

# Time-Dependent Modeling of Underwater Explosions by Convolution of Similitude Source with Bandlimited Impulse from the CASS/GRAB Model

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## LIST OF ABBREVIATIONS AND ACRONYMS

CASS/GRAB	Comprehensive Acoustic System Simulation/Gaussian Ray Acoustic Bundle
CW	Continuous wave
ESA	Endangered Species Act
FFT	Fast Fourier transform
FIREX	Firing exercise
MISSILEX	Missile exercise
MMPA	Marine Mammal Protection Act
NSPE	Navy Standard Parabolic Equation
OASES	Ocean Acoustics and Seismic Exploration Synthesis
OAML	Oceanographic and Atmospheric Master Library
PE	Parabolic equation
REFMS	Reflection and Refraction in Multilayered Ocean/Ocean Bottoms with Shear Wave Effects
SAFARI	Seismo-Acoustic Fast-Field Algorithm for Range-Independent
SEL	Sound exposure level
SINKEX	Sink exercise
SPL	Sound pressure level
TORPEX	Torpedo exercise

# **TIME-DEPENDENT MODELING OF UNDERWATER EXPLOSIONS BY CONVOLVING SIMILITUDE SOURCE WITH BANDLIMITED IMPULSE FROM THE CASS/GRAB MODEL**

## **INTRODUCTION**

The research documented in this report was conducted to develop a time-dependent pressure computational model for an explosive source in an underwater acoustic waveguide. This objective was accomplished by convolving a time-dependent (similitude) source with the farfield impulse response from the Comprehensive Acoustic System Simulation/Gaussian Ray Acoustic Bundle (CASS/GRAB) model<sup>1</sup>—a peer-reviewed, ray-theoretic model that is approved by the Oceanographic and Atmospheric Master Library (OAML).

The impulse response was found by performing an inverse fast Fourier transform (FFT) of the frequency-dependent, bandlimited transfer function (Green's function) from CASS/GRAB.<sup>2</sup> The bandlimited transfer function is determined using the CASS/GRAB amplitudes and phases over a range of frequencies. The result of this convolution is a time-dependent pressure field that satisfies the time-dependent hyperbolic wave equation with boundary conditions, which, in turn, produces a time series from the explosive source. In addition to describing the conduct of this investigation, this report presents the algorithm developed for the computational model and provides a comparison of measured and modeled data.

## **TECHNICAL BACKGROUND**

Time domain solutions can be obtained in two fundamental ways. First, the solution can be obtained by the Fourier synthesis method, where the medium transfer function is numerically sampled at Nyquist. The Fourier synthesis method, which involves taking the inverse Fourier transformation in the frequency domain, has a major disadvantage in that it requires many runs to sufficiently cover the frequency domain to properly model the resultant transfer function. This requirement can be problematic because the frequency responses at many frequencies must be independently calculated with a high degree of accuracy—leading to intensive computational calculations and long computational times when dealing with range-depth fields in analysis studies. Note also that the Fourier transform of the source must be taken in this process. For a similitude description of an explosive source, a well-behaved Fourier transform does not exist due to the instantaneous rise and lack of oscillation in the function itself.

Second, a time domain solution can be performed directly by convolving the medium impulse response (time-dependent Green's function) with a transient source. This method, referred to as "Duhamel convolution,"<sup>3</sup> is analogous to applying linear system theory to the

time-dependent inhomogeneous hyperbolic wave equation. For this research, the Duhamel convolution method was used and is explained in the next section.

When a short pulse is transmitted in the ocean, it is spread in time resulting in dispersion in the time domain. Dispersion in the time domain refers to environmental conditions that cause a finite pulse to be spread with respect to time. There are basically two types of time spreads: multipath and distortion dispersion. Multipath dispersion occurs when individual paths arrive at a receiver via different geometric travel times. These paths satisfy Snell's law: each path is not itself spread. Distortion dispersion is a wave effect and depends on frequency. Distortion dispersion is when the individual arrivals are spread in time, which can happen in ray-theoretic models like CASS/GRAB where the amplitudes and phases of the eigenrays depend on frequency. Distortion dispersion occurs in wave models like the Ocean Acoustics and Seismic Exploration Synthesis (OASES)<sup>4,5</sup> because of diffraction and scattering. Individual multipaths are resolved when the transmitted signal bandwidth  $B$  is greater than  $1/\Delta t$ , where  $\Delta t$  is the multipath separation. If, however, there is distortion dispersion, then the individual multipaths are spread due to diffraction/scattering and the entire received signal may appear as a single spread packet.

