Advanced Numerical Methods for Numerical Weather Prediction

Francis X. Giraldo
Naval Research Laboratory
Monterey, CA 93943-5502
phone: (831) 656-4882 fax: (831) 656-4769 e-mail: giraldo@nrlmry.navy.mil

Award #: N0001401WX20257

LONG-TERM GOAL

The long-term goal of this research is to construct the Navy's next generation NWP model using new numerical methods specifically tailored to take full advantage of distributed-memory computers. In order to take full advantage of the new computer architectures the spherical global domain must be partitioned into local sub-domains, or elements, which can then be solved independently on the multiple processors of these computers. The numerical methods used on these sub-domains must be not only local in nature but also high-order in accuracy and highly efficient. Thus the final objective of this project is to construct a new NWP model which is as accurate as the current spectral model while much more efficient thereby allowing for finer resolution forecasts.

OBJECTIVES

The objectives of this project are to construct high-order local methods for the Navy's next generation NWP model. The high-order accuracy of these methods will ensure that the new model yields the same accuracy as the current spectral model while the locality of these methods will ensure that the efficiency of the new model increases.

APPROACH

To meet our objectives we explore the three local high-order methods:

1. Continuous spectral element in space with an explicit Eulerian scheme in time (C-SEM),
2. Discontinuous spectral element in space with an explicit Eulerian scheme in time (D-SEM),
3. Spectral element in space with a semi-implicit semi-Lagrangian scheme in time (SESL).

These methods will be used to discretize the horizontal operators. For this reason we propose to implement these methods to the spherical shallow water equations as these equations contain all of the horizontal operators required in a full 3D NWP model. The power of continuous and discontinuous spectral element methods is that they are high order accurate, like spectral methods, yet are completely local in nature – meaning that the equations are solved independently within each individual element and processor. The reason for exploring discontinuous spectral element methods is they are more local than continuous methods thereby resulting in a potentially more efficient NWP model on distributed-memory computers. In addition, discontinuous methods are not only globally but also locally conservative which translates to proper handling of large gradient phenomena such as fronts and hurricanes. Semi-Lagrangian methods are also being considered because these methods, like high

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE	2. REPORT TYPE	3. DATES COVERED
30 SEP 2001		**00-00-2001 to 00-00-2001**

4. TITLE AND SUBTITLE
Advanced Numerical Methods for Numerical Weather Prediction

5a. CONTRACT NUMBER
5b. GRANT NUMBER
5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)

5d. PROJECT NUMBER
5e. TASK NUMBER
5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Naval Research Laboratory,,Monterey,,CA, 93943

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

13. SUPPLEMENTARY NOTES

14. ABSTRACT
The long-term goal of this research is to construct the Navy???s next generation NWP model using new numerical methods specifically tailored to take full advantage of distributed-memory computers. In order to take full advantage of the new computer architectures the spherical global domain must be partitioned into local sub-domains, or elements, which can then be solved independently on the multiple processors of these computers. The numerical methods used on these sub-domains must be not only local in nature but also high-order in accuracy and highly efficient. Thus the final objective of this project is to construct a new NWP model which is as accurate as the current spectral model while much more efficient thereby allowing for finer resolution forecasts.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a REPORT	b ABSTRACT	c THIS PAGE			
unclassified	**unclassified**	**unclassified**	**Same as Report (SAR)**	**7**	

order methods, have minimal dispersion errors. This property is important for properly capturing atmospheric phenomena. In addition, semi-Lagrangian methods offer vast improvements in computational efficiency due to the longer integration time steps that they permit.

Upon validation of the three horizontal discretization methods, the vertical integration scheme for the full 3D NWP model will be included. The final stage for the dynamics is to include performance-enhancing features such as Helmholtz solvers for the semi-implicit semi-Lagrangian method, as well as Message-Passing Interface (MPI) and Open MP implementations.

WORK COMPLETED

The three horizontal discretization methods (C-SEM, D-SEM, and SESL) have been applied to the shallow water equations on the sphere. These methods have been tested extensively and three papers have been written concerning these innovative approaches (Ref. [1,2,3]). The new spectral element 3D NWP model is undergoing extensive validation. Four test cases have been applied to the new model and this work was presented at a conference in Montreal (Ref. [4]).

