AFRL-AFOSR-VA-TR-2015-0277

# UNSTEADY OUTPUT-BASED ADAPTIVE SIMULATION OF SEPARATED AND TRANSITIONAL FLOWS

Krzysztof Fidkowski
UNIVERSITY OF MICHIGAN ANN ARBOR

03/19/2015
Final Report

Air Force Research Laboratory
AF Office Of Scientific Research (AFOSR)/ RTA2
Arlington, Virginia 22203
Air Force Materiel Command

## REPORT DOCUMENTATION PAGE

*Form Approved*
*OMB No. 0704-0188*

| | | |
|---|---|---|
| **1. REPORT DATE** *(DD-MM-YYYY)*<br>31-03-2015 | **2. REPORT TYPE**<br>Final | **3. DATES COVERED** *(From - To)*<br>15/05/2011 -- 14/12/2014 |

**4. TITLE AND SUBTITLE**
(YIP 2011) UNSTEADY OUTPUT-BASED ADAPTIVE SIMULATION OF SEPARATED AND TRANSITIONAL FLOWS

**5a. CONTRACT NUMBER**

**5b. GRANT NUMBER**
FA9550-11-1-0081

**5c. PROGRAM ELEMENT NUMBER**

**6. AUTHOR(S)**
Fidkowski, Krzysztof, J.

**5d. PROJECT NUMBER**

**5e. TASK NUMBER**

**5f. WORK UNIT NUMBER**

**7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)**
The Regents of the University of Michigan
University of Michigan, ORSP
3003 South State Street
Ann Arbor, MI 48109-1274

**8. PERFORMING ORGANIZATION REPORT NUMBER**

**9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)**
AFOSR/RSL
875 North Randolph Street
Suite 325, Room 3112
Arlington, VA 22203

**10. SPONSOR/MONITOR'S ACRONYM(S)**
AFOSR

**11. SPONSOR/MONITOR'S REPORT NUMBER(S)**

**12. DISTRIBUTION/AVAILABILITY STATEMENT**
Approval for public release; distribution is unlimited.

**13. SUPPLEMENTARY NOTES**

**14. ABSTRACT**

The goal of this project was to investigate and overcome the difficulties of extending output-based error estimation and mesh adaptation from steady to unsteady simulations of aerodynamics problems relevant to the U.S. Air Force. These methods have a direct impact on robustness, through estimates of numerical error on engineering outputs, and efficiency, through space-time mesh adaptive methodology, of unsteady simulations. The project led to significant advances in adjoint-based error estimates for large-scale, nonlinear, time-dependent problems on deformable domains, and in their use in driving efficient combined space-time mesh refinement. The methods were applied to several transient aerodynamics problems, culminating with a three-dimensional adaptive flapping-wing simulation. Approaches for reducing the cost of the methods were also investigated, including a hybridized discontinuous Galerkin discretization and entropy-adjoint unsteady error estimates. The findings were disseminated in a variety of presentations, conference proceedings, and archival journal publications.

**15. SUBJECT TERMS**

| **16. SECURITY CLASSIFICATION OF:** | | | **17. LIMITATION OF ABSTRACT** | **18. NUMBER OF PAGES** | **19a. NAME OF RESPONSIBLE PERSON** |
|---|---|---|---|---|---|
| **a. REPORT** | **b. ABSTRACT** | **c. THIS PAGE** | | | Dr. Krzysztof J. Fidkowski |
| U | U | U | UU | 10 | **19b. TELEPHONE NUMBER** *(Include area code)*<br>734-615-7247 |

Reset

**Standard Form 298** (Rev. 8/98)
Prescribed by ANSI Std. Z39.18
Adobe Professional 7.0

## INSTRUCTIONS FOR COMPLETING SF 298

**1. REPORT DATE.** Full publication date, including day, month, if available. Must cite at least the year and be Year 2000 compliant, e.g. 30-06-1998; xx-06-1998; xx-xx-1998.

**2. REPORT TYPE.** State the type of report, such as final, technical, interim, memorandum, master's thesis, progress, quarterly, research, special, group study, etc.

**3. DATES COVERED.** Indicate the time during which the work was performed and the report was written, e.g., Jun 1997 - Jun 1998; 1-10 Jun 1996; May - Nov 1998; Nov 1998.

**4. TITLE.** Enter title and subtitle with volume number and part number, if applicable. On classified documents, enter the title classification in parentheses.

**5a. CONTRACT NUMBER.** Enter all contract numbers as they appear in the report, e.g. F33615-86-C-5169.

**5b. GRANT NUMBER.** Enter all grant numbers as they appear in the report, e.g. AFOSR-82-1234.

**5c. PROGRAM ELEMENT NUMBER.** Enter all program element numbers as they appear in the report, e.g. 61101A.

**5d. PROJECT NUMBER.** Enter all project numbers as they appear in the report, e.g. 1F665702D1257; ILIR.

**5e. TASK NUMBER.** Enter all task numbers as they appear in the report, e.g. 05; RF0330201; T4112.

**5f. WORK UNIT NUMBER.** Enter all work unit numbers as they appear in the report, e.g. 001; AFAPL30480105.

**6. AUTHOR(S).** Enter name(s) of person(s) responsible for writing the report, performing the research, or credited with the content of the report. The form of entry is the last name, first name, middle initial, and additional qualifiers separated by commas, e.g. Smith, Richard, J, Jr.

**7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES).** Self-explanatory.

**8. PERFORMING ORGANIZATION REPORT NUMBER.** Enter all unique alphanumeric report numbers assigned by the performing organization, e.g. BRL-1234; AFWL-TR-85-4017-Vol-21-PT-2.

