Statistical Problems in Ocean Modeling and Prediction

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LONG-TERM GOALS

My project addresses statistical and stochastic problems in the following fields: Lagrangian prediction and Lagrangian data assimilation (1), estimating transport and mixing parameters from tracer observations (2), and ocean model validation (3). The long range scientific goals of this study comprise determining limits of predictability for Lagrangian motion in semi-enclosed seas and littoral zones on time scales of days and weeks, estimating mixing and transport parameters in the upper ocean to improve performance of numerical models , and constructing statistical tests for model validation based on realistic confidence intervals for estimated mean fields and appropriate quantitative misfit measures.

OBJECTIVES

The objectives for the third year of research were:

- developing model independent prediction algorithms for the Lagrangian trajectories,
- constructing and implementing Lagrangian data assimilation algorithms,
- investigating the predictability limit for the Lagrangian motion in the upper ocean.

APPROACH

I develop theoretical approaches to the Lagrangian prediction , transport estimation and model validation problems in context of statistical inference for random processes and fields covered by stochastic partial differential equations. I design computational algorithms realizing developed mathematical methods. A significant part of validating the algorithms is testing them via stochastic simulations. Such an approach implies an accurate error analysis. Together with my collaborators from Rosenstiel School of Marine and Atmospheric Research (RSMAS), we implement the algorithms in concrete ocean models such as QG and MICOM , as well as carry out a statistical analysis of different real data sets by means of new methods.

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14. ABSTRACT
My project addresses statistical and stochastic problems in the following fields: Lagrangian prediction and Lagrangian data assimilation (1), estimating transport and mixing parameters from tracer observations (2), and ocean model validation (3). The long range scientific goals of this study comprise determining limits of predictability for Lagrangian motion in semi-enclosed seas and littoral zones on time scales of days and weeks, estimating mixing and transport parameters in the upper ocean to improve performance of numerical models , and constructing statistical tests for model validation based on realistic confidence intervals for estimated mean fields and appropriate quantitative misfit measures.

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WORK COMPLETED

1. Lagrangian prediction.

A new algorithm was suggested for prediction of a Lagrangian particle position in a stochastic flow given observations of other particles , [5]. The algorithm does not use any model or prescribed parameters and it is based on linearization of the motion equations. A theoretical error analysis has been developed for the Brownian flow and a stochastic flow with memory. The asymptotic formulae were compared with simulation results to establish their applicability limits. Monte-Carlo simulations were carried out to compare the new algorithm with two others: the center of mass prediction and a Kalman filter type method. The algorithm was also tested on real data in the Tropical Pacific.

2. Lagrangian data assimilation.

First, the Bayesian approach to optimal combining observations and a model output, has been applied to construct four new algorithm for assimilation of Lagrangian data in Eulerian models , [6]. The simplest one-step local procedure accounts only current observations and implies a diagonal covariance matrix for observational errors. Three other procedures extend this mashinery to a non-local covariance matrix and accounting neighboring observations. The local one-step algorithm was tested on a 1.5 layer quasi-geostrophic (QG) model with the double-gyre configuration in 2000 km x 2000 km square domain with 100 x100 grid points. The standard twin experiment strategy was employed.

Then, analytical tools were developed for estimating parameters of deterministic currents from Lagrangian observations taking into account stochastic nature of the upper ocean Lagrangian motion , [4]. Using this new techniques, I developed an asymptotical error theory for the maximum likelihood estimators of Eulerian parameters given Lagrangian data.

Finally, a new approach to the assimilation problem has been started. The approach is based on discovered relations between equations of stochastic hydrodynamics in Eulerian variables and Lgrangian stochastic models.

3. Predictability limit.

We use the reciprocal of the Lyapunov exponent as a first approximation for a Lagrangian trajectory predictability limit. An exact expression for the finite size Lyapunov exponent was obtained for the Brownian flow in terms of the velocity variance and correlation radius. The finite size Lyapunov exponent allows to analyze the predictability properties and mixing processes in non-homogeneous environments when the eddy turbulence scale is comparable with basin sizes.
An analytical background for investigation of the top Lyapunov exponent for the second order Markov model has been developed. This would allow us to advance in studying predictability obstacles and mixing processes in high energetic zones where low order Lagrangian stochastic models fail.