RESULTS

Shallow Water Equations. To show the accuracy of the three horizontal discretization methods we show results for the methods for nonlinear zonal geostrophic flow (Case 2 in Ref. [1]). Figure 1 shows the spectral convergence of the 3 methods. It should be mentioned that while C-SEM and D-SEM, which are Eulerian in time, use a Courant-Friedrichs-Levy (CFL) restricted time step, SESL uses a time-step 10 times larger.

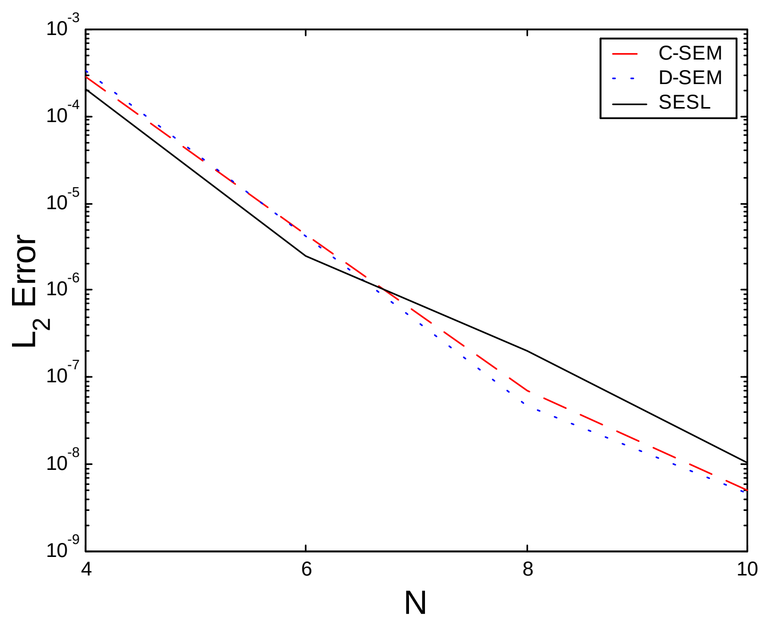

Figure 1: L2 error (y-axis) for the continuous spectral element method (C-SEM), discontinuous spectral element method (D-SEM), and spectral element semi-Lagrangian method (SESL) as a function of basis function order N (x-axis).

Note the high-order accuracy achieved by all three methods. In fact, all three methods converge at the same rate as the spectral method. This result confirms that all three methods are capable of yielding results equal to or better than the current spectral model.

In addition to SESL allowing larger time steps it has been proven that the error can in fact decrease with increasing time step. The error of SESL is bounded as follows

$$O\left(\Delta t^{K}+\frac{\Delta x^{N}}{\Delta t}\right)$$

where K corresponds to the order of integration of the particle trajectory equation

$$\frac{dx}{dt}=u(x,t)$$

and N corresponds to the order of interpolation and hence the order of the spectral element basis functions. The most amazing thing about this order of accuracy result is that as we increase N (as is done in the spectral element method) we can increase the time step without incurring any time discretization errors provided that K is sufficiently high. Figure 2 shows the L2 error versus time step for advection on the sphere for N=16 and K=2 and 4.

