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Science of Test
Measurement Accuracy - Data Sampling and Filter Selection during Data Acquisition

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David S. Kidman

AIR FORCE TEST CENTER
EDWARDS AFB, CA

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AIR FORCE TEST CENTER
EDWARDS AIR FORCE BASE, CALIFORNIA
AIR FORCE MATERIEL COMMAND
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14. ABSTRACT
Know how to specify our instrumentation requirements, especially the sample rates? We all know about the Nyquist frequency and choosing sample rates that are at least twice the Nyquist to prevent aliasing. But did you know that your data may contain high frequency content that can alias down and muddle the frequencies you care about? Once the aliased signal is digitized it can never be recovered. Your data might be ruined! Your instrumentation engineers know about this, and that is why they added anti-aliasing filters to your data acquisition system. What is an anti-alias filter? I don't know, but I'm sure my instrumentation engineer knows exactly what I need! Wrong. Your instrumentation engineer DOESN'T know what you need and probably chose a 6-pole butterworth filter with a specific cutoff because that's what they used last time. To achieve quality measurements with accurate magnitude and frequency content, the test must start by using comprehensive signal processing principles during initial data acquisition (e.g., correct data sample rates and anti-alias filter selection). To achieve quality measurements, the discipline engineer must verify proper data sampling and filtering principles have been applied during the data acquisition process. Example of how to determine sample rate and filter selection during data acquisition are provided.

15. SUBJECT TERMS data acquisition, Nyquist frequency, sampling rate, aliasing, filtering, butterworth, chebyshev, Bessel, PSD and Bode plots

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Section xx: Data Sampling and Filter Selection during Data Acquisition

References:

1) S. W. Smith, “Digital Signal Processing – A Practical Guide for Engineers and Scientists”, 2003
2) Nyquist Sampling Theorem appeared as early as 1959 in a book from his former employer, Bell Labs. Members of the Technical Staff of Bell Telephone Laboratories (1959). *Transmission Systems for Communications*. AT&T. pp. 26–4 (Vol.2).
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Background

In order to achieve quality pressure measurements with accurate magnitude and frequency content, the test must start by using comprehensive signal processing principles during initial data acquisition (e.g., correct data sample rates and anti-alias filter selection). Pressure measurements typically originate from steady state or dynamic pressure transducers as analog information. However, since most post-test data systems are digital, the analog signals must be converted to digital data prior to being recorded for storage and analysis. This analog-to-digital conversion process can be accomplished in many ways. This section will outline the most common and recommended approach in which the data are first anti-alias filtered and then digitally sampled based on frequencies of interest. Basic concepts associated with data sampling, aliasing, Nyquist frequency, and filter selection will be covered. Specific examples showing correct and incorrect approaches for determining sample rate and filter selection are also provided.

Data Sampling

Recognize that Sampling Theorem (Reference 3) states that least two data points per period are required to resolve the waveform of any analog signal. As a result, the maximum frequency that can be resolved from a data recording is one half of the data sampling frequency. For example: if your data recording system is sampling data at 1200Hz, then the maximum frequency that can be isolated from the analog signal is 600 Hz. This maximum resolvable frequency is called the Nyquist frequency. If the raw analog signal contains information content at frequencies above the Nyquist frequency, then the sampled signal will essentially map that higher frequency content information into the lower frequency domain through a process called aliasing. Aliasing has the undesirable effect of misrepresenting the true frequency content of the raw signal in the sampled data. Note that if the raw signal does not have information content at frequencies that are higher than the Nyquist frequency, then aliasing of the raw data signal will not occur. This knowledge provides an opportunity to eliminate the possibility of aliasing by using a low-pass filter to remove high frequency content on the analog data signal prior to digitally sampling the data. These concepts are discussed in more detail below.

Proper sampling is achieved if you can reconstruct the meaningful content of the original analog signal from the sampled data, then you have sampled the raw signal correctly. The data sampling rate to correctly reconstruct the magnitude and frequency is at least twice the highest frequency of interest. Additionally to avoid aliasing, the data acquisition systems must also using a low-pass filter to limit signal bandwidth above one-half of the sampling rate. Once aliasing has corrupted the information, the original signal cannot be reconstructed. Figure 1 shows a typical analog filter arrangement which is used prior to the analog-to-digital converter (A/D).

Figure 1.Typical anti-aliasing filter used in digital signal processing (Ref 1).

The only other way to avoid aliasing is to significantly oversample. However, the downside of oversampling is the increased cost in terms of providing bandwidth, storage and analysis of large data files. This is particularly true when there are many signals all with high bandwidth requirements (e.g., engine inlet rake data). Most data acquisition systems can easily accommodate large throughput (high sampling rates combined with a large number of channels). However, as throughput requirements increase data system costs rise exponentially. As a general rule, effort spent to limit the amount of data is well spent since it reduces overall cost.

Establishing Sampling Rate

To establish an appropriate sampling rate for data acquisition, the disciple engineer must understand system operating characteristics, test objectives and analysis approach. Clear test objectives and a fundamental understanding of the physical system help identify the appropriate sampling rate. For an aerodynamic assessment of inlet distortion on the engine compression system, flow field disturbances with persistence on the order of one rotor revolution may be considered important. There may also be a need to time correlate up to 40 independent pressures across the AIP. If data is used to evaluate aero-mechanical impacts, there may be a need to evaluate higher frequencies but have no need to time correlate with other pressures.

For an aerodynamic assessment of inlet distortion on the engine compression system, flow field disturbances that have persistence on the order of one rotor revolution may be considered important. However, if the data is to be used to evaluate aero-mechanical interactions, there may be a need to evaluate higher recording frequencies. A clear set of test objectives and a fundamental understanding of the physical system help to identify the appropriate sampling rate for the system and test of interest. It is important to note that uncertainty in system operating characteristics (engine surge sensitivity, expected fan speed, or inlet distortion characteristic) may lead to a requirement of data oversampling until better system understanding is available. The level of data oversampling (possibly 20 percent) should be commensurate the maturity of system understanding. Some general observations about sampling rate are provided here with more detail provided in the example calculation section that follows.

Discipline engineer must verify proper data sampling and filtering principles have been applied during the data acquisition process

Aliasing

In signal processing, aliasing refers to the effect that causes signals of higher frequency than the Nyquist frequency to become indistinguishable from lower frequency signals (or aliased) when improperly sampled, resulting in degraded signal information. Examples of proper and improper sampling for a 1 Hz signal are shown in Figure 2. If a continuous signal is sampled properly (at least twice the frequency of interest), the samples contain all the information needed to recreate the original waveform. Proper sampling is illustrated in Figures 2a and 2b. This is not always obvious, since the samples in Figure 2b do not appear to capture the shape of the original waveform. Nevertheless, each of these continuous signals forms a unique solution along with its sample rate, which guarantees that reconstruction can take place.

However, in Figure 2c, the frequency of the input analog sine wave is greater than the Nyquist frequency (defined as one-half the sampling rate). This results in aliasing, where the frequency of the reconstructed data is incorrect. In this last case, the input raw signal had a 1 Hz frequency. The sample rate was 1.9 Hz, which resulted in a Nyquist frequency of 0.95 Hz. The higher frequency signal “wraps” back into the lower frequency domain by using the Nyquist frequency as the pivot point: 1 Hz - 0.95 Hz = 0.05 Hz and 0.05 Hz subtracted from 0.95 Hz results in an aliased frequency of 0.90 Hz, the apparent frequency of the reconstructed signal.

Just as aliasing can change the frequency during sampling, it can also change the phase, as can be seen in the aliased signal in Figure 2c. The aliased digital signal is inverted from the original analog signal with a 180 deg phase shift. Note that only two phase shifts are possible: 0 deg (no phase shift) and 180 deg (inverted) and that the 0 or 180 deg phase shift is only valid at the starting point of the data stream. Phase shift is a concern when trying to time align peak inlet pressures in order to make inlet distortion or recovery calculations.

