ARL-TR-7380 • AUG 2015

US Army Research Laboratory

Inductance Calculations of Variable Pitch Helical Inductors

by Peter T Bartkowski and Paul R Berning

Approved for public release; distribution is unlimited.

NOTICES

Disclaimers

The findings in this report are not to be construed as an official Department of the Army position unless so designated by other authorized documents.

Citation of manufacturer's or trade names does not constitute an official endorsement or approval of the use thereof.

Destroy this report when it is no longer needed. Do not return it to the originator.

ARL-TR-7380 ● AUG 2015

US Army Research Laboratory

Inductance Calculations of Variable Pitch Helical Inductors

by Peter T Bartkowski and Paul R Berning
Weapons and Materials Research Directorate, ARL

Approved for public release; distribution is unlimited.

REPORT DOCUMENTATION PAGE					*Form Approved* *OMB No. 0704-0188*

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection information Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing the burden, to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302 Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number

PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.

1. REPORT DATE ***(DD-MM-YYYY)***	**2. REPORT TYPE**	**3. DATES COVERED (From - To)**
August 2015	Final	March 2014–May 2015

4. TITLE AND SUBTITLE	**5a. CONTRACT NUMBER**
Inductance Calculations of Variable Pitch Helical Inductors	
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	**5d. PROJECT NUMBER**
Peter T Bartkowski and Paul R Berning	
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)	**8. PERFORMING ORGANIZATION REPORT NUMBER**
US Army Research Laboratory ATTN: RDRL-WMP-E Aberdeen Proving Ground, MD 21005-5066	ARL-TR-7380
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	**10. SPONSOR/MONITOR'S ACRONYM(S)**
	11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES

14. ABSTRACT

A technique has been developed to calculate the inductance of variable pitch helicoil inductors of rectangular cross section that use a coaxial return feed along the center of the coil. The calculations discretize the coil into a series of rings of current that take into account the diffusion of current or skin depth. The inductance is a combination of each ring's self-inductance and the mutual inductances of all the current rings. To validate the technique, a series of coils were machined from aluminum tubes and then their inductances were measured at 3 different frequencies. The calculated inductances are compared to the measured inductances.

15. SUBJECT TERMS

inductance, helicoil, variable pitch, inductor, coil, current sheet, Rayleigh quadrature, skin depth

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON**
					Peter Bartkowski
a. REPORT	**b. ABSTRACT**	**c. THIS PAGE**			**19b. TELEPHONE NUMBER (Include area code)**
Unclassified	Unclassified	Unclassified	UU	32	410-278-0210

Standard Form 298 (Rev 8/98)
Prescribed by ANSI Std Z39 18

Contents

List of Figures

List of Tables

1. Introduction

Most of the derivations and formulae for the calculation of coil inductances are for the generic case of a conducting wire wound into a helix to form an inductor. Careful consideration must be made when calculating the inductance of alternate geometries, as the equations for simple wire-wrapped geometries may not prove appropriate. In this report, a technique for calculating the inductance of one such geometry, a variable pitch helical inductor (Fig. 1), is presented. The inductor is machined from a metallic tube on a 4-axis milling machine. This inductor has a non-uniform pitch with a conductor of rectangular cross section that varies in width. The gap between windings is constant. Current flows through the inductor and is returned via an end cap and interior coaxial cylinder. No simple equation can accurately predict the inductance of such geometry.

Fig. 1 Variable pitch helical inductor (front and cross-section views)

The technique presented here is not limited to this specific geometry and can be adapted to cover more complex geometries as needed.

2. Discrete Model

To begin, we split the geometry in Fig. 1 into 3 distinct regions. There is a section of concentric coaxial tubes on the left, a helical inductor in the middle with an interior tube, and another section of concentric coaxial tubes on the right. These 3 inductors are connected in series, thus we can calculate the total inductance:

$$L = L_{coax1} + L_{coil} + L_{coax2}\ . \tag{1}$$

Calculating the inductance of a set of coaxial tubes is relatively straightforward and will be covered in Section 3. Determining the inductance of the helical section is more difficult and cannot be done directly using analytical techniques. A common technique is to break the individual helical turns down into a series of rings of current (Fig. 2). By calculating the inductance of each ring and the mutual inductances of all the rings with respect to each other, we can determine the total inductance of the helical section[1]:

$$L_{coil} = \sum_{i=1}^{n}\sum_{j=1}^{n} M_{ij} \, , \tag{2}$$

where when $i = j$, M_{ii} = ring self-inductance.

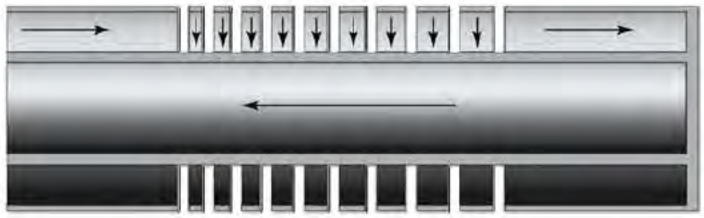

Fig. 2 The helical section replaced by discrete rings of current

A solenoid generates a nearly constant magnetic field internally. The central return cylinder shields this magnetic field from its interior volume. The shielding effect lowers the inductance by the ratio of shielded volume to total coil interior volume. Thus, we can modify Eq. 2 to account for the shielding effect:

$$L_{coil} = \frac{V_{coil} - V_{return}}{V_{coil}} \sum_{i=1}^{n}\sum_{j=1}^{n} M_{ij} = \frac{ID_{coil}^2 - OD_{return}^2}{ID_{coil}^2} \sum_{i=1}^{n}\sum_{j=1}^{n} M_{ij} = \frac{D_{coil}^2 - D_{ret}^2}{D_{coil}^2} \sum_{i=1}^{n}\sum_{j=1}^{n} M_{ij} \, . \tag{3}$$

There is no mutual inductance between the 2 coaxial sections and the center coil, as the direction of current and magnetic fields are nearly perpendicular to each other. The coaxial tubes have axial current flow (circumferential flux lines), and the coil has nearly circumferential current flow (axial flux lines). The flux lines are perpendicular to each other, yielding no magnetic coupling.