**9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES).** Enter the name and address of the organization(s) financially responsible for and monitoring the work.

**10. SPONSOR/MONITOR'S ACRONYM(S).** Enter, if available, e.g. BRL, ARDEC, NADC.

**11. SPONSOR/MONITOR'S REPORT NUMBER(S).** Enter report number as assigned by the sponsoring/ monitoring agency, if available, e.g. BRL-TR-829; -215.

**12. DISTRIBUTION/AVAILABILITY STATEMENT.** Use agency-mandated availability statements to indicate the public availability or distribution limitations of the report. If additional limitations/ restrictions or special markings are indicated, follow agency authorization procedures, e.g. RD/FRD, PROPIN, ITAR, etc. Include copyright information.

**13. SUPPLEMENTARY NOTES.** Enter information not included elsewhere such as: prepared in cooperation with; translation of; report supersedes; old edition number, etc.

**14. ABSTRACT.** A brief (approximately 200 words) factual summary of the most significant information.

**15. SUBJECT TERMS.** Key words or phrases identifying major concepts in the report.

**16. SECURITY CLASSIFICATION.** Enter security classification in accordance with security classification regulations, e.g. U, C, S, etc. If this form contains classified information, stamp classification level on the top and bottom of this page.

**17. LIMITATION OF ABSTRACT.** This block must be completed to assign a distribution limitation to the abstract. Enter UU (Unclassified Unlimited) or SAR (Same as Report). An entry in this block is necessary if the abstract is to be limited.

# Final Report

## (YIP 2011) UNSTEADY OUTPUT-BASED ADAPTIVE SIMULATION OF SEPARATED AND TRANSITIONAL FLOWS

Krzysztof J. Fidkowski

Aerospace Engineering Department
University of Michigan
Ann Arbor, 48109

March 17, 2015

Grant number: FA9550-11-1-0081

Dates covered: May 15, 2011 – December 14, 2014

### Abstract

The goal of this project was to investigate and overcome the difficulties of extending output-based error estimation and mesh adaptation from steady to unsteady simulations of aerodynamics problems relevant to the U.S. Air Force. These methods have a direct impact on robustness, through estimates of numerical error on engineering outputs, and efficiency, through space-time mesh adaptive methodology, of unsteady simulations. The project led to significant advances in adjoint-based error estimates for large-scale, nonlinear, time-dependent problems on deformable domains, and in their use in driving efficient combined space-time mesh refinement. The methods were applied to several transient aerodynamics problems, culminating with a three-dimensional adaptive flapping-wing simulation. Approaches for reducing the cost of the methods were also investigated, including a hybridized discontinuous Galerkin discretization and entropy-adjoint unsteady error estimates. The findings were disseminated in a variety of presentations, conference proceedings, and archival journal publications.

# 1 Introduction

Numerical error is inherent to unsteady Computational Fluid Dynamics (CFD) due to the finite nature of computation. To the extent that this error is not quantified, unsteady CFD simulations are not robust: using possibly-inaccurate results in a broader context such as design bears risk. As computations are performed with limited resources, of additional interest is efficiency, which is determined by the cost of a satisfactory solution. For complex unsteady applications, both spatial and temporal resolution are expected to have important effects on numerical error.

Quantification of this error is a difficult task for practitioners, even those with expert judgment. Standard error quantification techniques such as convergence studies are not always possible under resource constraints and can depend on the initial spatial and temporal resolutions. More robust are direct a posteriori estimates of the output error, which can also provide adaptive indicators for mesh refinement. This research developed practical output error estimates and mesh adaptation strategies for unsteady flows, with specific application to aerodynamics problems relevant to the Air Force.

# 2 Objectives

The goal of this research was to improve the robustness and efficiency of unsteady aerodynamics simulations by developing an output-based space-time adaptive methodology. The approach centered on unsteady discrete adjoint-based output error estimates for improving robustness and space-time adaptive mesh refinement for improving efficiency. The specific objectives were as follows:

- To investigate suitable error measures for unsteady simulations, including engineering output error and surrogate heuristics.
- To extend adjoint-based error estimation theory to unsteady simulations in a rigorous space-time variational setting.
- To implement an output error estimation procedure in a discontinuous Galerkin space-time finite element discretization.
- To develop space-time mesh adaptation mechanics for large-scale unsteady applications.
- To demonstrate the proposed error control framework for a target three-dimensional application problem.

To accomplish these objectives, the research was divided into three thrusts: high-order discretization, unsteady error estimation, and combined spatial/temporal mesh adaptation.

# 3 Accomplishments

During the period of performance of this award, we made multiple advances in the proposed research objectives related to unsteady output error estimation and mesh adaptation. The following sections outline the key contributions with references to relevant publications.

## 3.1 High-Order Finite Element Methods

Requirements of high accuracy in unsteady aerodynamics motivated the grounding of our research in high-order spatial and temporal discretizations. The discretization used, the discontinuous Galerkin (DG) finite-element method, was chosen because it offers several key advantages: it is unstructured, locally conservative, high-order accurate, stable for convection-dominated flows, low in dissipation and dispersion, optimally convergent, and amenable to $hp$ refinement. To make DG more robust for the chosen application problems, we developed a constrained pseudo-transient continuation solver [1, 5, 7] that enabled us to simulate challenging cases such as those in the drag-prediction workshop [6] and the international workshop on high-order CFD methods [17].