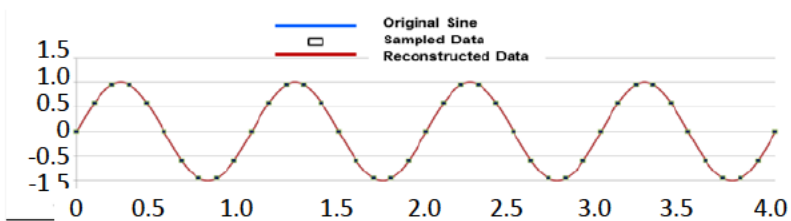


Figure 2a: Proper Sampling
- Frequency: 1 Hz
- Samples per cycle: 10
- Nyquist Frequency 5
- Reconstructed Frequency: 1 Hz
- Phase Distortion: 0 deg

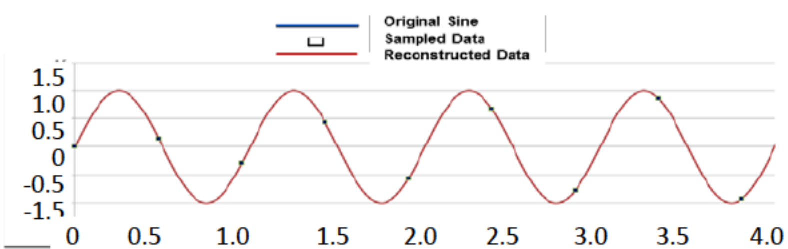


Figure 2b: Proper Sampling
- Frequency: 1 Hz
- Samples per cycle: 2.1
- Nyquist Frequency 1.05
- Reconstructed Frequency: 1 Hz
- Phase Distortion: 0 deg

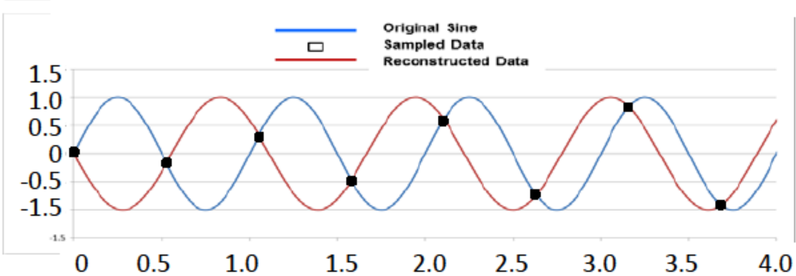


Figure 2c: Improper Sampling
- Frequency: 1 Hz
- Samples per cycle: 1.9
- Nyquist Frequency 0.95
- Reconstructed Frequency: 0.9 Hz
- Phase Distortion: -180 deg

Figures 2a, 2b, and 2c. Effects of proper and improper sampling

Nyquist frequency

The Nyquist frequency is the highest frequency that can be resolved from the raw signal at the chosen sample rate. The Nyquist frequency is defined as one-half the sample rate. This is known as the Nyquist sampling theorem, after the author of 1940s' papers on the topic (Ref 2 and 3). The Nyquist sampling theorem indicates that a continuous signal can be properly sampled only if it does not contain frequency components above one-half of the sampling rate.

Anti-Alias Filter Selection

Analog filtering is a critical portion of the typical data acquisition system which is designed to remove higher frequency information (above Nyquist) from the raw data signal to prevent undesirable aliasing of that information into the frequency range of interest. If a low-pass analog filter is not used, signals higher than half the sampling rate will be aliased into the observable frequency domain in the sampled digital data. Once a signal is aliased during the digitization process, it is impossible to differentiate between correctly resolved original signal content occurring in the observable frequency domain and undesirable higher frequency data that has been aliased into the observable frequency domain.

The characteristic of every digitized signal depends on the type of anti-alias filter used when it was acquired. If the nature of the anti-alias filter is not understood, the nature of the digital signal cannot be understood. Analog filters typically used during data acquisition include the Butterworth, Chebyshev and Bessel. Each of these filters is designed to optimize a different performance characteristic (e.g., low pass-band attenuation, sharp roll-off, or constant group delay).

Figure 3 shows the frequency response of the three low-pass filters with a 150Hz cutoff frequency. The Butterworth filter (Figure 3a) can be designed to have a quicker roll-off above the cutoff frequency by increasing the filter order (related to number of poles) without allowing ripple in the passband frequency range. The Chebyshev filter (Figure 3b) obtains its excellent sharp roll-off characteristic by allowing passband ripple. In comparison, the Bessel filter (Figure 3c) has no ripple in the passband, but roll-off at the cutoff frequency is far slower than the Butterworth or Chebyshev. Additionally, the Bessel filter suffers from significantly higher attenuation across the passband. Flat passband and quick roll-off are desirable for determining an individual peak pressures or stress

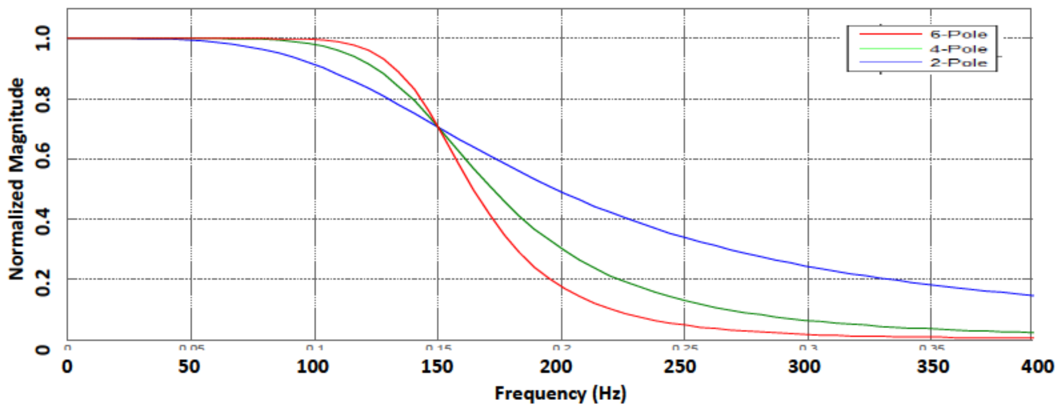


Figure 3a. Frequency response of 2-, 4-, 6-pole Butterworth filters shown with linear scales

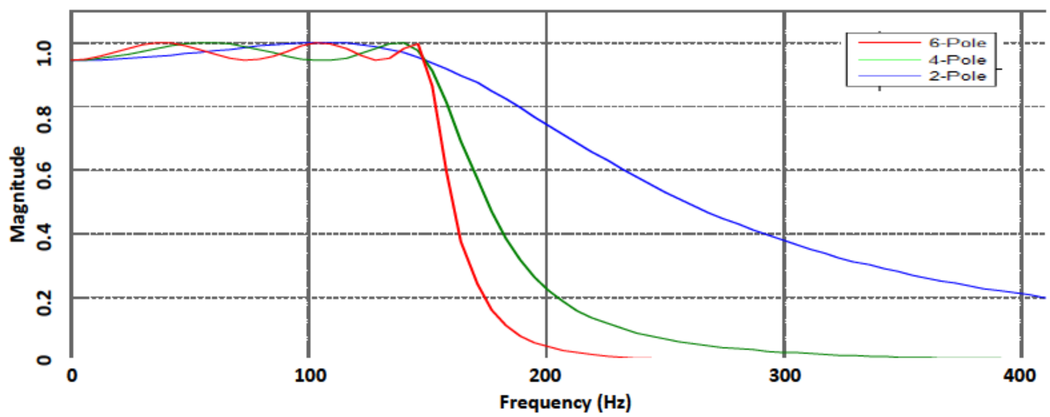


Figure 3b. Frequency response of 2-, 4-, 6-pole Chebyshev filters shown with linear scales

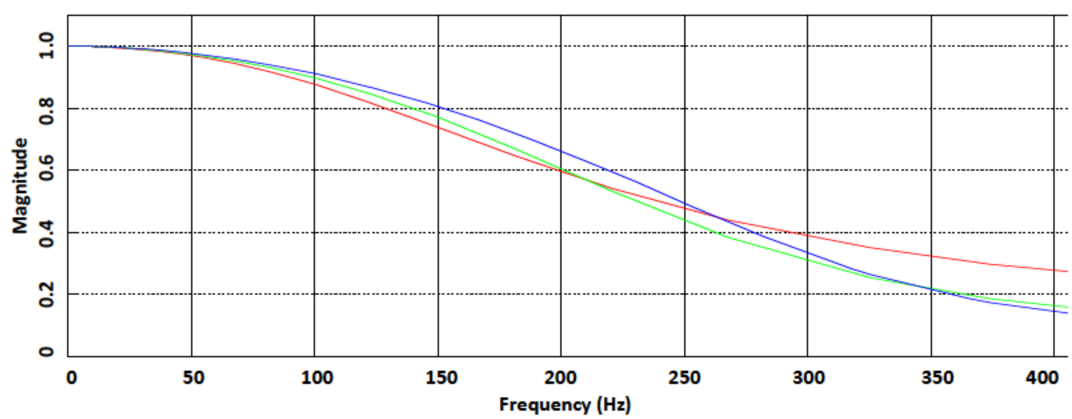


Figure 3c. Frequency response of 2-, 4-, 6-pole Bessel filters shown with linear scales

Figure 4 shows the step response of the same three filters with a 150-Hz cutoff frequency. As can be seen, both the Butterworth (Figure 4a) and Chebyshev (Figure 4b) both exhibit good response characteristic but show tendency to overshoot or ring, possibly leading to over-estimating max value. In comparison, the Bessel filter (Figure 4c) is quicker to respond and exhibits neither overshoots nor ringing, probably making it a better choice for rapidly changing pressures. A key property of the Bessel filter is that the rising and falling edges in the filter's output look similar; this is called linear phase and is an indicator of signal or time-distortion.