3. Coaxial Tube Section Inductance

The inductance per unit length of 2 concentric tubes (Fig. 3), where current flows through one and returns through the other, is straightforward[2]:

$$\frac{L_{coax}}{l} = \frac{\mu_o}{4\pi}\left[2\, ln\left(\frac{r_3}{r_2}\right) + \frac{r_4^2}{r_4^2 - r_3^2}\left[\frac{2r_4^2}{r_4^2 - r_3^2} ln\left(\frac{r_4}{r_3}\right) - 1\right] + \frac{r_1^2}{r_2^2 - r_1^2}\left[\frac{2r_1^2}{r_2^2 - r_1^2} ln\left(\frac{r_2}{r_1}\right) - 1\right]\right] \, . \tag{4}$$

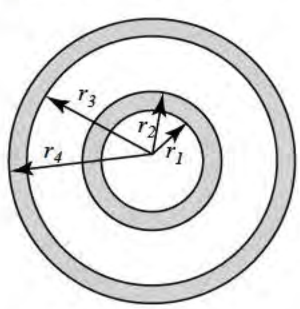

Fig. 3 Coaxial tube cross section

4. Self-Inductance Terms

To calculate the self-inductance of a single ring of current, we can use current sheet theory. A current sheet is an infinitesimally thin conducting sheet wrapped into a tube (Fig. 4). Current evenly flows circumferentially around the sheet, creating a magnetic field internally.

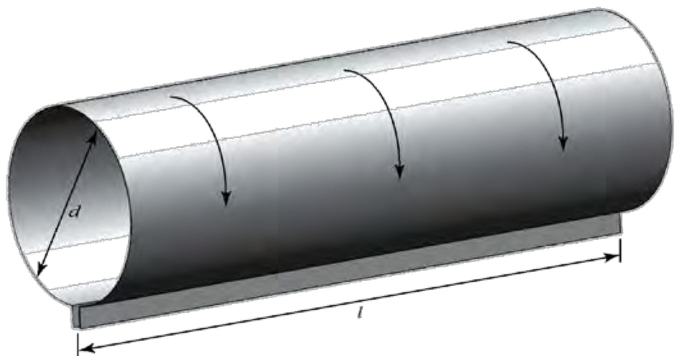

Fig. 4 Cylindrical current sheet

Its inductance can be calculated using Snow's formula (modified to MKS [meter, kilogram, second] units)[2]:

$$L_s = \frac{\mu_o}{3}\sqrt{l^2 + d^2}\left[K - E + \frac{d^2}{l^2}(E - k)\right], \tag{5}$$

where

$K =$ complete elliptic integral of the first kind $= \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-k^2\sin^2\theta}}$,

$E =$ complete elliptic integral of the second kind $= \int_0^{\frac{\pi}{2}} \sqrt{1 - k^2\sin^2\theta}\, d\theta$, and

$k = \frac{d}{\sqrt{l^2+d^2}}$.

Unfortunately, there are no analytical solutions for these 2 integrals. They can, however, be calculated numerically quite easily.

Another approach, presented by Welsby,[3] is much simpler to use and will produce acceptable results for all but the most demanding applications. For a current sheet where l >> *d*, the following formula will produce good results:

$$L = \mu_o \frac{A}{l} = \mu_o \frac{\pi d^2}{4l}, \tag{6}$$

where *A* = the cross-sectional area of the current sheet.

However, many applications, such as the one in Fig. 2, have rings where *d* > l. For this condition a correction factor must be applied to account for the end effects, which produce a nonuniform magnetic field within the volume encompassed by the current sheet. Our inductance equation now becomes

$$L = \mu_o \frac{\pi d^2}{4l} K_n \,, \tag{7}$$

where K_n is the so-called Nagaoka constant:

$$K_n = \frac{1}{1+\frac{0.9}{2}\left(\frac{d}{l}\right)-\frac{0.02}{4}\left(\frac{d}{l}\right)^2} \,. \tag{8}$$

The self-inductance of each ring of current in the inductor model can then be calculated by either numerically integrating the elliptical formulas in Eq. 5 or by using the simplified approximation of Eq. 7.

5. Mutual Inductance Terms

Direct calculation of the mutual inductances between rings of current is very difficult. A simplified approach is to apply the Rayleigh quadrature formula. It replaces the rings of current with several conducting filaments, and as long as current flows uniformly through the cross section of the ring, the mutual inductances between filaments can be calculated and then averaged to determine the overall mutual inductance between rings. Take 2 arbitrary rings of current, as depicted in Fig. 5. Filaments are placed at the numbered positions 1–5. The mutual inductances between each center filament and all filaments in the neighboring ring are calculated. The overall mutual inductance between the 2 rings is then calculated using the following averaging formula[4]:

$$M = \frac{1}{6}[(M_{12'} + M_{13'} + M_{14'} + M_{15'}) + (M_{1'2} + M_{1'3} + M_{1'4} + M_{1'5}) - 2M_{11'}] \,. \tag{9}$$

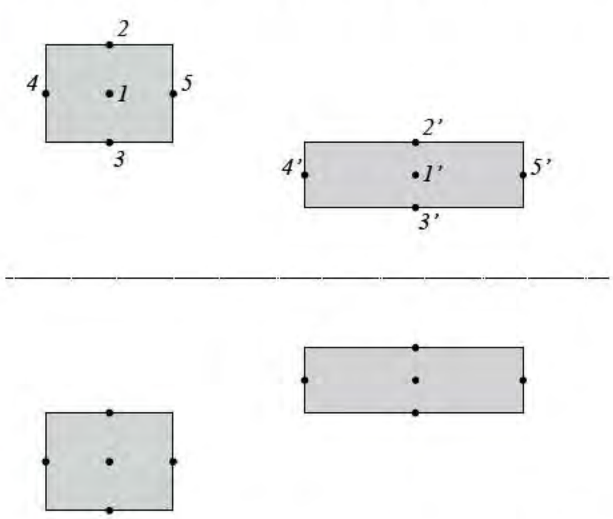

Fig. 5 Filament placement for mutual inductance calculation

For the simplified case where the 2 current rings have the same diameter and thickness (as in our case), we can reduce Eq. 9 to the following[4]:

$$M = \frac{1}{6}\left[2M_{11'} + M_{14'} + M_{15'} + M_{1'4} + M_{1'5}\right] \ . \tag{10}$$

Now that we have represented the 2 current rings by sets of filaments, we can use Turner's equation to calculate the mutual inductances between 2 circular filaments (Fig. 6).

