Seabed Variability and its Influence on Acoustic Prediction Uncertainty
Model and Data Variance and Resolution:
How Do We Quantify Uncertainty?

Robert I. Odom
Applied Physics Laboratory
College of Ocean and Fisheries Sciences
University of Washington
1013 N.E. 40th Street
phone: (206)685-3788 fax: (206)543-6785 email: odom@apl.washington.edu

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LONG-TERM GOALS

A basic tenet of the Office of Naval Research’s Uncertainty DRI is that, in any strategic situation, environmental parameters will never be known in complete enough detail to enable a perfectly accurate acoustic detection. In order to address the problem of unknown uncertainty this research is focused on two goals: 1. Assess and characterize seafloor variability in shelf areas. 2. Determine the impact of the seafloor variability on acoustic prediction uncertainty.

OBJECTIVES

The primary focus of this project will be to compute model and data sensitivities, investigate model parameter correlations, and optimal model parameterizations. The objectives can be stated as the answers to the following questions: Given the data we have available or can reasonably expect to be able to collect, which model parameters are most important? Can we resolve them? What will be the model variance? What additional data would be useful, if it were available? Which model parameters are essentially unresolvable (unconstrained)?

APPROACH

All our representations of the ocean/seafloor environment are, whether they are entries in a database or parameters supplied to a synthetic model, are under-parameterized versions of the true environment. Inverse problems that solve for environmental parameters are generally severely under-determined. This is because we attempt to represent what amount to continuous functions, e.g. water sound speed and bottom structure, with a finite number of discrete parameters. We hope that the parameters we choose capture most of the important features of the environment, but we are limited in the data we can collect by both practical considerations and physical constraints. Practically we can only collect a limited amount of data because of cost. Physical constraints make it impossible to obtain the kind of coverage available in a medical tomographic scan, for example. We simply do not have 360° coverage of our medium. Because of our limited data collecting capabilities and our finite model parameterization, we are forced to make trade-offs between variance reduction in our parameterized model and the resolution of our model (Menke, 1989).

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14. ABSTRACT

A basic tenet of the Office of Naval Research’s Uncertainty DRI is that, in any strategic situation, environmental parameters will never be known in complete enough detail to enable a perfectly accurate acoustic detection. In order to address the problem of unknown uncertainty this research is focused on two goals: 1. Assess and characterize seafloor variability in shelf areas. 2. Determine the impact of the seafloor variability on acoustic prediction uncertainty.

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The issue is that our model has some variance, this is the model uncertainty, which we wish to reduce. We collect acoustic or other data, which has its own variances, and combine this data in an inverse problem to reduce the variance in whatever starting model we have chosen to represent the environment. A common approach is to search the model space for a set of parameters, which produce a model that fits the data according to some criterion like aχ^2 statistic. Obtaining a model that fits the data is only half the problem. We still need to understand the variance and also the resolution of the model we have found. In fact the commonly used χ^2 statistic tells us nothing about the model variance, because it does not take the data variances into account. A low value of χ^2 tells us we have a model that fits the data, but it does not tell us how much of what we fit might be noise.

In addition to the model and data variance, we also need to know the resolution of both our data and our model. The variance and resolution are closely connected and we ultimately have to live with some compromise between the variance reduction in our model parameters and their resolution. We have a finite amount of data, some of which may be redundant. We can parameterize our environmental very finely, in which case we must use our limited data set to determine the values of many parameters. The result will be a model with many parameters, but with relatively large variances. On the other hand we could decide to parameterize our model very coarsely, and determine only a few parameters, with a consequent loss of resolution. However, because we have expended our data estimating only a few parameters, the variances of those parameters will be relatively lower. The choice of how to invest our data, whether to reduce the variance of a few parameters or to have a more finely parameterized model with larger variances, is ours to make.

It is possible to pre-compute measures of the model and data variance and resolutions. This allows us to determine the trade-offs available between the variance and resolution. The model and data variance and resolution matrices depend on partial derivatives of the pressure with respect to density and bulk modulus ($\partial p/\partial \rho$ and $\partial p/\partial \kappa$) (Tarantola, 1984). These derivatives, referred to as functional or Frechet derivatives in the literature, can be computed either by numerical differencing, a numerically intensive procedure, or by evaluation of very convenient analytical expressions (Pan, Phinney and Odom, 1986). These derivatives quantify the sensitivity of the model to perturbations in bulk modulus and density as a function of position. The two derivatives above are the most important. Other derivatives of interest can generally be constructed by application of the chain rule for differentiation. For example if we are interested in the sensitivity of the complex pressure field to perturbations in attenuation we can obtain it from $\partial p/\partial \kappa$ by making κ complex and then differentiating κ with respect to its imaginary part.

In addition to quantifying the model and data variances and resolutions, it is also important to understand correlations between model parameters. For example, sound speed and layer thickness are correlated (Schmidt and Baggeroer, 1995). This directly affects how we should parameterize our model. Bube et al. (1995) have provided quantitative guidelines for how to discretize the model to compute inverse solutions which are as accurate as possible in the features of the model which are well determined (resolved) by travel time data. In particular the sound speed model should not be discretized much coarsely than the reflectors as a way of stabilizing the inverse problem, because that may force the computed layer depths to try to match aspects of the data which are caused by features in the sound speed field.

WORK COMPLETED

The first year task of coding the Frechet derivatives has begun. The analytical expressions will be compared with the brute force numerical derivatives (e.g. $\Delta p/\Delta \kappa$) to assess their accuracy. These efforts are being coordinated with other members of the Seabed Variability Group.

RESULTS

Because the work has just begun, there are no results to report at this time.

IMPACT/APPLICATIONS

Answering the questions posed in the Objectives section of this report will provide quantitative bounds on what can be expected from an optimal experiment designed for environmental characterization, how much we are giving up for a non-optimal experiment, and which environmental parameters are best and least determined and determinable.

TRANSITIONS

In the short term, the results of this research will be utilized by the other members of the Seabed Variability Team. In the longer term, the results of this research will permit the quantification of the effects of sampling density, scale variability, and parameter sensitivity for inclusion in seabed environmental databases.

RELATED PROJECTS

This research is directly related to the other sub-projects in the "Seabed Variability and its Influence on Acoustic Prediction Uncertainty" group.

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