While most of our work was performed using DG, towards the end of the award we recognized that DG methods are consistently computationally expensive: a discontinuous high-order approximation in multiple dimensions creates a large number of coupled degrees of freedom per element. This shortcoming of DG motivated the transition of some of our research to hybridized DG (HDG) methods [9, 8] late in the award. Figure 1 illustrates the primary differences between DG and HDG, namely the introduction of additional degrees of freedom on faces that decouple elements from each other and yield a global system of smaller size at high order compared to DG. HDG also boasts optimal convergence of the gradient and a potential local postprocessing technique for superconvergent solution reconstruction.

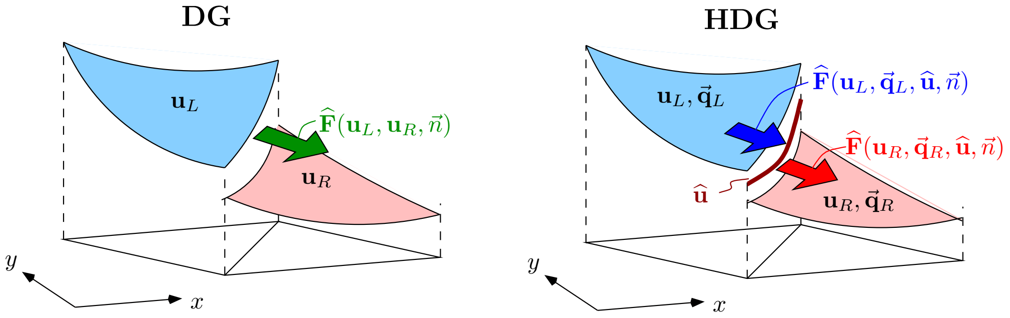


**Figure 1:** In the HDG method, additional unknowns on element interfaces allow elimination of the element-interior unknowns. This results in a global system in which the number of unknowns scales as $p^{\text{dim}-1}$ instead of $p^{\text{dim}}$ for DG.

During the award period, we added new capabilities to an in-house high-order finite-element library that was originally founded on DG. This library now supports both DG and HDG in the same framework. That is, the DG and HDG implementations share functions related to integration, residual calculation and linearization, and physics; the only differences is in short high-level functions that decide which weak-form terms contribute to the residual. Other discretizations, such as the embedded discontinuous Galerkin (EDG) will now be easy to introduce in future work. Research efforts supported by this code included: robustness improvements for the Reynolds-averaged Navier-Stokes (RANS) equations needed for the application problems; anisotropic $h$ and $p$ refinement for three-dimensional configurations; adaptive mesh motion using an ALE formulation; and efficiency and stability investigations with HDG. Figure 2 shows a sample comparison of adaptive runs using DG and HDG, with the result for this case that at high order, HDG indeed becomes more efficient than DG.

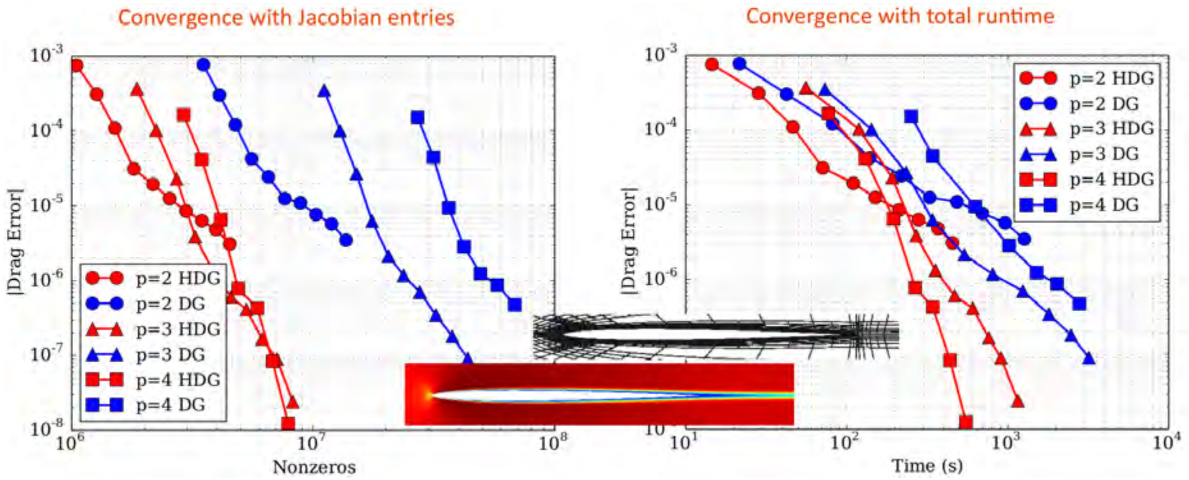


**Figure 2:** . Adaptive simulations of laminar viscous flow over a NACA 0004 airfoil at $M = 0.5$, $Re = 50,000$. A comparison of DG and HDG discretizations indicates a benefit of using HDG, especially at higher approximation orders.