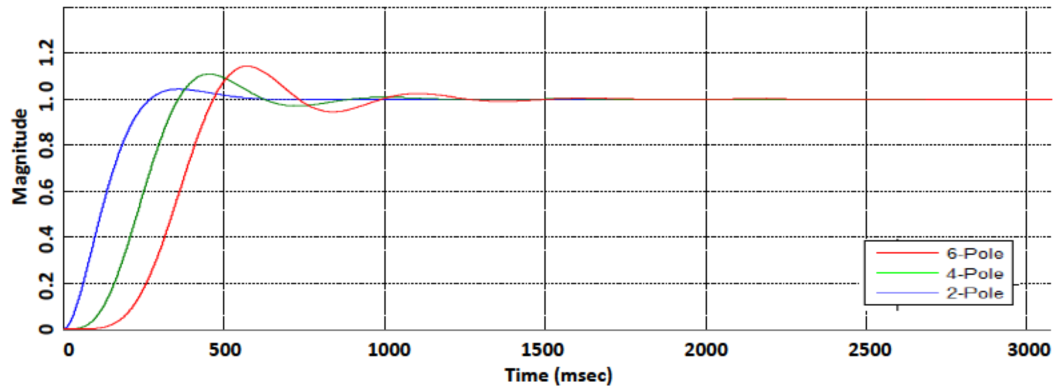


Figure 4a Step Response of 2-, 4-, and 6-Pole of Butterworth Filters

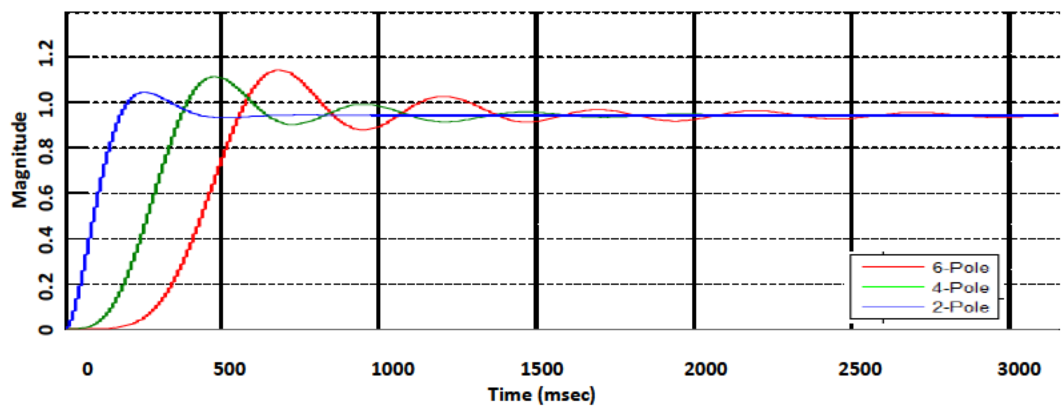


Figure 4b Step Response of 2-, 4-, and 6-Pole of Chebyshev Filters

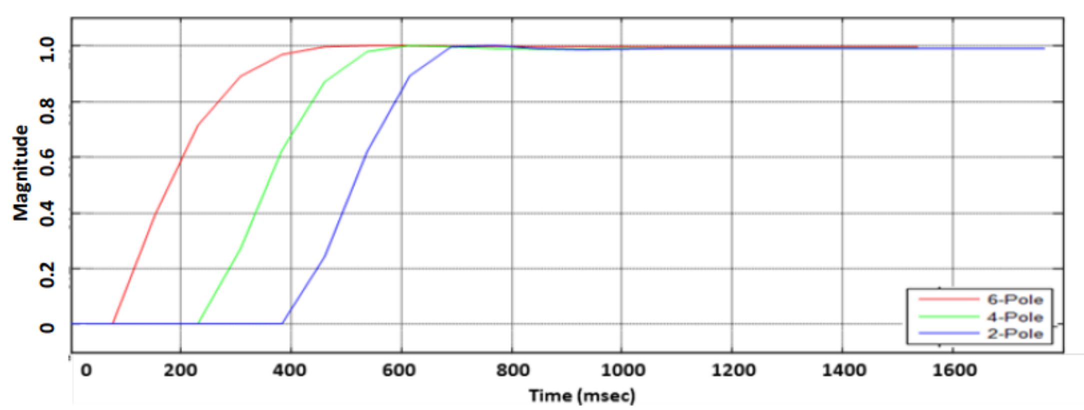


Figure 4c Step Response of 2-, 4-, and 6-Pole of Bessel Filters

The best indicator of signal or time-distortion is shown using group delay, Figure 5. Group delay is calculated by differentiating the phase response with respect to frequency. Group delay is a measure of the slope of the phase response at any given frequency. Variations in group delay cause signal distortion, just as deviations from linear phase cause distortion. Figure 5 shows the group delay for the same three filters (Butterworth, Chebyshev and Bessel) with a 150-Hz cutoff frequency. As can be seen, both the Butterworth (Figure 5a) and Chebyshev (Figure 5b) filters exhibit non-linear group delay in the passband while in comparison, the Bessel filter (Figure 5c) exhibits constant group delay in the passband. Also note that the higher the filter order, the more pronounced the non-linear group delay.

Constant group delay in the passband is preferred for processing engine inlet rake pressure data because it ensures the amplitude of the 40 measured pressures are not distorted in time. This is critical since pressure signals from 40 AIP pressure probes are typically combined in calculations of inlet distortion indices or recovery. Many applications (including engine inlet rake signal processing) cannot tolerate the amount of phase distortion associated with the higher order Chebyshev filter in Figure 5b.

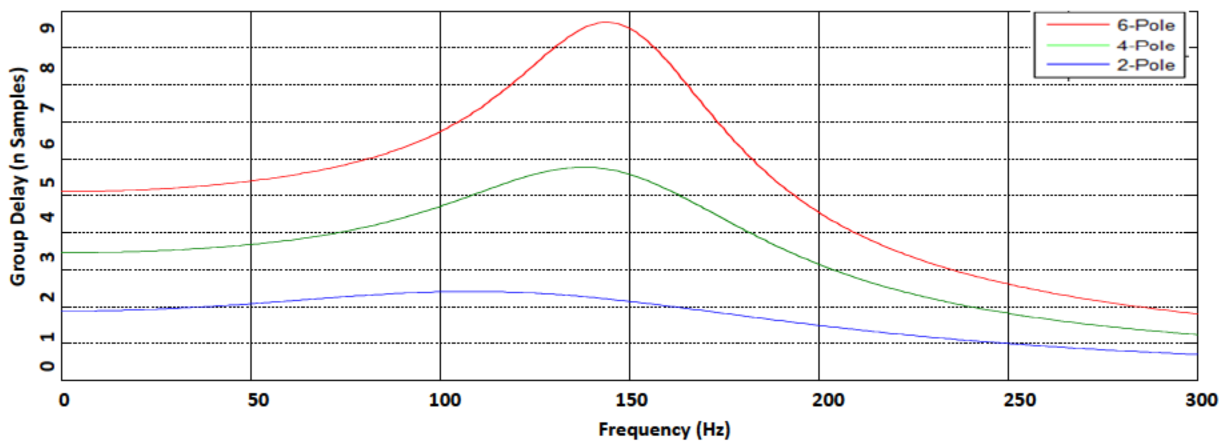


Figure 5a Group Delay of 2-, 4-, and 6-Pole of Butterworth Filters (Cutoff Freq= 150Hz)

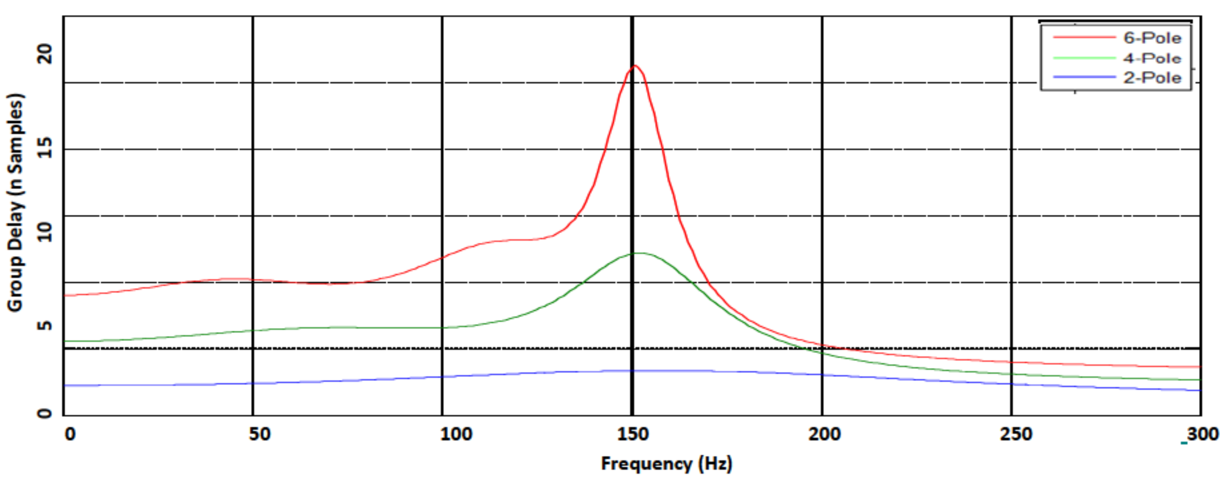


Figure 5b Group Delay of 2-, 4-, and 6-Pole of Chebyshev Filters (Cutoff Freq= 150Hz)

