the low- and high-order finite elements, spectral elements, isogeometric elements and the linear finite elements with reduced dispersion are determined and shown in Fig. 6. It should be mentioned that the two-stage time-integration approach with the filtering stage yields accurate convergent results at time and mesh refinements; e.g., see the numerical results for the 2-D impact problem in Fig. 7. Without the filtering stage, the error in velocities (or stresses) may exceed 300% and more; see Fig. 7.

# Coefficients $a_1$ and $a_2$ for different space-discretization methods [1]

(used for the calculation of the time increments at the filtering stage)

$$\Delta t = \alpha(N_1) \frac{\Delta x \Omega_{0.1}(N)}{c} \quad \alpha(N_1) = \alpha\left(\frac{cT}{\Delta x}\right) = a_1 \left(\frac{cT}{\Delta x}\right)^{a_2}$$

Types of Elements	Coefficients	1 <sup>st</sup>	2 <sup>nd</sup>	3 <sup>rd</sup>	4 <sup>th</sup>	5 <sup>th</sup>	10 <sup>th</sup>
Standard Finite Elements (Consistent mass matrix)	$a_1$	0.3574	0.3156	0.4485	0.5495	0.5115	
	$a_2$	0.3204	0.2364	0.1571	0.1111	0.1078	
Spectral Elements (Lumped mass matrix)	$a_1$	0.3342	0.448	0.5659	0.4790	0.4461	0.4317
	$a_2$	0.3363	0.1845	0.1139	0.1128	0.1073	0.0759
Isogeometric Elements (Consistent mass matrix)	$a_1$		0.2513	0.2311			
	$a_2$		0.2035	0.1508			
Linear Elements with Reduced Dispersion (Non-diagonal matrix)	$a_1$	0.2979					
	$a_2$	0.2074					
Linear Elements with Reduced Dispersion (Diagonal matrix)	$a_1$	0.3296					
	$a_2$	0.2180					

[1] A. V. Idesman, D. Pham, Accurate solutions of wave propagation problems under impact loading by the standard, spectral and isogeometric high-order finite elements. Comparative study of accuracy of different space-discretization techniques., *Finite Elements in Analysis and Design*, 88, 2014, pp. 67–89.

Fig. 6

## 2D impact axisymmetric problem: Convergence at mesh refinement using the new technique (with the filtering stage)

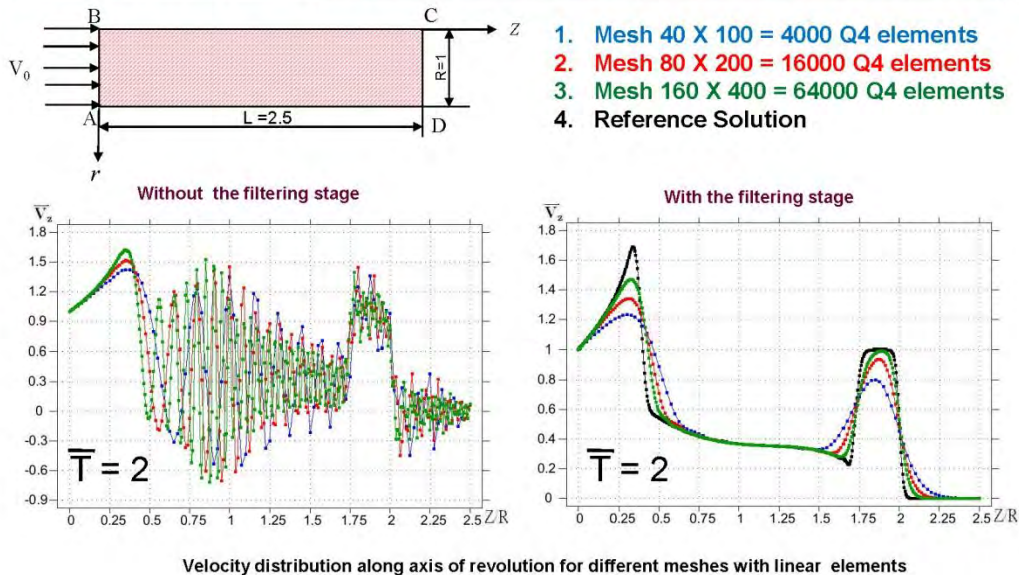


Fig. 7



Our publications with a detailed description of the results presented above.

1. Idesman A.V. , Accurate finite element modeling of wave propagation in composite and functionally graded materials, *Composite Structures*, 2014, 117, pp. 298-308.
2. Idesman A.V., Pham D., Foley J., Schmidt M. Accurate solutions of wave propagation problems under impact loading by the standard, spectral and isogeometric high-order finite elements. Comparative study of accuracy of different space-discretization techniques, *Finite Elements in Analysis and Design*, 2014, 88, pp. 67-89.
3. Idesman A.V., Pham D. , Accurate finite element modeling of acoustic waves, *Computer Physics Communications*, 2014, 185, pp. 2034-2045.
4. Idesman A.V., Mates S. P., Accurate finite element simulation and experimental study of elastic wave propagation in a long cylinder under impact loading, *International Journal of Impact Engineering*, 2014, 71, pp. 1-16.
5. Idesman A.V., Pham D., Finite element modeling of linear elastodynamics problems with explicit time-integration methods and linear elements with the reduced dispersion error, *Computer Methods in Applied Mechanics and Engineering*, 2014, 271 pp. 86-108.

## **2. New finite elements with the reduced dispersion error for explicit time-integration methods**

We have developed two finite element techniques with reduced dispersion for linear elastodynamics that are used with explicit time-integration methods. These techniques are based on the modified integration rule for the mass and stiffness matrices and on the averaged mass matrix approaches (see Fig. 8) that lead to the numerical dispersion reduction for linear finite elements from the second order to the fourth order. The analytical study of numerical dispersion for the new techniques has been carried out in the 1-D, 2-D and 3-D cases. The numerical study of the efficiency of the dispersion reduction techniques includes two-stage time-integration approach with the filtering stage that quantifies and removes spurious high-frequency oscillations from numerical results. We have found that in contrast to the standard linear elements with explicit time-integration methods and the lumped mass matrix, the finite element techniques with reduced dispersion yield more accurate results at small time increments (smaller than the stability limit) in the 2-D and 3-D cases. The recommendations for the selection of the size of time increments have been suggested. The new approaches with reduced dispersion can be easily implemented into existing finite element codes and lead to significant reduction in computation time by a factor of 10 – 1000 and more compared with the standard finite element formulations at a given accuracy. We should also mention that the formulations with reduced dispersion and the explicit time-integration methods can be directly and efficiently used (practically, without modifications) on parallel computers because the numerical algorithm includes the matrix and vector multiplications and can be performed at the element level without the solution of a system of algebraic equations.

## Modification of FE equations for explicit time-integration methods

$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{R}$  is replaced by  $\mathbf{D}\dot{\mathbf{V}} + \mathbf{K}\mathbf{U} = \mathbf{R}$ ,  $\mathbf{M}\mathbf{V} = \mathbf{D}\dot{\mathbf{U}}$   
or

$$\mathbf{D}\ddot{\mathbf{U}} + \mathbf{M}\mathbf{D}^{-1}\mathbf{K}\mathbf{U} = \mathbf{M}\mathbf{D}^{-1}\mathbf{R}$$

From the dispersion analysis (the central difference method):

□ Averaged mass matrix technique ([1])

$$\gamma = (3 - \bar{\tau}^2)/2$$

□ Modified integration rule technique ([1])

$$\alpha_M = \sqrt{(4 - \bar{\tau}^2)/3}, \quad \alpha_k = \sqrt{\frac{4(2\nu - 1)}{3(4\nu - 3)}}$$

where  $\bar{\tau} = c_2 \Delta t / dx$ ,  $c_2 = \sqrt{\frac{E}{2\rho(1+\nu)}}$ ,  $E, \nu, \rho$  are the material parameters

[1] Finite element modeling of linear elastodynamics problems with explicit time-integration methods and linear elements with the reduced dispersion error, Idesman A.V., Pham D., Computer Methods in Applied Mechanics and Engineering, 271, 2014, pp. 86-108.

