Determining the Stratification of Exchange Flows in Sea Straits

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LONG-TERM GOALS

Our long-term goal is to contribute to our understanding of the factors determining the stratification in sea straits (such as the Bab al Mandab) and in neighboring marginal seas and basins (such as the Red Sea and Gulf of Aden).

OBJECTIVES

Stratification in and around sea straits is determined by a number of factors, including hydraulic control, friction, mixing, and communication with neighboring basins. Although there exists some understanding of these factors in isolation, there are wide gaps in our understanding when they occur together. Our objective is to gain insight into how the stratification is formed when the factors come together, which is normally the situation in case studies.

APPROACH

Our approach emphasizes analytical and numerical modeling but has also involved some data analysis. In some cases modeling efforts are based on general and long-term mysteries, in other cases the modeling effort is motivated by specific observations. Analytical tools include various advanced techniques for finding the propagation speeds of long internal gravity waves for stratified shear flows in channels with nonuniform cross-sections. The numerical models include a 2D code for calculating time-dependent, nonhydrostatic flow over an obstacle and a 3D code for calculating the evolution of 1- and 2-layer flows in rotating channel/basin systems. Since a number of fundamental questions involving the interplay between hydraulic control and mixing remain unanswered, our calculations generally involve idealized settings rather than exact replication of field conditions. Much of the data

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14. ABSTRACT

Our long-term goal is to contribute to our understanding of the factors determining the stratification in sea straits (such as the Bab al Mandab) and in neighboring marginal seas and basins (such as the Red Sea and Gulf of Aden).

15. SUBJECT TERMS

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that has motivated our efforts comes from past and present fieldwork in the Bab al Mandab by Profs. Bill Johns (RSMAS) and Steve Murray (ONR).

WORK COMPLETED

Before reporting on specific work completed, we note that some little effort was directed toward the organization of a meeting entitled "The 2nd Meeting on the Physical Oceanography of Sea Straits". (Funding was also obtained from ONR/IFO). This meeting was organized and directed by L. Pratt with the help of an organizing committee (D. Farmer, M. Gregg, S. Murray, L. Prieur, and D. Smeed). The meeting took place in Villefranche-sur-Mer, France, over April 15-19. The meeting brought together 50 of the world's top authorities on sea straits. A volume of review papers and extended abstracts was produced and this document can be found on http://www.soc.soton.ac.uk/JRD/PROC/STRAIT/ . A meeting summary is available upon request to Pratt. A special issue of *Deep-Sea Research II* will be devoted to review papers based on the meeting. L. Pratt and D. Smeed are acting as guest reviewers. We expect this issue to appear in 2003.

The first work area involves the occurrence and detection of hydraulic control in the presence of continuous stratification, as exists in the Bab al Mandab. In order to better evaluate the hydraulic criticality of a flow at a narrows and sill, it was necessary to develop an extended Taylor-Goldstein equation (Pratt, et al., 2000). The new equation allows for the calculation of the phase speeds of long internal waves for a stratified shear flow in a channel of arbitrary cross-section. Theorems that proceed from this equation are potentially valuable in determining the conditions under which critical levels, instability, and consequential mixing can occur. Much of our work over the past year has been devoted to establishing these theorems. The results appear in a manuscript with C. Jones, J. Deng (both of Brown) and L. Howard (retired) (see Deng et al., 2002, and Pratt et al., 2002). An example of application of one of the theorems to the Bab al Mandab is discussed below.

Concerning our efforts to understand the effects of rotation and coupling with neighboring basins, K. Helfrich has developed a one-layer model that can accommodate special features including hydraulic jumps and bores that arise in hydraulically-driven flows. Further, the model can accommodate strait and basin geometries with general topography. Work has been completed on a study of linkage with an upstream basin containing a source-driven circulation (Helfrich and Pratt, 2002). Helfrich is continuing this study by generalizing the strait topography to be nonrectangular. A similar two-layer model is in the developmental stage and Pratt is completing an analytical two-layer theory for an exchange flow that will be used to test the numerical code.