The narrowband approximation fails to capture the frequency dependence of the amplitudes and phases; hence, only multipath dispersion is determined. No time spread (distortion dispersion) of the individual multipaths is allowed in the narrowband approximation. Distortion dispersion was considered in this research.

Note that the approach taken was in part motivated by the need to augment the undersea acoustic modeling capabilities of the U.S. Navy by creating a rigorous, well-understood model for the farfield acoustic pressure in the time domain for shock waves. One specific modeling priority for the Navy is modeling the impacts of underwater explosions from Navy training and testing activities to help determine the potential biological impacts on various marine species. This environmental modeling is necessary so the Navy can meet permitting requirements for all Navy activities. For example, training exercises such as firing exercises (FIREXs), missile exercises (MISSILEXs), torpedo exercises (TORPEXs), and sink exercises (SINKEXs) are activities that require permitting and include the modeling and analysis of the effects of underwater explosions on marine species to satisfy the Marine Mammal Protection Act (MMPA) and Endangered Species Act (ESA). Additional applications for an ocean waveguide explosive model like the method proposed in this report include modeling the acoustic effects of large air-gun arrays used for ocean-based seismic surveys and swimmer deterrence.

This model would add to the current OAML-approved models by extension into the time domain. It is believed that the present convolution method is a practical computational efficient method that should be considered as a rigorous addition to an OAML model. The CASS/GRAB Duhamel convolution method has been programmed in MATLAB as proof-of-concept and for comparison studies; however, the algorithm can be easily implemented into the CASS/GRAB FORTRAN and/or C++ source code, if desired, to extend CASS/GRAB results into the broadband domain explicitly for future broadband underwater acoustic propagation analysis efforts.

In the absence of an OAML-approved model, the Navy uses the Reflection and Refraction in Multilayered Ocean/Ocean Bottoms with Shear Wave Effects (REFMS)<sup>6</sup> for long-range underwater explosions modeling. REFMS uses the Cagniard<sup>7</sup> method coupled with the Spencer generalized ray theory<sup>8</sup> to determine the linear acoustics impulse response. REFMS is not OAML-approved. Unlike REFMS, the CASS/GRAB model (1) can utilize multiple sound speed environmental profiles that change depending on the distance from the explosive source and (2) is not dependent on homogeneous layers. The homogeneous layer requirement in REFMS is unrealistic; moreover, REFMS also calculates reflection at the interface of all homogeneous layers, using velocity and density in the calculations. Since the ocean has a non-constant density, reflection is calculated at every interface, leading to “trapped” ray paths in a single layer, which is not practical in approximating a waveguide with continuous parameters. Additionally, CASS/GRAB can include the effects of variance in bathymetry and sediment in the propagation loss calculations, whereas REFMS assumes a flat bottom and the same environmental conditions everywhere.

Regarding the acoustic effects of air guns, no standard procedure exists, but the usual method involves using a parabolic equation (PE) code, typically the Navy Standard Parabolic Equation (NSPE) model, and obtaining farfield transmission loss. An air-gun source can be easily modeled with the method described in this report.

## TECHNICAL APPROACH

This research used the Duhamel convolution to solve the time-dependent wave equation for reasons outlined previously. Duhamel’s formula is given for the problem of a point general source in an inhomogeneous environment.

A time-series pressure model is given here for an explosive source in an ocean channel based on convolving a bandlimited impulse from the CASS model with a similitude source. A bandlimited impulse response is a technique to allow for the frequency dependence of large bandwidth sources like explosives. The approach taken in this study was to keep the frequency dependence in the phases but to evaluate the amplitudes at the carrier frequency. The term *bandlimited* is understood to mean the source bandwidth since the coherent bandwidth of the medium, without scattering, is infinite; therefore, because the source bandwidth is finite; it acts as a filter on the medium bandwidth. The ocean, therefore, is bandlimited by physical sources that are bandlimited and thus act as a filter on the medium. This method is similar to that used by Jensen, Kuperman, et al.<sup>5</sup> in the section on frequency windowing.

This report provides a time-series pressure model for transient sources like explosives and air guns by convolving a time-dependent similitude source with the Green’s function impulse response from CASS/GRAB. The result of this convolution is a time-dependent pressure field that satisfies the time-dependent hyperbolic wave equation with boundary conditions. The impulse response is found by performing an inverse FFT of the frequency-dependent, bandlimited transfer function. This bandlimited transfer function is necessary to allow for the spread in time

(distortion dispersion) of each CASS/GRAB multipath. An inverse FFT is then performed on the bandlimited transfer function to obtain a bandlimited impulse function. The final time-dependent pressure for an explosive is determined by a convolution of the source with the bandlimited impulse response.