RESULTS

1. The developed regression prediction algorithm appears to be efficient for an initial tight cluster (the cluster radius is 4-5 times less than the Rossby radius) and small prediction time (not larger than 3-5 times the Lagrangian correlation time). Stochastic simulations showed that the regression algorithm (RA) performs better than a Kalman Filter type method (KF) in the presence of a deterministic linear

shear flow (Fig.1, the shear matrix is given in the figure legend). This is because it is based on the assumption of linear dependence of the particle current position on the initial position. If there is no shear in the mean flow the RA performs so as KF or little worse, however it presents several important simplifications with respect to KF: (i) this algorithm does not require any parameters, such as the Lagrangian parameters describing the characteristics of the underlying flow; the velocity correlation space scale and the Lagrangian correlation time scale, (ii) RA does not utilize the mean flow field, the calculation of which requires large data sets and the associated subgrid scale interpolation introduces further errors, (iii) RA does not need to be initialized with turbulent velocity fluctuations at the launch location, (iv) RA is not based on the integration of velocity field to estimate the particle position, which necessarily leads to accumulation of velocity errors as errors of drifter location, and (v) consequently RA is computationally far simpler than KF. Despite these simplifications, it is found on the basis of several oceanic clusters that RA outperforms CM, and that RA is as accurate as KF. In the example presented here, the predictand started from the cluster center of mass. In this case the RA and CM perform identically.

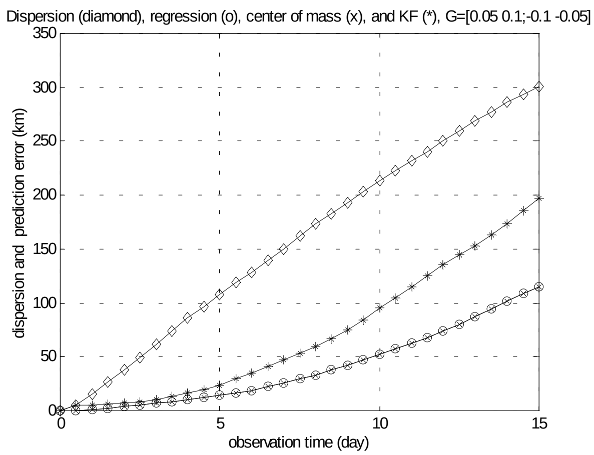
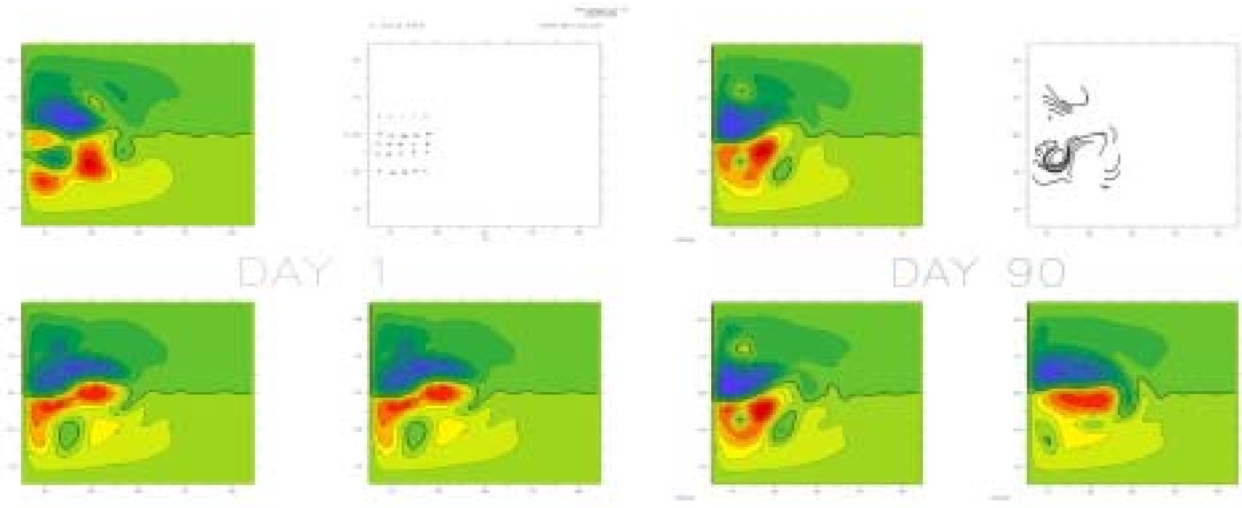