Fig. 8

Our publications with a detailed description of the results presented above.

1. Idesman A.V., Pham D., Finite element modeling of linear elastodynamics problems with explicit time-integration methods and linear elements with the reduced dispersion error, Computer Methods in Applied Mechanics and Engineering, 2014, 271 pp. 86-108.
2. Idesman A.V., Pham D., Foley J., Schmidt M. Accurate solutions of wave propagation problems under impact loading by the standard, spectral and isogeometric high-order finite elements. Comparative study of accuracy of different space-discretization techniques, Finite Elements in Analysis and Design, 2014, 88, pp. 67-89.
3. Idesman A.V., Pham D., Accurate finite element modeling of acoustic waves, Computer Physics Communications, 2014, 185, pp. 2034-2045.

### 3. Comparison of accuracy of different space-discretization techniques used for wave propagation

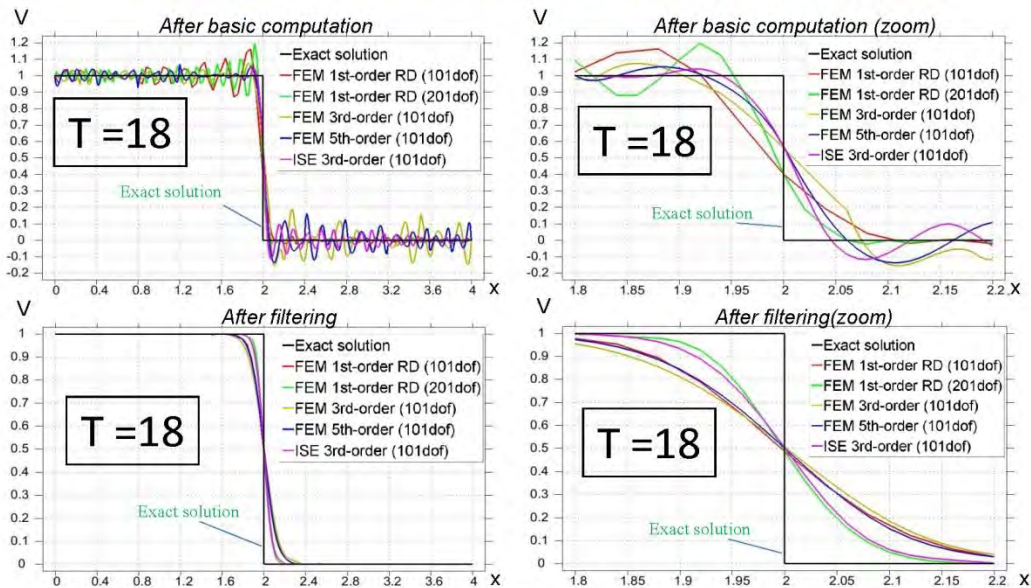
One issue with numerical solutions of wave propagation problems solved by different space-discretization techniques is the presence of spurious high-frequency oscillations. The range of spurious high frequencies is different for different space-discretization methods. We have not seen in the literature the numerical approaches that quantify and filter out all spurious oscillations from numerical solutions even solved by the popular space-discretization techniques

such as the standard, spectral and isogeometric high-order finite elements. We have resolved this issue by the application of the two-stage time-integration approach.

Another important result is related to the comparison of accuracy of different space-discretization techniques used for the solution of transient acoustics and elastodynamics problems. One way to compare the accuracy of these techniques is based on the evaluation of the numerical dispersion error. However, this error is defined for one selected mode of the elastodynamics system of equations and does not prescribe the combined effect of all modes on the accuracy. Moreover, the dispersion error does not estimate the effect of the observation time on the accuracy of numerical results. Therefore, for the comparison of the accuracy of different techniques, we solved the 1-D impact problem for which all modes are excited and which has a very simple analytical solution at short and long observation times. We should also mention that because the computational costs of different space-discretization techniques at the same number of degrees of freedom (dof) are different, then it is necessary to compare the efficiency of these techniques by the estimation of the computational costs at a given accuracy. The findings can be summarized as follows.

- The two-stage time-integration technique yields accurate numerical results for elastodynamics problems solved with different space-discretization approaches. For example, due to the spurious oscillations, we have not seen in the literature the accurate numerical solutions of elastodynamics problems at impact loading (or high-frequency loading) for the spectral and isogeometric elements as well as for the standard high-order finite elements even in the 1-D case. The applicability of the new approach to different space-discretization techniques is based on the fact that for all these techniques, lower frequencies are resolved more accurately than higher frequencies. Then, by the quantification of the inaccurate high frequencies (in terms of the coefficients  $a_1$  and  $a_2$ ) and by their filtering at the filtering stage, we obtain accurate numerical results without the spurious oscillations; see Figs. 9-10 (after filtering). The new coefficients  $a_1$  and  $a_2$  define different ranges of the spurious (inaccurate) frequencies for different space-discretization methods and are used for the filtering of the spurious oscillations in numerical solutions of the 1-D as well as multi-dimensional elastodynamics problems.
- As expected, at the same number of dof, the increase in the order of the standard finite elements, the spectral elements and the isogeometric elements leads to the increase in the accuracy of numerical results (however, the computational costs of higher-order elements are greater than that of lower-order elements at the same dof); see Figs. 9-10.
- Except the known case of the linear elements with the lumped mass matrix, other space-discretization techniques considered require small time-increments for time integration at the stage of basic computations. Moreover, the time increments should be decreased with the increase in the observation time. Even if for explicit time-integration methods the time increments close to the stability limit may yield accurate results at a small observation time, the time increments should be significantly decreased with the increase in the observation time; see Fig. 11. For example, for the 2nd-order time-integration methods, the size of time increments should be inversely proportional to the square root of the observation time (as predicted by the exact time error estimator developed in our paper); see Fig. 11. Similar to explicit time-integration methods, comparably small time

### Comparison of accuracy of different order elements with the consistent (non-diagonal) mass matrix that are used for elastodynamics

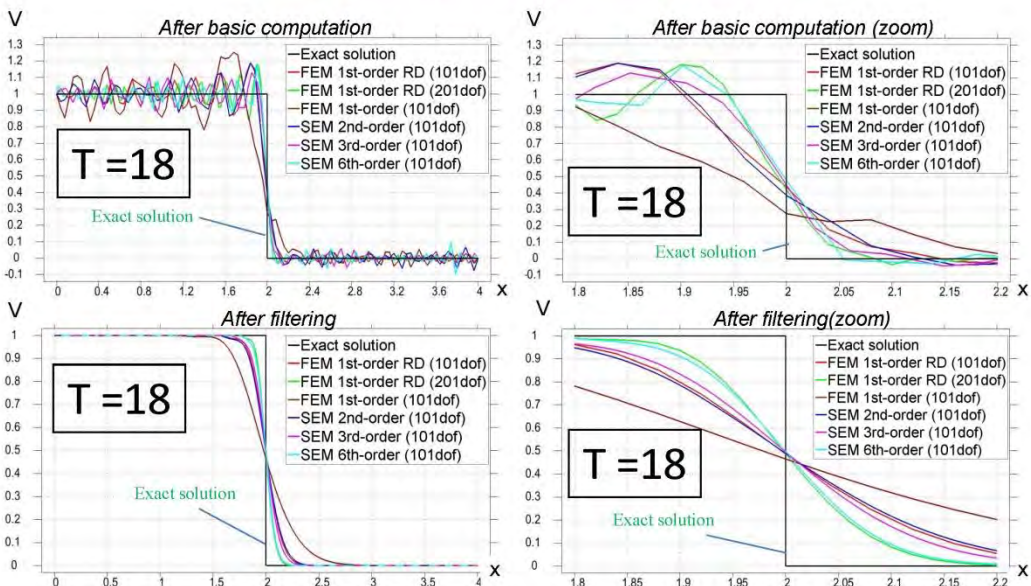


1D Impact Problem: Velocity distribution at time  $T=18$ .

Linear elements with reduced dispersion (RD) are more computationally effective than other.