If one makes the narrowband (quasi-monochromatic) approximation on the eigenrays from CASS/GRAB, then the time domain signal gives a sequence of amplitude modified replicas of the original waveform. The quasi-monochromatic approximation says that the bandwidth over the carrier frequency is less than one. While this is a good approximation for the eigenray amplitudes; it is not good for the eigenray phases. It is not possible to evaluate the amplitudes and phases of the eigenrays at Nyquist for bandwidths greater than 1000 Hz. A good approximation is to sample the phases within the bandwidth of the signal, and the amplitudes at the carrier frequency. Once this bandlimited medium impulse is found, then the received field is simply the convolution of the bandlimited impulse with the transmitted waveform. Each of the received eigenray paths is now spread in time.

The end result of this research is a model that is usable for transient shock waveforms; moreover, as a corollary of the effort, any general broadband waveform can be handled within this method—thus extending the model application to any broadband source modeled within the range-dependent ocean waveguide.

The transfer function (pressure Green's function) is determined from the CASS/GRAB model. The similitude source is based on the work of Hopkinson,<sup>9</sup> Arons,<sup>10</sup> Cole,<sup>11</sup> and Swisdak.<sup>12</sup> Arons, Cole, and Swisdak summarized most underwater explosive results after World War II. Most work on similitude is based on experimental data using dimensional analysis. Rogers<sup>13</sup> has derived the similitude equations from weak shock theory.

## **ALGORITHM TO IMPLEMENT THE DUHAMEL CONVOLUTION**

In this section, the inhomogeneous hyperbolic wave equation is solved using the Duhamel convolution integral for the time-dependent inhomogeneous hyperbolic wave equation. This treatment is more general than Fourier synthesis because there are signals that do not possess a Fourier transform. For example, for initial value problems, Fourier transforms do not exist. Additionally, for large bandwidths, it is computationally intensive to satisfy Nyquist in performing a Fourier synthesis. For example, the Seismo-Acoustic Fast Field Algorithm for Range Independent (SAFARI)<sup>14</sup> environments and OASES take hours to run for bandwidths greater than 1 kHz. The Duhamel convolution is the mathematical counterpart of the Huygens principle and is derived using the Green's function  $h(t - t', \bar{r})$  for a point time-dependent source in an inhomogeneous medium.

Modeling of a finite pulse in the ocean channel requires solving the time-dependent wave equation, as shown in equation (1):

$$\left( \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) P(r, z, t) = -S(t) \delta(\vec{r} - \vec{r}_0). \quad (1)$$

where  $S(t)$  is a real point-radiated signal ( $MT^{-2}$ ) at the channel (filter) input and the instantaneous pressure at the output is given by  $P(r, z, t)$ . The pressure is defined dimensionally by  $ML^{-1}T^{-2}$ .

The Laplacian  $\nabla^2$  is given in cylindrical coordinates (with variable density  $\rho$ ) as

$$\nabla^2 = \rho \nabla \cdot \left( \frac{1}{\rho} \nabla P(r, z, t) \right). \quad (1a)$$

Therefore equation (1) becomes (assuming azimuthal symmetry):

$$\left( \nabla^2 - \frac{1}{c(r, z)^2} \frac{\partial^2}{\partial t^2} \right) P(r, z, t) = -S(t) \delta(\vec{r} - \vec{r}_0) = -S(t) \frac{\delta(r) \delta(z - z_s)}{2\pi r}. \quad (1b)$$

Equation (1) can be symbolically written as

$$\mathfrak{L} \cdot P(r, z, t) = -S(t) \delta(\vec{r} - \vec{r}_0). \quad (2)$$

where the linear differential operator  $\mathfrak{L}$  is defined by

$$\mathfrak{L} = \left( \nabla^2 - \frac{1}{c(r, z)^2} \frac{\partial^2}{\partial t^2} \right). \quad (2a)$$

Now, define a kernel or Green's function impulse response  $h(t - t', \vec{r})$  for point source in space and time such that

$$\mathfrak{L}h(t - t', \vec{r}) = \delta(\vec{r} - \vec{r}_0) \delta(t - t'), \quad (3)$$

where the impulse Green's function  $h(t - t', \vec{r})$  depends only on relative time and distance. Note that all terms in equation (3) must be dimensionally homogeneous, with each term of dimension  $[L^{-3}T^{-1}]$  and the three-dimensional time-dependent impulse  $h(t - t', \vec{r})$  has dimensions of  $[L^{-1}T^{-1}]$ .

