

# Using Neural Network Parametrizations of the Nonlinear Energy Transfer for Application in Wave Models

Hendrik L. Tolman

Ocean Modeling Branch, Environmental Modeling Center,  
National Centers for Environmental Prediction, NWS, NOAA,  
5200 Auth Rd., Camp Springs, MD

phone: (301)763-8000 x 7253 fax: (301)763-8545 email: [Hendrik.Tolman@noaa.gov](mailto:Hendrik.Tolman@noaa.gov)

Dmitry V. Chalikov

Ocean Modeling Branch, Environmental Modeling Center,  
National Centers for Environmental Prediction, NWS, NOAA,  
5200 Auth Rd., Camp Springs, MD

phone: (301)763-8000 x 7239 fax: (301)763-8545 email: [Dmitry.Chalikov@noaa.gov](mailto:Dmitry.Chalikov@noaa.gov)

Vladimir M. Krasnopolsky

Ocean Modeling Branch, Environmental Modeling Center,  
National Centers for Environmental Prediction, NWS, NOAA,  
5200 Auth Rd., Camp Springs, MD

phone: (301)763-8000 x 7262 fax: (301)763-8545 email: [Vladimir.Krasnopolsky@noaa.gov](mailto:Vladimir.Krasnopolsky@noaa.gov)

Award Number: N00014-00-0332

<http://polar.wwb.noaa.gov/omb/papers/tn192/NNIA.html>

## LONG-TERM GOALS

Considering the crucial importance of nonlinear interaction  $S_{nl}$  for the development of third generation wave models the long-term goal of this work is to improve accuracy of calculating nonlinear interaction  $S_{nl}$  in wind wave models, and hence improving wave prediction in general.

## OBJECTIVES

The objective of this work is to develop a *computationally cheap yet accurate approximation* for  $S_{nl}$ .

## APPROACH

The approach is based on the neural networks (NN) technique. It is used to accelerate the calculations and improve the accuracy of the parameterization of nonlinear interaction  $S_{nl}$ . The nonlinear interaction source term can be considered as a nonlinear mapping between a source term  $S_{nl}$  and a spectrum  $F$

$$S_{nl} = T(F), \quad (1)$$

where  $T$  is the exact nonlinear operator given by the full Boltzmann interaction integral [1]. This algorithm is a factor  $10^4$  too expensive for use in operational wave models. We intend to use the NN technique to produce a cheap and accurate alternative approach.

# Report Documentation Page

Form Approved  
OMB No. 0704-0188

Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.

1. REPORT DATE <b>30 SEP 2001</b>		2. REPORT TYPE		3. DATES COVERED <b>00-00-2001 to 00-00-2001</b>	
4. TITLE AND SUBTITLE <b>Using Neural Network Parametrizations of the Nonlinear Energy Transfer for Application in Wave Models</b>				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S)				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) <b>Ocean Modeling Branch, Environmental Modeling Center,,National Centers for Environmental Prediction, NWS, NOAA,5200 Auth Rd.,,Camp Springs,,MD</b>				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT <b>Approved for public release; distribution unlimited</b>					
13. SUPPLEMENTARY NOTES					
14. ABSTRACT <b>Considering the crucial importance of nonlinear interaction S<sub>nl</sub> for the development of third generation wave models the long-term goal of this work is to improve accuracy of calculating nonlinear interaction S<sub>nl</sub> in wind wave models, and hence improving wave prediction in general.</b>					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT <b>Same as Report (SAR)</b>	18. NUMBER OF PAGES <b>7</b>	19a. NAME OF RESPONSIBLE PERSON
a REPORT <b>unclassified</b>	b ABSTRACT <b>unclassified</b>	c THIS PAGE <b>unclassified</b>			

NNs are a generic tool for fast and accurate approximation of continuous mappings and, therefore, can be used to replace the exact algorithm. In order to convert the mapping (1) to a continuous mapping of two finite vectors (independent of the actual spectral discretization), the spectrum  $F$  and source function  $S_{nl}$  are expanded using systems of two-dimensional functions each of which ( $\Phi_i$  and  $\Psi_q$ ) creates a complete and orthogonal two-dimensional basis

$$F \approx \sum_{i=1}^n x_i \Phi_i, \quad S_{nl} \approx \sum_{q=1}^m y_q \Psi_q, \quad (2)$$

where for  $x_i$  and  $y_q$  we have

$$x_i = \iint F \Phi_i, \quad y_q = \iint S_{nl} \Psi_q, \quad (3)$$

where the double integral identifies integration over the spectral space. Because both sets of basis functions  $\{\Phi_i\}_{i=1,\dots,\infty}$  and  $\{\Psi_q\}_{q=1,\dots,\infty}$  are complete, increasing  $n$  and  $m$  in (2) improves the accuracy of approximation, and any spectrum  $F$  and source function  $S_{nl}$  can be approximated by (2) with a required accuracy. Substituting (2) into Eq. (3) we can get