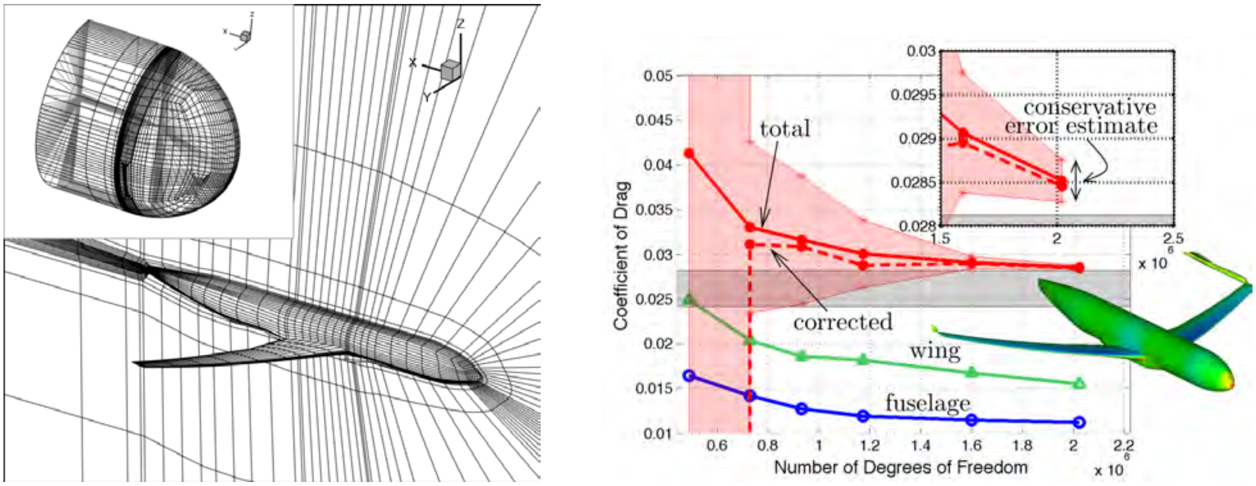
## 3.2 Error Estimation and Adaptation for Steady-State RANS Flows

As a precursor to unsteady problems, in the early part of this award we verified our steady-state RANS error estimates and adaptive mesh refinement, and we extended them to complex three-dimensional problems in preparation for the unsteady application. The key added adaptive capability was $hp$-refinement of quadrilateral and hexahedral meshes in two and three dimensions using a discrete-choice optimization approach [2, 3]. This required anisotropy detection through element-patch-local sampling of refinement options and a unified output-error driven selection procedure for combined $p$ and anisotropic-$h$ refinement options. We demonstrated the results on large three-dimensional wing and wing-body configurations, using the RANS equations. A sample steady adaptive result is illustrated in Figure 3, which shows a test case from the fifth Drag Prediction Workshop organized by the American Institute for Aeronautics and Astronautics in 2012 [4, 6]. The case involved predicting the drag on an aircraft (wing-body) at transonic cruise flight conditions. Figure 3 shows results obtained using our $hp$-adaptive capability, driven by drag error estimates. Note that the output error estimates shown in Figure 3b serve as numerical error bars that improve solution robustness. Towards the end of the award, we extended our error estimation and adaptation framework to simulations requiring output trim constraints [16]. This effort also built on our mesh motion capabilities developed as part of this award and discussed in Section 3.4.

## 3.3 Output-Based Adaptive Space-Time Methods

Whereas output-based adaptive methods are becoming fairly mature for steady problems, unsteady applications are less common because they pose challenges in the cost of error estimates and in the complexity of adaptive mechanics. As part of this grant, we extended our error estimation and adaptation methods from steady-state to unsteady simulations. Relevant contributions include:

- Combined temporal and spatial mesh $h$-refinement with a static geometry and mesh [12].
- Static and dynamic spatial order refinement on static geometries [15, 10].
- Combined temporal and dynamic-order spatial refinement on deformable domains [14].

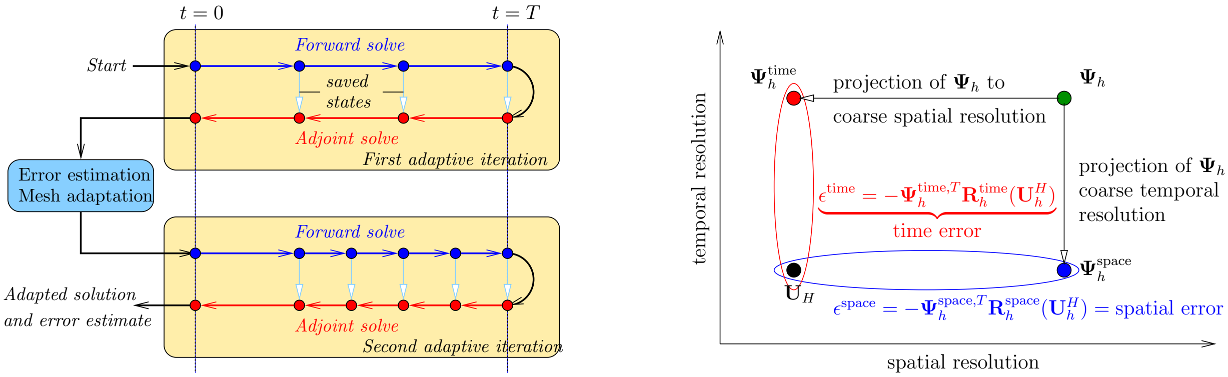


(a) Initial mesh

(b) Convergence of the drag output

**Figure 3:** Adaptive results for the wing body Common Research Model (CRM), the subject of the fifth AIAA Drag Prediction Workshop. Results were obtained using steady output-based $hp$ refinement capabilities of an in-house (UM) discontinuous Galerkin finite element code.

This work employed a space-time discontinuous Galerkin finite element discretization using time slabs and an approximate space-time solver [12]. For nonlinear problems, the adjoint system at each adaptive iteration is solved backwards in time using primal states saved from the forward run. Figure 4(a) shows a schematic of this process. Primal checkpointing was not needed for any of the runs but could be implemented if needed in the future. An important contribution to adaptive efficiency was in the form of a space-time anisotropy measure using projection of the fine adjoint to semi-refined spaces, illustrated in Figure 4(b). This enabled decisions of whether spatial or temporal refinement would most efficiently reduce the output error on a space-time element.