Fig. 9

### Comparison of accuracy of different order elements with the diagonal mass matrix that are used for elastodynamics



1D Impact Problem: Velocity distribution at time  $T=18$ .

Linear elements with reduced dispersion (RD) are more computationally effective than other.

Fig. 10



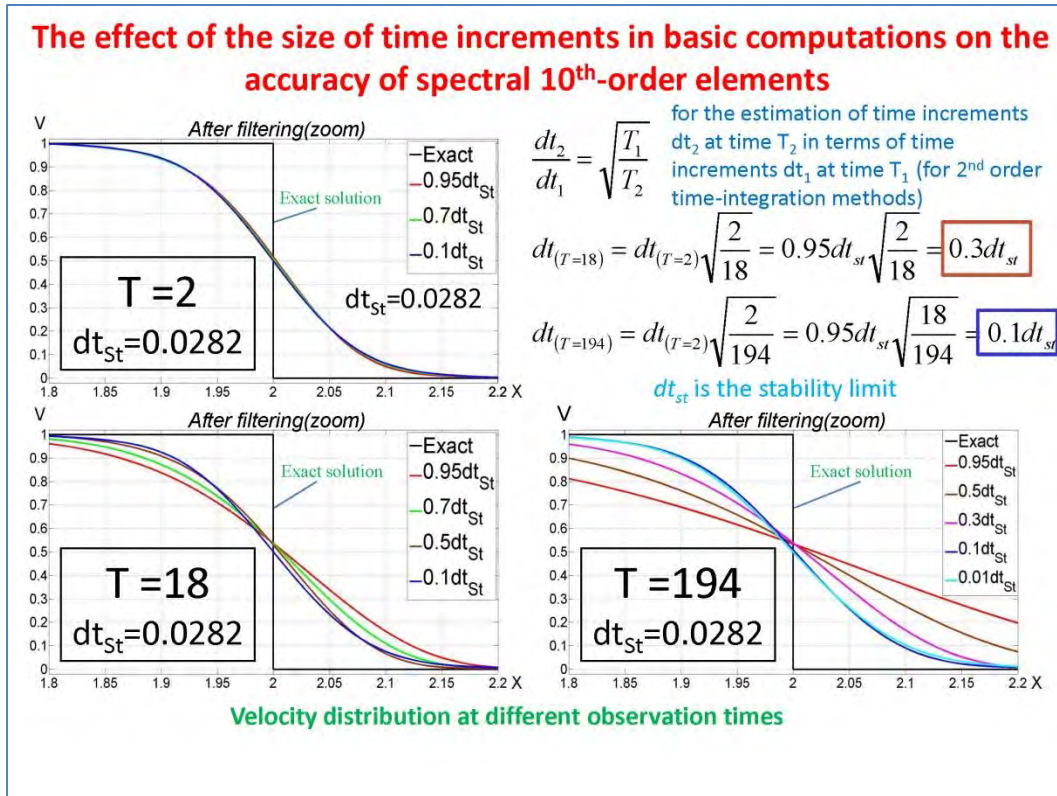


Fig. 11

increments should be used with implicit time-integration methods for the 1-D impact problem. According to our results, the typical statement in finite element textbooks, that for explicit methods a time increment should be close to the stability limit is not true, because the size of a time increment should depend on the observation time and should be much smaller than the stability limit at large observation times.

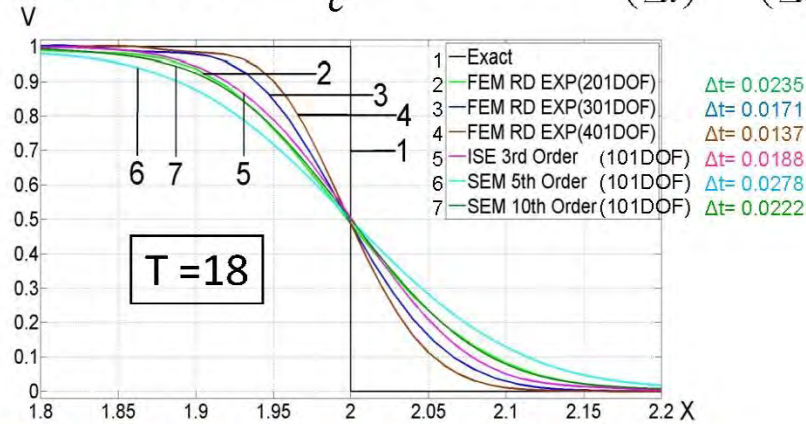
- The comparison of the space-discretization techniques based on the non-diagonal mass matrices (used with implicit time-integration methods) show that at the same number of dof, the isogeometric elements yield more accurate results compared with the standard high-order finite elements and the linear elements with reduced dispersion; see Fig. 9. However, when we compare the computational costs at a given accuracy, the numerical results show that the linear elements with reduced dispersion are more computationally efficient than other space-discretization techniques. We should also mention that compared with the isogeometric 3rd-order elements, the computational efficiency of the linear elements with reduced dispersion decreases with the increase in the observation time.
- The comparison of the space-discretization techniques based on the diagonal mass matrices (used with explicit time-integration methods) show that at the same number of dof, the spectral high-order elements yield more accurate results compared with the standard linear and quadratic finite elements and the linear elements with reduced dispersion; see Fig. 10. However, when we compare the computational costs at a given accuracy, the numerical results show the linear elements with reduced dispersion are

### Comparison of accuracy of different space-discretization techniques used for elastodynamics

The size  $\Delta t$  of time increments at the filtering stage as an indicator of accuracy

Smaller  $\Delta t \rightarrow$  Greater range of actual frequencies  $\rightarrow$  Greater accuracy  
(similar to the modal decomposition)

$$\Delta t(\Delta x, T) = \alpha(N_1) \frac{\Delta x \Omega_{0.1}(N)}{c}; \quad \alpha(N_1) = \alpha\left(\frac{cT}{\Delta x}\right) = a_1 \left(\frac{cT}{\Delta x}\right)^{a_2}$$



1D Impact Problem: Velocity distribution at time  $T=18$  after the filtering stage.

Fig. 12

more computationally efficient than the spectral high-order elements. We should also mention that compared with the spectral 10th-order elements, the computational efficiency of the linear elements with reduced dispersion decreases with the increase in the observation time.

- It is interesting to note that the size of time increments at the filtering stage of the two-stage time-integration technique (this size is calculated according the special formulas) defines the range of actual frequencies used in numerical solutions and can serve as a quantitative measure for the comparison and the prediction of the accuracy and the computational efficiency of different space-discretization techniques; see Fig. 12.
- The estimation and comparison of accuracy of different space-discretization techniques obtained for impact problems for which all frequencies of the semi-discrete equations are excited are also valid for any transient acoustics or elastodynamics problem for which only a part of frequencies of the semi-discrete equations is excited. We showed that the two-stage time-integration approach can be equally applied to wave propagation problems under impact loading as well as under low- and high-frequency loading (the same range of spurious high-frequencies should be filtered independent of applied loading). The comparative study of other space-discretization techniques (similar to that considered here) will help us determine the most computationally efficient technique for elastodynamics and acoustics.

Our publications with a detailed description of the results presented above.

1. Idesman A.V., Pham D., Foley J., Schmidt M. Accurate solutions of wave propagation problems under impact loading by the standard, spectral and isogeometric high-order finite elements. Comparative study of accuracy of different space-discretization techniques, *Finite Elements in Analysis and Design*, 2014, 88, pp. 67-89.
2. Idesman A.V., Pham D. , Accurate finite element modeling of acoustic waves, *Computer Physics Communications*, 2014, 185, pp. 2034-2045.
3. Idesman A.V., Pham D., Finite element modeling of linear elastodynamics problems with explicit time-integration methods and linear elements with the reduced dispersion error, *Computer Methods in Applied Mechanics and Engineering*, 2014, 271 pp. 86-108.