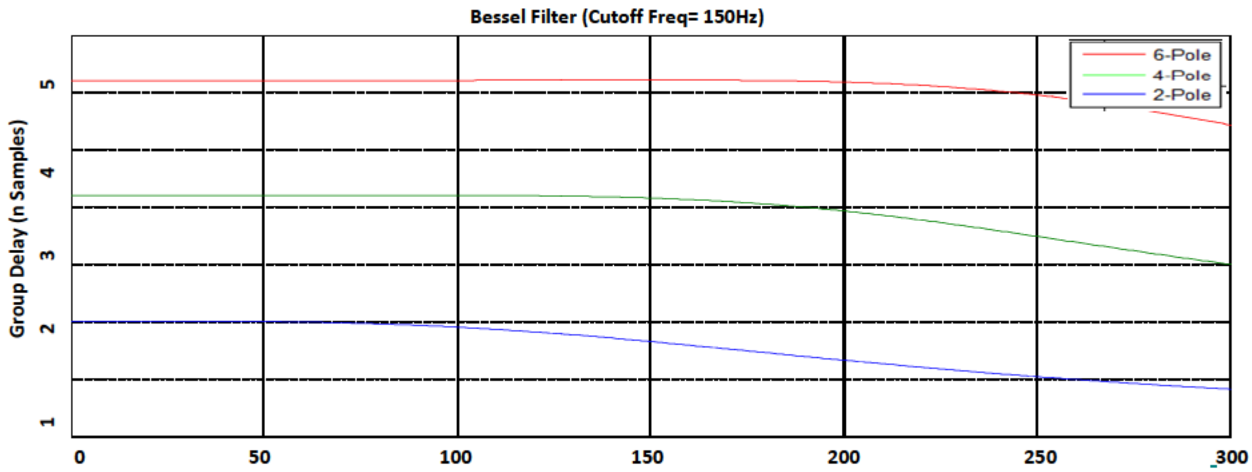


Figure 5c Group Delay of 2-, 4-, and 6-Pole of Bessel Filters (Cutoff Freq= 150Hz)

In summary, the three most common filters used in data acquisition include the Butterworth, Chebyshev and Bessel with each having their own strengths and weaknesses. The Butterworth optimizes the passband flatness but is slower to respond to rapid changes, has ripple and has non-linear group delay in higher order filters. The Chebyshev optimizes quickness in roll-off but is slower to respond to rapid changes, has ripple and has a non-linear group delay in higher order filters. The Bessel optimizes quickness to respond, eliminates passband ripple and has linear group delay and but suffers from poor roll-off quickness.

Ultimately, selection of the anti-alias filter depends on application and test goals. AIP inlet pressures depend almost entirely on the ability to accurately recreate pressure magnitudes and their alignment in time. As a result, inlet analysis places an emphasis on linear group delay and eliminating ripple in the passband making the Bessel the best choice for anti-alias during the A/D conversion. For determining peak pressures or stress of individual measurements at specific frequencies, flat passband and quick roll-off are more desirable making Butterworth or Chebyshev more desirable

Examples – Determining Data Sample Rate and Filter Selection

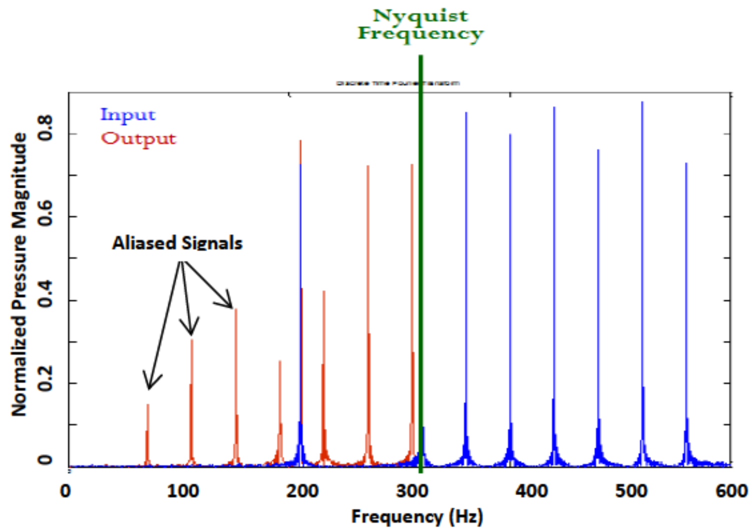
Data sample rate and filter selection during data acquisition can adversely impact magnitude and frequency content of pressure signals used for engine inlet compatibility analysis. The following section will provide a hands-on example showing the effects of using incorrect and then correct data sampling and filtering techniques. Assume test goal is to acquire quality pressure data with frequencies up to 200 Hz and remove signal content above 200Hz. Note this example focuses on the process of determining sample rate and filter selection for determining peak pressures up to the frequency of interest and did not attempt to control or account for time distortion.

Bad Example – Determining Data Sample Rate and Filter Selection

This first example will show the effect of incorrect data sampling and filter selection. Assume the test effort had frequencies of interest up to 200Hz but used the following data sample rates and filtering during data acquisition:

- Data sample rate for AIP dynamic pressures was 631 samples per second (sps)
- A/D converter used a 6-pole Butterworth filter with low-pass cutoff at 450Hz

The problem is that if the frequencies of interest were up to 200Hz, then the sample rate and filter selection were not optimal. To demonstrate, the following example uses an input signal constructed of 200Hz with additional pulses every 40 Hz from 320 to 560Hz plus noise. As shown in Figure 6, the output signal magnitude was not significantly impacted by aliasing at 200Hz exactly. However, aliasing did significantly affect energy content at multiple discrete frequencies below 200Hz.This aliased energy content would definitely impact the distortion calculation and make it difficult or impossible to discern the actual distortion magnitude or actual frequency content. Once filtered and digitally sampled, the original signal cannot be reconstructed.



Input Signal: 200Hz + pulses from 320Hz to 560Hz (every 40Hz) + noise.
Output Signal: 450Hz 6-pole low-pass Butterworth filter and sampled at 631sps

Figure 6 PSD plot showing the effect of aliasing due to incorrect sampling and filter selection

Good Example – Determining Data Sample Rate and Filter Selection

The next example shows steps needed to ensure the correct data sample rate and filter are chosen to avoid signal aliasing:

1) Assume 200Hz and below are the frequencies of interest.

2) Assume a 6-pole Butterworth filter was chosen for the A/D converter with a goal of attenuating criteria of 5-percent of signal amplitude at the maximum frequency of interest. The ratio of flat to within 5-percent at the 95-percent attenuation frequency is shown in Table 1. Ratios of flat to within 10-, 2-, and 1-percent are also shown in Table 1. The Nyquist frequency is typically defined at the 95-percent attenuation frequency, or in this case (200Hz/0.5043) = 397Hz.

3) To avoid aliasing, sample rate should be at least twice the Nyquist frequency (397Hz*2) = 794 sps.

Table 1 Ratio of flat to within 1-, 2-, 5-, and 10-percent of the 95-percent attenuation frequencies.

Filter Type	Order	Flat to within			
		10%	5%	2%	1%
Butterworth	2nd	15.57	12.83	10.08	8.45
Butterworth	4th	39.46	35.82	31.75	29.06
Butterworth	6th	53.80	**50.43**	46.54	43.87
Butterworth	8th	62.82	59.85	56.35	53.91
Bessel	2nd	10.03	7.11	4.51	3.20
Bessel	4th	18.39	12.91	8.13	5.74
Bessel	6th	20.67	14.47	9.09	6.42
Bessel	8th	21.08	14.74	9.26	6.53
0.5dB Chebyshev	2nd	21.23	10.03	5.67	3.93
0.5dB Chebyshev	4th	57.67	16.07	8.81	6.06
0.5dB Chebyshev	6th	76.24	14.62	7.97	5.48
0.5dB Chebyshev	8th	85.29	12.40	6.75	4.63

4) To guide filter cutoff frequency, recall frequencies of interest were 200Hz and below and that a 6-pole Butterworth low-pass filter was selected for the aircraft A/D converter use. Also note that filter characteristics are usually defined based on the cutoff frequency at the 3dB down point. As a result, the ratios of 3dB (29.3 percent loss) to 95-percent attenuation frequencies are shown in Table 2. The filter 3dB cutoff frequency is calculated as follows (Nyquist 397Hz)(0.60709) = 240Hz. Figure 7 shows the Bode plot response for this filter selection. As can be seen, filter response is flat in the passband, allows 5 percent attenuation at 200 Hz and 95 percent attenuation at the Nyquist frequency of 397 Hz. Another useful way to view filter performance is through the use of a PSD plot for comparing input and output signals. As can be seen in Figure 8, when the correct filter and sample rates were selected aliasing was eliminated and magnitude at frequencies of interest was accurate.

Table 2 shows ratio of 3dB to 95-percent attenuation frequencies.