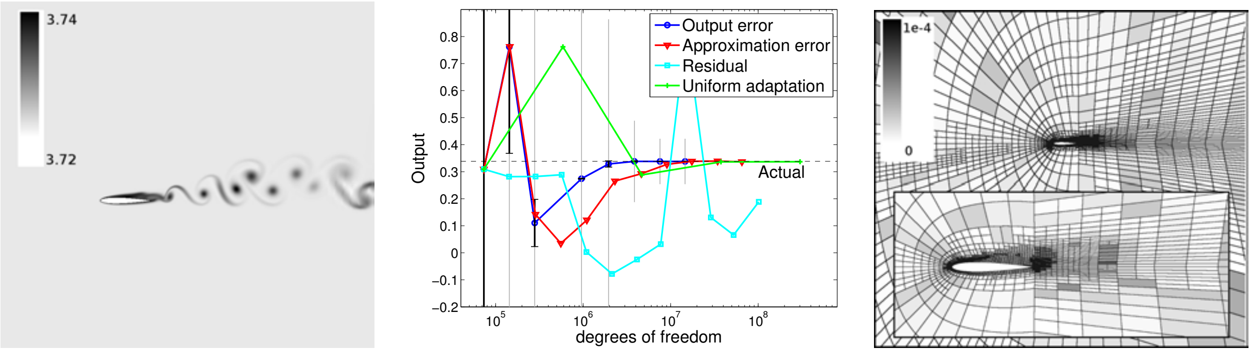


(a) Unsteady output-based adaptive solution process

(b) Space-time anisotropy via semi-refined spaces

**Figure 4:** Schematic of an adaptive primal and adjoint solution procedure for output-based unsteady simulations. Decisions of refinement in space and time are driven by an anisotropy measure that projects a fine-space adjoint down to semi-refined spaces.

Using this anisotropy measure, we investigated several adaptive schemes, including time slab bisection, time node redistribution, static $h$ spatial mesh refinement, and static/dynamic spatial order refinement. Figure 5 shows a static $h$ refinement result for a laminar vortex shedding simulation [12]: output-based refinement beats uniform and heuristic refinements in degrees of freedom for a strict error tolerance, and it comes equipped with an error estimate.



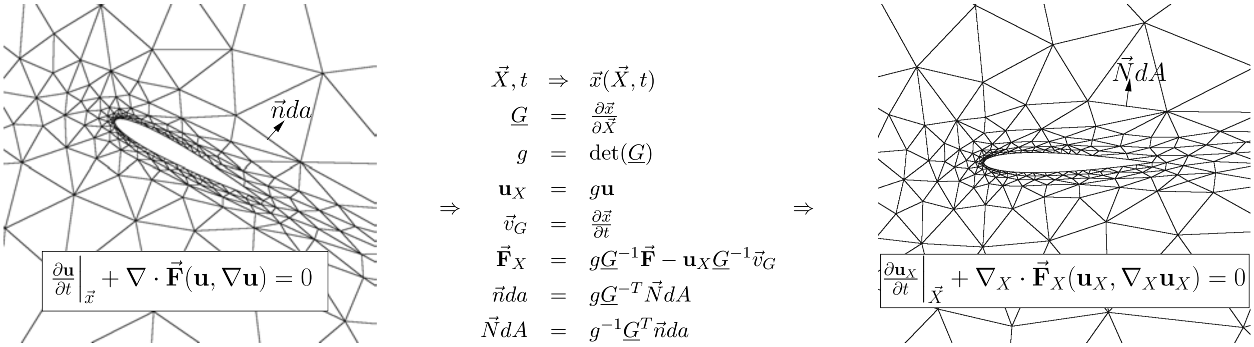
(a) Entropy contours (b) Output convergence (c) Output-adapted mesh

**Figure 5:** Unsteady output-adaptive result for a laminar vortex shedding simulation using static spatial meshes and DG discretizations in space and time; comparisons show performance versus other, heuristic, indicators.

### 3.4 Deformable Domains

In addition to unsteady problems on fixed domains, we also considered domains deforming due to motion, such as that induced by flapping. These simulations have far-reaching applications, from bio-inspired flight to aircraft maneuver and flutter analysis. The runs are generally computationally intensive and the resulting solutions are often rich in features.

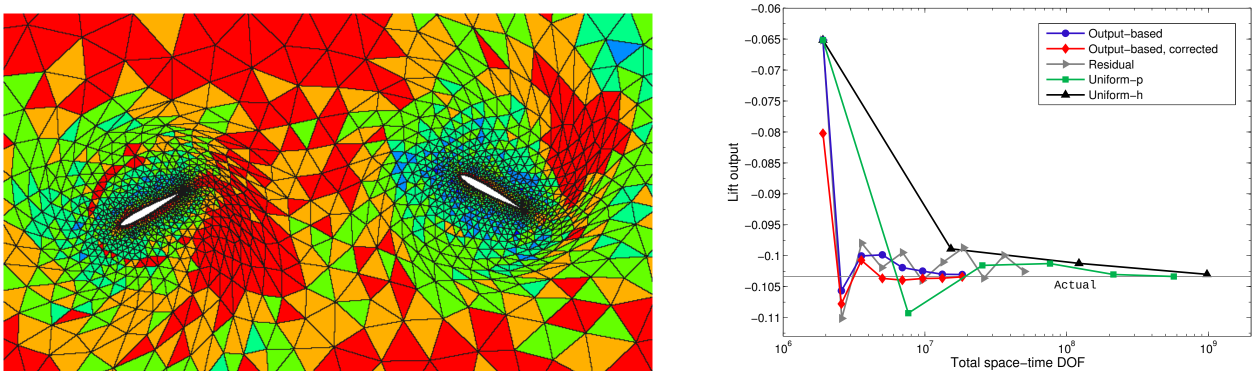
During the award period, we implemented unsteady mesh-motion capability into our solver and extended our output-based theory and methods to problems involving deformable domains. The mesh motion was performed using an arbitrary Lagrangian-Eulerian (ALE) formulation, illustrated in Figure 6. In this formulation, error estimation and adaptation are done on the static reference space. As such,much of the output-based theory carries over in a straightforward manner. The geometric conservation law (GCL) does require special attention for the adjoint solve, error estimate, and adaptive indicator, and we studied the effect of the GCL in our works [13, 14]. Figure 7