Pratt, Helfrich and visiting scientist Morten Holtegaard Nielsen are completing a study of the effects of entrainment on the density distribution downstream of a sill. The study makes use of the nonhydrostatic model, which allows for continuous stratification and mixing. A major issue at stake is whether such effects can be reproduced by a layered hydraulic model with entrainment velocities.

RESULTS

A major result from the study by Helfrich and Pratt (2002) on coupling with an upstream basin circulation is the following finding. If fluid is fed into the upstream basin at a fixed rate but variable location the upstream circulation undergoes drastic changes while the characteristics of the flow in the strait remain remarkably fixed. Further, the state observed in the strait is consistent with the maximization of potential energy in the upstream basin. This finding suggests that there may be certain preferred stratifications that may occur in the strait and which are to some extent independent of changes in the upstream basin. The finding also has important implications for how the outflow from large basins can be monitored.

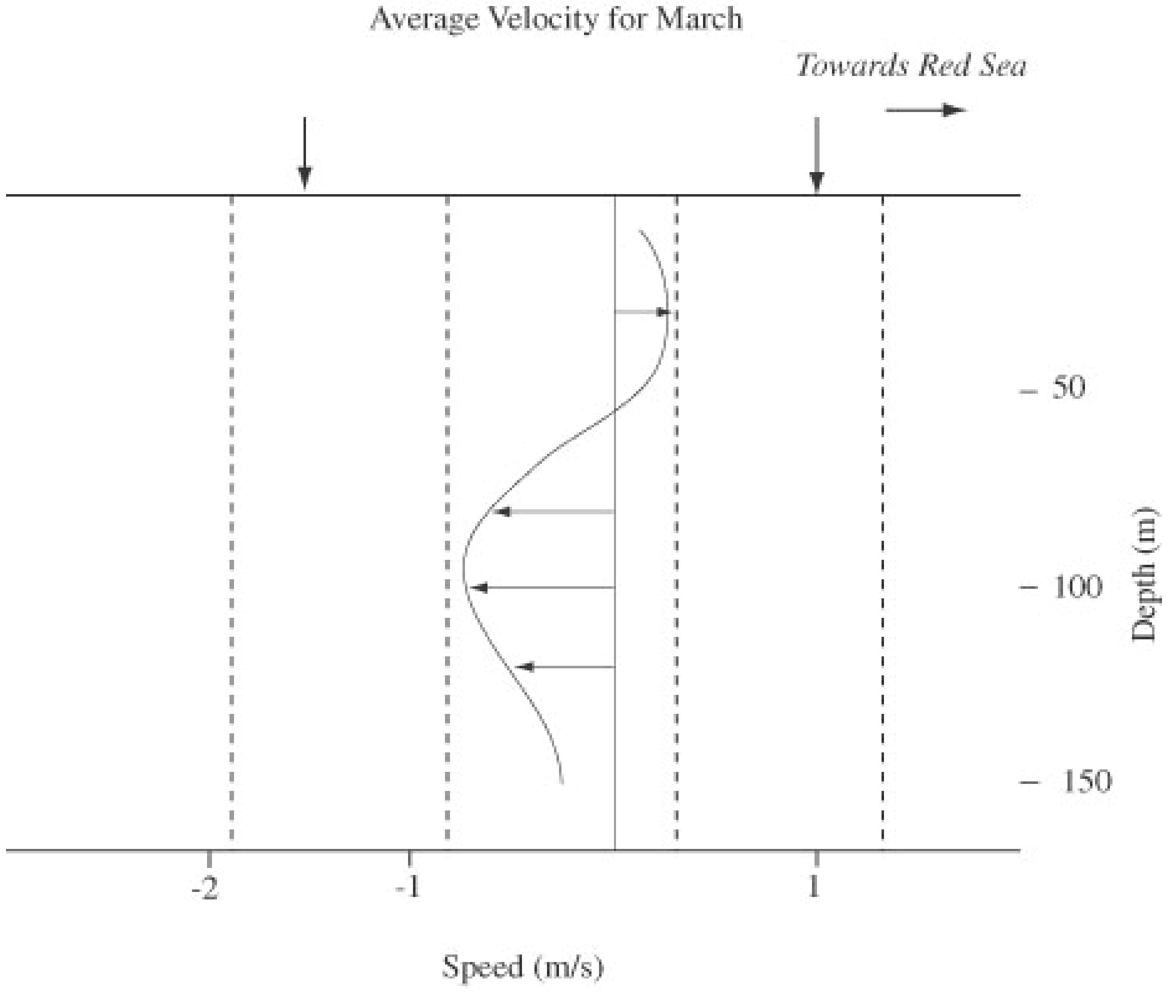
A major finding from the theoretical work on the hydraulics of continuously stratified flows (Deng, et al., 2002) is shown in the accompanying figure. The figure illustrates the practical use of a theorem involving the calculation of the speeds of long, internal waves propagating through a strait and under the influence of continuous stratification and shear, and complicated cross-sectional topographic variations. Knowledge of these speeds is crucial to the understanding of the hydraulics of the flow. If the maximum value $N_{\max}$ of the buoyancy frequency is known along with the minimum and maximum values $U_{\min}$ and $U_{\max}$ of the horizontal velocity, then the phase speeds c_i of all possible waves in question must lie within bands determined by