Figure 1

The real data comparison of different prediction algorithms are in good qualitative agreement with the simulation results. Even the prediction error values are of the same order as our simple error theory concluded. Deviations are related to oversimplifications accepted in the considered stochastic model such as the linearity of the mean shear flow and linearity and isotropy of the fluctuations.

2. Regarding to the data assimilation we first focused on the QG model. The initial conditions and results of the 90 day runs are presented in Fig. 2a and 2b respectively. For each picture the upper left panels, the upper right, the lower left, and the lower right ones show the true stream function, the "observed" Lagrangian trajectories, the assimilated run, and the control run respectively. The assimilation and control runs have the initial conditions corresponding to a spin-up of 21 years. The assimilation and control runs were carried out using a different wind from the truth: 1.5 times the wind of truth. The different wind was maintained during all the assimilation and control runs. in 90 days. We used the assimilation frequency Δt = 1.5 hour and varied it up to 20 days. It was found that the assimilation is very efficient up to Δt = 2 – 3 days with the assimilation error about 0.2, then it starts to deteriorate. For Δt = 5 days the error is around 0.5. It is interesting that for too small Δt (less than 6 hours) and very small ratio of the observational and modeling errors the models blows up.

We underscore that our approach uses essentially the Lagrangian character of observations. In particular, this requires accurate computing the variational derivative of the Lgrangian velocity with respect to the Eulerian one. As our experiments showed, the assimilation error is remarkably increasing if data first are transformed to Eulerian form. A quantitative description of the assimilation deterioration was given.



a **b**

Figure 2

3. The analysis of the Lyapunov exponent in the framework of higher order Markov models showed that the predictability limit in high energetic zones is mostly determined by the time scale defined as ratio of mean square velocity and acceleration fluctuations rather than by the Lagrangian correlation time. Another significant factor is the velocity space correlation radius which can be taken as the Rossby radius in first approximation.

The exact expression for the finite size Lyapunov exponent in an idealized model of the upper ocean Lagrangian turbulence resulted in an explicit relation between the predictability limit and the Rossby radius for a finite initial separation of two particles.

IMPACT/APPLICATIONS

1. The suggested prediction algorithm is much simpler and performs not worse than the Kalman filter type algorithm. Thus it can be effectively used for real forecasting of lost objects in the sea based on observations of drifters floating in the same area.

2. Probably it is the first time that assimilation algorithms take into account the Lagrangian nature of data explicitly. The developed assimilation procedures can be and will be applied in high resolution models of oceanic circulation such as MICOM. The obtained analytical results open a way to developing other assimilation algorithms based on the Kalman filter ideas and the maximum likelihood approach.

3. The investigated dependence of the Lyapunov exponent on the physical parameters can serve as a guidance for approximate estimating the predictability limit for a Lagrangian particle position based on field measurements.

TRANSITIONS

The developed prediction and assimilation algorithms were and will be used in RSMAS to study the Lagrangian predictability for other synthetic and real data sets.

RELATED PROJECTS

1. “Predictability of Particle Trajectories in the Ocean”, ONR, PI T.Ozgokmen (RSMAS),

2. “Lagrangian Data Analysis in Mesoscale Prediction and Model Validation Studies”, ONR, PI A.Griffa (RSMAS)

3. “Circulation of Marginal and Semi-Enclosed Seas (Modeling Support for CREAMS II in the Japan (East) Sea”, ONR, PI C.Mooers

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