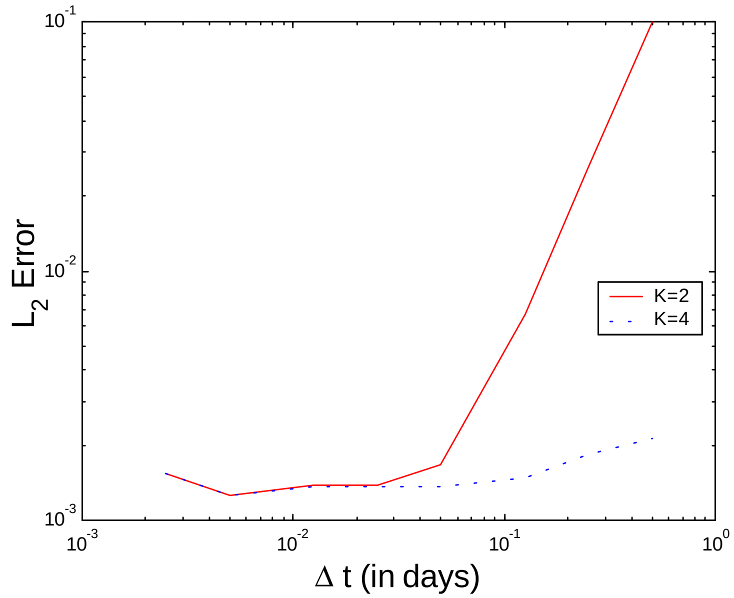

Figure 2: L2 error (y-axis) for the spectral element semi-Lagrangian method (SESL) with N=16 as a function of time-step (x-axis) for K=2 and K=4.

We can see that for K=2 the error increases linearly with time step after 0.005 days. By increasing K to 4 we see that the error remains flat even for time-steps of 0.5 days. The point of this study is to show that the spectral element semi-Lagrangian method (SESL) will not only yield exponentially accurate solutions but it can do so using very large time steps thereby offering increased accuracy and efficiency over any other numerical method.

3D NWP Equations. As a first test to validate the new 3D NWP model, a still, isothermal atmosphere was run for a 100 day integration. The results, as expected, was that very little happened. This test was used to check the stability of the algorithms and to ensure that no "numerical sources" arise.

As a second test, we used Case 1 with a conical mountain embedded along the Equator and ran the code for 100 days. The model showed completely symmetric flow patterns thereby confirming that no biasing is introduced by either the numerical scheme or the grid.

As a third test, we ran Case 2 with the mountain located at 30N degrees latitude. Figure 3 shows the results at the surface for N=10 with L=10 vertical levels after 100 days.

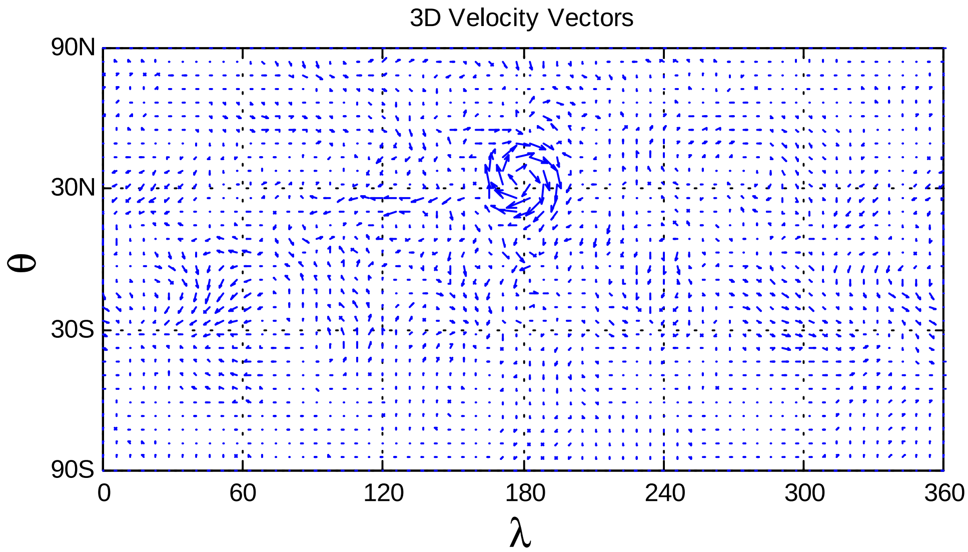

Figure 3: Velocity vectors for flow over a mountain with L=N=10 as a function of longitude (lambda) and latitude (theta) at the surface for a 100 day integration.

As a final case, we ran the Held-Suarez test case. This test case is the primary test run by all new atmospheric models because it mimics a realistic atmosphere with full physics. We ran this case for 100 days and measured conservation of mass and energy. Figure 4 shows the contour plot of the mean zonally-averaged zonal velocity as functions of latitude and vertical coordinates for N= 4 and L=20 vertical levels.