(a) Physical space (b) Mapping (c) Reference space

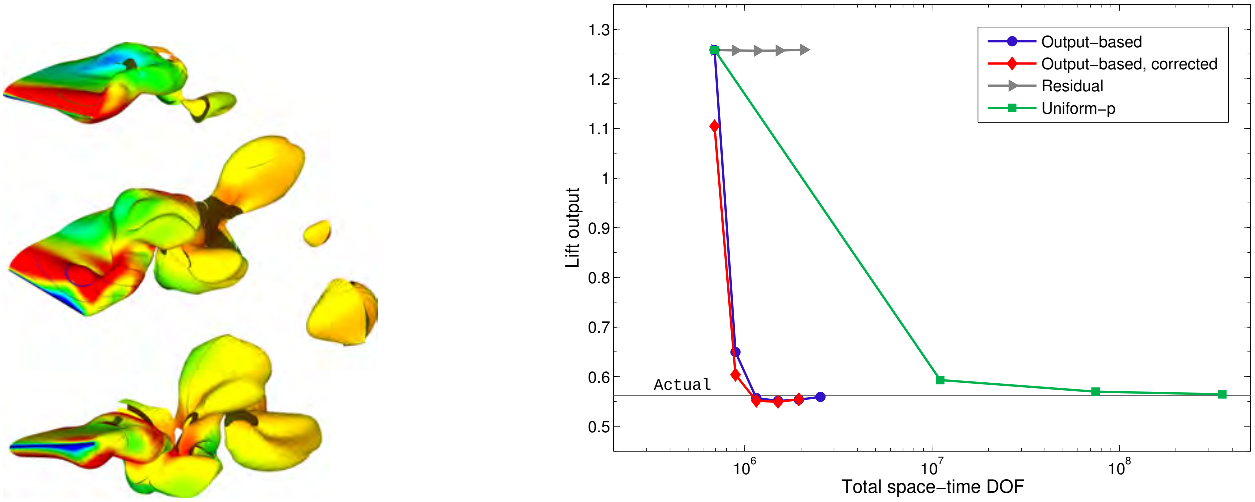
**Figure 6:** Arbitrary Lagrangian Eulerian mapping used for simulations on deforming domains. Error estimation and adaptation is done on the transformed equations in reference space.

shows dynamic-order adaptive results for two- and three-dimensional problems with mesh motion. Output-based refinement clearly beats uniform order refinement and dynamic order refinement driven by a residual indicator. In our implementation, this advantage also extends to computational time.



(a) 2D order snapshot

(b) 2D output convergence



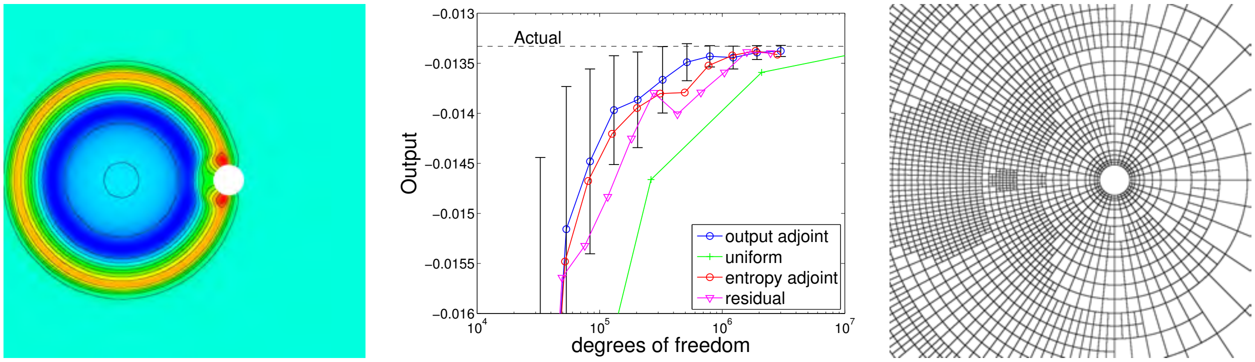
(c) 3D solution snapshots

(d) 3D output convergence

**Figure 7:** Comparison of output-based and heuristic space-time refinement strategies for a two-dimensional problem with staggered airfoils and a three-dimensional wing undergoing prescribed flapping motion [14]. In both cases, the output is a lift coefficient at/near the final time of the simulation.

## 3.5 The Entropy Adjoint Indicator

The entropy adjoint technique for steady-state adaptation was introduced in collaborative work prior to this award. The key idea behind this approach is that readily available entropy variables are in fact adjoint solutions for an entropy balance output, and adapting on an indicator computed from this cheap adjoint targets areas of spurious entropy generation. Work under the AFOSR project elucidated connections to output-based drag adaptation [11] and extended the technique to unsteady problems [15] in a variational space-time setting. Figure 8 shows an adaptive unsteady simulation of the shallow water equations using DG in space and time: the entropy adjoint indicator yields an error convergence plot most similar to the engineering-output (force) indicator. This result shows that we can significantly reduce the cost of unsteady output-based adaptation. Though the output is specific to an entropy balance, we have found that for many problems, areas of spurious entropy generation correlate with areas important for engineering outputs.