#### **4. Propagation of acoustic waves**

We have analyzed in more detailed the application of the new numerical approach to propagation of acoustic waves. For the space-discretization, we have used the standard linear finite elements as well as the linear elements with reduced dispersion. However, any other space-discretization techniques such as the spectral elements, the isogeometric elements, the boundary element method, different meshless methods and many others can be applied with the new approach suggested. The new findings can be summarized as follows.

In contrast to the results known from the literature, we use the dispersion analysis of the reduced dispersion technique and the standard finite element approach for transient acoustic problems in order to determine the effect of the size of time increments on the dispersion error. We have found that for linear finite elements the decrease in the size of time increments leads to the increase in the dispersion error for both techniques. Therefore, the time increments close to the stability limit in basic computations yield the most accurate numerical solutions. We should mention that these results are totally different from those for multi-dimensional elastodynamics problems for which smaller time increments in basic computations yield more accurate solutions for the linear elements with reduced dispersion.

- We have shown that for a suddenly applied load even the reduced dispersion technique with the time increments close to the stability limit leads to divergent results at mesh refinement; e.g., see Fig 14 (after basic computations). This is explained by the increase in the amplitudes of spurious oscillations at mesh refinement. Therefore, in order to get convergent results, we have modified and applied the two-stage time integration technique to acoustic problems. This technique allows the quantification of the range of spurious oscillations and their filtering from numerical solutions for different space-discretization methods at the filtering stage. For the first time we have shown that the application of the two-stage time integration approach yields accurate convergent results

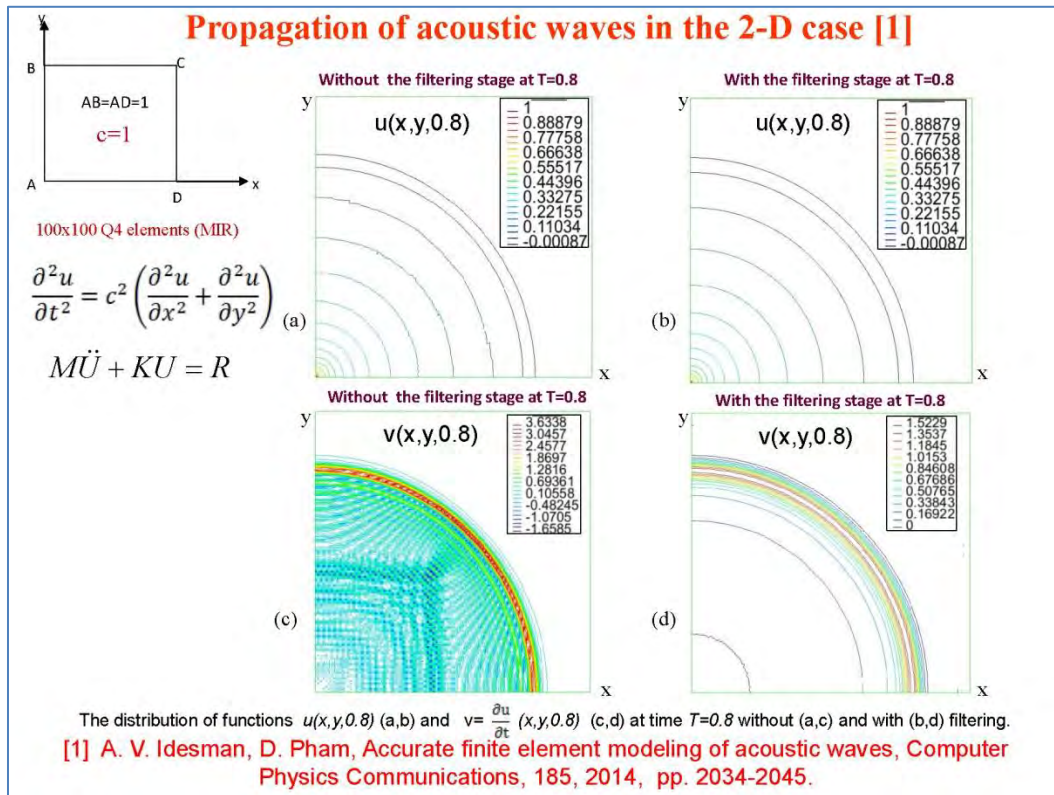


Fig. 13

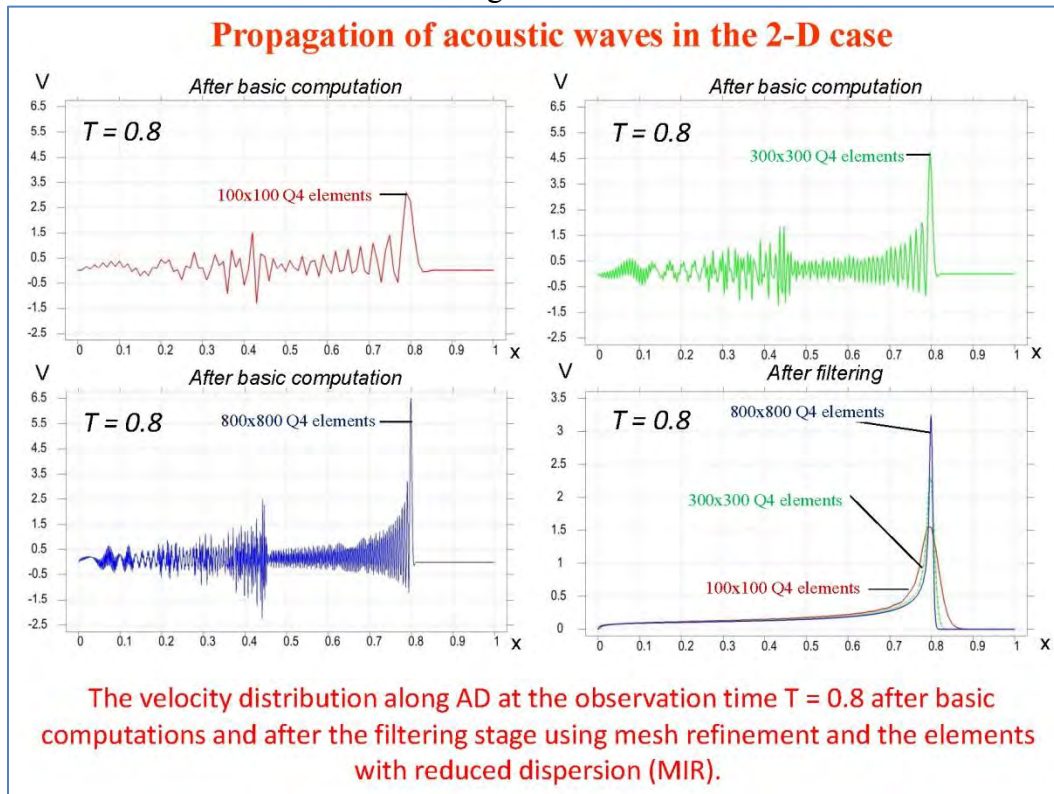


Fig. 14



at mesh refinement; e.g., see 14 (after filtering). It is also interesting to note that along with the damping of spurious oscillations, the filtering stage significantly reduces the numerical anisotropy of solutions; e.g., see Fig. 13.

- In paper by B. Yue, M.N. Guddati, J. Acoust. Soc. Am. 118 (2005) 2132–2141, the reduced dispersion technique and the standard technique with the lumped mass matrix are compared at the same time increments (corresponding to the Courant number  $\tau=0.75$ ) which are close to the stability limit for the reduced dispersion technique but are smaller than the stability limit ( $\tau=1$ ) for the standard approach. Therefore, this comparison cannot be considered as the illustration of the advantage of the reduced dispersion technique because the time increments close to the stability limit ( $\tau=1$ ) significantly improve the accuracy of the standard approach. We have compared these techniques for the time increments close to the stability limit for each technique. It is interesting to note that along the Cartesian axes, the reduced dispersion technique and the standard technique yield approximately the same accuracy. However, the numerical solution for the standard technique is inaccurate in other directions as predicted by the dispersion analysis. After the filtering of spurious oscillations in all directions, we showed that the reduced dispersion technique is much more accurate than the standard approach.

