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Smoothness-Increasing Accuracy-Conserving (SIAC) Filters for Post-Processing Unstructured Discontinuous Galerkin Fields

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14. ABSTRACT
The purpose of this proposal is to develop smoothness-increasing accuracy-conserving filters that respect the mathematical properties of the data while providing levels of smoothness so that commonly used visualization tools can be used appropriately, accurately, and efficiently. The goals of this effort are to define, investigate, and address the technical obstacles inherent in visualization of data derived from high-order discontinuous Galerkin methods and to provide robust and easy to use algorithms to overcome the difficulties that arise due to lack of smoothness. The goal of this proposal is to construct smoothness-increasing accuracy-conserving (SIAC) filters for discontinuous Galerkin solutions on fully-unstructured triangular and tetrahedral meshes. In Particular, we propose to contribute both mathematically and algorithmically to the class of smoothness-increasing and accuracy-conserving (SIAC) methods to provide a robust and freely available software solution to the high-order simulation community.

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SMOOTHNESS-INCREASING ACCURACY-CONSERVING (SIAC) FILTERS FOR POST-PROCESSING UNSTRUCTURED DISCONTINUOUS GALERKIN FIELDS

FINAL REPORT

AFOSR FA9550-12-1-0428

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Abstract

Although discontinuous and continuous Galerkin methods have advantages mathematically and computationally, they suffer from one "feature" that can in turn become a disadvantage – they do not require high levels of smoothness at the element boundaries. Lack of smoothness across elements can hamper simulation post-processing like feature extraction and visualization. The purpose of this proposal is to develop *smoothness-increasing accuracy-conserving filters* that respect the mathematical properties of the data while providing levels of smoothness so that commonly used visualization tools can be used appropriately, accurately, and efficiently. The goals of this effort are to define, investigate, and address the technical obstacles inherent in visualization of data derived from high-order discontinuous Galerkin methods and to provide robust and easy to use algorithms to overcome the difficulties that arise due to lack of smoothness. The goal of this proposal is to construct smoothness-increasing accuracy-conserving (SIAC) filters for discontinuous Galerkin solutions on fully-unstructured triangular and tetrahedral meshes. In particular, we propose to contribute both mathematically and algorithmically to the class of smoothness-increasing and accuracy-conserving (SIAC) methods and to provide a robust and freely available software solution to the high-order simulation community.

This work is done in active collaboration with Dr. Jennifer K. Ryan, formerly at Delft University of Technology and now at University of East Anglia (UK), who is sponsored by the Air Force Office of Scientific Research, Air Force Material Command, USAF, under grant number FA8655-13-1-3017.

Status/Progress

Funding from AFOSR for the current grant started in the Fall of 2012. A new student, Mr. Amit Roy, joined the project. This was his first year as a PhD student at the University of Utah. His focus for this first year of the grant was to take our previous CPU and GPU post-processing codes (for 1D segments and 2D quadrilaterals) and make them available for public dissemination. He completed a package containing the codes, compilation directions, execution instructions and examples. At the conclusion of the Spring Semester 2013, he decided to switch to another advisor and is no longer on the project. Currently, Dr. Mahsa Mirzargar is working on the project as part of her post-doc work here at SCI. Mahsa is trying to use her skills in computation and approximation theory to help us bridge the gap between unstructured meshing, polynomial order, and resolution. Our hope is to establish a way of determining a characteristic length for filtering that works across various mesh types and polynomial orders.