(a) Primal solution (depth)

(b) Output convergence

(c) Entropy-adjoint adapted mesh

**Figure 8:** Adaptive refinement of a 2D shallow-water equation simulation using an entropy adjoint indicator: performance is nearly identical to an output-based method, which requires a separate, backwards-in-time adjoint calculation.

# 4 Acknowledgment/Disclaimer

This work was sponsored (in part) by the Air Force Office of Scientific Research, USAF, under grant/contract number FA9550-11-1-0081. The views and conclusions contained herein are those of the authors and should not be interpreted as necessarily representing the official policies or endorsements, either expressed or implied, of the Air Force Office of Scientific Research or the U.S. Government.

# 5 Personnel Supported

| | | | |
|---|---|---|---|
| Krzysztof Fidkowski | Principal Investigator | Aerospace Eng. | U. Michigan |
| Marco Ceze | Ph.D. student/postdoctoral associate | Aerospace Eng. | U. Michigan |
| Steven Kast | Ph.D. student | Aerospace Eng. | U. Michigan |
| Benjamin Rothacker | Ph.D. student | Aerospace Eng. | U. Michigan |
| Loc Khieu | Postdoctoral associate | Aerospace Eng. | U. Michigan |
| Derek Dalle | Postdoctoral associate | Aerospace Eng. | U. Michigan |

# 6 Publications

The publications resulting from this work are the complete set listed in the references section, 7.

# 7 References

[1] M. A. Ceze and K. J. Fidkowski. A robust adaptive solution strategy for high-order implicit CFD solvers. AIAA Paper 2011-3696, 2011.

[2] M. A. Ceze and K. J. Fidkowski. Output-based $hp$-adaptation applied to aerodynamic flows. Tenth World Congress on Computational Mechanics WCCM-18038, 2012.

[3] M. A. Ceze and K. J. Fidkowski. An anisotropic hp-adaptation framework for functional prediction. *American Institute of Aeronautics and Astronautics Journal*, 51:492–509, 2013.

[4] M. A. Ceze and K. J. Fidkowski. Drag prediction using adaptive discontinuous finite elements. AIAA Paper 2013-0051, 2013.

[5] M. A. Ceze and K. J. Fidkowski. Pseudo-transient continuation, solution update methods, and CFL strategies for DG discretizations of the RANS-SA equations. AIAA Paper 2013–2686, 2013.

[6] M. A. Ceze and K. J. Fidkowski. Drag prediction using adaptive discontinuous finite elements. *AIAA Journal of Aircraft*, 51(4):1284–1294, 2014.

[7] M. A. Ceze and K. J. Fidkowski. Constrained pseudo-transient continuation. *International Journal for Numerical Methods in Engineering*, 2015. Accepted.

[8] J. P. Dahm and K. J. Fidkowski. Error estimation and adaptation in hybridized discontinous Galerkin methods. AIAA Paper 2014–0078, 2014.

[9] K. Fidkowski. High-order output-based adaptive methods for steady and unsteady aerodynamics. In H. Deconinck and R. Abgrall, editors, *37th Advanced CFD Lectures series; Von Karman Institute for Fluid Dynamics (December 9–12, 2013)*. von Karman Institute for Fluid Dynamics, 2013.

[10] K. J. Fidkowski. An output-based dynamic order refinement strategy for unsteady aerodynamics. AIAA Paper 2012-77, 2012.

[11] K. J. Fidkowski, M. A. Ceze, and P. L. Roe. Entropy-based drag error estimation and mesh adaptation in two dimensions. *AIAA Journal of Aircraft*, 49(5):1485–1496, September-October 2012.

[12] K. J. Fidkowski and Y. Luo. Output-based space-time mesh adaptation for the compressible Navier-Stokes equations. *Journal of Computational Physics*, 230:5753–5773, 2011.

[13] S. M. Kast, M. A. Ceze, and K. J. Fidkowski. Output-adaptive solution strategies for unsteady aerodynamics on deformable domains. Seventh International Conference on Computational Fluid Dynamics ICCFD7-3802, 2012.

[14] S. M. Kast and K. J. Fidkowski. Output-based mesh adaptation for high order Navier-Stokes simulations on deformable domains. *Journal of Computational Physics*, 252(1):468–494, 2013.

[15] S. M. Kast, K. J. Fidkowski, and P. L. Roe. An unsteady entropy adjoint approach for adaptive solution of the shallow-water equations. AIAA Paper 2011-3694, 2011.

[16] B. A. Rothacker, M. A. Ceze, and K. J. Fidkowski. Adjoint-based error estimation and mesh adaptation for problems with output constraints. AIAA Paper 2014–2576, 2014.

[17] Z. Wang, K. Fidkowski, R. Abgrall, F. Bassi, D. Caraeni, A. Cary, H. Deconinck, R. Hartmann, K. Hillewaert, H. Huynh, N. Kroll, G. May, P.-O. Persson, B. van Leer, and M. Visbal. High-order CFD methods: Current status and perspective. *International Journal for Numerical Methods in Fluids*, 2013. DOI: 10.1002/fld.3767.

1.