$$M_{ij} = \frac{2\mu_o r_i r_j}{\sqrt{l_{ij}^2+(r_i-r_j)^2}}\left[\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\cos^2\theta + {k_c}^2\sin^2\theta}} - 2\int_0^{\frac{\pi}{2}} \frac{\sin^2\theta\, d\theta}{\sqrt{\cos^2\theta + {k_c}^2\sin^2\theta}}\right], \tag{11}$$

where

$$k_c = \frac{{l_{ij}}^2+(r_i+r_j)^2}{{l_{ij}}^2+(r_i-r_j)^2} .$$

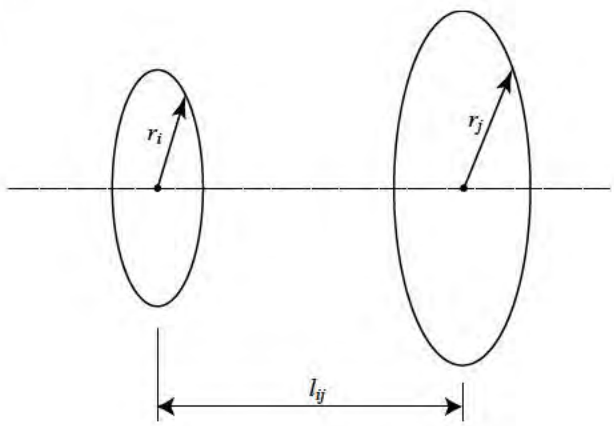

Fig. 6 Conducting filaments

We can simplify these equations further, where the radius and thickness of our conductors are constant, $r_i = r_j = r_m$:

$$M_{ij} = \frac{2\mu_o r_m^2}{l_{ij}}\left[\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\cos^2\theta + {k_c}^2\sin^2\theta}} - 2\int_0^{\frac{\pi}{2}} \frac{\sin^2\theta\, d\theta}{\sqrt{\cos^2\theta + {k_c}^2\sin^2\theta}}\right], \tag{12}$$

where

$$k_c = \frac{{l_{ij}}^2 + 4r_m^2}{{l_{ij}}^2} = 1 + \frac{4r_m^2}{{l_{ij}}^2} .$$

Again, we are presented with solving elliptical integrals, which have no analytical solution. We must resort to using numerical integration techniques. In this case, for each mutual inductance calculation, we must calculate 10 elliptical integrals. For a 9-turn coil, we have 72 mutual inductances to calculate, which will require a total of 720 elliptical integrals to be solved. Modern computing hardware can perform these calculations to sufficient accuracy quickly.

6. Skin Depth

There are limitations to the formulas presented previously for self-inductance and mutual inductance. The self-inductance formulas assume current flows through an infinitesimally thin conducting sheet while the mutual inductance formulas assume current flows uniformly through the conducting ring's cross section. Neither assumption is entirely correct. To reconcile the 2 disparate assumptions, we introduce the concept of classical skin depth.

When current begins flowing through a conductor, it starts at the surface and diffuses with time into the interior volume of the conducting material. If the magnetic field is produced by alternating current, the current (and subsequently the magnetic field) is continually diffusing in and out of the conductors but always within a finite thickness or depth referred to as the skin depth region. Because the diffusion process has a finite rate, at higher frequencies the current conducts through a thinner cross section of conductor than at lower frequencies. At very high frequencies, the self-inductance calculations will be correct using the current sheet equations. Conversely, at very low frequencies, the mutual inductance calculations will be correct. Adjustments can be made to both inductance calculations to account for the actual skin depth being used by the current in our application, increasing the accuracy of our inductance calculations.

For convenience, we introduce the concept of Classical Skin Depth. All diffusion processes in nature have a characteristic exponential decay with distance. Classical Skin Depth simplifies this exponential decay by replacing it with a finite depth of uniform current. Using the classical skin depth definition, we can adjust the effective diameters used to calculate the inductances. The classical skin depth can be calculated using the following formula[5]:

$$\delta = \sqrt{\frac{1}{\pi f \sigma_o \mu_o}}, \tag{13}$$

where

f = frequency,

σ_o = conductor conductivity, and

μ_o = magnetic permability of free space = $4\pi \times 10^{-7}\ \mathrm{H}/\mathrm{m}$.

The coaxial tube inductance formula is based on current flowing uniformly through the conducting cross section of the tubes. But we know now that current is only flowing through a thin layer on the inner diameter of the outer conductor and a thin layer on the outer diameter of the return conductor (Fig. 7). Thus, we modify the following 2 radii to reflect current flowing only through the area encompassed by the skin depth of conductor:

$$r_1 = r_2 - \delta$$
$$r_4 = r_3 + \delta \,. \tag{14}$$

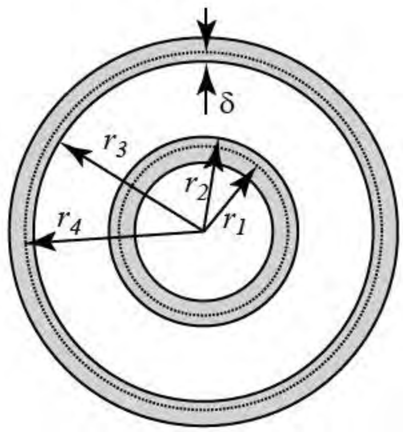

Fig. 7 Skin depth in coaxial conductors

Since the ring self-inductance formula is accounting for the magnetic field over all space, not just inside the conducting rings, we must also adjust our diameter of the current sheet to include the skin depth of the current (Fig. 8). Subsequently, we can use the following effective current sheet diameter for calculating self-inductance:

$$D_{coil} = ID_{coil} + 2\delta \,. \tag{15}$$

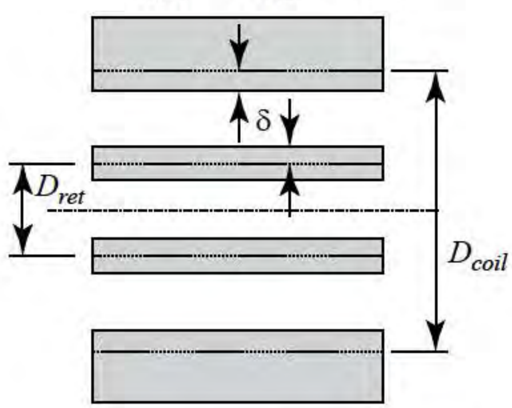

Fig. 8 Adjustment to diameter for skin depth of current in conducting ring

The skin depth effect also applies to the return conductor. We modify its diameter by subtracting the thickness of the skin depth in the following formula:

$$D_{ret} = OD_{return} - 2\delta \; . \tag{16}$$

For our mutual inductance calculations, instead of having the current flow evenly through the conducting ring cross section, we assume it flows through only the skin depth area on the inside of the ring. Now, instead of using the average diameter of the conducting ring for our filament model, we use the diameter corresponding to half the through thickness of the skin depth, as shown in Fig. 9. The diameter/radius of the filaments is determined to be