Filter Type	Order	Ratio 3dB to 95% attenuation point
Butterworth	2nd	0.22375
Butterworth	4th	0.47302
Butterworth	6th	**0.60709**
Butterworth	8th	0.68776
Bessel	2nd	0.17781
Bessel	4th	0.32317
Bessel	6th	0.36861
Bessel	8th	0.37784
0.5dB Chebyshev	2nd	0.26073
0.5dB Chebyshev	4th	0.61124
0.5dB Chebyshev	6th	0.78286
0.5dB Chebyshev	8th	0.86580

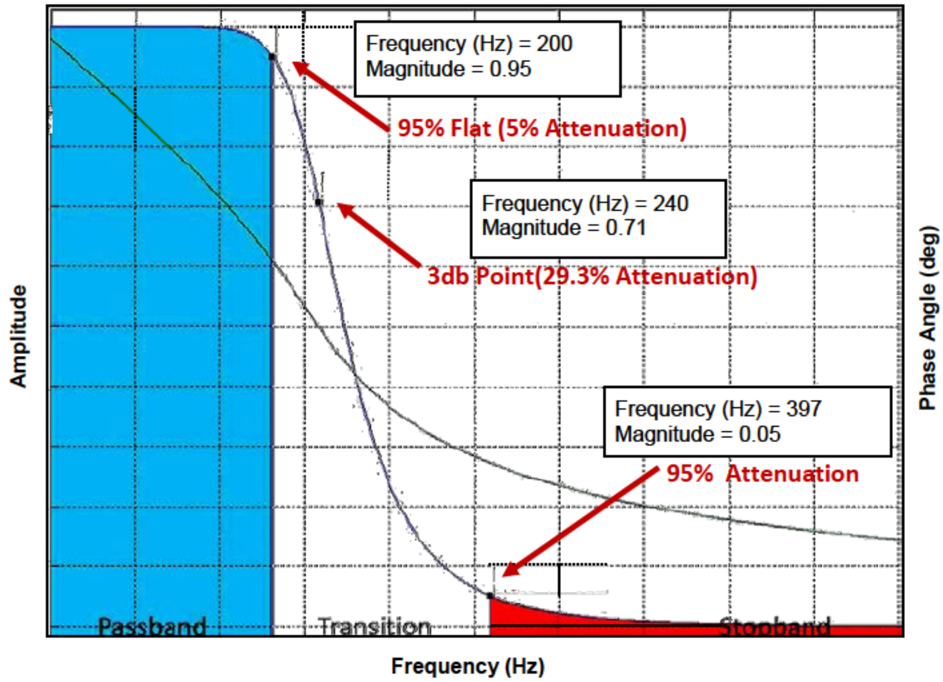
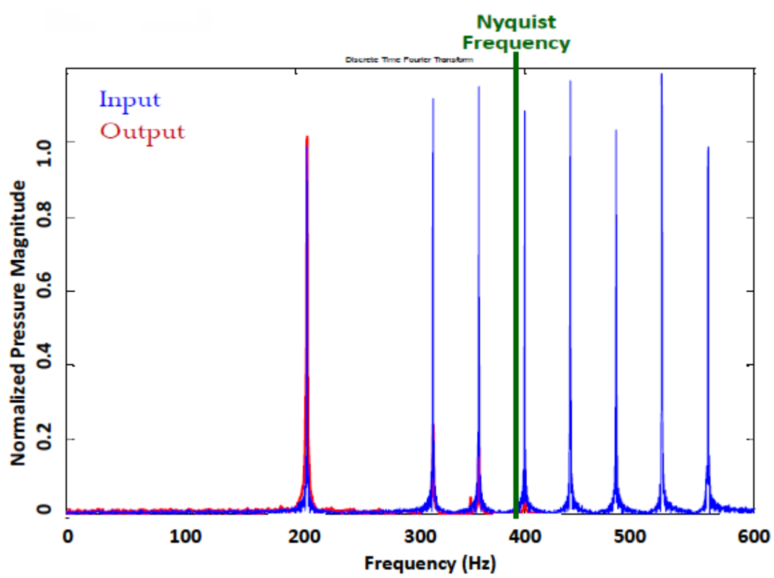


Figure 7 Bode plot showing magnitude and phase characteristics of the 240 Hz low-pass 6-pole Butterworth filter



Input Signal: 200Hz + pulses from 320Hz to 560Hz (every 40Hz) + noise.
Output Signal: 240Hz 6-pole low-pass Butterworth filter and sampled at 794 sps

Figure 8 PSD plot showing the effect of correct sampling and filter selection

In summary, to achieve quality pressure measurements with accurate magnitude and frequency resolution, the discipline engineer must understand system operating characteristics, test objectives and analysis approach. Additionally, the discipline engineer must verify proper data sampling and filtering principles have been applied during the data acquisition process.

Our example focused on the process of determining sample rate and filter selection for determining peak pressures up to the frequency of interest and did not attempt to control or account for time distortion. We were able to ensure a maximum of 5-percent attenuation for frequencies of interest up to 200Hz. The Nyquist frequency was determined to be of 397Hz (using 95-percent attenuation) and the minimum data sample rate was determined to be 794 Hz. Using a low-pass 6-pole Butterworth filter, cutoff was determined to be 240Hz at the 3dB down point. Prior to filter implementation, it is highly recommended to view filter characteristics on both Bode and PSD plots.

SCIENCE OF TEST

Measurement Accuracy - Data Sampling and Filter Selection during Data Acquisition

David Kidman
Air Force Test Center
412th Test Wing, Propulsion Integration Flight
Edwards AFB CA, 93524
david.kidman@us.af.mil

Outline

Background

Data acquisition concepts

- Test Requirements
- Data sampling, Aliasing and Nyquist frequency
- Filter selection for A/D conversion

Examples showing correct and incorrect data sampling and filtering

Background

Measurements start **analog** but converted to **digital** prior to being recorded

Quality data (Accurate magnitude and Frequency content) starts with careful **attention to setup of data acquisition system** (e.g. data sample rates and anti-alias filter selection)

If acquisition setup is not understood, then recorded signal can not be understood !

Understanding Requirements

Discipline **Engineer Responsibilities**:

1) **Ultimately responsible** for data quality for evaluating system under test
2) **Guide instro setup** by understanding system operating characteristics, test objectives and analysis approach
   - -- If data is used to determine aero-mechanical impacts, analysis may emphasize accurate magnitude and frequency but less concerned with time correlation
   - -- If data is used for inlet distortion impact on engine stability, need accurate magnitudes and frequency but analysis emphasizes time correlation of 40 independent pressures
3) **Verify proper techniques** (data sampling and A/D filtering) were applied

Uncertainty in system operation may require additional frequency content (≈ 20 percent) until better system understanding is available

Data Sampling and Aliasing

Sampling Theorem states “…at least two data points per period are required to recreate waveform”

Correct sampling (> 2x), then can recreate the original waveform, Figures 2a and 2b

Incorrect sampling (< 2x), then aliasing results, Figure 2c shows changing the frequency and phase

Note: Two phase shifts possible: 0 deg (no phase shift) and 180 deg (inverted)

Once aliased, the original signal cannot be reconstructed

Figure 2a: Proper Sampling

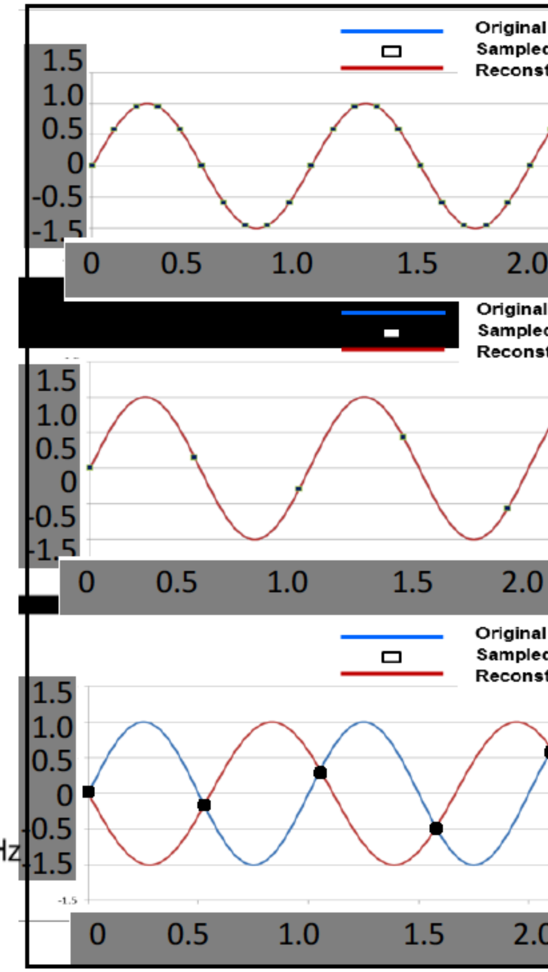
- Frequency: 1 Hz
- Samples per cycle: 10
- Nyquist Frequency 5
- Reconstructed Frequency: 1 Hz
- Phase Distortion: 0 deg

Figure 2b: Proper Sampling

- Frequency: 1 Hz
- Samples per cycle: 2.1
- Nyquist Frequency 1.05
- Reconstructed Frequency: 1 Hz
- Phase Distortion: 0 deg