Multiply both sides by  $S(t')$  and integrate with respect to  $t'$ :

$$\int \mathfrak{L}h(t - t', \vec{r}) S(t') dt' = - \int \delta(\vec{r} - \vec{r}_0) \delta(t - t') S(t') dt'. \quad (4)$$

Time integrals are assumed to have limits  $[0, \infty)$ . If all initial conditions are assumed to be zero, then the order of differentiation and integration can be interchanged since  $\mathfrak{L}$  was defined to be linear in order to give

$$\mathfrak{L} \int h(t - t', \vec{r}) S(t') dt' = -S(t) \delta(\vec{r} - \vec{r}_0). \quad (5)$$

Comparison of equation (5) with equation (2) gives the following Duhamel integral for the  $\mathfrak{L}$  operator using the impulse Green's function  $h(t - t', \vec{r})$ :

$$P(r, z, t) = \int_{causal} S(t') h(r, z, t - t') dt'. \quad (6)$$

By applying properties of Fourier transforms, one obtains the usual Fourier synthesis solution:

$$P(r, z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega) H(r, z, \omega) e^{-i\omega t} d\omega. \quad (7)$$

assuming the Fourier transform of the signal  $S(t)$  exists.

$S(\omega)$  is the spectrum of the source  $S(t)$  defined as

$$S(\omega) = \int_{-\infty}^{\infty} S(t) e^{i\omega t} dt. \quad (8)$$

$H(r, z, \omega)$  is the spatial Green's transfer<sup>15</sup> function satisfying the following elliptic Helmholtz wave equation:

$$[\nabla^2 + k^2(r)] H(r, z, \omega) = -\frac{\delta(r) \delta(z - z_s)}{2\pi r}. \quad (9)$$

$H(r, z, \omega)$  has dimensions of  $[L^{-1}]$  and

$$h(t - t', r) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(r, z, \omega) e^{-i\omega t} d\omega, \quad (10)$$

so that the impulse response  $h(t - t', r)$  is determined from the CASS/GRAB transfer function  $H(r, z, \omega)$  by the inverse Fourier transform.

## BANDLIMITED CASS TRANSFER FUNCTION FROM THE CASS EIGENRAYS

The Green's function pressure  $H(r, z, \omega)$  can be expressed as a traveling wave form for the CASS/GRAB model as follows:

$$H(r, z, \omega) = \sum_{n=1}^N A_n e^{i\omega\tau_n + i\theta_n}, \quad (11)$$

where  $A_n$  is path amplitude,  $\tau_n$  is the delay travel time for the various eigenray paths, and  $\theta_n$  is the eigenray phase associated with boundary and caustic phases shifts.

The Green's function impulse function  $h(t - t', r)$  is defined as the Fourier transform of the transfer function  $H(r, z, \omega)$ :

$$h(t - t', r) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(r, z, \omega) e^{-i\omega t} d\omega. \quad (12)$$

If the amplitudes  $A$ 's and phases are evaluated at the carrier frequency (quasi-monochromatic approximation), then one can obtain the impulse function  $h(t - t', r)$ :

$$h(t - t', r) = \sum_{n=1}^N A_n \delta(t - \tau_n) e^{i\theta_n}. \quad (13)$$

## SIMILITUDE DESCRIPTION OF EXPLOSION SOURCE

The initial source description of an explosion can be represented by equation (14):

$$P(r, z, t) = P_m e^{-\left(\frac{t}{\theta}\right)}, \quad (14)$$

where  $P_m$  is the initial maximum pressure and  $\theta$  is the time constant. The time constant is the time over which the pressure-time history can be approximated with an exponential decay. Over practical ranges of interest, it has been empirically established that shock-wave pressures decay at later times more slowly than those of an exponential decay.

The maximum pressure  $P_m$  and the time constant  $\theta$  are defined by

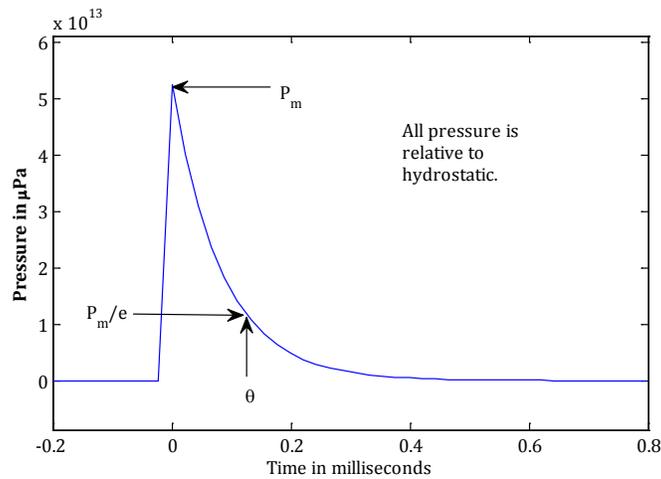
$$P_m = K \cdot \left( \frac{\sqrt[3]{W}}{R} \right)^\alpha \text{ MPa}, \quad (15)$$

and

$$\theta = K_2 \cdot \sqrt[3]{W} \cdot \left( \frac{\sqrt[3]{W}}{R} \right)^{\alpha_2} \text{ ms}, \quad (16)$$

where distance  $R$  is in meters (m), pressure  $P$  is in megaPascals (MPa), time constant  $\theta$  is in milliseconds (ms), and the explosive weight  $W$  is in kilograms (kg).