$$D_M = ID_{coil} + \delta \; , \tag{17}$$

or

$$r_M = \frac{ID_{coil}+\delta}{2} \; . \tag{18}$$

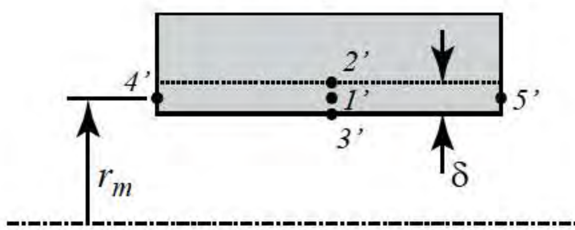

Fig. 9 Filament locations for skin depth current

7. Preliminary Results

The previously described technique was used to calculate the inductances of a set of coils machined from aluminum tubes, as shown in Fig. 1. Keeping the length of the coil constant, the number of coil turns was varied (i.e., 3, 5, 7, 9, 11, and 13 turns). The inductances for the coax end pieces and the coil section were also calculated. For comparison, the self-inductances of each ring were calculated using both techniques above (elliptical integrals and shortcut formula). The equations were entered into a Microsoft Excel spreadsheet for calculation (see Appendix A). Visual Basic subroutines (see Appendix B) were written in Excel to perform the numeric elliptical integrals. The subroutines used Simpson's rule with 1,000 intervals to calculate the integrals. This number of intervals was chosen as a good compromise between accuracy and calculation speed. Inductance measurements were also performed in the lab using an inductance meter to verify the accuracy of the calculations at 3 different frequencies: 1, 10, and 100 kHz. The results are listed in Table 1 and Fig. 10.

Table 1 Measured and calculated system inductances using integrals

No. of Turns	Measured Inductances (μH)			Calculated Inductances (μH)		
	1 kHz	10 kHz	100 kHz	1 kHz	10 kHz	100 kHz
3	0.381	0.318	0.301	0.400	0.328	0.305
5	0.917	0.810	0.760	0.989	0.815	0.760
7	1.673	1.502	1.421	1.830	1.513	1.414
9	2.665	2.420	2.300	2.922	2.420	2.262
11	3.878	3.536	3.361	4.267	3.539	3.310
13	5.355	4.912	4.671	5.879	4.881	4.565

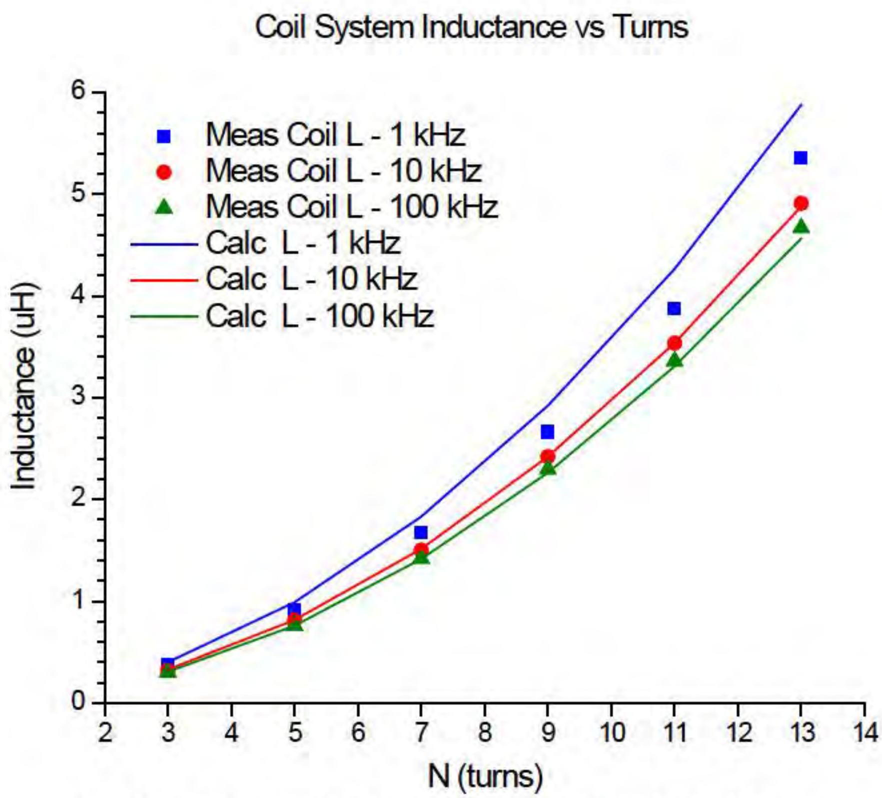

Fig. 10 Coil system inductance measurements and calculations

As can be seen in Fig. 10, we have very good agreement between measurements and calculations at the higher frequencies of 10 and 100 kHz. The 1-kHz calculations are predicting higher inductances than what was measured. A closer look at the calculations reveal that the classical skin depth at 1 kHz is nearly identical to the conductor thickness of the coil. Initially, this might seem fortuitous, as we are efficiently using all of the conductor thickness to conduct current; however, we are not. The definition of classical skin depth is an approximation that assumes that all the current is flowing evenly within the region encompassed by the skin depth. The current is actually diffusing into the conductor with the highest levels at the inner surface and decreasing exponentially through the conductor thickness. Equation 19 gives us the current density versus depth:

$$J(x) = J_s e^{-x/\delta} , \tag{19}$$

where

J_s = field strength at the conductor surface,

x = distance from the conductor surface, and

δ = classical skin depth.

If we plot this equation through the thickness of our coil for each frequency, we get the plot displayed as Fig. 11:

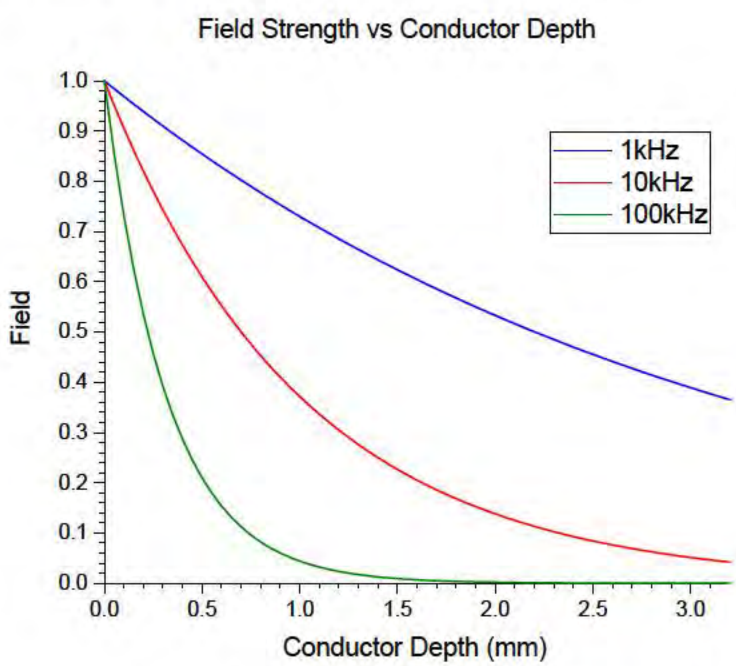

Fig. 11 Current/field strength in coil conductor

At 100 kHz, we see that 100% of the potential field is inside the conductor. At 10 kHz, nearly all of the potential field is inside the conductor. At 1 kHz, however, we find that a large portion of the exponential tail is missing. The classical skin depth assumes that all of the potential field is within the conductor; at the lower frequencies we find this is not true. We can make a correction to our classical skin depth calculation to account for this missing field. If we integrate Eq. 19 over the conductor thickness, we can calculate the following ratio:

$$\int_0^x J_s e^{-\frac{x}{\delta}} dx = -\delta\, J_s \left[e^{-x/\delta}\right]_0^x = \delta\, J_s \left[e^0 - e^{-x/\delta}\right] = \delta\, J_s \left[1 - e^{-x/\delta}\right]. \quad (20)$$