$$U_{\min} - \frac{DN_{\max}}{(\pi^2 + \frac{1}{4}T_{\min}^2 D^2)^{1/2}} < c_{-1} < c_{-2} < c_{-3} \cdots < U_{\min} < U_{\max} < \cdots c_3 < c_2 < c_1 < U_{\max} + \frac{DN_{\max}}{(\pi^2 + \frac{1}{4}T_{\min}^2 D^2)^{1/2}}$$

where $T(z)$ is a function of depth determined by the topography and D is the maximum depth. In figure below, the phase speeds must lie in the bands indicated by the dashed lines, drawn along side a particular velocity profile. The data was collected by Murray and Johns (1997) for the Bab al Mandab sill. According to the figure, internal long waves can propagate in either direction (towards or away from the Red Sea in this case), meaning that the flow is hydraulically 'subcritical'. The theorem in this case allows the hydraulic character of the flow to be classified (as subcritical, supercritical, or critical) without the need to actually calculate the phase speed. Since direct calculation of the phase speed is quite difficult in the presence of vertically varying shear and stratification, the theorem may provide a useful tool in the interpretation of the hydraulic character of observed strait flows.

TRANSITIONS

As part of the establishment of stability conditions based on the extended Taylor-Goldstein equation, we have developed a new mathematical approach to proving stability. It is not clear yet how general this approach is, but it may prove useful in other stability problems. The theorem illustrated in the accompanying figure should also be of general use to investigators of sea straits. The various numerical codes (the rotating, single-layer, shock-capturing code, the 2D nonhydrostatic code, and a code for solving the extended Taylor-Goldstein equation) are completed and will be available to the community at large after they are tested. The single-layer code has already been used by others in the community, including G. Yuan of Brown University. The nonhydrostatic code is being used by E. Jarosz of LSU to simulate baroclinic tide generation in the Bab al Mandab. A code for solving the extended Taylor-Goldstein equation is being used by Jarosz and by D. Smeed of Southampton.



REFERENCES

Deng, J., L. J. Pratt, L. Howard, and C.K.R.T. Jones, 2002. On stratified shear flow in straits having arbitrary cross sections, *Studies in Applied Math*, submitted.

Helfrich, K. R. and L. J. Pratt, 2002. Rotating hydraulics and upstream basin circulation. *J. Phys. Oceanogr.*, submitted.

Murray, S. P. and W. Johns, 1997. Direct observations of seasonal exchange through the Bab al Mandab Strait. *Geophys. Res. Lett.,* **24**, 2557-2560.

Pratt, L. J., H. Deese, S. Murray, and W. Johns, 2000. Continuous dynamical modes in Straits Having Arbitrary Cross Sections, with Applications to the Bab al Mandab. *J. Phys. Oceanogr.,* **30**, 2515-2534.

Pratt, L. J., 2002. Recent progress in understanding the effects of rotation in sea straits in: *The 2nd Meeting on the Physical Oceanography of Sea Straits*. (Unpublished document.)