Our publications with a detailed description of the results presented above.

1. Idesman A.V., Pham D. , Accurate finite element modeling of acoustic waves, Computer Physics Communications, 2014, 185, pp. 2034-2045.
2. Idesman A.V., Pham D., Foley J., Schmidt M. Accurate solutions of wave propagation problems under impact loading by the standard, spectral and isogeometric high-order finite elements. Comparative study of accuracy of different space-discretization techniques, Finite Elements in Analysis and Design, 2014, 88, pp. 67-89.
3. Idesman A.V., Pham D., Finite element modeling of linear elastodynamics problems with explicit time-integration methods and linear elements with the reduced dispersion error, Computer Methods in Applied Mechanics and Engineering, 2014, 271 pp. 86-108.

## 5. Wave propagation in inhomogeneous materials.

We have extended the new technique to wave propagation in inhomogeneous materials including composite and functionally graded materials and we have shown the main differences for the modeling of wave propagation in inhomogeneous and homogeneous materials. Below we will summarize the main findings.

- For the special design of finite element meshes in the 1-D case, we can obtain very accurate results for wave propagation problems in inhomogeneous materials if we use the standard linear elements with the lumped mass matrix and the explicit central difference method with the time increments equal to the stability limit. In this case, there are no spurious oscillations in the numerical solutions after basic computations (the filtering stage is not required) and a discontinuity in a solution (if presented) is spread over three nodes. In contrast to the finite element formulations with graded finite elements used in the literature for such problems, we use a piecewise constant variation of materials properties (they are constant within any finite element). Nevertheless, even for functionally graded materials the numerical results converge very fast to the exact solution at mesh refinement. It is interesting to mention that in the 1-D case the analytical solution to wave propagation in functionally graded materials can be obtained by the approach developed by Chiu T-C, Erdogan F. (One-dimensional wave propagation in a functionally graded elastic medium. *J Sound Vib* 1999;222(3):453–87). However, this analytical solution is based on the infinite series and may lead to spurious oscillations due to Gibbs phenomena when a finite number of terms is used in the series. In contrast to this case, the numerical solutions based on the special non-uniform meshes with standard linear finite elements, on the lumped mass matrix and on the explicit central-difference method with the time increments equal to the stability limit do not include spurious oscillations at any number of degrees of freedom. They can be used as reference solutions for testing computer codes as well as for the analysis of wave propagation phenomena under high-frequency and impact loadings.
- Except one case described above, in all other cases (e.g., uniform meshes or the non-lumped mass matrix or smaller time increments or other time-integration methods or high-order finite elements or the 2-D and 3-D problems etc.) numerical solutions obtained by existing methods may include large spurious high-frequency oscillations especially under impact loading. However, the new numerical approach based on the two-stage time-integration technique (with basic computations and the filtering stage) quantifies and removes the spurious oscillations for these cases. For the first time we have obtained accurate finite element solutions of wave propagation problems in composite and functionally graded materials without spurious oscillations and without the interaction between user and computer code; see Figs. 15-17.