The coefficients ( $K$ ,  $K_2$ ,  $\alpha$ , and  $\alpha_2$ ) are specific to a given explosive type. The inverse scaled range  $\sqrt[3]{W} \cdot R^{-1}$ , the explosive weight divided by the slant range ( $R$ ), is an important term: it is the famous Hopkinson “cube root” scaling rule.<sup>9</sup> The cube-root scaling rule can be obtained using similitude analysis; it allows the comparison of differing explosive weights and provides the means to scale the pressure, energy, and effects on acoustic receivers from an underwater explosion. A typical shock wave for an explosion is shown in figure 1.



**Figure 1. Similitude Time Series**

## SHOCK-WAVE PRESSURE TIME SERIES USING QUASI-MONOCROMATIC APPROXIMATION FROM EQUATION (11)

The simple case of only multipath dispersion is given by convolving the similitude source  $S(t)$  shown in equation (17) at a distance of 1 meter:

$$S(t) = 52.4 \cdot (\sqrt[3]{W})^{1.13} \cdot e^{-\left(\frac{t}{\theta}\right)} \cdot 10^{12} \mu Pa, \quad (17)$$

where the weight of the charge  $W$  is in Kg; the nonlinear time constant is given by  $\theta$ ; and  $R^{-0.13}$  is the similitude correction at path distance  $R$ . This similitude correction is thought to account for the losses associated with energy dissipated at the shock front as well as the usual absorption losses associated with linear acoustics.<sup>16</sup> (Clay and Medwin<sup>17</sup> attribute the similitude correction to “excess attenuation at the shock front.”)

$S(t)$  is convolved here with the impulse response  $h(t - t', r)$  from equation (13); that is,  $P(r, z, t) = h(t - t', r) \otimes S(t)$  to obtain:

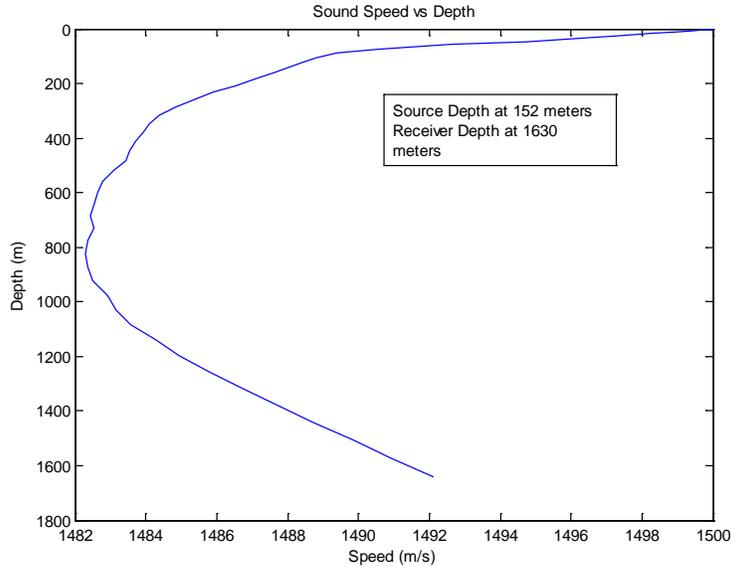
$$P(r, z, t) = 52.4 \cdot \sqrt[3]{W}^{1.13} \cdot e^{-\left(\frac{t}{\theta}\right)} \cdot 10^{12} \otimes \sum_{n=1}^N A_n \delta(t - \tau_n) e^{i\theta_n} \cdot R^{-0.13} \mu Pa. \quad (18)$$

In general, the convolution given by equation (6) will be implemented with a bandlimited impulse response using the eigenray outputs from CASS/GRAB. Frequency is included in the path phases but not the amplitudes, which is the same as assuming the amplitudes satisfy the quasi-monochromatic approximation stated by Born.<sup>18</sup> Another underlying assumption is that the variance in phase as a function of frequency can be approximated in a linear fashion.