For a semi-infinite thickness, evaluate this over the range of x = 0 – ∞, to get

$$\delta\, J_s[1 - e^{-\infty}] = \delta\, J_s[1 - 0] = \delta\, J_s\,. \quad (21)$$

Comparing this result to the finite thickness calculation for the conductors, we get:

$$\frac{\delta\, J_s\left[1-e^{-x/\delta}\right]}{\delta\, J_s} = \left[1 - e^{-x/\delta}\right]. \quad (22)$$

At 1 skin depth (δ) we get 0.632 or 63%, at 2 skin depths we get 87%, and 3 skin depths yields 95%. We will then reduce the skin depth by multiplying by the factor calculated by Eq. 22 to account for the field not present because of a finite thickness conductor. Equation 13 now becomes:

$$\delta = \sqrt{\frac{1}{\pi f \sigma_o \mu_o}} \left[1 - e^{-x/\delta}\right]. \quad (23)$$

Using the new skin depth calculated for each conductor, we repeat the calculations for inductance (Table 2 and Fig. 12).

Table 2 Measured and using integrals and corrected skin depth

No. of Turns	Measured Inductances (μH)			Calculated Inductances (μH)		
	1 kHz	**10 kHz**	**100 kHz**	**1 kHz**	**10k Hz**	**100 kHz**
3	0.381	0.318	0.301	0.373	0.328	0.305
5	0.917	0.810	0.760	0.924	0.815	0.760
7	1.673	1.502	1.421	1.712	1.513	1.414
9	2.665	2.420	2.300	2.736	2.420	2.262
11	3.878	3.536	3.361	3.998	3.539	3.310
13	5.355	4.912	4.671	5.51	4.881	4.565

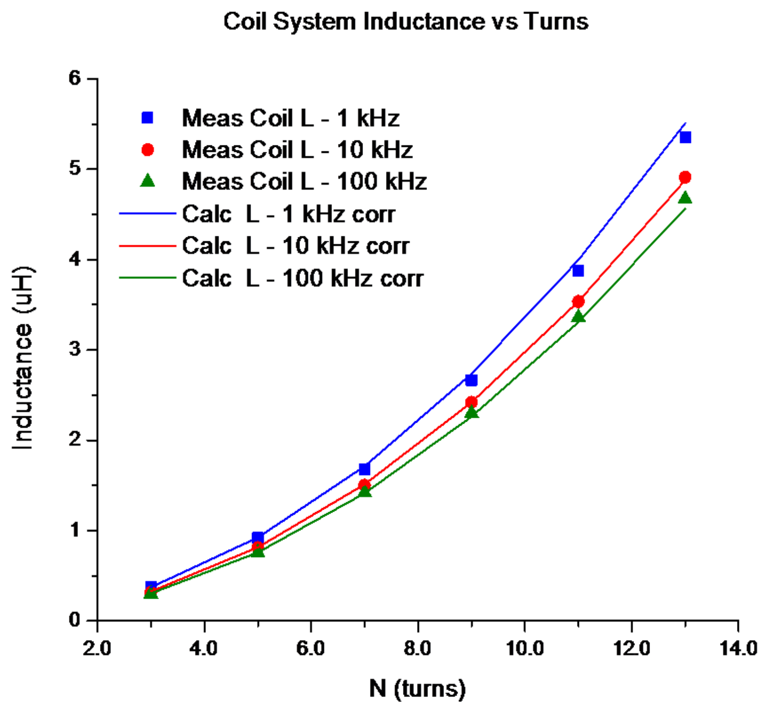

Fig. 12 Coil system inductance measurements and calculations with corrected skin depth

The corrected skin depth does a much better job in predicting inductance at the lower 1-kHz frequency with a maximum error of only 2.9% for the 13-turn coil. A close look at the calculations show that the influence of the coaxial ends to inductance is small compared that of the coil. The 89-mm-long coaxial ends add a total of 0.018, 0.015, and 0.013 µh of inductance at 1-, 10-, and 100-kHz frequencies.

The individual inductance calculations for self-inductance and mutual inductance are given in Table 3. For comparison, the self-inductances were also calculated using the much easier Nagaoka correction to the current sheet inductance formula. This produces a maximum error of only 1.75% in calculating self-inductance or 0.6% in total inductance. This error may be very acceptable in many situations.

Table 3 Comparison of self- and mutual inductance calculations

No. of Turns	Calculated Inductances (µH) @ 1 kHz			Calculated Inductances (µH) @ 10 kHz			Calculated Inductances (µH) @ 100 kHz		
	Self (Integrals)	Self (Nagaoka)	Mutual	Self (Integrals)	Self (Nagaoka)	Mutual	Self (Integrals)	Self (Nagaoka)	Mutual
3	0.250	0.251	0.105	0.220	0.221	0.093	0.205	0.206	0.087
5	0.515	0.513	0.391	0.453	0.452	0.347	0.423	0.422	0.325
7	0.824	0.816	0.870	0.727	0.720	0.772	0.678	0.672	0.722
9	1.171	1.152	1.547	1.033	1.017	1.372	0.964	0.949	1.285
11	1.554	1.524	2.426	1.372	1.345	2.153	1.280	1.256	2.016
13	1.975	1.942	3.517	1.744	1.714	3.122	1.628	1.600	2.924

It’s also worth noting that coils with few turns are dominated by self-inductance while the many turn coils are dominated by the mutual inductance between windings.

8. Conclusions

The technique described earlier for calculating the inductance of variable pitch helicoils of rectangular cross section has produced results accurate to a few percent over a range of frequencies and number of turns for a sample coil. Taking into account the effects of skin depth into the inductance calculations is important for coils with a coaxial return, such as those described here. The volume of field contained in the coil conductors can be a large fraction of the overall field. Not incorporating this field will yield inaccurate inductance predictions. Larger-diameter coils and smaller-diameter coaxial returns will have inductances that are less affected by the field contained within the conductors.