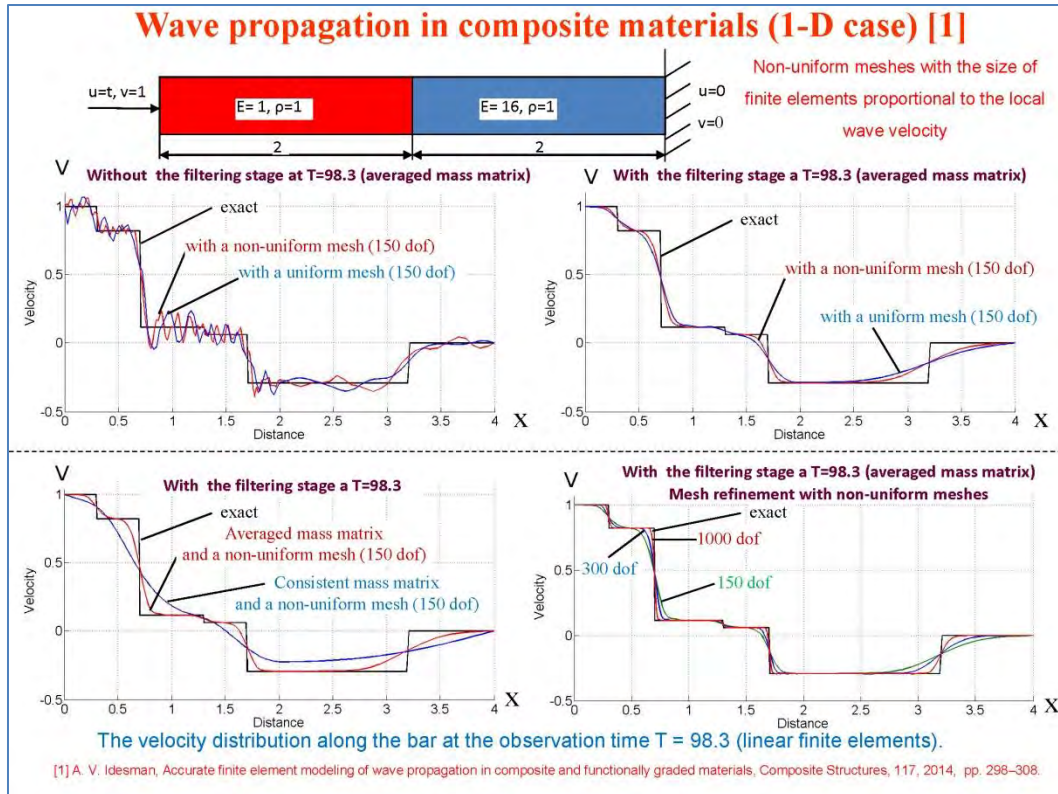


Fig. 15

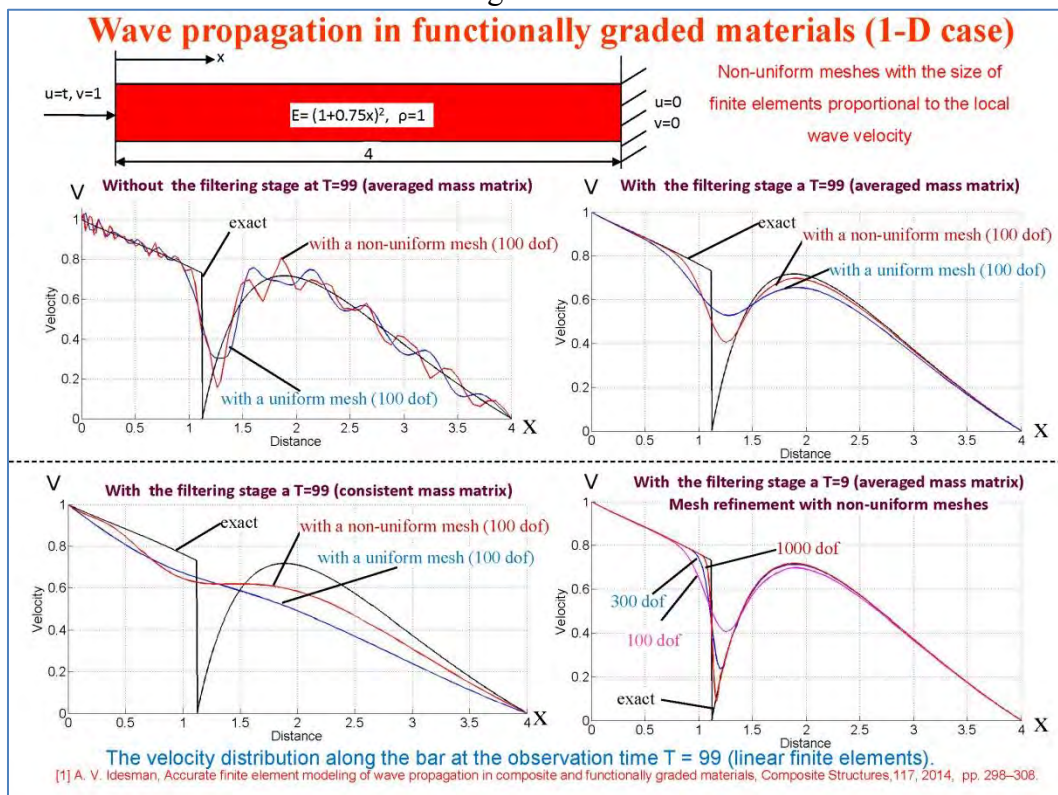


Fig. 16

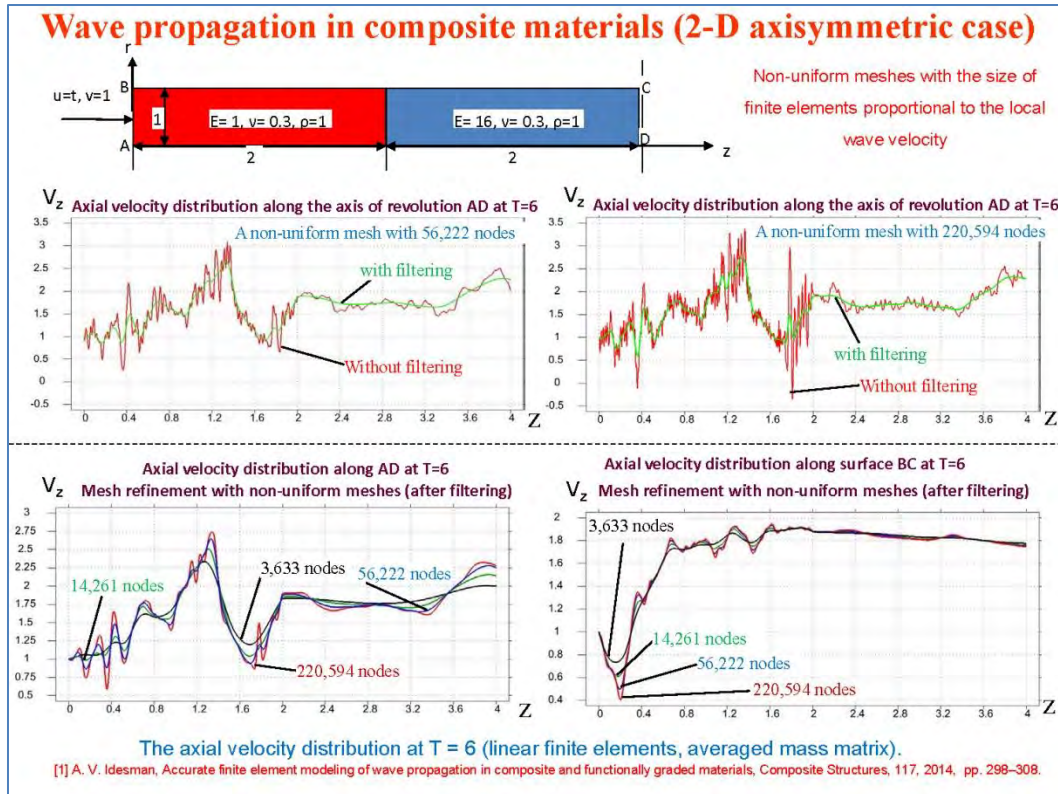


Fig. 17

- In many known papers on the simulation of the dynamic response of structures made of inhomogeneous materials, uniform meshes or meshes independent of material properties are used with the standard approaches with explicit or implicit time-integration methods. We have shown that despite uniform meshes are justified for homogeneous materials, non-uniform meshes with the constant ratios of the local wave velocity to the size of the finite element yield much more accurate results than uniform meshes do; see Figs. 15-16.
- We have shown that for inhomogeneous materials (similar to homogeneous materials), the finite element formulation with reduced dispersion (based on the averaged mass matrix and the linear elements) yields much more accurate results than the standard formulations with the lumped or consistent mass matrices do; see Figs. 15-16. There is one exception for the case of the lumped mass matrix with the non-uniform mesh with linear elements and the explicit central difference method with the time increments equal to the stability limit. However, this exception is valid in the 1-D case only.
- We have shown that the size of time increments at the filtering stage of the two-stage time-integration technique defines the range of actual frequencies used in numerical solutions and can serve as a quantitative measure for the comparison and the prediction of the accuracy and the computational efficiency of different space-discretization techniques used for wave propagation in inhomogeneous materials. The smaller size of time increments at the filtering stage calculated according to the special formulas corresponds to more accurate numerical solutions.



Our publications with a detailed description of the results presented above.

1. Idesman A.V. , Accurate finite element modeling of wave propagation in composite and functionally graded materials, Composite Structures, 2014, 117, pp. 298-308.

## 6. Propagation of high-frequency pulses in the Hopkinson Pressure Bar

We have considered the modeling of wave propagation in the components of the Hopkinson Bar. For the first time, detailed accurate numerical solutions for elastic wave propagation in a long axisymmetric elastic bar under impact loading are obtained using the new finite element technique. In contrast to known numerical techniques, the new numerical approach quantifies and removes spurious high-frequency oscillations which may invalidate numerical results in impact loading simulations. The comparison of the accurate experimental results (by Dr. Mates from NIST) for the impact of striker and incident bars with the corresponding accurate numerical results (see Fig. 18) allows us to explain some details of elastic wave propagation in long bars.

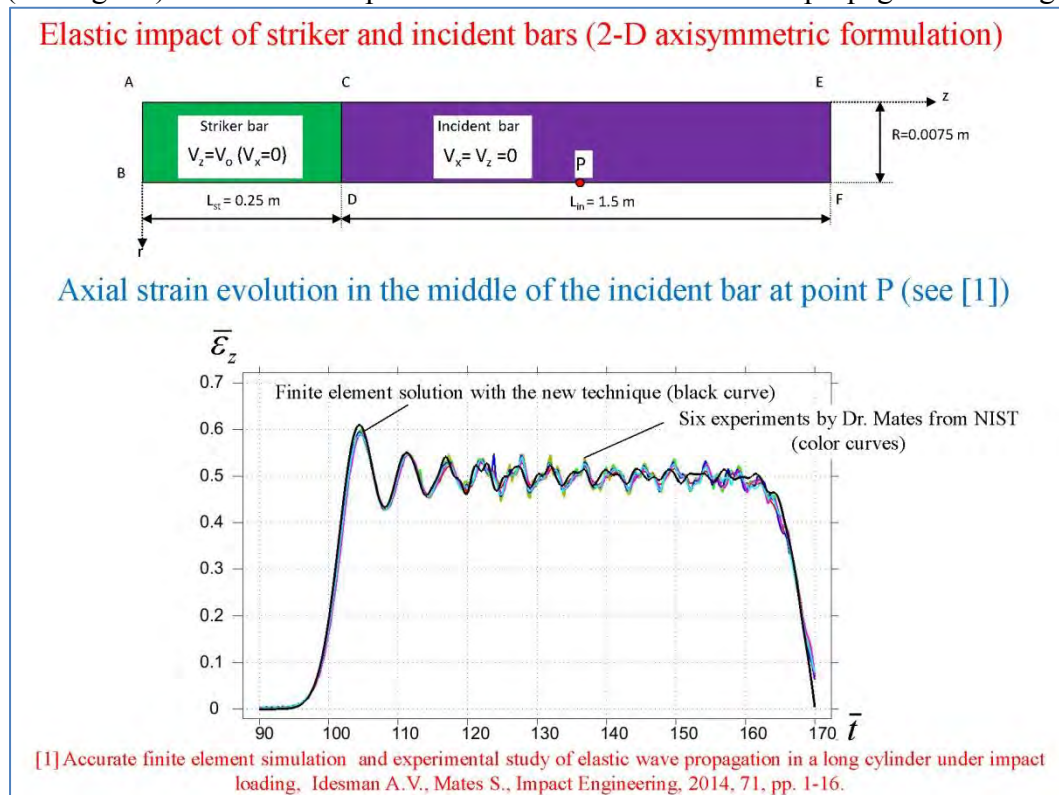


Fig. 18

For example, due to the absence of very high frequencies in the obtained experimental results, the mathematical formulation of the problem should include physical damping for the corresponding range of high frequencies. This range can be defined by the filtering stage of the new approach in terms of the number of finite elements along the radial direction of the bar. By the variation of this number we can fit the experimental curves with the numerical results obtained by the new numerical technique. However, for the accurate numerical solution of the impact problem with zero physical damping, the number of elements in the radial direction

should be large. By the comparison of the numerical and experimental data, we can accurately determine the longitudinal wave velocity from the experiments. The accurate numerical solutions also allow the analysis of the uniformity of the different strain and velocity components across the radius at different distances from the impact face. The validity of some assumptions used in the 1-D theory for wave propagation in long bars has been also checked by the use of the accurate numerical solution. We have also shown that at the elastic impact the known dispersion-correction technique used for the description of the shape of the wave pulse at different locations along the axisymmetric bar is inaccurate for the prediction of pulses close to the impact face.