Figure 2c: Improper Sampling

- Frequency: 1 Hz
- Samples per cycle: 1.9
- Nyquist Frequency 0.95
- Reconstructed Frequency: 0.9 Hz
- Phase Distortion: -180 deg



Aliasing

If analog signal contains information above Nyquist, the digitized **signal will alias** higher frequency content into the lower frequency domain

Aliasing can be avoided by removing higher frequency content from analog signal prior to digitizing by **using a low-pass filter**

Typical anti-alias filters used during data acquisition include the **Butterworth, Chebyshev and Bessel**. Each designed to optimize a different performance characteristic (e.g. pass-band flatness, quick roll-off, or low phase distortion)

Anti-Alias Filter Selection – Frequency Response

Figure 3 shows frequency response of three low-pass filters with a 150Hz cutoff. **<u>Note</u>**: Cutoff frequency typically defined at 3dB (~30pct) down point

Butterworth filters (Figure 3a) have a flat passband and can be designed to have a quicker roll-off at cutoff by increasing filter order

Chebyshev filters (Figure 3b) obtains its sharp roll-off by allowing some passband ripple

Bessel filters (Figure 3c) have significantly higher attenuation across the passband and slow roll-off

Flat passband and quick roll-off are desirable for determining peak magnitude at specific frequencies for individual measurements

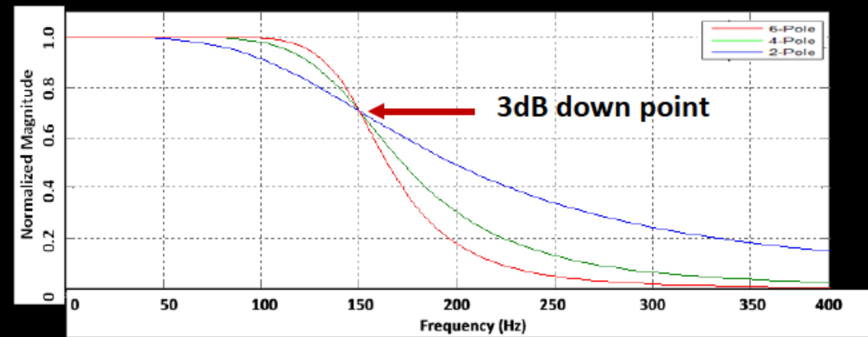


Figure 3a. Frequency response of 2-, 4-, 6 pole Butterworth filters shown with linear scales

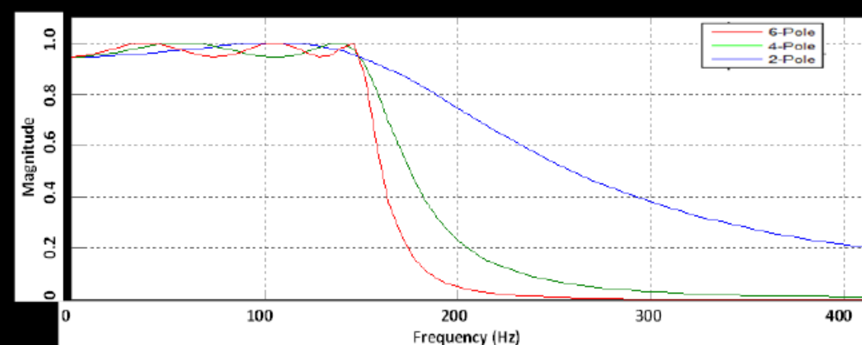


Figure 3b. Frequency response of 2-, 4-, 6 pole Chebyshev filters shown with linear scales

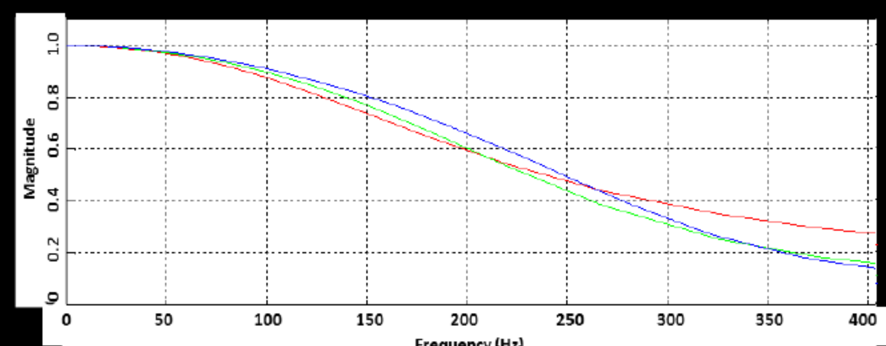


Figure 3c. Frequency response of 2-, 4-, 6 pole Bessel filters shown with linear scales

Anti-Alias Filter Selection – Step Response

Figure 4 shows the step response for same three filters with a 150-Hz cutoff. Fast response to step input shows how filter would respond to rapidly changing values

Butterworth (Figure 4a) and **Chebyshev** (Figure 4b) both exhibit good response characteristic but higher-order tends to overshoot, leading to over-estimate

Bessel filter (Figure 4c) is quickest to respond and doesn't overshoot, probably making it a better choice for rapidly changing magnitudes

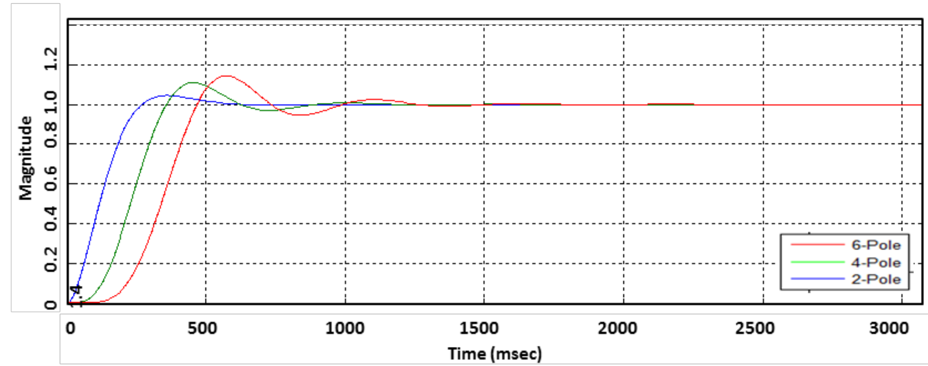


Figure 4a Step Response of 2-, 4-, and 6-Pole of Butterworth Filters

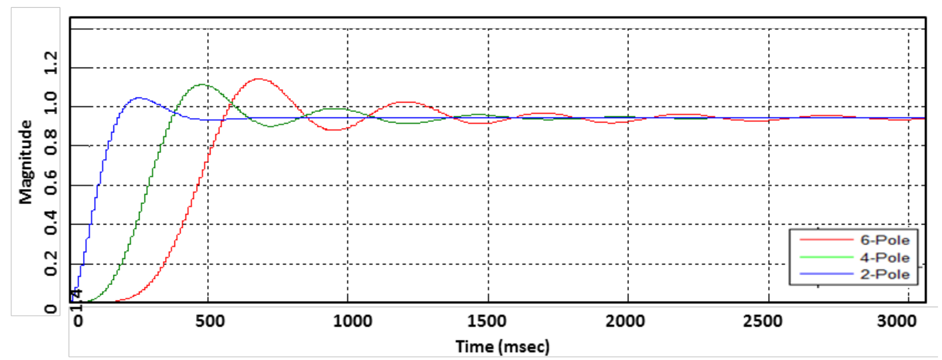


Figure 4b Step Response of 2-, 4-, and 6-Pole of Chebyshev Filters

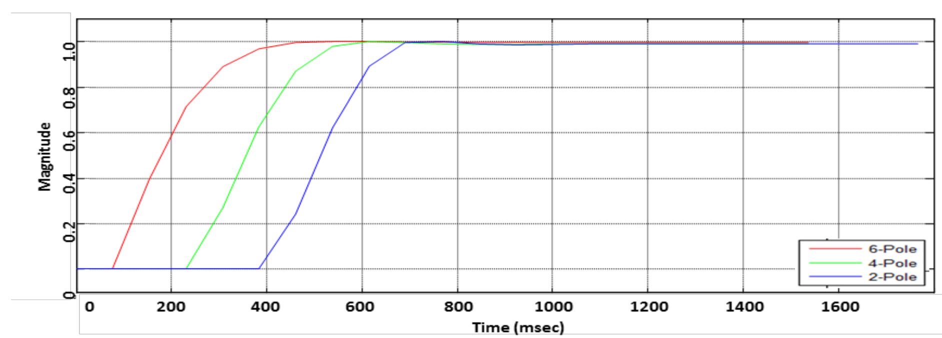


Figure 4c Step Response of 2-, 4-, and 6-Pole of Bessel Filters

Anti-Alias Filter Selection – Group Delay

Figure 5 shows Group delay (best indicator of signal/time distortion) for the same three filters

Higher-order **Butterworth** (Figure 5a) filters exhibit non-linear Group delay in the passband

Chebyshev (Figure 5b) exhibits significant non-linear Group delay near cutoff

Bessel (Figure 5c) exhibits constant group delay in the passband

Constant group delay in the passband is preferred for analysis requiring correlation of multiple parameters (e.g. inlet distortion)