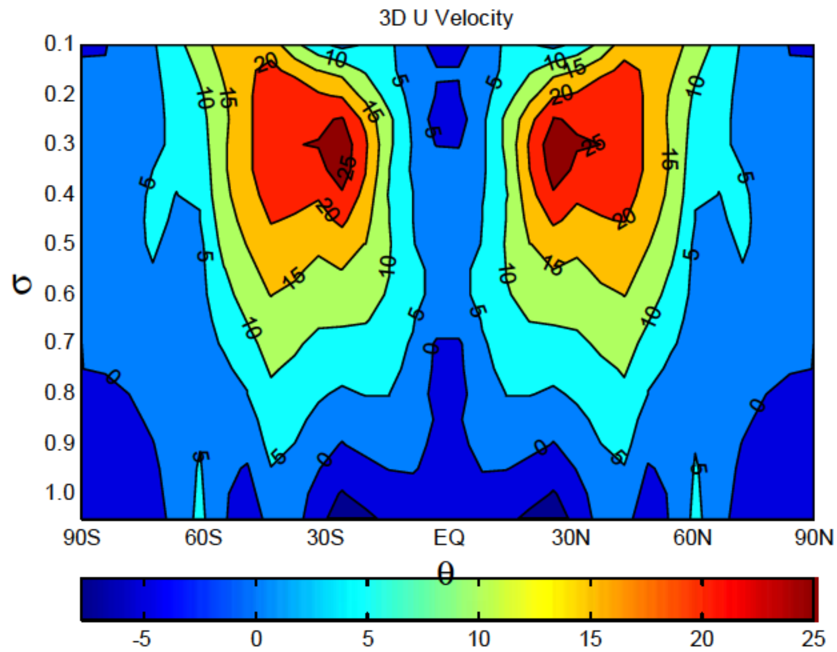

Figure 4: Zonally-averaged zonal velocity (U) for the Held-Suarez test case with N=4, L=20 as a function of latitude (theta) and vertical coordinate (sigma) for a 100 day integration.

Figure 4 compares well with the results from Held and Suarez and clearly shows the formation of the jet stream.

IMPACT

The Navy Operational Global Atmospheric Prediction System (NOGAPS) is run operationally by the Fleet Numerical Meteorology and Oceanography Center (FNMOC) and is the heart of the Navy's operational weather prediction support to nearly all DOD users worldwide. Our work here targets the next generation of this system for the next generation of computer architectures. These architectures are expected to be distributed memory, commodity based systems with enormous theoretical computational power. However, exploiting this capability will require drastically redesigning many important model algorithms.

TRANSITIONS

Improved algorithms for model processes will be transitioned to the 6.4 Large-Scale Atmospheric Models project (PE 0603207N, CAPT Clark, SPAWAR, Project Manager) as they are ready, and will ultimately be transitioned to FNMOC with future NOGAPS upgrades.

RELATED PROJECTS

Some of the technology developed for this project will be used immediately to improve the current spectral formulation of NOGAPS in conjunction with the 6.2 project: Improved Subgrid-scale Physical

Parameterizations for Global Modeling Global (NRL BE-35-2-18). In addition, the work herein will complement the work done on constructing an efficient message passing NOGAPS as is being done in the 6.3 project: Development of Two Complete Numerical Weather Prediction Systems for Heterogeneous Scalable Computing Environments (PE 0603704N).

SUMMARY

We have constructed a semi-implicit spectral element semi-Lagrangian (SESL) and discontinuous spectral element methods for the horizontal operators. In addition we have continued to validate the 3D spectral element NWP model. The results of this year have shown the feasibility of our approach and confirm that the long-term goals of this project will be realized on schedule.

REFERENCES/PUBLICATIONS

1. Giraldo, F. X., 2001: A Spectral Element Shallow Water Model on Spherical Geodesic Grids. *International Journal for Numerical Methods in Fluids, Vol.* 35, pp. 869-901.

2. Giraldo, F. X., 2001: A Spectral Element NWP Model. *Workshop on the Solution of Partial Differential Equations on the Sphere*, Montreal, Canada (May 2001).

3. Giraldo, F. X., Perot J.B., 2001: A Spectral Element Semi-Lagrangian Method for the Spherical Shallow Water Equations. *Journal of Computational Physics,* submitted.

4. Giraldo, F. X., Hesthaven J.S., Warburton, T.E., 2001: Nodal High-Order Discontinuous Galerkin Methods for the Spherical Shallow Water Equations. *Journal of Computational Physics,* submitted.