These computations are done as a follow-on process in MATLAB by formatting the CASS/GRAB output with replicated matrices for the linear terms representing amplitude, frequency, and phase over the respective bandwidth. The result of this replicated system of matrices yields a bandlimited transfer function that can be inverse Fourier transformed into a bandlimited impulse response. Then the bandlimited impulse response is convolved with the similitude equations to provide the following quantities: (1) instantaneous pressure time series, (2) peak pressure (sound pressure level (SPL)), (3) positive impulse, and (4) total energy spectra (sound exposure level (SEL)).

## CASS CONVOLUTION WITH AND WITHOUT DISTORTION DISPERSION

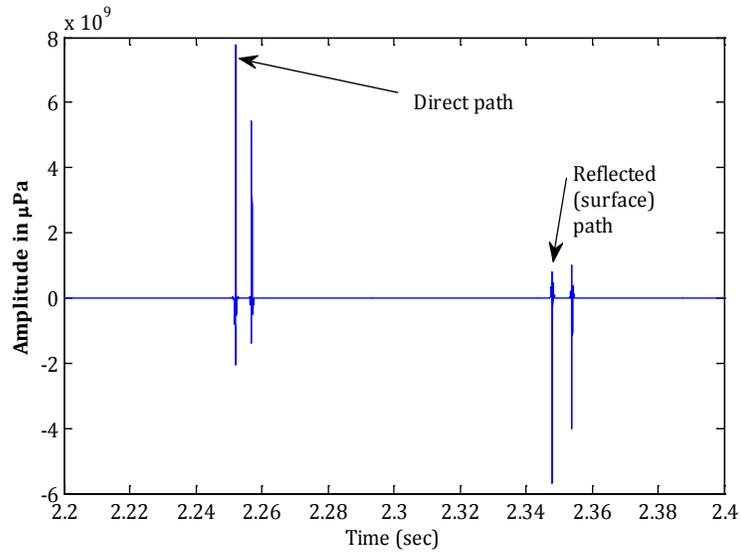
This section provides a graphical comparison of the pressure time series with only multipath dispersion (equation (11)) and with distortion dispersion, which is allowed in the CASS/GRAB frequency-dependent eigenrays (equation (9)). The frequency sampling rate used in this example is  $2^{12}$  Hz. The sound speed profile used in this modeling is shown in figure 2.



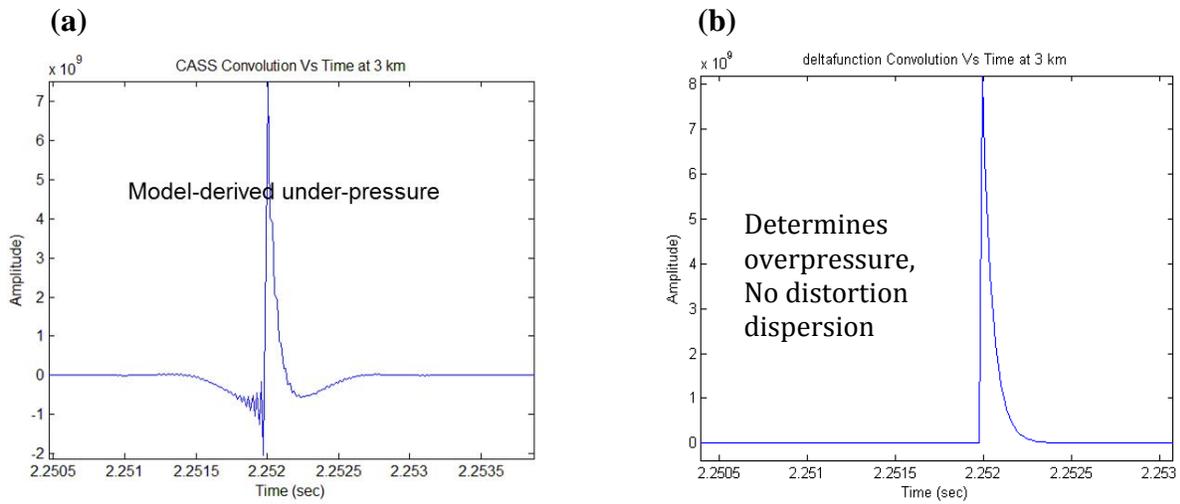
**Figure 2. Sound Speed Profile**

Figure 3 shows the multipath eigenray travel time structure for the sound speed profile shown in figure 2. The arrival times compare within a hundredth of a second with those of the output of a comparable run in a wavenumber integration model, OASES. The model was run at a distance of 3 km from the source.

