Weak Turbulence in Ocean Waves

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LONG-TERM GOALS

Further develop weak turbulence theory, which is used to predict the evolution of spectral energy density in ocean surface waves and internal waves. Isolate, identify and quantify sources of possible discrepancies between numerical solutions of weak turbulence modeling and observations of ocean waves. Obtain theoretical predictions of forms of steady state spectral energy distributions for surface and internal ocean waves.

OBJECTIVES

(A) Apply weak turbulence methodology using rigorous mathematical techniques to identify possible sources of discrepancies between theory and observation of surface gravity waves.

(B) Calculate corrections to the surface wave kinetic equations arising from these discrepancies.

(C) Construct the form of the stationary spectral energy density of wind driven surface gravity waves in view of the modified kinetic equation.

(D) Use weak turbulence theory to predict the stationary spectral energy density of internal waves in the ocean. Compare the results with experimental observations.

(E) Study the coupling between ocean surface and internal waves, in particular the coupling of long gravity waves to internal waves.

APPROACH

Weak (or wave) turbulence is a universal theory used for the statistical description of an ensemble of weakly interacting waves. It has been used for the description of ocean waves since pioneering works of Hasselmann[1] and Zakharov[2].

The key feature of the weak turbulence description is the derivation of the statistical equation for the time evolution of spectral energy density of wave fields. Such an equation is called a kinetic equation. Derivation of the kinetic equation is in turn based on the Hamiltonian structure of the waves in question and multiple time scale expansions of time evolution equations of the statistical averages.

Some key assumptions used for the derivation of the kinetic equations are:

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14. ABSTRACT

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(A) Weak nonlinear interactions between waves.

(B) Random phases of the waves.

(C) Exactly resonant interactions between wave triads, quartets or quintets.

Violation of any of these assumptions or other implicit assumptions used to derive kinetic equations will lead to discrepancies between theory and experiments.

Principal collaborators on this project are my graduate student Boris Pokorni and Prof. Esteban Tabak from Courant Institute. I also work with RPI Prof. Peter Kramer and RPI VIGRE postdoc Joseph Biello on general wave turbulence methodology. I have started to collaborate on internal waves measurements with Dr. Kurt Polzin from WHOI and Prof. Raffaele Ferrari from MIT.

WORK COMPLETED

(A) The Hamiltonian structure for surface waves was known since work by Zakharov [2] and Miles. Following Zakharov's original arguments, the implicit Hamiltonian was expanded in a series in powers of assumed-small wave steepness. The boundary value problem for each order was solved and its solutions were used in the expression for kinetic energy to obtain the Hamiltonian in Fourier space. Fourier space is natural for a weak turbulence description since to leading order waves do not interact, and thus their Fourier coefficients are constants. The downside of Zakharov's approach is that the resulting expressions for interaction coefficients are lengthy.

(B) Alternatively, one can use the smallness of wave steepness to approximate the kinematic equations for the free surface, and then write a Hamiltonian in terms of the canonically conjugated variables, as was done by W. Choi [4]. We have generalized his Hamiltonian for two-dimensional waves. The resulting Hamiltonian is specified in physical space and in fact is much more compact than Zakharov's expressions.

(C) We have verified that the Hamiltonian structures developed by Zakharov and Choi are in fact equivalent. Although the equivalence is intuitive, this statement is not easy to prove.

(D) After establishing the Hamiltonian structure of the waves, the next step in the weak turbulence methodology is to derive the kinetic equation for the spectral energy density evolution. For surface gravity waves in deep water, the interactions of three waves are nonresonant and thus could be excluded by the appropriate near-identity canonical transformation [3]. However, there are triads for which three-wave interactions are *nearly* resonant, so that the resulting transformation is no longer near-identity. Then the question emerges of how to deal with these nearly resonant interactions, and how to write a kinetic equation for them. To address this question a simple model system was considered, and the weak turbulence theory was generalized to include near-resonant interactions [7].

(E) Research on wave-wave interactions in internal waves was continued. In our recent work a novel canonical Hamiltonian structure for long internal waves with frequencies well above the inertial frequency was developed [5]. To achieve that goal, potential flows were assumed, i.e. Coriolis effects were neglected. We have now generalized that Hamiltonian structure to include Coriolis force effects. To achieve this goal the velocity field was decomposed into gradient flow (vorticity-free) and vortex components, and conservation of potential vorticity was used. As a result, novel canonical Hamiltonian structure for internal waves in rotating environment superimposed into arbitrary vertical distribution of potential vorticity was developed [6].

(F) Following weak turbulence methodology we have derived a weak turbulence kinetic equation for the spectral energy density of internal waves in a rotating ocean [6]. The steady state solution in the high frequency limit of this equation was found and it was shown that it is not far from the high frequency limit of the Garrett and Munk spectra of internal waves.

(G) In order to verify that our theory gives realistic distributions of spectral energy density, active collaborations with Dr. Kurt Polzin [WHOI] and Raffaele Ferrari [MIT] were established. We are currently looking into the possibility of experimental verification of weak turbulence predictions.

RESULTS

- The validity of Zakharov's form of the weak turbulence Hamiltonian was reestablished.
- The form of the surface-wave Hamiltoniian in physical space was established, and it was found to coincide with generalization of Choi's Hamiltonian[3]. The resulting Hamiltonian is simpler and more compact than the one in [2].
- The equivalence of these two Hamiltonian structures was established.
- The weak turbulence formalism was generalized for a simple model to include kinetic equations arising from near-resonant interaction of triads of waves [7].
- The canonical Hamiltonian structure for long internal waves in hydrostatic balance in a rotating environment was derived [6].
- The kinetic equation for the spectral energy density evolution of internal waves was found, and its solution in the high frequency limit was derived. This solution is not far from the Garrett and Munk spectra of internal waves in the ocean [6].

IMPACT/APPLICATIONS

Continuing results from this project will significantly enhance our understanding of nonlinear wave interactions in shallow- and deep-water environments and consequently will lead to improved forecasting and prediction for Naval and civilian applications.

TRANSITIONS

RELATED PROJECTS

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PUBLICATIONS

- Published: [5].
- Submitted: [6], [7].