**1. Report Type**

Final Report

**Primary Contact E-mail**

**Contact email if there is a problem with the report.**

kfid@umich.edu

**Primary Contact Phone Number**

**Contact phone number if there is a problem with the report**

734-615-7247

**Organization / Institution name**

University of Michigan

**Grant/Contract Title**

**The full title of the funded effort.**

(YIP 2011) UNSTEADY OUTPUT-BASED ADAPTIVE SIMULATION OF SEPARATED AND TRANSITIONAL FLOWS

**Grant/Contract Number**

**AFOSR assigned control number. It must begin with "FA9550" or "F49620" or "FA2386".**

FA9550-11-1-0081

**Principal Investigator Name**

**The full name of the principal investigator on the grant or contract.**

Krzysztof J. Fidkowski

**Program Manager**

**The AFOSR Program Manager currently assigned to the award**

Dr. Fariba Fahroo

**Reporting Period Start Date**

05/15/2011

**Reporting Period End Date**

12/14/2014

**Abstract**

The goal of this project was to investigate and overcome the difficulties of extending output-based error estimation and mesh adaptation from steady to unsteady simulations of aerodynamics problems relevant to the U.S. Air Force. These methods have a direct impact on robustness, through estimates of numerical error on engineering outputs, and efficiency, through space-time mesh adaptive methodology, of unsteady simulations. The project led to significant advances in adjoint-based error estimates for large-scale, nonlinear, time-dependent problems on deformable domains, and in their use in driving efficient combined space-time mesh refinement. The methods were applied to several transient aerodynamics problems, culminating with a three-dimensional adaptive flapping-wing simulation. Approaches for reducing the cost of the methods were also investigated, including a hybridized discontinuous Galerkin discretization and entropy-adjoint unsteady error estimates. During the course of the project, we made the following accomplishments: (1) We developed a constrained pseudo-transient continuation solver to make the discontinuous Galerkin (DG) method more robust for the chosen application

problems; (2) we applied output-based methods to the novel hybridized discontinuous Galerkin (HDG) method; (3) we developed an hp-adaptive framework based on discrete choice optimization for quadrilateral and hexahedral elements; (4) we extended error estimation and adaptation to steady-state problems requiring trim constraints; (5) we extended steady-state output methods to time-dependent problems, using unsteady discrete adjoints; (6) we implemented mesh deformation and applied output error estimation and order adaptation to simulations on deformable domains; (7); we developed an unsteady entropy-adjoint indicator for mesh refinement without engineering-output adjoints. The findings were disseminated in a variety of presentations, conference proceedings, and archival journal publications.

**Distribution Statement**

**This is block 12 on the SF298 form.**

Distribution A - Approved for Public Release

**Explanation for Distribution Statement**

**If this is not approved for public release, please provide a short explanation. E.g., contains proprietary information.**

**SF298 Form**

**Please attach your SF298 form. A blank SF298 can be found here. Please do not password protect or secure the PDF The maximum file size for an SF298 is 50MB.**

AFD-070820-035.pdf

**Upload the Report Document. File must be a PDF. Please do not password protect or secure the PDF . The maximum file size for the Report Document is 50MB.**

main.pdf

**Upload a Report Document, if any. The maximum file size for the Report Document is 50MB.**

**Archival Publications (published) during reporting period:**

M. A. Ceze and K. J. Fidkowski. An anisotropic hp-adaptation framework for functional prediction. American Institute of Aeronautics and Astronautics Journal, 51:492–509, 2013.

M. A. Ceze and K. J. Fidkowski. Drag prediction using adaptive discontinuous finite elements. AIAA Journal of Aircraft, 51(4):1284–1294, 2014.

K. Fidkowski. High-order output-based adaptive methods for steady and unsteady aerodynamics. In H. Deconinck and R. Abgrall, editors, 37th Advanced CFD Lectures series; Von Karman Institute for Fluid Dynamics (December 9–12, 2013). von Karman Institute for Fluid Dynamics, 2013.

K. J. Fidkowski, M. A. Ceze, and P. L. Roe. Entropy-based drag error estimation and mesh adaptation in two dimensions. AIAA Journal of Aircraft, 49(5):1485–1496, September-October 2012.

K. J. Fidkowski and Y. Luo. Output-based space-time mesh adaptation for the compressible Navier-Stokes equations. Journal of Computational Physics, 230:5753–5773, 2011.

S. M. Kast and K. J. Fidkowski. Output-based mesh adaptation for high order Navier-Stokes simulations on deformable domains. Journal of Computational Physics, 252(1):468–494,2013.

Z. Wang, K. Fidkowski, R. Abgrall, F. Bassi, D. Caraeni, A. Cary, H. Deconinck, R.

Hartmann, K. Hillewaert, H. Huynh, N. Kroll, G. May, P.-O. Persson, B. van Leer, and M. Visbal. High-order CFD methods: Current status and perspective. International Journal for Numerical Methods in Fluids, 2013. DOI: 10.1002/fld.3767

**Changes in research objectives (if any):**

**Change in AFOSR Program Manager, if any:**

**Extensions granted or milestones slipped, if any:**

A 7-month extension was granted from the original end-date of May 14, 2014 to December 14, 2014 due to personnel acquisition delays.

**AFOSR LRIR Number**

**LRIR Title**

**Reporting Period**

**Laboratory Task Manager**

**Program Officer**

**Research Objectives**

**Technical Summary**

**Funding Summary by Cost Category (by FY, $K)**

| | Starting FY | FY+1 | FY+2 |
|---|---|---|---|
| Salary | | | |
| Equipment/Facilities | | | |
| Supplies | | | |
| Total | | | |

**Report Document**

**Report Document - Text Analysis**

**Report Document - Text Analysis**

**Appendix Documents**

## 2. Thank You

**E-mail user**

Mar 17, 2015 15:13:09 Success: Email Sent to: kfid@umich.edu