Figure 4 (a) and figure 4 (b) show the effect of dispersion on the first multipath. Figure 4 (a) illustrates model-derived under-pressure and distortion (time spread) dispersion due to the frequency dependence of the phase in equation (9). Figure 4 (b) shows the lack of distortion dispersion using equation (11) where the impulse  $h(t - t', r)$  is proportional to the delta function  $\delta(t - \tau_n)$ . Also note that the results shown in figure 4 (b) violate conservational physical laws.



**Figure 3. CASS/GRAB Multipath Eigenray Travel Times**



**Figure 4. Dispersion Comparison: (a) Model-Derived Under Pressure and Distortion Dispersion and (b) No Distortion Dispersion**

## COMPARISON OF SILVER STRAND SHALLOW-WATER DATA WITH THE CASS CONVOLUTION MODEL

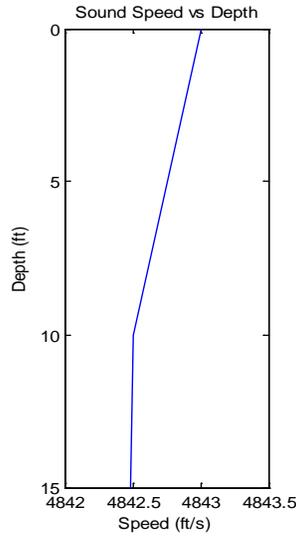
The shock-wave time series experimental data from the Silver Strand<sup>19, 20</sup> experiment were compared with time series from the CASS/GRAB convolution model. The Silver Strand data are a collection of experimental data in very shallow water explosion tests at Naval Amphibious Base, Coronado, CA, and San Clemente Island, CA.<sup>19</sup>

Two cases were considered for this comparison: shot 5209 and shot 5211. Pressure time-series predictions were calculated using a CASS/GRAB convolution model. Peak pressure was then calculated and compared with the Silver Strand peak pressure data. The results show that the peak pressure in pounds per square inch were the same as the data for shot 5209 and within 1 psi for shot 5211. Peak pressure is important because it is used in calculating SPL. Because it is difficult to determine how long to integrate the impulsive time series, the usual definition for SPL is not practical for impulsive sources. The Silver Strand report compares data with the REFMS model and an isovelocity model.<sup>20</sup> The isovelocity model is an Excel spreadsheet that calculates peak pressure, energy flux density, and impulse based on the similitude equations in an isovelocity ocean without boundaries. The isovelocity peak pressure was consistently greater than that in the experimental data and the convolution CASS model because the isovelocity model does not include the frequency dependence necessary for distortion dispersion. The distortion dispersion causes each multipath to be time spread and causes a decrease in the multipath amplitudes, hence a lower peak.

The MATLAB-implemented convolution was run with a center frequency of 2048 Hz, a bandwidth of 4096 Hz, and a sampling frequency of 8192 Hz. This bandlimited impulse was used in the convolution of the source with the medium impulse response to allow for time dispersion. It was found that the term  $(1/R)^{0.13}$  was necessary to account for nonlinear attenuation effects and for nonlinear range effects on the propagating shock wave.

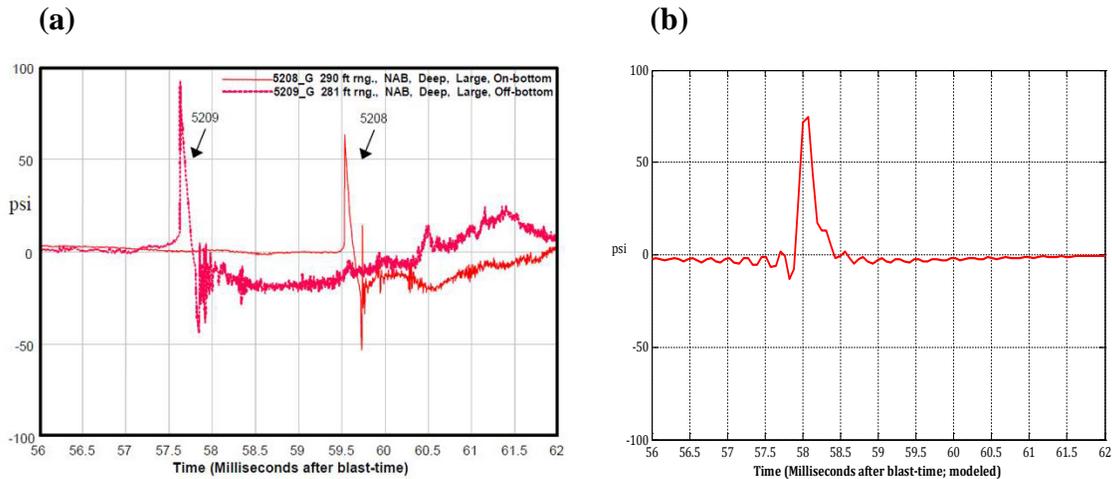
The convolution CASS/GRAB model was run with the experimental sound speed profile (see figure 5), the volume attenuation model developed by François,<sup>1</sup> and a center frequency of 2048 Hz. The source was 2 feet off bottom, and the receiver was at 7.5 feet.