$$\mathbf{Y} = T(\mathbf{X}), \quad (4)$$

which represents a continuous mapping of the finite vectors  $\mathbf{X} \in \mathfrak{R}^n$  and  $\mathbf{Y} \in \mathfrak{R}^m$ , and where  $T$  still represents the full nonlinear interaction operator. As described in the previous section, this operator can be approximated with a NN with  $n$  inputs and  $m$  outputs and  $k$  neurons in the hidden layer

$$\mathbf{Y} \approx T_{NN}(\mathbf{X}). \quad (5)$$

The accuracy of this approximation ( $T_{NN}$ ) is determined by  $k$ , and can generally be improved by increasing  $k$ .

To train the NN approximation  $T_{NN}$  of  $T$ , a training set has to be created which consists of pairs of vectors  $\mathbf{X}$  and  $\mathbf{Y}$ . To create this training set, a representative set of spectra  $F_p$  has to be generated with corresponding (exact) interactions  $S_{nl,p}$ . For each pair  $(F, S_{nl})_p$ , the corresponding vectors  $(\mathbf{X}, \mathbf{Y})_p$  are determined using Eq. (3). All pairs of vectors are then used to train the NN to obtain  $T_{NN}$ .

After  $T_{NN}$  has been obtained by training, the resulting NN Interaction Approximation (NNIA) algorithm consists of three steps :

- (1) Decompose  $F$  by applying Eq. (3) to calculate  $\mathbf{X}$ .
- (2) Estimate  $\mathbf{Y}$  from  $\mathbf{X}$  using Eq. (5).
- (3) Compose  $S_{nl}$  from  $\mathbf{Y}$  using Eq. (2).

## WORK COMPLETED

This year we completed a study, which addressed the basic feasibility of the NNIA approach. We (1) select basis functions  $\Phi_i$  and  $\Psi_q$  and the number of each ( $n, m$ ); (2) designed a NN topology (number of neurons  $k$ ); (3) constructed a representative training set; and (4) selected training strategies. The first three issues all have a significant impact on both accuracy and economy of a NNIA. Unfortunately, there is no pre-defined way to tackle these issues. It is therefore unavoidable that the development of a NNIA involves many iterations. This year we completed the first iteration. The major requirement of an NNIA to be potentially useful in operational wave modeling, is that the exact interactions  $S_{nl}$  are closely reproduced for computational costs comparable to that of the DIA. The feasibility study we have completed showed the potential of this approach with the design of a simple ad-hoc NNIA.

## RESULTS

We have considered an NNIA to estimate the nonlinear interactions  $S_{nl}(f, \theta)$  as a function of frequency  $f$  and direction  $\theta$  from the corresponding spectrum  $F(f, \theta)$  in deep water only. To train and test this NNIA, we used a set of about 20,000 simulated realistic spectra for  $F(f, \theta)$ , and the corresponding exact estimates of  $S_{nl}(f, \theta)$  [2]. Simulation has been performed using a generator that calculated a spectral function composed of several Pierson-Moskowitz spectra for different peak frequencies oriented randomly in  $[0, 2\pi]$  interval. Comparison of simulated spectra with spectra simulated by WAVEWATCH model [3, 4] shows that this approach allowed us to simulate reasonably realistic and complicated spectra describing a broad range of wave systems. Spectra with four peaks were used in calculations below. Separate data sets have been generated for training and validation.

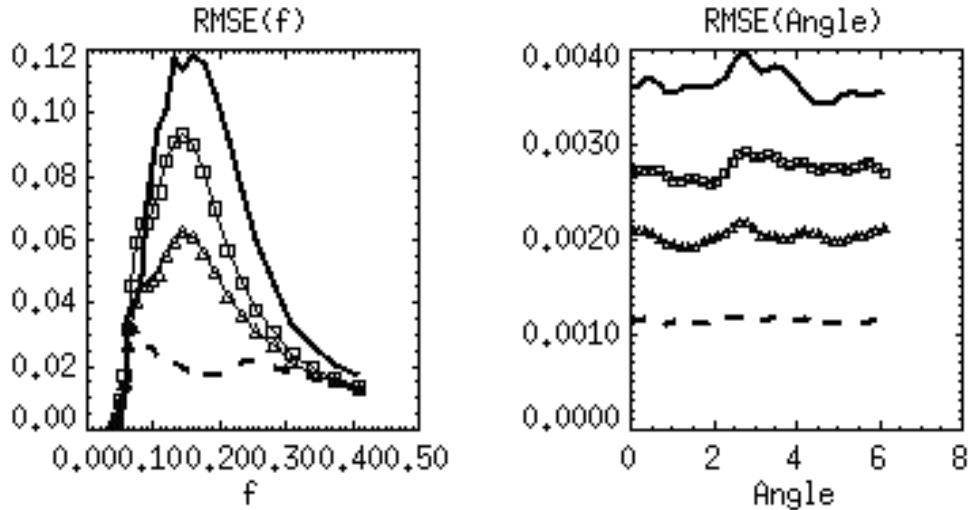
As is common in parametric spectral descriptions, we choose separable basis functions where frequency and angular dependence are separated. For  $\Phi_i$  this implies