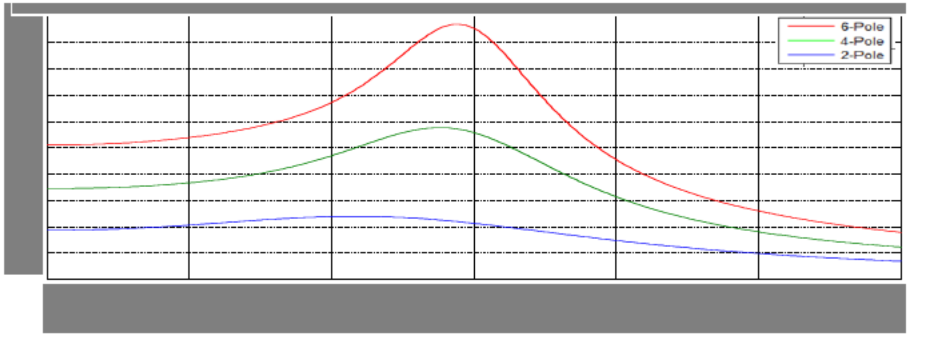


Figure 5a Group Delay of 2-, 4-, and 6-Pole of Butterworth Filters (Cutoff Freq= 150Hz)

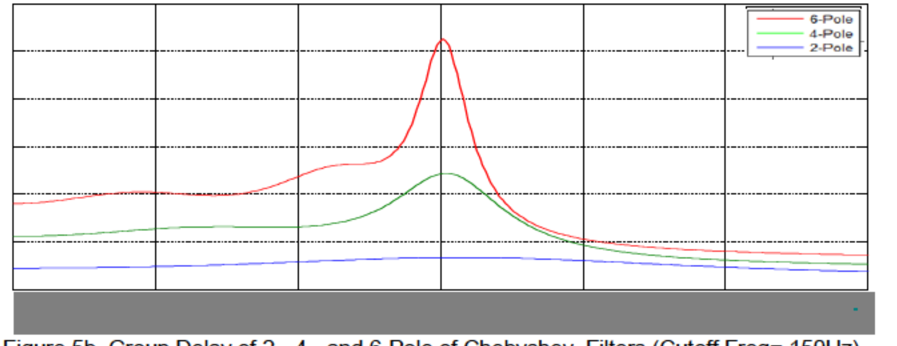


Figure 5b Group Delay of 2-, 4-, and 6-Pole of Chebyshev Filters (Cutoff Freq= 150Hz)

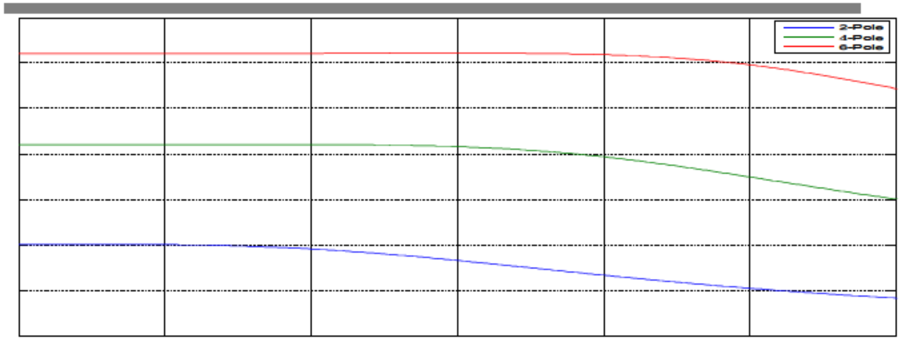


Figure 5c Group Delay of 2-, 4-, and 6-Pole of Bessel Filters (Cutoff Freq= 150Hz)

Takeaway

Most common filters used in data acquisition include the **Butterworth, Chebyshev** and **Bessel**, each having their own strengths and weaknesses

Butterworth optimizes the passband flatness but has some ripple to step input and has non-linear group delay in higher order filters

Chebyshev optimizes quickness in roll-off but is slowest to respond to rapid changes, has most ripple and has significant non-linear group delay in higher order filters

Bessel optimizes quickness to respond, has no passband ripple and has constant group delay and but suffers from poor passband flatness and slow roll-off

Ultimately, **selection of the anti-alias filter depends on** test objectives and analysis goals

- Engine inlet analysis places an emphasis on eliminating time distortion and quickness to respond to rapidly changing conditions, probably making the **Bessel** the best choice for anti-alias during the A/D conversion

- Accurate magnitudes of individual measurements at specific frequency, flat passband and quickness in roll-off are most important making **Butterworth** the best choice

<u>Good and Bad Examples</u> –
Data Sampling and Filter Selection

Assume test objective is to acquire accurate pressure magnitude from individual transducer with frequencies of interest up to 200 Hz

> <u>Note</u>: Since example is for single transducer, no attempt to control time distortion

<u>Bad Example</u> – Data Sampling and Filter Selection

Assume: Frequencies of interest up to 200Hz, A/D converter used a 6-pole Butterworth filter with low-pass cutoff at 450Hz,digitized data at 631 sps

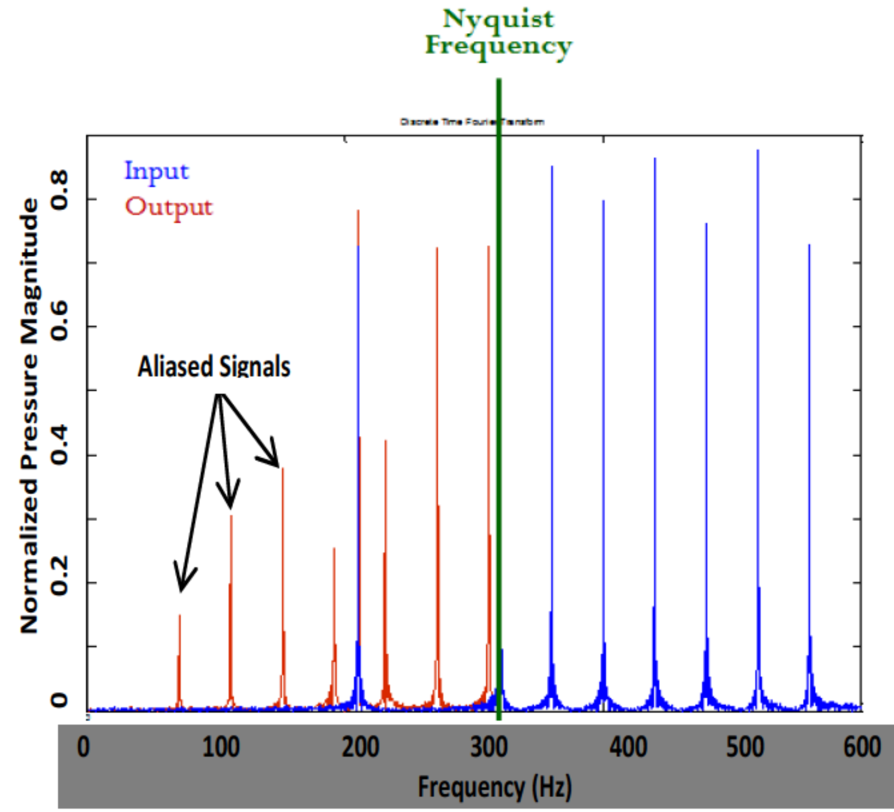
Problem: Sample rate and filter not optimal

Demonstrate: Input signal constructed of 200Hz with additional pulses every 40 Hz from 320 to 560Hz plus noise

Figure 6 PSD shows **aliasing** significantly effected output **at multiple frequencies below 200Hz**

Aliasing makes it impossible to discern actual pressure magnitude or frequency

Once filtered and digitally sampled, the **original signal cannot be reconstructed**



Input Signal: 200Hz + pulses every 40Hz (320Hz to 560Hz) + noise
Output Signal: 450Hz 6-pole low-pass Butterworth then sampled at 631sps

Figure 6 PSD plot showing the effect of aliasing due to incorrect sampling and filter selection

Good Example – Data Sampling and Filter Selection

Assume: Frequencies of interest up to 200Hz, A/D converter uses 6-pole Butterworth with max attenuation of 5-percent at 200Hz

Data Sample Rate Selection: Table 1 shows ratio of flat to within 5-percent at the 95-percent attenuation frequency

Nyquist = $Freq\ Reqd / (\frac{5pct}{95pct})$

= (200Hz/0.5043) = 397Hz

To avoid aliasing, sample rate should be at least twice the Nyquist frequency

Sample Rate = (397Hz∗2) = 794 sps

Table 1 Ratio of flat to within 1-, 2-, 5-, and 10-percent with 95-percent attenuation at ω=1