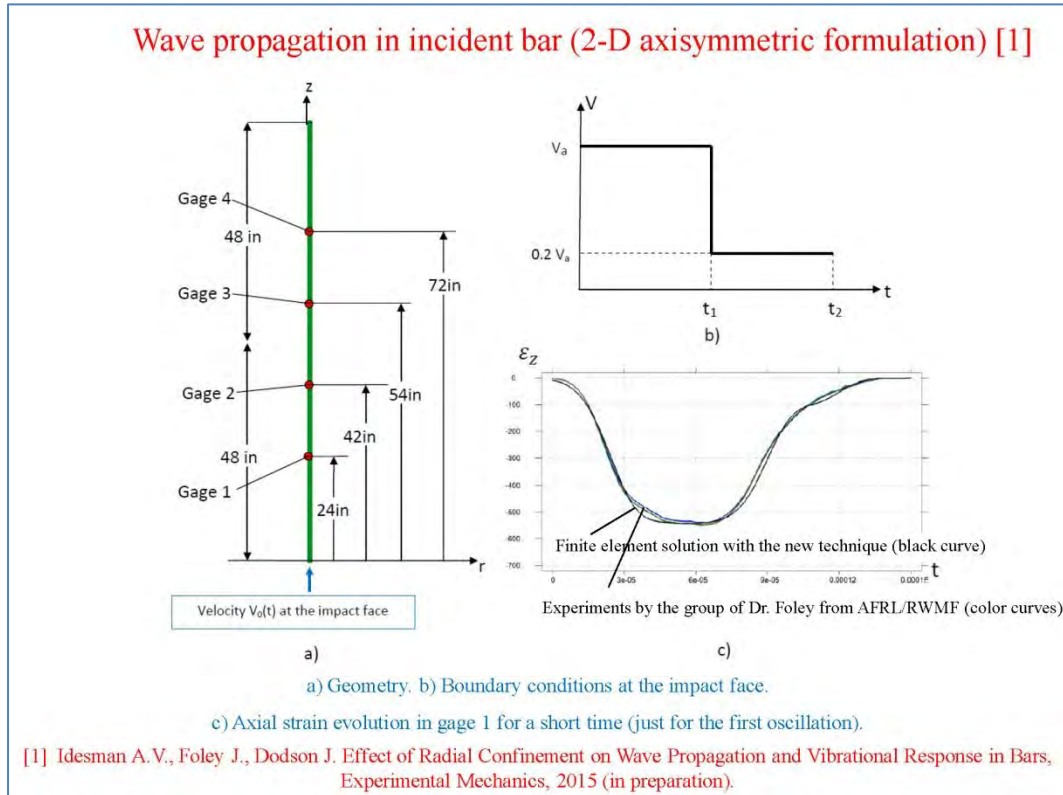


Fig. 19

We have also simulated the experiments on wave propagation in long bars under impact loading. These experimental data have been provided by the group of Dr. Foley from AFRL/RWMF, Eglin. The experimental results included the evolution of the axial velocity at four gages located along an incident bar; see Figs. 19, 20. In order to simplify the simulations, we have not considered the contact interaction of a striker bar with the long incident bar. The action of the striker bar we have replaced by the following boundary conditions at the impact face: the normal velocity  $v_z(r,0,t) = V_0(t)$  and zero tangential tractive forces are applied until time  $t < t_2$  (we matched the numerical and experimental results for the first oscillation in gage 1 by an appropriate selection of  $V_0(t)$ ; see Fig. 19) and the zero normal and tangential tractive forces are applied after time  $t > t_2$ . The problem was solved by the new numerical technique. We have obtained a surprisingly good agreement between the experiments and the calculations not only for large oscillations but also for the following small oscillations; see color and black curves in Fig. 20. We should emphasize that in our approach we assigned the velocity  $V_0(t)$  at the impact face that allowed us to fit the experimental and numerical results just for the first big oscillations in gage 1 for a short time; see Fig. 19. Then, all other numerical results at time  $t > 0.00015$  s in

gage 1 and the results in gages 2-4 represent an accurate numerical solution of the wave propagation problem for the given boundary conditions; see Fig. 20. By the numerical modeling we could explain such interesting experimental results as a small increase in the amplitudes of big oscillations as well as the appearance of new small oscillations with the increase in the observation time (the effect of high frequencies and physical dispersion); see Fig. 20.

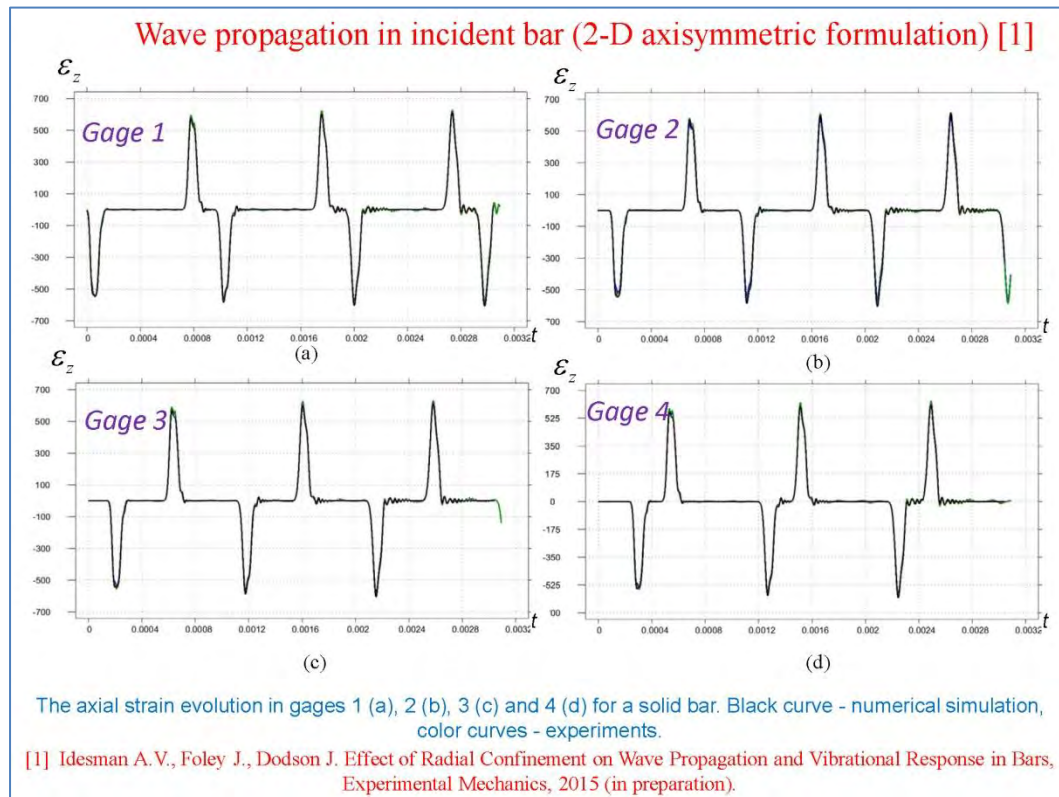


Fig. 20

Our publications with a detailed description of the results presented above.