Most Significant Accomplishments

This grant had three major thrusts. The first was to make our previously published software available. As previously noted, it is now packaged for general distribution and use. As a spin-off of our efforts, we were able to publish a Computer Science paper [4] documenting the general algorithms we employed. Secondly, we continued to push on three scientific areas related to the grant – all of which have led to published refereed journal papers: Filtering on Unstructured Triangular Meshes [1,2], Filtering On Structured Tetrahedral Meshes [3], and One-sided Filtering On Uniform Meshes [5]. In [1], we presented a study of this mesh scaling and how it factors into the theoretical errors. To accomplish the large volume of post-processing necessary for the study, commodity streaming multiprocessors were used; we demonstrate for structured meshes up to a 50 times speed up in the computational time over traditional CPU implementations of the SIAC filter. In [2], motivated by the ideas in [1], we moved to fully unstructured triangle meshes. As the assumption of any sort of regularity including the translation invariance of the mesh is a hindrance towards making the SIAC filter applicable to real life simulations, we demonstrated in [2] for the first time, the behavior and complexity of the computational extension of this filtering technique to fully unstructured tessellations. We considered different types of unstructured triangulations and showed that it is indeed possible to get order improvement and accuracy enhancement through a proper choice of kernel scaling. Moreover, we provided results that show we are able to introduce smoothness back to our original DG solution. These results are promising as they pave the way towards a more generalized SIAC filtering technique. In [3], we attempted to address the potential usefulness of smoothness-increasing accuracy-conserving (SIAC) filters when applied to real-world simulations – in particular, 3D tetrahedral meshes. As tetrahedral meshes are often the type considered in more realistic simulations, we contributed to the class of SIAC post-processors by demonstrating the effectiveness of SIAC filtering when applied to structured tetrahedral meshes. Lastly, in [5, 7, 8], we introduced a computationally efficient position-dependent one-sided Smoothness-Increasing Accuracy-Conserving (SIAC) filter that, in general, numerically enhances the accuracy and increases the smoothness of approximations obtained using discontinuous Galerkin (DG) methods. This is a significant improvement, in particular computationally, over the previously developed position-dependent one-sided SIAC filter available in the literature. This computationally efficient position-dependent filter can be applied up to domain boundaries, near a discontinuity in the solution, or at interface of different mesh sizes. This work [5, 7, 8] denotes one of two significant breakthroughs that came as a consequence of this grant. Through this sequence of papers, the relationship between mesh spacing, polynomial order and filter support was identified, and many corresponding properties were either proved and/or demonstrated numerically.

In the final year of the grant, a new research direction was sparked through the aforementioned work. In [6], we focused on establishing the theoretical connection between quasi-interpolation SIAC Filtering. We present the theoretical results that establish the connection between SIAC filtering to long-standing concepts in approximation theory such as quasi-interpolation and polynomial reproduction. This connection bridges the gap between the two related disciplines and provides a decisive advancement in designing new filters and mathematical analysis of their properties. In particular, we derived a closed formulation for convolution of SIAC kernels with polynomials. We also compared and contrasted cardinal spline functions as an example of filters designed for image processing applications with SIAC filters of the same order, and studied their properties. This paper is the second significant breakthrough that comes as a consequence of this grant, and provides the catalyst for future work currently submitted to the AFOSR Computational Mathematics Program.

Acknowledgment/Disclaimer

This work was sponsored (in part) by the Air Force Office of Scientific Research, USAF, under grant/contract number FA9550-12-1-0428. The views and conclusions contained herein are those of the authors and should not be interpreted as necessarily representing the official policies or endorsements, either expressed or implied, of the Air Force Office of Scientific Research or the U.S. Government.

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[1] James King, Hanieh Mirzaee, Jennifer K. Ryan and Robert M. Kirby. "Smoothness-Increasing Accuracy-Conserving (SIAC) Filtering for discontinuous Galerkin Solutions: Improved Errors versus High-Order Accuracy", *Journal of Scientific Computing*, Vol. 53, Issue 1, pages 129-149, 2012.

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Superconvergence and Optimal Accuracy”, Submitted to the *SIAM Journal on Scientific Computing*, 2015.

Personnel Supported During Duration of Grant

Amit Roy	Graduate Student, University of Utah
Mahsa Mirzargar	Post-doctoral Student, University of Utah
Robert M. Kirby	Associate Professor, University of Utah

Personnel Involved In The Work But Supported By Other Grants

Xiaozhou Li	Graduate Student, Delft University. Funded by AFOSR (FA8655-13-1-3017)
James King	Graduate Student, University of Utah. Funded by DOE-NETL.

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Abstract

In the final year of the grant, a new research direction was sparked through the aforementioned work. In [6], we focused on establishing the theoretical connection between quasi-interpolation SIAC Filtering. We present the theoretical results that establish the connection between SIAC filtering to long-standing concepts in approximation theory such as quasi-interpolation and polynomial reproduction. This connection bridges the gap between the two related disciplines and provides a decisive advancement in designing new filters and mathematical analysis of their properties. In particular, we derived a closed formulation for convolution of SIAC kernels with polynomials. We also compared and contrasted cardinal spline functions as an example of filters designed for image processing applications with SIAC filters of the same order, and studied their properties. This paper is the second significant breakthrough that comes as a consequence of this grant, and provides the catalyst for future work currently submitted to the AFOSR Computational Mathematics Program.

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