$$\Phi_i(f, \theta) \Rightarrow \Phi_{ij} = \phi_{f,i}(f) \phi_{\theta,j}(\theta) \quad (6)$$

A similar separation is used for  $\Psi_q$ . Considering the strongly suppressed behavior of  $F$  and  $S_{nl}$  for  $f \rightarrow 0$ , and the exponentially decreasing asymptotic for  $f \rightarrow \infty$ , generalized Laguerre's polynomials are used to define  $\phi_f$  and  $\psi_f$ . Considering that no directional preferences exist in  $F$  and  $S_{nl}$ , a Fourier decomposition is used for  $\phi_\theta$  and  $\psi_\theta$ . The number of base functions is chosen to be  $n = 51$  and  $m = 64$  to keep the accuracy of approximation for  $F$  on average better than 2% and for  $S_{nl}$  - better than 5-6%. The number of hidden neurons was taken  $k = 30$  which allows a satisfactory approximation (5) for the mapping (4).

**Table 1. RMSE statistics for 10,000  $S_{nl}$**

	Mean RMSE	$\sigma_{\text{RMSE}}$	Max RMSE
DIA	0.0133	0.0111	0.104
NNIA	0.0068	0.0063	0.065

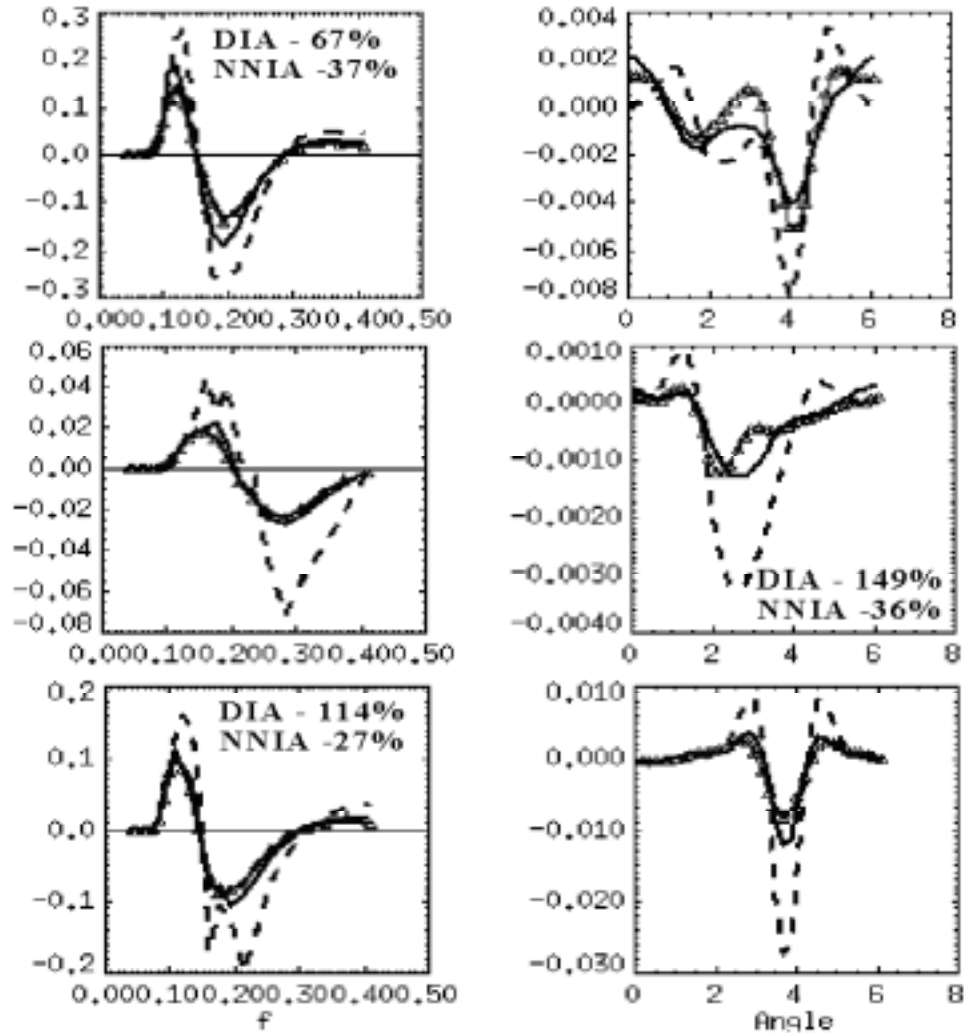


*Figure 1. RMSE as functions of frequency  $f$  and angle. Dashed line – error of approximation (lower bound for all other errors). Solid line – DIA, line with squares – NNIA (51:20:64), and line with triangles – NNIA (51:30:64)*

Table 1 compares three important statistics for source function RMS errors (with respect to exact solution) calculated using DIA and NNIA for 10,000 spectra (independent validation set). NNIA improves accuracy about twice as compared with DIA.

Figure 1 shows mean RMSE as function of the frequency  $f$  (left) and the angle  $\theta$  (right). Numbers in Table 1 correspond to NNIA with NN with 30 neurons in the hidden layer (51:30:64).

Figure 2 shows 3 pairs (one row in the figure corresponds to one pair) of one dimensional, integrated over  $\theta$ , source functions  $S_{nl}(f)$  (left column) and one dimensional, integrated over  $f$ , **source functions  $S_{nl}(\theta)$  (right column) from the validation data set. Thick solid curves** correspond to the exact  $S_{nl}$ . Dashed curves correspond to DIA of  $S_{nl}$ . Curves with triangles correspond to the NNIA estimate of  $S_{nl}$ . Numbers inside the panels show DIA and NNIA errors in percents with respect to exact solution.



*Figure 2. See explanations in the text above.*