This technique for calculating inductance can be applied to other more complex forms of geometry, including tapered coils, by simply using the more general forms of the self- and mutual inductance formulae.

9. References

1. Turner GR. Inductance calculations for helical magnetocumulative generators. Journal of Electromagnetic Phenomena. July 2003;3:392–396.
2. Snow C. Formulas for computing capacitance and inductance. In: National bureau of standards circular 544. Washington (DC): Government Printing Office (US); 1954 Sep. pp. 31 and 39.
3. Welsby VG. The theory and design of inductance coils. Great Britain: John Wiley and Sons, Inc.; 1960. p. 40–44.
4. Terman FE. Radio engineer's handbook. New York (NY): McGraw-Hill Book Company, Inc.; 1943. p. 72–73.
5. Knoepfel H. Pulsed high magnetic fields. New York (NY): American Elsevier Publishing Company, Inc.; 1970. p. 46–72.

Appendix A. Sample MS Excel Spreadsheet Calculations

This appendix appears in its original form, without editorial change.

Coil Parameters:										
Material =	Al	Al or Cu				Classic Skin D =	0.0032	m	R1 =	0.0322	m
Frequency =	1	kHz				Eff Coil Skin D =	0.0020	m	R2 =	0.0353	m
Coil N turns =	9	turns				Eff Ret Skin D =	0.0028	m	R3 =	0.0560	m
Coil Helix cut length =	0.1969	m							R4 =	0.0592	m
Coil P initial cut =	0.0125	m/turn				Coil P final cut =	0.0269	m/turn	Coax 1 L =	0.089	m
Winding Gap =	0.0064	m							Coax 2 L =	0.089	m
Coil ID =	0.1080	m	Coil OD =	0.1143	m	Eff Coil OD =	0.1120	m	Coax1 =	9.0	nH
Return OD =	0.0762	m	Return ID =	0.0635	m	Eff Return ID =	0.0707	m	Coax2 =	9.0	nH

Coax Calculations: (R1–R4, Coax 1 L, Coax 2 L, Coax1, Coax2 above)

				Self Inductances (nH)		Mutual Inductances (nH)													
				Cur Sheet	Elliptic	1	2	3	4	5	6	7	8	9	10	11	12	13	Mutual
Coil #	Pitch (m)	Width (m)	Pos (m)	w Kn	Integrals	0.0132	0.0279	0.0440	0.0615	0.0805	0.1009	0.1227	0.1460	0.1707	0.1969	0.0000	0.0000	0.0000	Sum
1	0.0139	0.0076	0.0132	149.7	152.3	0.0	60.9	33.2	19.6	12.0	7.4	4.7	3.1	2.0	0.0	0.0	0.0	0.0	142.9
2	0.0154	0.0090	0.0279	141.8	144.9	61.3	0.0	57.5	30.4	17.5	10.4	6.4	4.0	2.6	0.0	0.0	0.0	0.0	189.9
3	0.0168	0.0105	0.0440	135.6	138.6	33.4	57.9	0.0	54.4	27.8	15.6	9.1	5.5	3.4	0.0	0.0	0.0	0.0	207.1
4	0.0182	0.0119	0.0615	130.5	133.1	19.8	30.6	54.8	0.0	51.6	25.6	14.0	8.0	4.8	0.0	0.0	0.0	0.0	209.0
5	0.0197	0.0133	0.0805	126.1	128.3	12.0	17.6	28.0	52.0	0.0	49.0	23.6	12.6	7.1	0.0	0.0	0.0	0.0	201.9
6	0.0211	0.0148	0.1009	122.1	124.0	7.5	10.5	15.7	25.8	49.4	0.0	46.6	21.8	11.3	0.0	0.0	0.0	0.0	188.6
7	0.0226	0.0162	0.1227	118.6	120.1	4.8	6.4	9.2	14.1	23.7	47.0	0.0	44.5	20.1	0.0	0.0	0.0	0.0	169.9
8	0.0240	0.0176	0.1460	115.4	116.5	3.1	4.0	5.5	8.1	12.7	21.9	44.8	0.0	42.5	0.0	0.0	0.0	0.0	142.6
9	0.0254	0.0191	0.1707	112.4	113.2	2.0	2.6	3.4	4.8	7.1	11.4	20.3	42.8	0.0	0.0	0.0	0.0	0.0	94.5
10	0.0000	0.0000	0.1969	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.0000	0.0000	0.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.0000	0.0000	0.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.0000	0.0000	0.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Rings Self L =	**1152**	**1171**	**nH**
Total L (Coil+Coax) =	**2717**	**2736**	**nH**

Sum Ring Mutual Inductances = 1547 nH

Coil Parameters:									
Material =	Al	Al or Cu				Classic Skin D =	0.0010	m	R1 =	0.0361	m
Frequency =	10	kHz				Eff Coil Skin D =	0.0010	m	R2 =	0.0371	m
Coil N turns =	9	turns				Eff Ret Skin D =	0.0010	m	R3 =	0.0549	m
Coil Helix cut length =	0.1969	m							R4 =	0.0559	m
Coil P initial cut =	0.0125	m/turn				Coil P final cut =	0.0269	m/turn	Coax 1 L =	0.089	m
Winding Gap =	0.0064	m							Coax 2 L =	0.089	m
Coil ID =	0.1080	m	Coil OD =	0.1143	m	Eff Coil OD =	0.1099	m	Coax1 =	7.3	nH
Return OD =	0.0762	m	Return ID =	0.0635	m	Eff Return ID =	0.0742	m	Coax2 =	7.3	nH

Coax Calculations:

				Self Inductances (nH)		Mutual Inductances (nH)													
				Cur Sheet	Elliptic	1	2	3	4	5	6	7	8	9	10	11	12	13	Mutual
Coil #	Pitch (m)	Width (m)	Pos (m)	w Kn	Integrals	0.0132	0.0279	0.0440	0.0615	0.0805	0.1009	0.1227	0.1460	0.1707	0.1969	0.0000	0.0000	0.0000	Sum
1	0.0139	0.0076	0.0132	132.1	134.5	0.0	54.2	29.5	17.4	10.5	6.5	4.1	2.7	1.8	0.0	0.0	0.0	0.0	126.7
2	0.0154	0.0090	0.0279	125.2	127.9	54.6	0.0	51.2	26.9	15.5	9.2	5.6	3.5	2.2	0.0	0.0	0.0	0.0	168.7
3	0.0168	0.0105	0.0440	119.8	122.3	29.7	51.6	0.0	48.4	24.7	13.8	8.0	4.8	3.0	0.0	0.0	0.0	0.0	183.9
4	0.0182	0.0119	0.0615	115.2	117.5	17.5	27.1	48.8	0.0	45.9	22.7	12.4	7.1	4.2	0.0	0.0	0.0	0.0	185.5
5	0.0197	0.0133	0.0805	111.3	113.2	10.6	15.6	24.9	46.3	0.0	43.6	20.9	11.1	6.2	0.0	0.0	0.0	0.0	179.1
6	0.0211	0.0148	0.1009	107.8	109.4	6.6	9.3	13.9	22.8	44.0	0.0	41.5	19.3	10.0	0.0	0.0	0.0	0.0	167.3
7	0.0226	0.0162	0.1227	104.7	105.9	4.2	5.6	8.1	12.5	21.0	41.8	0.0	39.5	17.8	0.0	0.0	0.0	0.0	150.6
8	0.0240	0.0176	0.1460	101.8	102.7	2.7	3.5	4.9	7.1	11.2	19.4	39.9	0.0	37.7	0.0	0.0	0.0	0.0	126.5
9	0.0254	0.0191	0.1707	99.2	99.8	1.8	2.3	3.0	4.2	6.3	10.1	18.0	38.1	0.0	0.0	0.0	0.0	0.0	83.7
10	0.0000	0.0000	0.1969	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.0000	0.0000	0.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.0000	0.0000	0.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.0000	0.0000	0.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Rings Self L =	**1017**	**1033**	**nH**
Total L (Coil+Coax) =	**2404**	**2420**	**nH**

Sum Ring Mutual Inductances = 1372 nH

Coil Parameters:									**Coax Calculations:**		
Material =	Al	Al or Cu				Classic Skin D =	0.0003	m	R1 =	0.0375	m
Frequency =	100	kHz				Eff Coil Skin D =	0.0003	m	R2 =	0.0378	m
Coil N turns =	9	turns				Eff Ret Skin D =	0.0003	m	R3 =	0.0543	m
Coil Helix cut length =	0.1969	m							R4 =	0.0546	m
Coil P initial cut =	0.0125	m/turn				Coil P final cut =	0.0269	m/turn	Coax 1 L =	0.089	m
Winding Gap =	0.0064	m							Coax 2 L =	0.089	m
Coil ID =	0.1080	m	Coil OD =	0.1143	m	Eff Coil OD =	0.1086	m	Coax1 =	6.5	nH
Return OD =	0.0762	m	Return ID =	0.0635	m	Eff Return ID =	0.0756	m	Coax2 =	6.5	nH

				Self Inductances (nH)		Mutual Inductances (nH)													
				Cur Sheet	Elliptic	1	2	3	4	5	6	7	8	9	10	11	12	13	Mutual
Coil #	Pitch (m)	Width (m)	Pos (m)	w Kn	Integrals	0.0132	0.0279	0.0440	0.0615	0.0805	0.1009	0.1227	0.1460	0.1707	0.1969	0.0000	0.0000	0.0000	Sum
1	0.0139	0.0076	0.0132	123.3	125.6	0.0	50.9	27.6	16.2	9.8	6.1	3.8	2.5	1.6	0.0	0.0	0.0	0.0	118.7
2	0.0154	0.0090	0.0279	116.9	119.4	51.3	0.0	48.0	25.2	14.4	8.6	5.2	3.3	2.1	0.0	0.0	0.0	0.0	158.1
3	0.0168	0.0105	0.0440	111.8	114.2	27.8	48.4	0.0	45.4	23.1	12.9	7.5	4.5	2.8	0.0	0.0	0.0	0.0	172.3
4	0.0182	0.0119	0.0615	107.5	109.6	16.4	25.4	45.8	0.0	43.1	21.2	11.5	6.6	3.9	0.0	0.0	0.0	0.0	173.8
5	0.0197	0.0133	0.0805	103.9	105.6	9.9	14.5	23.3	43.4	0.0	40.9	19.5	10.4	5.8	0.0	0.0	0.0	0.0	167.7
6	0.0211	0.0148	0.1009	100.6	102.0	6.1	8.6	13.0	21.4	41.2	0.0	38.9	18.0	9.3	0.0	0.0	0.0	0.0	156.6
7	0.0226	0.0162	0.1227	97.7	98.8	3.9	5.3	7.5	11.6	19.7	39.3	0.0	37.1	16.7	0.0	0.0	0.0	0.0	141.0
8	0.0240	0.0176	0.1460	95.0	95.8	2.5	3.3	4.5	6.6	10.4	18.2	37.4	0.0	35.4	0.0	0.0	0.0	0.0	118.4
9	0.0254	0.0191	0.1707	92.5	93.1	1.7	2.1	2.8	3.9	5.8	9.4	16.8	35.7	0.0	0.0	0.0	0.0	0.0	78.2
10	0.0000	0.0000	0.1969	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.0000	0.0000	0.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.0000	0.0000	0.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.0000	0.0000	0.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Rings Self L =	**949**	**964**	**nH**
Total L (Coil+Coax) =	**2247**	**2262**	**nH**

Sum Ring Mutual Inductances = 1285 nH

19

Appendix B. Visual Basic Subroutines

This appendix appears in its original form, without editorial change.

Visual Basic Subroutine for Complete Elliptical Integral of the First Kind

```
Function Elliptic1(l, d) As Double
'
' Integral solution using Simpson's Rule
'
Dim i As Integer
Dim Pi As Double, uo As Double, kc As Double
Dim a As Double, amax As Double, da As Double
Dim steps As Integer
Dim func1a As Double, func1b As Double
'
On Error GoTo err_TorisV1
steps = 1000
Pi = 3.14159
uo = 4 * Pi * 0.0000001
'
'---------------------------------------------------------
' Procedure:
' Calculate mutual inductance between two nearby circular
' fillaments or radius ri, rj with distance between d
'
' Proceedure calculates the two elliptical integrals necessary
' to calculate the mutual inductance
'
' Uses simpsons rule for the numerical integration
'
'
'---------------------------------------------------------
'


amax = Pi / 2
da = amax / steps
kc = d / Sqr(l ^ 2 + d ^ 2)

a = 0
'simpsons rule, first term
Elliptic1 = 1 / Sqr((Cos(a)) ^ 2 + kc ^ 2 * (Sin(a)) ^ 2) * da

For i = 1 To (steps - 1) / 2
   a = a + da
   'simpons rule, 4y terms
   func1a = 4 / Sqr(1 - kc ^ 2 * (Sin(a)) ^ 2) * da

   a = a + da
   'simpsons rule, 2y terms
   func1b = 2 / Sqr(1 - kc ^ 2 * (Sin(a)) ^ 2) * da
```

```
    Elliptic1 = Elliptic1 + func1a + func1b

   Next i
'simsons rule, last term
a = a + da
Elliptic1 = 1 / 3 * (Elliptic1 + 1 / Sqr(1 - kc ^ 2 * (Sin(a)) ^ 2) * da)

Exit Function
'
err_TorisV1:
Elliptic1 = 0
'
End Function
```