The Silver Strand experiment was performed with two sizes of composition C4 charges (2 lb and 15 lb). Only the 15-lb charge was modeled at the mid-range of 281 feet and at the larger range of 892 feet. The data obtained were for on-bottom and off-bottom charge depths. Modeling a transient source in the sediment would typically require a finite-element approach. The OASES model can input a continuous wave (CW)-type source on the ocean bottom, but, because it uses Fourier transforms, OASES cannot be used.



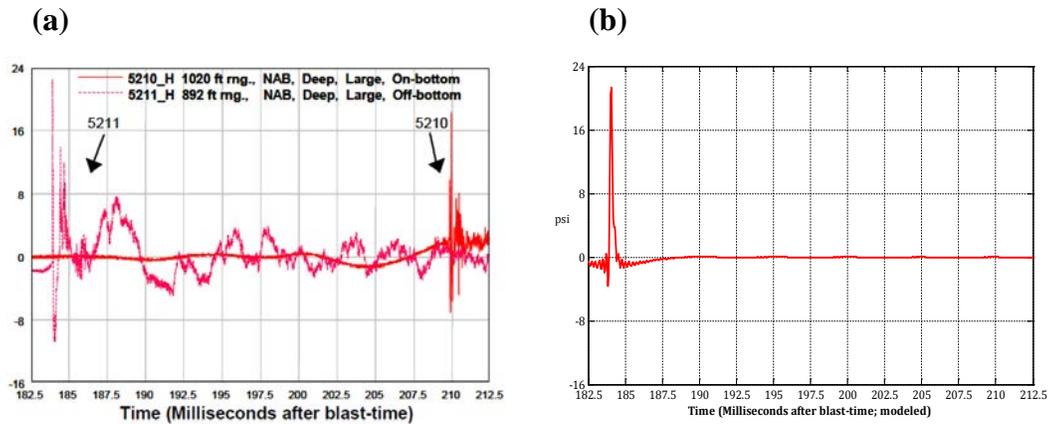
**Figure 5. Measured Sound Speed Profile for Silver Strand Experiments**

Figures 6 (a) and 6 (b) compare data measured in the Silver Strand study (6 (a)) for shots 5208 and 5209 and the CASS/GRAB convolution method (6 (b)) for shot 5209. The measured peak from the Silver Strand data is 75 psi with an arrival time of approximately 57.6 msec. Using the same charge weight (15 lb), range (85.6 m), source depth (4.1 m), and receiver depth (3.5 m), the CASS/GRAB convolution modeled a peak level of 74.75 psi at an arrival time of 58.07 msec.



**Figure 6. Comparison of (a) Measured Pressure Data for Shots 5208 and 5209 in the Silver Strand Study and (b) Modeled Pressure Data for Shot 5209 Using CASS/GRAB Convolution Method**

Figures 7 (a) and 7 (b) show comparisons of data measured in the Silver Strand study (7 (a)) for shots 5210 and 5211 and the CASS/GRAB convolution method (7 (b)) for shot 5211. The measured peak from the Silver Strand data is 23 psi with an arrival time of approximately 184 msec. Using the same charge weight (15 lb), range (272 m), source depth (4.1 m), and receiver depth (3.5 m), the CASS/GRAB convolution modeled a peak level of 21.3 psi at an arrival time of 184.1 msec.



**Figure 7. Comparison of (a) Measured Pressure Data for Shots 5210 and 5211 in the Silver Strand Study and (b) Modeled Pressure Data for Shot 5211 Using CASS/GRAB Convolution Method**

## SUMMARY

This research successfully developed a time-dependent pressure computational model for an explosive source in an underwater acoustic waveguide. The model was accomplished by convolving a time-dependent (similitude) source with the farfield impulse response from the CASS/GRAB model. The impulse response was found by performing an inverse fast Fourier transform of the frequency-dependent, bandlimited transfer function from CASS/GRAB. The bandlimited transfer function was determined using the CASS/GRAB amplitudes and phases over a range of frequencies. The result of this convolution is a time-dependent pressure field that satisfies the time-dependent hyperbolic wave equation with boundary conditions, which, in turn, produces a time series from the explosive source. This report presents the algorithm developed for the computational model and provides a comparison of measured and modeled data.

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