The results in Fig. 2 are fairly representative for the validation data set. In general, the NNIA reproduces the exact  $S_{nl}$  accurately. Even if clear oscillations are present in the decomposed spectrum (e.g., line in middle panel on left), the NNIA shows no spurious oscillations, and gives reasonable results. Note that many DIA source functions exhibit complicated behavior and spurious oscillations. Major peaks in these functions coexist with more or less random small-scale fluctuations. These fluctuations are probably an artifact produced by a simplified nature of DIA. Exact interactions are the result of averaging over much larger number of resonant sets of wave numbers, and are therefore much smoother than the results of the DIA.

## IMPACT/APPLICATIONS

The NNIA under development is intended for implementation in operational wave models such as WAVEWATCH.

## TRANSITIONS

The resulting NNIA algorithm will be transitioned to other members of the AWPP-SNL group upon completion.

## RELATED PROJECTS

AWPP-SNL group

## REFERENCES

- [1] Hasselmann, S, and K. Hasselmann, 1985: Computations and parametrizations of the nonlinear energy transfer in a gravity wave spectrum. Part I: a new method for efficient computations of the exact nonlinear transfer integral. *J. Phys. Oceanogr.*, **15**, 1369-77  
-----, -----, J.A. Allender, and T.P. Barnet, 1985: Computations and parametrizations of the nonlinear energy transfer in a gravity wave spectrum. Part II: parametrization of the nonlinear transfer for application in wave models. *J. Phys. Oceanogr.*, **15**, 1378-91
- [2] Van Veldder, G.Ph., T.H.C. Herbers, B. Jensen, D.T. Resio, and B. Tracy: Modelling of nonlinear quadruplet wave-wave interactions in operational coastal wave models. Abstract, accepted for presentation at ICCE 2000, Sydney
- [3] Tolman, H. L., 1999: User manual and system documentation of WAVEWATCH-III version 1.18. NOAA / NWS / NCEP /OMB technical note 166, 110 pp.
- [4] Tolman, H.L., and D.V. Chalikov, 1996: Source terms in a third-generation wind wave model. *J. Phys. Oceanogr.*, **26**, 2497-2518.

## PUBLICATIONS

Krasnopolsky V.M., D.V. Chalikov, and H.L. Tolman, 2001: Using Neural Network for Parameterization of Nonlinear Interactions in Wind Wave Models, in Proceedings of INNS-IEEE International Joint Conference on Neural Networks, 15-19 July 2001, Washington DC

Krasnopolsky V.M., D.V. Chalikov, and H.L. Tolman, 2001: Use of Neural Networks to Improve Computational Efficiency of Environmental Numerical Models, OMB/EMC/NCEP Technical Note, No. 199, pp.37