1. Idesman A.V., Mates S. P., Accurate finite element simulation and experimental study of elastic wave propagation in a long cylinder under impact loading, *International Journal of Impact Engineering*, 2014, 71, pp. 1-16.
2. Idesman A.V., Foley J., Dodson J. Effect of Radial Confinement on Wave Propagation and Vibrational Response in Bars, *Experimental Mechanics*, 2015 (in preparation).

## 7. Activities supported by the grant

### Students

Three PhD students (one of them graduated in December 2013).

### Publications

1. Idesman A.V. , Accurate finite element modeling of wave propagation in composite and functionally graded materials, *Composite Structures*, 2014, 117, pp. 298-308.
2. Idesman A.V., Pham D., Foley J., Schmidt M. Accurate solutions of wave propagation problems under impact loading by the standard, spectral and isogeometric high-order finite elements. Comparative study of accuracy of different space-discretization techniques, *Finite Elements in Analysis and Design*, 2014, 88, pp. 67-89.
3. Idesman A.V., Pham D. , Accurate finite element modeling of acoustic waves, *Computer Physics Communications*, 2014, 185, pp. 2034-2045.
4. Idesman A.V., Mates S. P., Accurate finite element simulation and experimental study of elastic wave propagation in a long cylinder under impact loading, *International Journal of Impact Engineering*, 2014, 71, pp. 1-16.
5. Idesman A.V., Pham D., Finite element modeling of linear elastodynamics problems with explicit time-integration methods and linear elements with the reduced dispersion error, *Computer Methods in Applied Mechanics and Engineering*, 2014, 271 pp. 86-108.
6. Idesman A.V., A New Numerical Technique with Reduced Dispersion for Wave Propagation Problems. Application to Isogeometric Elements, *Computer Methods in Applied Mechanics and Engineering*, 2015, pp. 1-21 (under review).
7. Idesman A.V., Dey A., A new approach for the reduction of the dispersion error in high-order numerical methods for wave propagation problems, *Computational Physics*, 2015 (under review)
8. Idesman A.V., Foley J., Dodson J. Effect of Radial Confinement on Wave Propagation and Vibrational Response in Bars, *Experimental Mechanics*, 2015 (in preparation).
9. Idesman A.V., Bhuiyan A., Foley J. Accurate Finite Element Modeling of Stresses for Stationary Cracks Under Impact Loading, *International Journal of Solids and Structures*, 2015 (in preparation).

### Presentations

- ***Semi-plenary lecture:*** at the 5th ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering, Greece, May 26, 2015.
- ***Three Keynote lectures:*** at the 11th World Congress on Computational Mechanics (July 21, 2014)
- ***at the 12<sup>th</sup> and 13th U.S. National Congress on Computational Mechanics*** (July 24, 2013, July 29, 2015),
- ***Twelve Invited lectures:*** U.S. Army Engineer Research and Development Center, Vicksburg, MS (August 20, 2015), Air Force Research Lab, Eglin (July 2, 2015 and



June 21, 2014), **ABAQUS**, Providence (October 10, 2014), **Stanford University** (March 31, 2015), **University of California, Berkeley** (March 30, 2015), **U.S. Army Research Laboratory**, Aberdeen (May 20, 2014), **University of Texas at Austin** (March 27, 2014), **U.S. Army Engineer Research and Development Center, Hanover, NH** (August 7, 2013), **Naval Undersea Warfare Center**, Newport, RI (August 6, 2013), **Los Alamos National Laboratory** (June 25, 2013), **Ruhr University, Bochum, Germany**, (January 7, 2013)

- Organizer (co-organizer) of five mini-symposia at: **the 12<sup>th</sup> and 13th U.S. National Congress on Computational Mechanics** (July 2013 and July 2015), **the 11th World Congress on Computational Mechanics, Spain** (July 2014), **the 4<sup>th</sup> and 5<sup>th</sup> ECCOMAS Thematic Conferences on Computational Methods in Structural Dynamics and Earthquake Engineering, Greece** (June 2013 and May 2015)

1.

**1. Report Type**

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**Grant/Contract Title**

The full title of the funded effort.

A New Fast, Accurate and Non-Oscillatory Numerical Approach for Wave Propagation Problems in Solids.  
Application to High-frequency Pulse Propagation in the Hopkinson Pressure Bar.

**Grant/Contract Number**

AFOSR assigned control number. It must begin with "FA9550" or "F49620" or "FA2386".

FA9550-12-1-0324

**Principal Investigator Name**

The full name of the principal investigator on the grant or contract.

DR ALEXANDER IDESMAN

**Program Manager**

The AFOSR Program Manager currently assigned to the award

DR JEAN-LUC CAMBIER

**Reporting Period Start Date**

06/15/2012

**Reporting Period End Date**

06/14/2015

**Abstract**

We have developed an effective two-stage time-integration technique for elastodynamics and acoustic wave propagation problems solved with explicit and implicit time-integration methods and different space-discretization methods. For the first time, we have quantified the range of spurious oscillations for different space-discretization methods and have effectively filtered the spurious oscillations at the filtering stage. We have also developed an analytical procedure for the comparison and prediction of accuracy of different space-discretization methods used for wave propagation. The study of the effect of time increments on the accuracy of numerical results obtained by different space-discretization methods showed that except one particular case, the time increments should be much smaller than the stability limit at large observation times and the result can be predicted by the exact time error estimator. These results contradict the typical statement in finite element textbooks that the time increments should be close to the stability limit. We have developed new finite elements with reduced dispersion for explicit time-integration methods as well as an analytical procedure for the selection of the size of time increments for the stage of basic computations and the filtering stage for the new finite elements with reduced numerical dispersion. The solution of 1D, 2D and 3D benchmark wave propagation problems showed that the new technique yields accurate and non-oscillatory results without interaction between computer code and user and significantly reduces the

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computation time by a factor of 10–1000 and more compared with the standard finite element approaches. A surprisingly good agreement between the experiments by group of Dr. Foley from AFRL/RWMF, Eggen and the simulations with the new numerical technique has been obtained for wave propagation under impact loading in the components of the Hopkinson Pressure Bar. The numerical results also explain the effect of high frequencies on wave propagation in these experiments.

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1. Idesman A.V., Accurate finite element modeling of wave propagation in composite and functionally graded materials, *Composite Structures*, 2014, 117, pp. 298-308.
2. Idesman A.V., Pham D., Foley J., Schmidt M. Accurate solutions of wave propagation problems under impact loading by the standard, spectral and isogeometric high-order finite elements. Comparative study of accuracy of different space-discretization techniques, *Finite Elements in Analysis and Design*, 2014, 88, pp. 67-89.
3. Idesman A.V., Pham D., Accurate finite element modeling of acoustic waves, *Computer Physics Communications*, 2014, 185, pp. 2034-2045.
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6. Idesman A.V., A New Numerical Technique with Reduced Dispersion for Wave Propagation Problems. Application to Isogeometric Elements, *Computer Methods in Applied Mechanics and Engineering*, 2015, pp. 1-21 (under review).
7. Idesman A.V., Dey A., A new approach for the reduction of the dispersion error in high-order numerical methods for wave propagation problems, *Computational Physics*, 2015 (under review)
8. Idesman A.V., Foley J., Dodson J. Effect of Radial Confinement on Wave Propagation and Vibrational Response in Bars, *Experimental Mechanics*, 2015 (in preparation).
9. Idesman A.V., Bhu Yan A., Foley J. Accurate Finite Element Modeling of Stresses for Stationary Cracks Under Impact Loading, *International Journal of Solids and Structures*, 2015 (in preparation).

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**Changes in research objectives (if any):**

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Yes. Former: DR FARIBA FAHROO

Current: DR JEAN-LUC CAMBIER

**Extensions granted or milestones slipped, if any:**

NO

**AFOSR LRIR Number**

**LRIR Title**

**Reporting Period**

**Laboratory Task Manager**

**Program Officer**

**Research Objectives**

**Technical Summary**

**Funding Summary by Cost Category (by FY, \$K)**

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Equipment/Facilities			
Supplies			
Total			

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