Visual Basic Subroutine for Complete Elliptical Integral of the Second Kind:

```
Function Elliptic2(l, d) As Double
'
' Integral solution using Simpson's Rule
'
Dim i As Integer
Dim Pi As Double, uo As Double, kc As Double
Dim a As Double, amax As Double, da As Double
Dim steps As Integer
Dim func2a As Double, func2b As Double
Dim num As Double, den As Double
'
On Error GoTo err_TorisV1
steps = 1000
Pi = 3.14159
uo = 4 * Pi * 0.0000001
'
'---------------------------------------------------------
' Procedure:
' Calculate mutual inductance between two nearby circular
' fillaments or radius ri, rj with distance between d
'
' Proceedure calculates the two elliptical integrals necessary
' to calculate the mutual inductance
'
' Uses simpsons rule for the numerical integration
'
'
'---------------------------------------------------------
```

```
'

amax = Pi / 2
da = amax / steps
kc = d / Sqr(l ^ 2 + d ^ 2)

a = 0
'simpsons rule, first term
Elliptic2 = Sqr(1 - kc ^ 2 * (Sin(a)) ^ 2) * da

For i = 1 To (steps - 1) / 2
    a = a + da
    'simpons rule, 4y terms
    func1a = 4 * Sqr(1 - kc ^ 2 * (Sin(a)) ^ 2) * da

    a = a + da
    'simpsons rule, 2y terms
    func1b = 2 * Sqr(1 - kc ^ 2 * (Sin(a)) ^ 2) * da

    Elliptic2 = Elliptic2 + func1a + func1b

   Next i
'simsons rule, last term
a = a + da
Elliptic2 = 1 / 3 * (Elliptic2 + Sqr(1 - kc ^ 2 * (Sin(a)) ^ 2) * da)


Exit Function
'
err_TorisV1:
Elliptic2 = 0
'
End Function
```

Visual Basic Subroutine for Mutual Inductance between Filaments:

```
Function MutualI(d, ri, rj) As Double
'
' Integral solution using Simpson's Rule
'
Dim i As Integer, steps As Integer
Dim kc As Double, Pi As Double, uo As Double
Dim Elliptic1 As Double
Dim Elliptic2 As Double
Dim a As Double, amax As Double, da As Double
```

```
Dim func1a As Double, func2a As Double, func1b As Double, func2b As Double

'
On Error GoTo err_TorisV1
steps = 1000
Pi = 3.14159
uo = 4 * Pi * 0.0000001
'
'----------------------------------------------------------
' Procedure:
' Calculate mutual inductance between two nearby circular
' fillaments or radius ri, rj with distance between d
'
' Proceedure calculates the two elliptical integrals necessary
' to calculate the mutual inductance
'
' Uses simpsons rule for the numerical integration
'
'
'----------------------------------------------------------
'


amax = Pi / 2
da = amax / steps
kc = Sqr((d ^ 2 + (ri + rj) ^ 2) / (d ^ 2 + (ri - rj) ^ 2))

a = 0
'simpsons rule, first term
Elliptic1 = 1 / Sqr((Cos(a)) ^ 2 + kc ^ 2 * (Sin(a)) ^ 2) * da
Elliptic2 = (Sin(a)) ^ 2 / Sqr((Cos(a)) ^ 2 + kc ^ 2 * (Sin(a)) ^ 2) * da

For i = 1 To (steps - 1) / 2
   a = a + da
   'simpons rule, 4y terms
   func1a = 4 / Sqr((Cos(a)) ^ 2 + kc ^ 2 * (Sin(a)) ^ 2) * da
   func2a = 4 * (Sin(a)) ^ 2 / Sqr((Cos(a)) ^ 2 + kc ^ 2 * (Sin(a)) ^ 2) * da

   a = a + da
   'simpsons rule, 2y terms
   func1b = 2 / Sqr((Cos(a)) ^ 2 + kc ^ 2 * (Sin(a)) ^ 2) * da
   func2b = 2 * (Sin(a)) ^ 2 / Sqr((Cos(a)) ^ 2 + kc ^ 2 * (Sin(a)) ^ 2) * da

   Elliptic1 = Elliptic1 + func1a + func1b
   Elliptic2 = Elliptic2 + func2a + func2b

  Next i
```

```
'simsons rule, last term
a = a + da
Elliptic1 = Elliptic1 + 1 / Sqr((Cos(a)) ^ 2 + kc ^ 2 * (Sin(a)) ^ 2) * da
Elliptic2 = Elliptic2 + (Sin(a)) ^ 2 / Sqr((Cos(a)) ^ 2 + kc ^ 2 * (Sin(a)) ^ 2) * da

MutualI = 2 * uo * ri * rj / Sqr(d ^ 2 + (ri - rj) ^ 2) * (1 / 3 * Elliptic1 - 2 / 3 * El-
liptic2)
Exit Function
'
err_TorisV1:
MutualI = 0
'
End Function
```

1 (PDF) DEFENSE TECHNICAL
INFORMATION CTR
DTIC OCA

2 (PDF) DIRECTOR
US ARMY RESEARCH LAB
RDRL CIO LL
IMAL HRA MAIL & RECORDS
MGMT

1 (PDF) GOVT PRINTG OFC
A MALHOTRA

41 (PDF) DIR USARL
RDRL WMP
D LYON
S SCHOENFELD
RDRL WMP A
P BERNING
S BILYK
M COPPINGER
J FLENIKEN
C HUMMER
T KOTTKE
M MCNEIR
A PORWITZKY
G THOMSON
W UHLIG
C WOLFE
RDRL WMP C
R MUDD
RDRL WMP D
J RUNYEON
M KEELE
N BRUCHEY
R DONEY
M DUFFY
F MURPHY
M ZELLNER
R FREY
D KLEPONIS
A BARD
S SCHRAML
B SCOTT
K STOFFEL
G VUNNI
V WAGONER
W CLARK
RDRL WMP E
P SWOBODA
M BURKINS
S BARTUS
B CHAMISH
D GALLARDY
W GOOCH
D HACKBARTH
P BARTKOWSKI
D HORNBAKER
E HORWATH
C KRAUTHAUSER

INTENTIONALLY LEFT BLANK.