Filter Type	Order	Flat to within			
		10%	5%	2%	1%
Butterworth	2nd	15.57	12.83	10.08	8.45
Butterworth	4th	39.46	35.82	31.75	29.06
Butterworth	6th	53.80	**50.43**	46.54	43.87
Butterworth	8th	62.82	59.85	56.35	53.91
Bessel	2nd	10.03	7.11	4.51	3.20
Bessel	4th	18.39	12.91	8.13	5.74
Bessel	6th	20.67	14.47	9.09	6.42
Bessel	8th	21.08	14.74	9.26	6.53
0.5dB Chebyshev	2nd	21.23	10.03	5.67	3.93
0.5dB Chebyshev	4th	57.67	16.07	8.81	6.06
0.5dB Chebyshev	6th	76.24	14.62	7.97	5.48
0.5dB Chebyshev	8th	85.29	12.40	6.75	4.63

Good Example – Data Sampling and Filter Selection

Filter Selection: Frequencies of interest up to 200Hz and 6-pole Butterworth lowpass filter selected

Recall filter characteristics usually defined based on the cutoff frequency at 3dB down point. Table 2 shows ratios of 3dB to 95-percent attenuation point

Filter 3dB cutoff frequency:

$$\text{Filter freq} = Nyquist\left(\frac{3dB}{95pct}\right)$$

$$= (397\text{Hz})(0.60709) = 240\text{Hz}$$

Table 2 shows ratio of 3dB to 95-percent attenuation frequencies

Filter Type	Order	Ratio 3dB to 95% attenuation point
Butterworth	2nd	0.22375
Butterworth	4th	0.47302
Butterworth	6th	0.60709
Butterworth	8th	0.68776
Bessel	2nd	0.17781
Bessel	4th	0.32317
Bessel	6th	0.36861
Bessel	8th	0.37784
0.5dB Chebyshev	2nd	0.26073
0.5dB Chebyshev	4th	0.61124
0.5dB Chebyshev	6th	0.78286
0.5dB Chebyshev	8th	0.86580

<u>Good Example</u> – Data Sampling and Filter Selection

Figure 7 Bode plot shows magnitude characteristics of lowpass 6-pole Butterworth filter with 240 Hz cutoff

Filter designed is flat in the passband, allows 5-percent attenuation at 200Hz and 95-percent attenuation at the Nyquist frequency of 397Hz

Filter exhibits quick roll-off characteristic

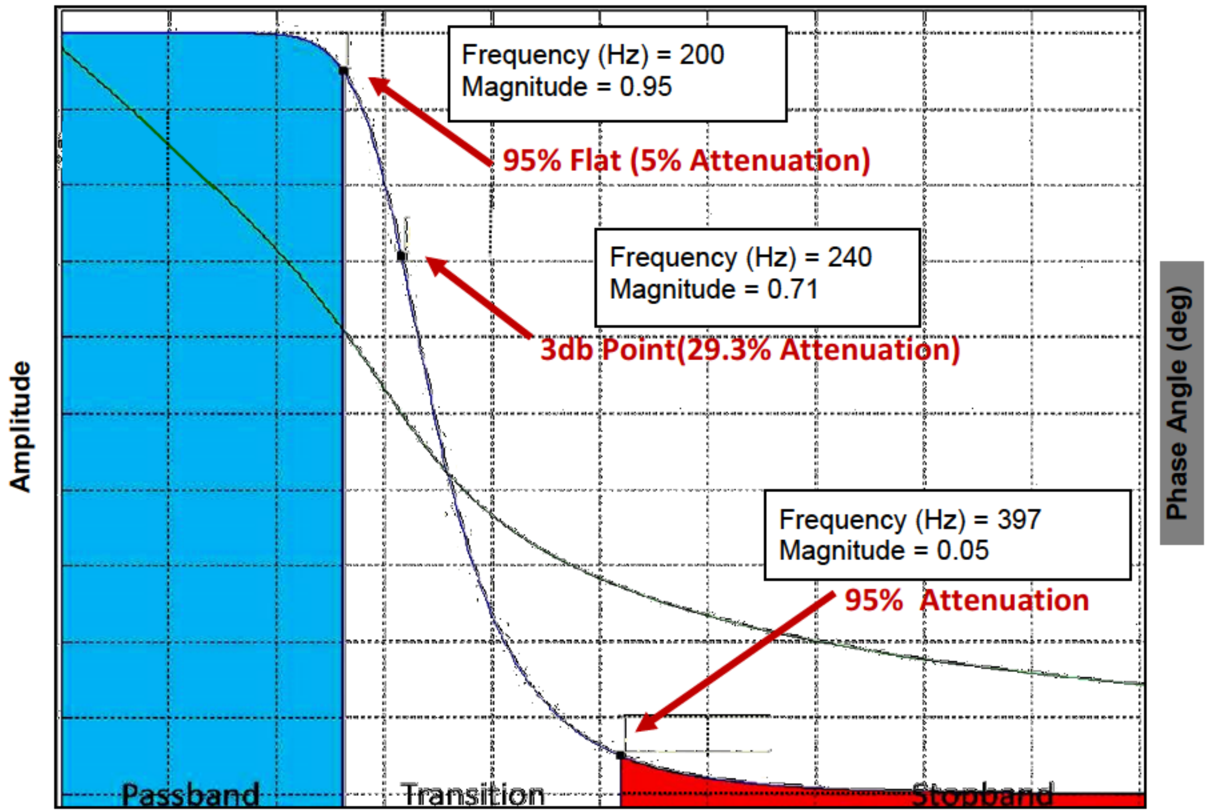
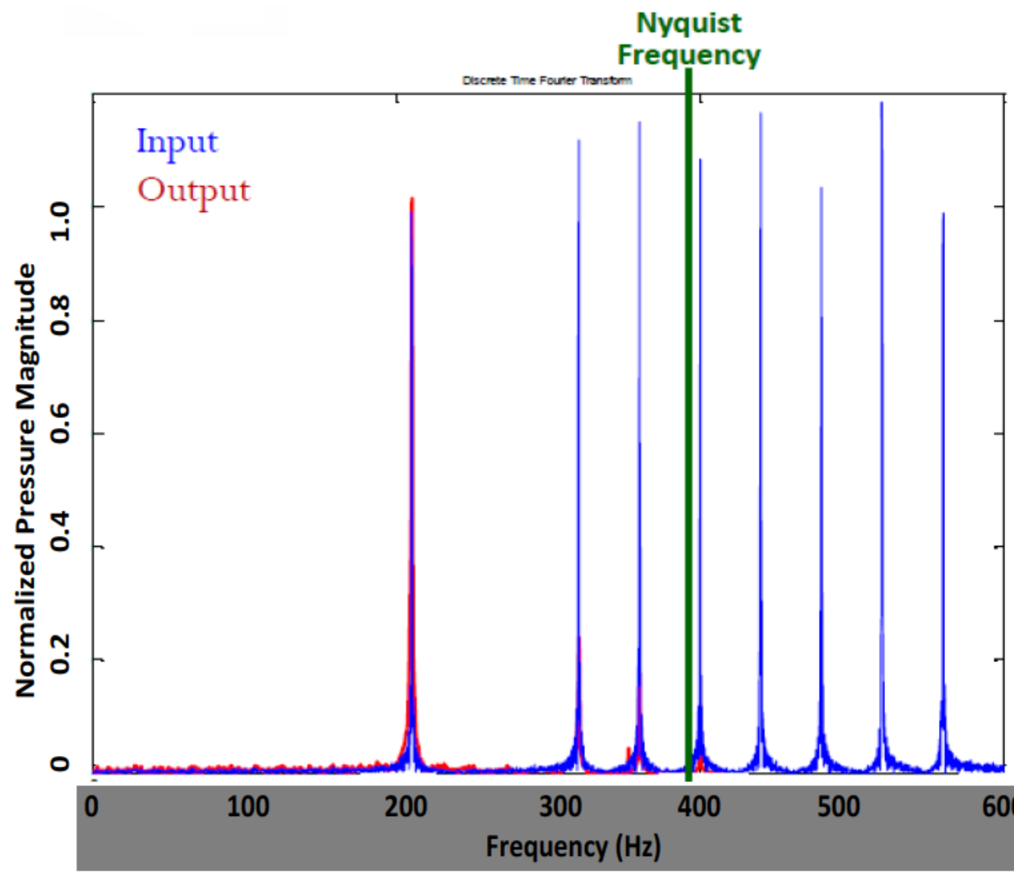


Figure 7 Bode plot showing magnitude characteristics of the 240 Hz low-pass 6-pole Butterworth filter

<u>Good Example</u> – Data Sampling and Filter Selection

Figure 8 PSD plot showing comparison of input and output signals for correct filter selection

Correct filter eliminates aliasing and magnitudes are accurate at 200Hz and below



Input Signal: 200Hz + pulses from 320Hz to 560Hz (every 40Hz) + noise.

Output Signal: 240Hz 6-pole low-pass Butterworth filter and sampled at 794 sps

Summary

<u>Discipline Engineers:</u>

1) **Ultimately responsible** for data quality for evaluating system under test
2) **Guide instro setup** by understanding system operating characteristics, test objectives and analysis approach
3) **Verify proper data acquisition techniques** were applied

<u>Recommend</u>: Use both Bode and PSD plots to evaluate filter and sample rate effects prior to implementation

<u>Reference</u>: The Scientist and Engineer's Guide to Digital Signal Processing, by Steven W. Smith, Ph.D. **http://www.dspguide.com/pdfbook.htm**

Questions ???